The Black Hole Information Paradox, and its resolution in string theory

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Hawking 1974:

General relativity predicts black holes

Quantum mechanics around black holes is INCONSISTENT

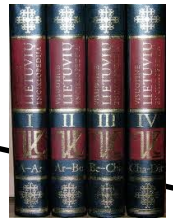
This is known as the black hole information paradox

Strings 2000 (Michigan): Top 10 problems for the new millenium (Duff, Gross, Witten)

8: What is the resolution of the black hole information paradox?

In 2004, Stephen Hawking surrendered his bet to John Preskill ...

Stephen Hawking



John Preskill

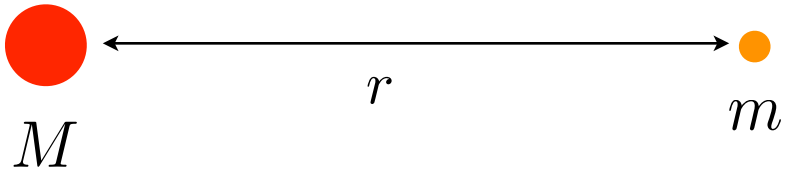
Kip Thorne

But Kip Thorne did not agree to surrender the bet ...

What is going on ?

A first pass

Gravity is an attractive force



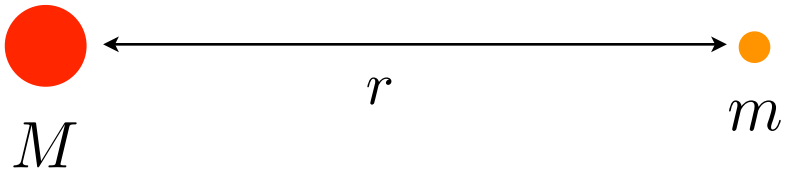
$$PE = -\frac{GMm}{r}$$

By itself, the small mass has an intrinsic energy

$$E = mc^2$$

When it is placed near the larger mass, what energy should we assign ?
Let us start with the Newtonian approximation ...

$$E = mc^2 - \frac{GMm}{r}$$

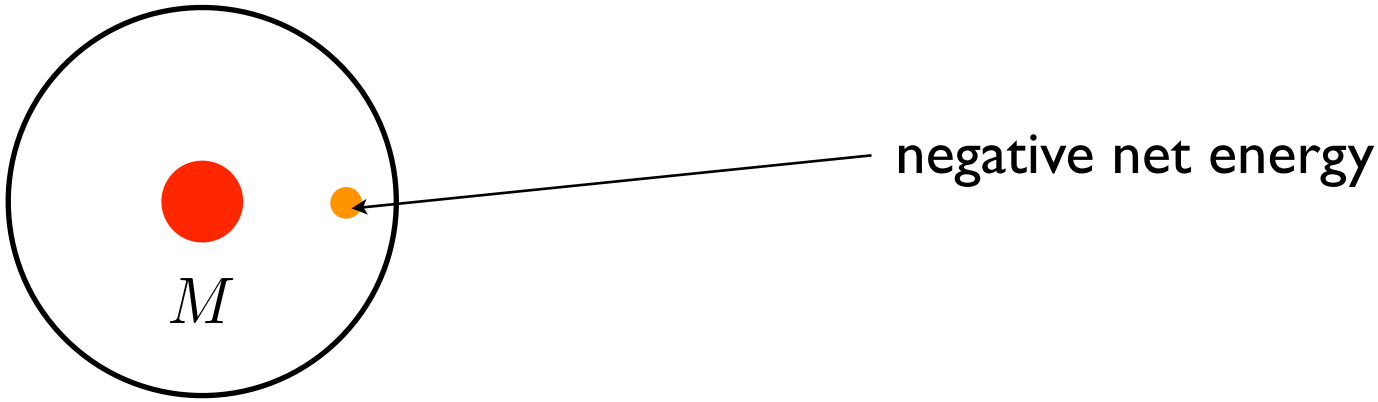


$$E = mc^2 - \frac{GMm}{r}$$

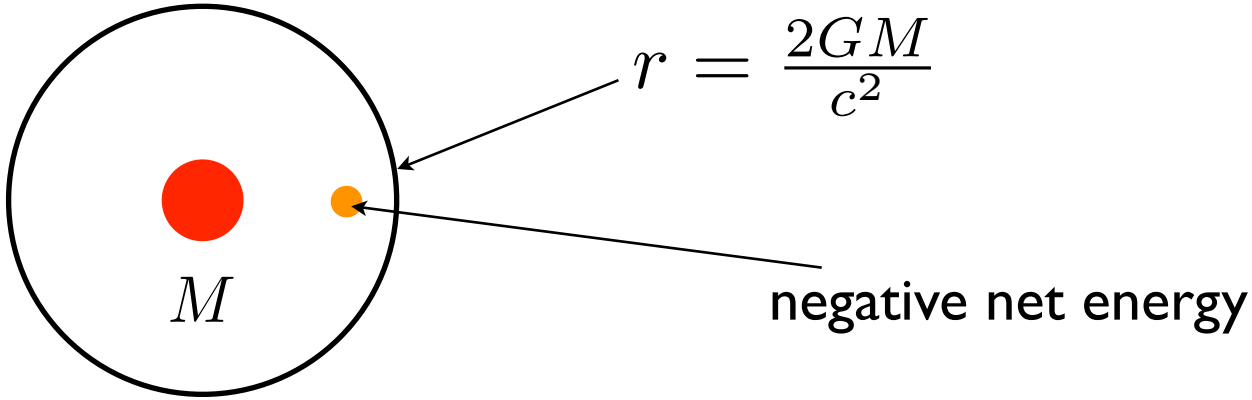
We see that the total energy of m becomes zero at

$$r = \frac{GM}{c^2}$$

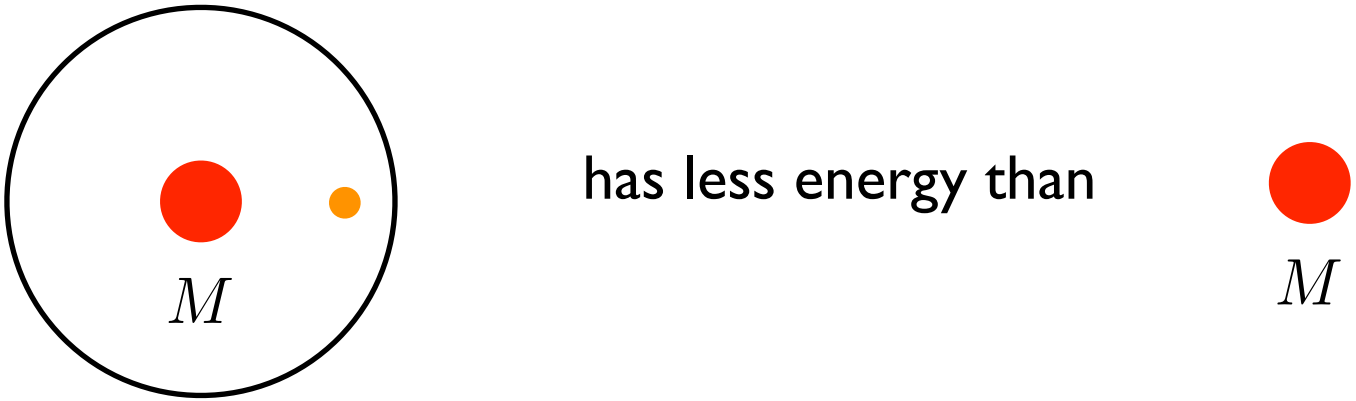
and for smaller r it is negative

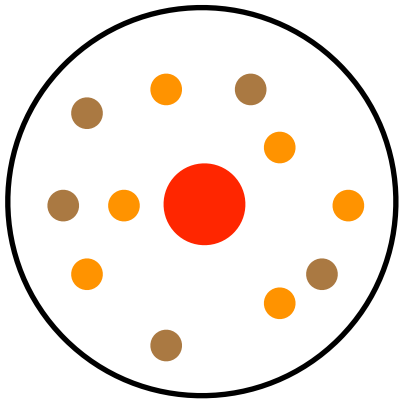


Doing this properly with general relativity does not change the answer much



So we see that

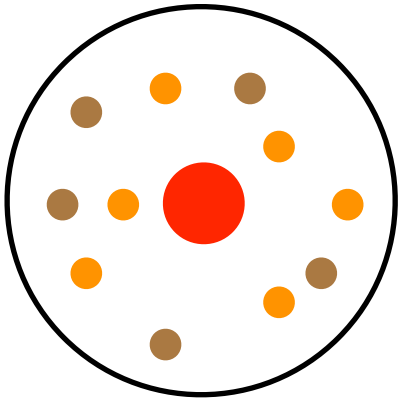




Suppose we keep placing more and more masses inside the horizon radius, until the mass comes to zero (or close to zero; it may stop at the planck mass)

Then we have a low mass object with a lot of internal structure. Such objects are called remnants

The problem is that we can make an infinite number of remnants ...



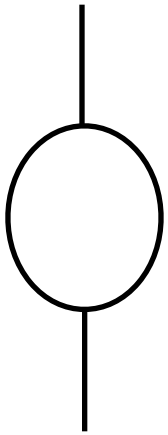
(a) There are many ways to choose the mass M

(b) There are many ways to rearrange the masses m
If there are N masses, then there are 2^N arrangements

(c) We can take bigger and bigger masses M
and cancel their mass down to zero

Because of (c), there will be an infinite number of remnants ....

This is a strange situation …

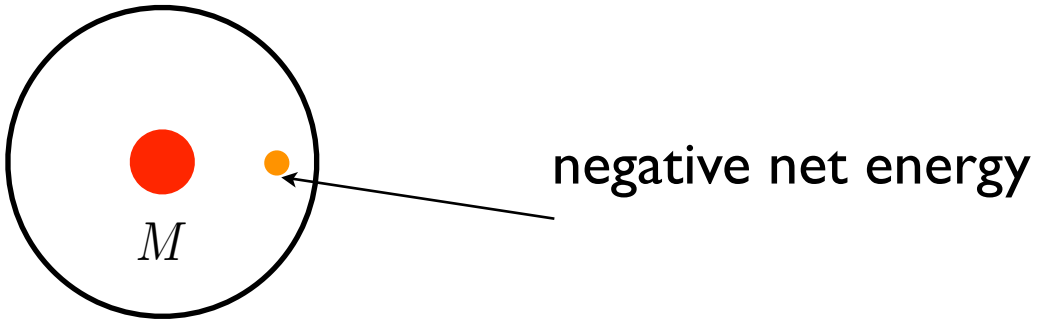


For example, in quantum field theory, we have loop diagrams

If there are an infinite number of remnants, then how do we avoid an infinite contribution to all quantum processes ?

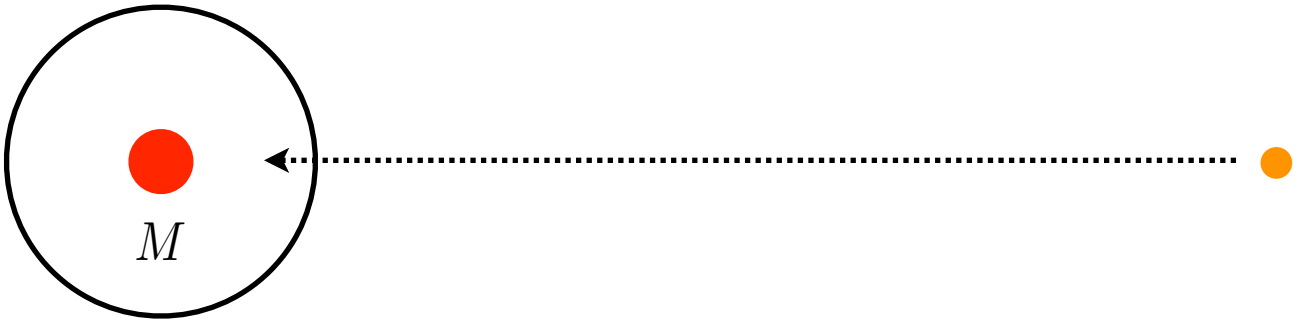
A second pass ... The Hawking effect

How can we make remnants ?



$$E = mc^2 - \frac{GMm}{r}$$

(A) Suppose we let the particle fall in from far away ...



$$E = mc^2 - \frac{GMm}{r} + KE = mc^2$$

so this method does not work

(B) Suppose we lower the particle with a rope ...

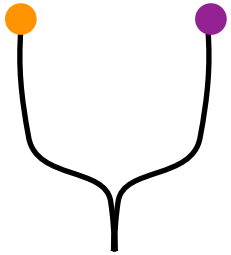
$$E = mc^2 - \frac{GMm}{r}$$

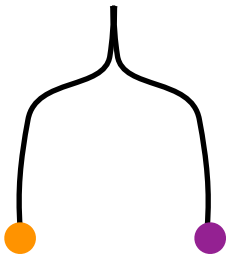
If we do this properly using general relativity, then we find that we cannot prevent the particle from falling in once it is inside the horizon

So this method does not work either ...

Hawking's discovery: Even though we seem to not be able to make remnants using classical physics, they are automatically created once we use quantum mechanics ...

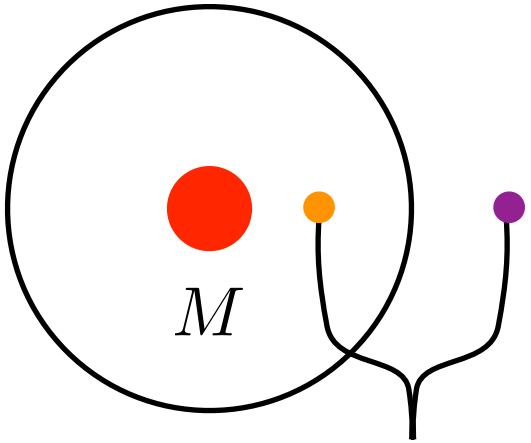
In quantum mechanics, the vacuum can have fluctuations which produce a particle-antiparticle pair





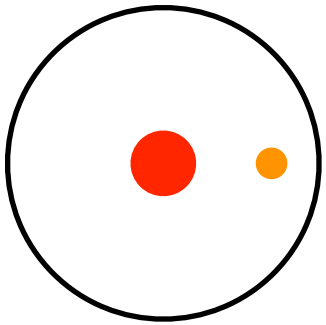
$$\Delta E \; \Delta t \sim \hbar$$

But if a fluctuation happens near the horizon, the particles do not have to re-annihilate



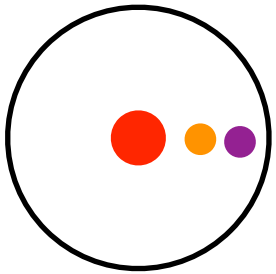
$$\Delta E = 0 \quad \rightarrow \quad \Delta t = \infty$$

Thus the negative energy particle gets automatically placed in the correct position inside the horizon



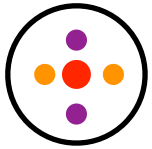
The outer particle drifts off to infinity as ‘Hawking radiation’

The mass of the hole has gone down, so the horizon shrinks slightly



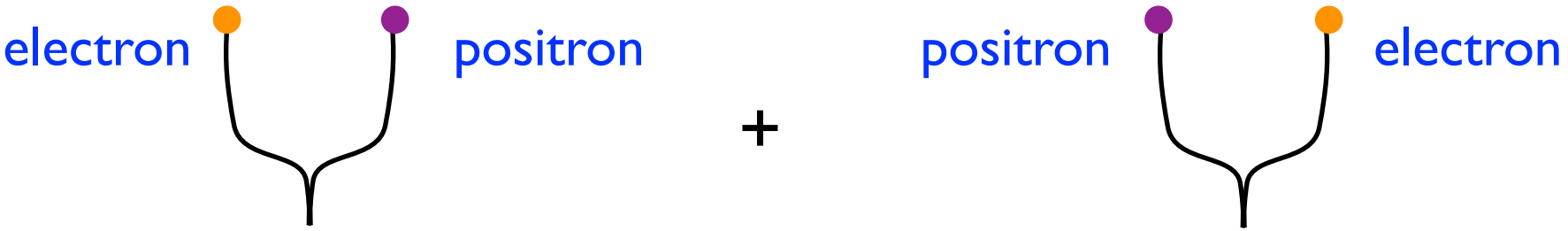
The process repeats, and another particle pair is produced

The energy of the hole is now in the radiation

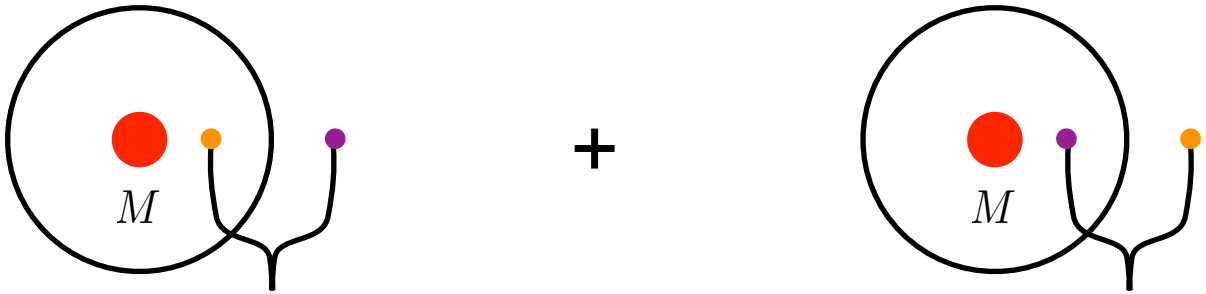


A massless (or planck mass) remnant is left

The crucial issue now has to do with 'entanglement'



Vacuum fluctuations typically produce entangled states ...

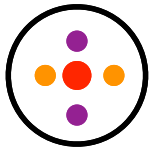


So the state of the radiation is entangled with the state of the remnant

The amount of this entanglement is very large ...

If N particles are emitted, then there are 2^N possible arrangements

We can call an electron a 0 and a positron a 1



	111111	000000
	....	....
+	010011	101100
+	000000	111111

Now there are two possibilities:

(a) Information loss: The evaporation goes on till the remnant has zero mass. At this point the remnant simply vanishes

vacuum

000000

....

101100

111111

The radiation is entangled, but there is nothing that it is entangled WITH

The radiation cannot be assigned ANY quantum state ... it can only be described by a density matrix ... this is a violation of quantum mechanics(Hawking 1974)

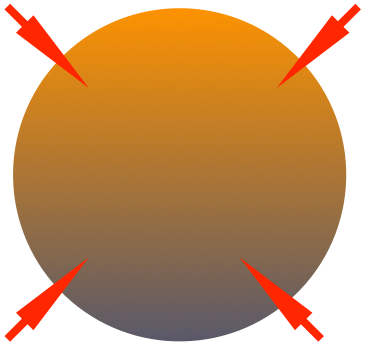
(b) We assume the evaporation stops when we get to a planck sized remnant.

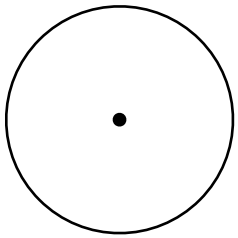
The remnant must have at least 2^N internal states

	111111	000000
	....	....
+	010011	101100
+	000000	111111

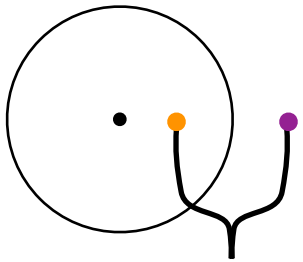
But how can we hold an unbounded number of states in planck volume with energy limited by planck mass?

The black hole information paradox





General Relativity:
Black holes form



Quantum mechanics:
entangled pairs are created

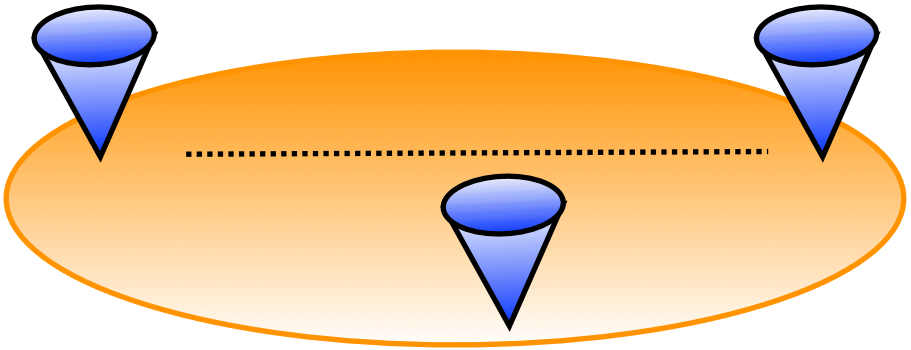
There is a problem near the endpoint of evaporation

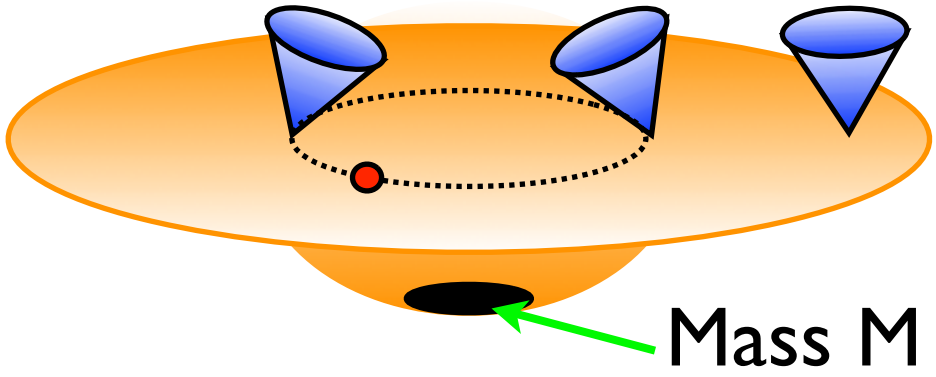
Can we imagine a different structure for the black hole?

General relativity

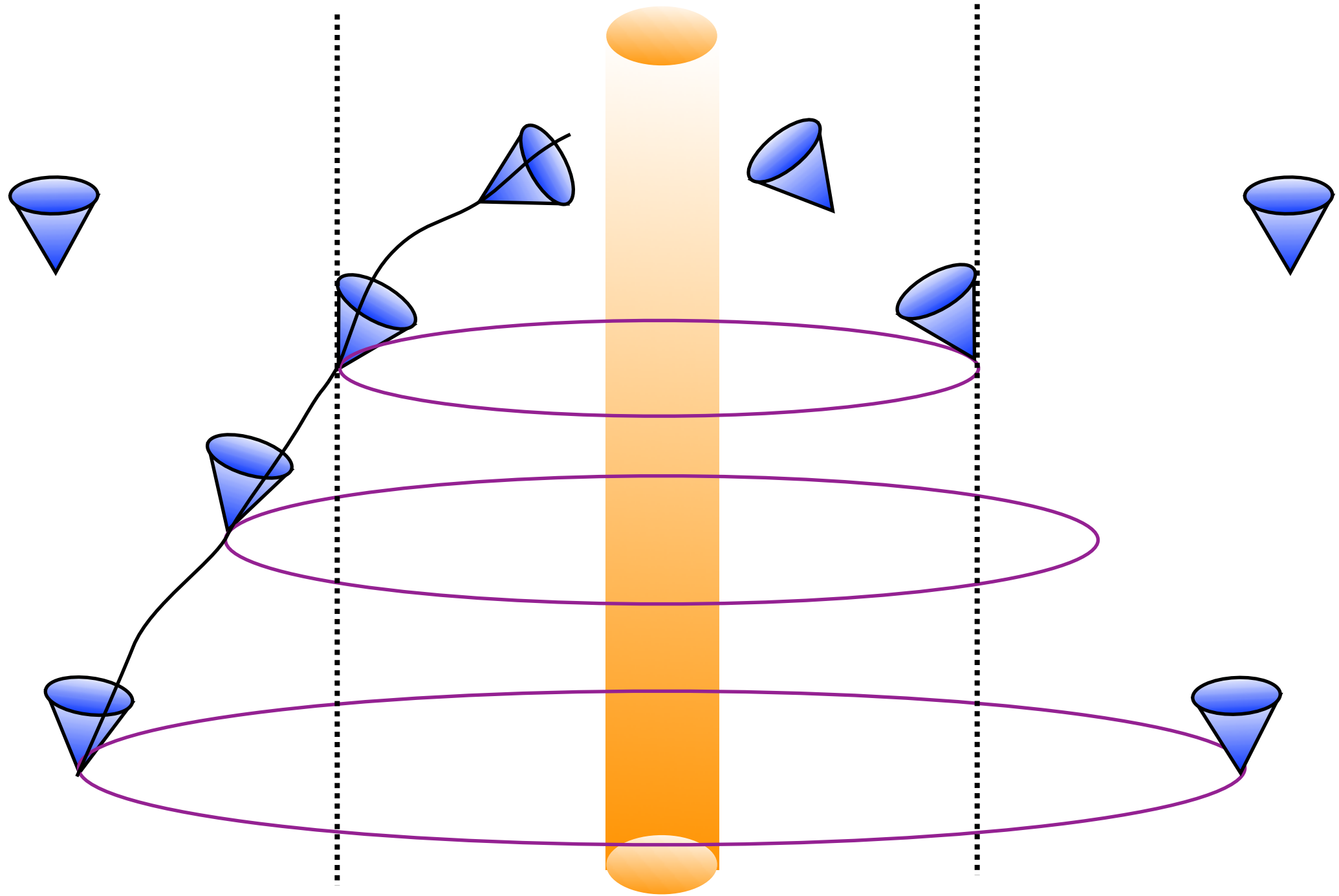
Mass curves spacetime

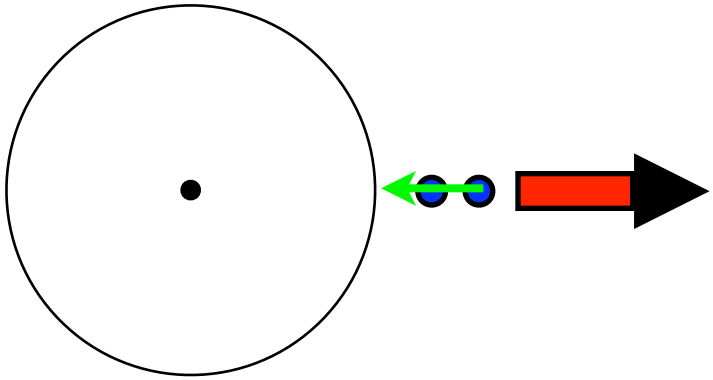
All the ‘force’ of gravity is encoded
in this curvature of spacetime

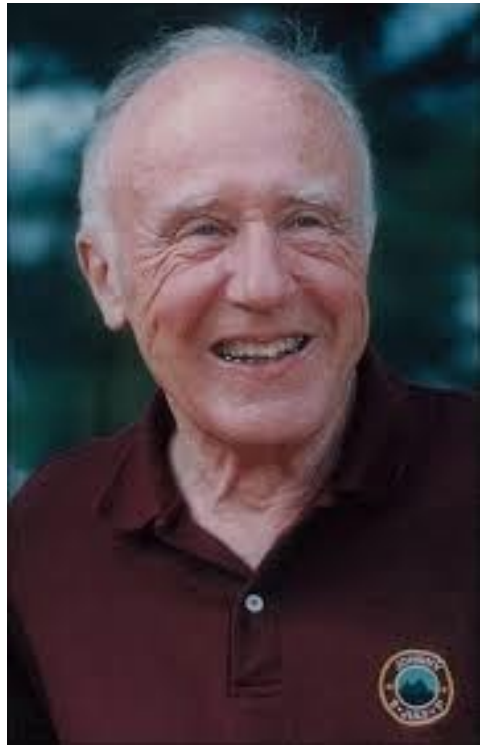




The Black Hole





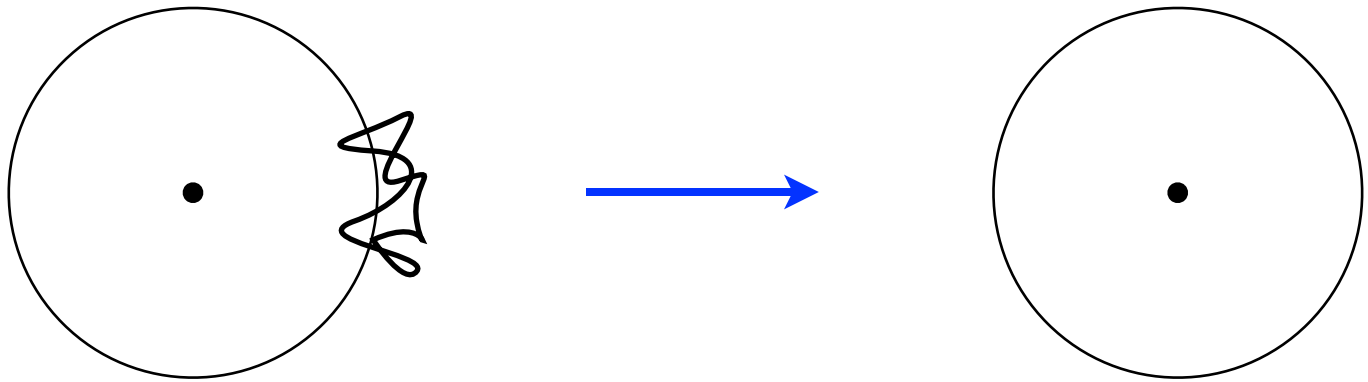


It is very hard to stay near the horizon:
any structure there falls in

The black hole then reduces back to its standard shape:

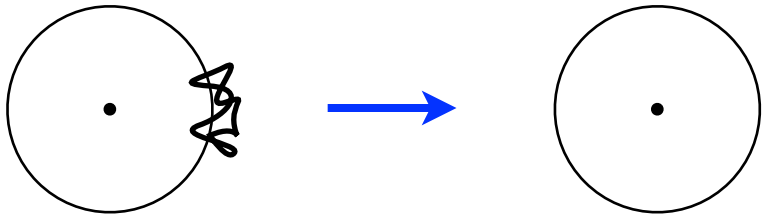
"Black holes have no hair"

If you place a string near the horizon, it will fall in,
so just having string theory does not solve anything

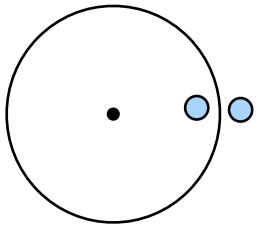


So the information paradox is a combination of two observations:

(1) The no-hair 'theorems' tell us the black hole tends to quickly settle down to a state where the region around the horizon is vacuum



(2) The vacuum creates entangled pairs by the Hawking process



But we will now see that in string theory there is indeed a way that the no hair 'theorem' gets bypassed …

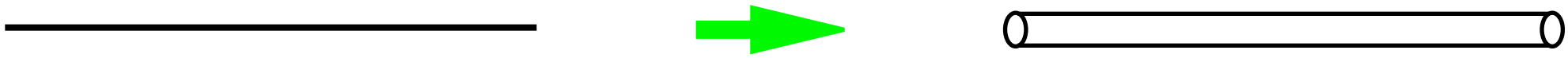
Fuzzballs

Avery, Balasubramanian, Bena, Carson, Chowdhury, de Boer, Gimon, Giusto, Hampton, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Niehoff, Park, Peet, Potvin, Puhm,Ross, Ruef, Saxena, Simon, Skenderis, Srivastava, Taylor, Turton, Vasilakis, Warner ...

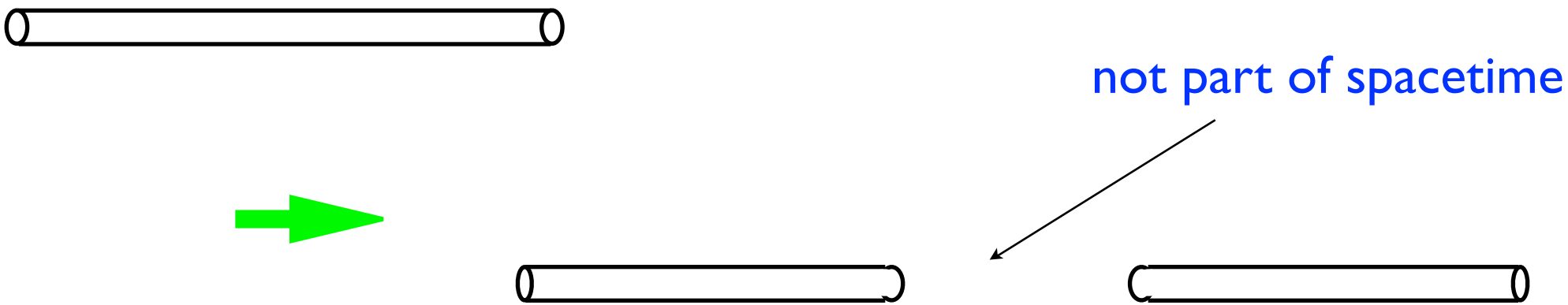
First consider a rough analogy …

Witten 1982: ‘Bubble of nothing’

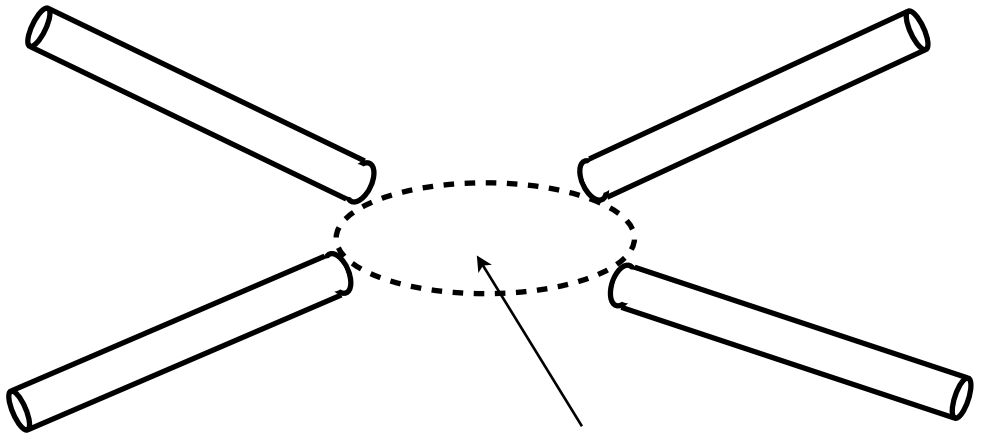
Consider Minkowski space with an extra compact circle



This space-time is unstable to tunneling into a ‘bubble of nothing’



In more dimensions :

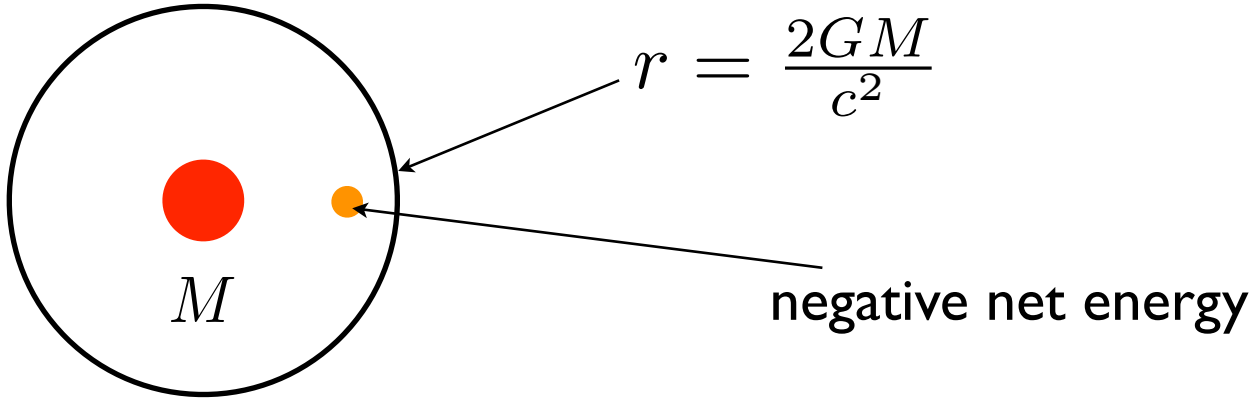


not part of spacetime

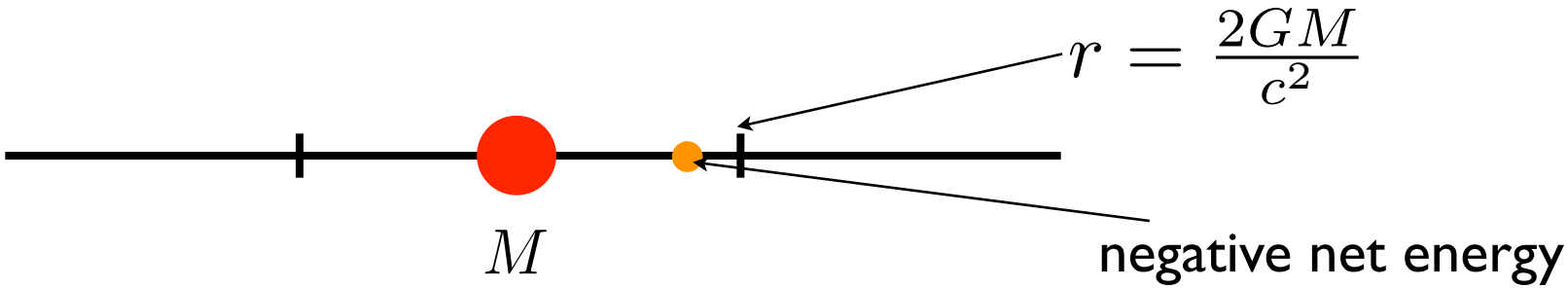
People did not worry about this instability too much, since it turns out that fermions cannot live on this new topology without having a singularity in their wave function …

But now consider the black hole …

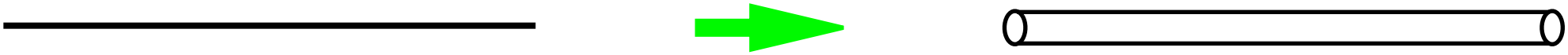
We live in 3 space and 1 time dimension. Recall the black hole ...



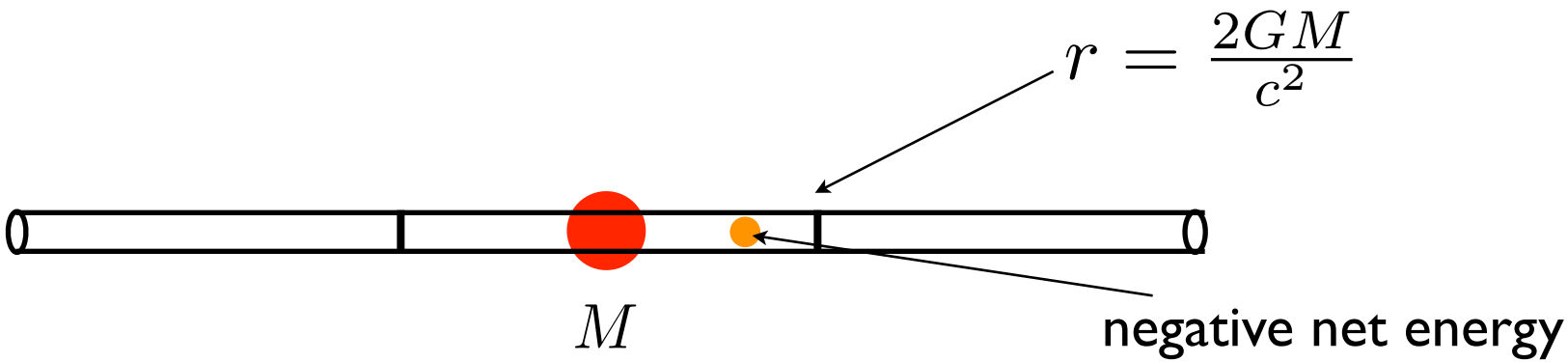
Let us draw just one space direction for simplicity



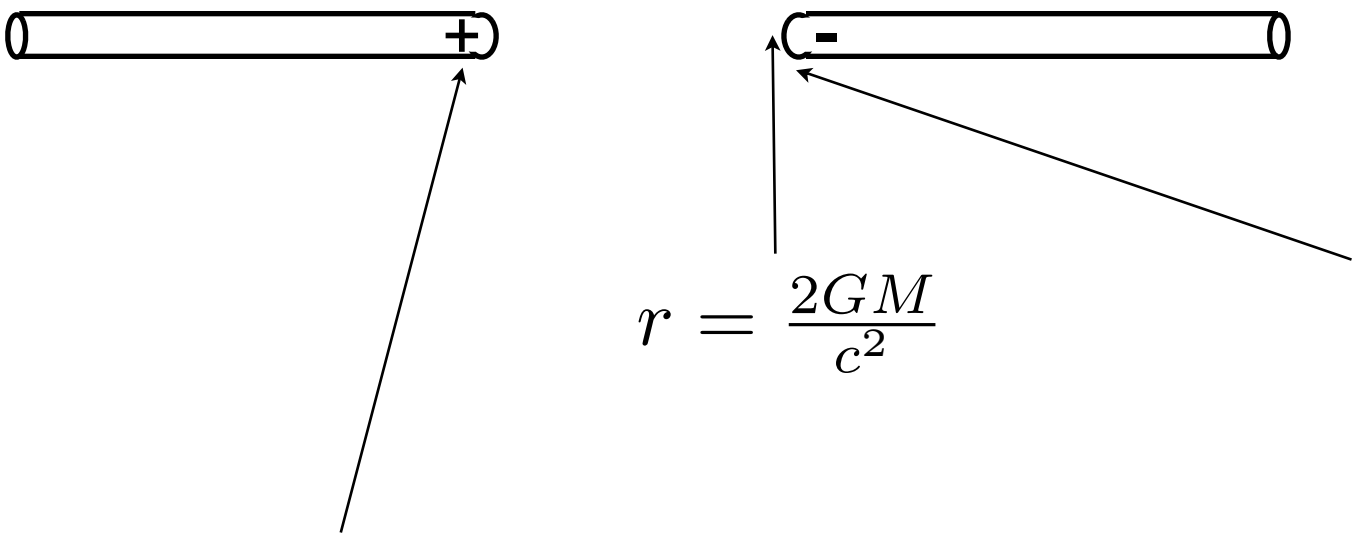
Now suppose there was an extra dimension (e.g., string theory has 6 extra dimensions)



People have thought of extra dimensions for a long time, but they seemed to have no particular significance for the black hole problem



But there is a completely different structure possible with compact dimensions ...



No place to put particles with net negative energy

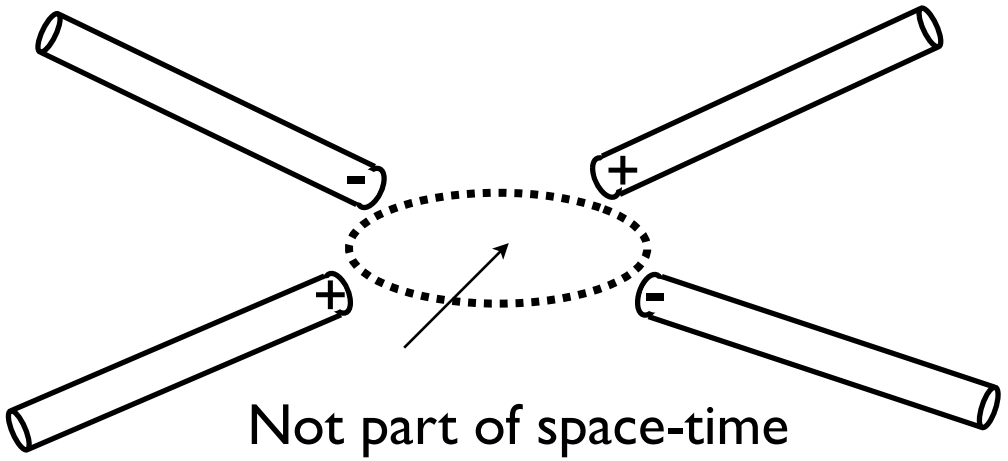
The mass M is captured by the energy in the curved manifold

There is an extra 'twist' in the space-time which makes it consistent to have both boson and fermion wave functions

(Kaluza Klein monopoles and anti-monopoles)

1-dimension

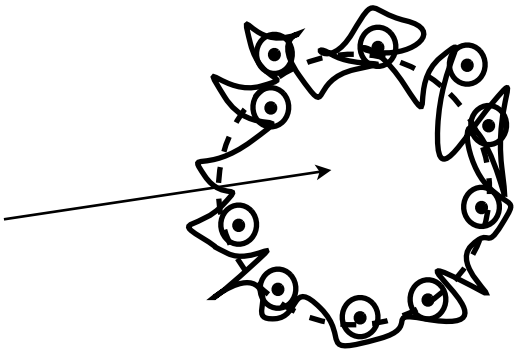
In more dimensions :



Not part of space-time
(no horizon forms)

We will draw only the structure near the horizon :

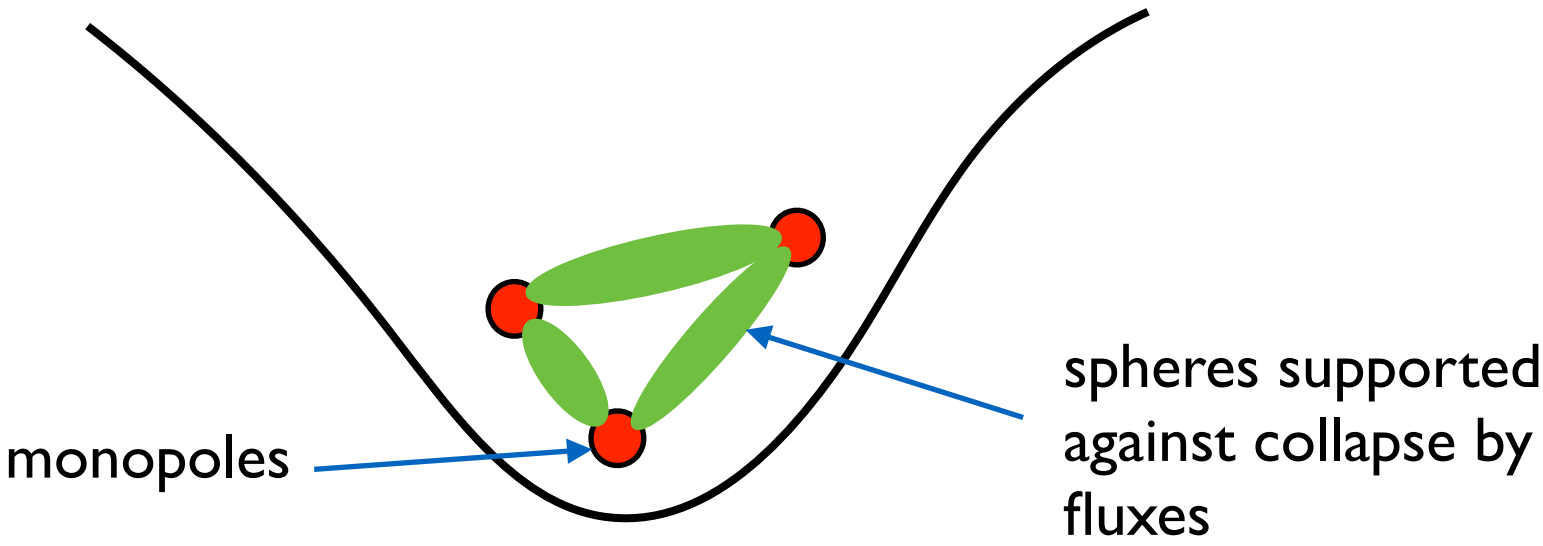
not part of spacetime



"Fuzzball"

Nothing can fall into the hole because spacetime ends just outside the horizon

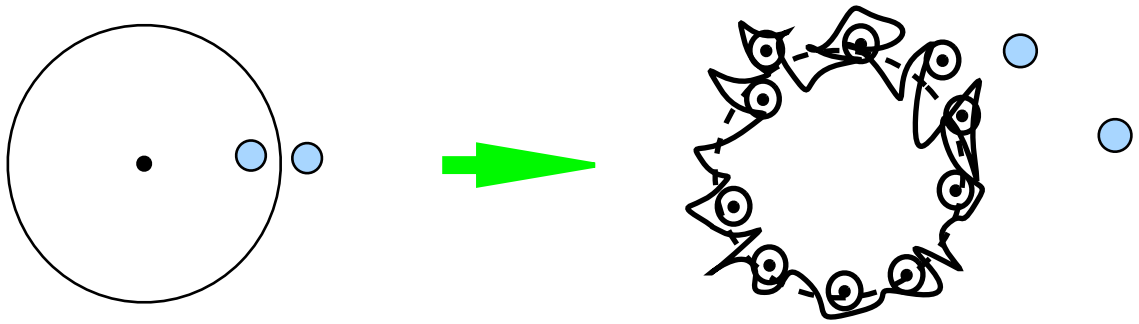
Explicit solutions of 10-d string theory :



Gibbons - Warner:

The new features of string theory allow us to bypass the belief "Black holes have no hair" …

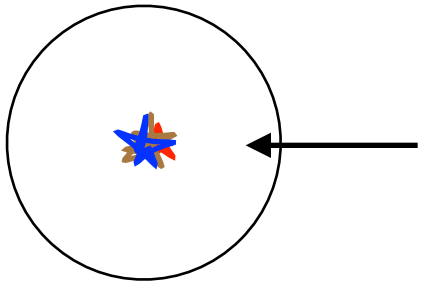
The traditional arguments for " no-hair" do not work when we have extra dimensions and these extra dimensions are involved in new topological structures …



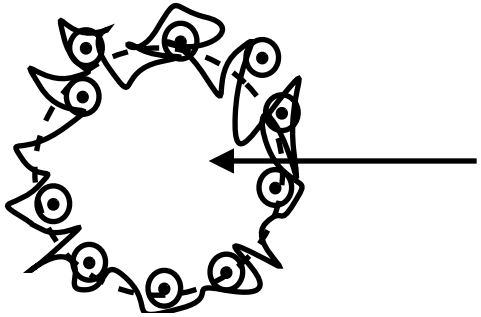
The ‘fuzzball’ radiates from its surface just like a piece of coal, so there is no information paradox

All states investigated so far have a fuzzball structure (extremal, near extremal, neutral with max rotation …)

Fuzzball conjecture: no state in string theory has a traditional horizon



vacuum to leading order



no horizon or interior

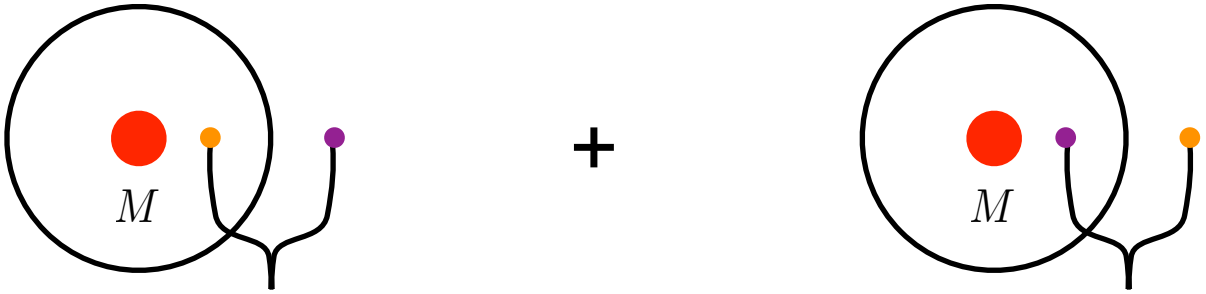
The small corrections theorem

But this solution was not immediately accepted by everyone, because many string theorists believed there was a simpler resolution …..

The cumulative effect of small corrections

(Maldacena 2001, Hawking 2004)

Let us see what this idea was …



$$|0\rangle + |1\rangle$$

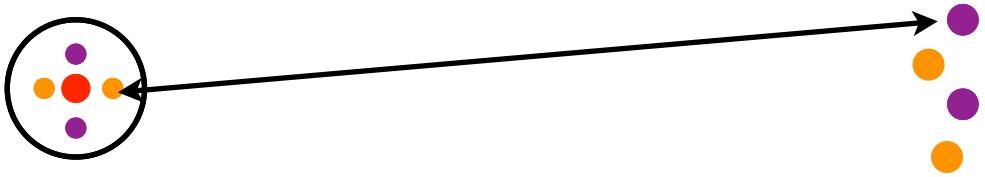
Leading order
Hawking computation

?? $\quad (1+\epsilon)\,|0\rangle + (1-\epsilon)\,|1\rangle$

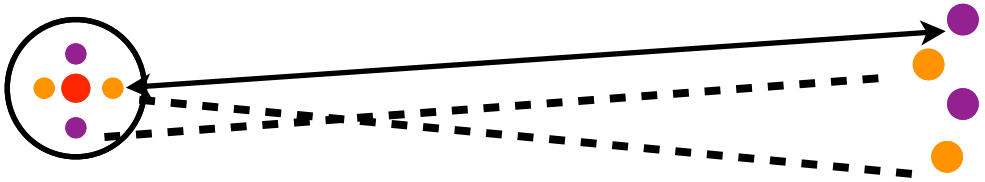
Small corrections, perhaps due to gravitational instanton effects

ϵ is very small, perhaps of order $Exp[-(M/m_p)^2]$

But the number of radiated quanta is very large ….

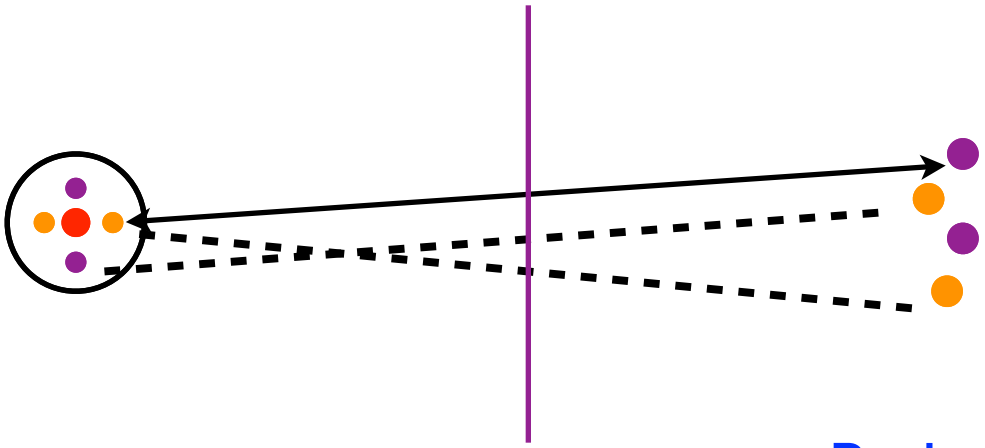


leading order



leading order +
subleading effects

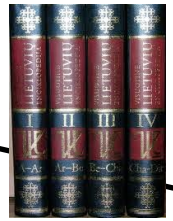
Number of emitted quanta is very large $\sim (M/m_p)^2$



Perhaps with all these corrections,
the entanglement goes down to zero …

In 2004, Stephen Hawking surrendered his bet to John Preskill using similar arguments ...

Stephen Hawking



John Preskill

Kip Thorne

But Kip Thorne did not agree to surrender the bet ...

Who is correct ?

In 2009 an inequality was derived which showed that NO set of small corrections could reduce the entanglement

$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

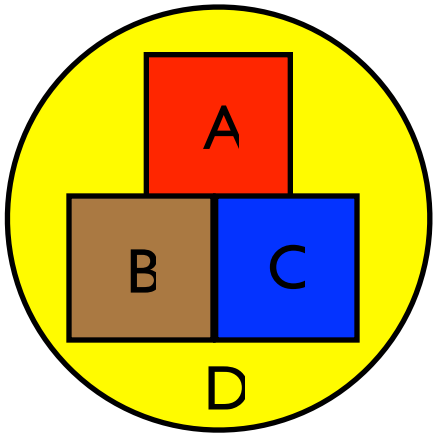
(SDM 2009)

The nontrivial power came from something called the strong sub-additivity theorem for quantum entanglement entropy

This was derived by Lieb and Ruskai in 1973 ..

(No elementary proof is known …)

$$S(A+B) + S(B+C) \geq S(A) + S(C)$$

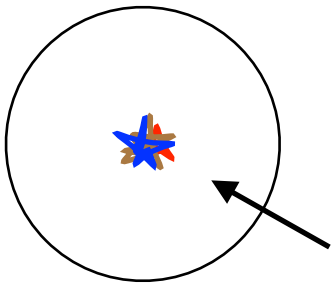


With this 'small corrections theorem', the Hawking 1975 argument for the information problem became a rigorous theorem:

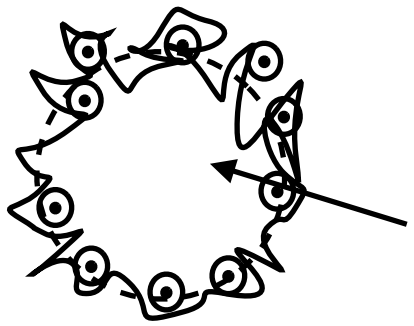
If the physics at the horizon is like the physics in this room to leading order, then there is no solution to the information problem.

In other words, we need a correction of order unity to low energy physics at the horizon (SDM 2009)

This established fuzzballs as a logical resolution of the black hole information paradox …



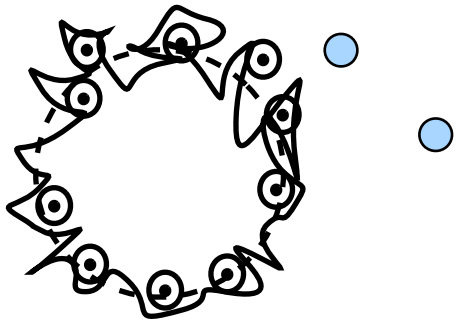
vacuum to
leading order



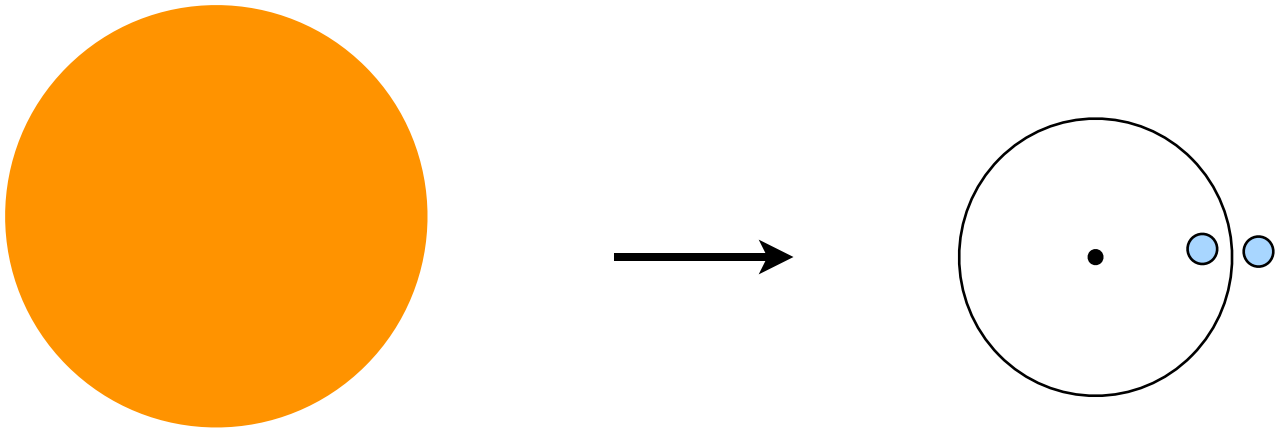
no horizon
or interior

How does the classical expectation get violated so dramatically ?

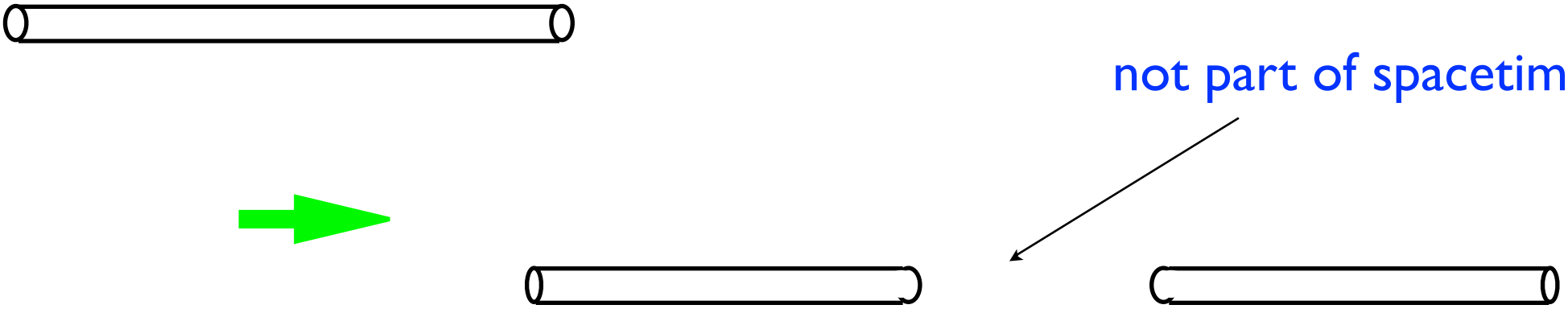
The fuzzball construction seems to be the only correct solution to the paradox ...



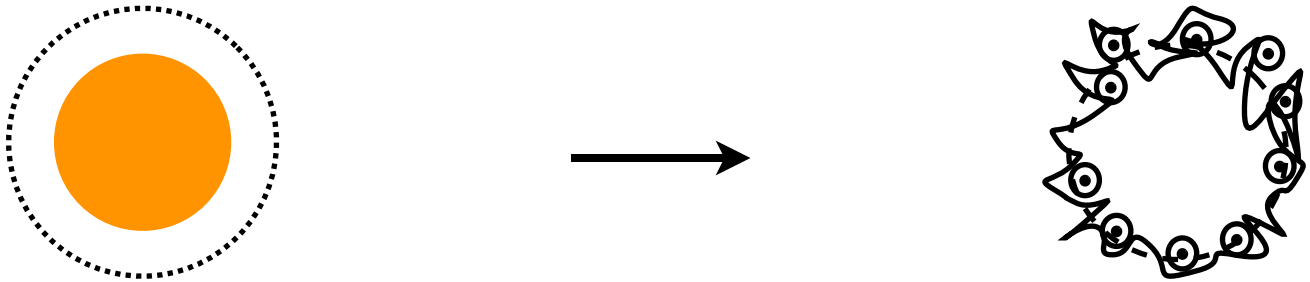
But if a star collapses, then the physics looks quite classical, and so one seems to make the usual black hole with a smooth horizon ...

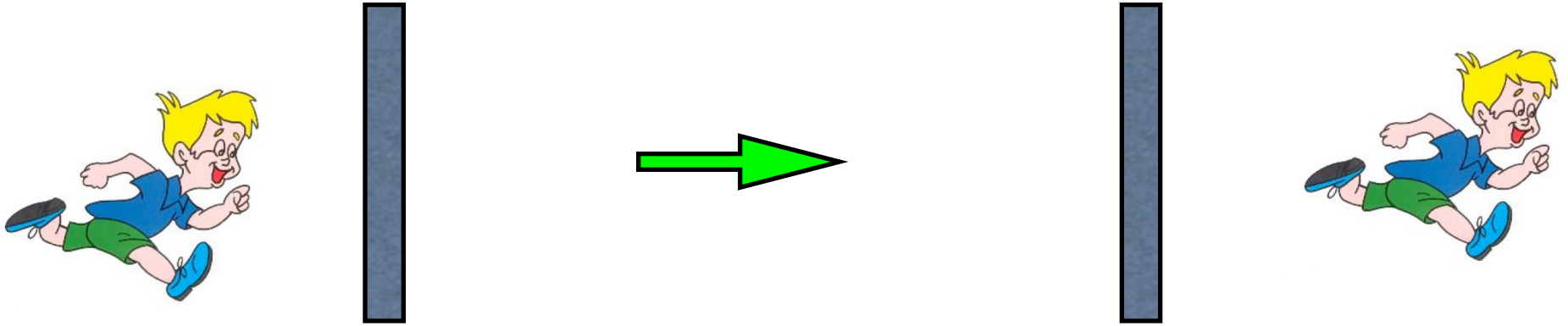


Recall Witten's 'bubble of nothing' where Minkowski space tunnels into a new topology …



It turns out that a collapsing shell can tunnel into a fuzzball state …





There is always a small probability that an object can tunnel ...

But this probability is usually ignorable for a macroscopic object ...

Is there something special about a black hole ?

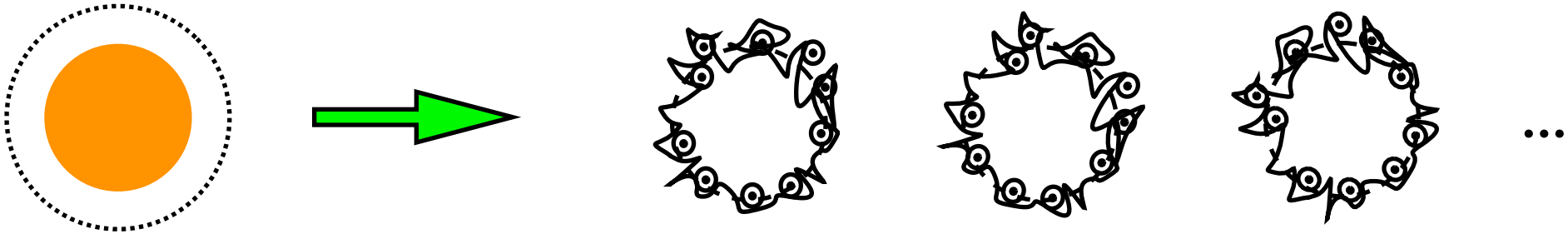
In 1972, Bekenstein taught us that black holes have an entropy

$$S = \frac{c^3}{\hbar}\frac{A}{4G} \sim \frac{A}{l_p^2}$$

This means that a solar mass black hole has $\sim 10^{10^{144}}$ states

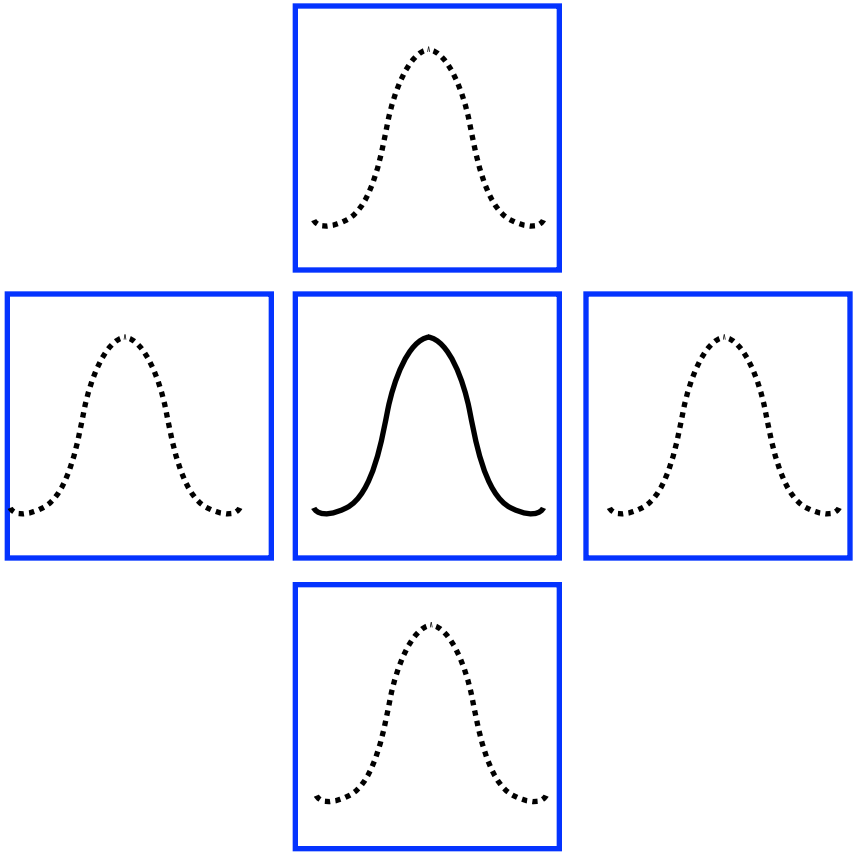
This is far larger than the number of states of normal matter with the same energy

We must multiply the (small) amplitude of tunneling by the (large) number of fuzzball states that we can tunnel to …



Toy model

Small amplitude to tunnel to a neighboring well, but
there are a correspondingly large number of adjacent wells

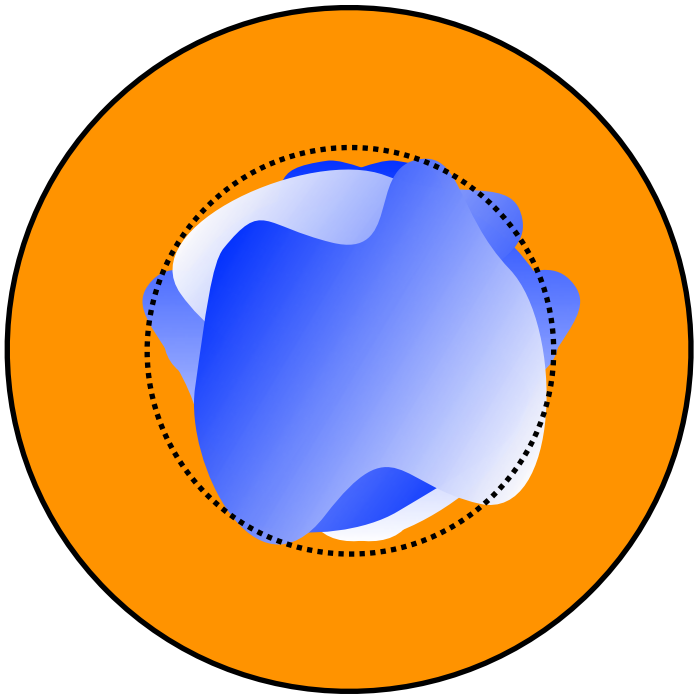


In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells

The smallness of the tunneling amplitude can be cancelled by the largeness of the number of fuzzball states …

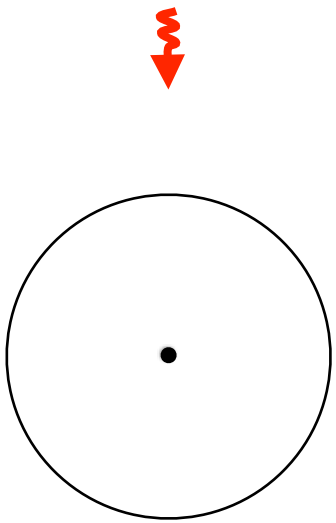
Thus the collapsing shell is immediately UNSTABLE to tunneling into fuzzballs

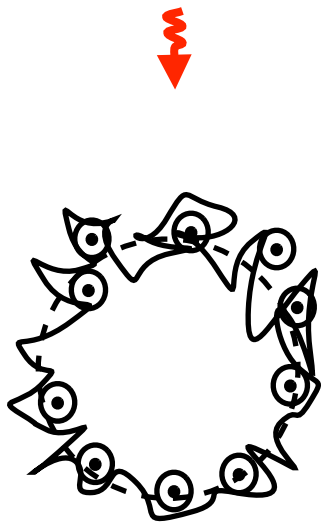
This suggests that the entire black hole is a very quantum object … and semiclassical physics is invalid …



What happens when you fall onto the fuzzball?

What happens if an energetic photon falls towards the hole ?





In the old picture, it would fall in

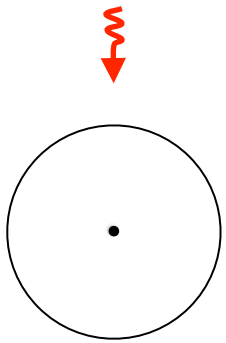
In the fuzzball picture, there is no interior of the hole to fall into

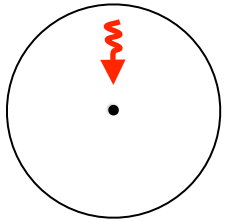
One might think that the photon has hit a “brick wall” or a “firewall”

But there is a second, more interesting, possibility ….

→ ***The idea of fuzzball complementarity***

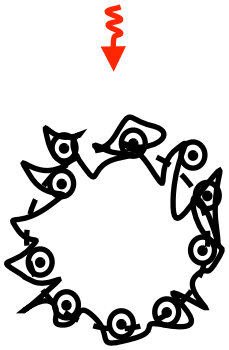
The dynamics of infall into a black hole are described by some frequencies

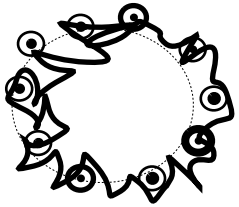




$$\nu_1^{bh}, \nu_2^{bh}, \nu_3^{bh}, \ldots \nu_n^{bh}$$

Oscillations of the fuzzball are also described by some frequencies





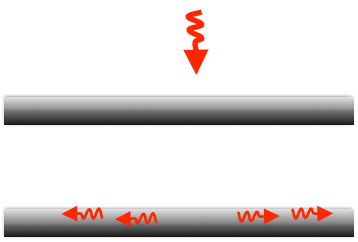
$$\nu_1^{fb}, \nu_2^{fb}, \nu_3^{fb}, \ldots \nu_n^{fb}$$

What if

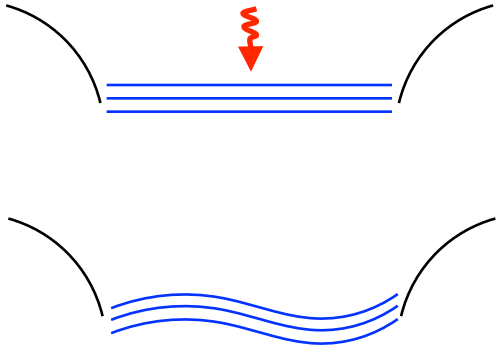
$$\nu_1^{bh}, \nu_2^{bh}, \nu_3^{bh}, \ldots \nu_n^{bh} \quad \approx \quad \nu_1^{fb}, \nu_2^{fb}, \nu_3^{fb}, \ldots \nu_n^{fb} \quad ?$$

In that case falling onto the fuzzball will feel (approximately) like falling into a classical horizon …

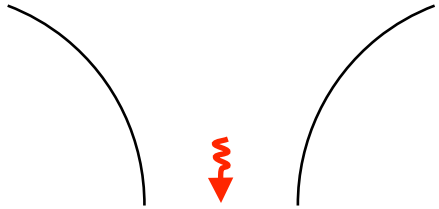
This may seem strange, but something like this happened with AdS/CFT duality …



Create random excitations



D-branes oscillate with some frequencies

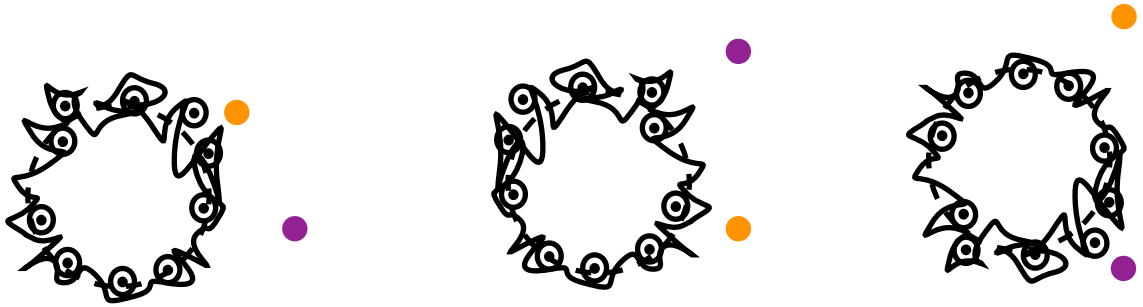


Gravitons in AdS space have the same frequency spectrum

Maldacena 97

In our case, the frequencies of the traditional hole and of the fuzzball can be only approximately equal, since the fuzzballs are all a little different from each other …

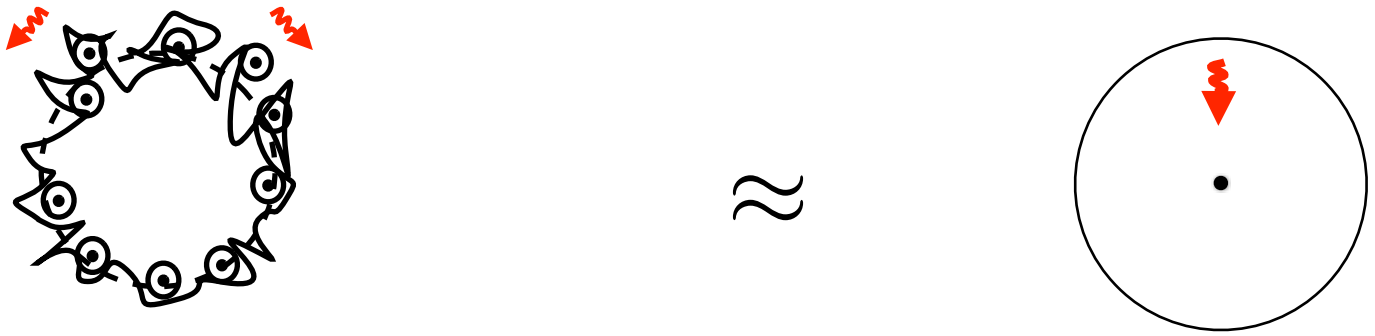
This is crucial, since this is what allows information to escape !!



Low energy radiation
($E \sim T$)
is different between different fuzzballs, carries information

High energy impacts ($E \gg T$) give a near-universal set of frequencies, which reproduces the frequencies of classical infall

Thus we recover information, and also preserve, approximately, our classical intuition !!



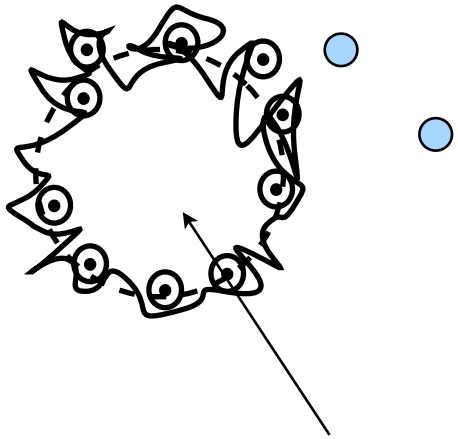
The surface of the fuzzball behaves approximately like the membrane of the membrane paradigm, but this time with real degrees of freedom at the horizon, and spacetime does really end at this 'membrane'

(SDM+PLumberg 2011)

Summary

(1) Hawking's information paradox has been made into a rigorous theorem, (using strong subadditivity of entanglement entropy), so that it is stable against all subleading corrections (SDM 2009)

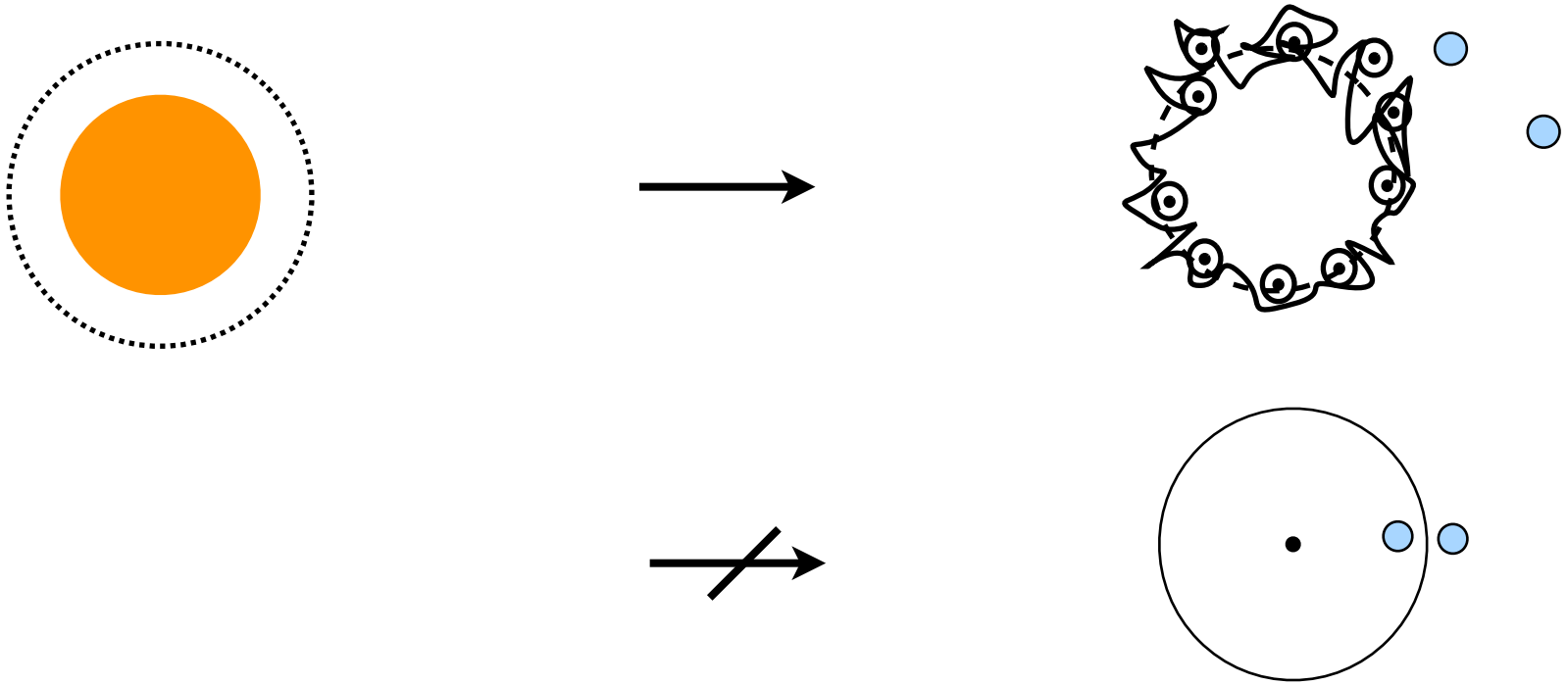
(2) Individual microstates of black holes in string theory are found, in all cases that have been worked out, to have a 'fuzzball structure'



not part of spacetime

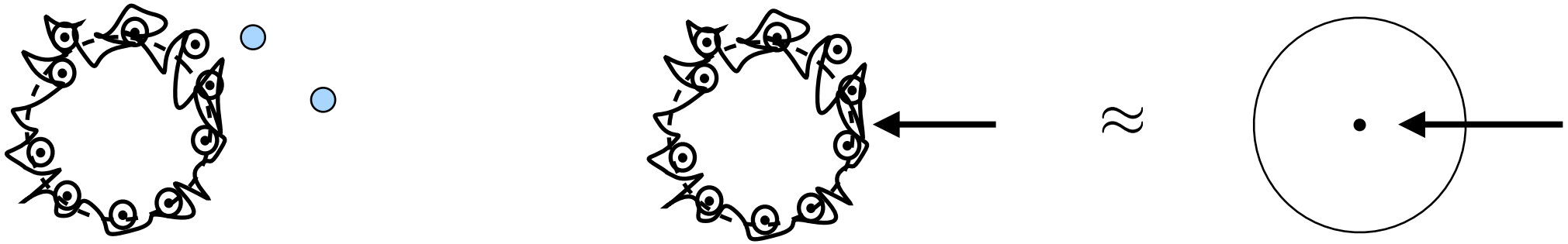
Assuming this holds for all
micro states of all holes,
we resolve the information paradox

(3) An estimate shows that a collapsing shell is unstable into tunneling into a linear combination of fuzzball states; this happens because the small amplitude of tunneling can be offset by the large number of fuzzball states (Exponential of the Bekenstein entropy)



(4) The conjecture of Fuzzball complementarity (SDM+Plumberg 2011):

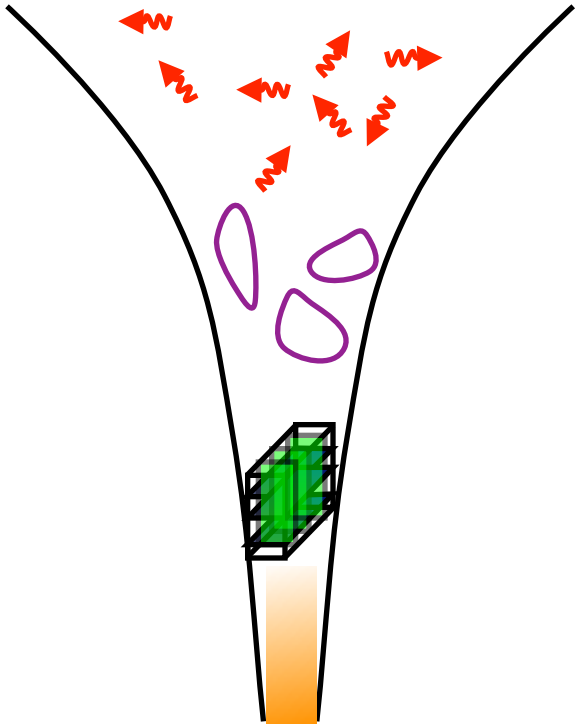
Real structure at horizon. E~T quanta carry information, while E>>T quanta generate universal oscillations that holographically encode the equations of free infall



Looking forward:

Singularity of black hole is resolved by tunneling into fuzzballs

What about the singularity of the early Universe ??



Many results suggest a universal formula
for the entropy density of the early Universe:

$$s \sim \sqrt{\frac{\rho}{G}}$$

An interesting new set of ideas emerge from this …

THANK YOU !!