Emergent gravity

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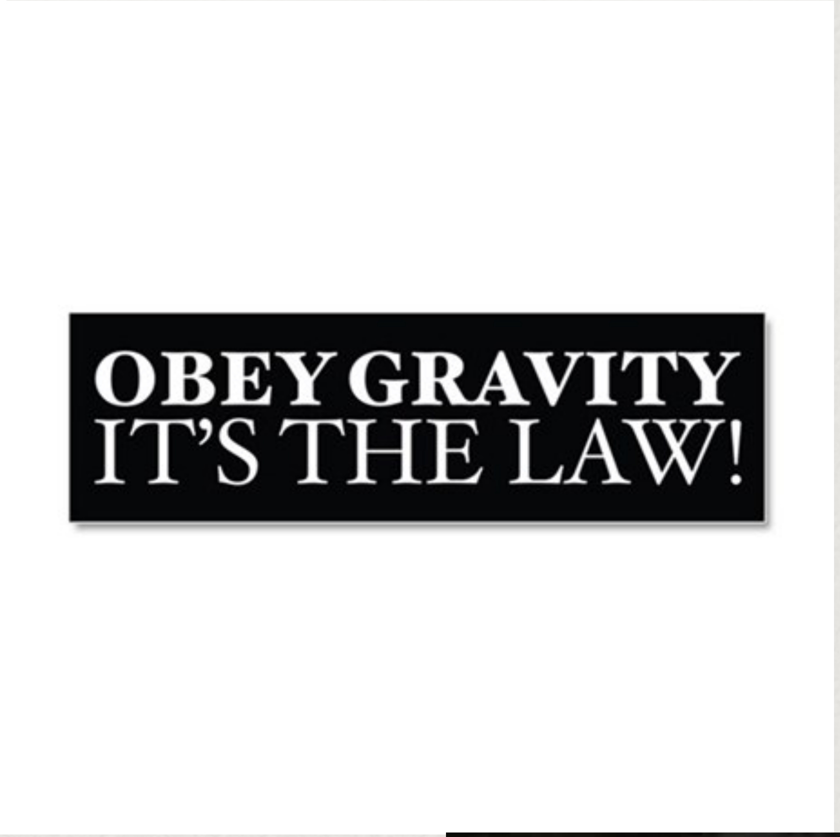
Wile E. Coyote's
GRAVITY
LESSONS


OBEY GRAVITY
IT'S THE LAW!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Gravity Gets
Me Down

What is gravity?

Newton, 1686: Universal gravitational attraction law

$$F = G\frac{M_1 M_2}{R_{12}^2}$$

Einstein, General Relativity, 1915:
Gravity is a manifestation of the geometry/curvature of spacetime.

$$R_{\mu\nu} - g_{\mu\nu}R = 8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Is gravity one of the fundamental forces, besides electromagnetism,
weak and strong nuclear forces?

Fundamental forces are described by fields. For gravitation, the field is the metric of spacetime $g_{\mu\nu}$.

Distances between spacetime points $x^\mu, x^\mu + dx^\mu$ *are measured by*

$$ds^2 = dx^\mu dx^\nu g_{\mu\nu}(x)$$

which is invariant under a change of coordinates

$$x^\mu \longrightarrow \xi^\mu(x)$$

$$g_{\mu\nu}(x) \longrightarrow \tilde{g}_{\alpha\beta}(\xi)$$

$$g_{\mu\nu}(x) = \tilde{g}_{\alpha\beta}(\xi)\frac{\partial \xi^\alpha}{\partial x^\mu}\frac{\partial \xi^\beta}{\partial x^\nu} .$$

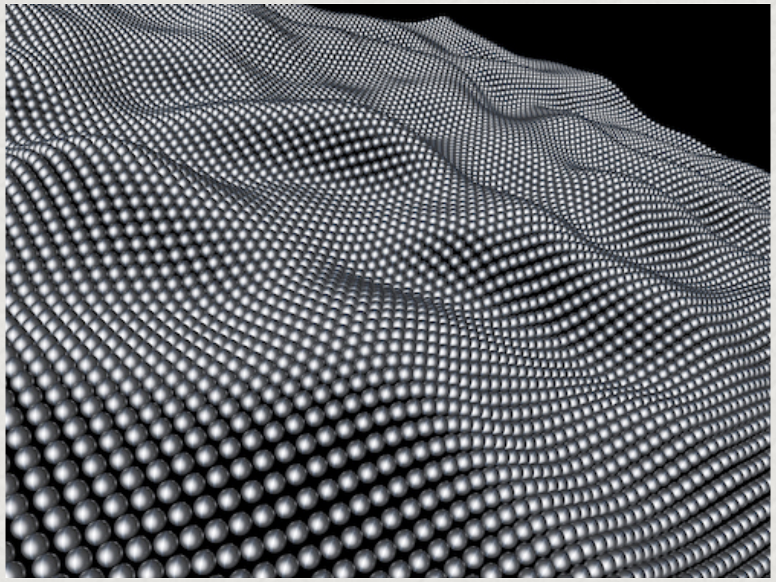
diffeomorphisms/general coordinate/Einstein transformations

This is the symmetry of General Relativity.

Weak field limit of Einstein's equation:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

metric fluctuation



Linearized Einstein equation, without sources:

$$\Box\, h_{\mu\nu} + \partial_\mu \partial_\nu h - \partial_\mu \partial^\rho h_{\rho\nu} - \partial_\nu \partial^\rho h_{\mu\rho} = 0$$

is invariant under Einstein transformations

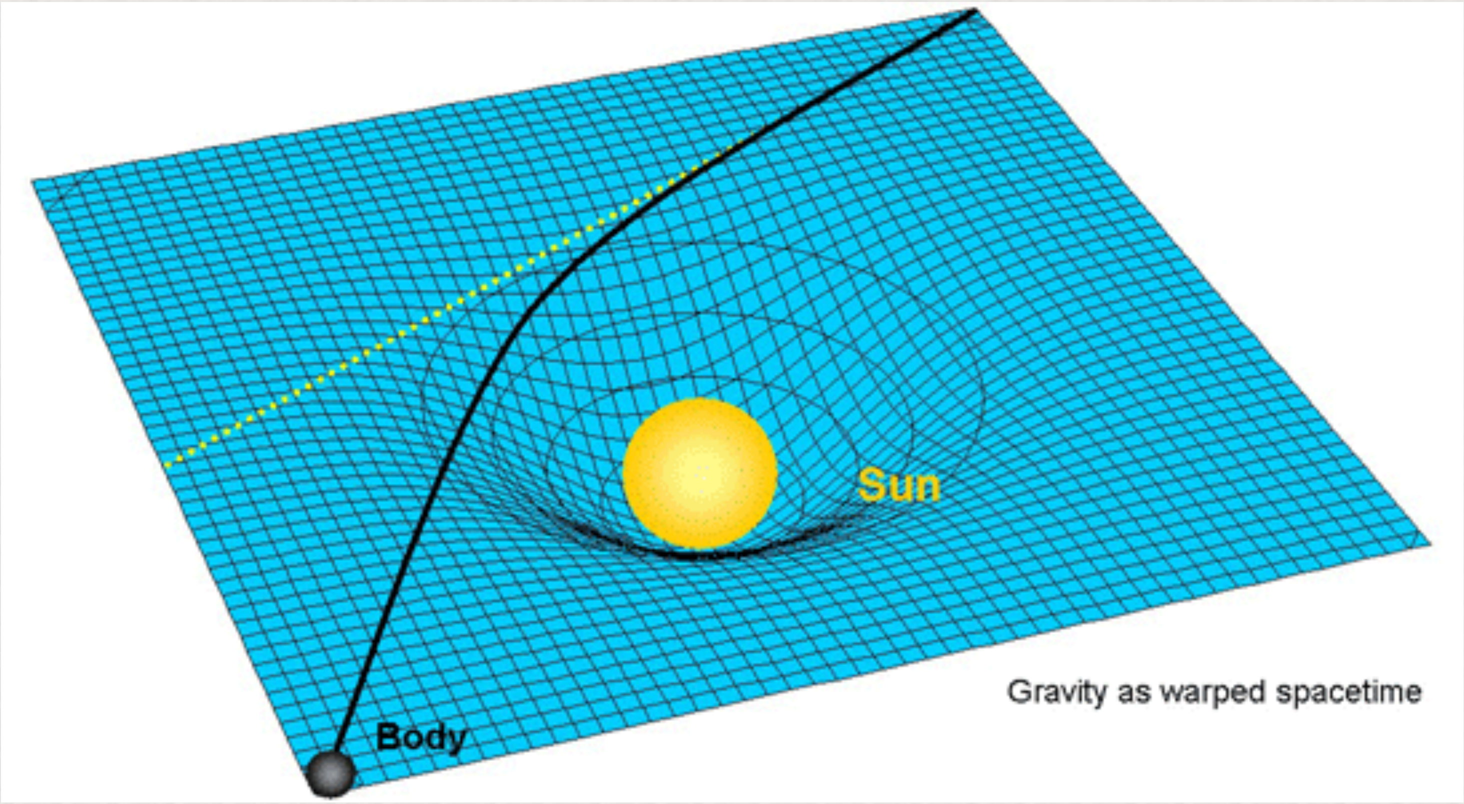
$$\delta h_{\mu\nu} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$$

Fix a gauge: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h,\qquad \partial^\rho \bar{h}_{\rho\nu} = 0$

$\Rightarrow\quad \Box\, \bar{h}_{\mu\nu} = 0 \quad\Rightarrow\quad$ *Gravitational waves*

Weak field limit, with sources

Massive objects curve spacetime.



$$R_{\mu\nu} - g_{\mu\nu}R = 8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu}$$

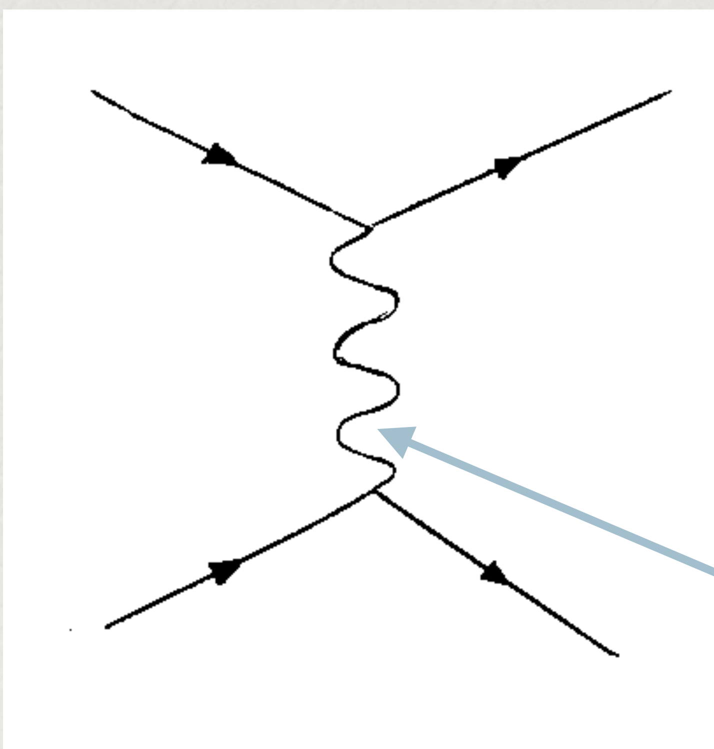
In a curved spacetime free particles/planets move on geodesics.

$\Rightarrow$ *Gravity!*

Weak field limit, with sources

Gravity couples to the energy-momentum tensor

$$S = S_{\text{matter}}[\phi] + S_{\text{gravity}}[h_{\mu\nu}] - \int \frac{1}{2} T_{\mu\nu}(x)\, h^{\mu\nu}(x) + \ldots$$



With $T^{00} = M_1 \delta(\vec{r})$, $T^{00} = M_2 \delta(\vec{r} - \vec{R})$

$$\Longrightarrow \quad V(R) = -\,G \frac{M_1 M_2}{R}$$

Graviton (massless spin 2 particle) exchange/propagator

Electromagnetism analogy:

Currents couple to the electromagnetic potential/field

$$\int_x j^\mu A_\mu\,,\quad j^\mu = (\text{charge density}, \overrightarrow{\text{current density}})$$

The field sourced by a particular current

Massless pole in q-space

$$A_\mu(x) = \int_y Propagator_{\mu\nu}(x,y)j^\nu(y)$$

An electric charge would interact with the electromagnetic field of the other charge

$$\int_{x,y} J^\mu(x)\text{Propagator}_{\mu\nu}(x,y)j^\nu(y) \Rightarrow$$ *Coulomb potential*

Faraday, 1850: "On the possible relation of gravity to electricity"

"Gravity. Surely this force must be capable of an experimental relation to electricity, magnetism, and other forces, so as to bind it up with them in reciprocal action and equivalent effect."

"I have been arranging certain experiments in reference to the notion that gravity itself may be practically or directly related by experiment to the other powers of matter, and this morning proceeded to make them."

"Here end my trials for the present. The results are negative. They do not shake my strong feeling of the existence of a relation between gravity and electricity, though they give no proof that such a relation exists."

Weinberg-Witten: A NO-GO Theorem

Can massless spin two states (gravitons) arise as bound states in a quantum field theory of particles with spin less or equal to one? If yes, then gravity is emergent.

Weinberg-Witten Theorem: There can be no massless states with helicity greater than one in theories with a conserved, Lorentz covariant energy-momentum tensor $T_{\mu\nu}$ *, with nonzero values of the energy and/or momentum.*

Corollary: the Standard Model (or any relativistic theory) cannot accommodate a composite massless graviton.

Emergent gravity from the vanishing of $T_{\mu\nu}$

Consider the following possibility to evade the WW theorem: a field theory which is diffeomorphism invariant, in particular a theory for which

$$T_{\mu\nu} = 0$$

Carone, Erlich, DV: consider N+D scalars coupled to an auxiliary metric $g_{\mu\nu}$

$$S = \int d^D x \sqrt{|g|} \left[\frac{1}{2} g^{\mu\nu} \left(\sum_{a=1}^{N} \partial_\mu \phi^a \, \partial_\nu \phi^a + \sum_{I=0}^{D-1} \partial_\mu X^I \partial_\nu X^J \eta_{IJ} \right) - V(\phi^a) \right]$$

The metric is non-dynamical. The energy momentum tensor is zero.

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \quad \Longrightarrow \quad T_{\mu\nu} = 0$$

Solve for the auxiliary metric from the vanishing of $T_{\mu\nu}$.

Substitute into the action. Obtain:

$$S = \int d^D x \left(\frac{D-2}{2V(\phi)} \right)^{D/2-1} \sqrt{\left| \det \left(\sum_{a=1}^{N} \partial_\mu \phi^a \partial_\nu \phi^a + \sum_{I,J=0}^{D-1} \partial_\mu X^I \partial_\nu X^J \eta_{IJ} \right) \right|}$$

Reminiscent of the action for a free particle

$$S = -\, m \int d\tau \sqrt{|\dot{X}^\mu(\tau) \dot{X}^\nu(\tau) \eta_{\mu\nu}|}$$

or, the Nambu-Goto string action

$$S = -\, \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{\left| \det \left((\partial_\alpha X^\mu(\xi) \partial_\beta X^\nu(\xi)\, \eta_{\mu\nu} \right) \right|}$$

Emergent gravity model: non-metric action,

$$S = \int d^D x \left(\frac{D-2}{2V(\phi)}\right)^{D/2-1} \sqrt{\left|\det\left(\sum_{a=1}^{N} \partial_\mu \phi^a \partial_\nu \phi^a + \sum_{I,J=0}^{D-1} \partial_\mu X^I \partial_\nu X^J \eta_{IJ}\right)\right|}$$

which is invariant under reparametrizations, just like the free particle/string actions.

Matter fields

Clock-and-ruler fields

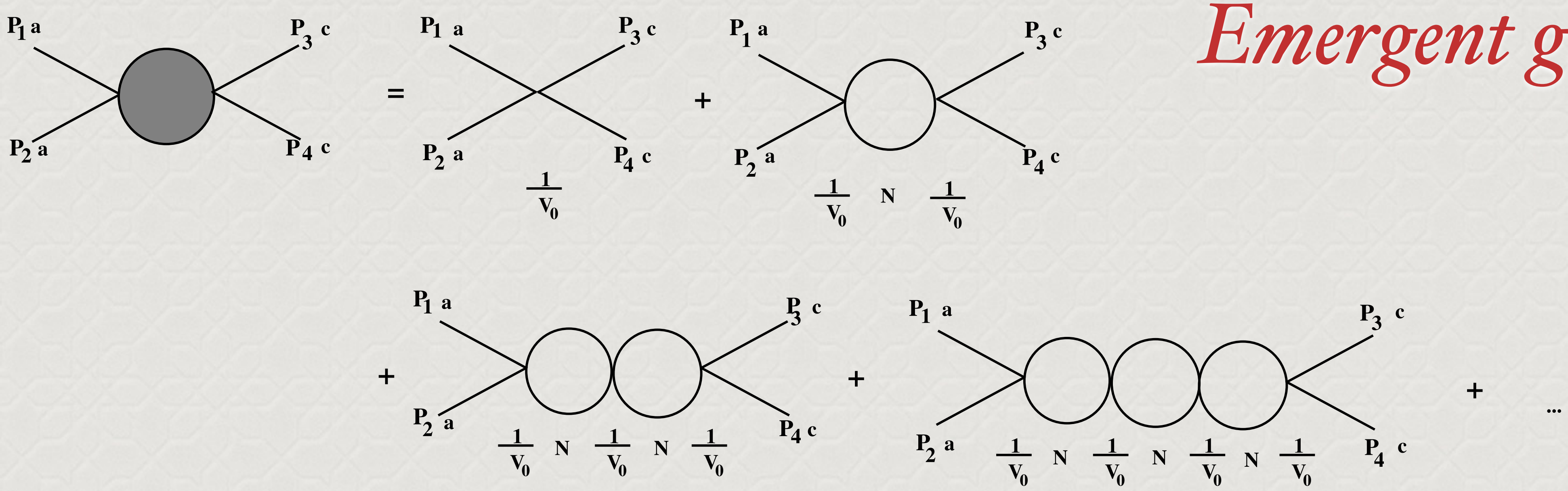
Gauge fix: $X^I \propto x^\mu \delta^I_\mu$

A large energy scale

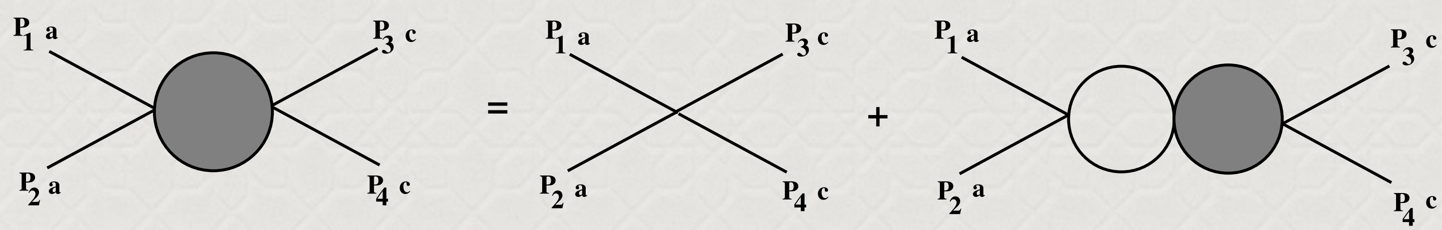
$$S = S_{\text{free}}[\phi] + \int_x \frac{1}{V_0} t_{\mu\nu} \Pi^{\mu\nu|\rho\sigma} t_{\rho\sigma} + \dots$$

Energy-momentum tensor of free scalar fields

Emergent gravitons

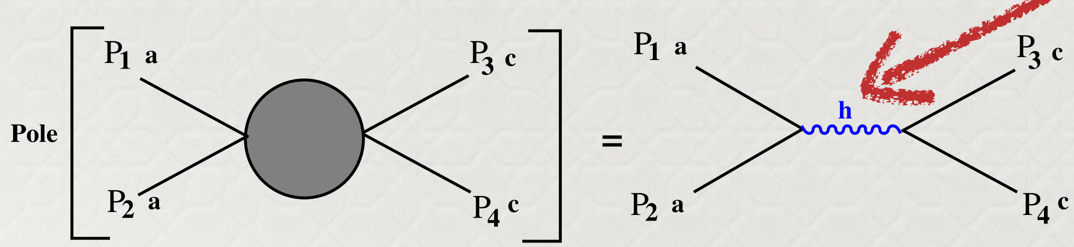


(a)



(b)

Graviton exchange
in a 2-2 scalar scattering process

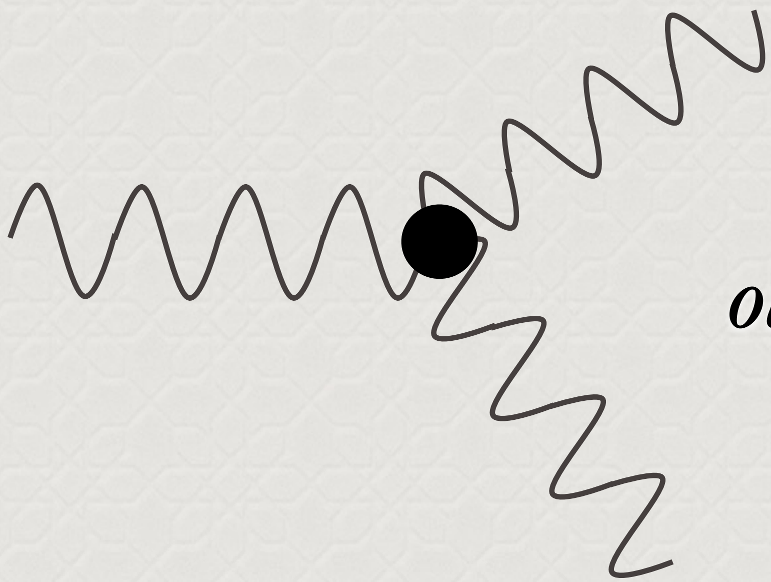


$$= t_{\mu\nu}(q)\frac{\cdots}{q^2}t_{\rho\sigma}(q)$$

(c)

Planck mass/Newton's constant is determined in terms of the UV regulator.

What about emergent graviton self-interactions?

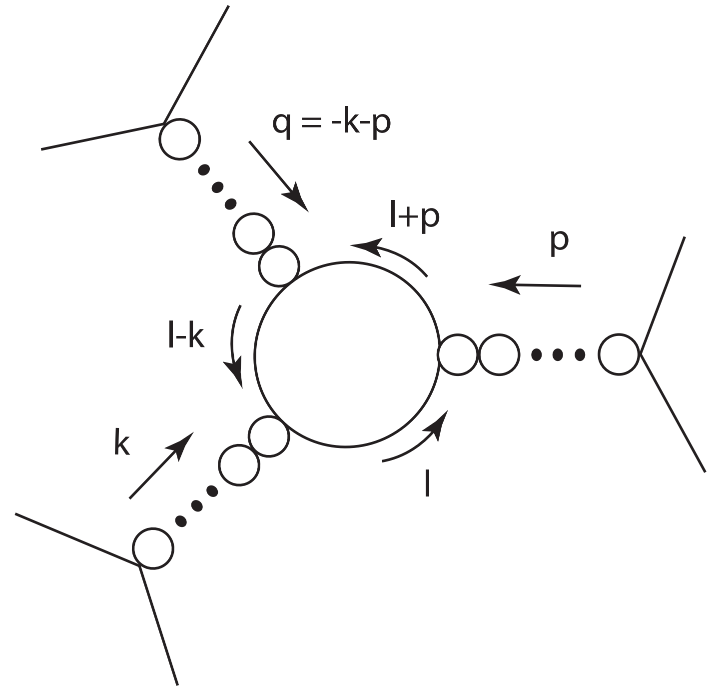


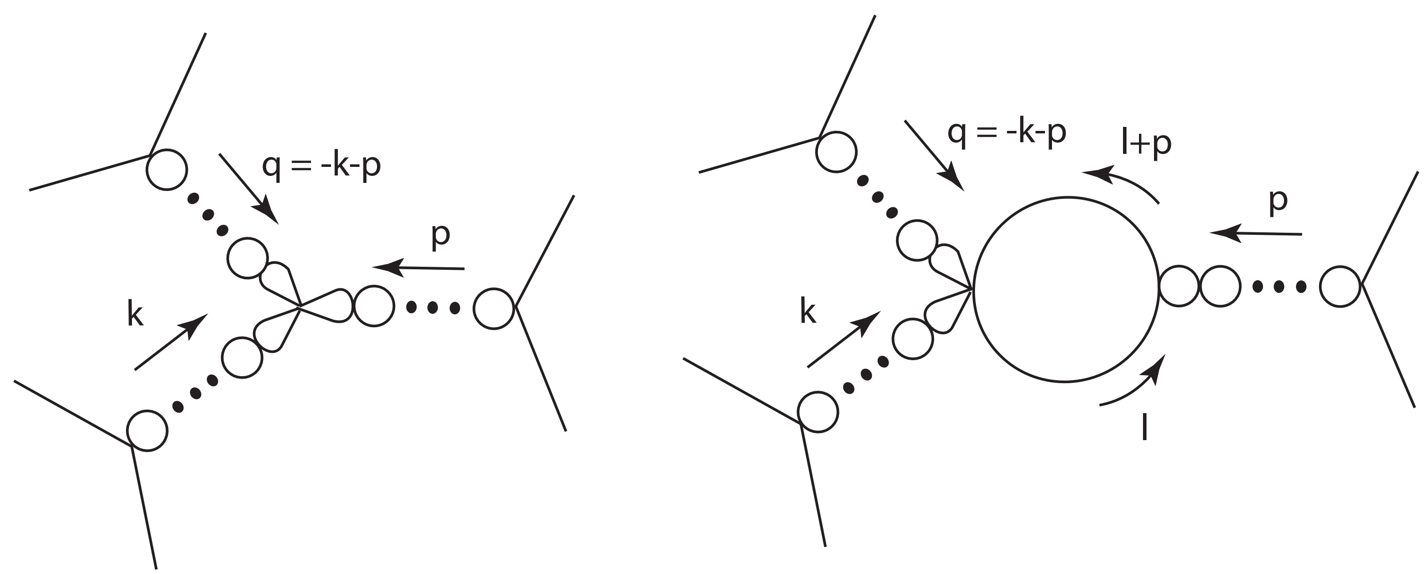
Cubic graviton interaction obtained from Einstein's action:

$$\frac{1}{16\pi G}\int d^4x\sqrt{g}R$$

Carone, Claringbold, DV:

This is accounted for by expanding the scalar action to 6th order in fields and computing a 6-scalar amplitude.





(a) (b)

Emergent graviton cubic interactions match those of GR.

Beyond flat space-time linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad\qquad \langle h_{\mu\nu} \rangle = 0$$

Want to have a geometry which is non-flat?

Energy momentum tensor in flat spacetime

Add a conserved source to the non-metric action

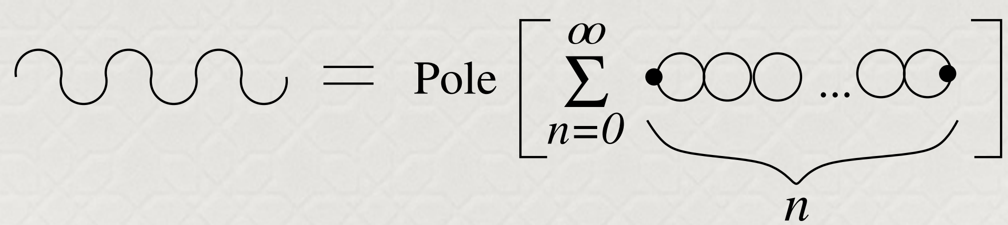
$$\delta S = \int d^D x J^{\alpha\beta}(x) P_{\alpha\beta|\mu\nu} t^{\mu\nu}[\phi]$$

A Minkowski metric bi-tensor

$$\langle h_{\mu\nu} \rangle_J =$$

Cubic graviton vertex

(a)



(b)

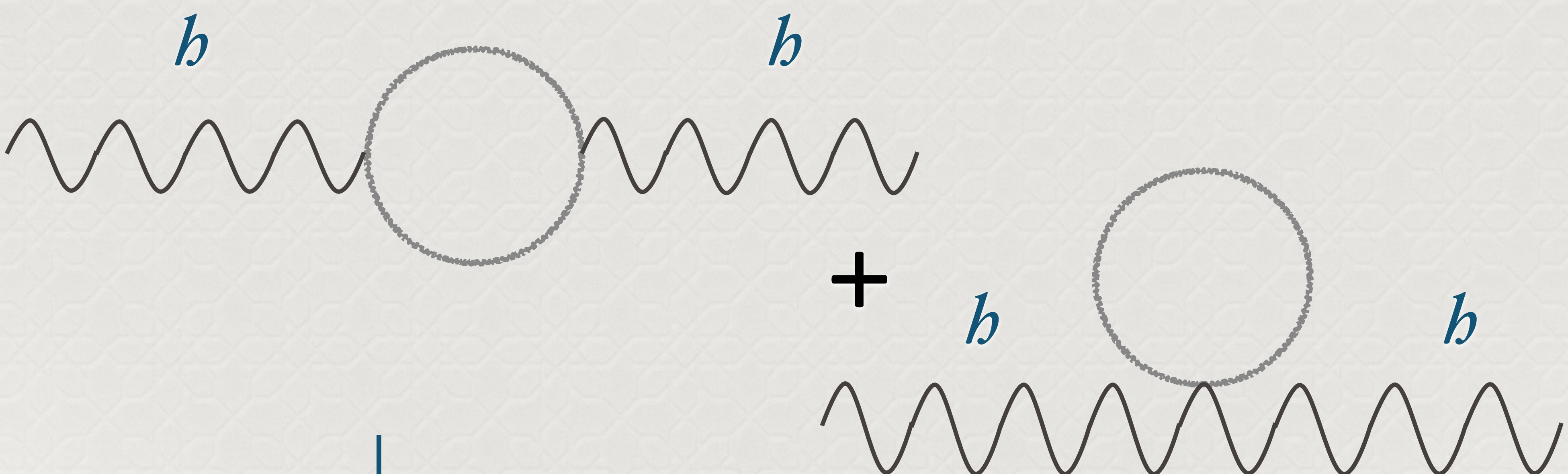
Past attempts at composite gravitons

Start with some generic quartic interaction T.. T.. and constrain such that a massless spin 2 pole is found. Moving beyond linearized gravity was a problem.

Here we start with a theory that has the same symmetries as General Relativity. It was predetermined that the composite graviton, if found at the linearized level, will have the same interactions as in GR.

Sakharov: Induced gravity

1967: Sakharov considered the possibility that gravity is like elasticity of spacetime. He considered matter fields in a curved spacetime. The quantum dynamics of matter fields generates an effective action for the background metric, which includes the GR Einstein action.



$$= S_{\text{effective}}[h]\Big|_{2nd\ order\ h} \qquad = S_{\text{Einstein-Hilbert}} + \text{cosmo const}\Big|_{2nd\ order\ h}$$

Bjorken: Emergent electromagnetism

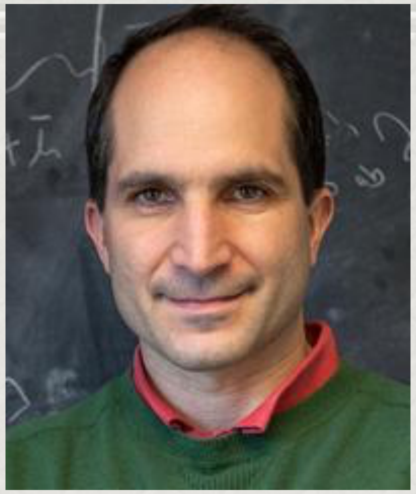
1963: "A dynamical origin for the electromagnetic field"

Electromagnetism is not a fundamental force: start with a Nambu Jona-Lasinio model with a 4-fermi interaction. Introduce an auxiliary 4-vector field A and do a Hubbard-Stratonovich transformation. Integrate out the fermions. Get an effective action for the auxiliary field

$$S_{\text{effective}}[A] = S_{Maxwell} + \int \#A \cdot A + \#(A \cdot A)^2$$

Then break Lorentz symmetry by giving a vev to the field A. The massless Nambu-Goldstone mode is be interpreted as the photon.

Gravity from entanglement in AdS/CFT



AdS/CFT correspondence/holography

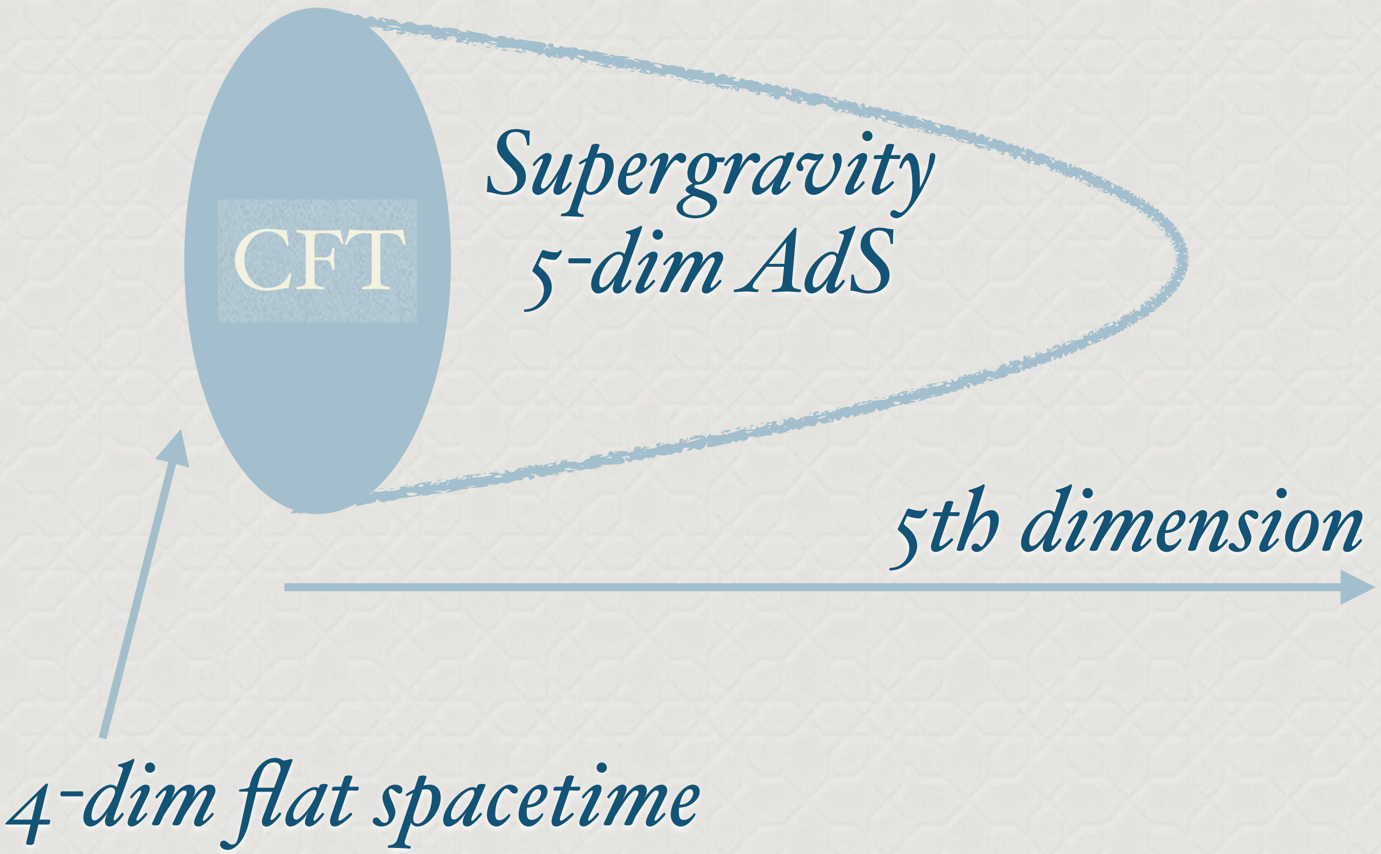
1998: Maldacena formulated a duality between a strongly coupled field theory (of spin less or equal to one) and a weakly coupled gravity/superstring theory in a curved space time with one extra infinite-extent direction, called the radial or holographic direction. The flat-space field theory lives at the boundary of the gravity bulk space.

Dictionary:

AdS = Anti de Sitter space

$$-X_{-1}^2 - X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = -R^2$$

CFT: conformal field theory $SO(2,4)$



Escher

What is it used for?

Compute correlation functions of operators in the boundary theory using the supergravity partition function.

Entanglement

Have the field theory be described by some density matrix/operator state ρ .

Define a subregion A of the field theory space-time. Perform measurements of observables with support only in A.

$$\langle O_A \rangle = Tr(\rho O_A) = Tr_A(\rho_A O_A)$$

ρ_A : *reduced density matrix* $\qquad \rho_A = Tr_{A^c}(\rho)$

Entanglement entropy: measure of the information loss regarding the degrees of freedom in A^c

$$S_A = -Tr(\rho_A \ln \rho_A)$$

Example: $\rho = \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|)$

$$\rho_1 = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$$

Focus on the reduced density matrix and define the entanglement Hamiltonian:

$$\rho_A = \exp(-H_A)$$

Then, for a spherical region A in a CFT,

Casini, Huerta, Myers

Energy density

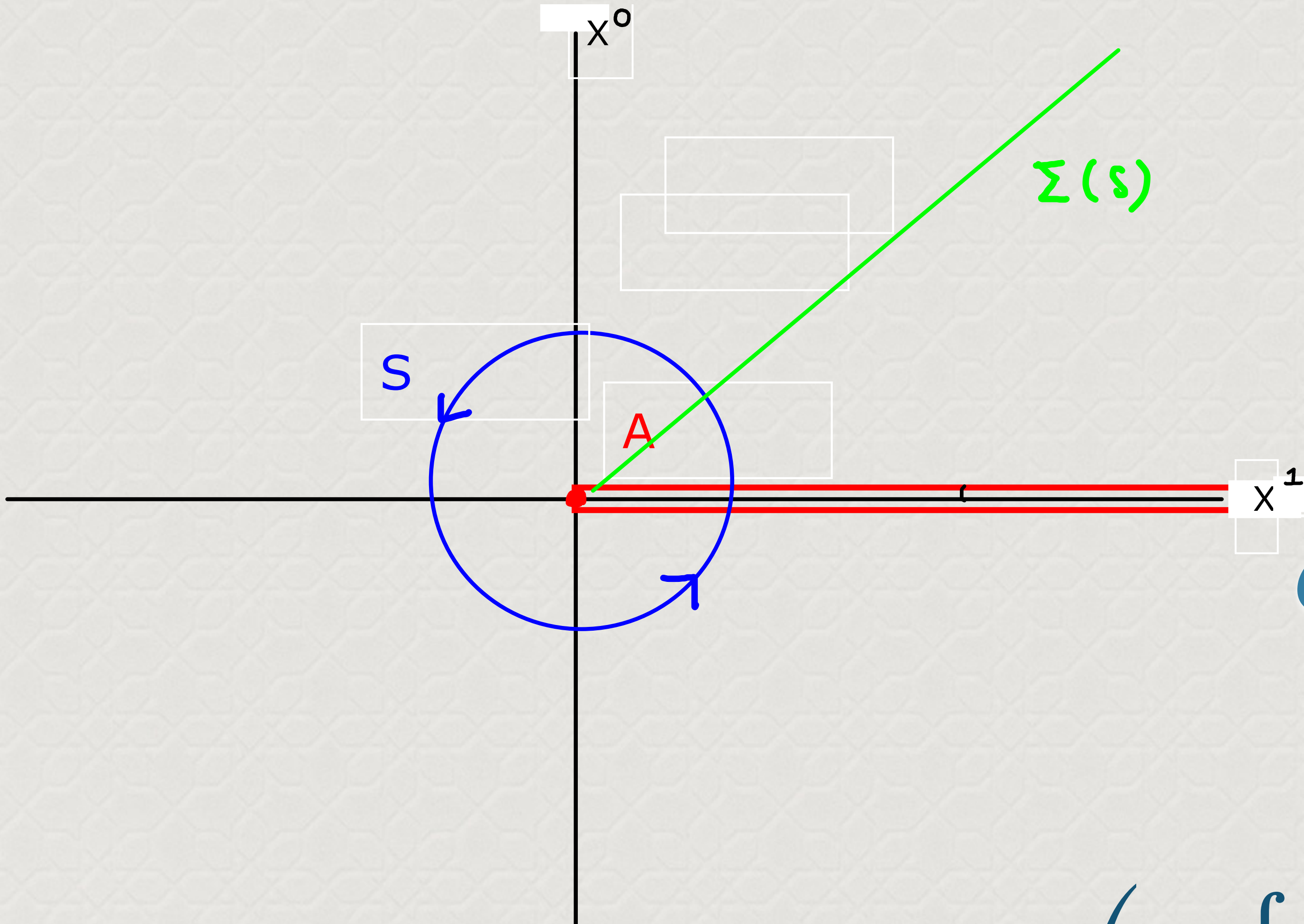
$$H_A = \int_A \beta(x) T_{00}(x)$$

Inverse entanglement temperature

Entanglement entropy $$S_A = \int_A \frac{c\pi}{3\beta(x)}$$

Proof of the CHM formula

Klich, Wong, Pando-Zayas, DV



Generator of s-evolution

$$\langle \phi(x)^- \,|\, \rho_A \,|\, \phi(x)^+ \rangle = Z_A^{-1} \exp\left(- \int_s K(s) \right)$$

2dim QFT, A=half-line :

$$K = \text{ang mom } = \int_\Sigma (x^1 T_{00} - x^0 T_{01}) = \int_A x^1 T_{00} \quad \Rightarrow \quad \beta = 2\pi x^1$$

Just for curiosity's sake, $$S_A = \int dx \frac{c\pi}{6x} = \frac{c}{6} \ln \frac{L}{\epsilon}$$

Entanglement entropy of excited states and first law for EE

Consider a small change $\rho \longrightarrow \rho + \delta\rho$

The entropy change is

$$\delta S_A = -\,Tr(\delta\rho_A \ln(\rho_A)) = \int_A \beta(x)\delta\langle T_{00}(x)\rangle$$

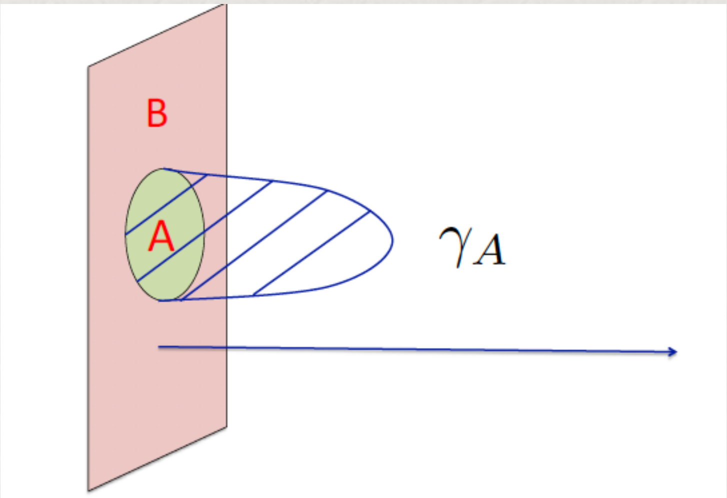
For a small region A $$\delta S = \beta_0\, \delta E_A$$

$$TdS = dE$$

Klich, Wong, Pando-Zayas, DV

Ryu-Takayanagi formula for holographic EE

2006: The entanglement entropy for a CFT state, in a subregion A, is computed in the holographic dual by the minimal area of a surface which is anchored at the boundary onto ∂A .



$$S_A = \frac{Area(\gamma_A)}{4G}$$

γ_A *is the bulk geometry minimal surface*

Entanglement entropy is directly related to the bulk metric.

Constraints on bulk gravity from EE

Bulk metric expansion at the boundary

$$ds^2 = \frac{1}{z^2}(dz^2 + dx^\mu dx_\mu + z^D H_{\mu\nu}(x,z) dx^\mu dx^\nu)$$

Holography: $$\langle T_{\mu\nu}(x)\rangle = \frac{D}{16\pi G} H_{\mu\nu}(x, z=0)$$

Consider a first order change to the area due to a small change to the boundary metric

$$\delta S_A = \frac{1}{8G}\int_{\gamma_A} \delta g_{ab}\, g^{ab}\sqrt{g} = \int_A \beta\langle \delta T_{00}\rangle$$

True if the bulk metric fluctuation obeys Einstein's equation.

Other examples of emergent gravity

Verlinde, 2010 *Gravity is an entropic force.*

Imagine a holographic screen; a particle approaching the screen will change the entropy stored by 1 unit if at Compton length away.

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \delta x$$

An entropic force will be felt by the particle if the screen has some temperature

$$F\delta x = T\Delta S$$

Relate temperature to acceleration $k_b T = \frac{\hbar}{c} a$

Get Newton's second law. $F = ma$

And gravity?

Consider a mass M inside a spherical holographic screen.

On the screen store N bits of information

$$N = \frac{Ac^3}{G\hbar} = \frac{4\pi R^2 c^3}{G\hbar}$$

Relate energy and temperature on the screen

$$E = \frac{1}{2} N k_B T = Mc^2$$

Use the F(orce)-S(entropy) relation for a test particle approaching the screen

$$F\delta x = T\Delta S = 2\pi k_B T \frac{mc\delta x}{\hbar}$$

$$\Rightarrow \quad F = G\frac{Mm}{R^2}$$

Thank you!