PATH TO BUILDING QUANTUM SPIN LIQUIDS AND TOPOLOGICAL QUBITS WITH EXISTING QUANTUM HARDWARE

Dmitry Green

Claudio Chamon (BU)

Zhi-Cheng Yang (Maryland)

arXiv: 1908.04791

October 2019

Summary

- We solve the outstanding problem of how to build **topological quantum spin liquids** with physically accessible interactions. One of the applications is to build **topological qubits**.
- Theorists have been studying **"multi-spin"** interactions for 50+ yrs. However, these interactions do not exist in nature. We have discovered that they can be effectively **realized exactly** by **programming existing quantum hardware** (for example, D-Wave).
- So, if nature does not give us the appropriate interactions, we will **build** them, instead.

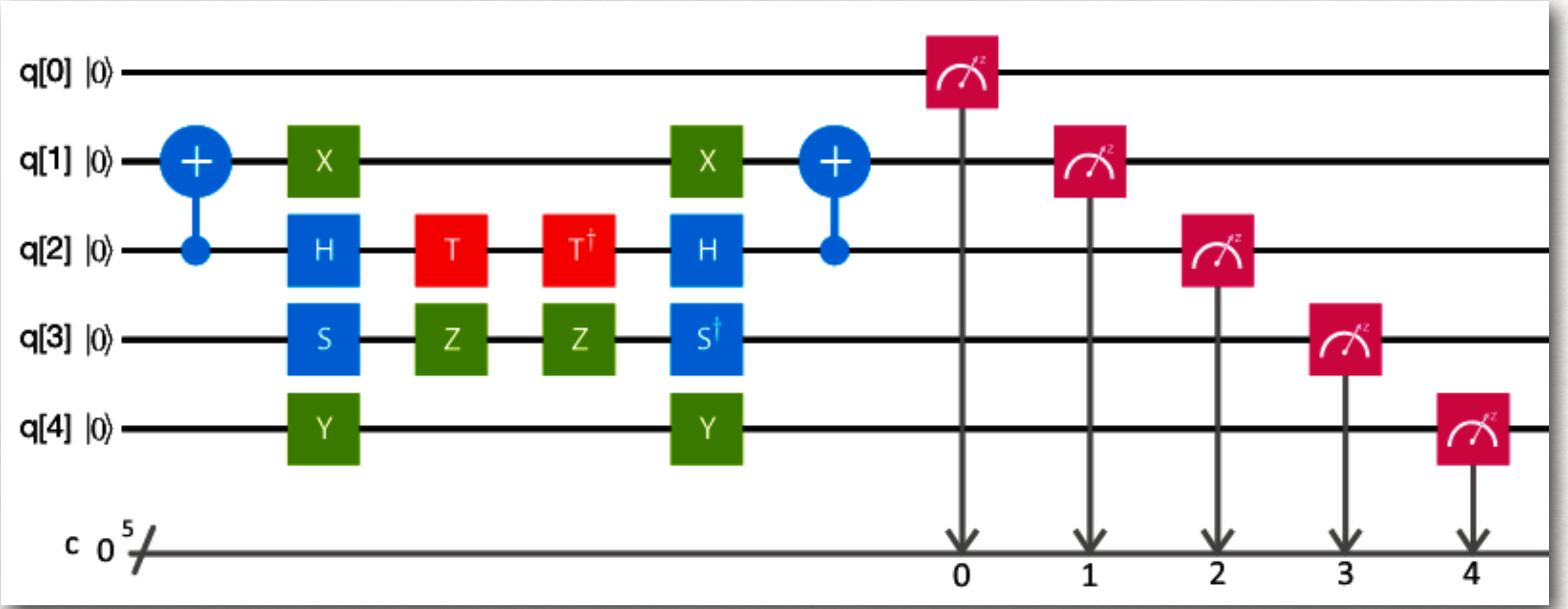
Contents

Motivation: current state of quantum computing & simulation

Three main approaches today:

1. Quantum Gate Array
2. Topological Quantum Computing
3. Quantum Annealers

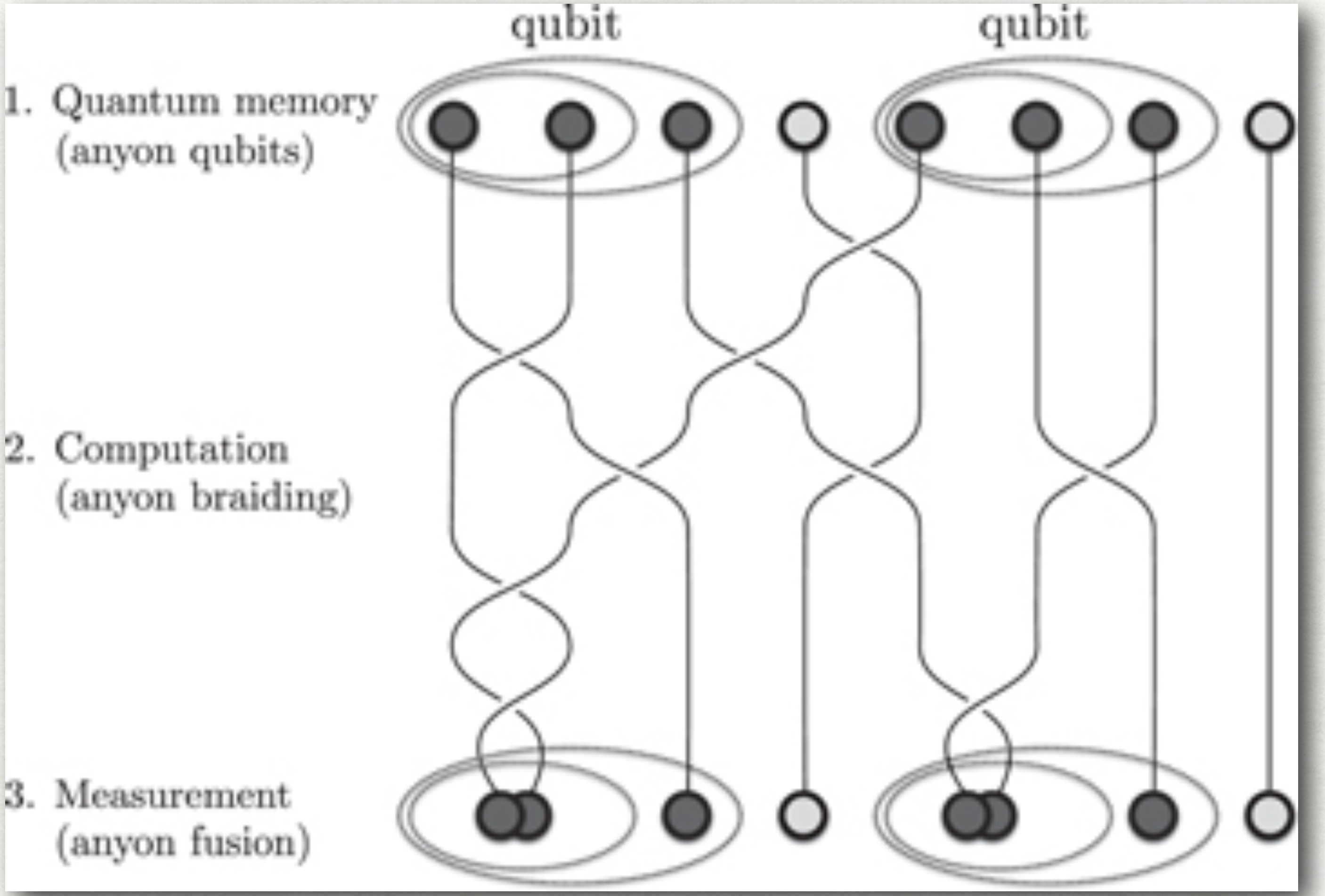
1. Quantum Gate Array: standard but few qubits



- **Computation method:** unitary operations on fixed qubits
- **Major players:** IBM, Google, Intel, Rigetti, and many more
- **Max number of qubits:** ~ 50 - 70

Source: IBM Quantum Composer documentation

2. Topological Quantum Computing: powerful but no qubits

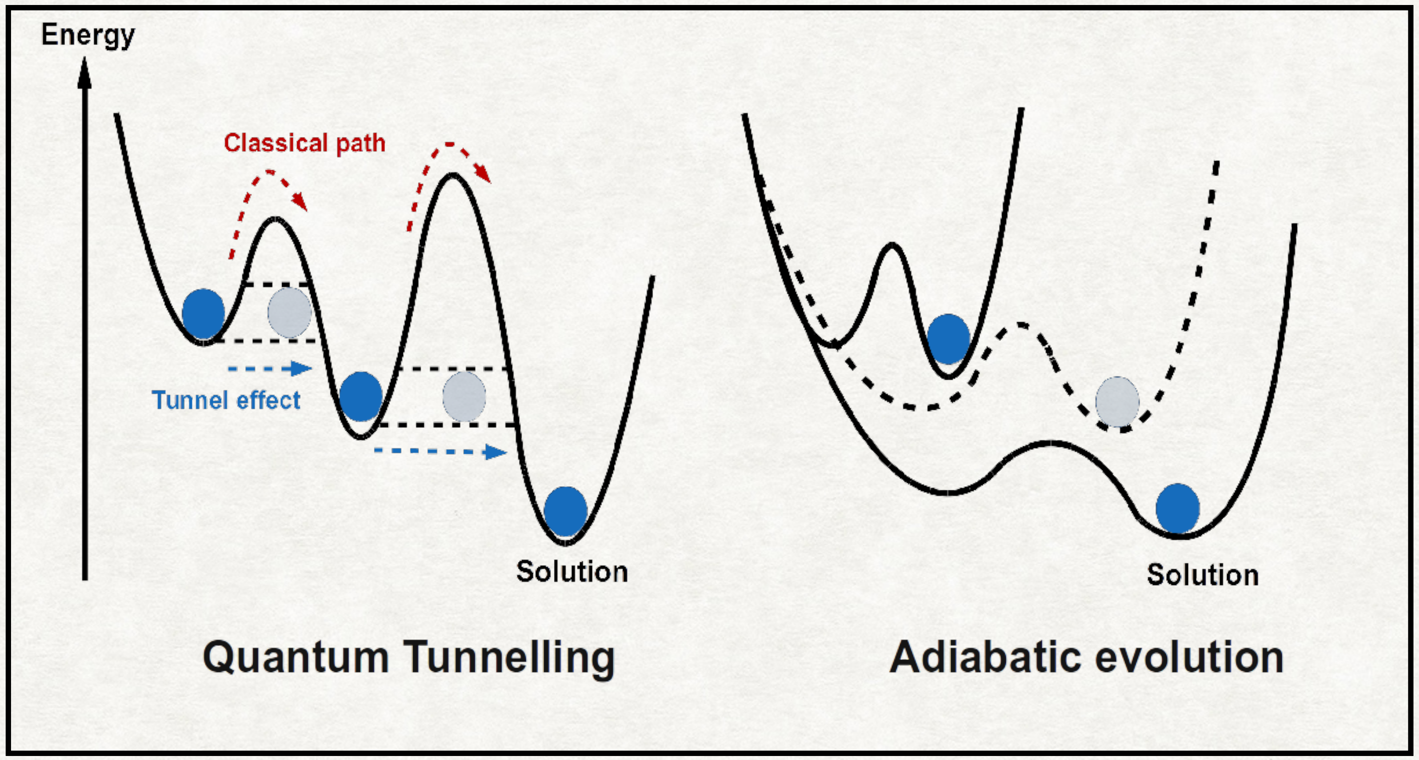


- **Computation method:** physical braiding of anyons
- **Major player:** Microsoft
- **Max number of qubits:** 0 - 1

Source: Field and Simula (2018)

3. Quantum Annealers: limited use but lots of qubits

Classical: $\sum_{i,j} J_{ij}\, \sigma_i^z\, \sigma_j^z$ + Quantum: $\Gamma \sum_i \sigma_i^x$



- **Computation method:** minimize energy with quantum fluctuations
- **Major player:** D-Wave
- **Max number of qubits:** 2,000 - 5,000

Source: D-Wave

3. Quantum Annealers: conventional view

Quantum Annealer

The quantum annealer is least powerful and most restrictive form of quantum computers. It is the easiest to build, yet can only perform one specific function. The consensus of the scientific community is that a quantum annealer has no known advantages over conventional computing.

APPLICATION
Optimization Problems

GENERALITY
Restrictive

COMPUTATIONAL POWER
Same as traditional computers

Source: IBM (2015)

Question

Can we build a computationally interesting physical system using only the 2-body classical Ising $J\sigma^z\sigma^z$ interaction and one uniform transverse field $\Gamma\sigma^x$?

Answer

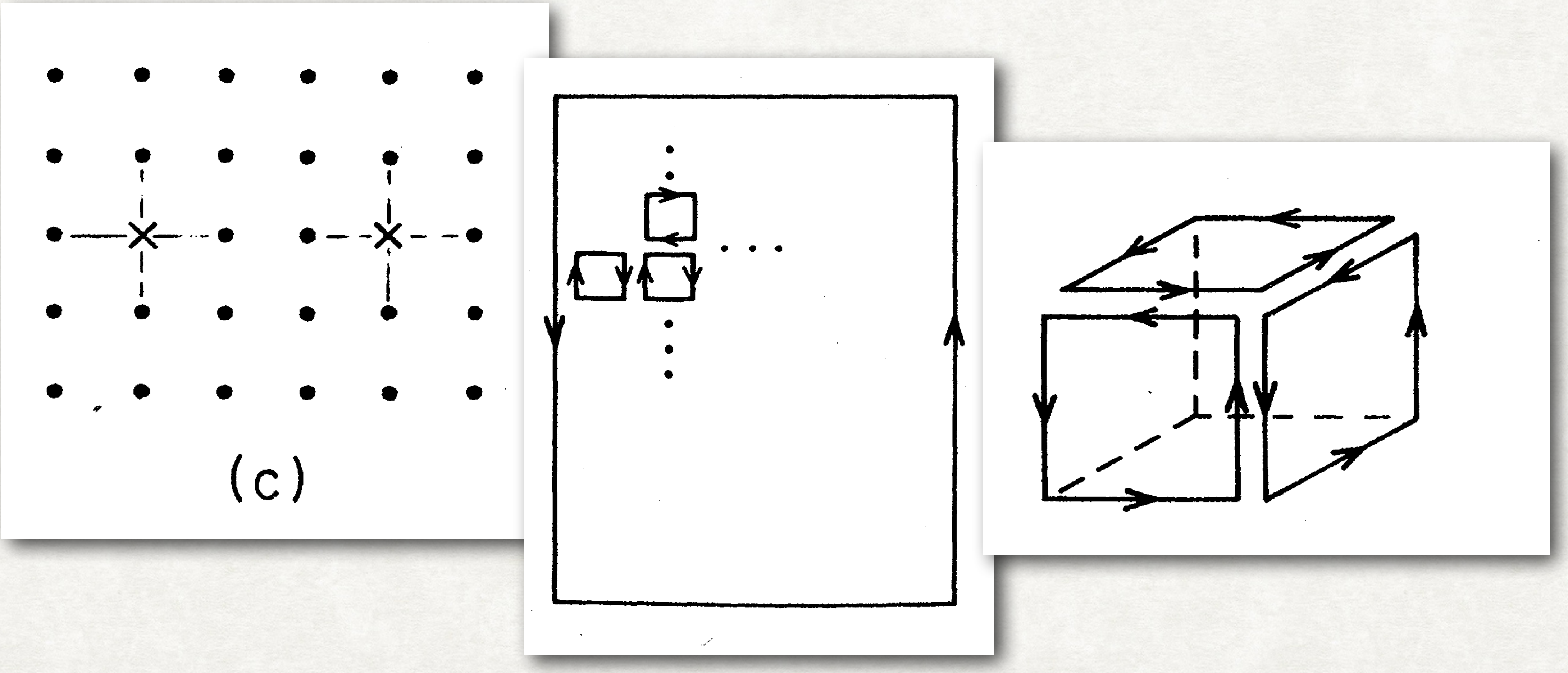
Yes!

A Quantum Spin Liquid

Which can be used to implement Topological QC

Digression: QSL date back to 1970's

- Introduced by Wegner (1971). $\mathbb{Z}_2$ Gauge Theory
- Considered by Anderson (1973) in context of superconductivity
- Rich theoretical results, but no experimental signature to-date



Source: Kogut (1979)

Modern understanding of QSL

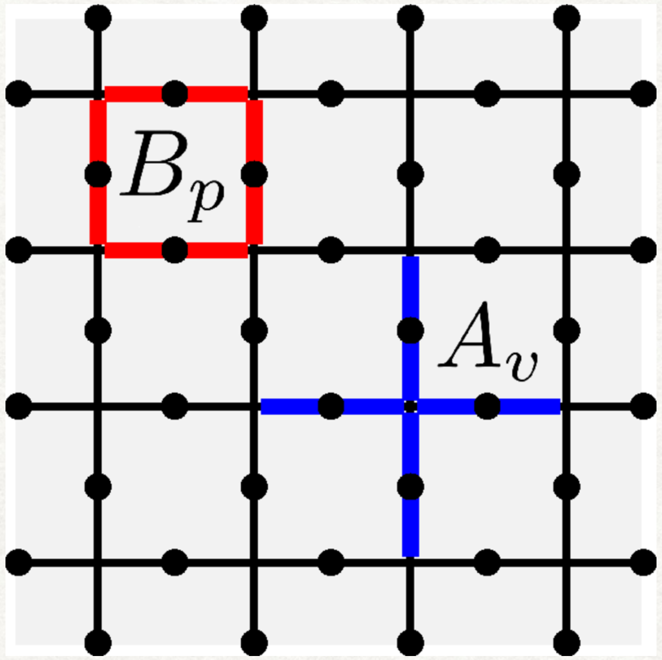
Approximate Definition

Quantum paramagnet that does not break any degeneracies of the microscopic Hamiltonian

Features

- No long range order
- Local gauge symmetry
- Topological degeneracy of the ground state (with energy gap)
- Fractionalization of quasiparticles
- Anyonic statistics

QSL Example: Toric Code (limit of $\mathbb{Z}_2$)



$$A_v = J \prod_{i \in v} \sigma_i^z \qquad B_p = \Gamma \prod_{i \in p} \sigma_i^x$$

$$H = -\sum_v A_v - \sum_p B_p$$

RECALL:

$\sigma^z \,|\uparrow\rangle = |\uparrow\rangle$

$\sigma^z \,|\downarrow\rangle = -\,|\downarrow\rangle$

$\sigma^x \,|\uparrow\rangle = |\downarrow\rangle$

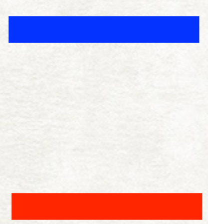
$\sigma^x \,|\downarrow\rangle = |\uparrow\rangle$

A. Kitaev (2003)

Magic of the four-spin interaction

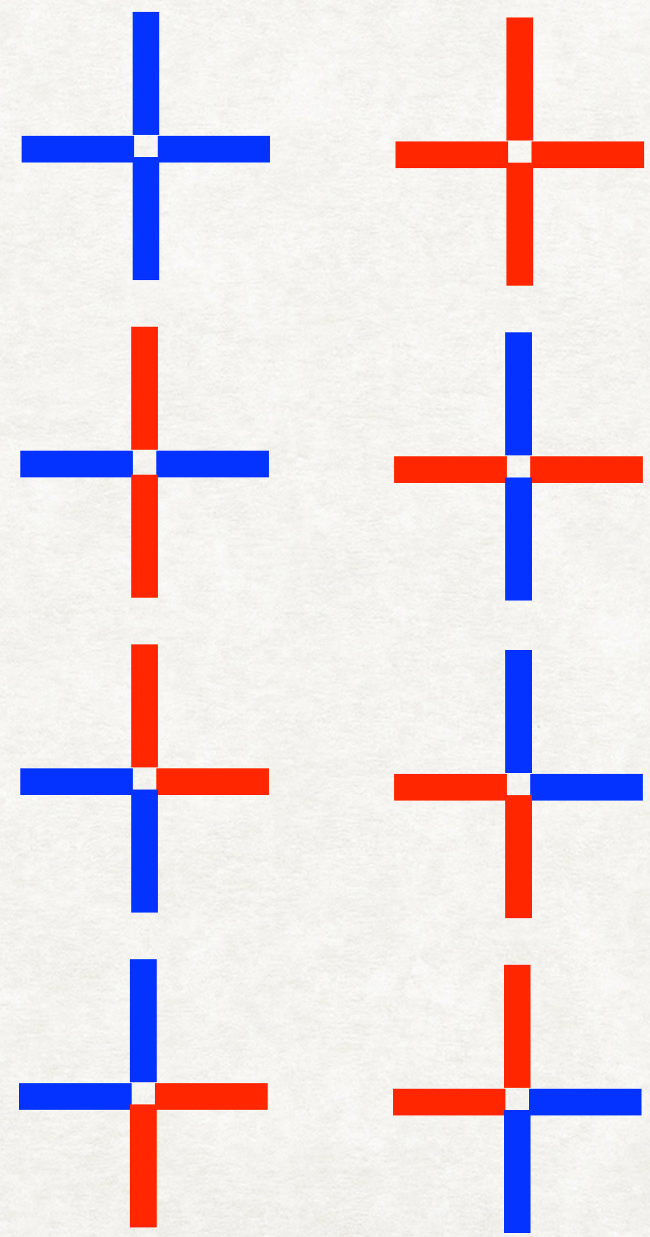
$$A_v = J \prod_{i \in v} \sigma_i^z$$

8 degenerate star states. Parity P=+1 example.

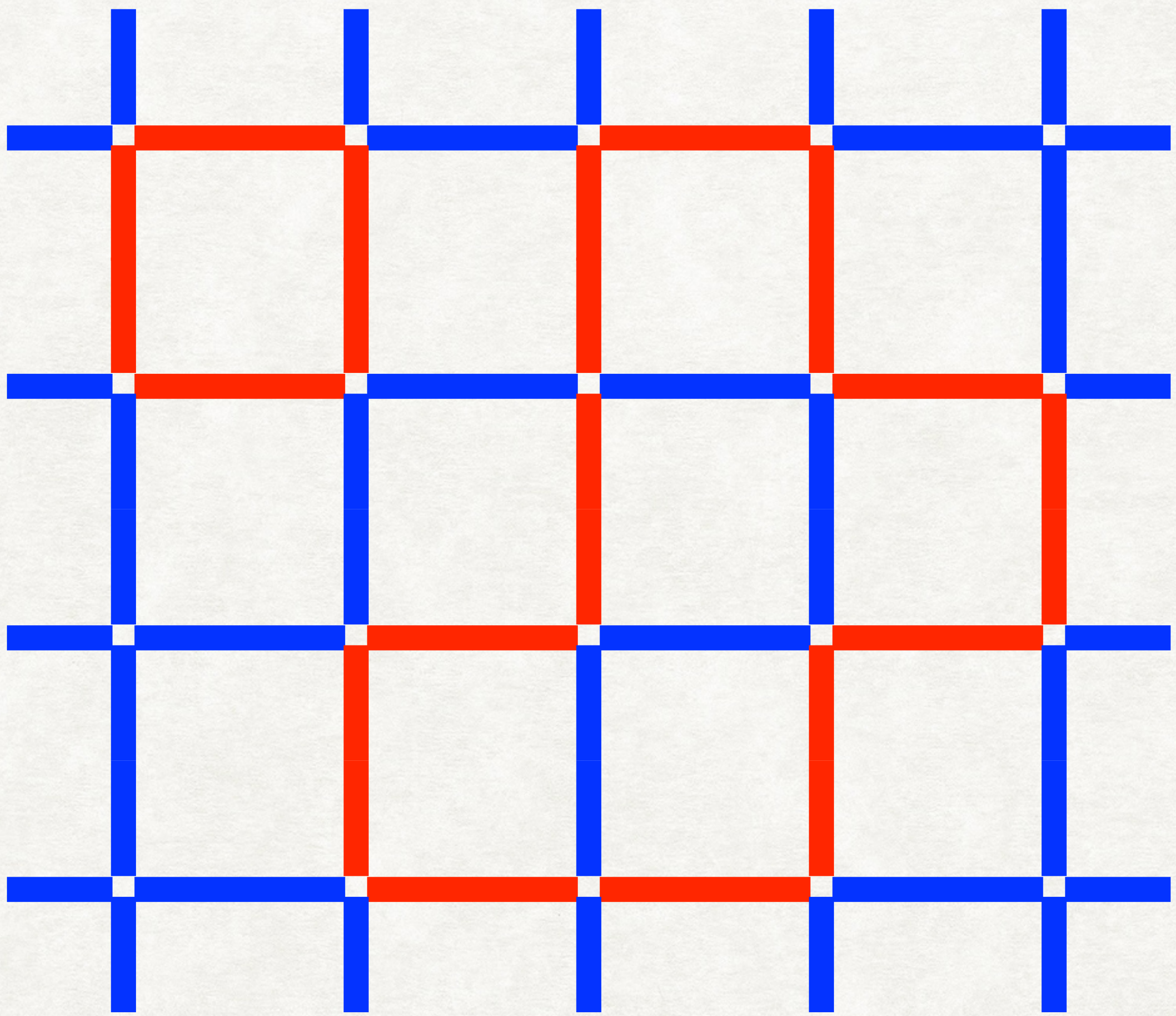


$= |\uparrow\rangle$

$= |\downarrow\rangle$



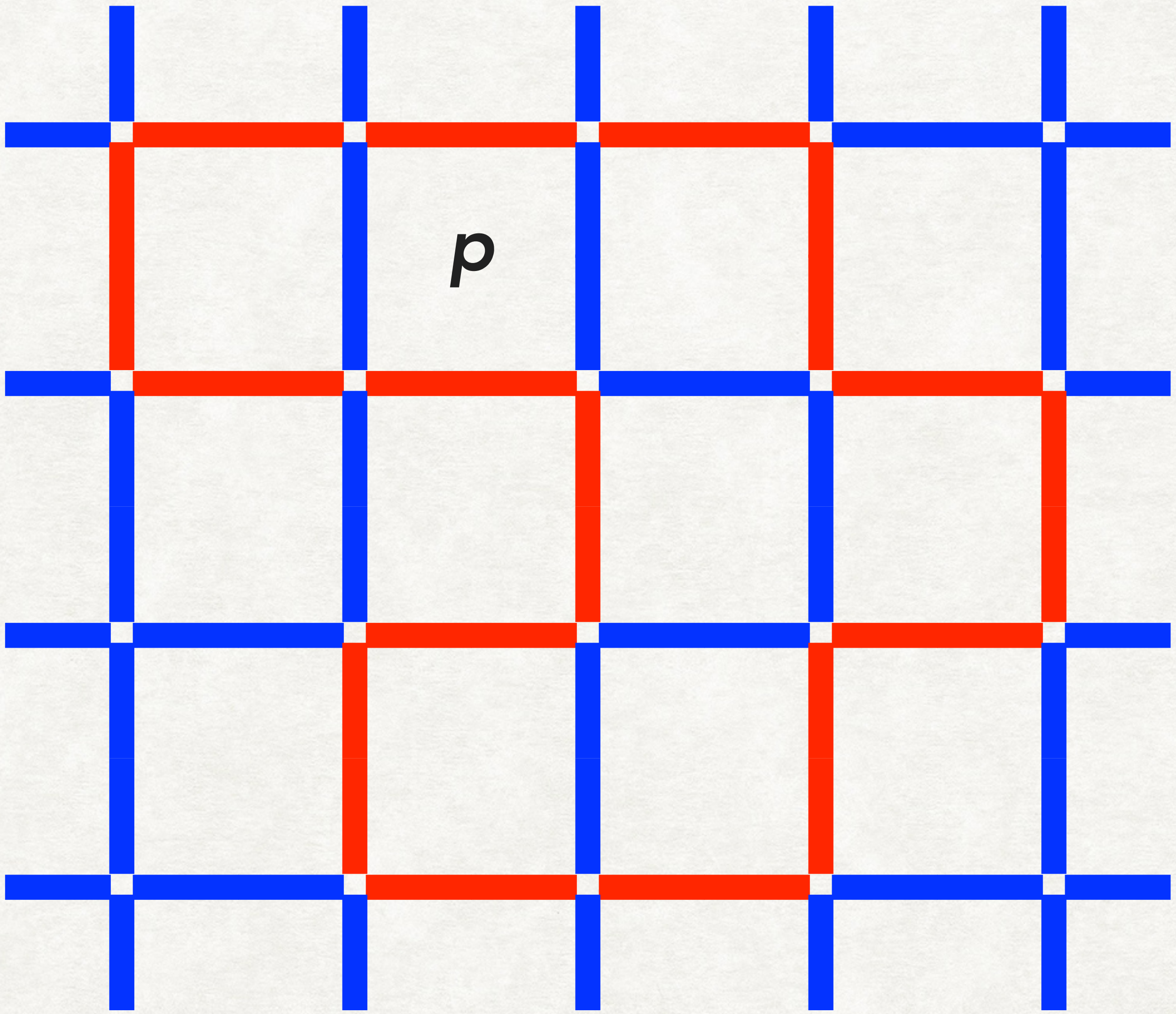
The ground state is a “loop gas”



- All stars have P=+1
- The ground state is a superposition of all possible loops

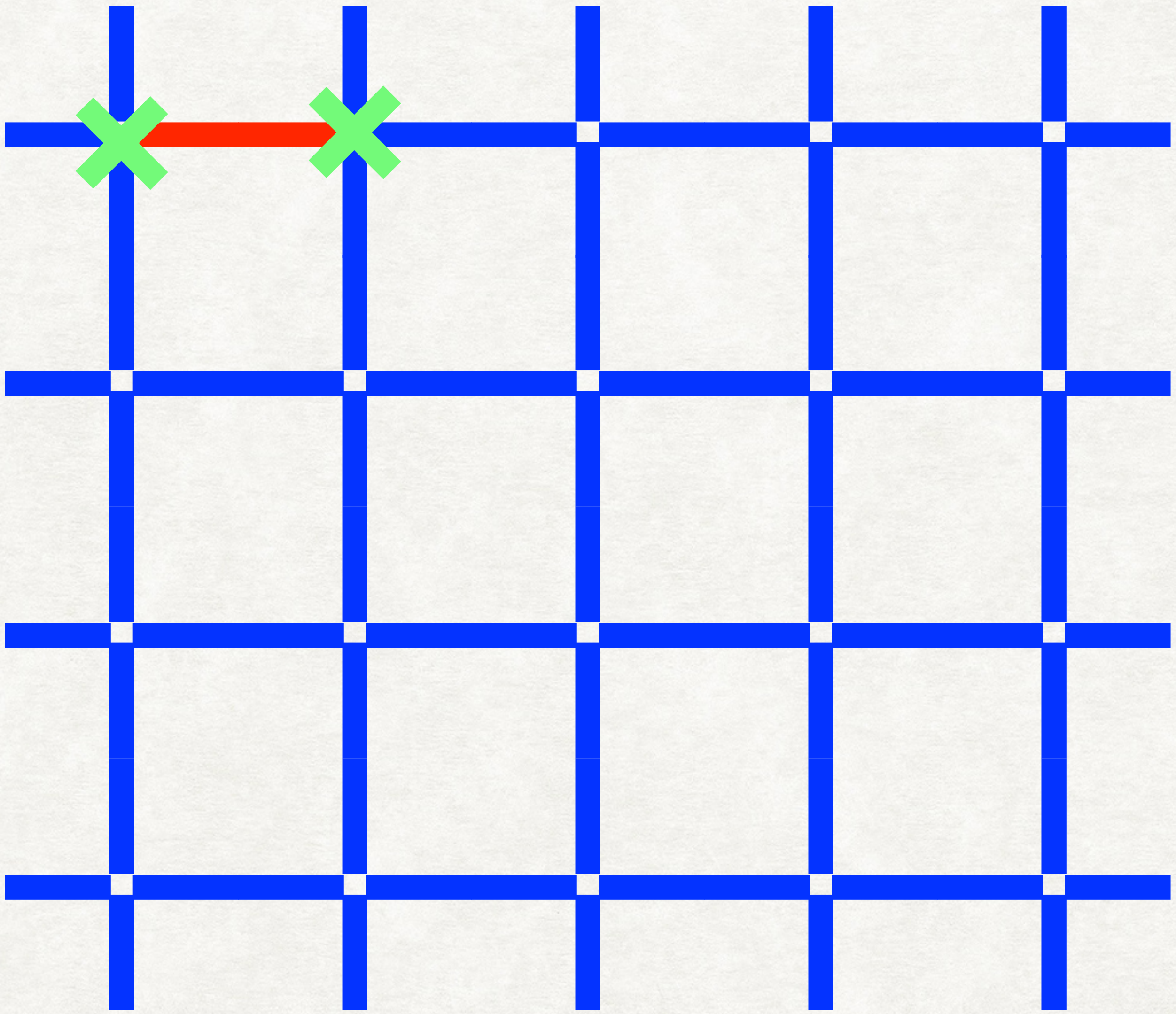
Loop "surgery"

$$B_p = \Gamma \prod_{i \in p} \sigma_i^x$$



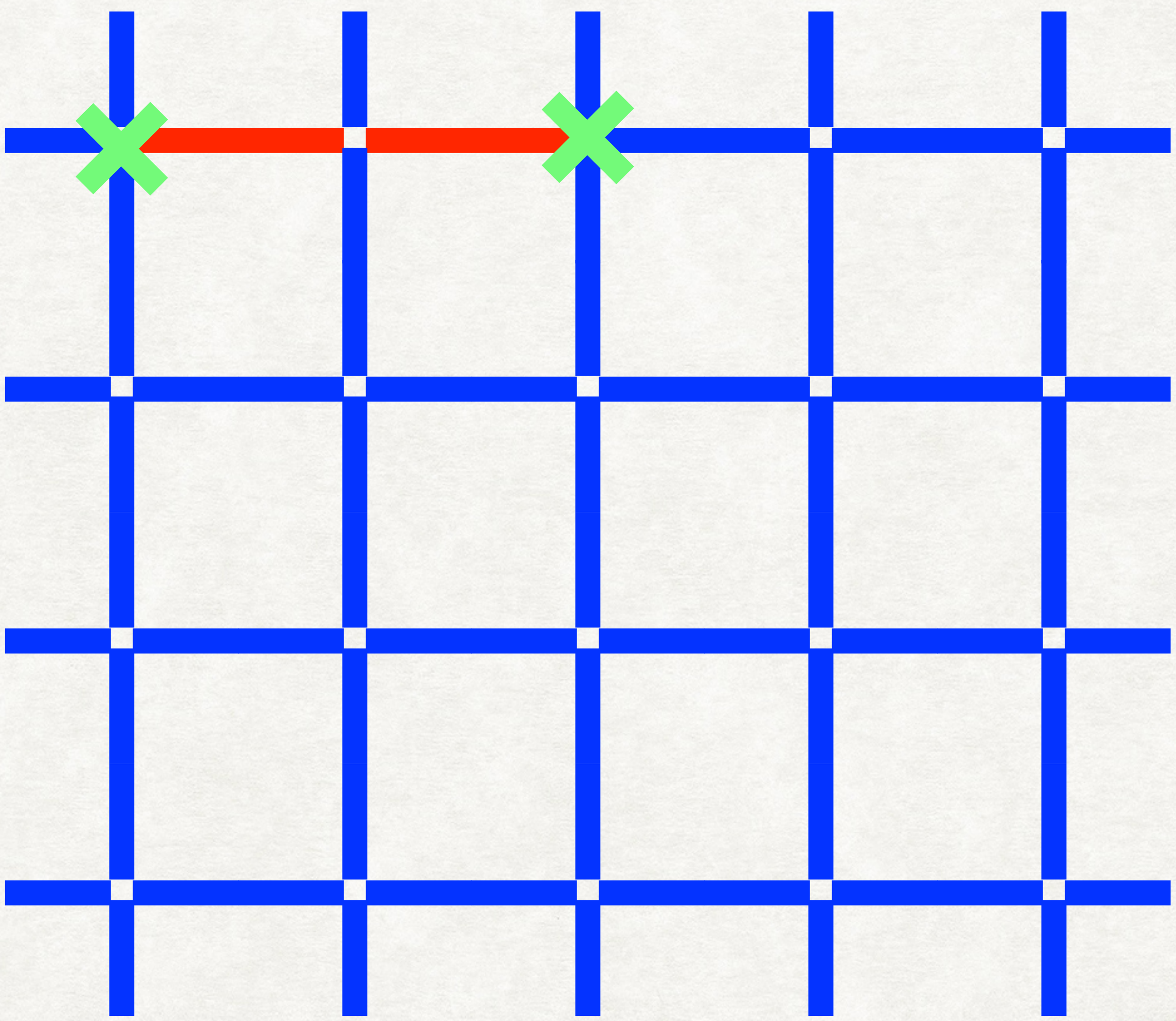
- Plaquette operator flips all spins around a plaquette
- Does not change ground state energy: "gauge symmetry"

Fractional excitations: created in pairs

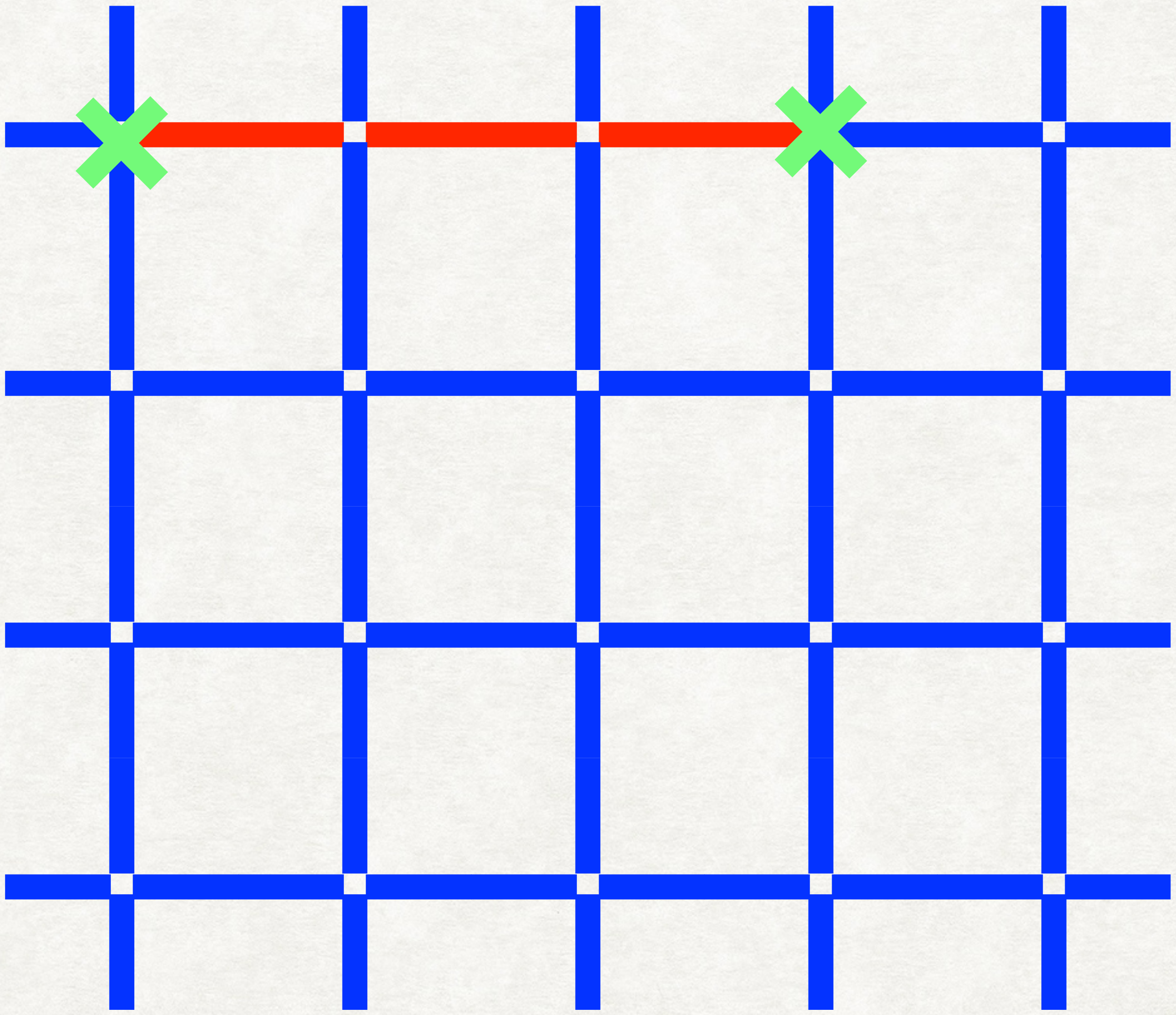


- Apply σ^x to a link - breaks two stars (P=-1)
- Transport it along a path by applying sequence of σ^x

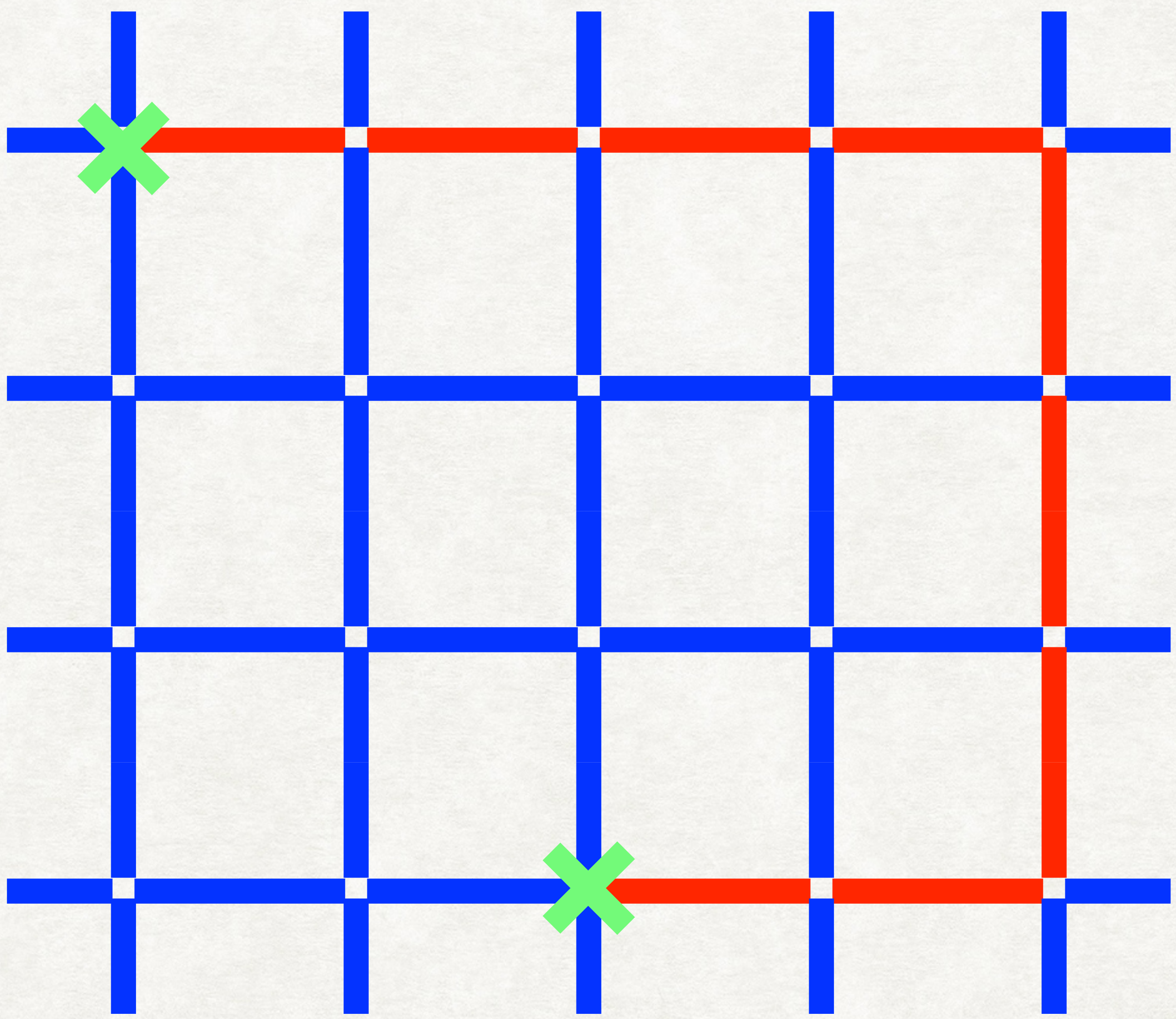
Fractional excitations: created in pairs



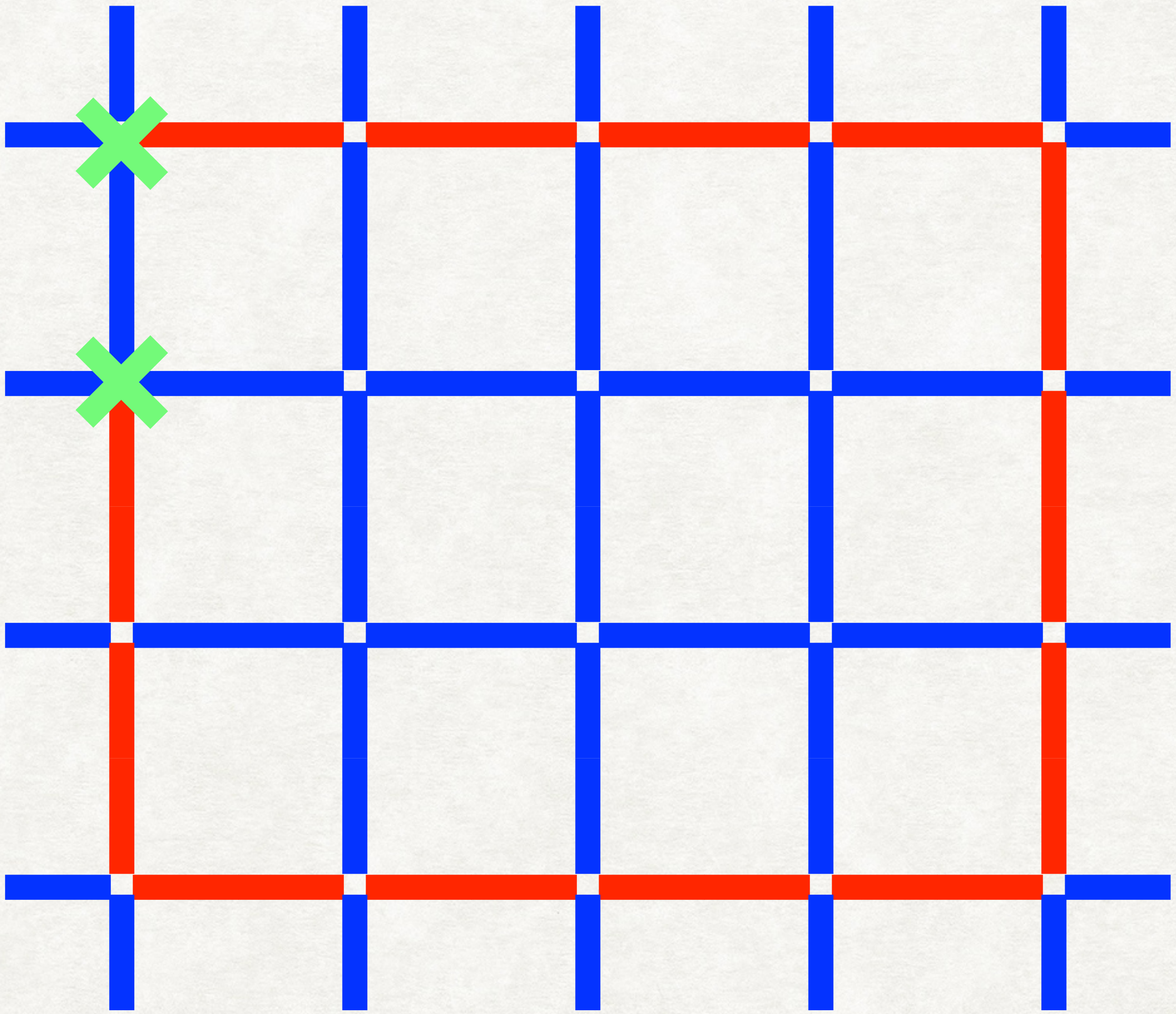
Fractional excitations: created in pairs



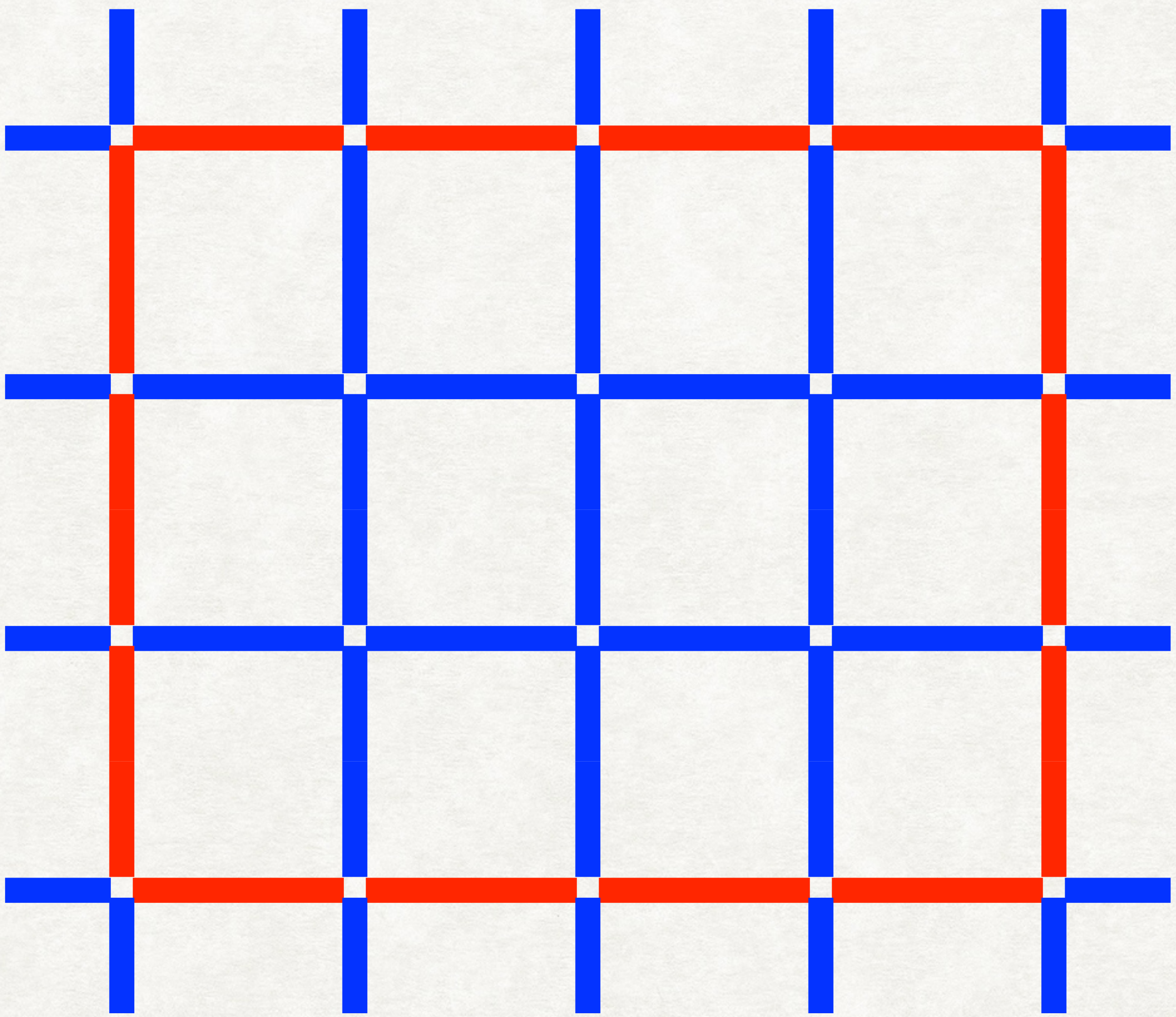
Fractional excitations: created in pairs



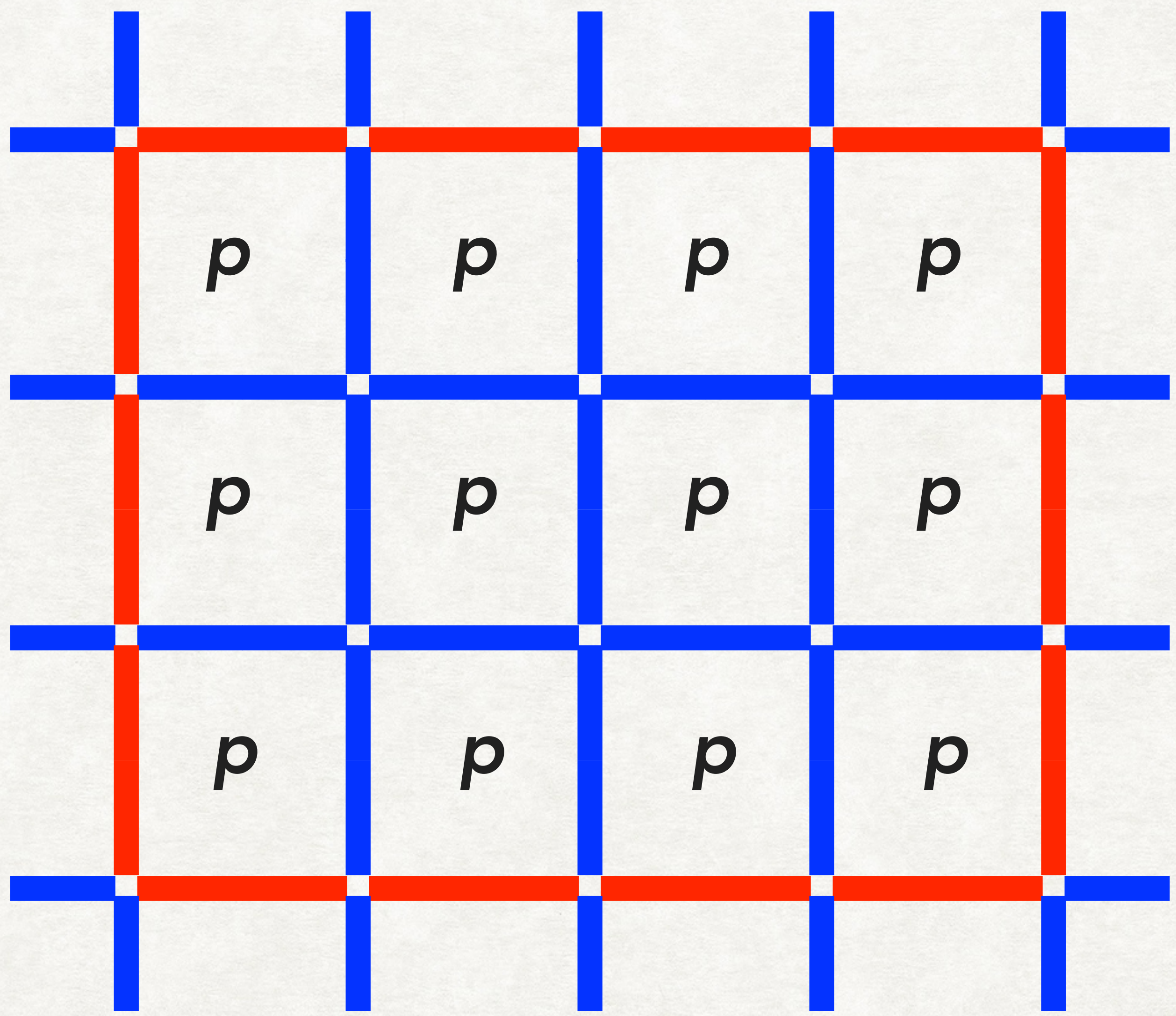
Fractional excitations: created in pairs



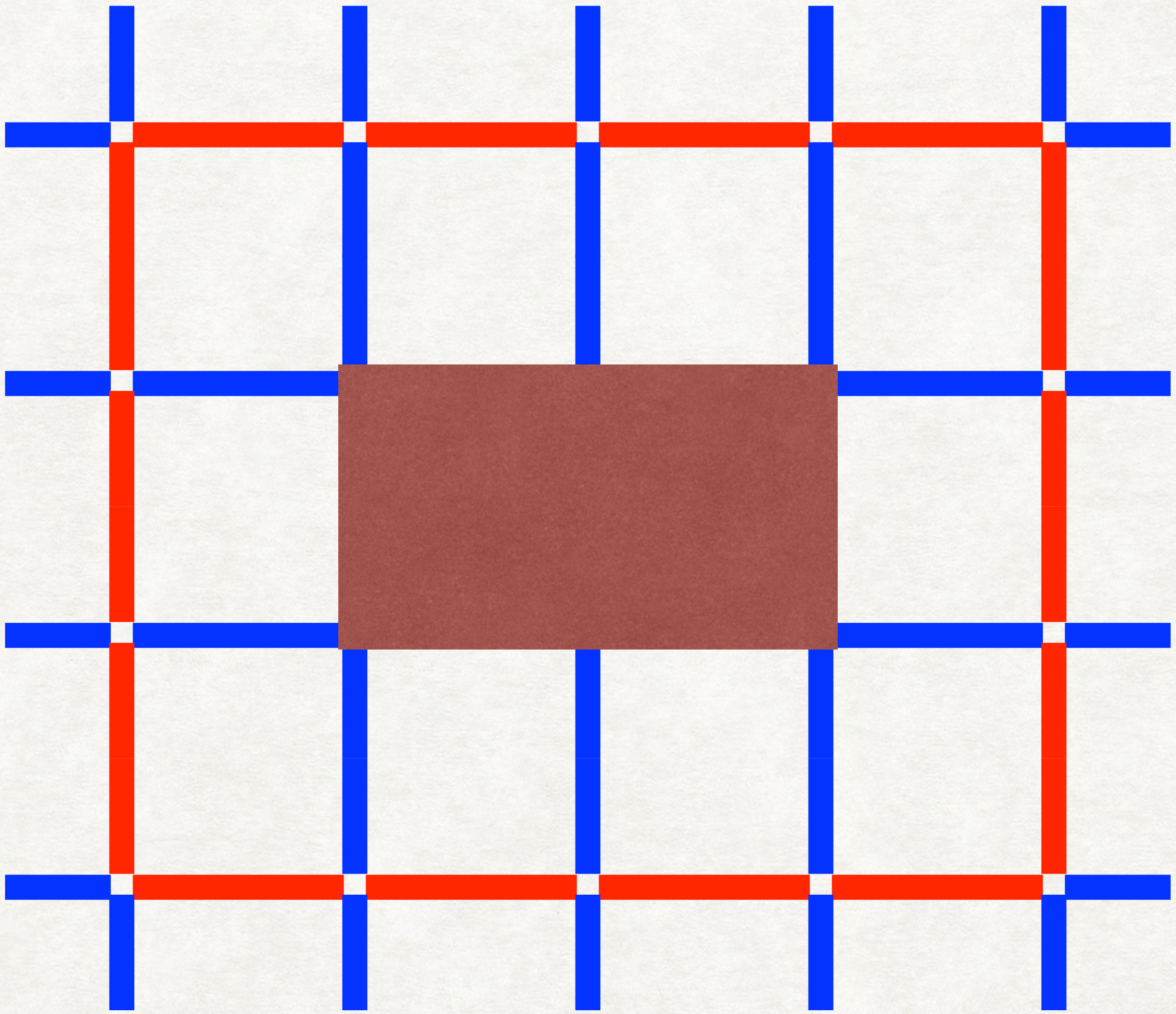
Excitation annihilated...



…and loop can be contracted to zero…



...unless there is a hole (topology)

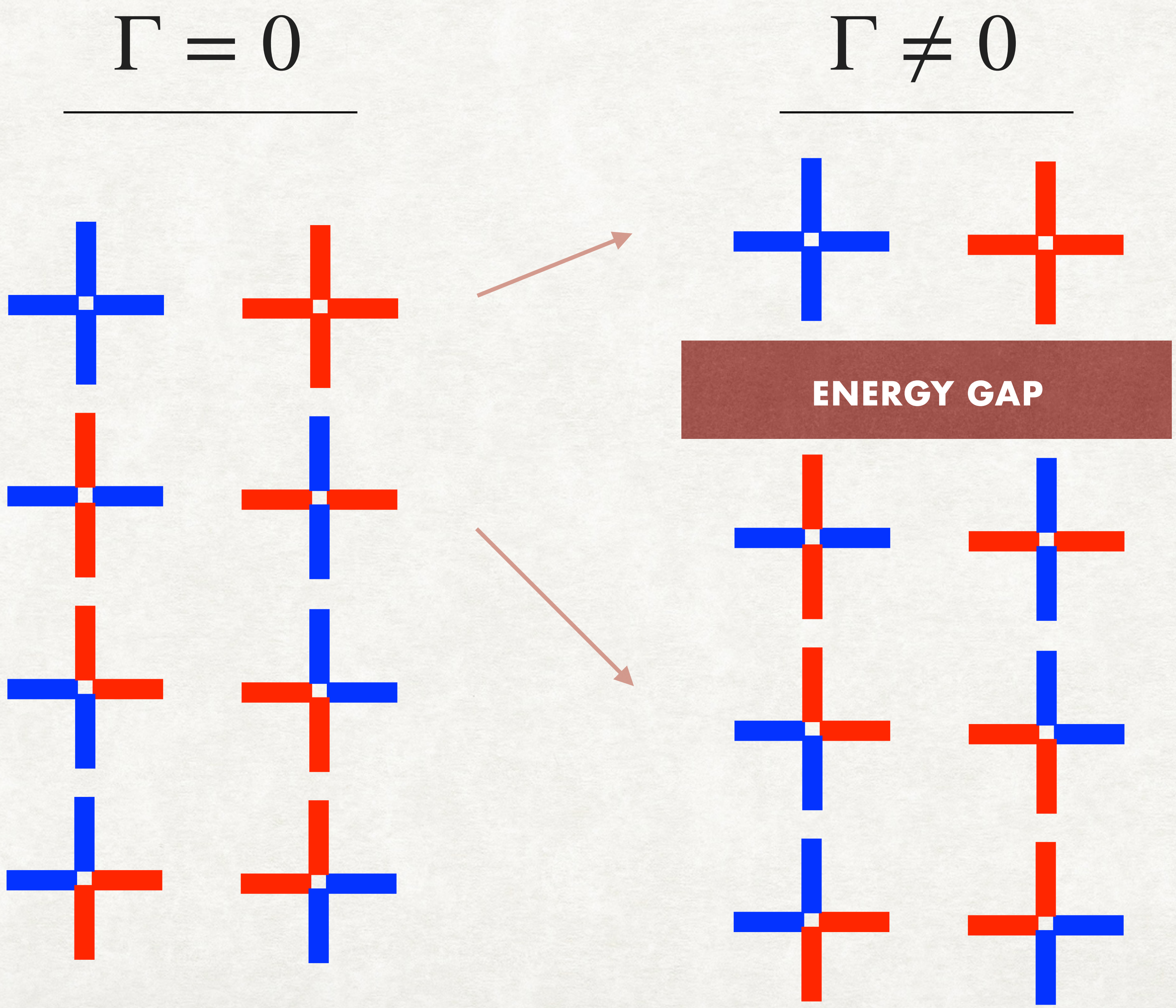


Problem #1

- Four-spin interactions have not been found in nature (yet)
- QSLs typically rely on four-spin (or more) interactions
- Only known two-body exception is the Kitaev honeycomb model, but it also has not been observed ($\sigma^x\sigma^x$, $\sigma^y\sigma^y$, $\sigma^z\sigma^z$ interactions)

Problem #2

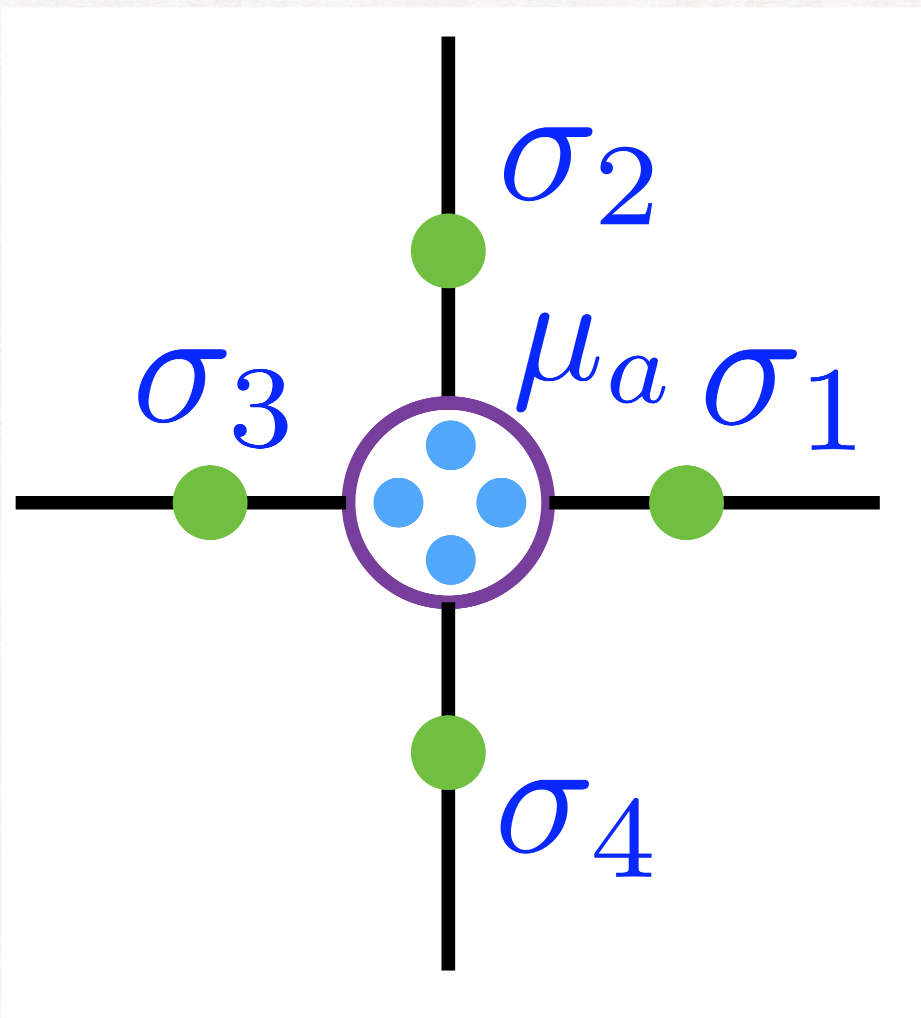
Usual attempts to build the four-body out of two-body interactions only work in the classical limit (no transverse field), *i.e.*, no quantum effects

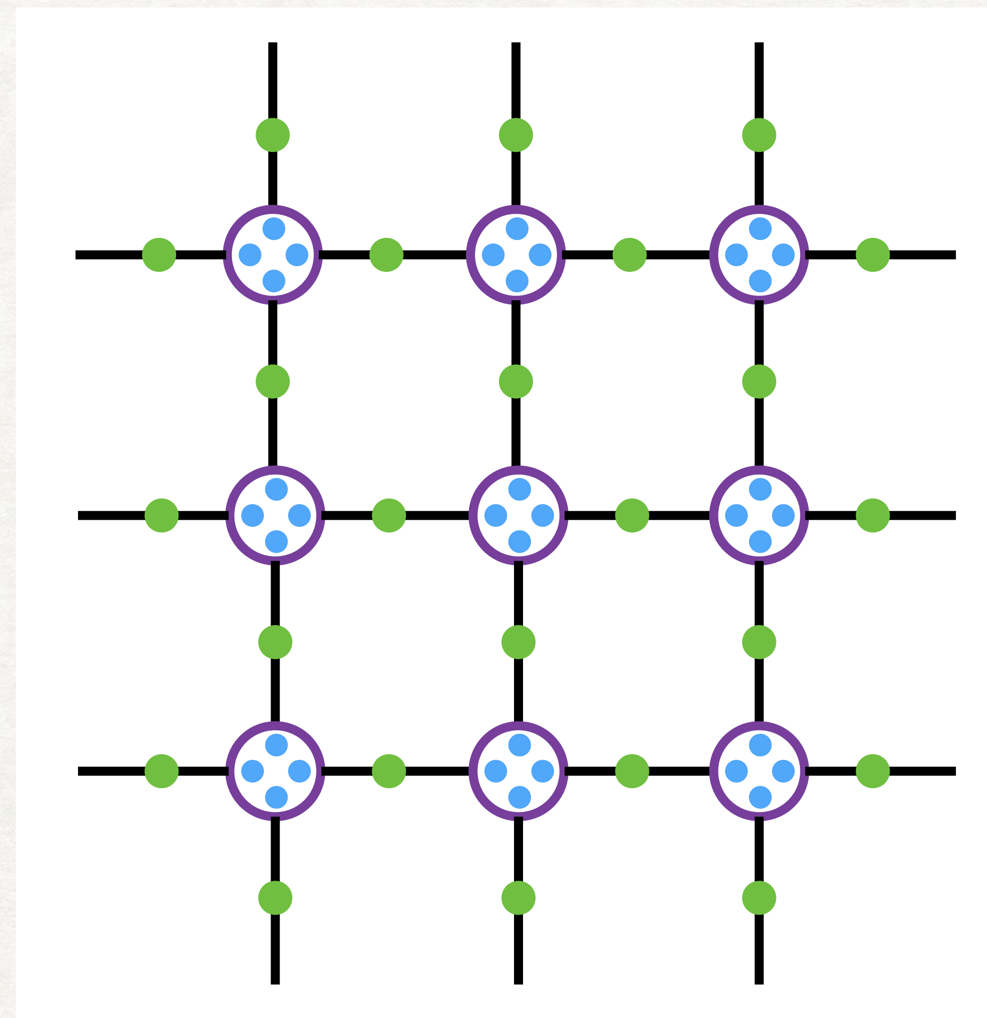


- No "magic"
- Lattice state is crystalline ("order-by-disorder")

Our solution

- Introduce four "matter" spins μ at each vertex
- The "gauge" spins σ couple to other sites, but matter spins do not

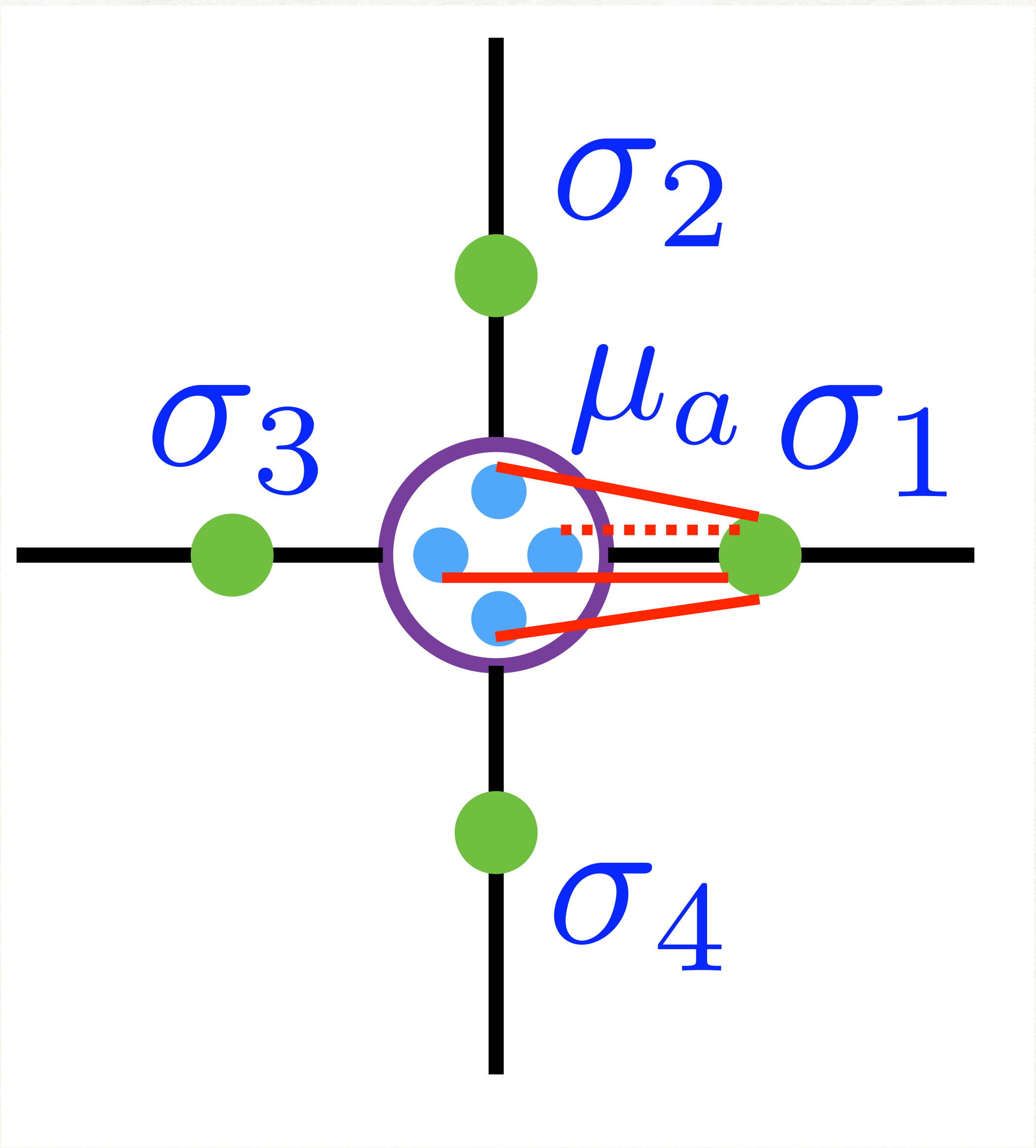




Two-body gauge-matter interaction

Anti-ferromagnetic

Ferromagnetic

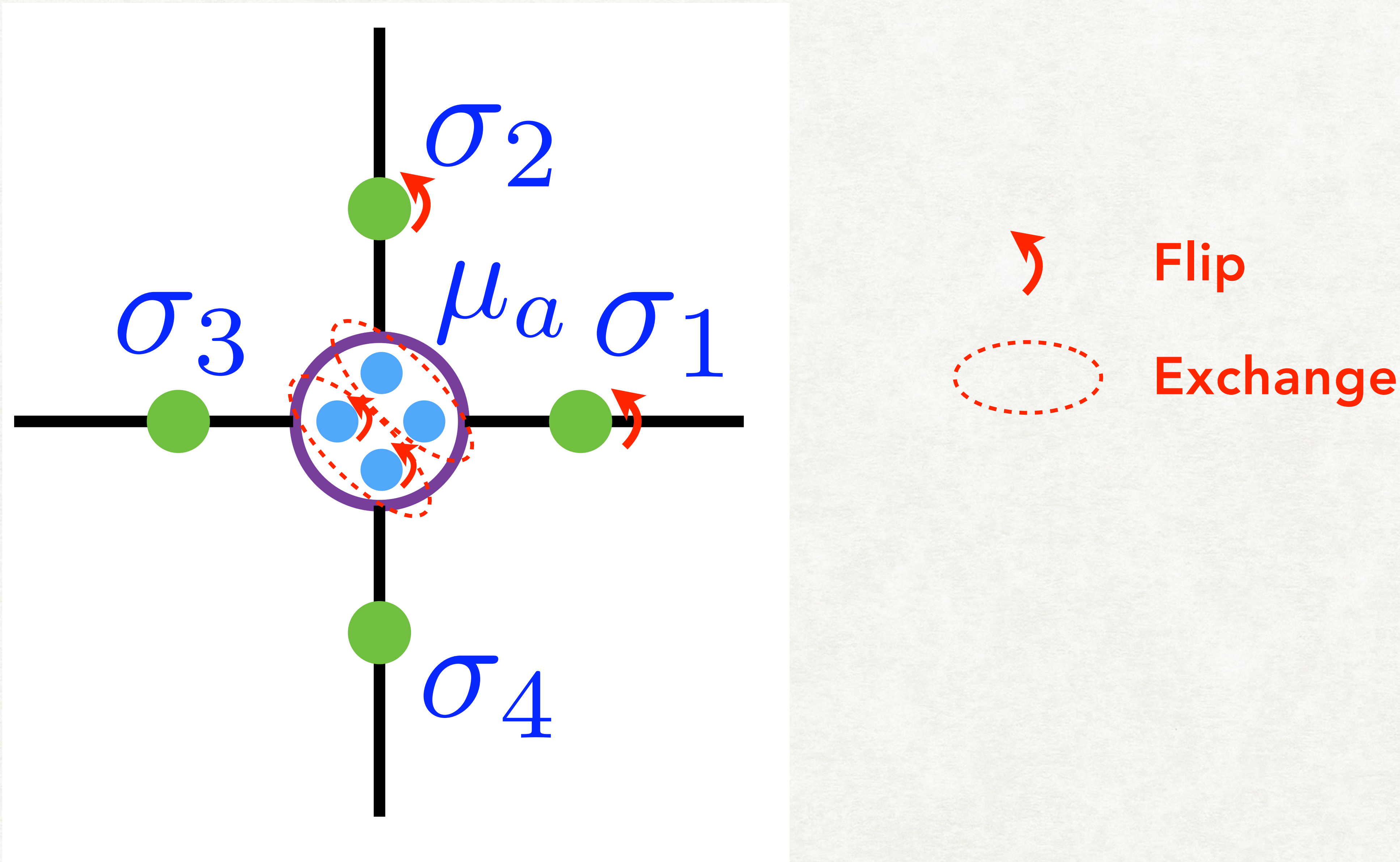


$$H_0 = -J \sum_{a=1}^{4} \left(\sum_{i=1}^{4} W_{ai}\, \sigma_i^z \right) \mu_a^z$$

$$W = \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

4 x 4 Hadamard matrix

"Combinatorial Gauge Symmetry"



8-fold degeneracy is maintained for <u>all J and Γ</u>

(2 gauge spin flips keep P=+1)

Mathematically: matter "slaved" to gauge spins

$$L^{-1}WR = W$$

Automorphism*

$$\begin{pmatrix} 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Monomial matrix

$$\begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

Monomial matrix

$$\mu_a^z \to \sum_{b=1}^{4} \mu_b^z (L^{-1})_{ba}$$

$$\sigma_i^z \to \sum_{j=1}^{4} R_{ij}\sigma_j^z$$

*Hadamard matrices have been around since 1860s. Only ± 1 elements with $W^T W = 1$.

Why does this work?

Full Hamiltonian:

$$H = -\sum_s \left[J \sum_{a\in s,\ i\in s} W_{ai}\ \sigma_i^z \mu_a^z + \Gamma \sum_{a\in s} \mu_a^x \right] - \widetilde{\Gamma} \sum_i \sigma_i^x$$

2-spin Ising interaction

Transverse field: invariant under spin flips <u>and</u> permutations

Monomial transformations preserve spin algebra

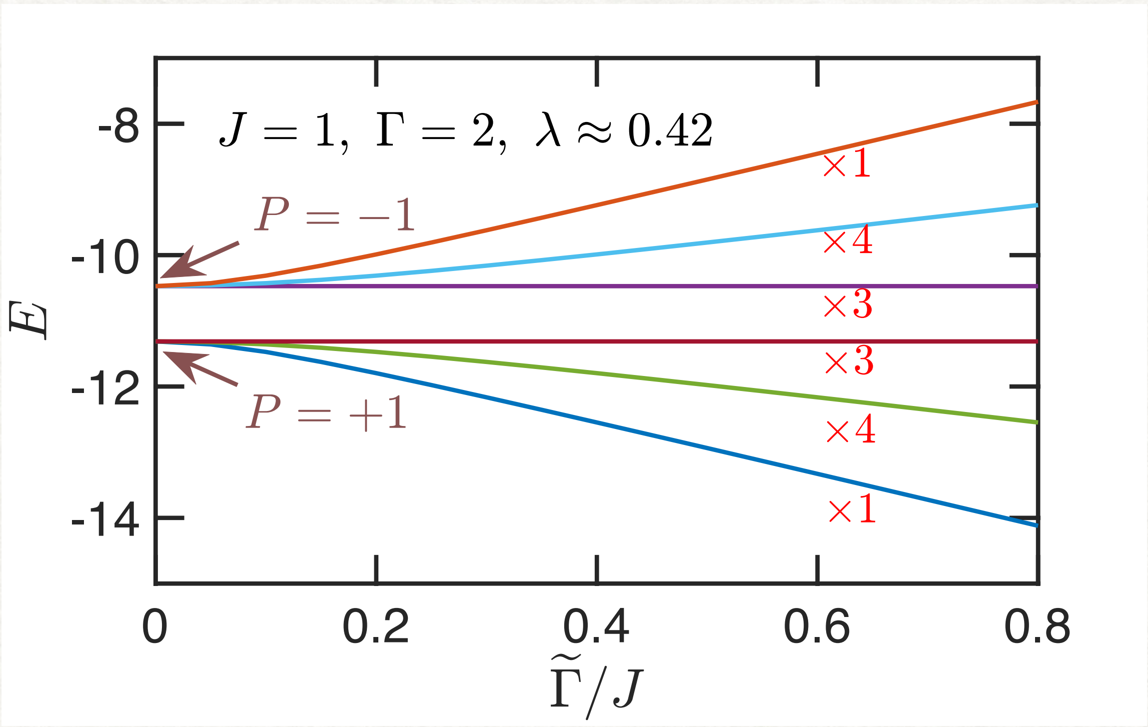
$\Gamma < \widetilde{\Gamma}$ result of embedding in D-Wave

[more later]

Numerical test #1: spectrum of single star

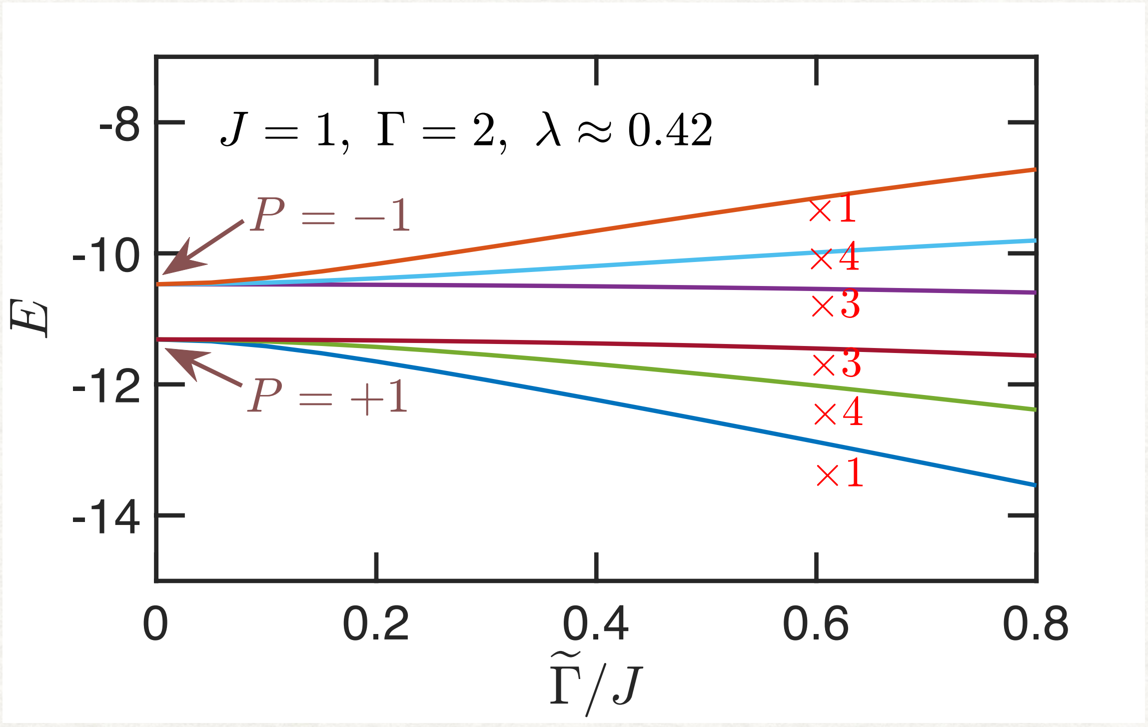
Exact star

$$H = -\lambda\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z - \widetilde{\Gamma}\sum_i \sigma_i^x$$



2-body star

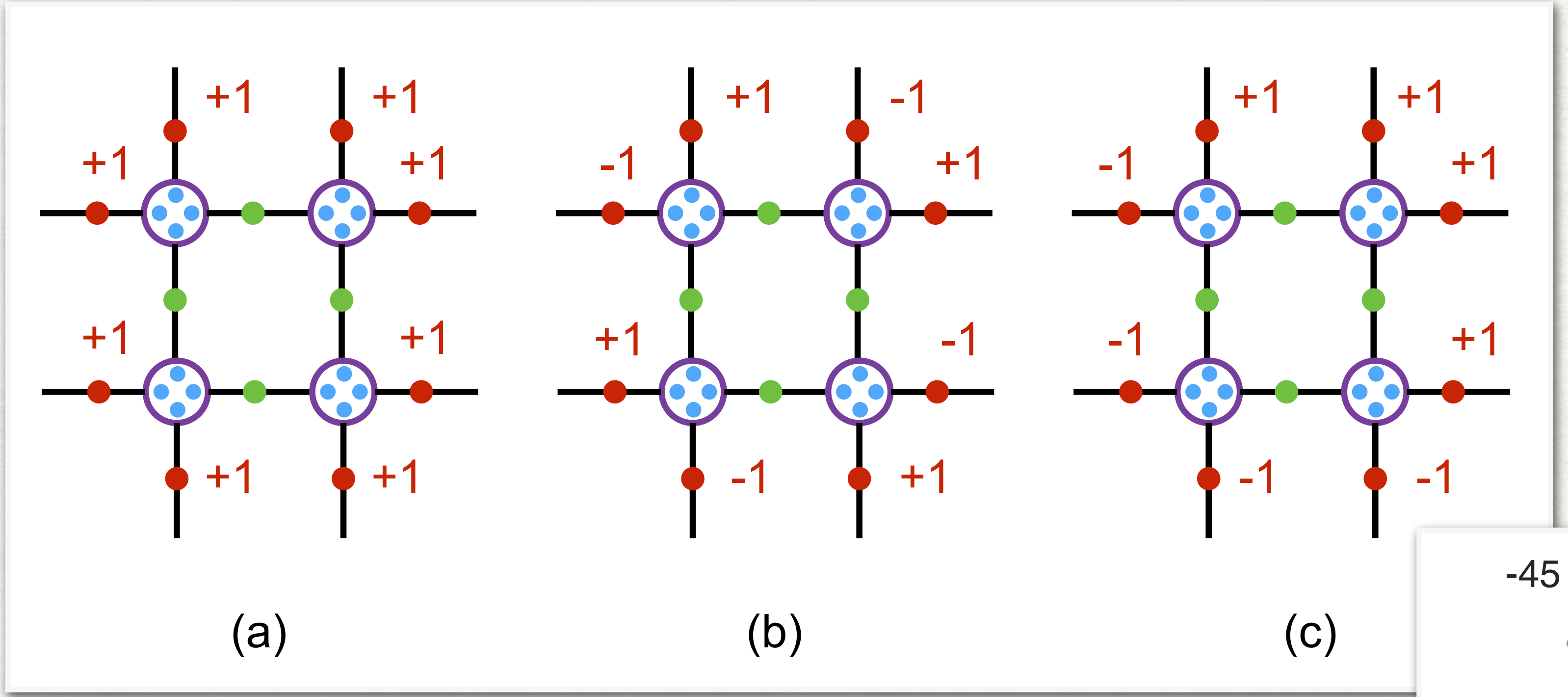
$$H = -J\sum_{a,i} W_{ai}\sigma_i^z\mu_a^z - \Gamma\sum_a \mu_a^x - \widetilde{\Gamma}\sum_i \sigma_i^x$$



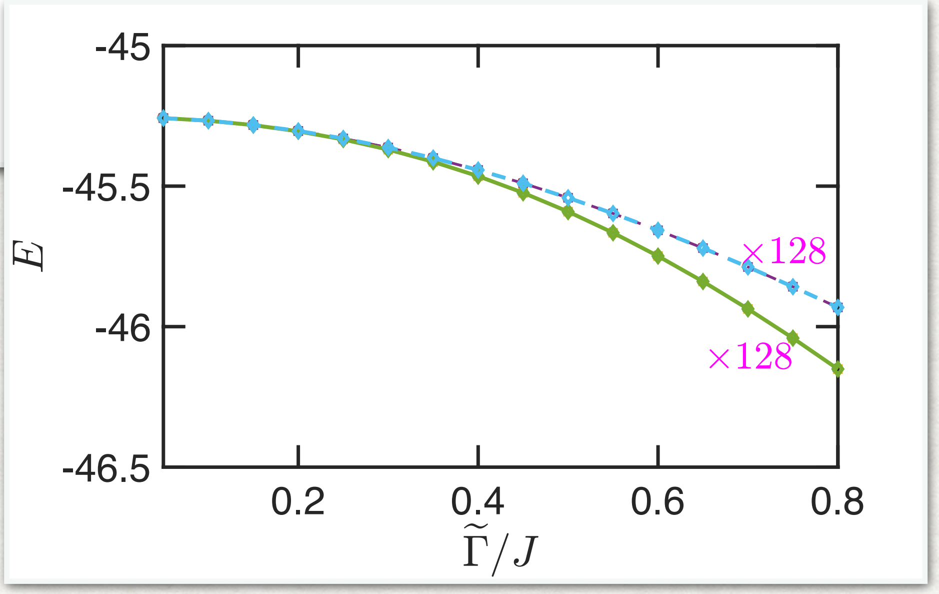
Degeneracy matches

Numerical test #2: plaquette

Spectrum as function of external configuration (legs)



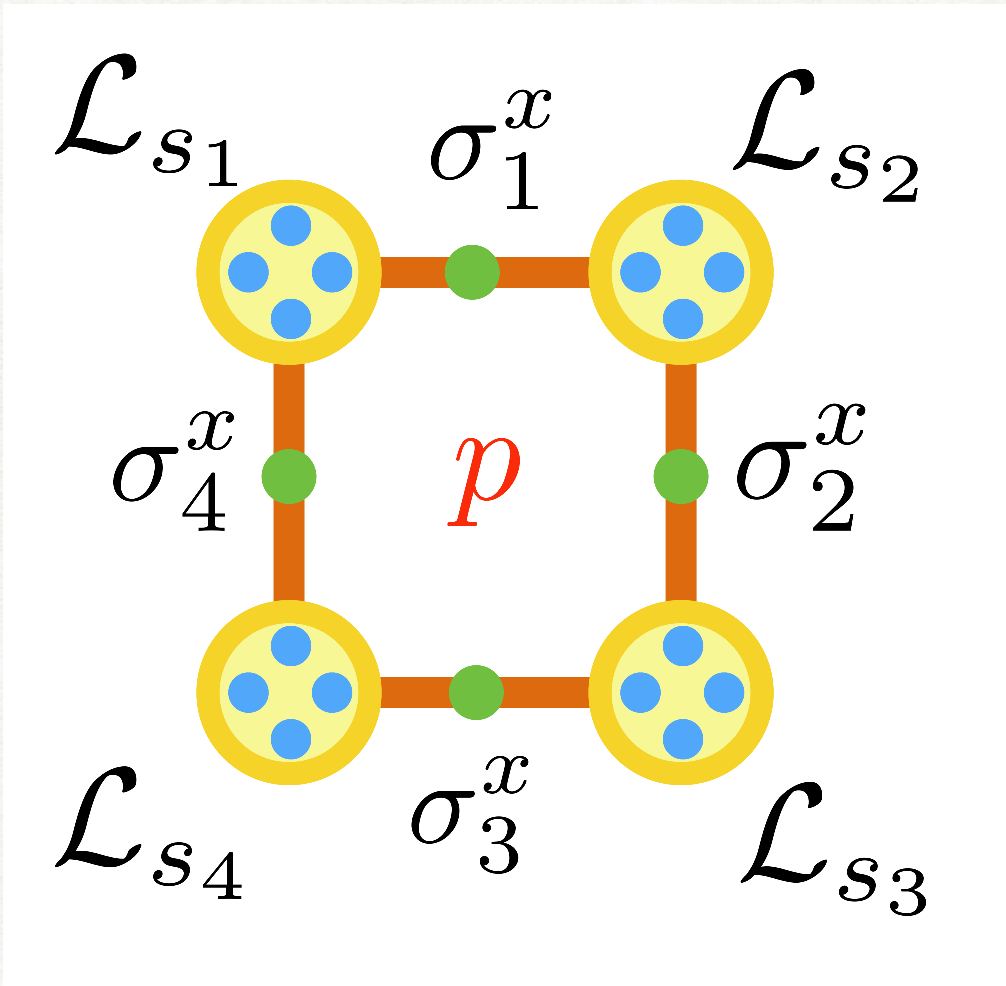
Two symmetric / anti-symmetric sets of states



Spectrum of plaquette is independent of its environment

Mathematically: exact local gauge symmetry

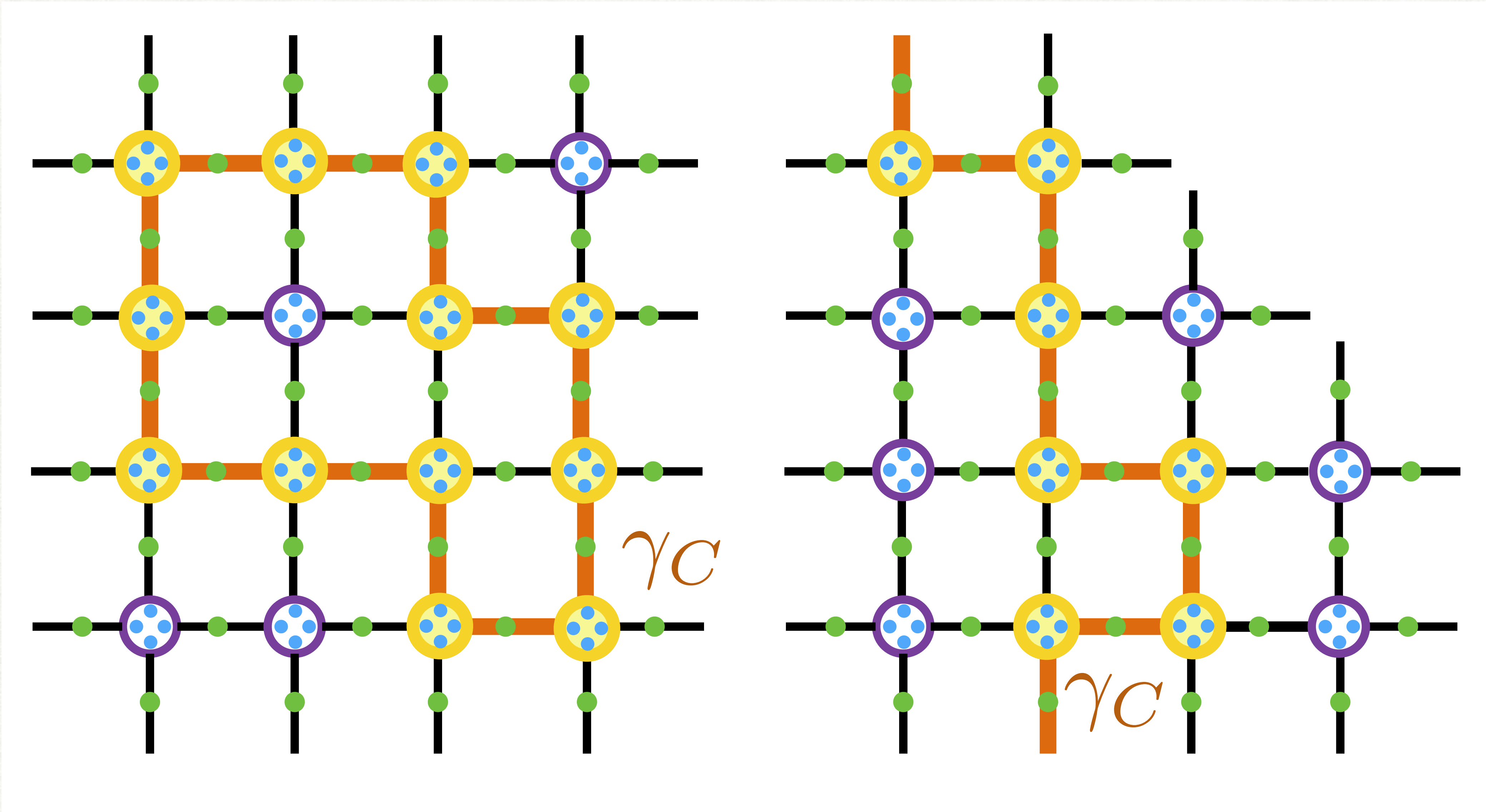
Plaquette operator



- σ^x around the plaquette flips all gauge spins, like the usual Toric Code
- $\mathscr{L}_{s_i}$ flips and permutes matter spins according to the Hadamard automorphism of W

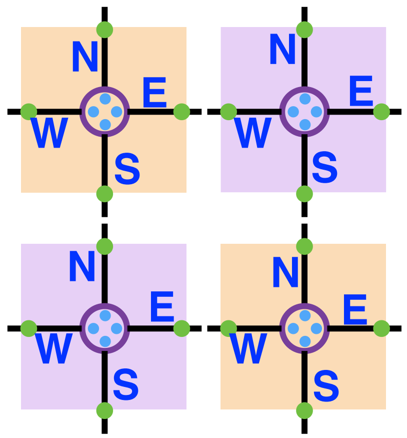
$$G_p = \prod_{s \in p} \mathcal{L}_s \prod_{i \in p} \sigma_i^x \qquad [H, G_p] = 0 \qquad [G_p, G_{p'}] = 0$$

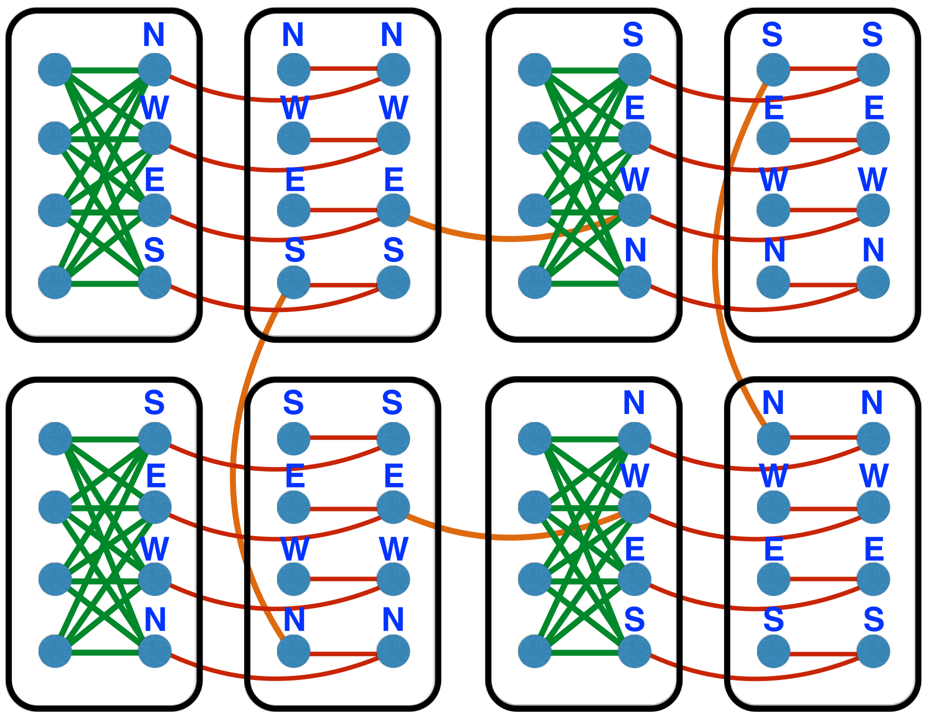
Loops and paths just like in the $\mathbb{Z}_2$ gauge theory



QSL using only two body interactions!

Embedding in D-Wave





- D-Wave qubits can connect to 5 others; we need 8 (D-Wave 2000Q chimera architecture)
- Therefore require copies of gauge spins* $\rightarrow$ effective $\widetilde{\Gamma}$
- Unit cell: 2048 / 16 x 2 = 256 gauge spins

* 6 copies of each gauge spin

SUMMARY

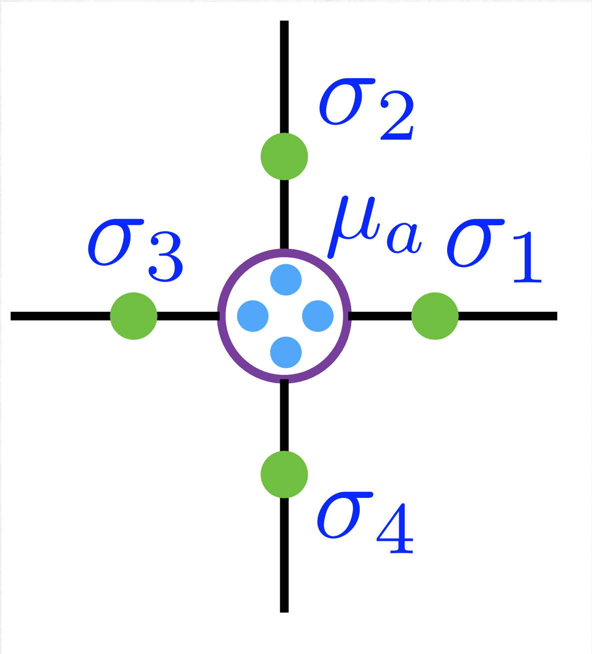
Blueprint for building QSL in programmable devices &

Alternative path to Topological QC

- Convert quantum annealer to "Emulator of Topological States" (ETS). Contingent on noise.
- Explore other systems, *e.g.*, Rydberg atoms
- Extend to Non-Abelian excitations, *e.g.*, by twists in lattice
- Optimize quantum annealer architectures, *e.g.*, 3D

Appendix

Some intuition - single star



- Consider each μ_a in an effective field of σ_i^z:

$$H = -J \sum_{a=1}^{4} \left(\sum_{i=1}^{4} W_{ai} \sigma_i^z \right) \mu_a^z - \Gamma \sum_{a=1}^{4} \mu_a^x$$

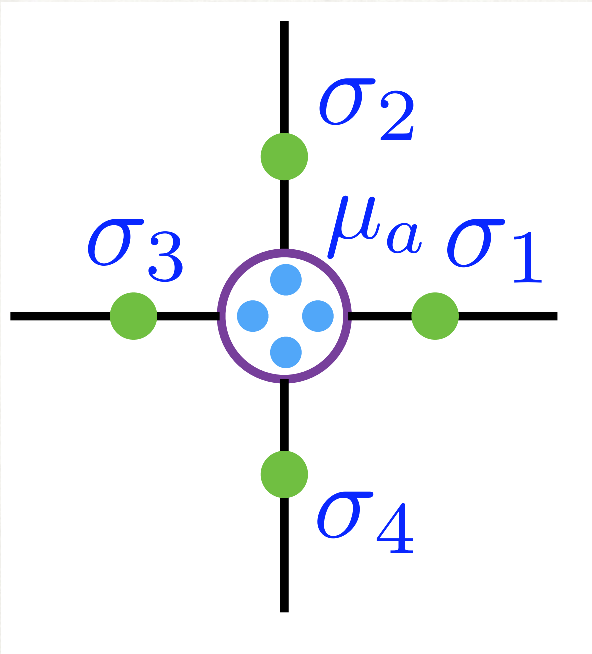
- Diagonalize each μ_a exactly:

$$E_a^{(\pm)}(\sigma_1^z, \sigma_2^z, \sigma_3^z, \sigma_4^z) = \pm \left[J^2 \left(\sum_{i=1}^{4} W_{ai} \sigma_i^z \right)^2 + \Gamma^2 \right]^{1/2}$$

- Ground state:

$$H_{(-)} = \sum_{a=1}^{4} E_a^{(-)}$$

Some intuition - single star (cont.)



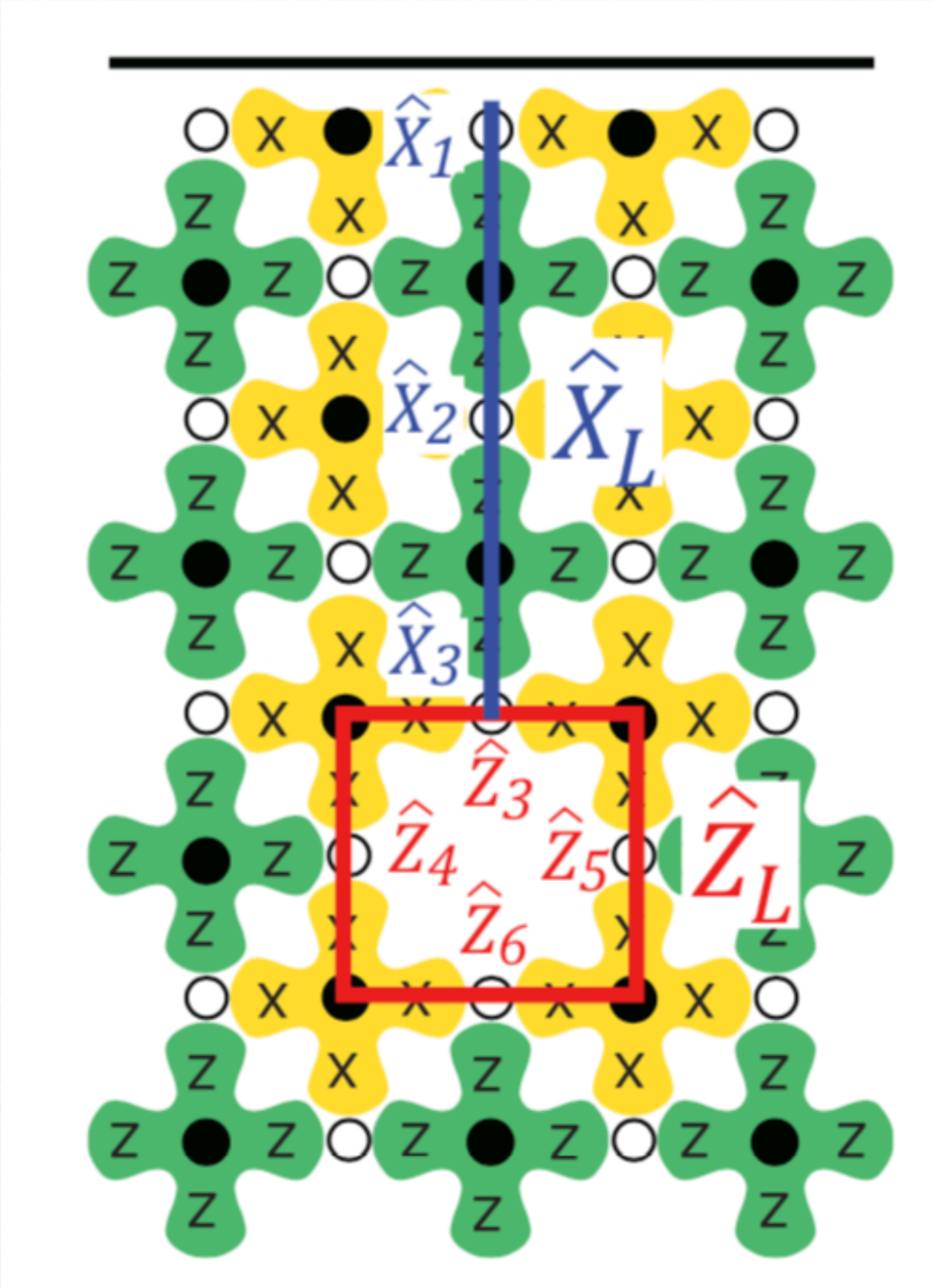
- The ground state is EXACTLY the 4-spin interaction:

$$H_{(-)} = \gamma - \lambda\, \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$$

- γ and λ are functions of J and Γ

- Holds for <u>all</u> J and Γ due to symmetries of W

Toric Code can be used for Quantum Computation

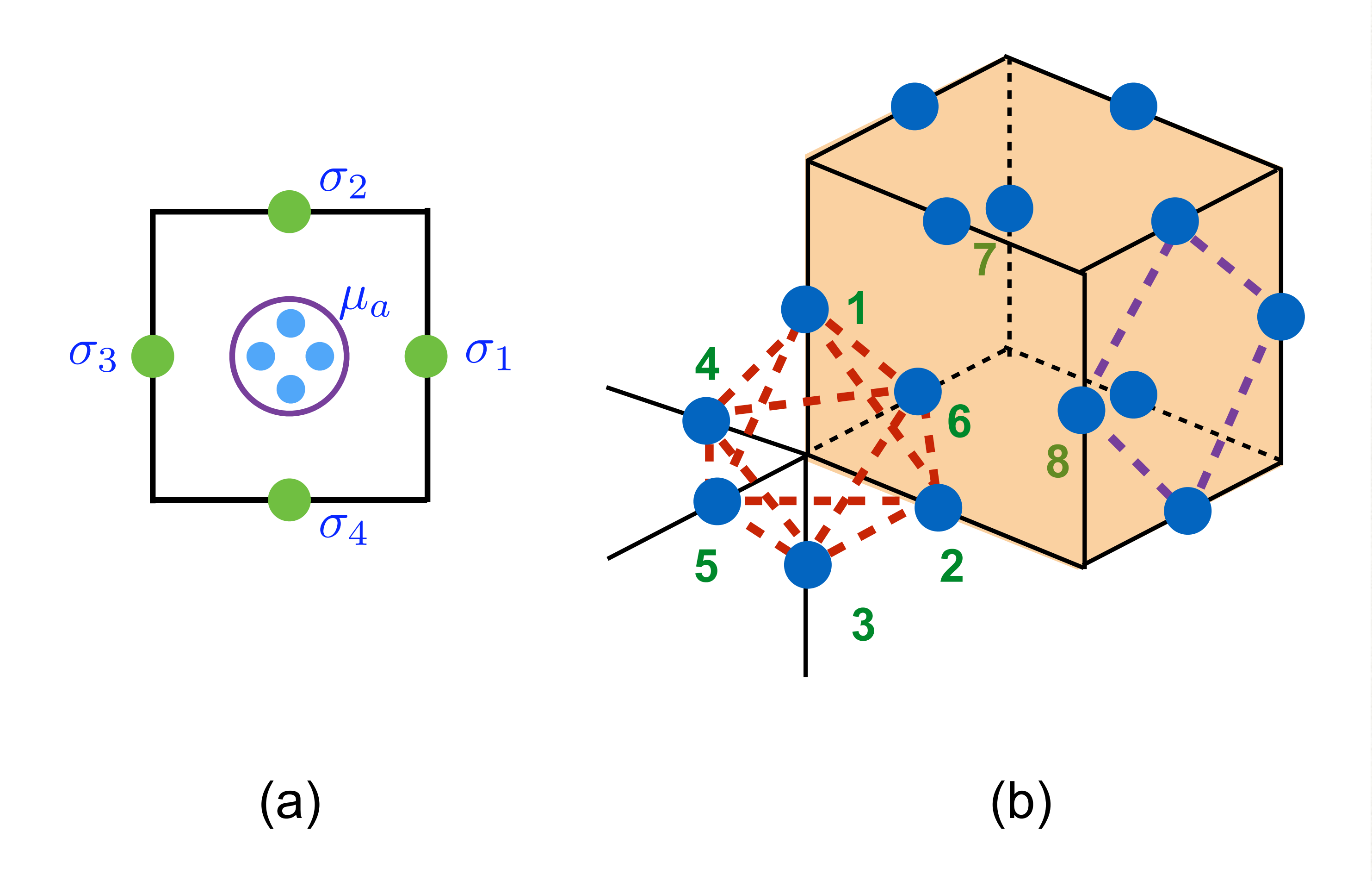


A. Fowler et al. (2012)

- Encode logical qubits by creating "holes"

Generalization: 3D toric code

Place matter spins at the center of plaquette (not vertex)



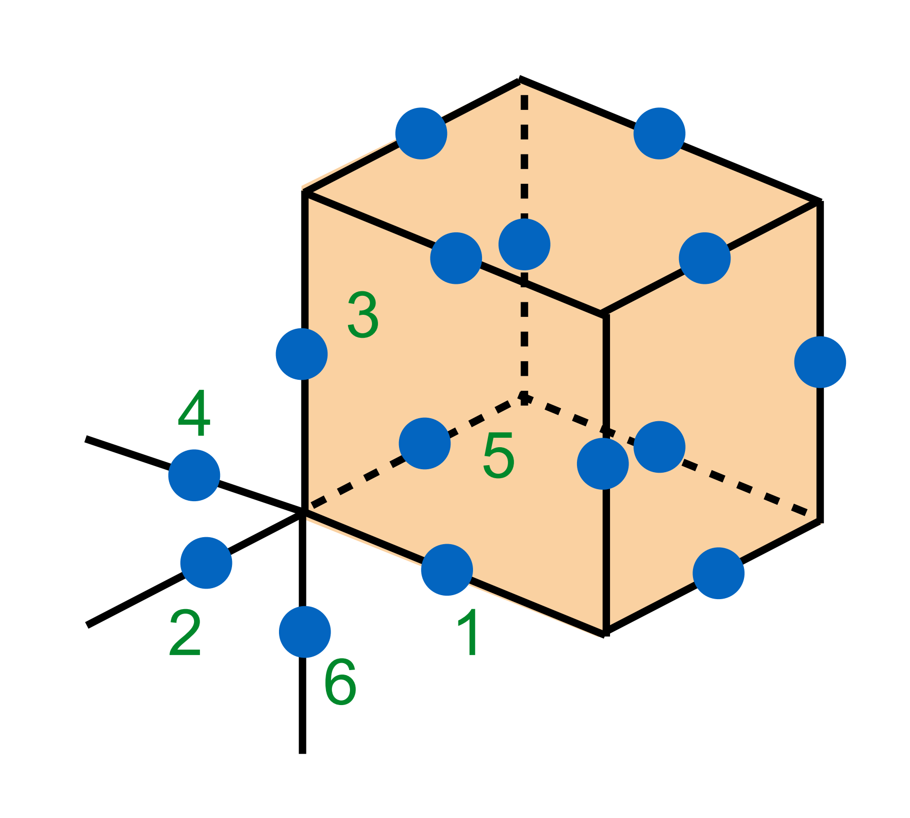
$$B_p = \prod_{i \in p} \sigma_i^z \qquad A_v = \prod_{i \in v} \sigma_i^x$$

$$B = \sigma_1^z \sigma_2^z \sigma_8^z \sigma_7^z$$

$$A = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \sigma_5^x \sigma_6^x$$

Generalization: X-Cube model

3 types of vertex operators, 12 matter spins



$$B^{xy} = \sigma_1^z \sigma_2^z \sigma_4^z \sigma_5^z$$
$$B^{yz} = \sigma_2^z \sigma_3^z \sigma_5^z \sigma_6^z$$
$$B^{xz} = \sigma_1^z \sigma_3^z \sigma_4^z \sigma_6^z$$

$$H_{\tau=-1} = -\lambda \sum_s (B_s^{xy} + B_s^{yz} + B_s^{xz}) - \widetilde{\Gamma} \sum_i \sigma_i^x$$

S. Vijay, J. Haah and L. Fu (2016)

Model exhibiting fracton topological order, sub-extensive entropy