Astromaterial Science

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November 15, 2016

U.S. DEPARTMENT OF ENERGY | Office of Science

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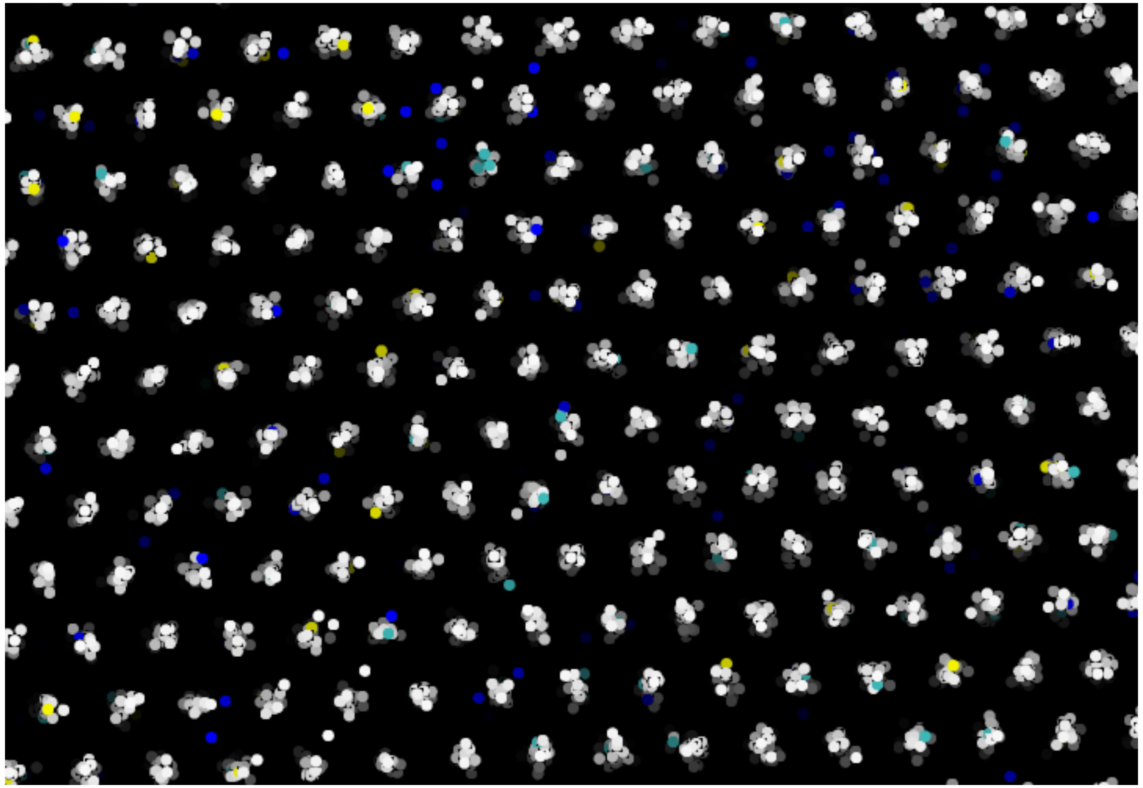
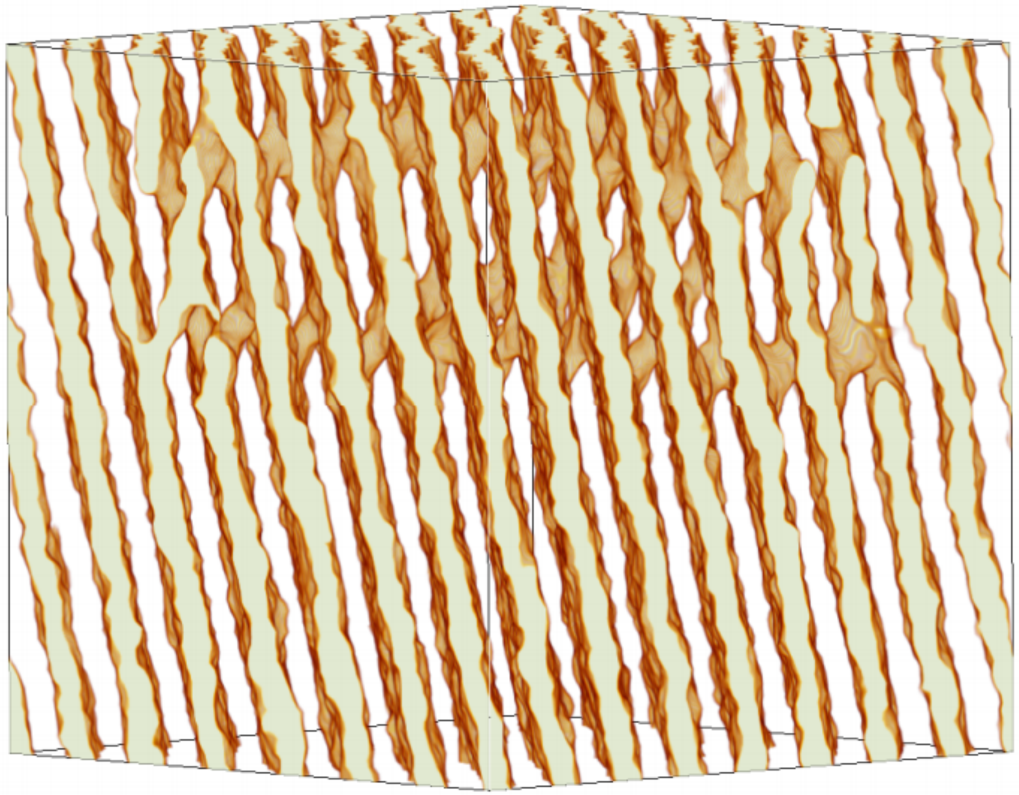
Astromaterials

- Stars freeze. But not all stars. Only parts of some stars freeze.



HARD



SOFT

Shameless Self-promotion

Astromaterial Science and Nuclear Pasta

M. E. Caplan, C. J. Horowitz

(Submitted on 12 Jun 2016)

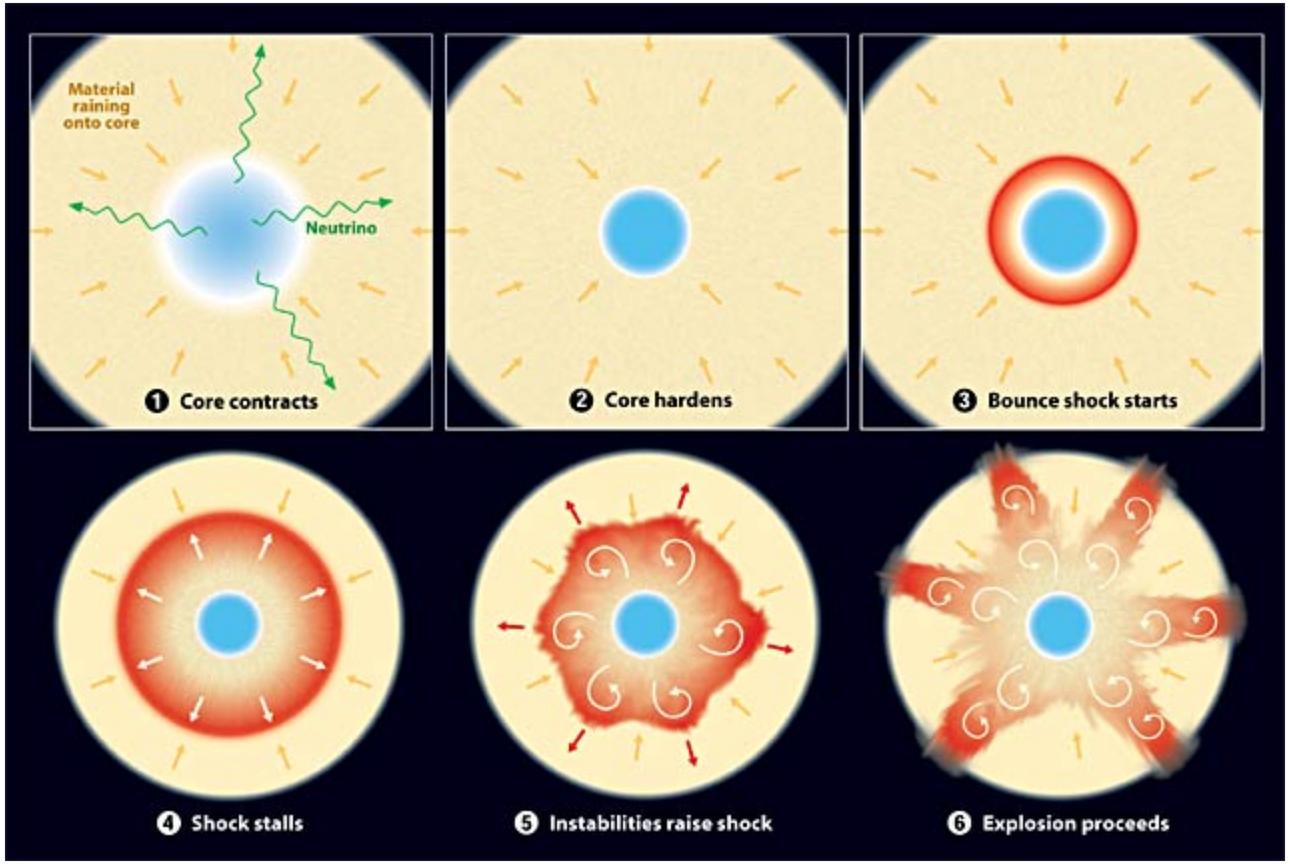
The heavens contain a variety of materials that range from conventional to extraordinary and extreme. For this colloquium, we define Astromaterial Science as the study of materials, in astronomical objects, that are qualitatively denser than materials on earth. Astromaterials can have unique properties, related to their density, such as extraordinary mechanical strength, or alternatively be organized in ways similar to more conventional materials. The study of astromaterials may suggest ways to improve terrestrial materials. Likewise, advances in the science of conventional materials may allow new insights into astromaterials. We discuss Coulomb crystals in the interior of cold white dwarfs and in the crust of neutron stars and review the limited observations of how stars freeze. We apply astromaterial science to the generation of gravitational waves. According to Einstein's Theory of General Relativity accelerating masses radiate gravitational waves. However, very strong materials may be needed to vigorously accelerate large masses in order to produce continuous gravitational waves that are observable in present detectors. We review large-scale molecular dynamics simulations of the breaking stress of neutron star crust that suggest it is the strongest material known, some ten billion times stronger than steel. Nuclear pasta is an example of a soft astromaterial. It is expected near the base of the neutron star crust at densities of ten to the fourteen grams per cubic centimeter. Competition between nuclear attraction and Coulomb repulsion rearrange neutrons and protons into complex non-spherical shapes such as flat plates (lasagna) or thin rods (spaghetti). We review semi-classical molecular dynamics simulations of nuclear pasta. We illustrate some of the shapes that are possible and discuss transport properties including shear viscosity and thermal and electrical conductivities.

Comments: 13 pages, 7 figures
Subjects: **High Energy Astrophysical Phenomena (astro-ph.HE)**; Materials Science (cond-mat.mtrl-sci); Soft Condensed Matter (cond-mat.soft); Nuclear Theory (nucl-th)
Cite as: **arXiv:1606.03646 [astro-ph.HE]**
(or **arXiv:1606.03646v1 [astro-ph.HE]** for this version)

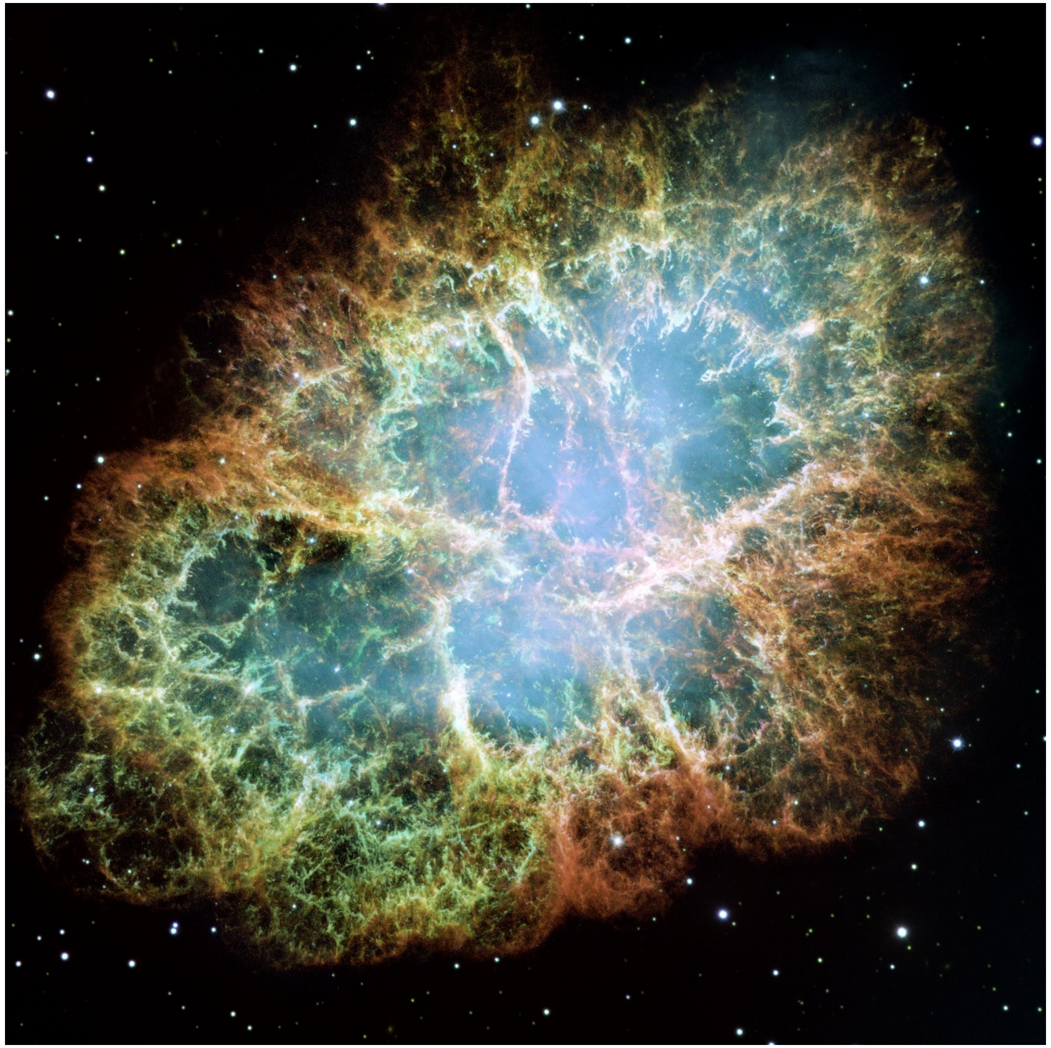
Neutron Stars

Supernova

- The star implodes
- Outer layers rebound off of the core (*bounce*)
- Neutrinos heat and push the outer shell off
- *Kablowy*!

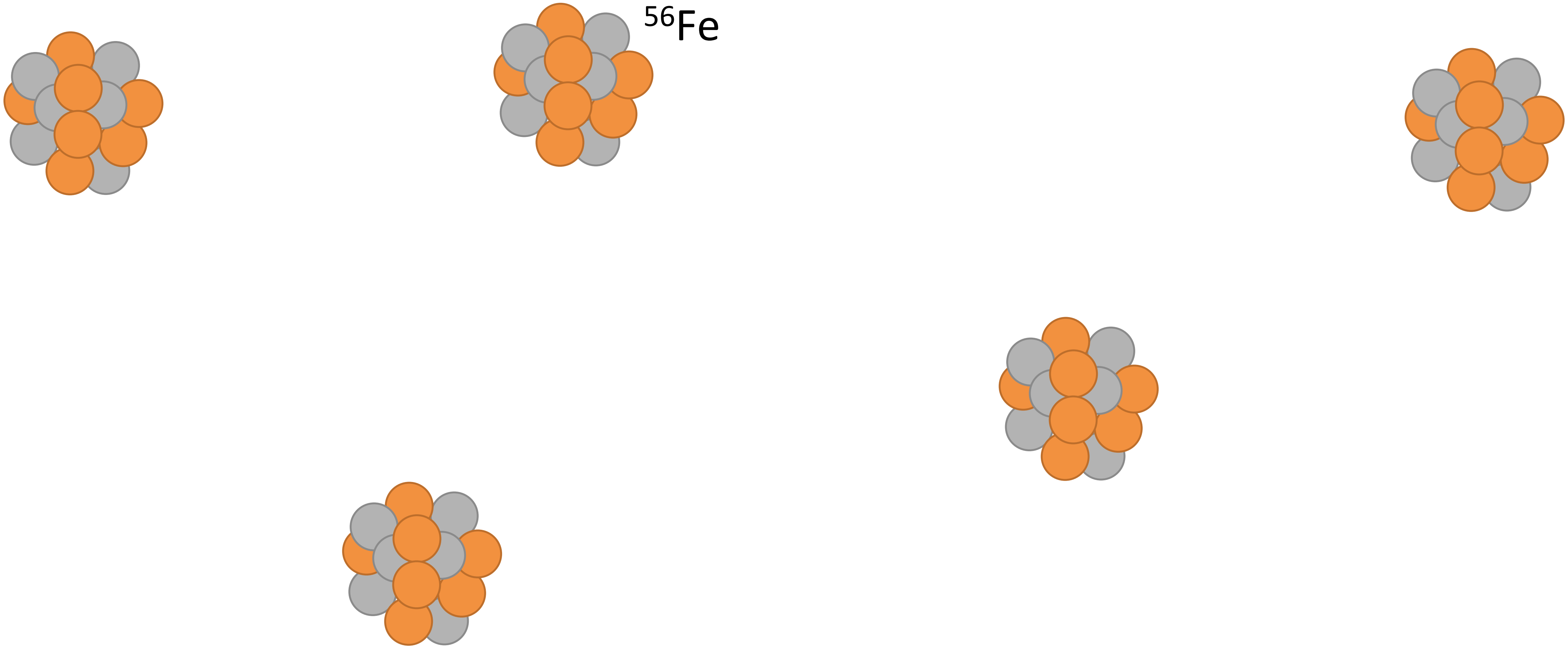


Supernova



Collapse

- Without the pressure from fusion to support the core, it will collapse



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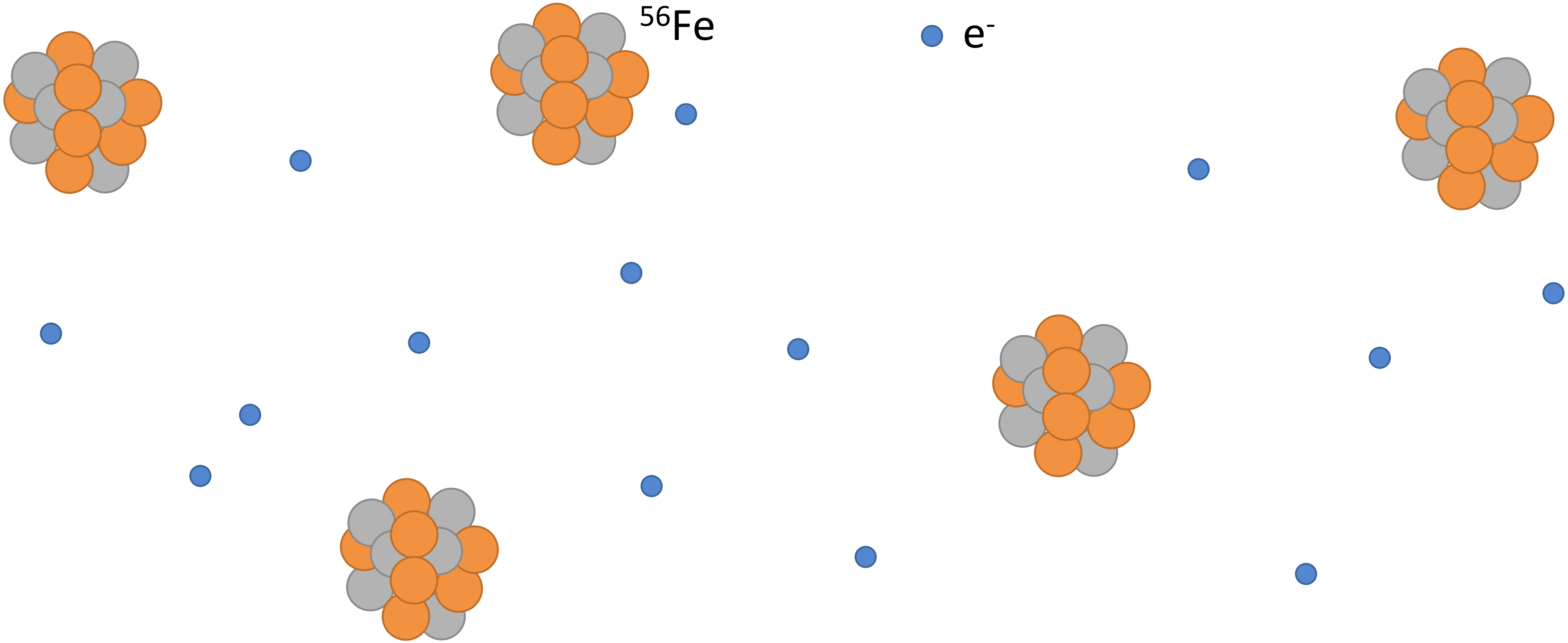

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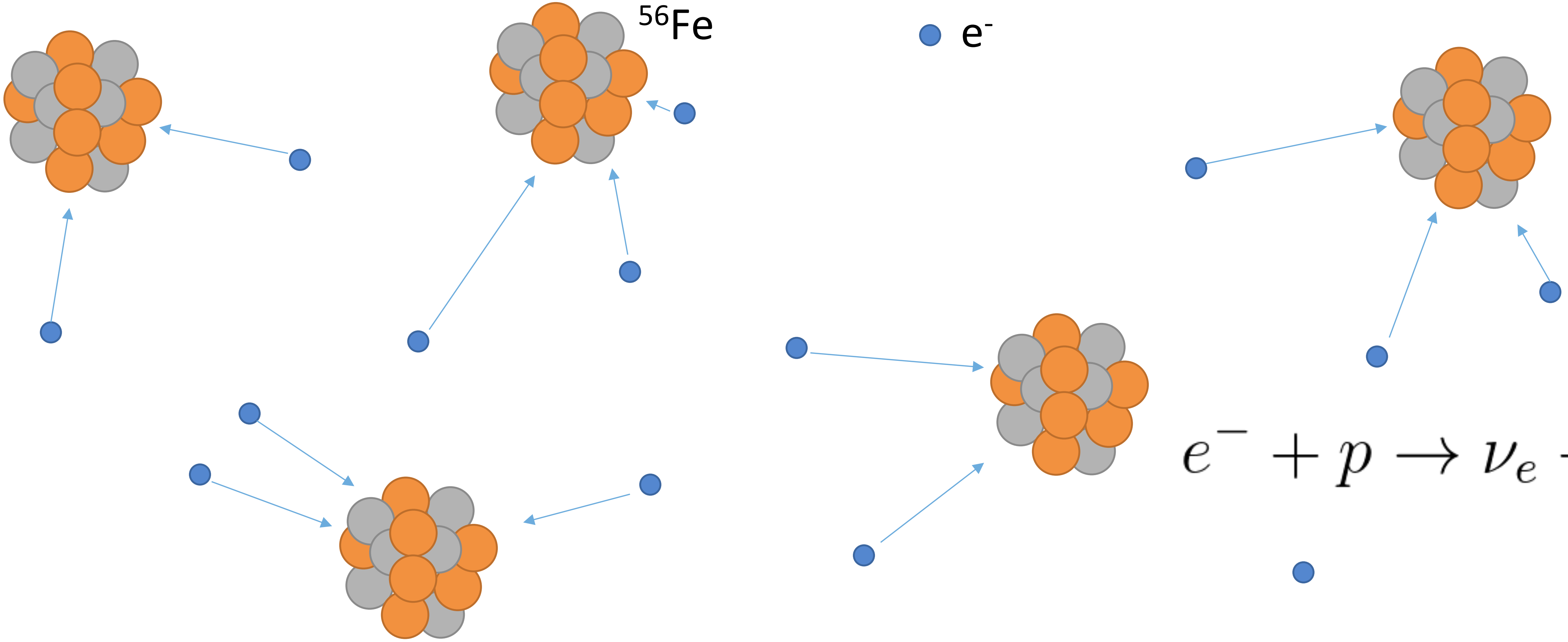

$$e^- + p \rightarrow \nu_e + n$$

Collapse

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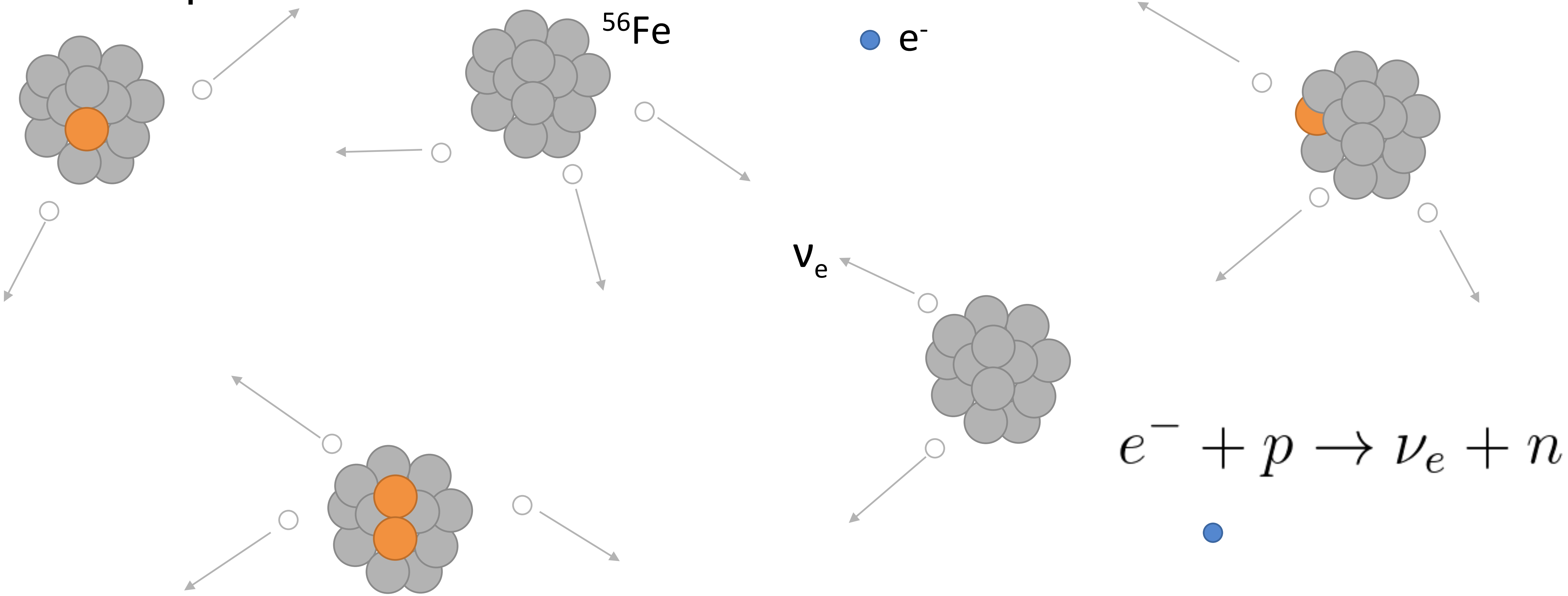

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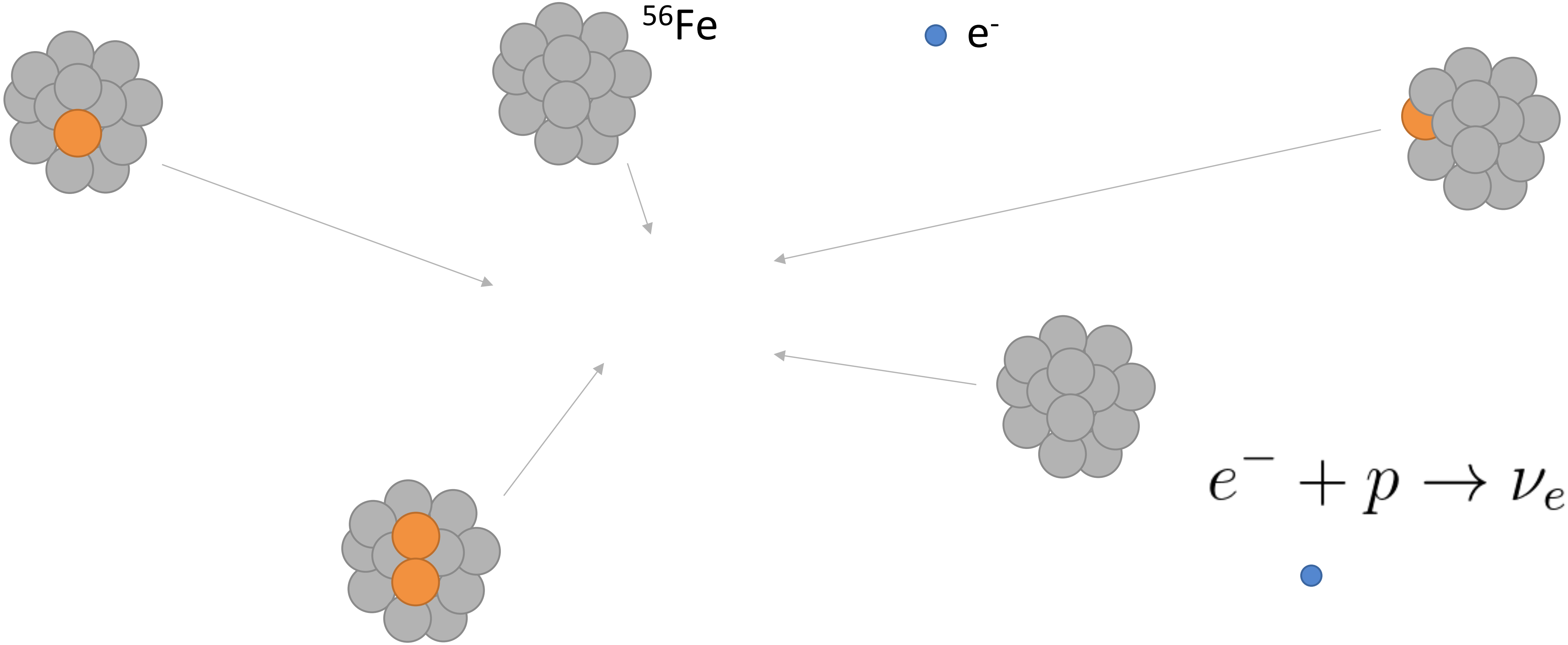

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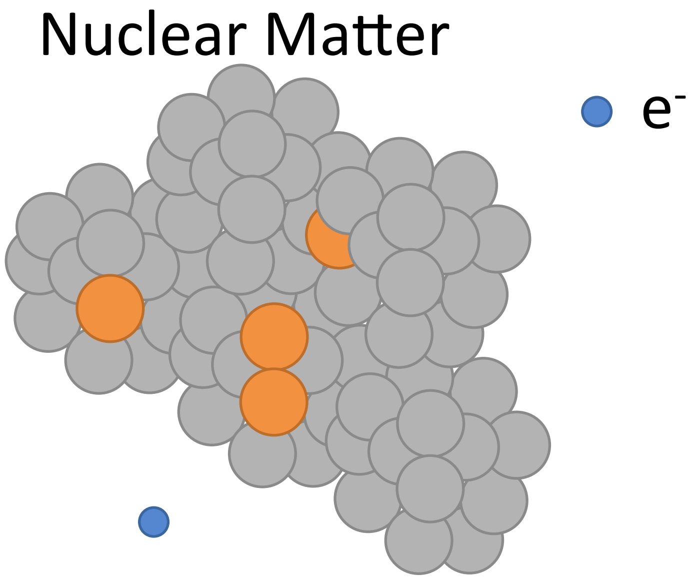
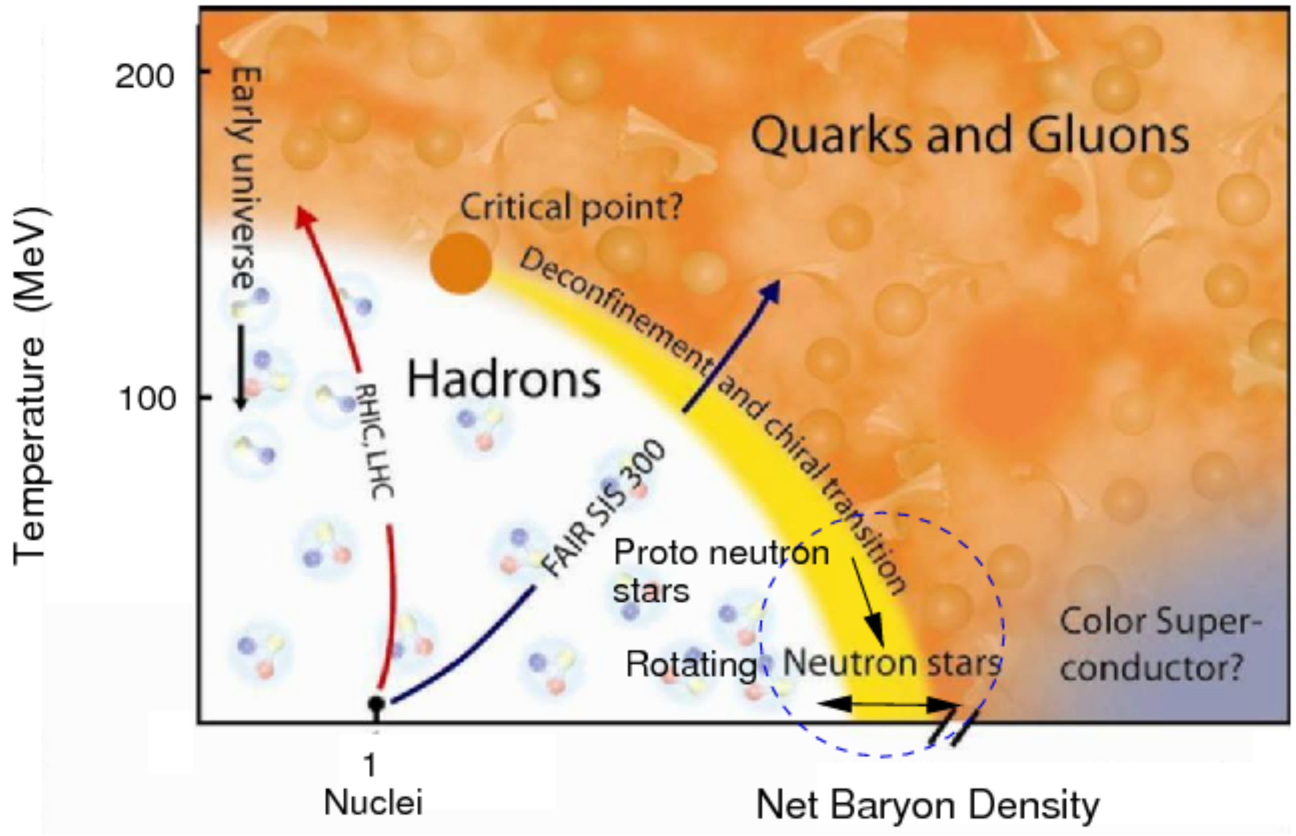


$$e^- + p \rightarrow \nu_e + n$$

Phase Diagram



Neutron Stars

- How much does the volume of the star change?
- Nucleus: R ~ 10^{-15} m
- Atom: R ~ 10^{-10} m

Image Credit: Google maps

Neutron Stars

- Neutron stars are so dense that Mt. Everest would fit in a cup of coffee
- If you dropped a solar mass neutron star on the Rotunda…

Image Credit: Google maps

Neutron Stars

- Neutron stars are so dense that Mt. Everest would fit in a cup of coffee
- If you dropped a solar mass neutron star on the Rotunda…
  it wouldn't even reach Shenandoah Natl. Park

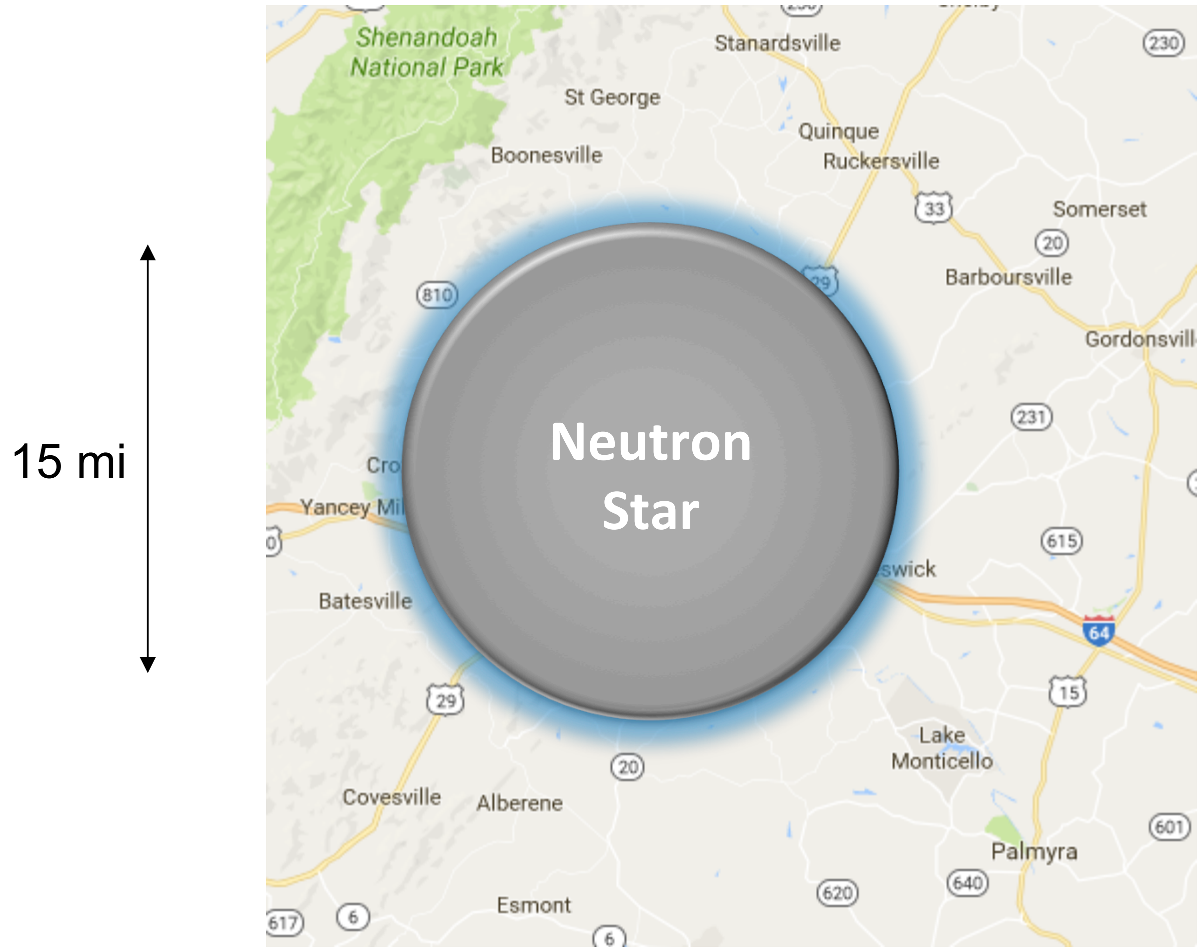


Image Credit: Google maps

Neutron stars

- So what physics changes after collapse?

$$R \to 10^{-5} R$$

Neutron stars

- So what physics changes after collapse? $R \to 10^{-5} R$

(1) Rotation: Cons of Ang Mom: $L = Iw \qquad I = MR^2$

$$L_1 = L_2$$

$$MR^2 w_1^2 = M(10^{-5}R)^2 w_2^2$$

Neutron stars

- So what physics changes after collapse?

$$R \to 10^{-5} R$$

(1) Rotation: Cons of Ang Mom: $L = I w \qquad I = M R^2$

$$L_1 = L_2$$

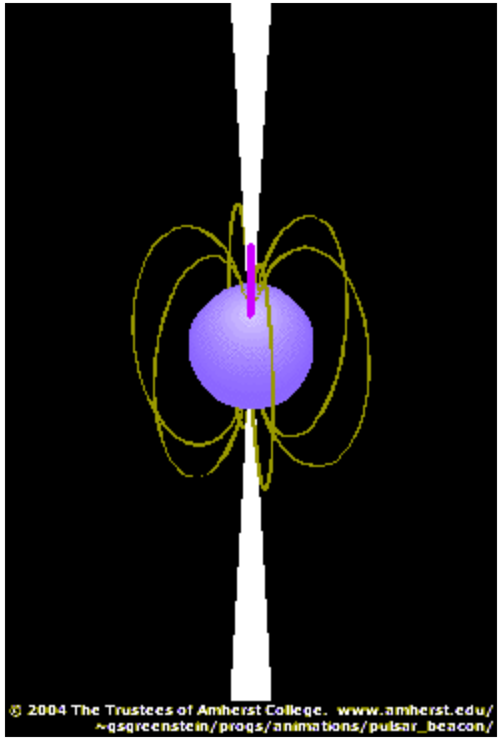
$$M R^2 w_1^2 = M (10^{-5} R)^2 w_2^2$$

$$10^{10} w_1 = w_2$$

$$T_2 = 10^{-10} T_1$$

$$T_1 = \text{A few days?}$$

$$T_2 = \text{A few milliseconds}$$

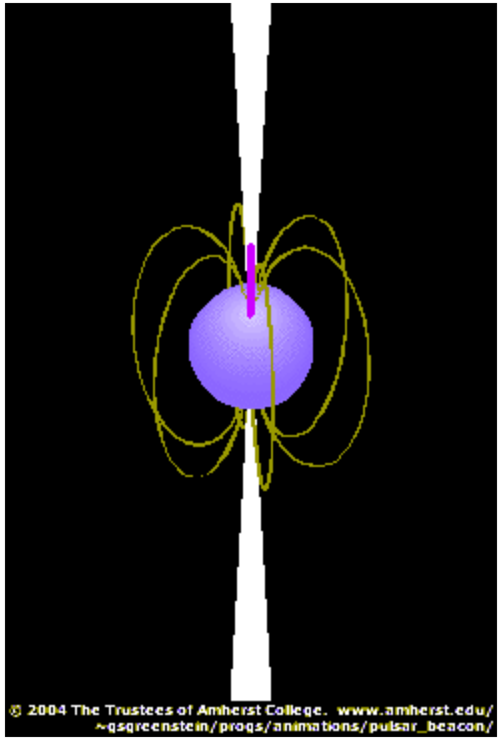


Neutron stars

$$R \to 10^{-5} R$$

- So what physics changes after collapse?

(1) Rotation: Millisecond pulsars!



Neutron stars

- So what physics changes after collapse?

$$R \to 10^{-5} R$$

(1) Rotation: Millisecond pulsars!

(2) Escape Velocity:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$M_\odot = 2 \times 10^{30}\ \mathrm{kg}$$

$$R = 12\ \mathrm{km}$$

$$v_{esc} = 0.5c$$

Neutron stars

- So what physics changes after collapse?

$$R \to 10^{-5} R$$

(1) Rotation: Millisecond pulsars!

(2) Escape Velocity: Half the speed of light!

Neutron stars

- So what physics changes after collapse?

$$R \to 10^{-5} R$$

(1) Rotation: Millisecond pulsars!

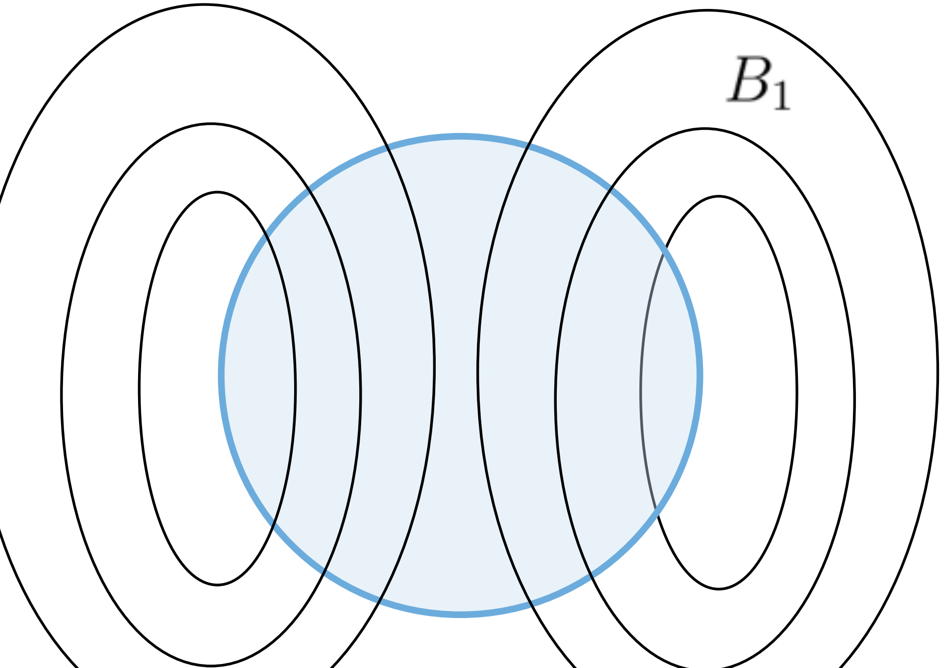
(2) Escape Velocity: Half the speed of light!

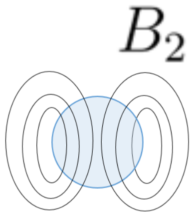
(3) Magnetic Field: Conserve flux:

$$\Phi_B = BA$$

$$B_1 R^2 = B_2 (10^{-5} R)^2$$

$$10^{10} B_1 = B_2$$





$$B_1 = 10^3 \text{ G}$$

$$B_2 = 10^{13} \text{ G}$$

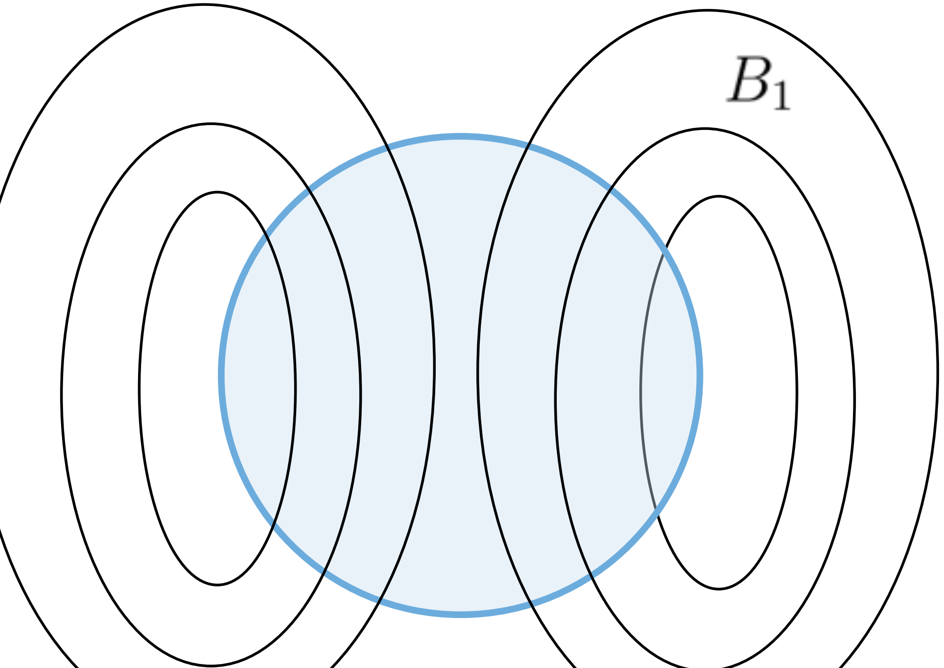
Neutron stars

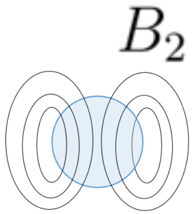
- So what physics changes after collapse?

(1) Rotation: Millisecond pulsars!

(2) Escape Velocity: Half the speed of light!

(3) Magnetic Field: Literally nothing for comparison…





$$B_1 = 10^3 \text{ G}$$
$$B_2 = 10^{13} \text{ G}$$

The Four Forces

- Neutron stars are the only objects in the universe where all four forces play notable roles!

Weak Force:
Neutrinos

Electromagnetism:
Strong B field

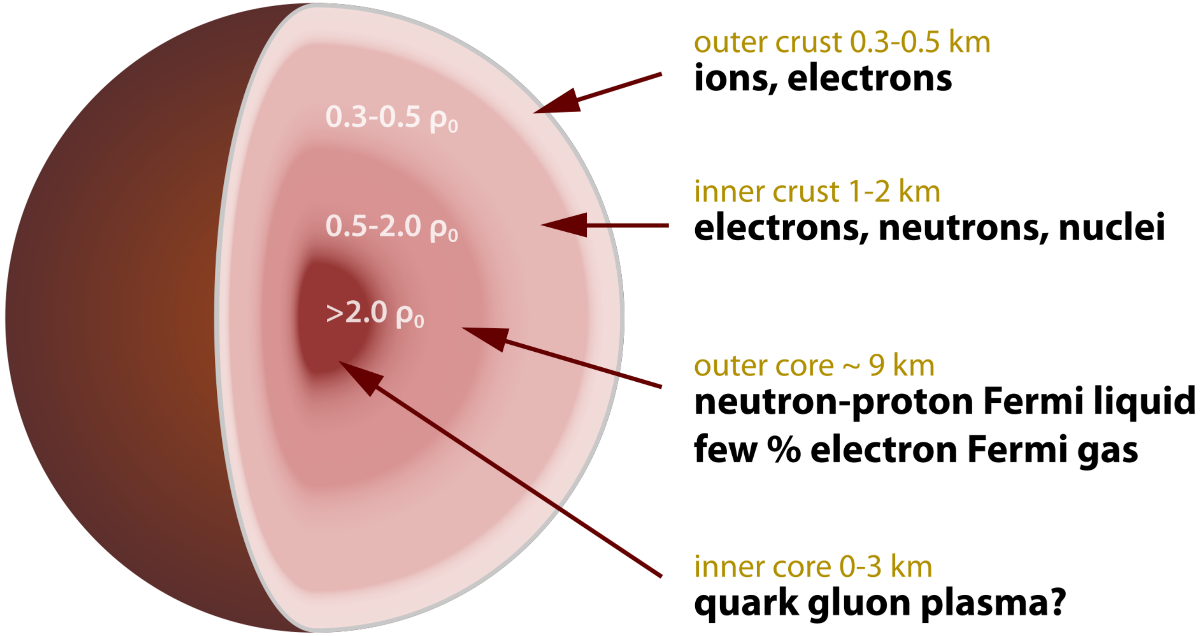
Strong Force:
Nuclear interactions

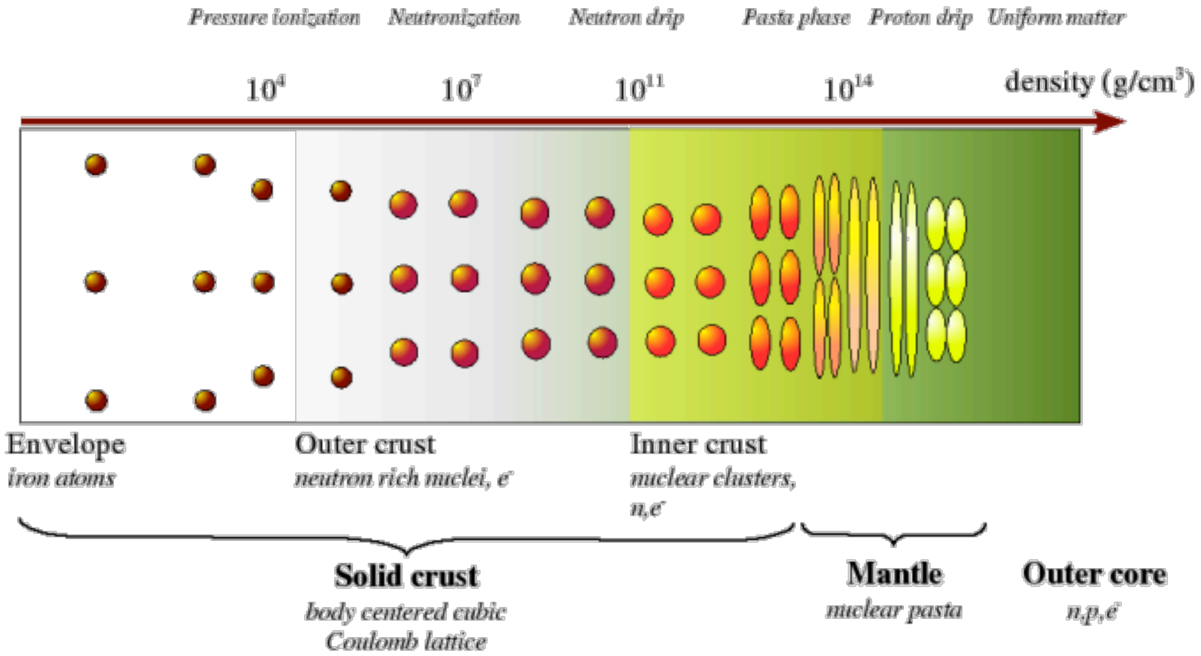
Gravity:
So much gravity.

What’s inside a neutron star?

Neutron Star Structure

- What's inside a neutron star?

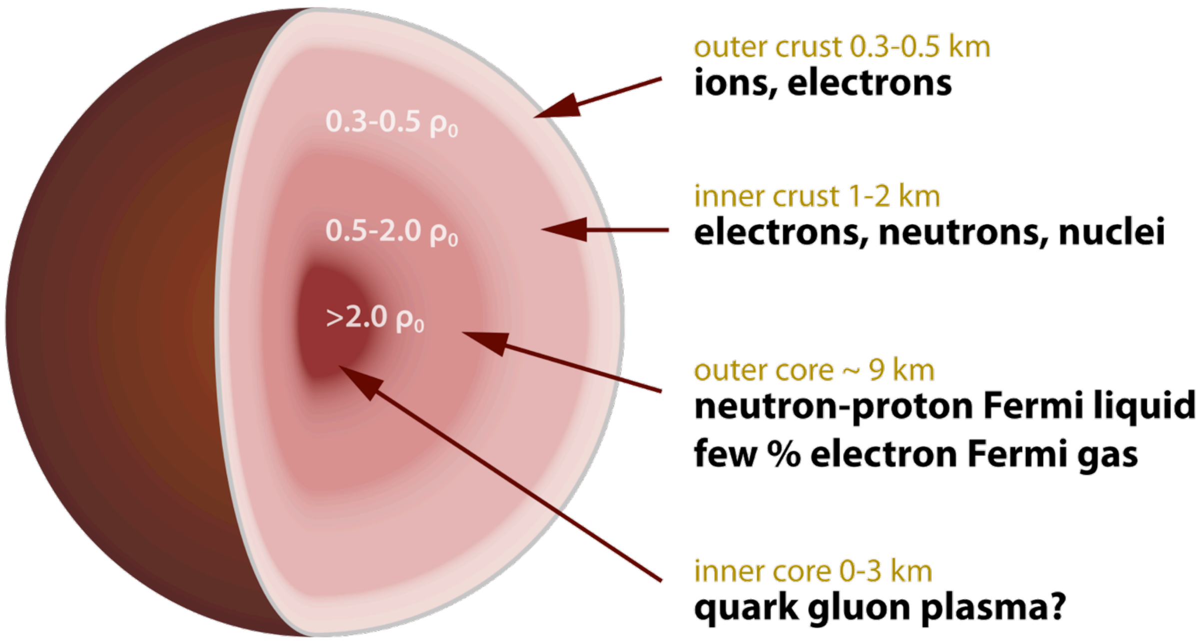


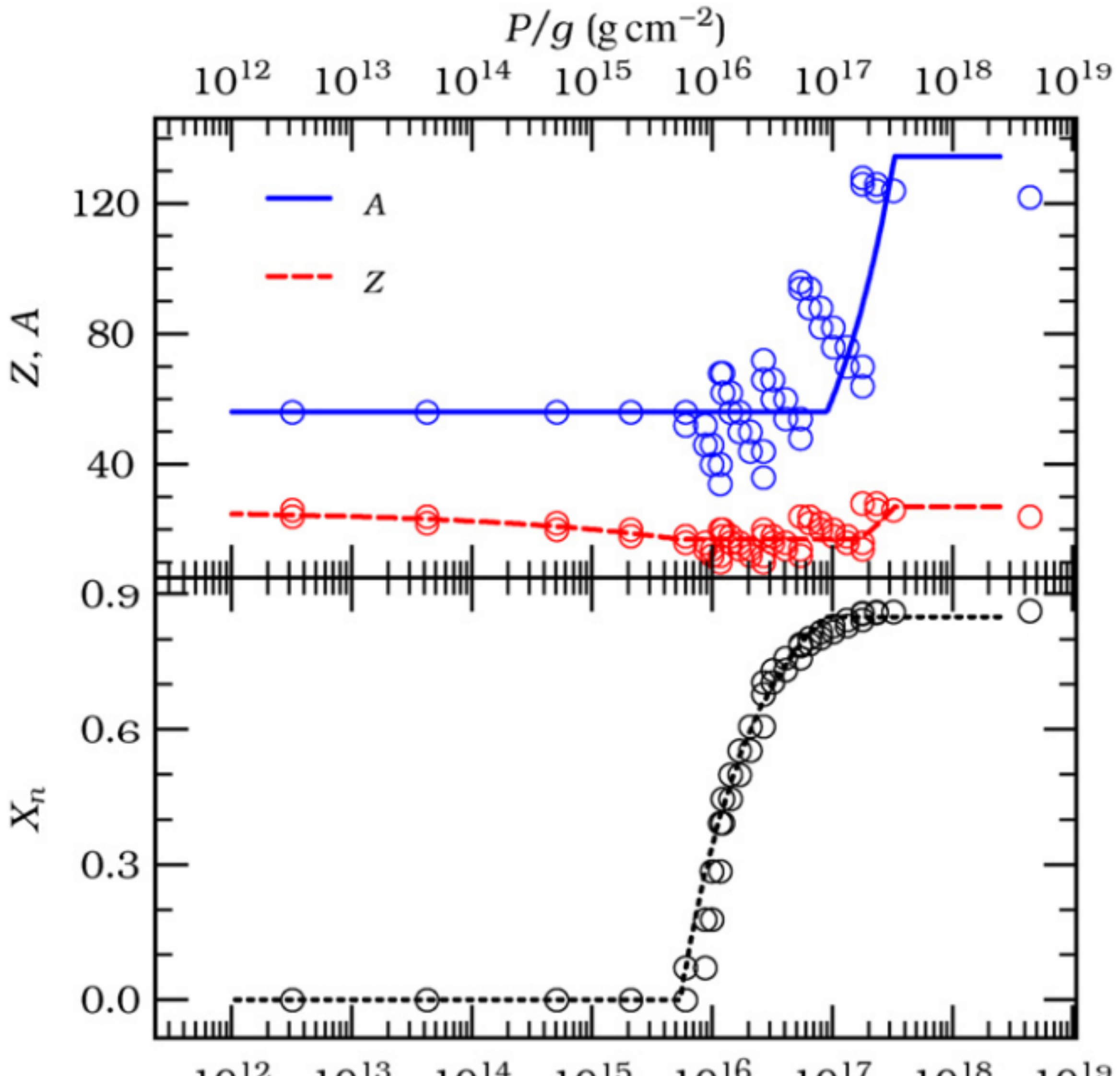


Not just a "giant nucleus in space!"

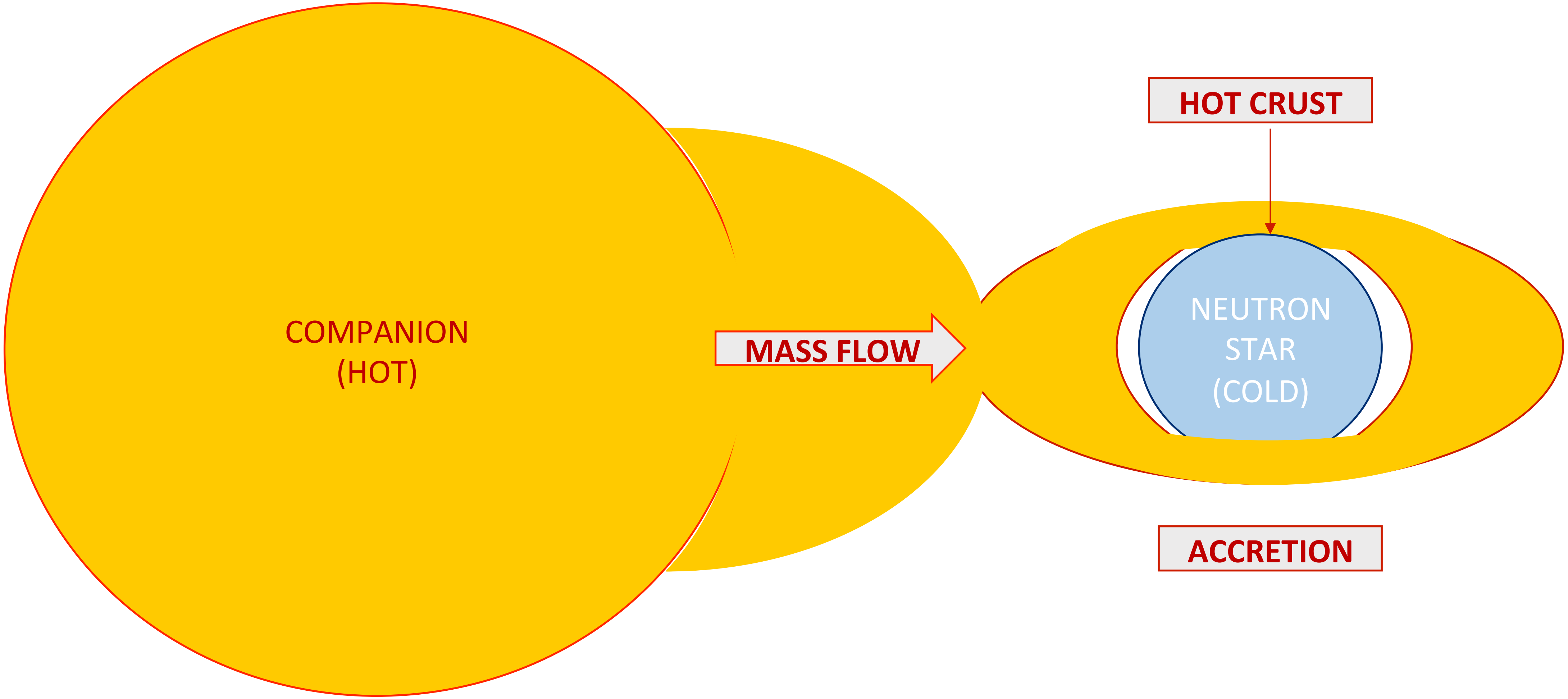
Neutron Star Structure

- What's inside a neutron star?



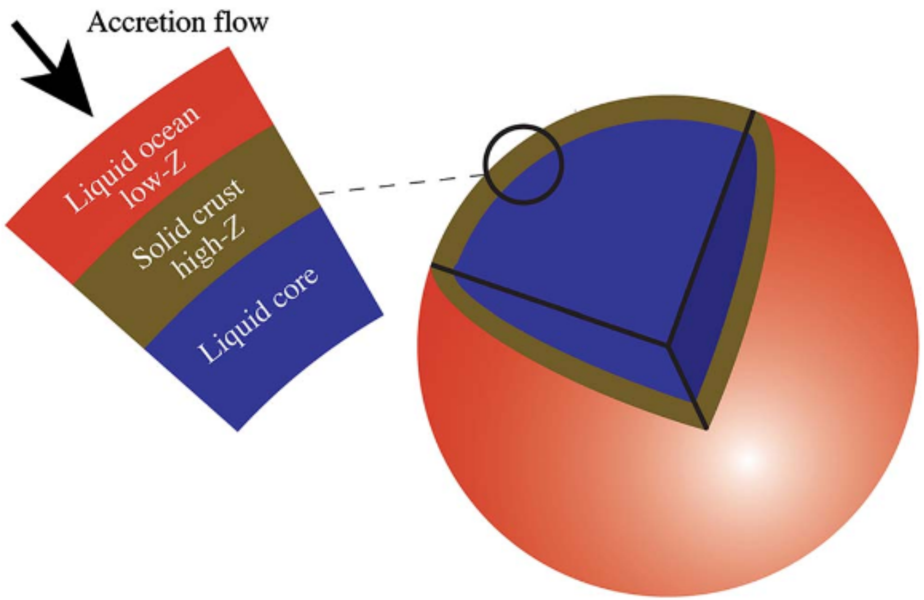


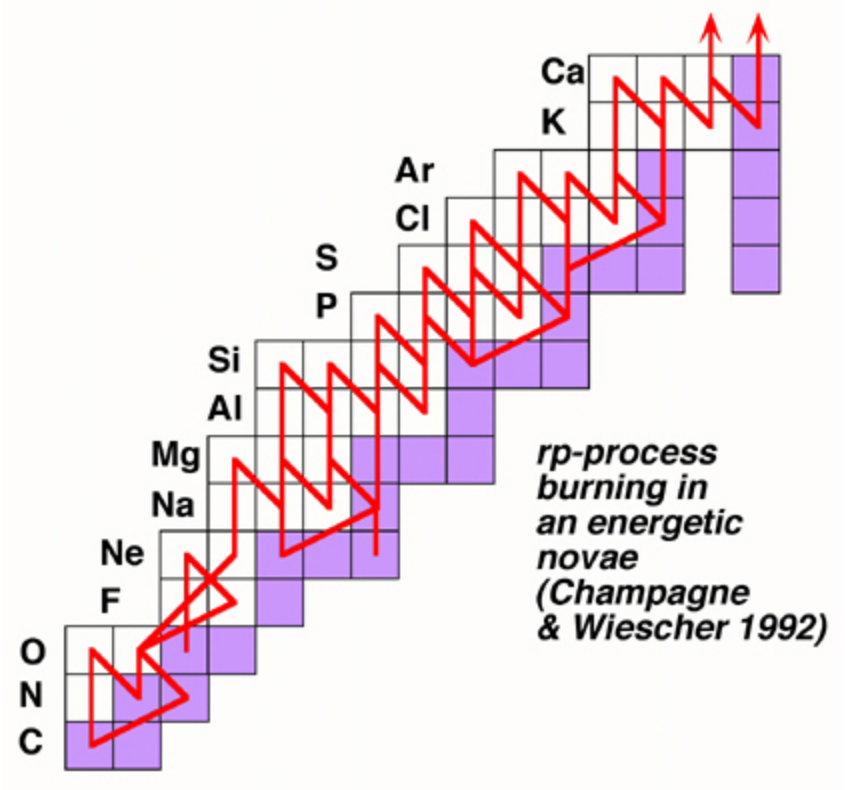
Low Mass X-Ray Binaries



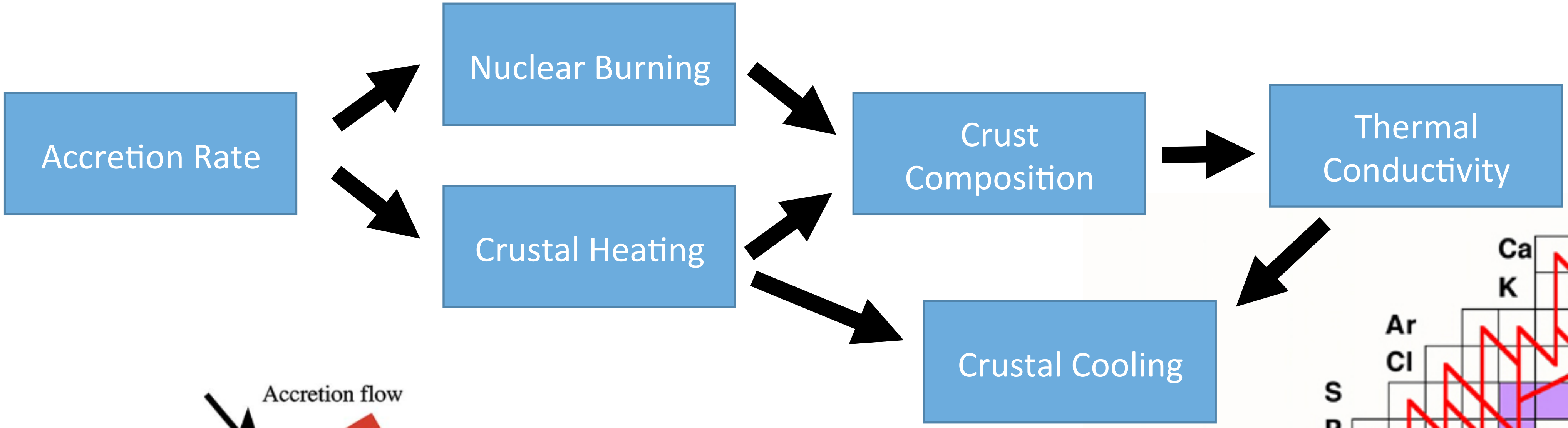
X-ray bursts

- As matter accretes, it is compressed, buried, and heated
- Explosive nuclear burning produces a mix of heavy nuclei (rp-process)
- Ash is buried, and crystallizes

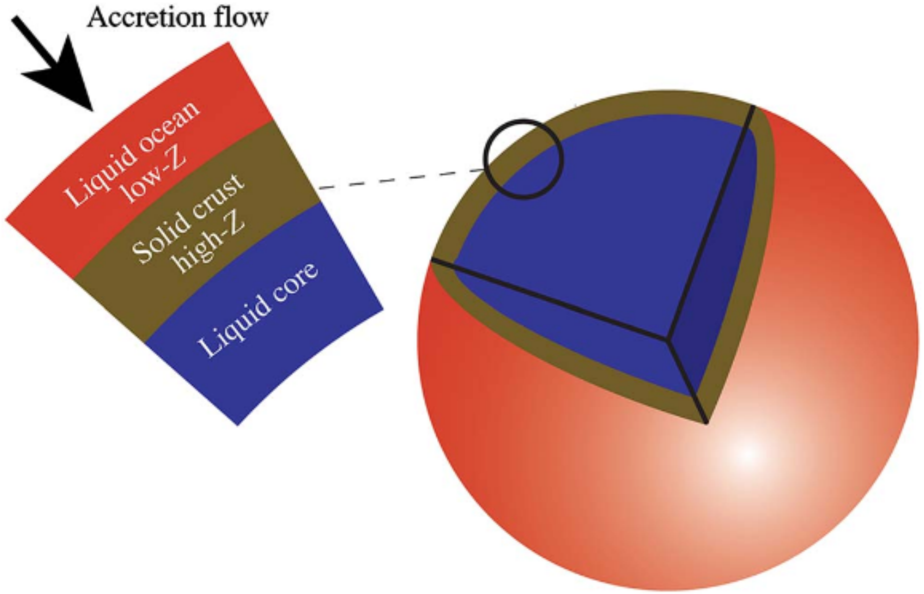




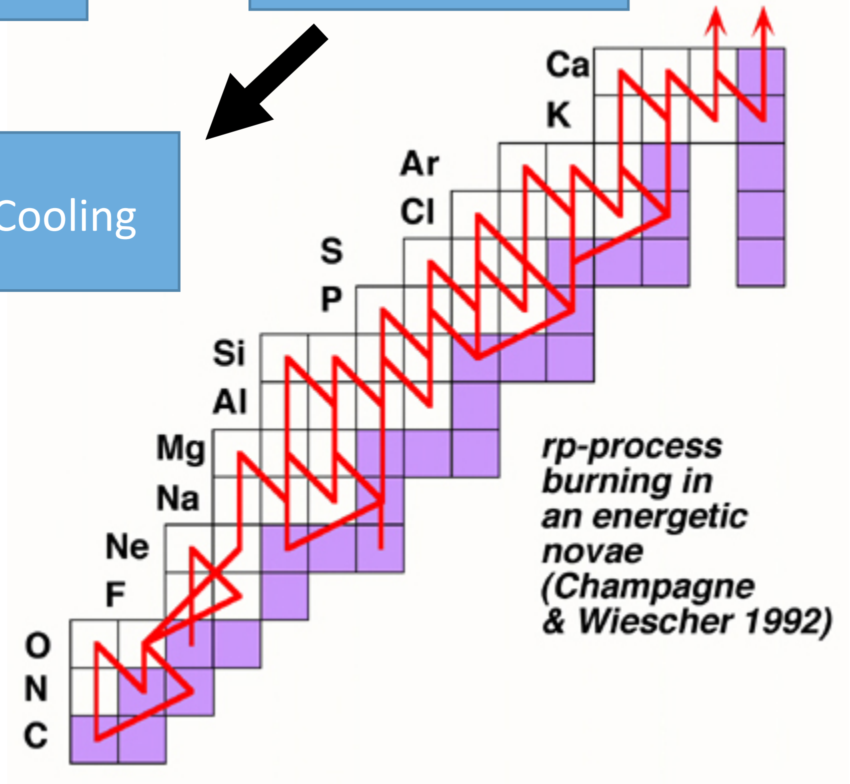
X-ray bursts



Accretion Rate
Nuclear Burning
Crustal Heating
Crust Composition
Thermal Conductivity
Crustal Cooling



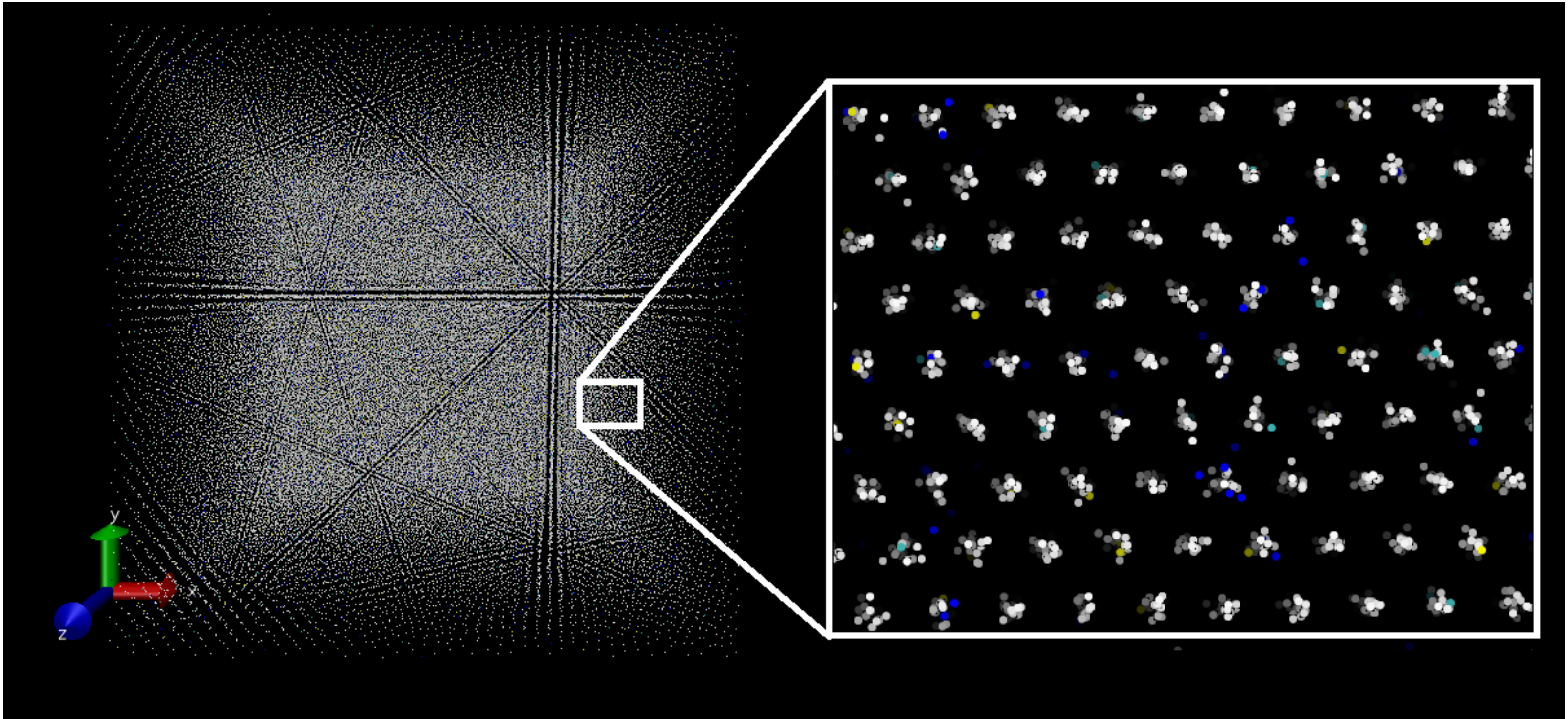
Accretion flow
Liquid ocean low-Z
Solid crust high-Z
Liquid core



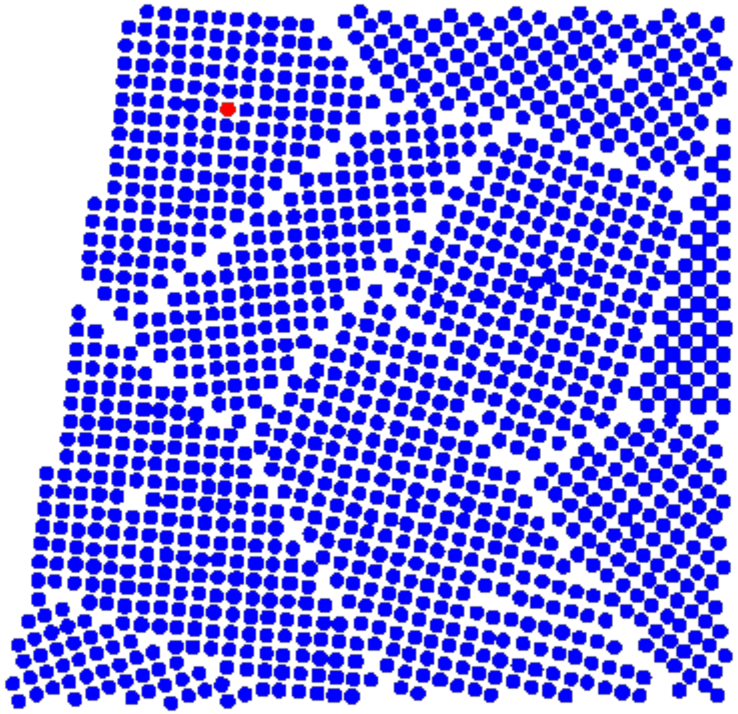
Ca
K
Ar
Cl
S
P
Si
Al
Mg
Na
Ne
F
O
N
C
rp-process burning in an energetic novae (Champagne & Wiescher 1992)

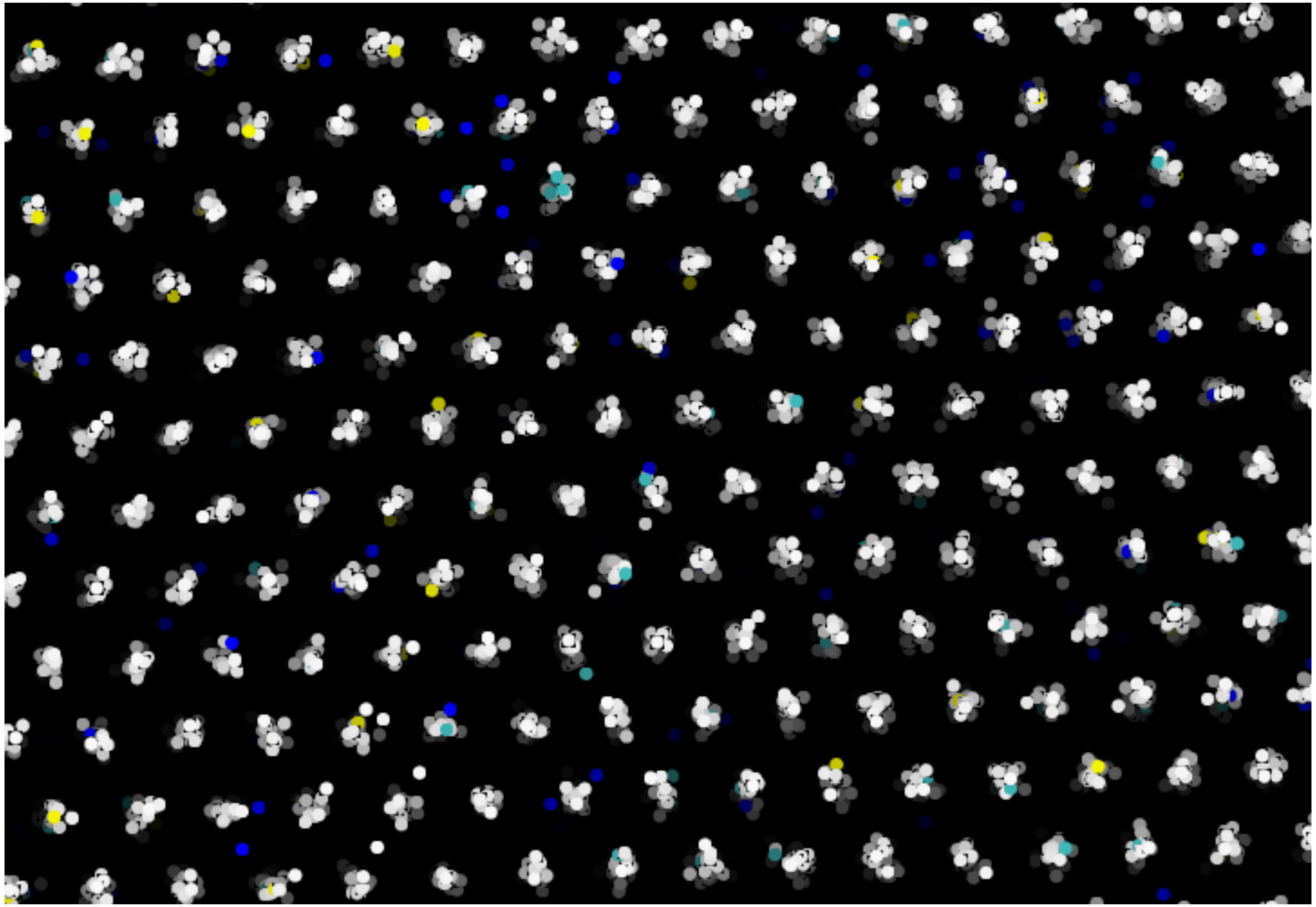
Hard Astromaterials

Crystallization



Crystallization





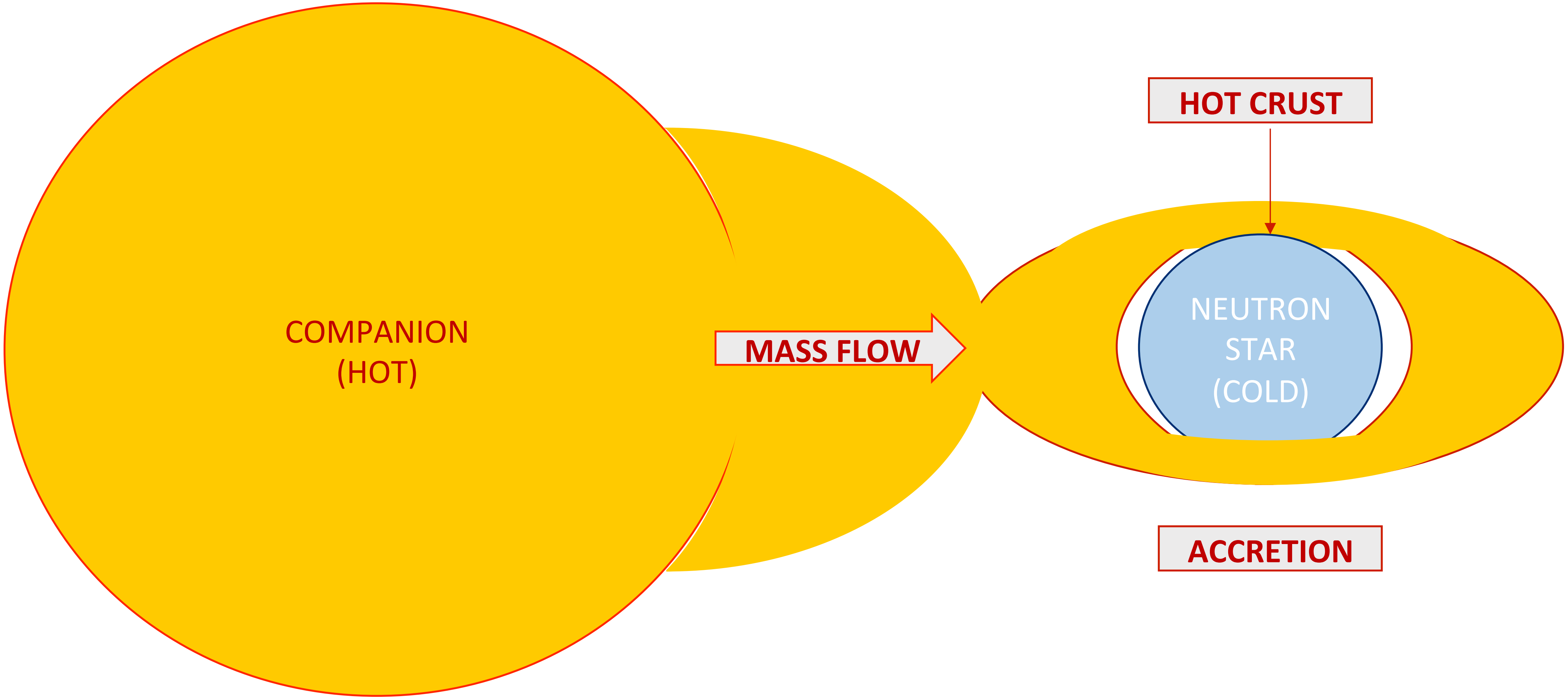
Crystallization

- Physics is set by an ‘impurity parameter’ $$Q_{\rm imp} \equiv n_{\rm ion}^{-1} \sum_i n_i (Z_i - \langle Z \rangle)^2$$

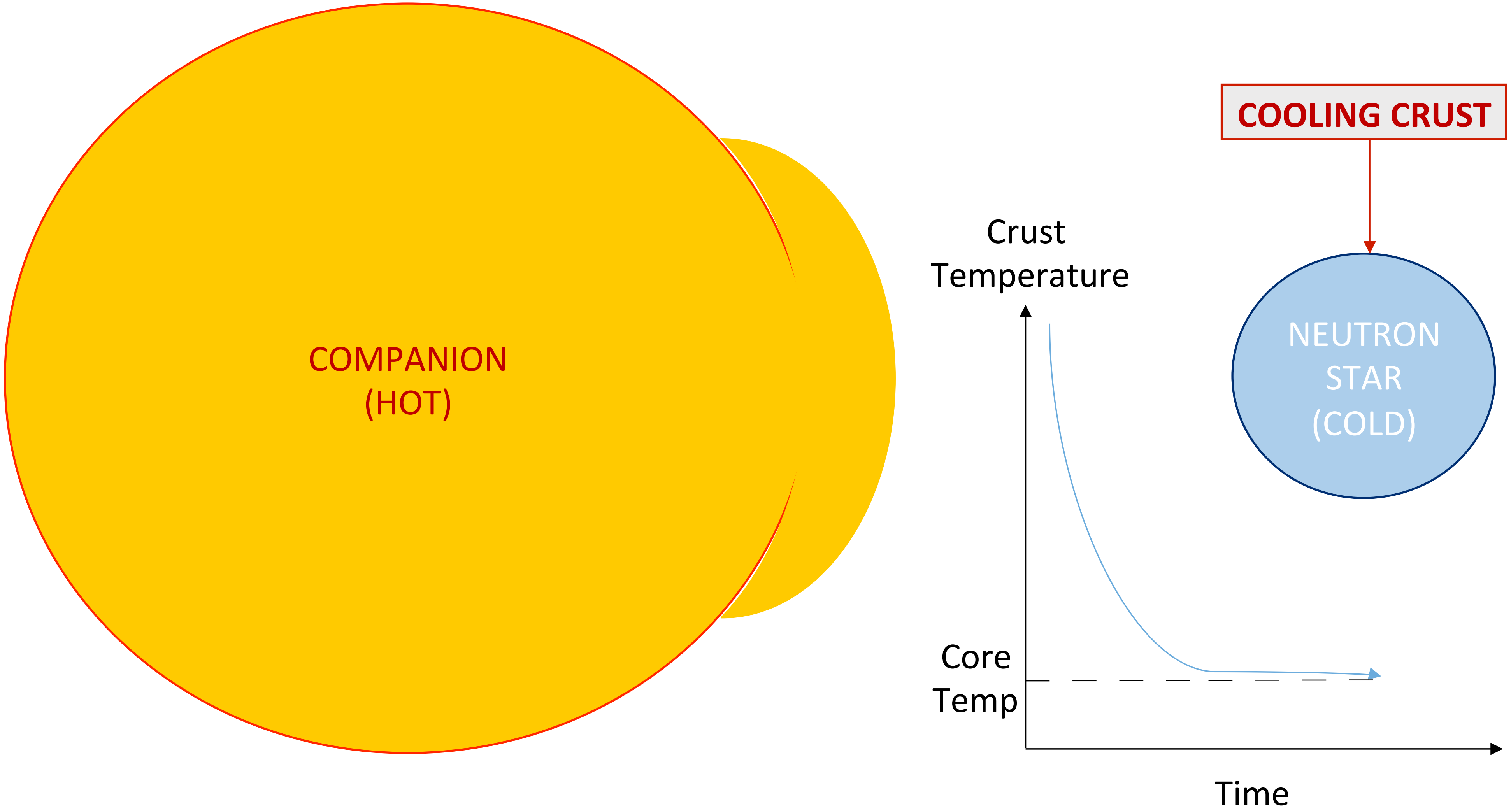
- Low impurity parameter implies thermally conductive crust
- High impurity parameter implies thermally resistive crust

- rp-ash has a large impurity parameter (30-50)
- What does observation favor?

Low Mass X-Ray Binaries



Low Mass X-Ray Binaries

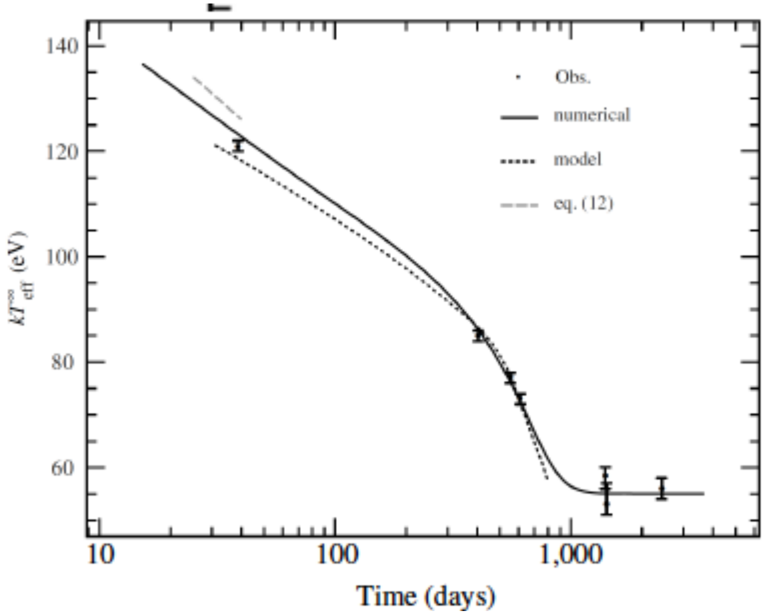


Observables – Thermal Properties

- Find an effective impurity parameter and try to fit neutron star cooling curves
- Cooling curves: low mass X-ray binary MXB 1659-29

$$Q_{\rm imp} \equiv n_{\rm ion}^{-1} \sum_i n_i (Z_i - \langle Z \rangle)^2$$

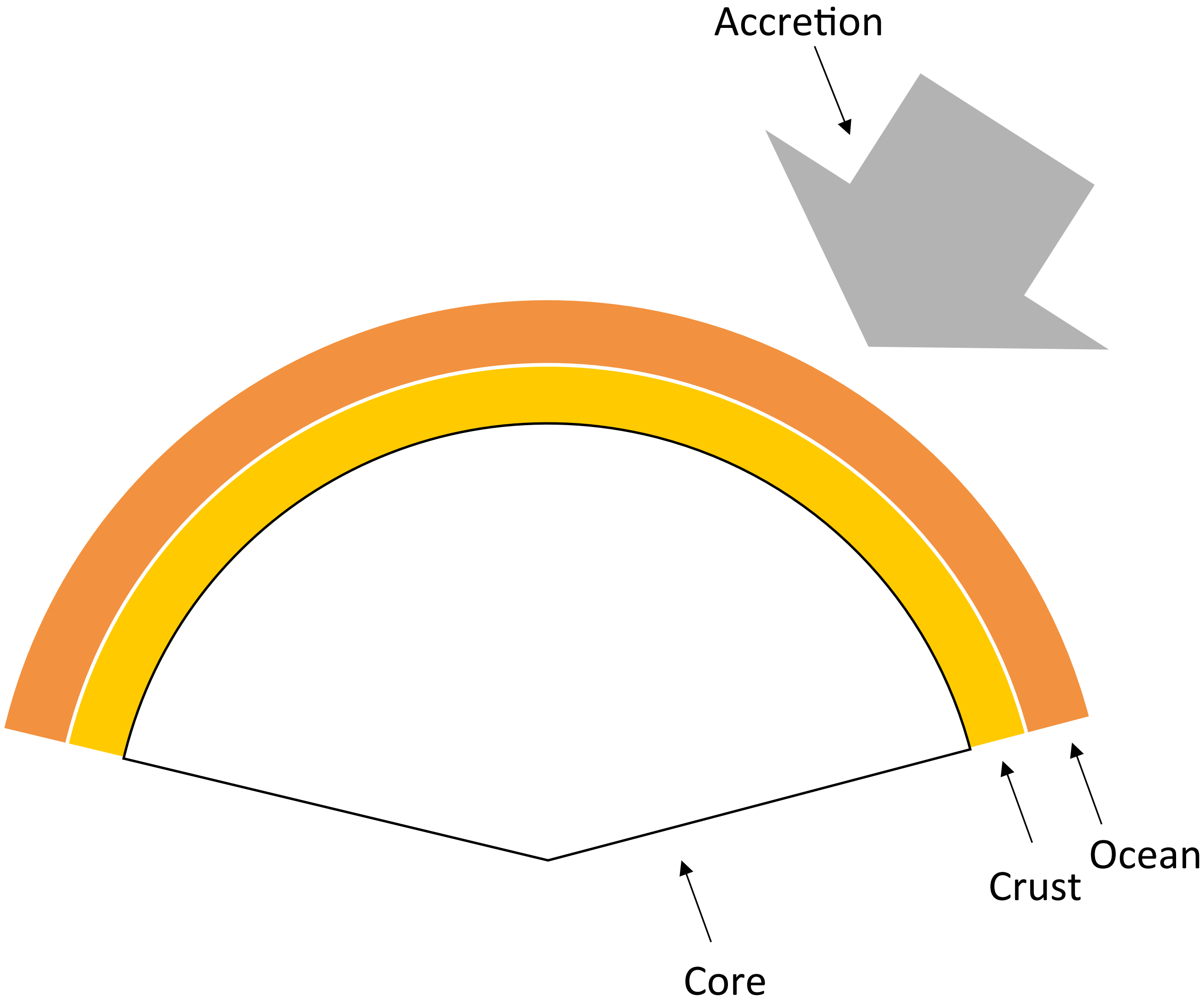
  - Blue: Conductive crust
    $Q_{imp} = 3.5$
    $T_c = 3.05 \times 10^7$ K



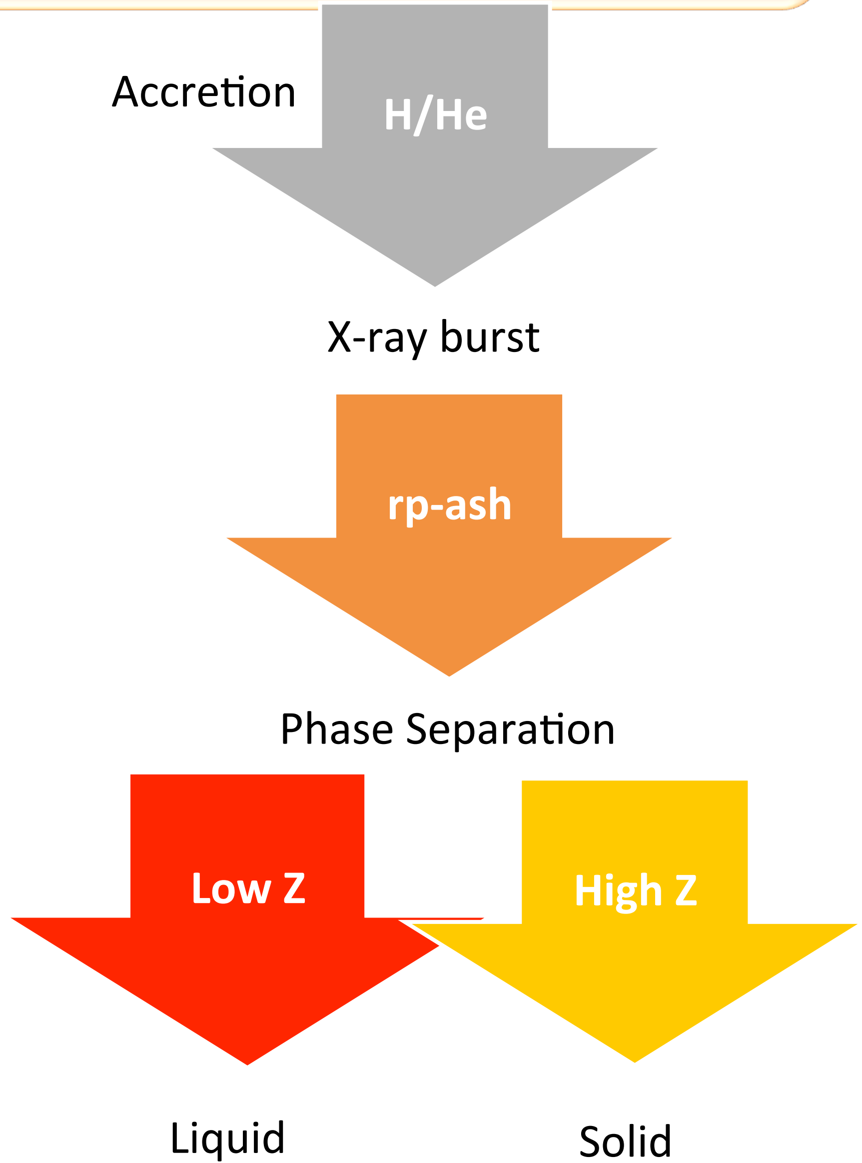
A problem?

- rp-ash has a large impurity parameter (30-50) while observation favors a low impurity parameter (<10)
- How do we reconcile this? Purify the crust with phase separation! (Mckinven et al 2016)

A problem?

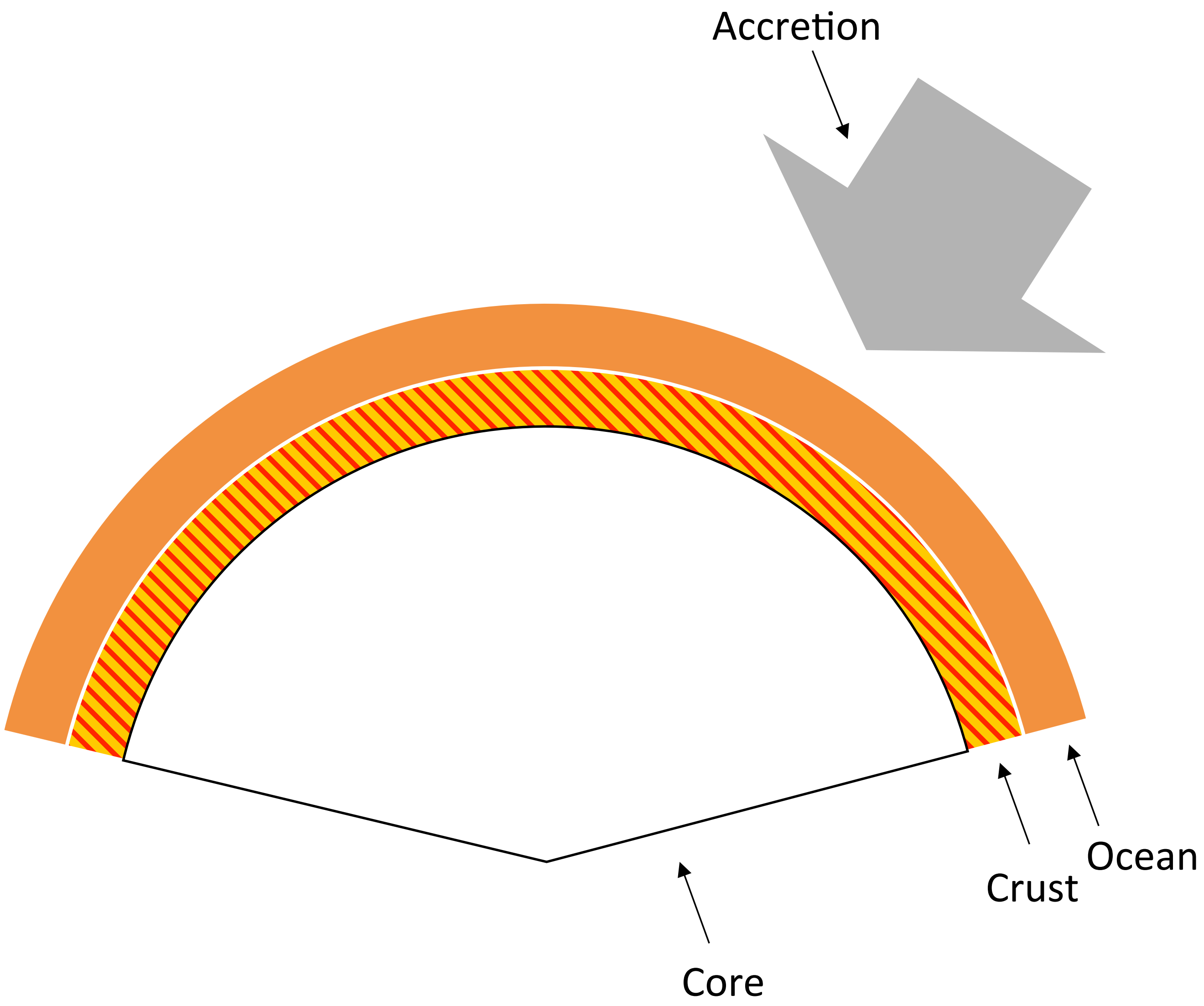


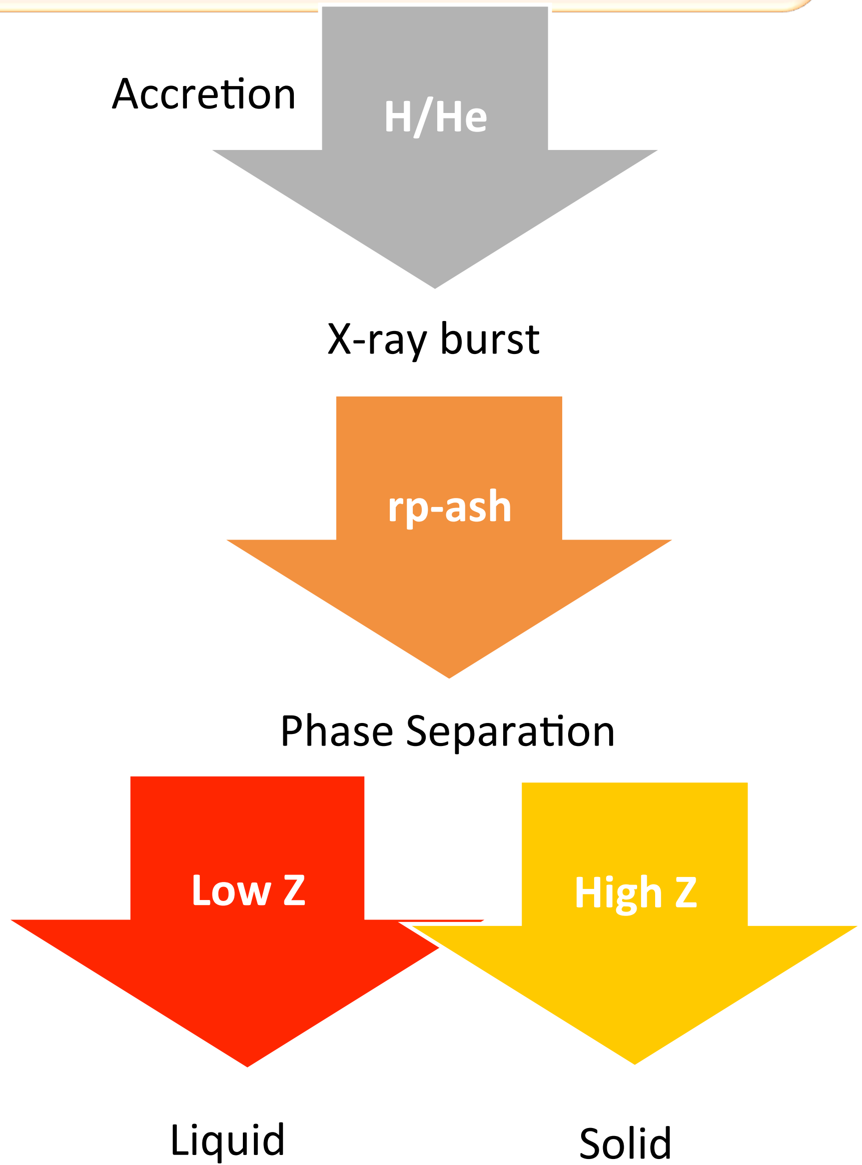
Accretion
Core
Crust
Ocean



Accretion
H/He
X-ray burst
rp-ash
Phase Separation
Low Z
High Z
Liquid
Solid

A problem?


Accretion
Crust
Ocean
Core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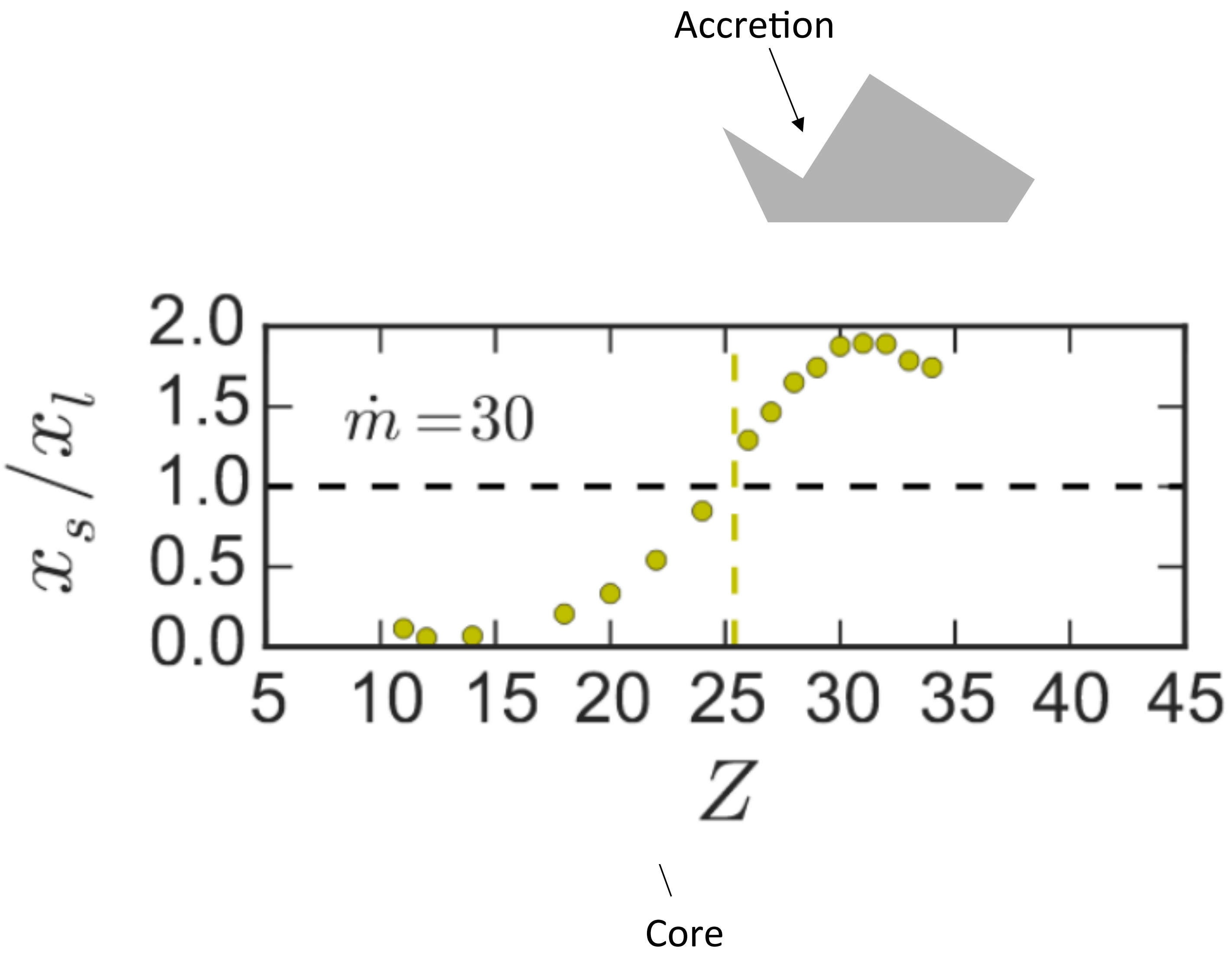
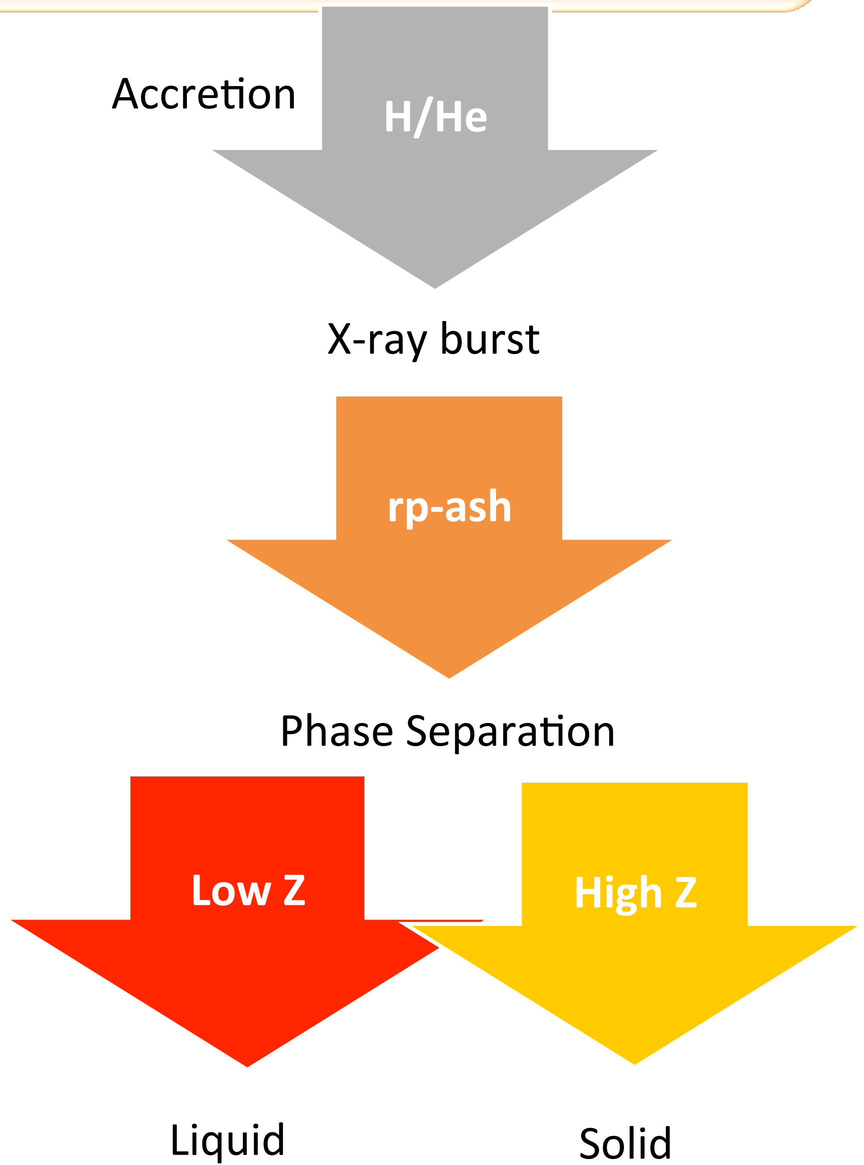


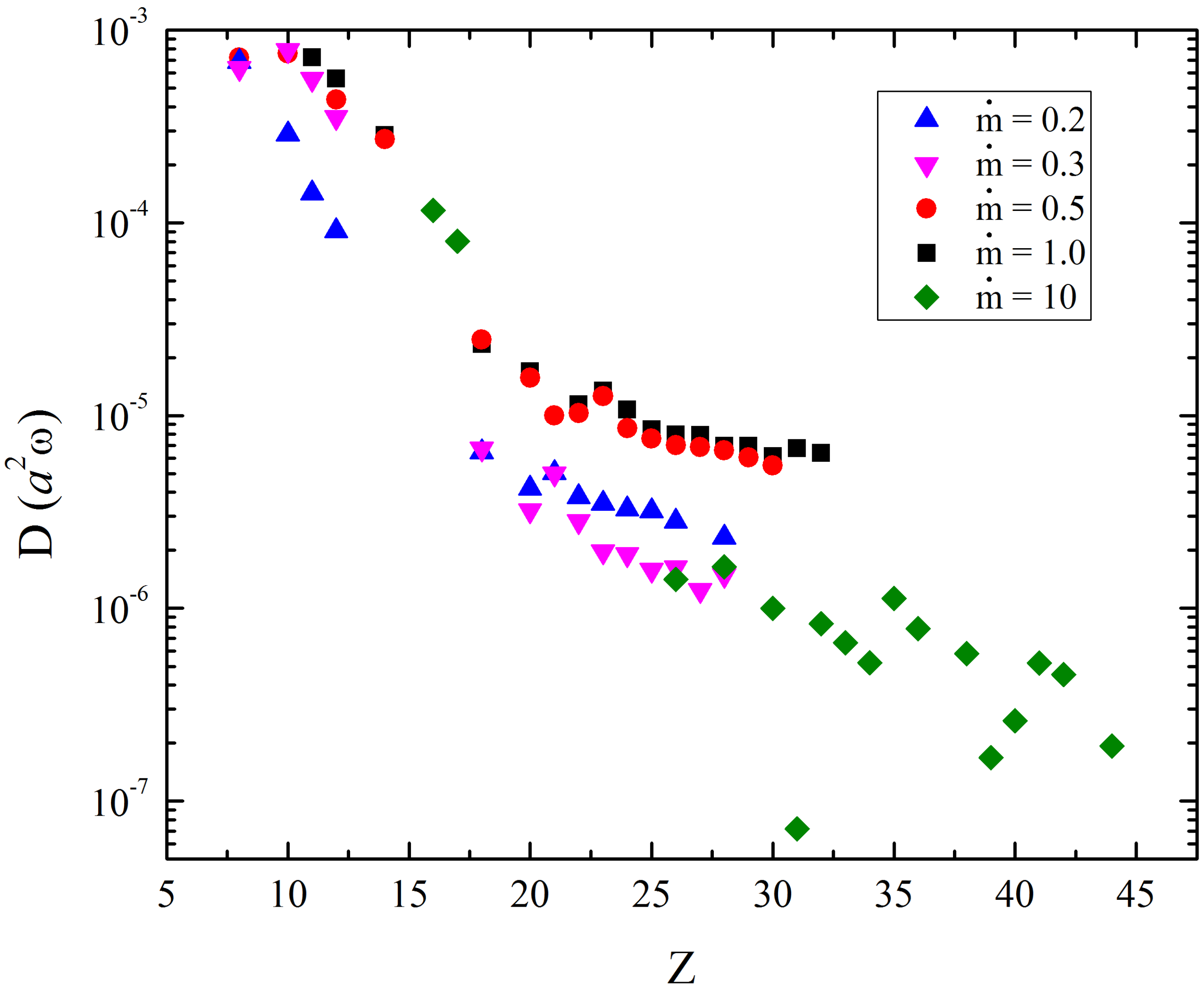
Accretion
$\dot{m} = 30$
x_s / x_l
2.0
1.5
1.0
0.5
0.0
5
10
15
20
25
30
35
40
45
Z
Core



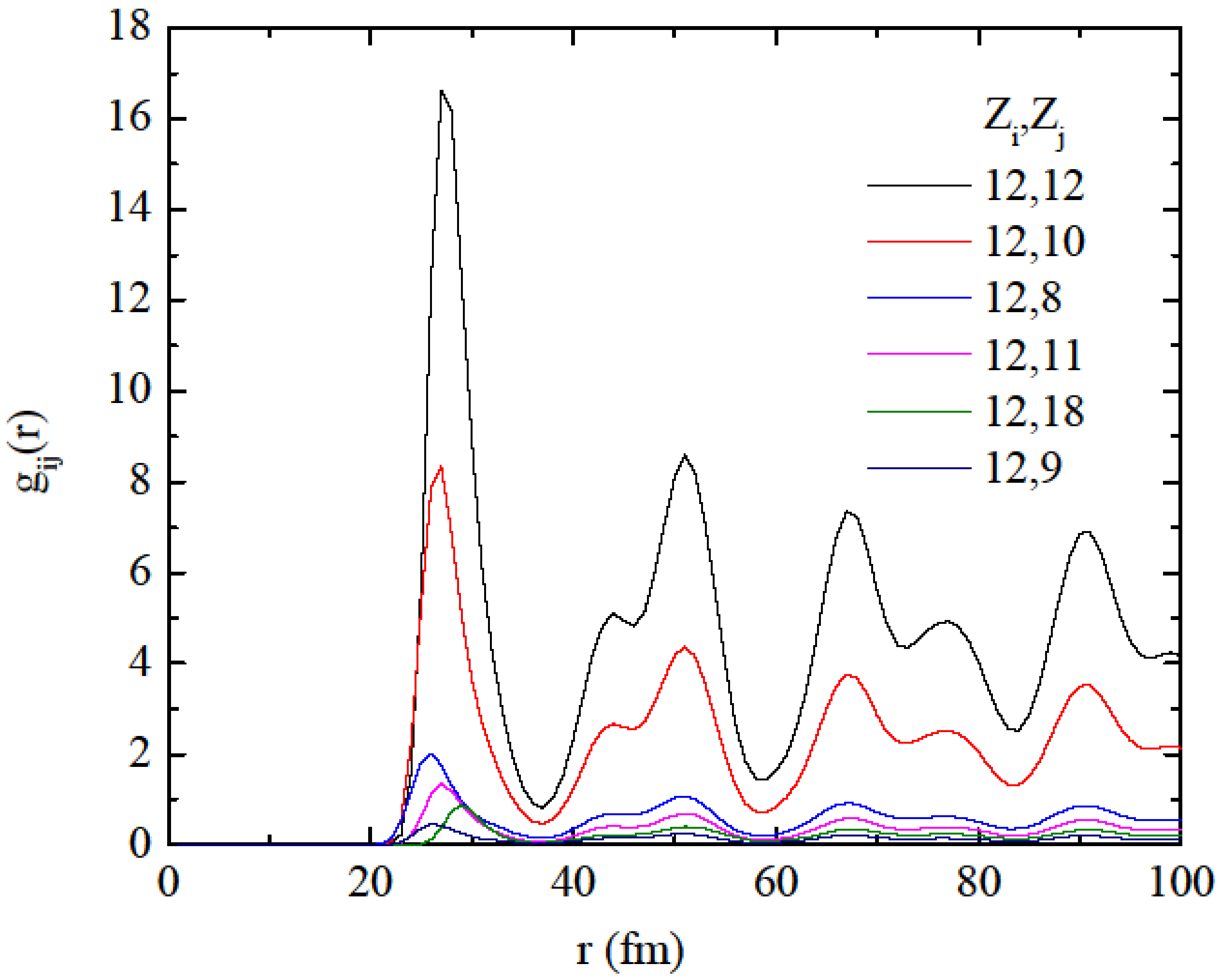
Accretion
H/He
X-ray burst
rp-ash
Phase Separation
Low Z
High Z
Liquid
Solid

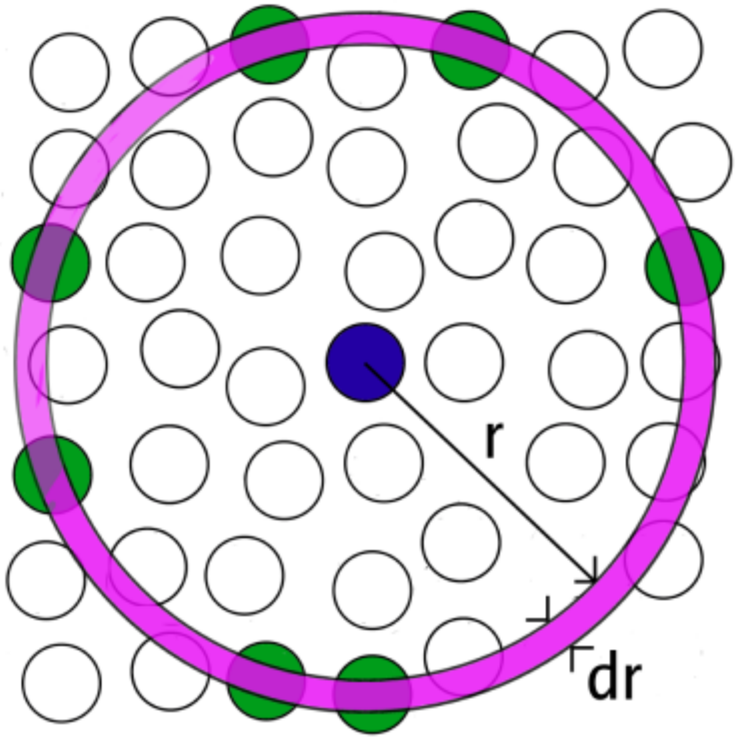
Diffusion Coefficients

- Lattice site hops in simulations of the crystal allow for diffusion near the melting temperature
- Broken power law in D(Z)
- If $D=10^{-6}\, w_p\, a^2$, then 3 cm layers with 100 yrs accretion (Mckinven et al 2016)

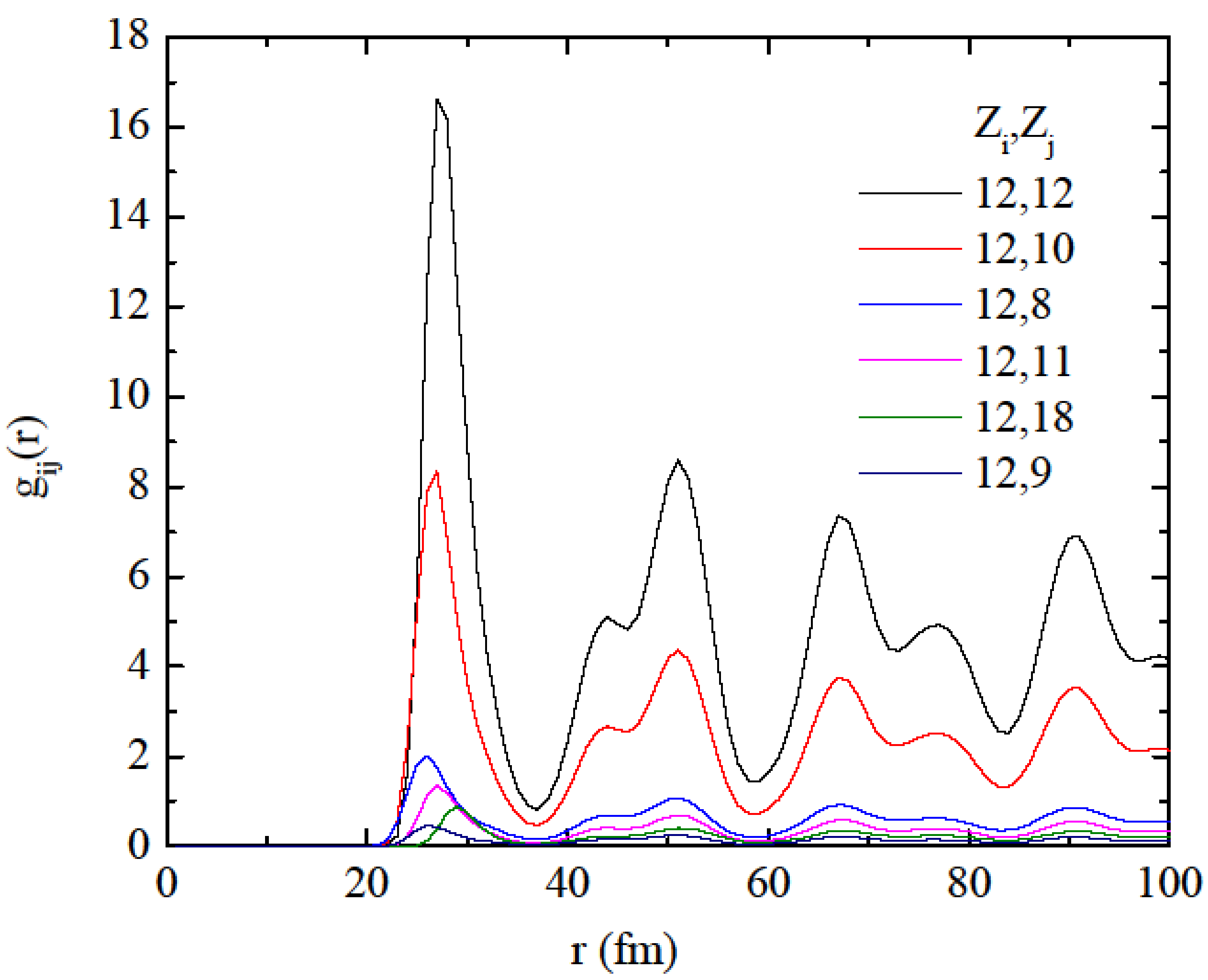


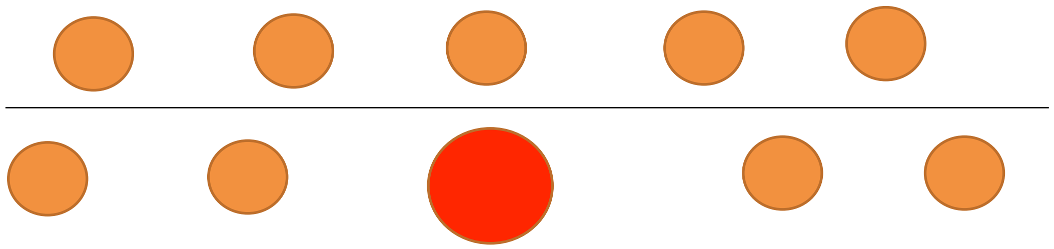
Radial Distribution Functions



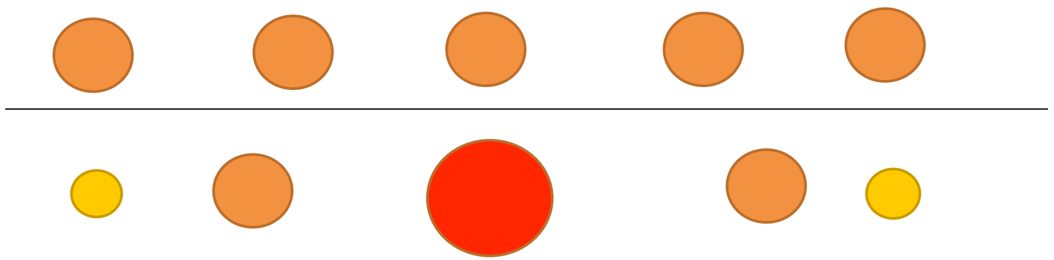


Radial Distribution Functions





First peak in g(r) shifts depending on Coulomb force
Second peaks are not seen to shift
Screening of impurities? Long range correlations?



Soft Astromaterials

Neutron stars

- The crust is a crystalline lattice, while the core is uniform nuclear matter, like a solid nucleus. What's in between these two phases?

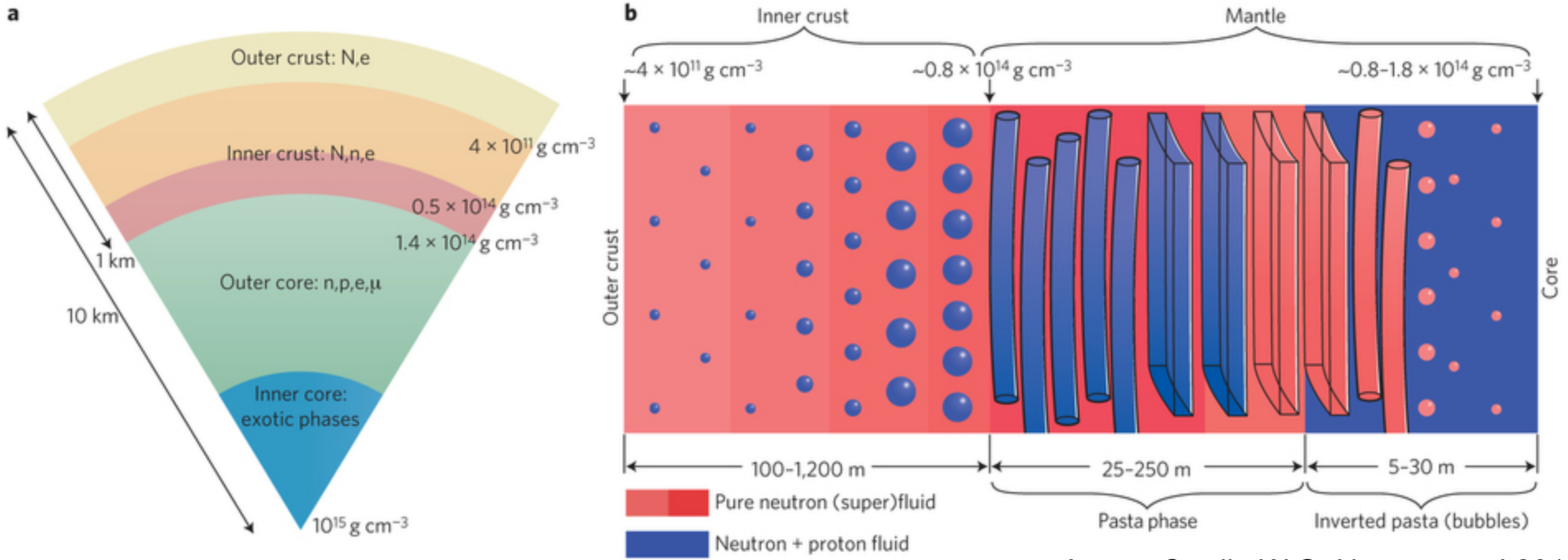


Image Credit: W.G. Newton et al 2012

Non-Spherical Nuclei

- First theoretical models of the shapes of nuclei near n_0
  - 1983: Ravenhall, Pethick, & Wilson
  - 1984: Hashimoto, H. Seki, and M. Yamada
- *Frustration*: Competition between proton-proton Coulomb repulsion and strong nuclear attraction
- Nucleons adopt non-spherical geometries near the saturation density to minimize surface energy

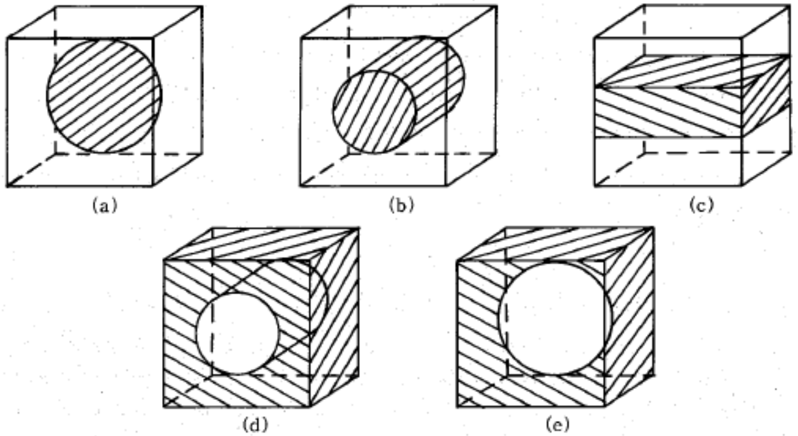
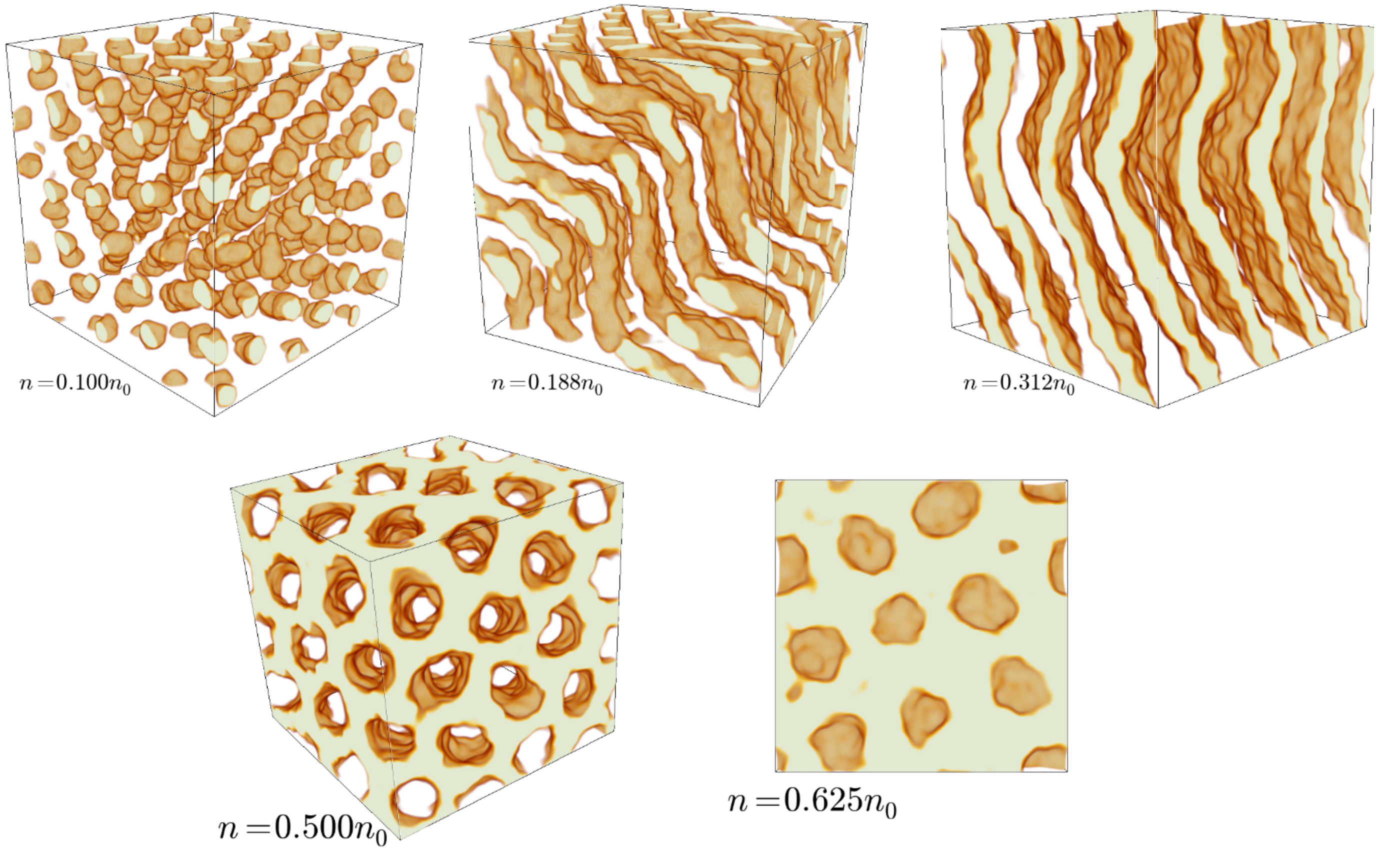


Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are, (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).

Nuclear Pasta



Shape of Nuclei in the Crust of Neutron Star

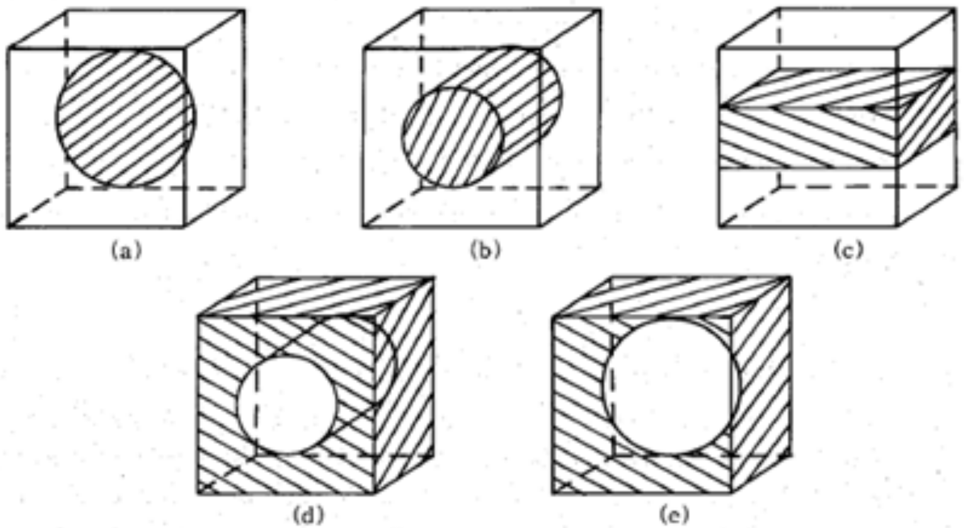
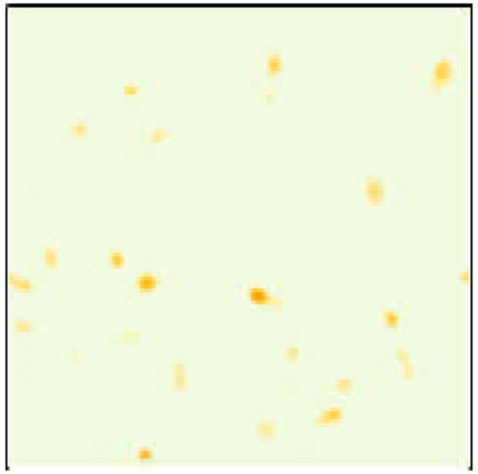


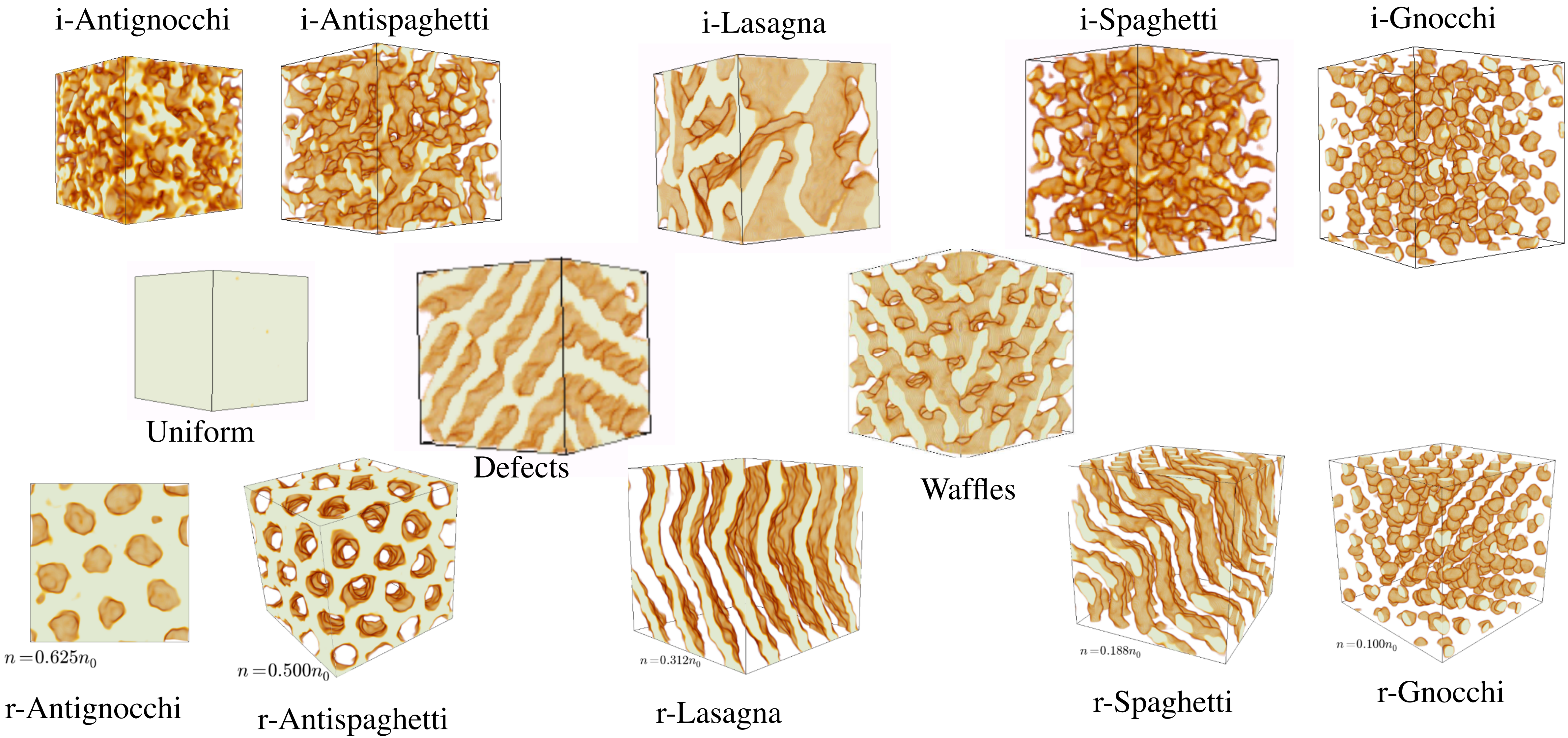
Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are, (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).

Gold Nucleus
For Scale



$$n = 0.1200 \mathrm{fm}^{-3}$$

Phases


i-Antignocchi
i-Antispaghetti
i-Lasagna
i-Spaghetti
i-Gnocchi
Uniform
Defects
Waffles
$n = 0.625 n_0$
$n = 0.500 n_0$
$n = 0.312 n_0$
$n = 0.188 n_0$
$n = 0.100 n_0$
r-Antignocchi
r-Antispaghetti
r-Lasagna
r-Spaghetti
r-Gnocchi

Classical Pasta Formalism

- **Classical Molecular Dynamics** with IUMD on Big Red II

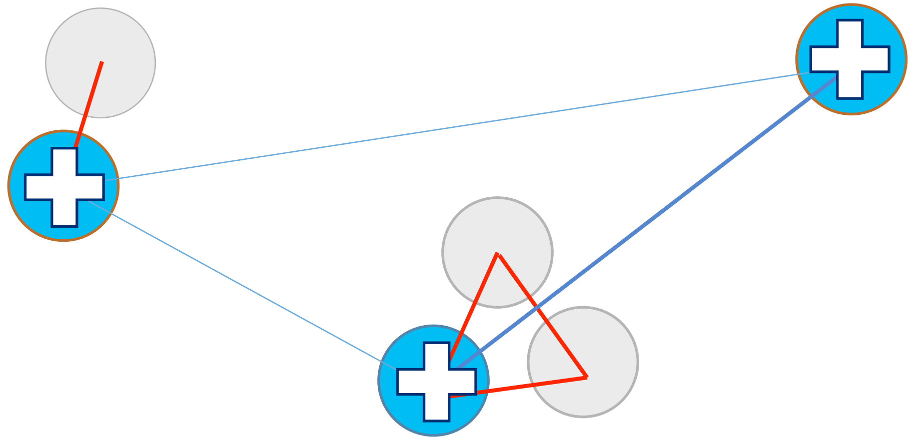
a	b	c	Λ	λ
110 MeV	-26 MeV	24 MeV	1.25 fm^2	10 fm

$$V_{np}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b - c] e^{-r_{ij}^2/2\Lambda}$$

$$V_{nn}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2\Lambda}$$

$$V_{pp}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2\Lambda} + \frac{\alpha}{r_{ij}} e^{-r_{ij}/\lambda}$$

- Short range nuclear force
- Long range Coulomb force



Nucleus	Monte-Carlo $\langle V_{tot} \rangle$ (MeV)	Experiment (MeV)
^{16}O	-7.56±0.01	-7.98
^{40}Ca	-8.75±0.03	-8.45
^{90}Zr	-9.13±0.03	-8.66
^{208}Pb	-8.2 ±0.1	-7.86

Classical Pasta Form…alism

- **Classical Molecular Dynam…** …IUMD on Big Red II

$$V_{np}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b - c] e^{-r_{ij}^2/2\Lambda}$$

$$V_{nn}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2\Lambda}$$

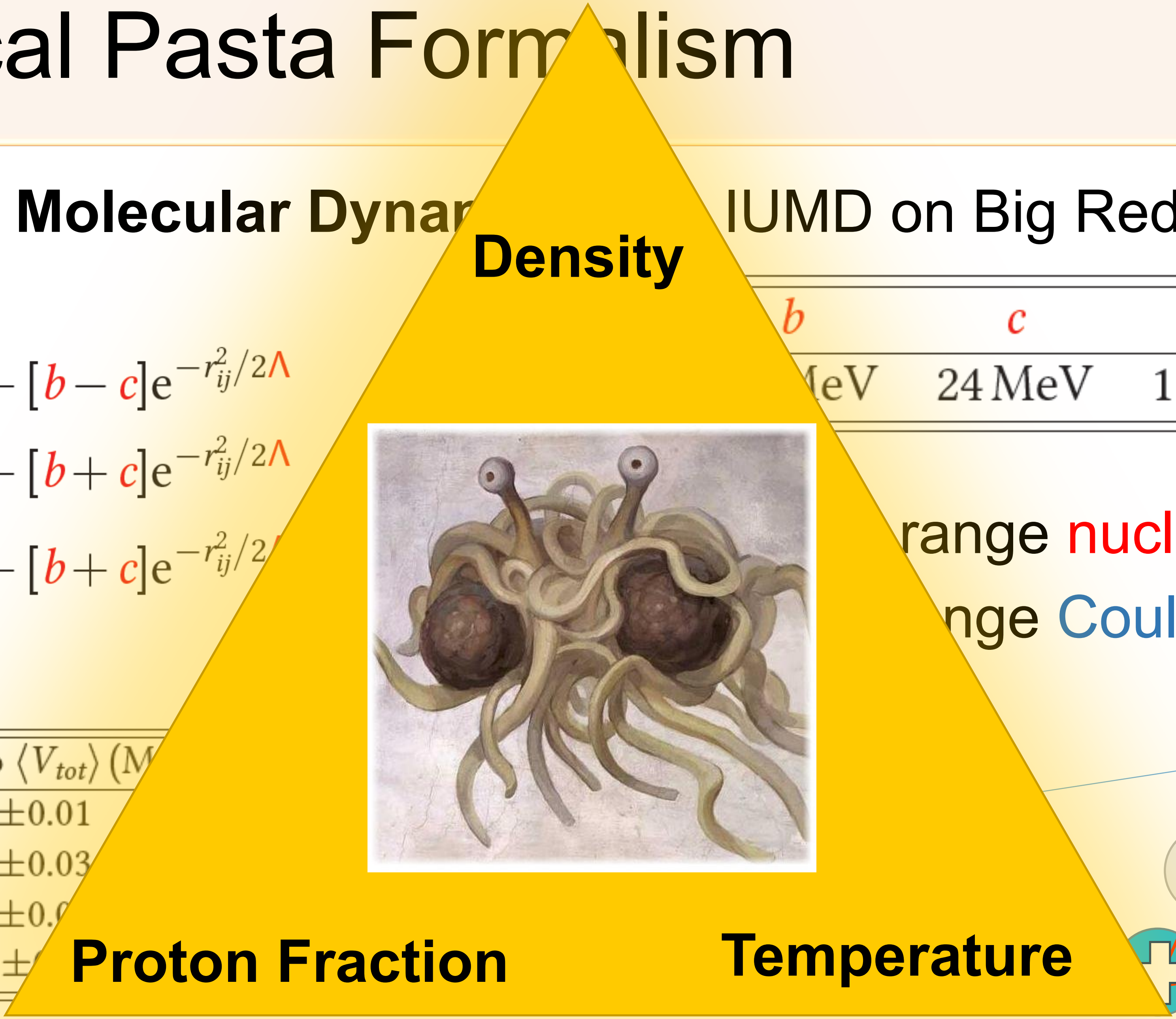
$$V_{pp}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2…}$$

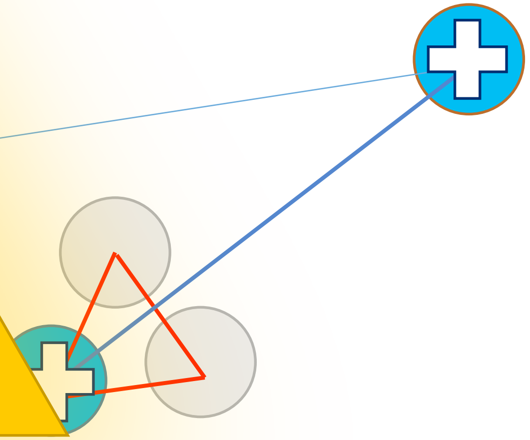
b	c	Λ	λ
…eV	24 MeV	1.25 fm^2	10 fm

…range **nuclear** force

…nge **Coulomb** force

Nucleus	Monte-Carlo $\langle V_{tot} \rangle$ (M…
^{16}O	-7.56±0.01
^{40}Ca	-8.75±0.03…
^{90}Zr	-9.13±0.…
^{208}Pb	-8.2 ±…

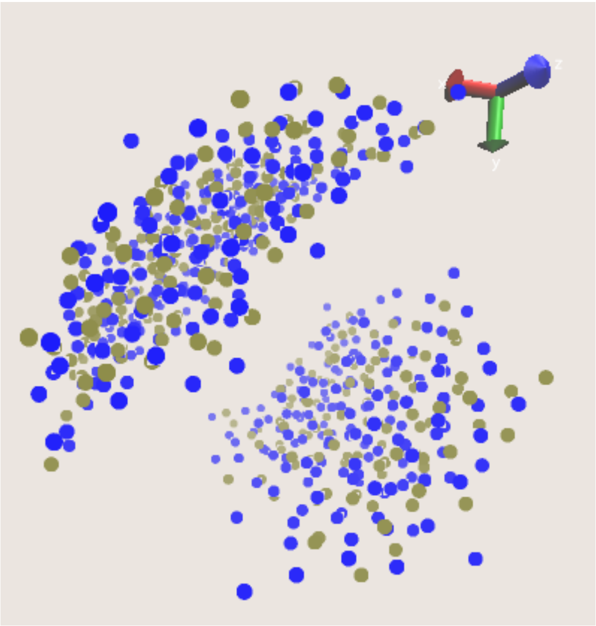


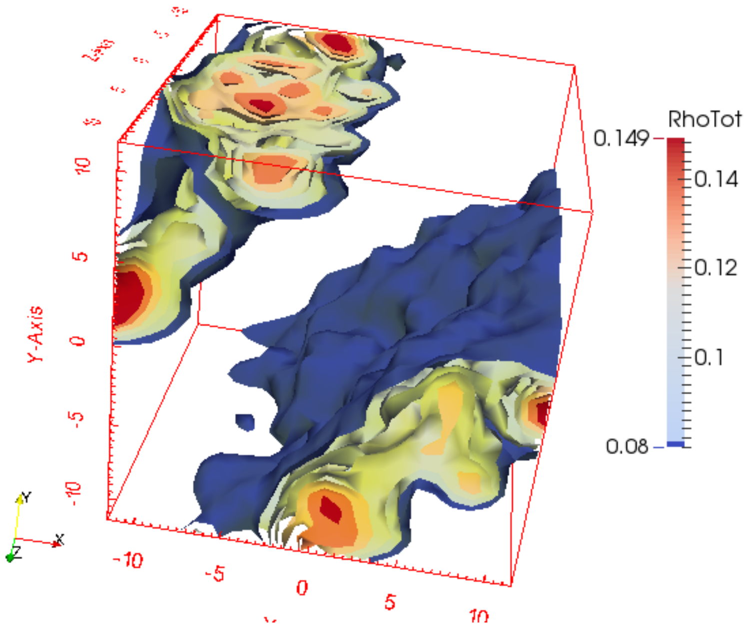


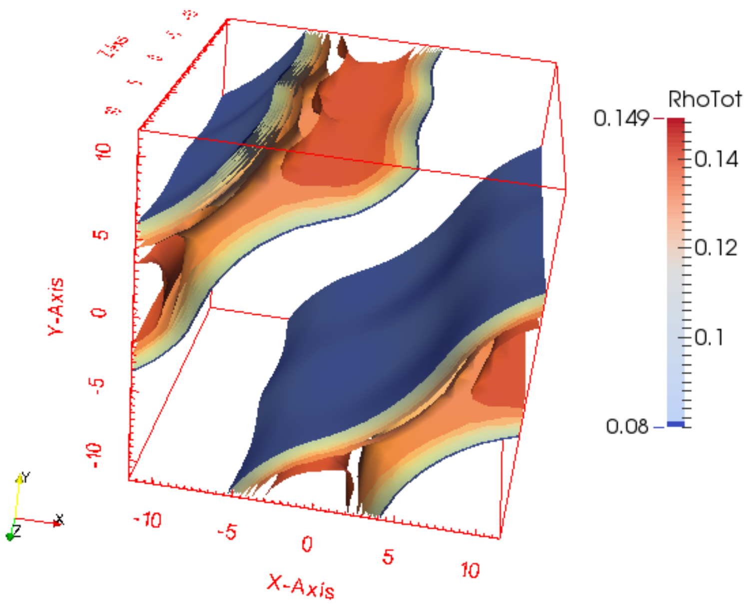
Classical and Quantum MD

- We can use the classical pasta to initiate the quantum codes
- Classical structures remain stable when evolved via Hartree-Fock

Classical Points → **Folded with Gaussian** → **Equilibrated Wavefunctions**

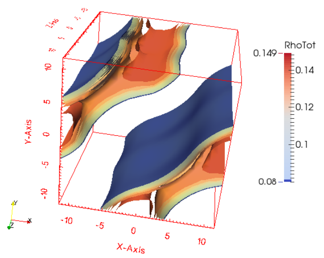






Classical and Quantum MD

800 nucleons
24 fm



Classical and Quantum MD

800 nucleons
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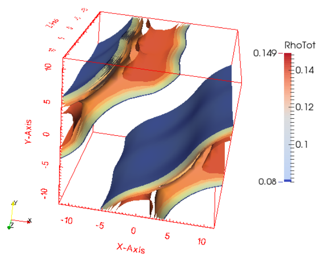
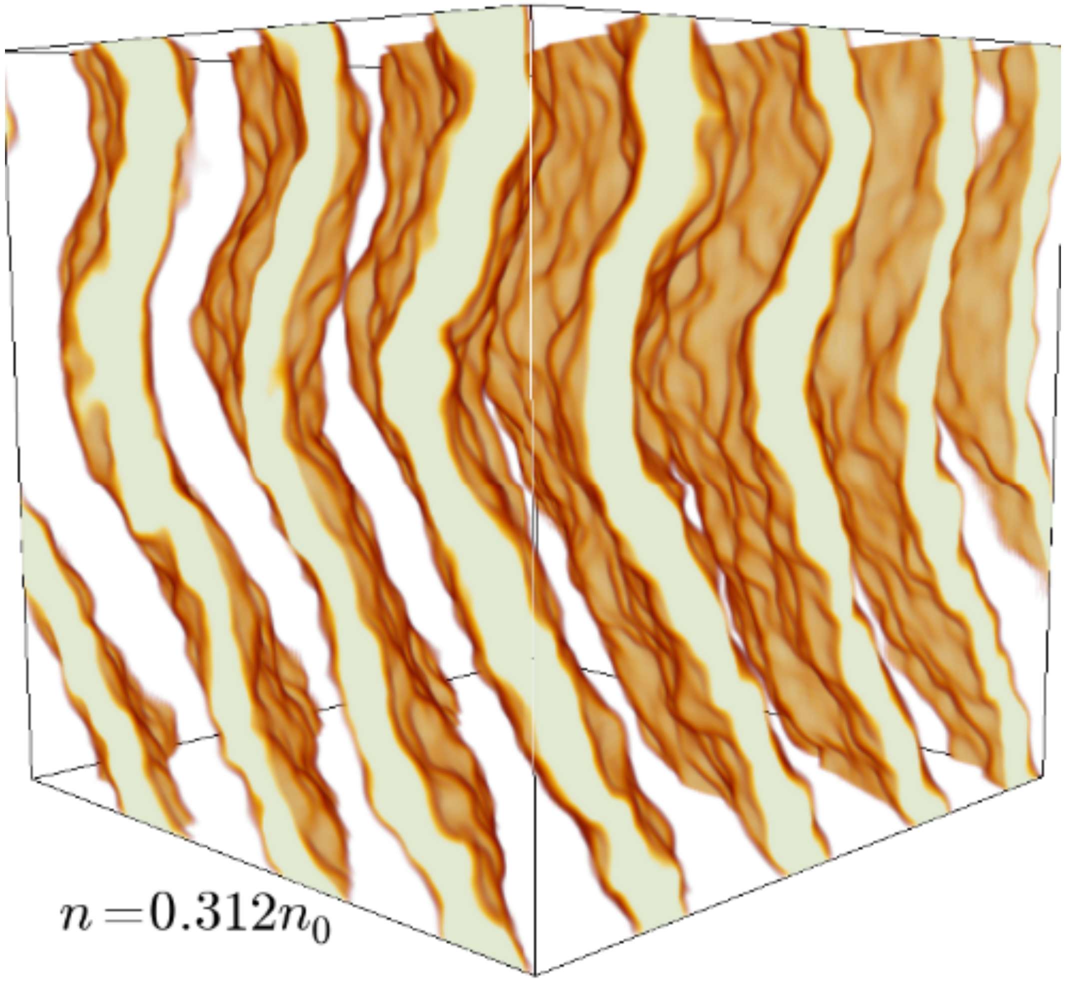


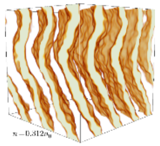
51,200 nucleons

100 fm

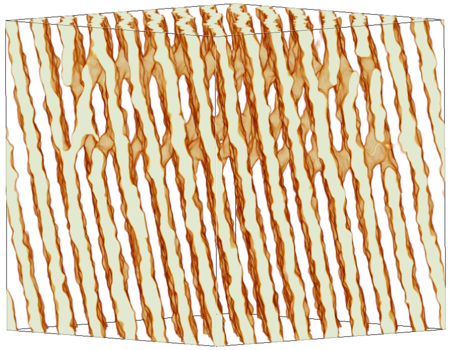


Molecular Dynamics

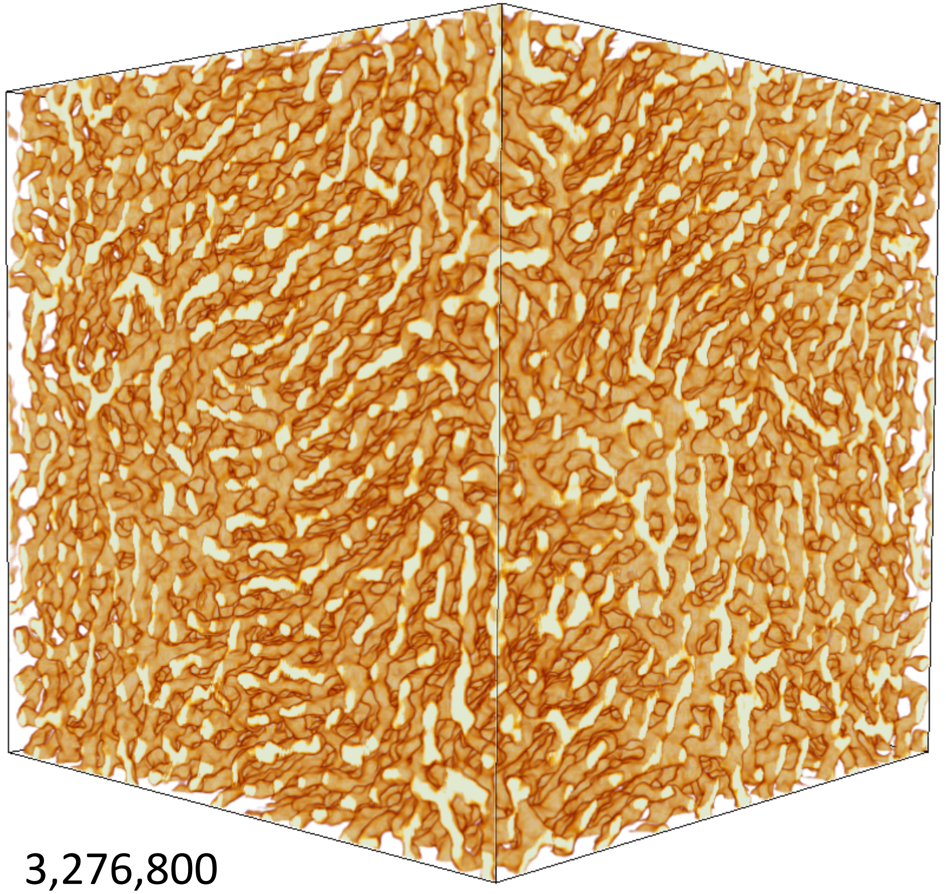
- We have evolved simulations of 409,600 nucleons, 819,200 nucleons, 1,638,400 nucleons, and 3,276,800 nucleons



51,200



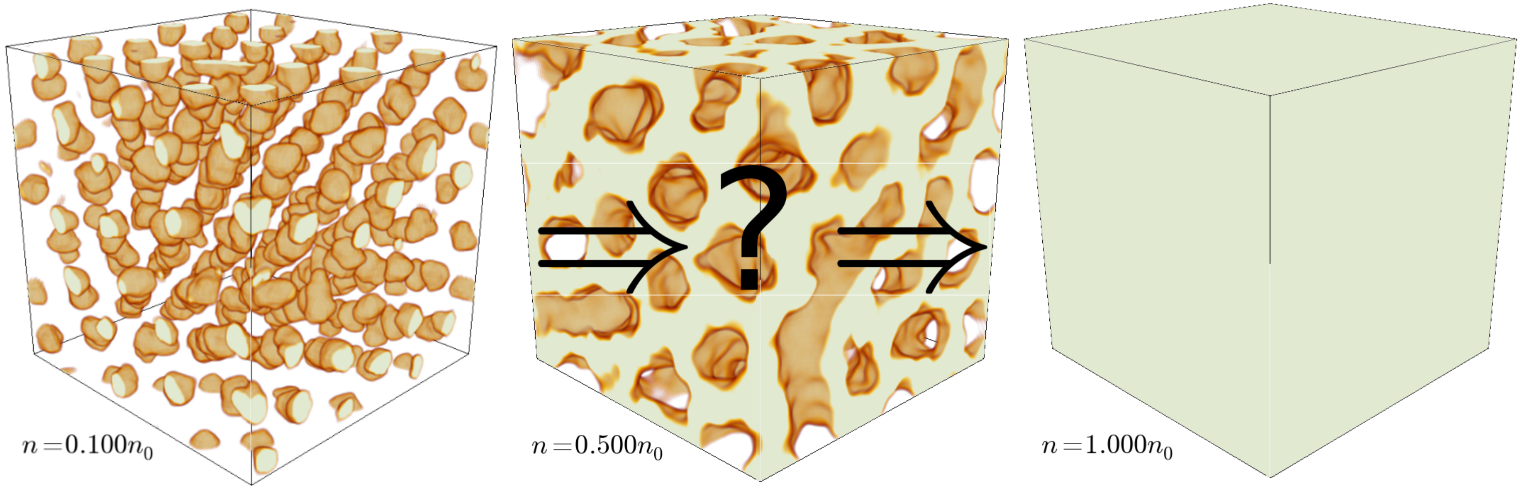
409,600



3,276,800

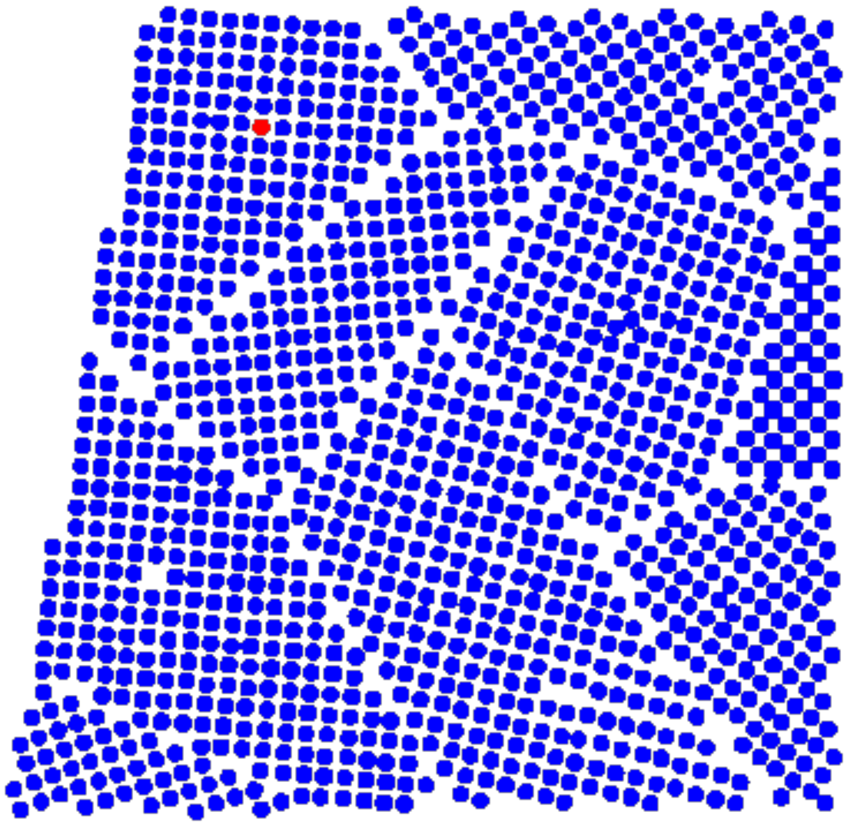
Nuclear Pasta

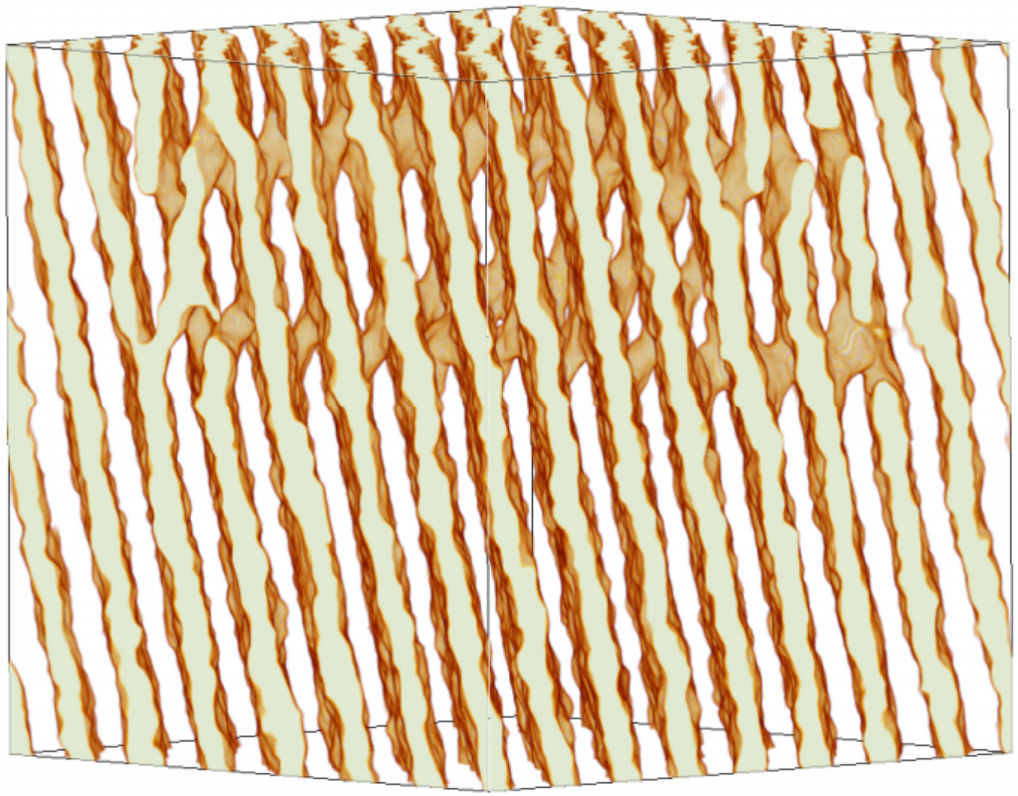
- Important to many processes:
  - Thermodynamics: Late time crust cooling
  - Magnetic field decay: Electron scattering in pasta
  - Gravitational wave amplitude: Pasta elasticity and breaking strain
  - Neutrino scattering: Neutrino wavelength comparable to pasta spacing
  - R-process: Pasta is ejected in mergers



Defects

- In the same way that crystals have defects, pasta does too!
- Electrons don't scatter from *order*, they scatter from *disorder*

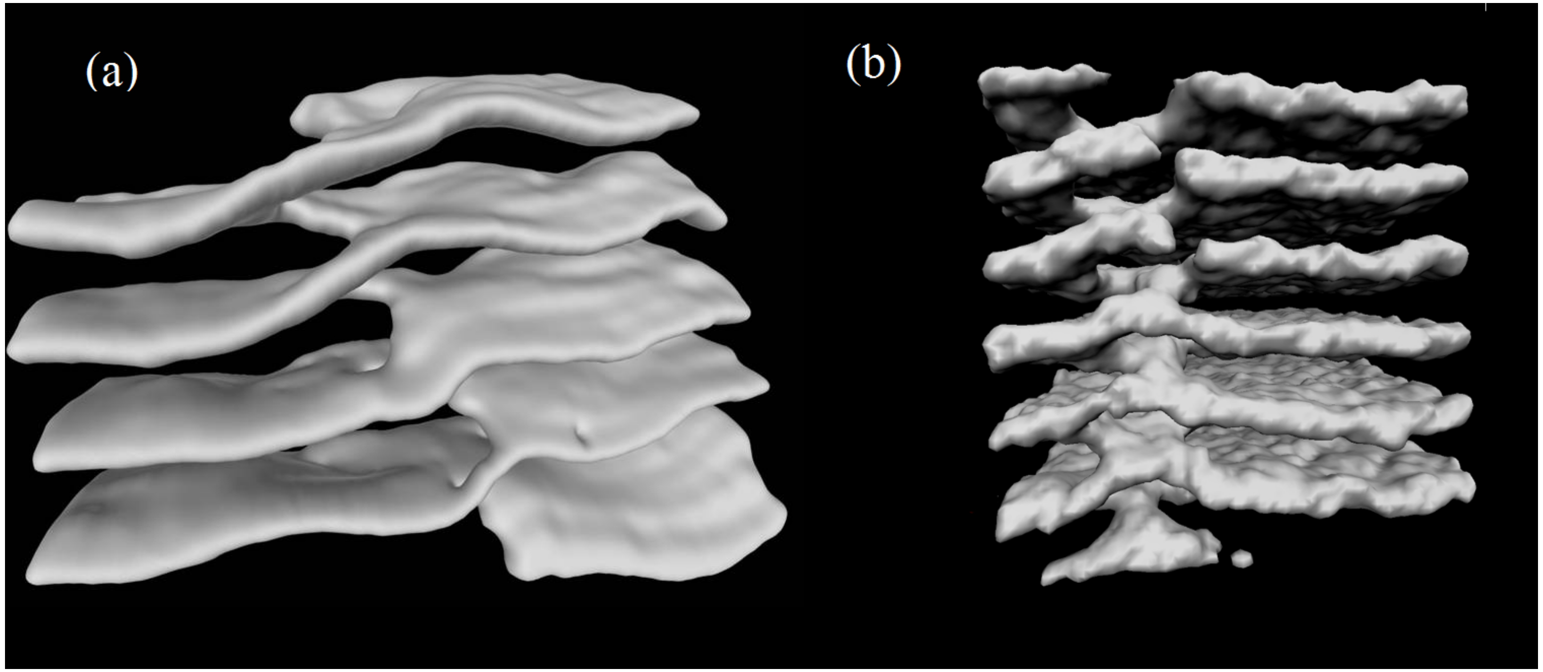




- Horowitz et al, PRL.114.031102 (2015)

Self Assembly

- Left: Electron microscopy of helicoids in mice endoplasmic reticulum



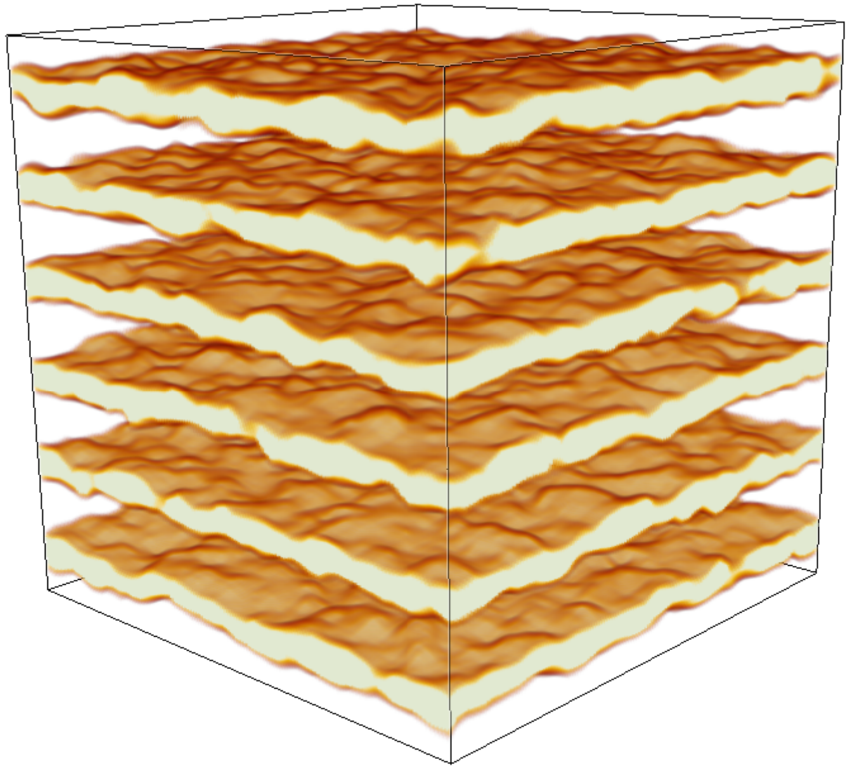
Terasaki et al, Cell 154.2 (2013)   Horowitz et al, PRL.114.031102 (2015)

- Right: Defects in nuclear pasta MD simulations

Parking Garage Structures in astrophysics and biophysics (Berry et al **Phys. Rev. C 94, 055801**) 2016

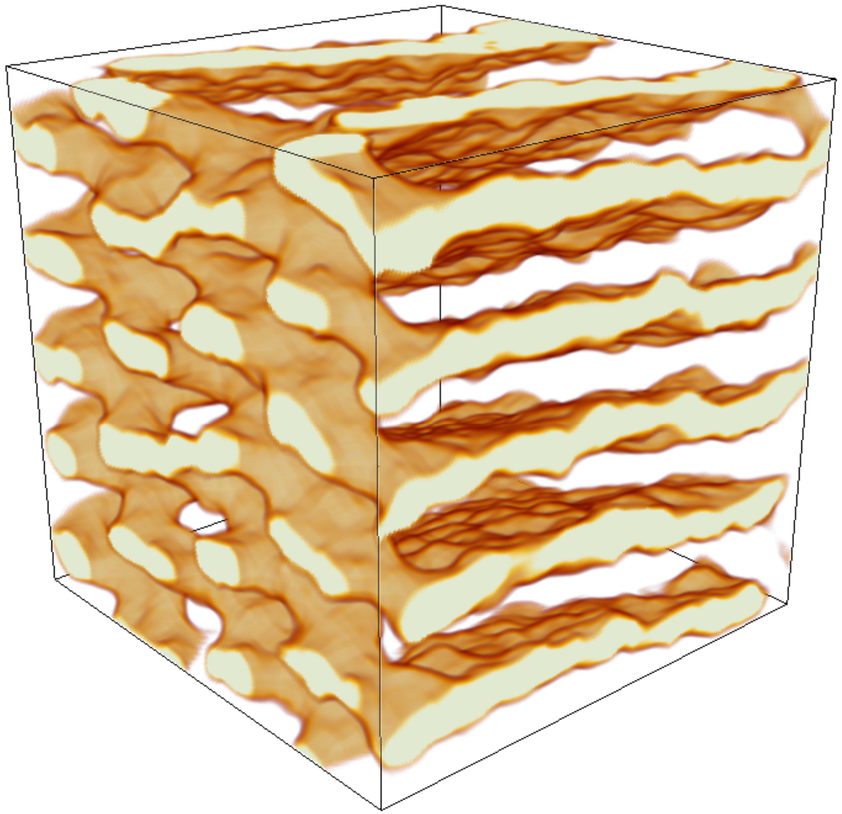
Pasta Defects

- Defects act as a site for *scattering*



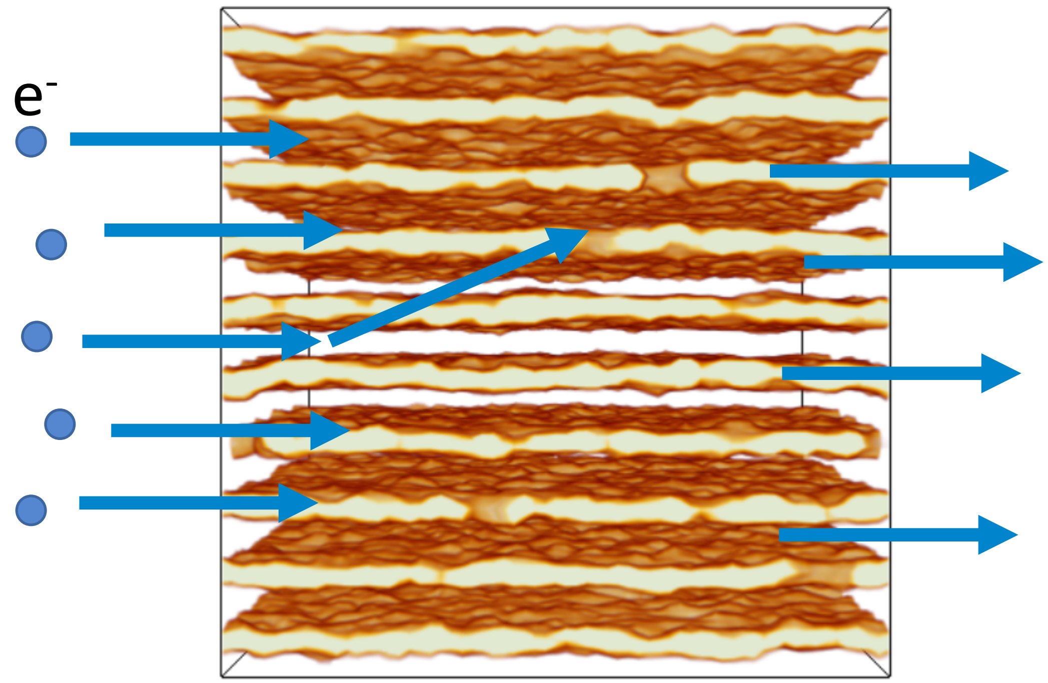
← Perfect

Defects →



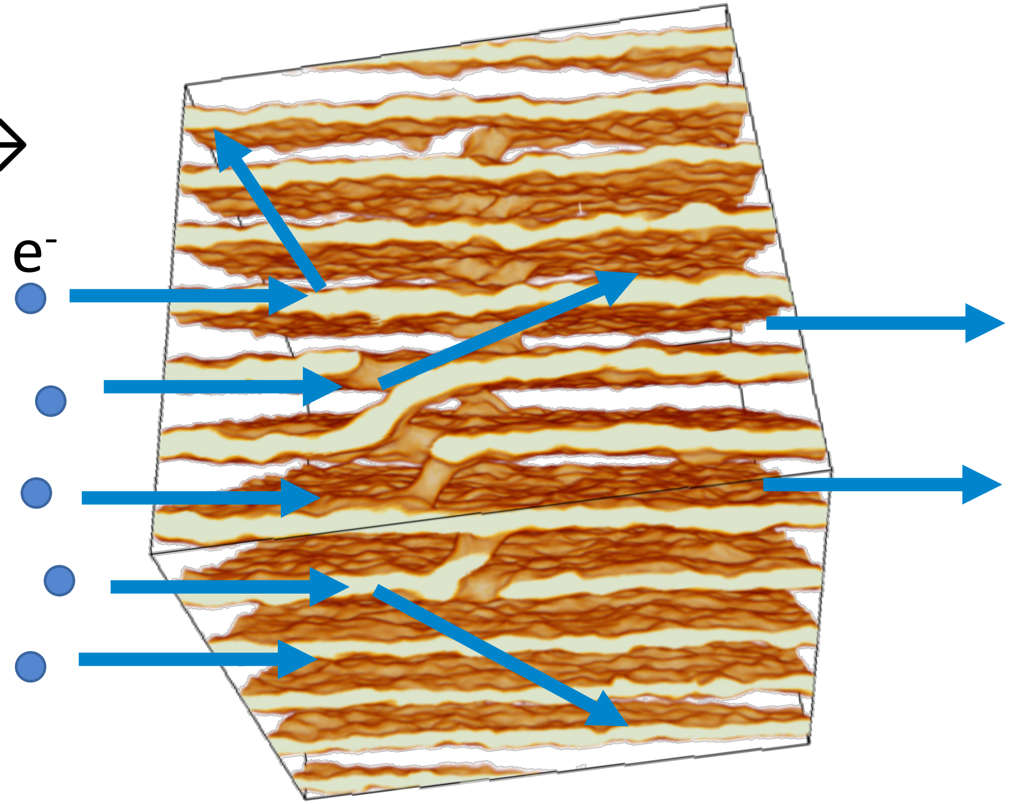
Pasta Defects

- The magnetic field decays within about 1 million years, indicating that there is an electrically resistive layer in neutrons stars (Pons et al 2013)



← Perfect

Defects →



Lepton Scattering

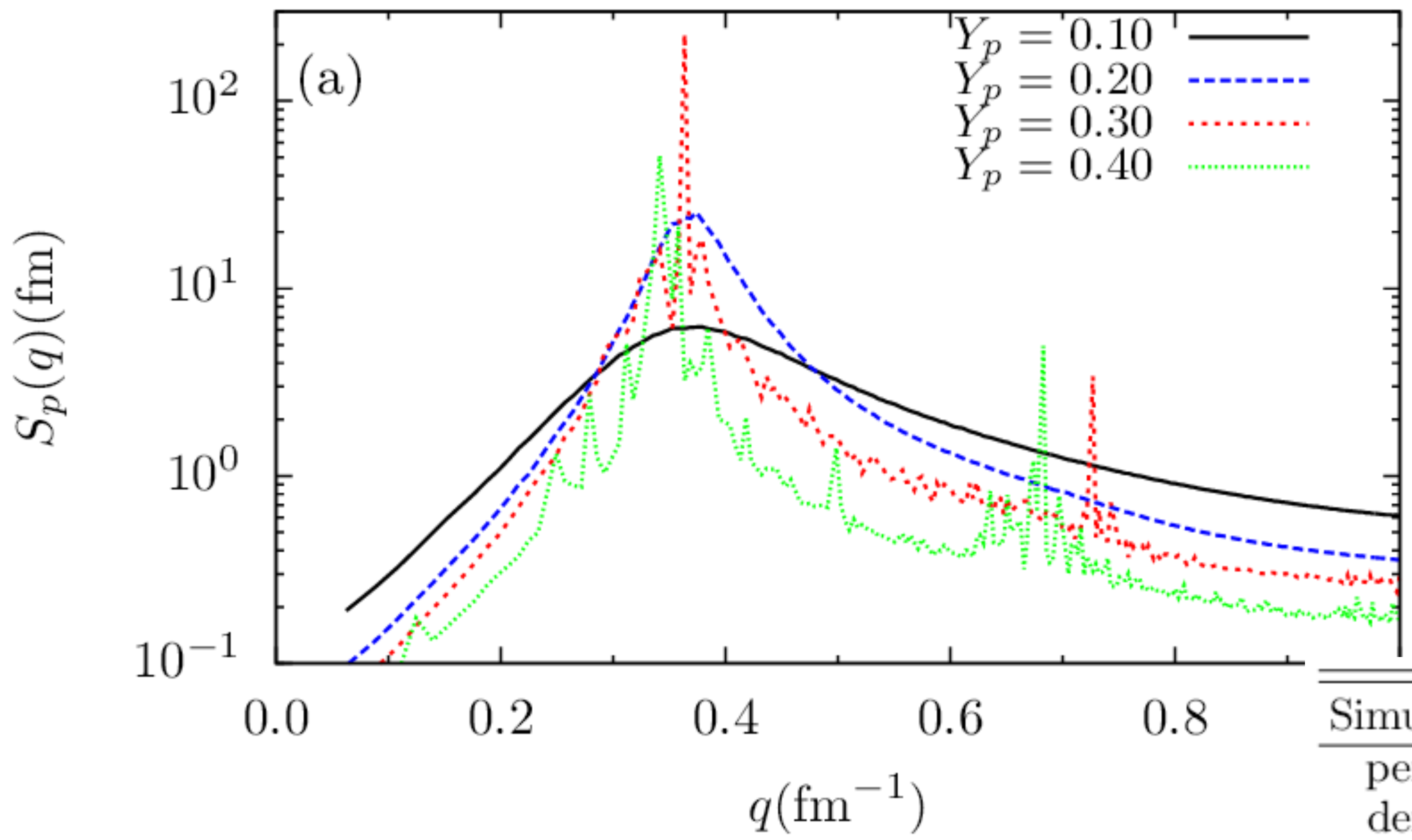
- Lepton scattering from pasta influences a variety of transport coefficients:
- Shear viscosity: $$\eta = \frac{\pi v_F^2 n_e}{20\alpha^2 \Lambda_{\rm ep}^{\eta}},$$
- Electrical conductivity: $$\sigma = \frac{v_F^2 k_F}{4\pi\alpha\Lambda_{\rm ep}^{\sigma}}$$ $$\Lambda_{\rm ep}^{\eta} = \int_0^{2k_F} \frac{dq}{q\epsilon^2(q)} \left(1 - \frac{q^2}{4k_F^2}\right)\left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q)$$
- Thermal conductivity: $$\kappa = \frac{\pi v_F^2 k_F k_B^2 T}{12\alpha^2 \Lambda_{\rm ep}^{\kappa}}.$$ $$\Lambda_{\rm ep}^{\kappa} = \Lambda_{\rm ep}^{\sigma} = \int_0^{2k_F} \frac{dq}{q\epsilon^2(q)} \left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q).$$

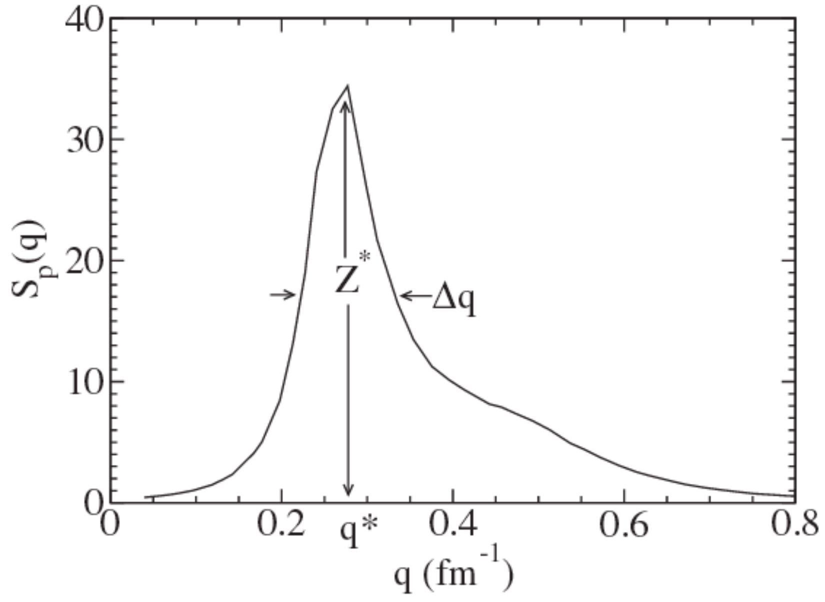
Lepton Scattering

$$S_i(\boldsymbol{q}) = \langle \rho_i^*(\boldsymbol{q},t)\rho_i(\boldsymbol{q},t)\rangle_t - \langle \rho_i^*(\boldsymbol{q},t)\rangle_t \langle \rho_i(\boldsymbol{q},t)\rangle_t$$

$$\rho_i(\boldsymbol{q},t) = N_i^{-1/2} \sum_{j=1}^{N_i} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_j(t)}$$

$$\Lambda_{ep} \approx \frac{\Delta q^* Z^*}{q^*}$$

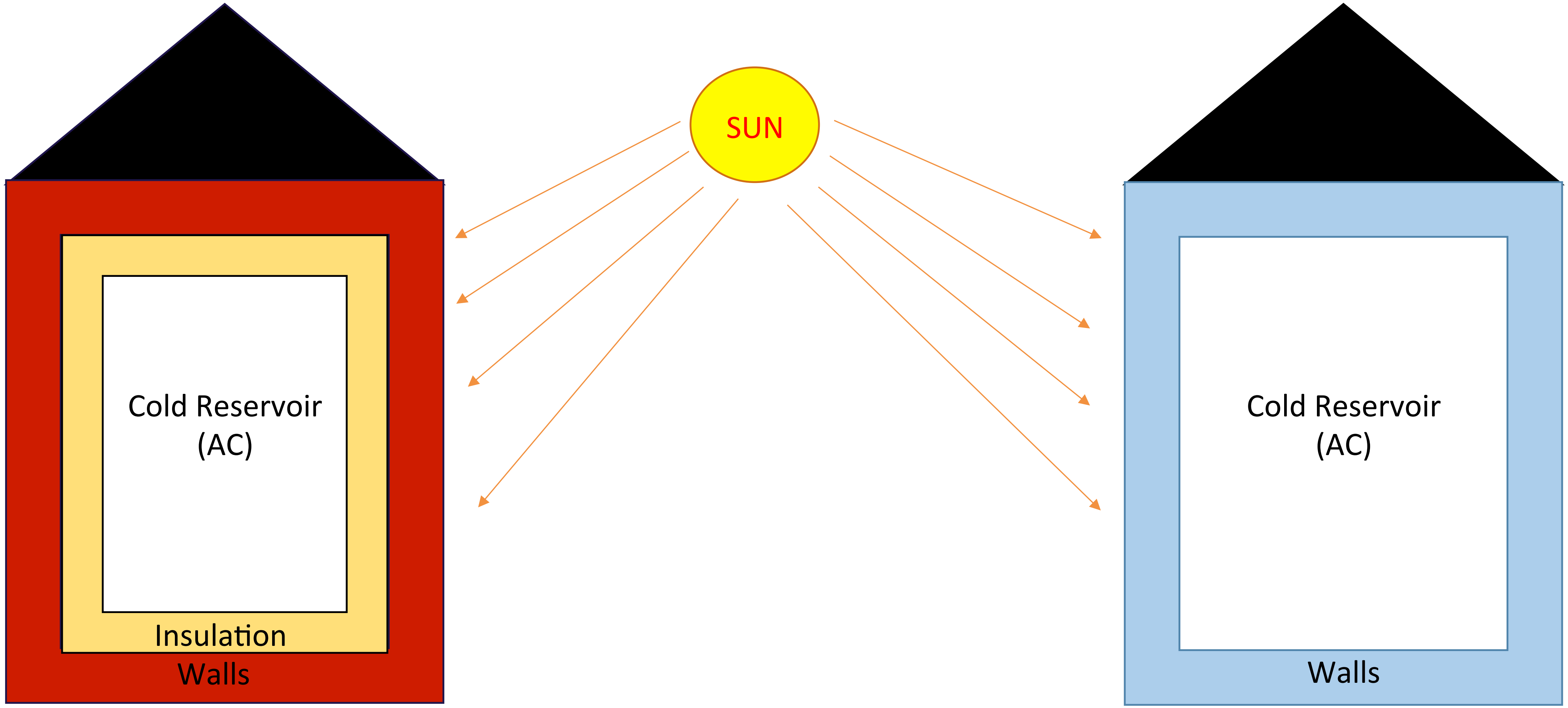




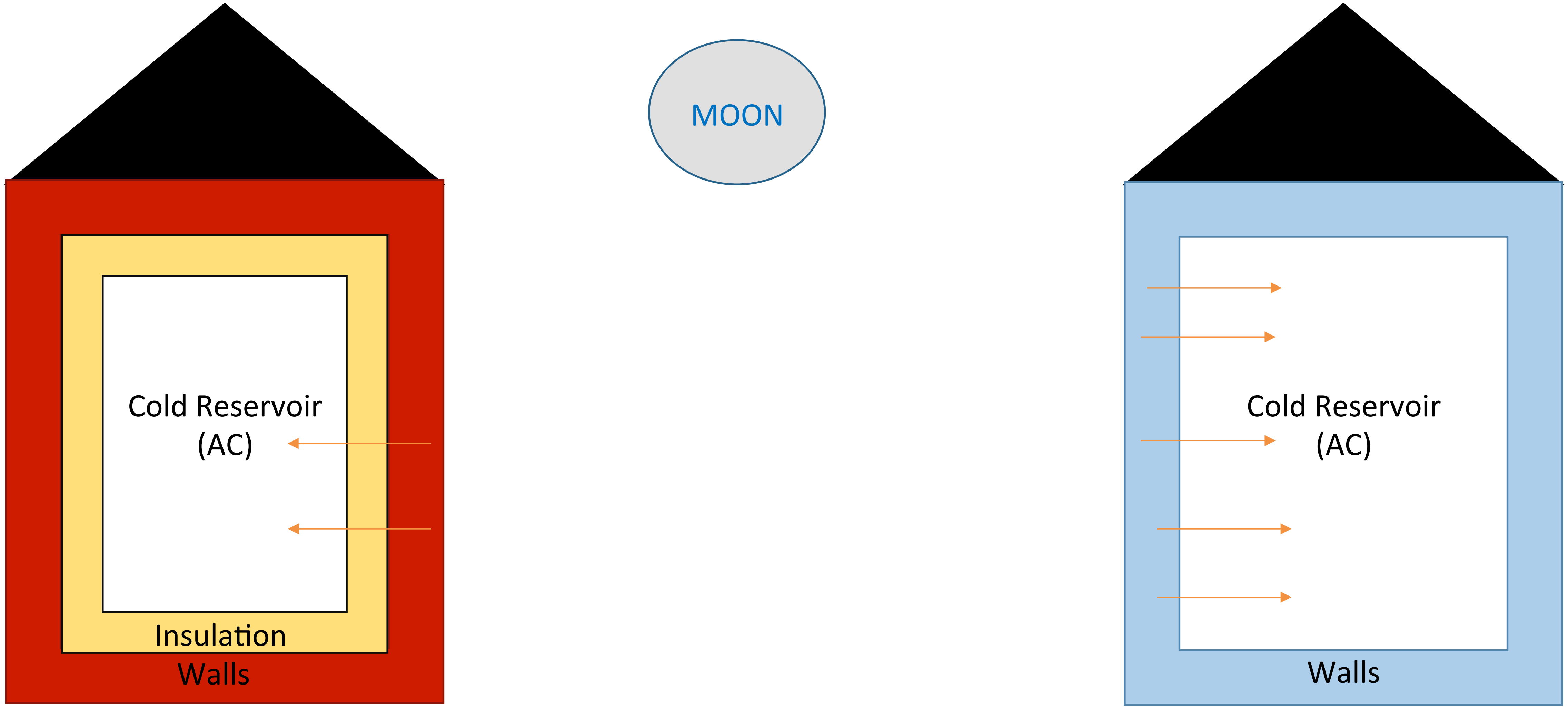
Simulation	$\bar{\eta}(\mathrm{fm}^{-3})$	$\bar{\kappa}(10^3 k_B\ \mathrm{MeV/fm})$	$\bar{Z}^*$
perfect	87.7	6.66	5.5
defects	55.5	4.15	50.2

Crust Cooling

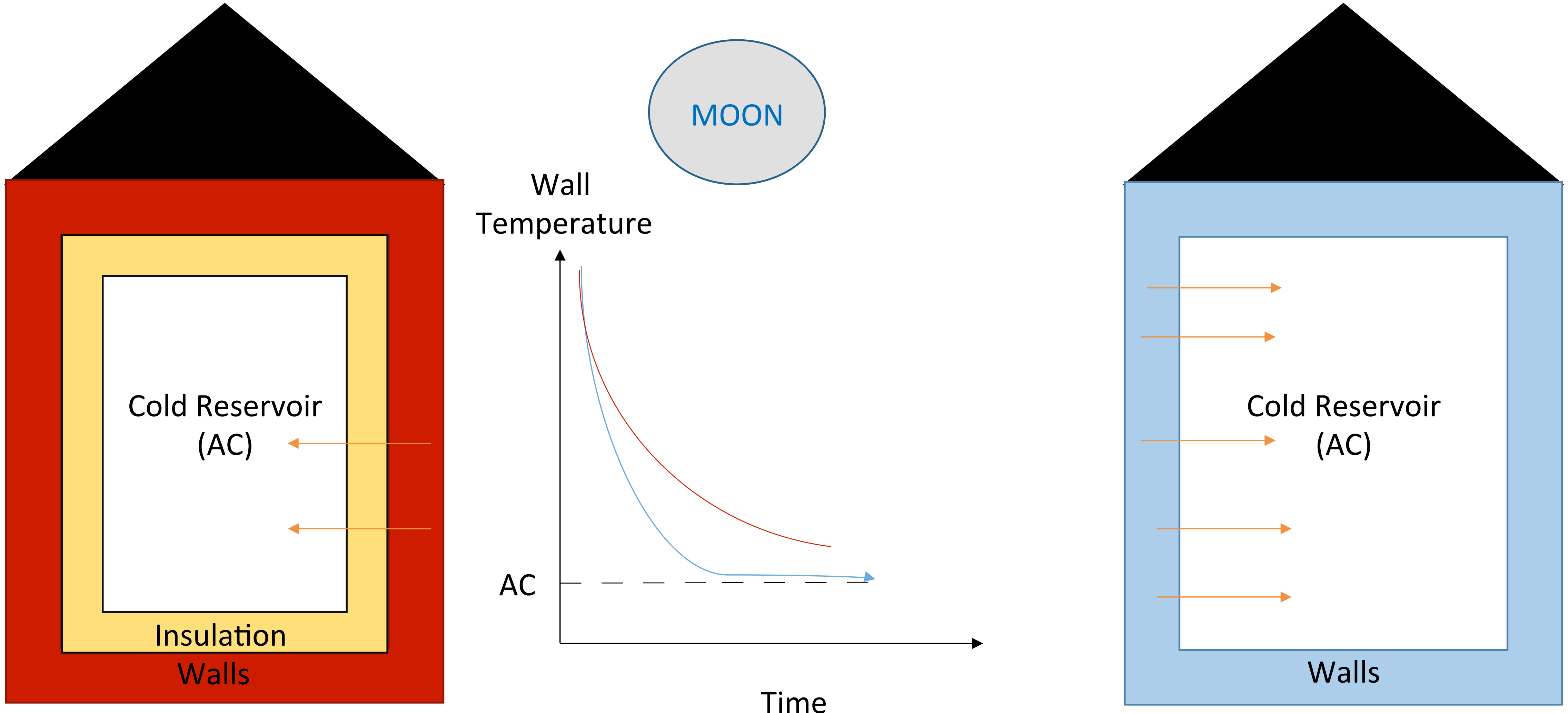
An Example


Cold Reservoir
(AC)
Insulation
Walls
SUN
Cold Reservoir
(AC)
Walls

An Example

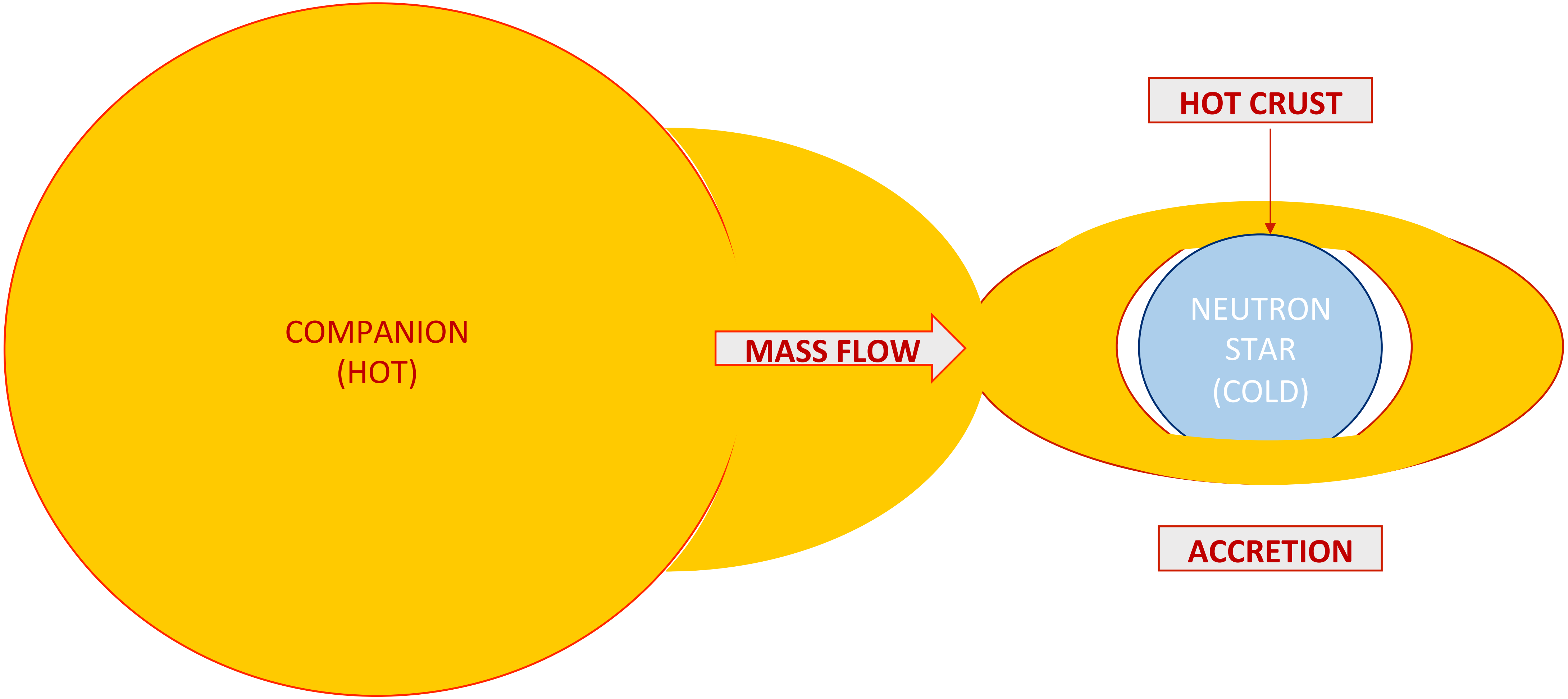

MOON
Cold Reservoir
(AC)
Insulation
Walls
Cold Reservoir
(AC)
Walls

An Example

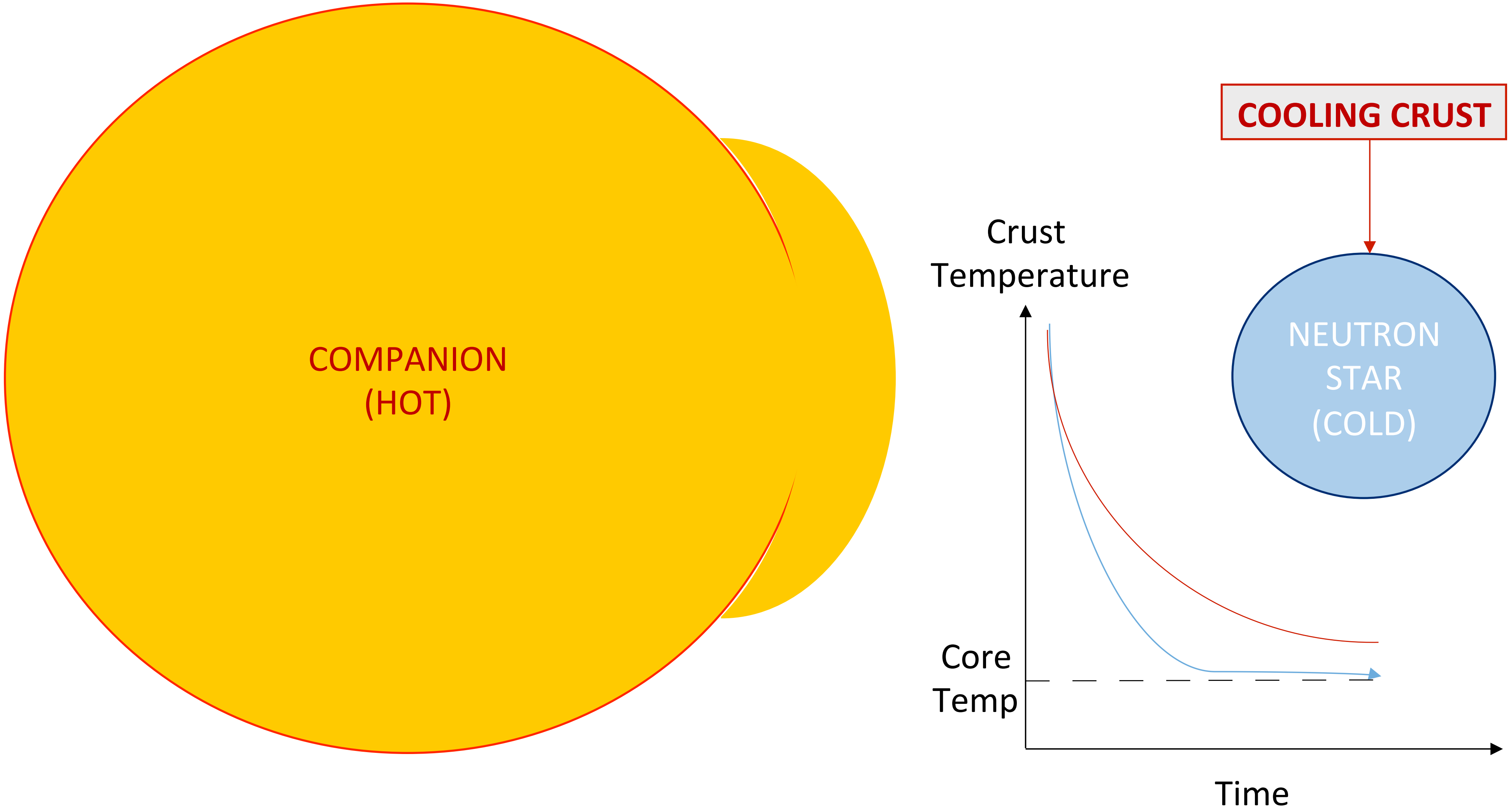


Cold Reservoir
(AC)
Insulation
Walls
MOON
Wall
Temperature
AC
Time
Cold Reservoir
(AC)
Walls

Low Mass X-Ray Binaries



Low Mass X-Ray Binaries

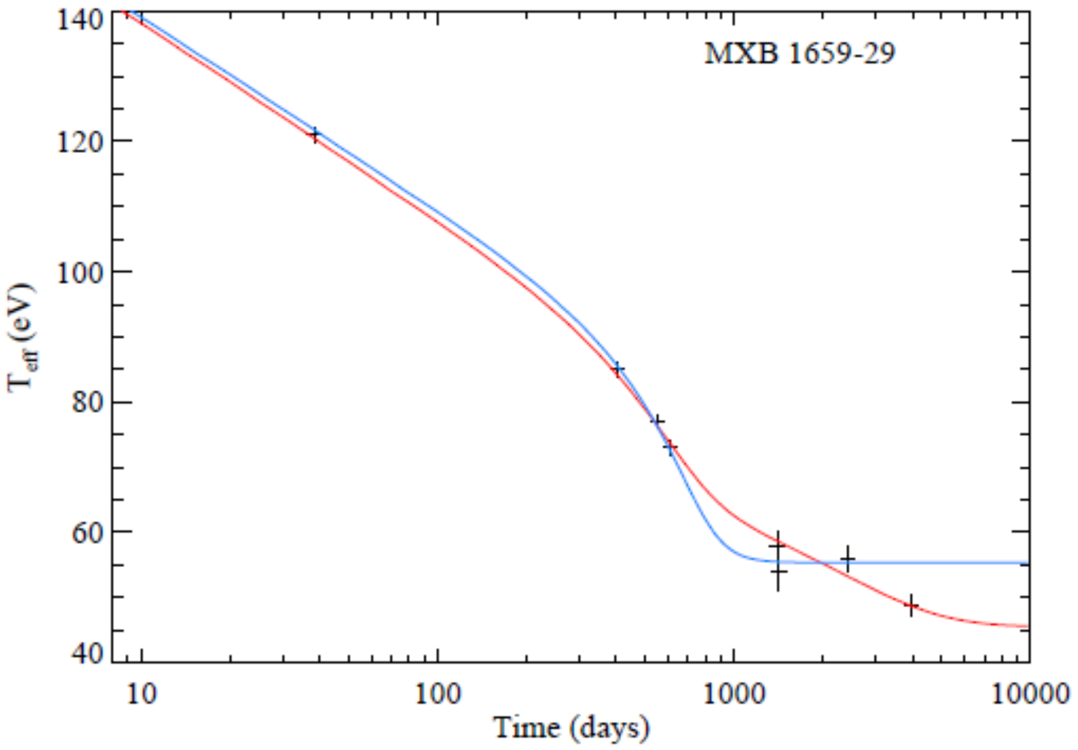


Observables – Thermal Properties

- Guess an effective impurity parameter for defects and try to fit neutron star cooling curves

$$Q_{\rm imp} \equiv n_{\rm ion}^{-1} \sum_i n_i (Z_i - \langle Z \rangle)^2$$

- Cooling curves: low mass X-ray binary MXB 1659-29
  - Blue: normal isotropic matter
    $Q_{imp} = 3.5$
    $T_c = 3.05 \times 10^7$ K
  - Red: impure pasta layer
    $Q_{imp} = 1.5$ (outer crust)
    $Q_{imp} = 30$ (inner crust)
    $T_c = 2 \times 10^7$ K



Summary

To interpret observations of neutron stars, we much first develop microscopic models of their interiors. By simulating the kinds of matter we expect to find in the crust we can calculate properties of the star, and potentially constrain fundamental physics.

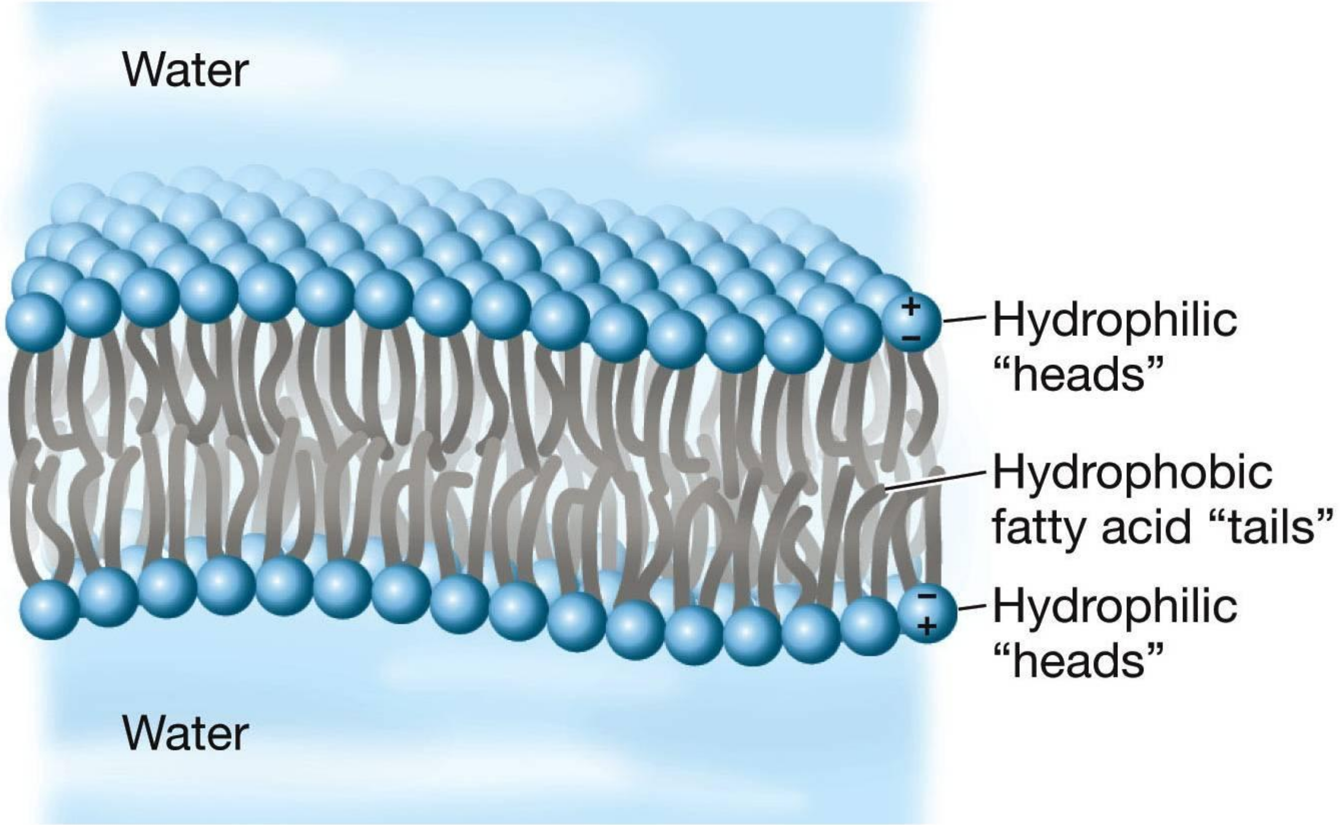
Backup Slides!

Self Assembly

Self Assembly

- Well studied in phospholipids: hydrophilic heads and hydrophobic tails self assemble in an aqueous solution

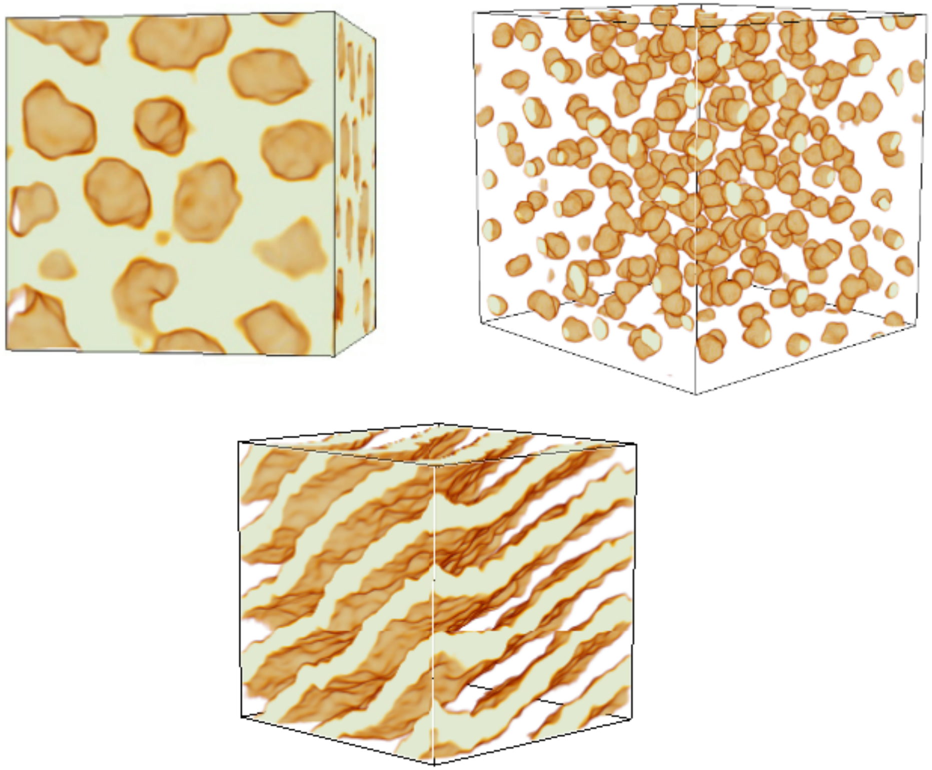
(B) Phospholipid bilayer

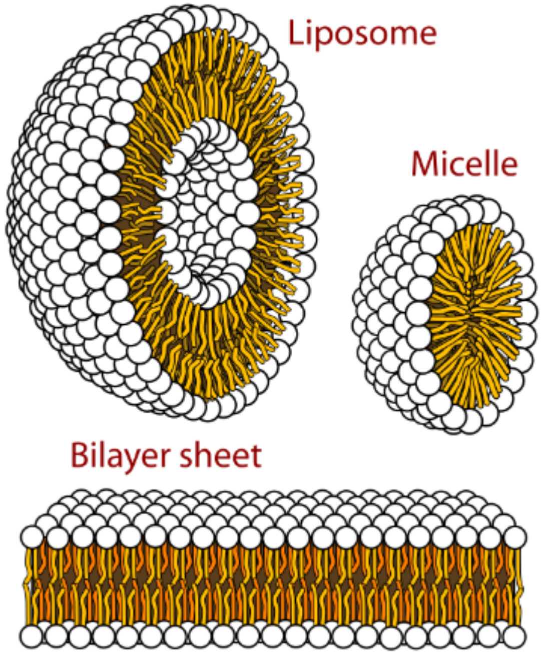


***LIFE* 8e, Figure 3.20 (Part 2)**

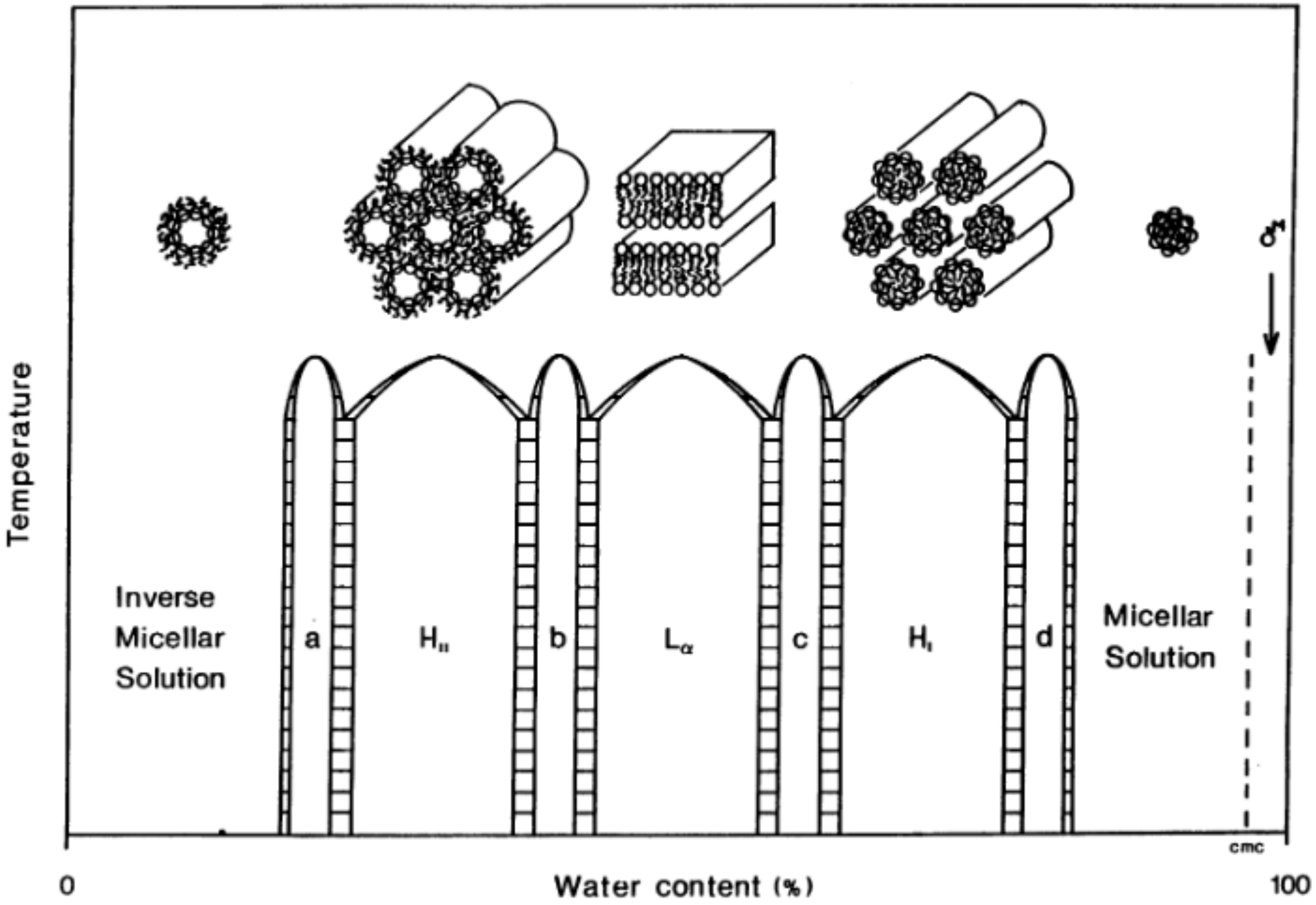
Self Assembly

- Well studied in phospholipids: hydrophilic heads and hydrophobic tails self assemble in an aqueous solution





Self Assembly



Seddon, BBA 1031, 1–69 (1990)

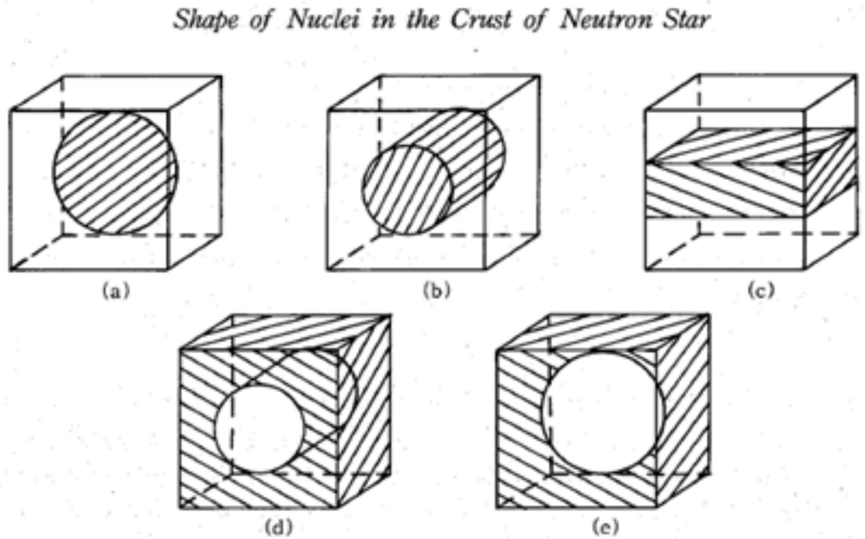
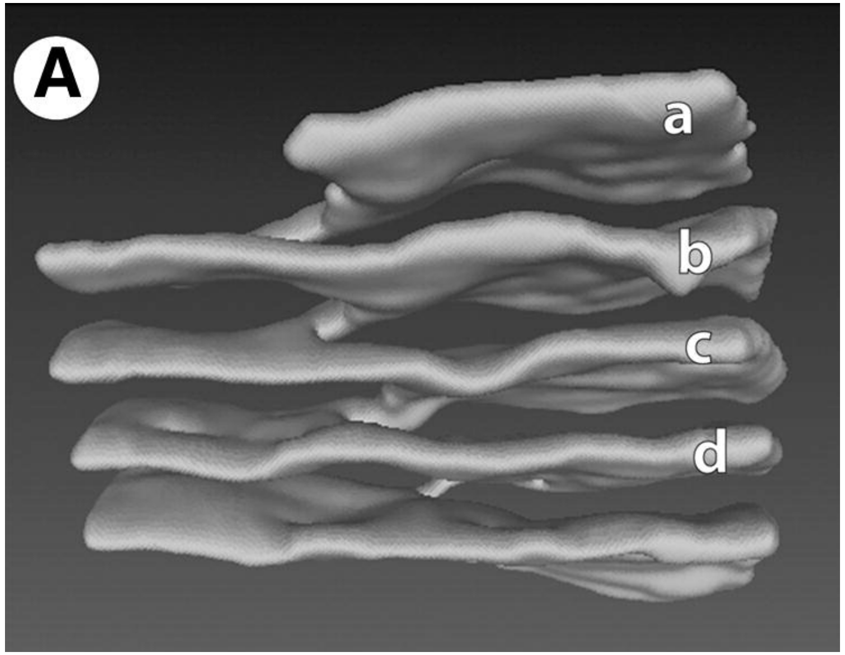


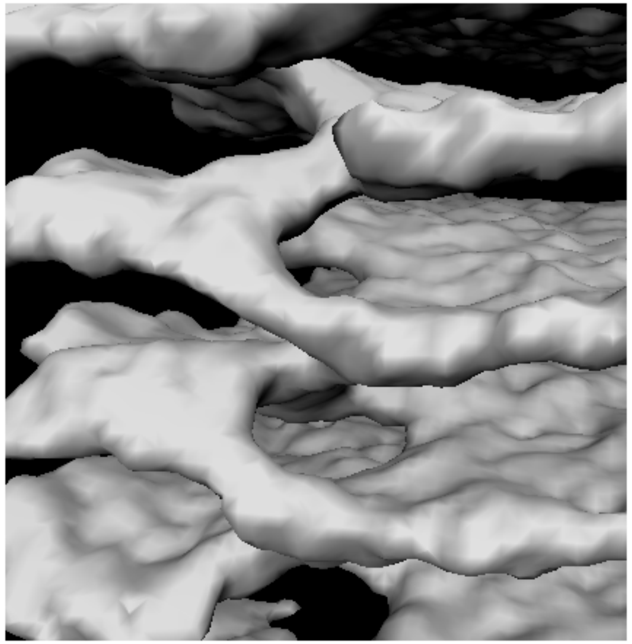
Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are, (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).

Self Assembly

- Left: Electron microscopy of helicoids in mice endoplasmic reticulum



Terasaki et al, Cell 154.2 (2013)



Horowitz et al, PRL.114.031102 (2015)

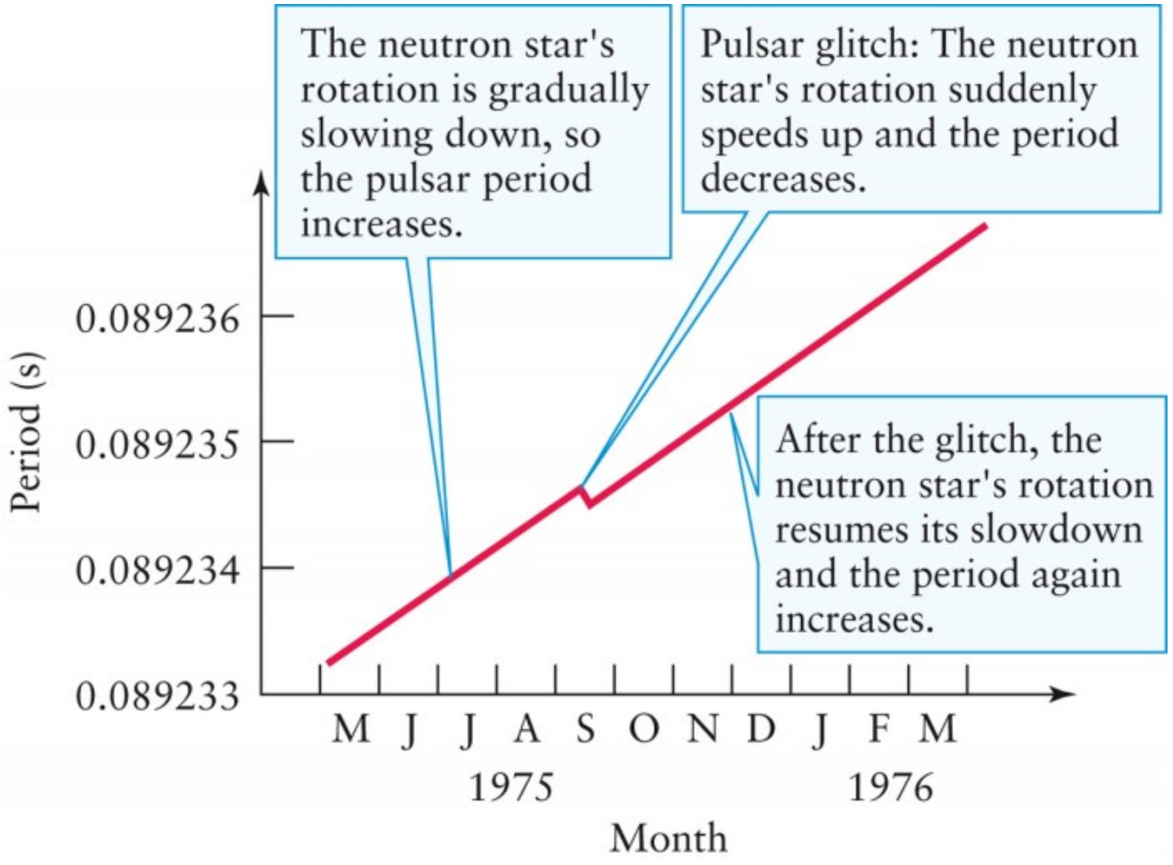
- Right: Defects in nuclear pasta MD simulations

Parking Garage Structures in astrophysics and biophysics (arXiv:1509.00410)

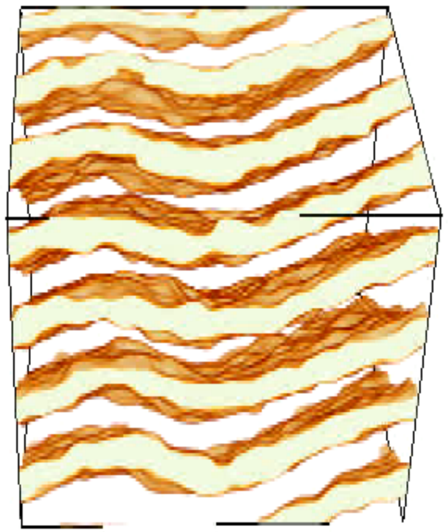
Crust Breaking

Pulsar Glitches (Astroseismology)

- Pulsars slowly *spin down,* meaning their period gets longer
- Occasionally, they 'glitch' and start to spin faster
- Is this crust breaking? Is this a *starquake*?
- The breaking strain of the crust determines the frequency and 'intensity' of glitches



Linear Elasticity



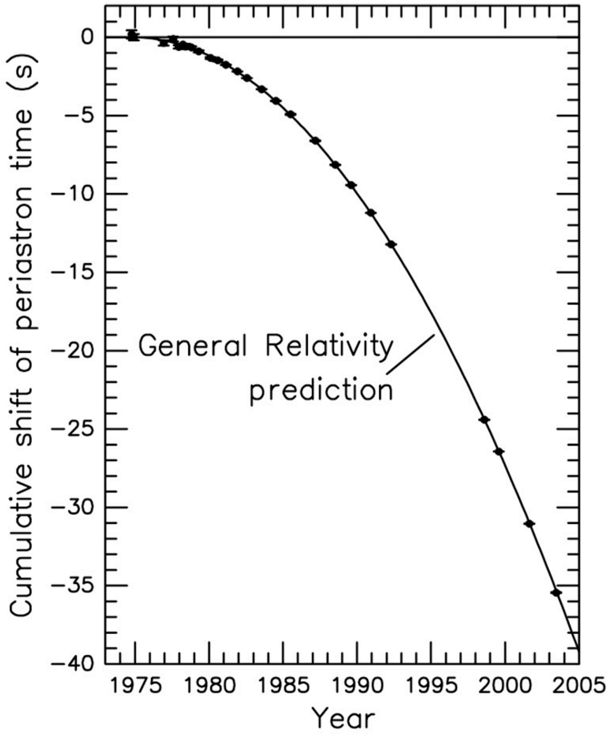
$l_z = 100.80\ \text{fm}$

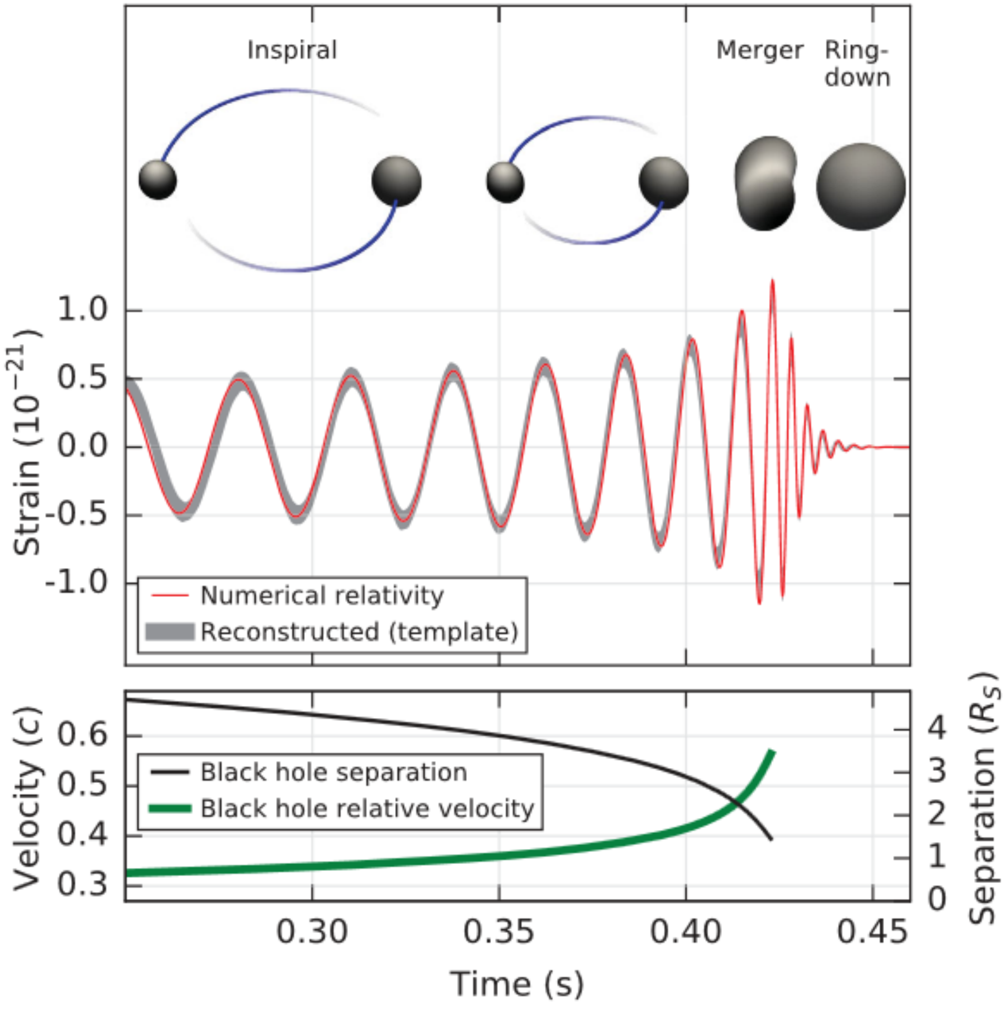
$l_x = l_y = 100.80\ \text{fm}$

Gravitational Waves

Gravitational Waves

- LIGO has confirmed that direct detection is viable!
- First detected via a binary pulsar:





Neutron star mergers

Event Rate

TABLE II: Compact binary coalescence rates per Milky Way Equivalent Galaxy per Myr.

Source	R_{low}	R_{re}	R_{high}	R_{max}
NS-NS (MWEG^{-1} Myr^{-1})	1 [1][a]	100 [1][b]	1000 [1][c]	4000 [16][d]
NS-BH (MWEG^{-1} Myr^{-1})	0.05 [18][e]	3 [18][f]	100 [18][g]	
BH-BH (MWEG^{-1} Myr^{-1})	0.01 [14][h]	0.4 [14][i]	30 [14][j]	
IMRI into IMBH (GC^{-1} Gyr^{-1})			3 [19][k]	20 [19][l]
IMBH-IMBH (GC^{-1} Gyr^{-1})			0.007 [20][m]	0.07 [20][n]

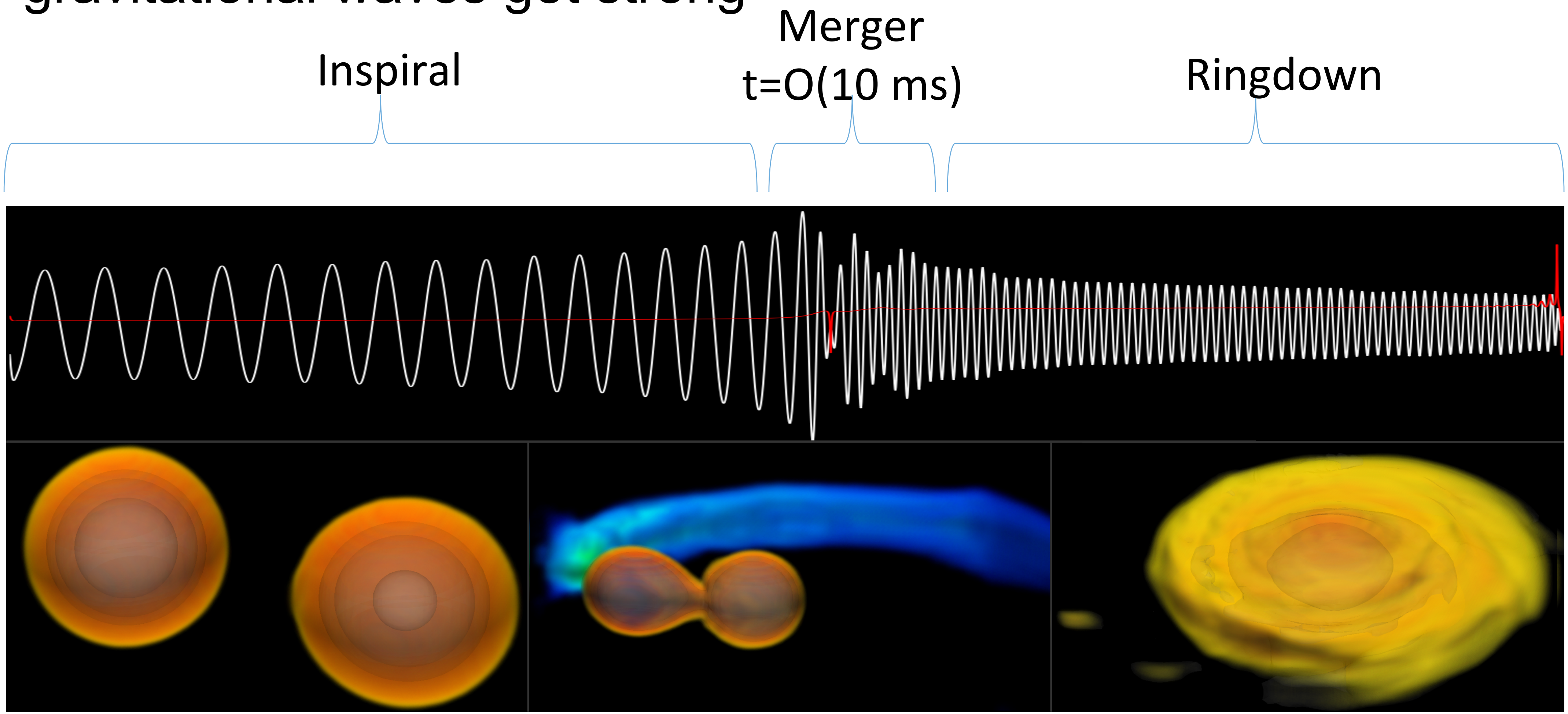
TABLE V: Detection rates for compact binary coalescence sources.

IFO	Source[a]	$\dot{N}_{\mathrm{low}}$ yr^{-1}	$\dot{N}_{\mathrm{re}}$ yr^{-1}	$\dot{N}_{\mathrm{high}}$ yr^{-1}	$\dot{N}_{\mathrm{max}}$ yr^{-1}
	NS-NS	2×10^{-4}	0.02	0.2	0.6
	NS-BH	7×10^{-5}	0.004	0.1	
Initial	BH-BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			< 0.001[b]	0.01[c]
	IMBH-IMBH			10^{-4}[d]	10^{-3}[e]
	NS-NS	0.4	40	400	1000
	NS-BH	0.2	10	300	
Advanced	BH-BH	0.4	20	1000	
	IMRI into IMBH			10[b]	300[c]
	IMBH-IMBH			0.1[d]	1[e]

(Abadie, 2010)

Neutron star mergers

- When the binary separation is similar to the neutron star radius, gravitational waves get strong

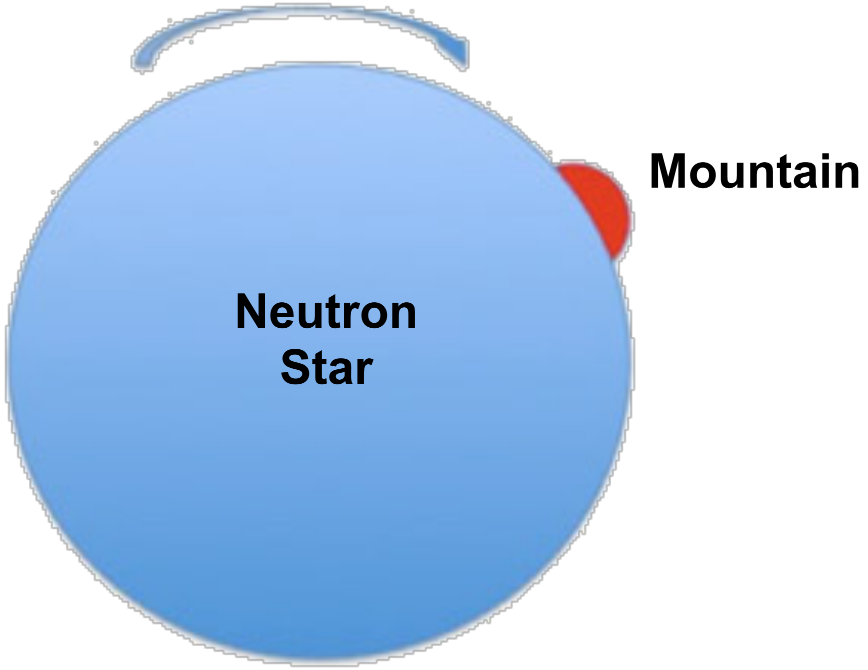


Mountains

- What if the surface is lumpy? Are there mountains?
- Dense, fast lump produces ripples in spacetime

- How big can they be? A few centimeters?
- How long do they last?

- The pasta is the densest stuff, therefore, it's the stiffest. Could pasta support mountains?

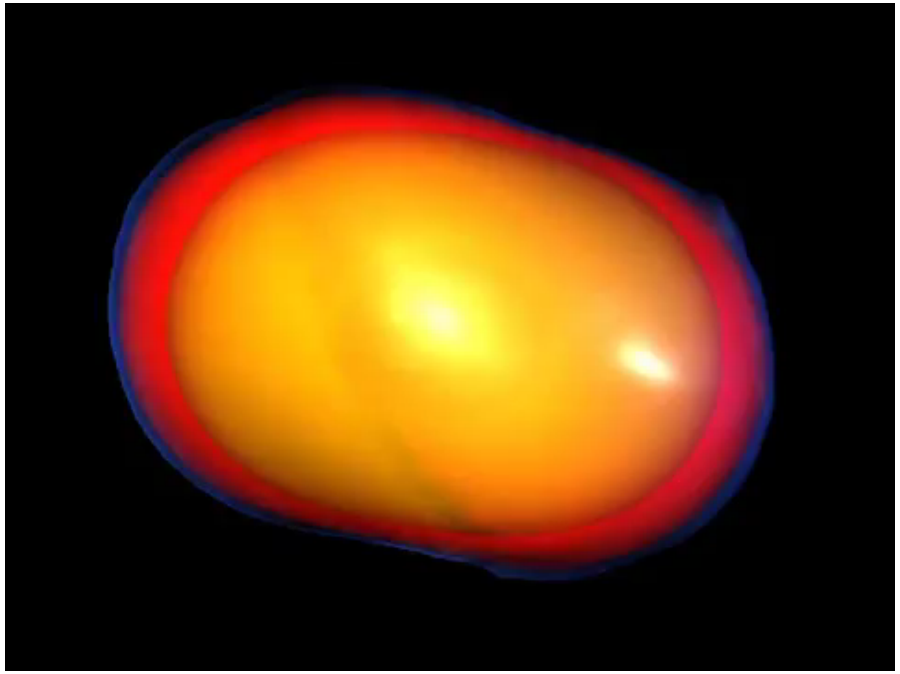


R-mode instability

- *Rotational*-mode – toroidal oscillation of neutron star that is unstably driven by gravitational wave emission

$$\vec{u} = (w_l \hat{r} \times \nabla Y_{ll} + v_{l+1} \nabla Y_{l+1\ l} + u_{l+1} Y_{l+1\ l} \hat{r})\, e^{i\omega t}$$

- Primarily the *l*=*m*=2 mode
- Solution: Is the damping from the crust enough to stabilize the star?



Youtube: LSU_Astrophysics

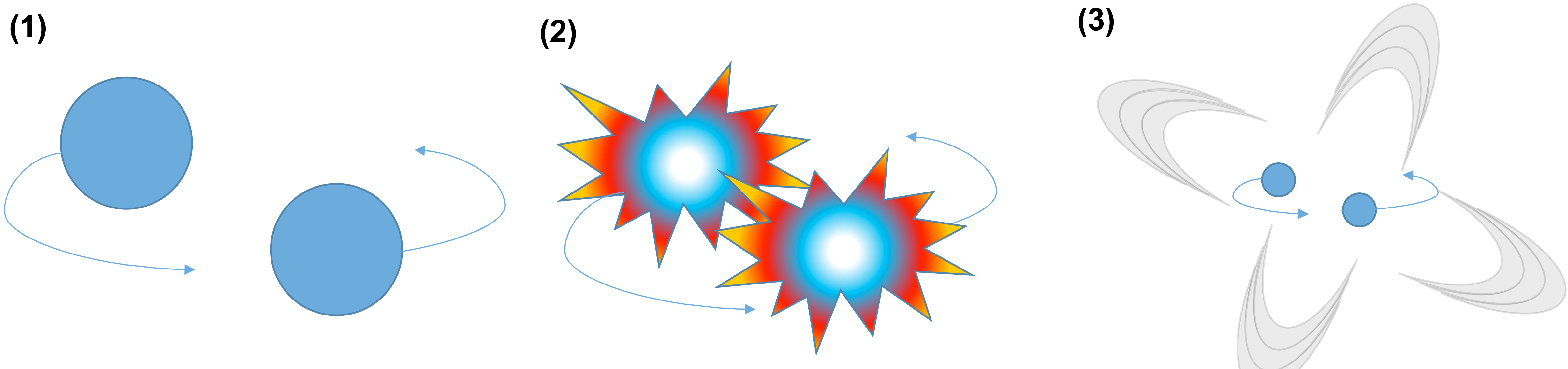
Nucleosynthesis

Recipe: Neutron star mergers

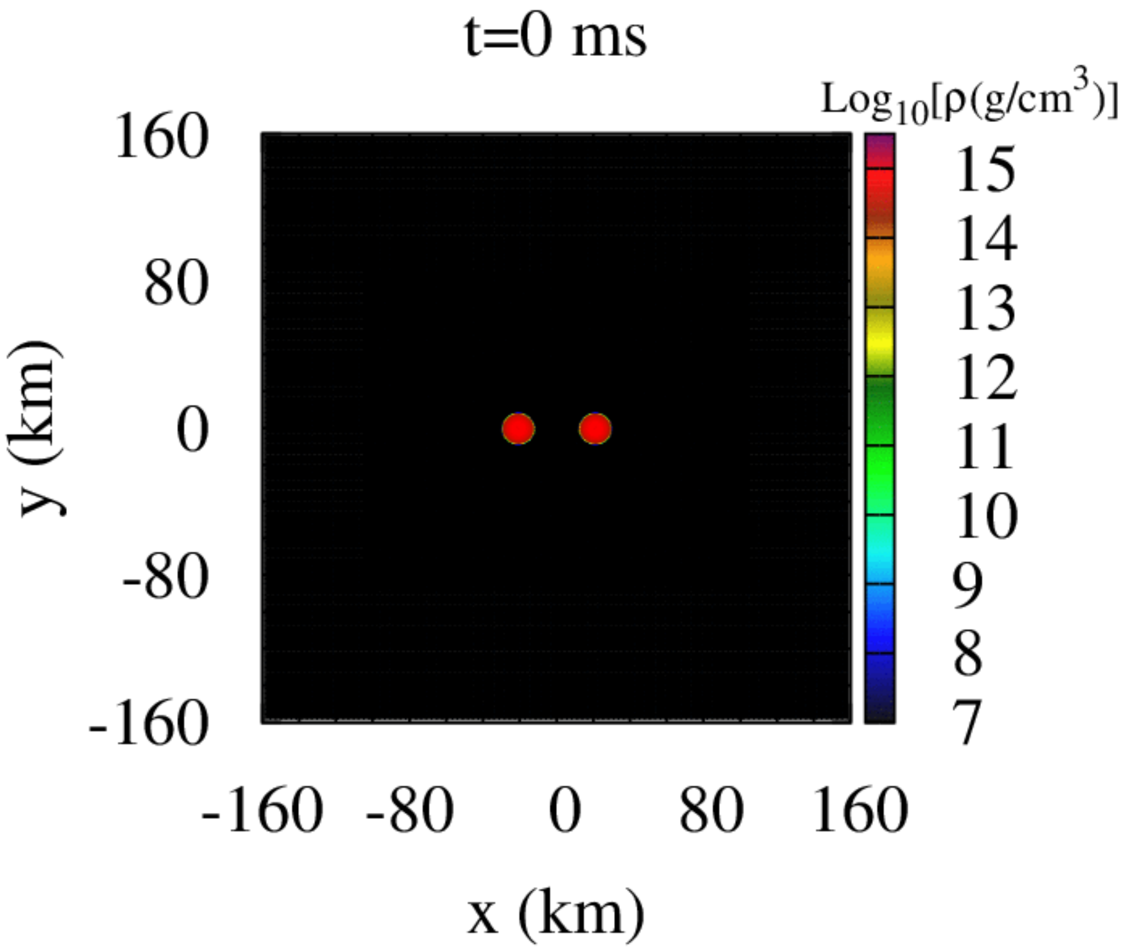
(1) Start with a binary of massive stars

(2) Make them supernova

(3) Merge the neutron stars' by radiating gravitational waves



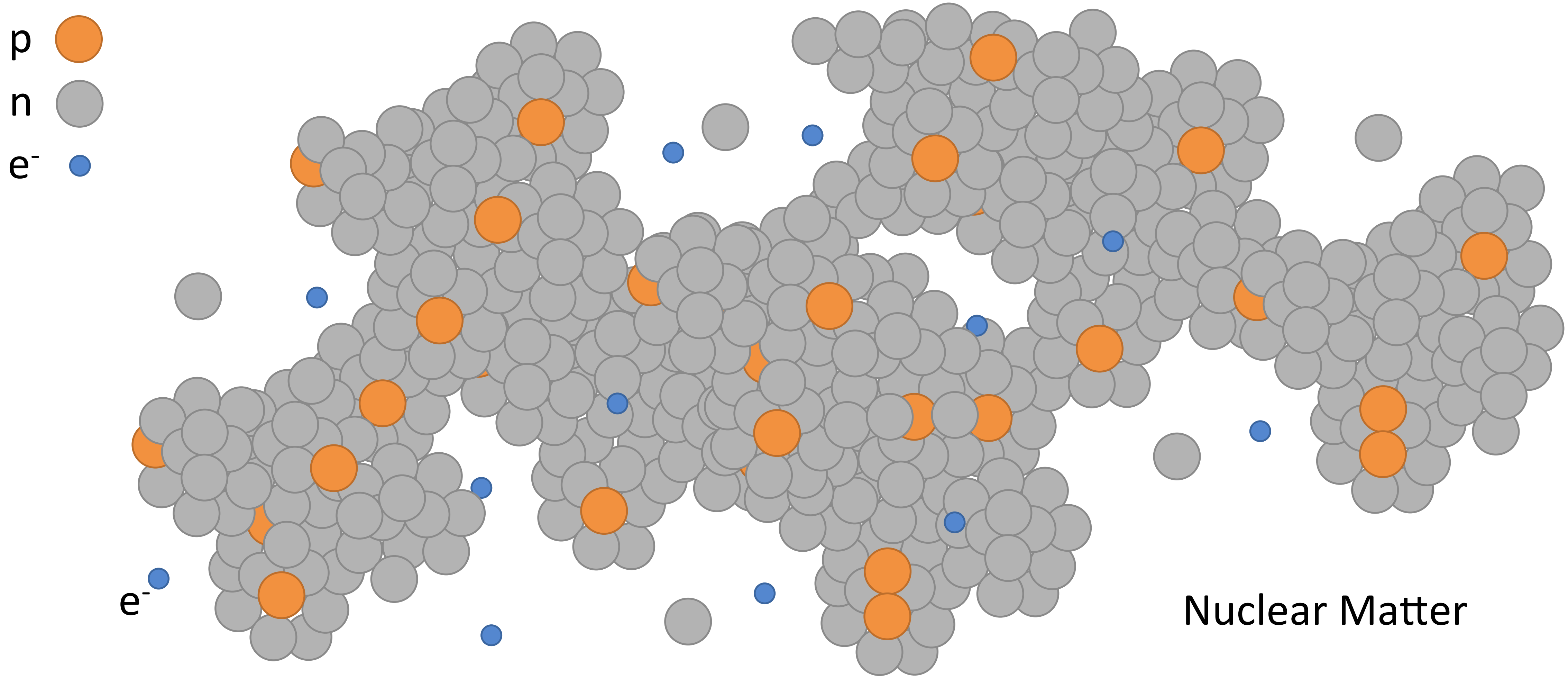
Neutron star mergers



- Makes a LOT of observables:
  - Short Gamma Ray Burst
  - Gravitational Waves
  - Black Hole
  - Neutron Rich Ejecta?
  - Kilonova?

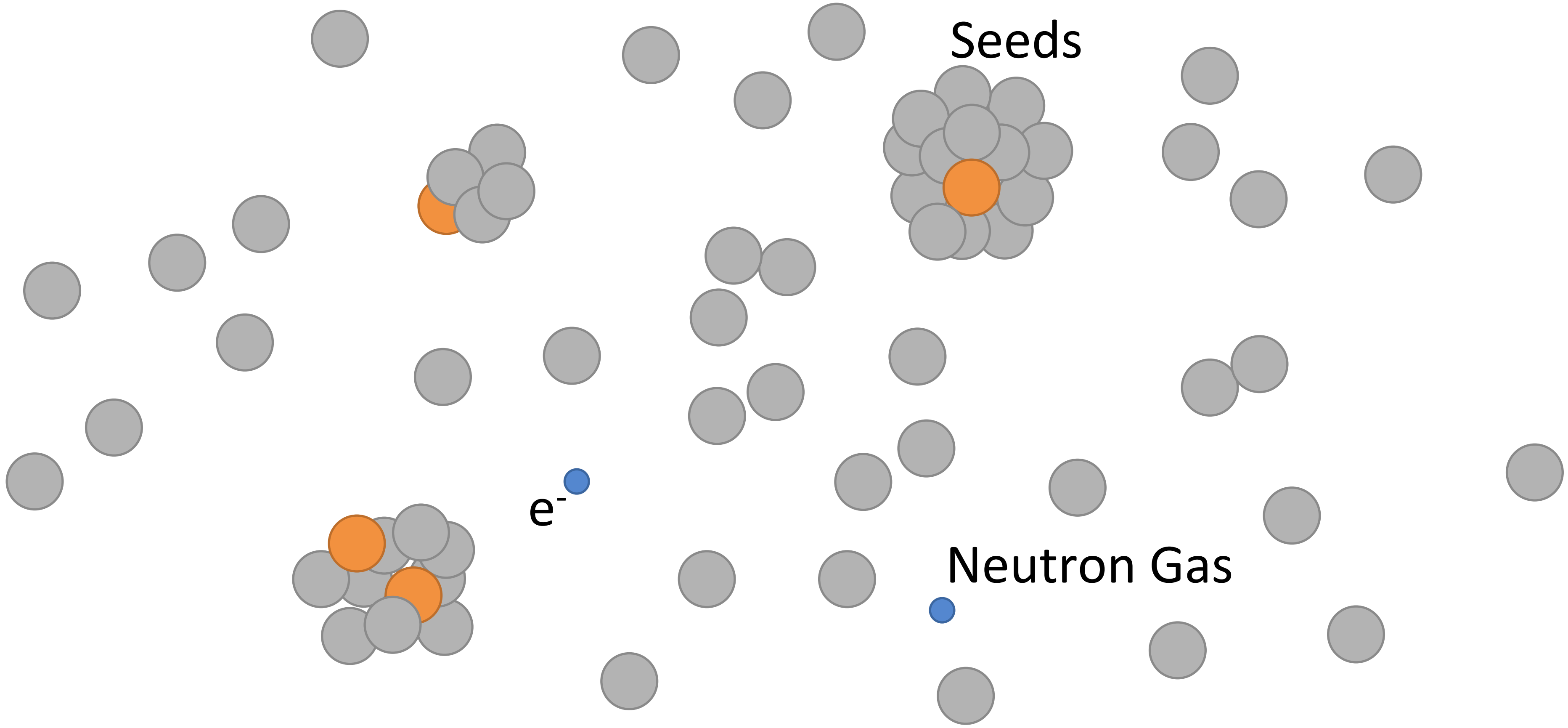
Ejecta Evolution

- Nuclear matter is ejected from the crust and decompresses



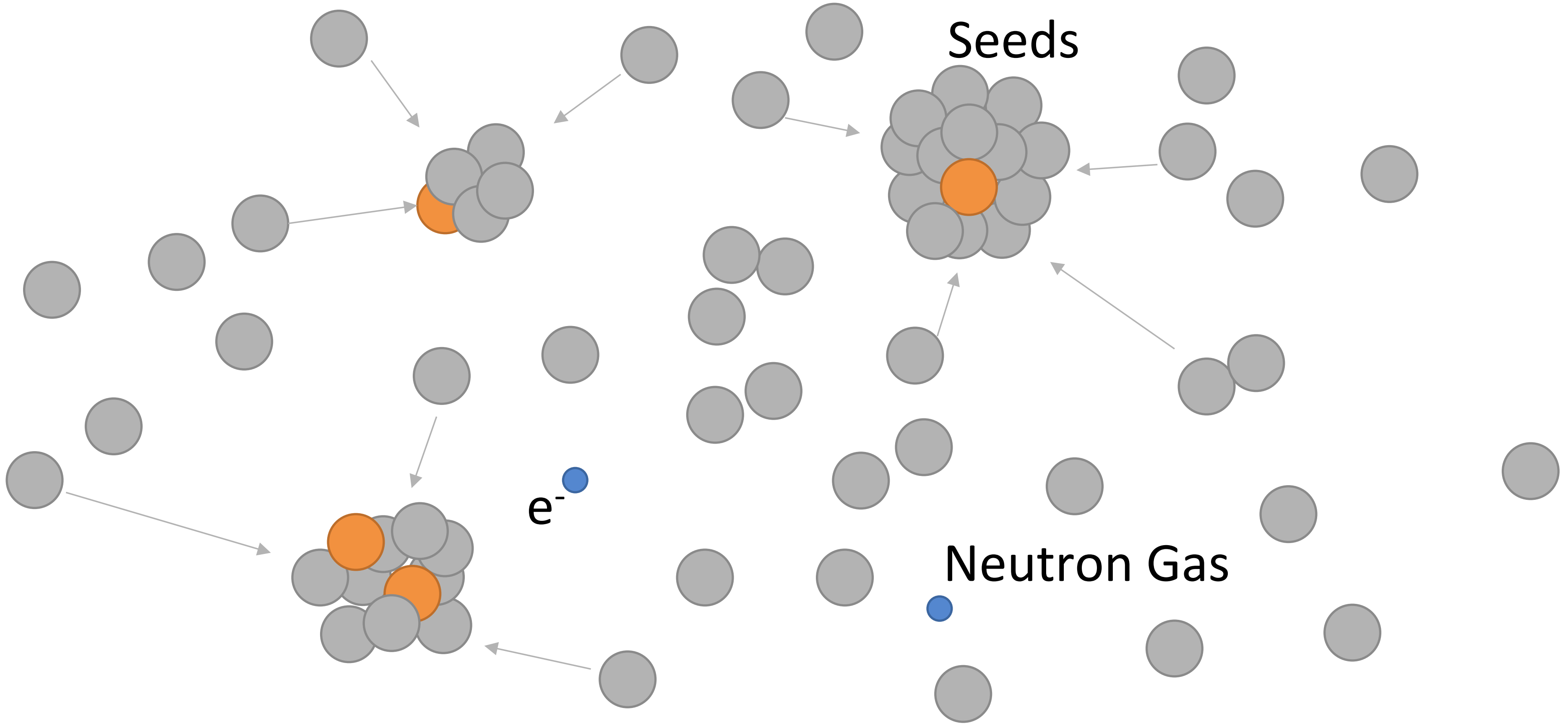
Ejecta Evolution

- The protons form small clusters which 'seed' the neutron gas



Ejecta Evolution

- Neutrons capture onto the seeds, forming neutron rich isotopes



Ejecta Evolution

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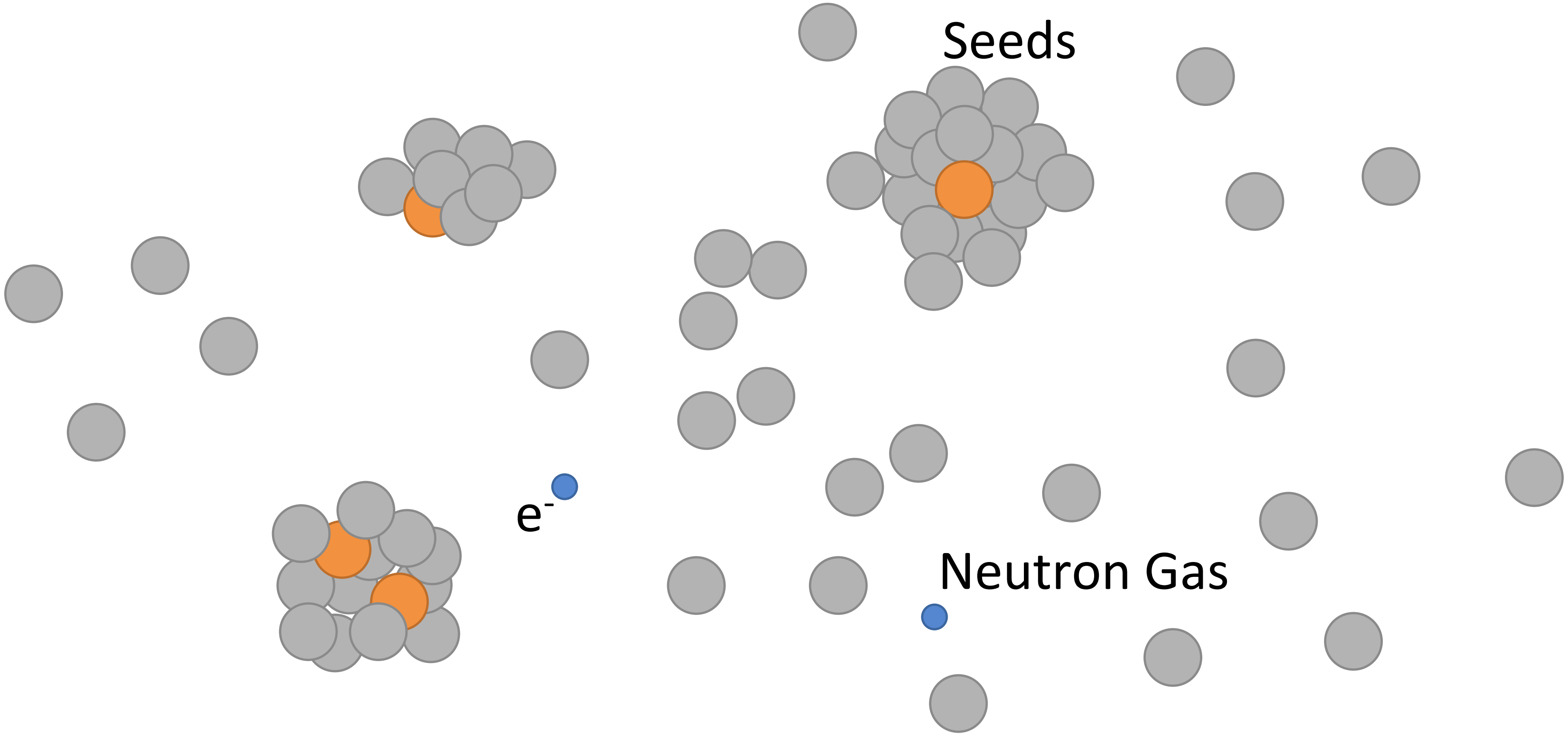
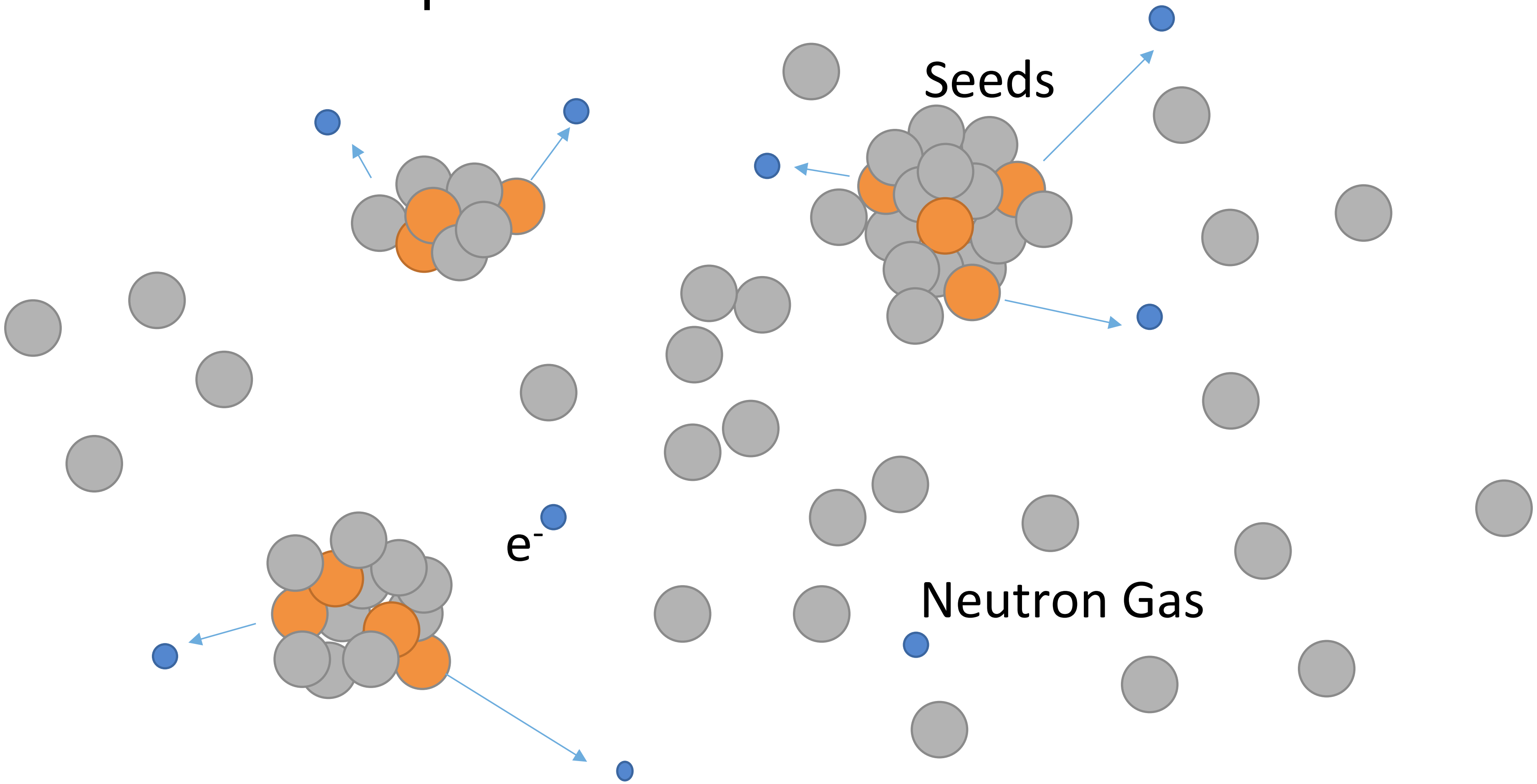


Ejecta Evolution

- These seeds beta decay: $n \rightarrow p + e^{-} + \bar{\nu}_e$
  and continue to capture neutrons



Ejecta Evolution

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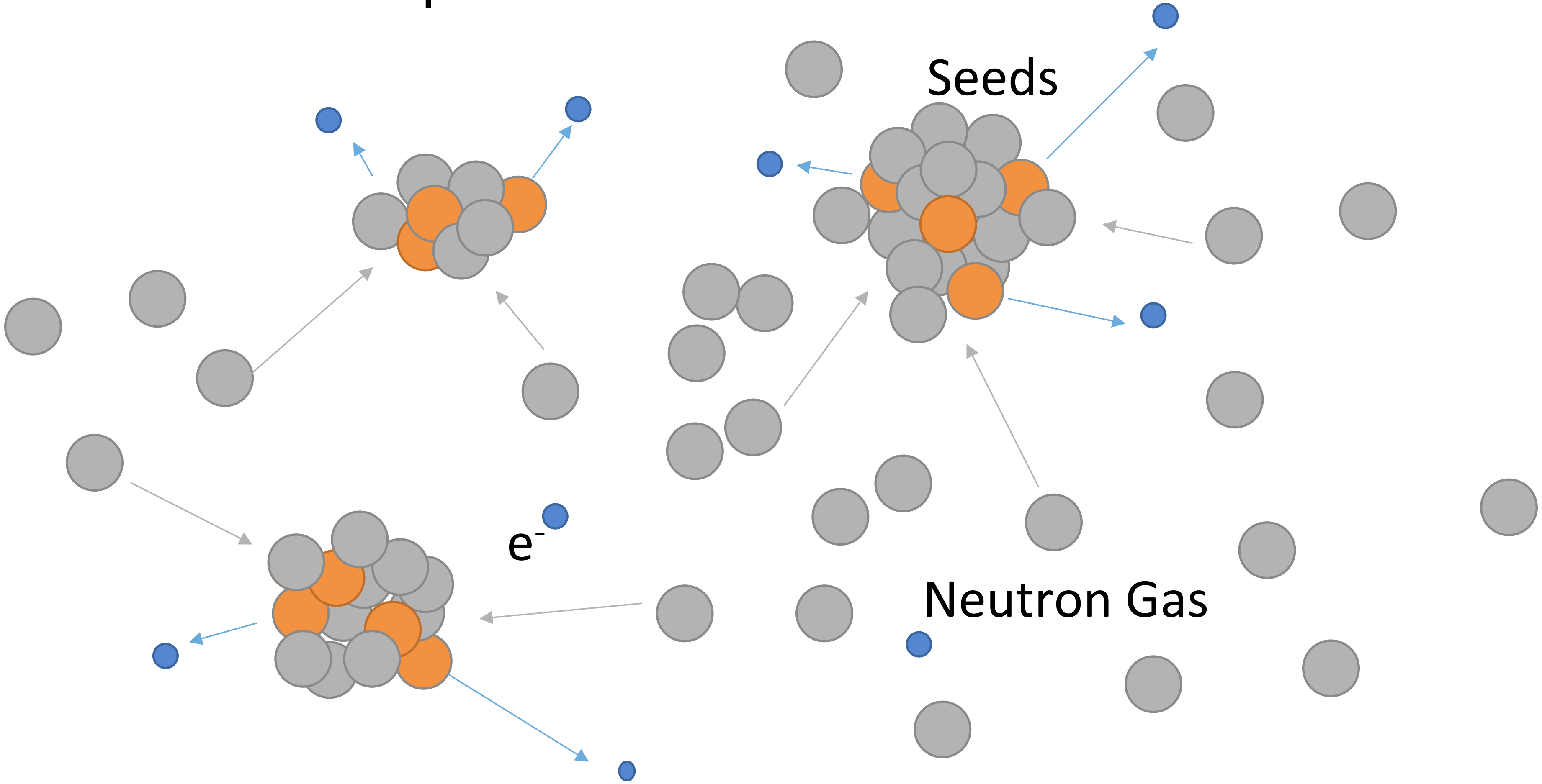
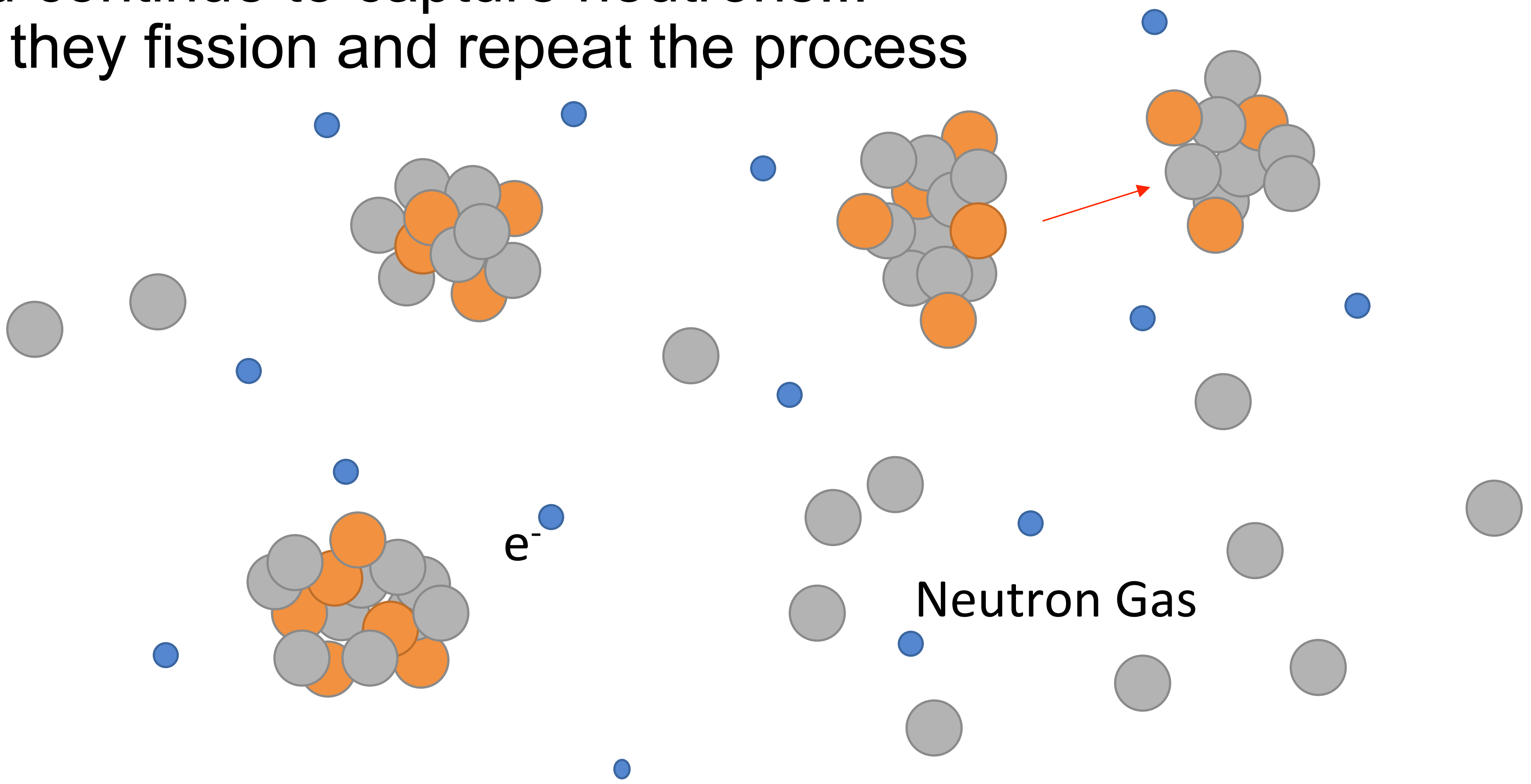


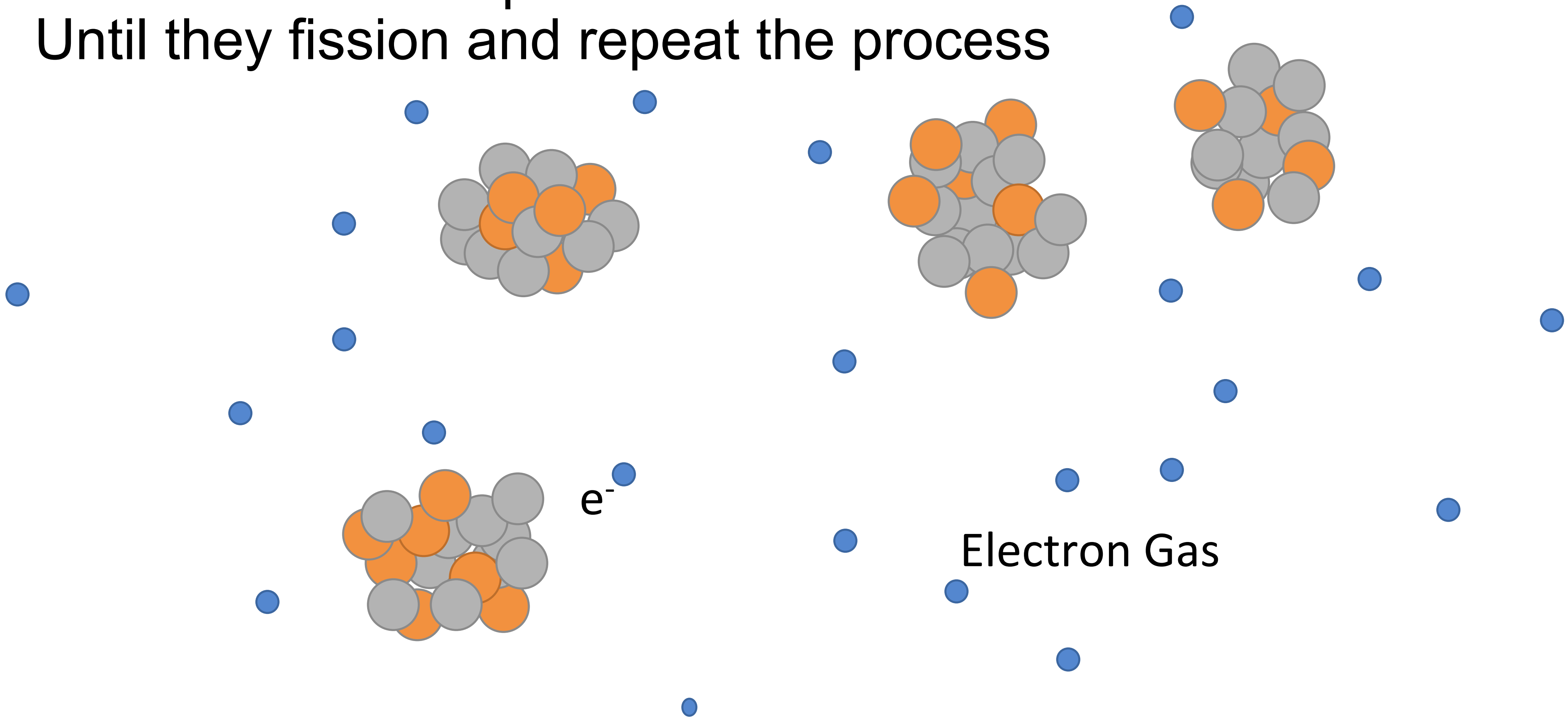
Ejecta Evolution

- These seeds beta decay: $n \rightarrow p + e^{-} + \bar{\nu}_e$
  and continue to capture neutrons...
  ntil they fission and repeat the process



Ejecta Evolution

- These seeds beta decay: $n \rightarrow p + e^- + \bar{\nu}_e$
  and continue to capture neutrons...
  Until they fission and repeat the process



Ejecta Evolution

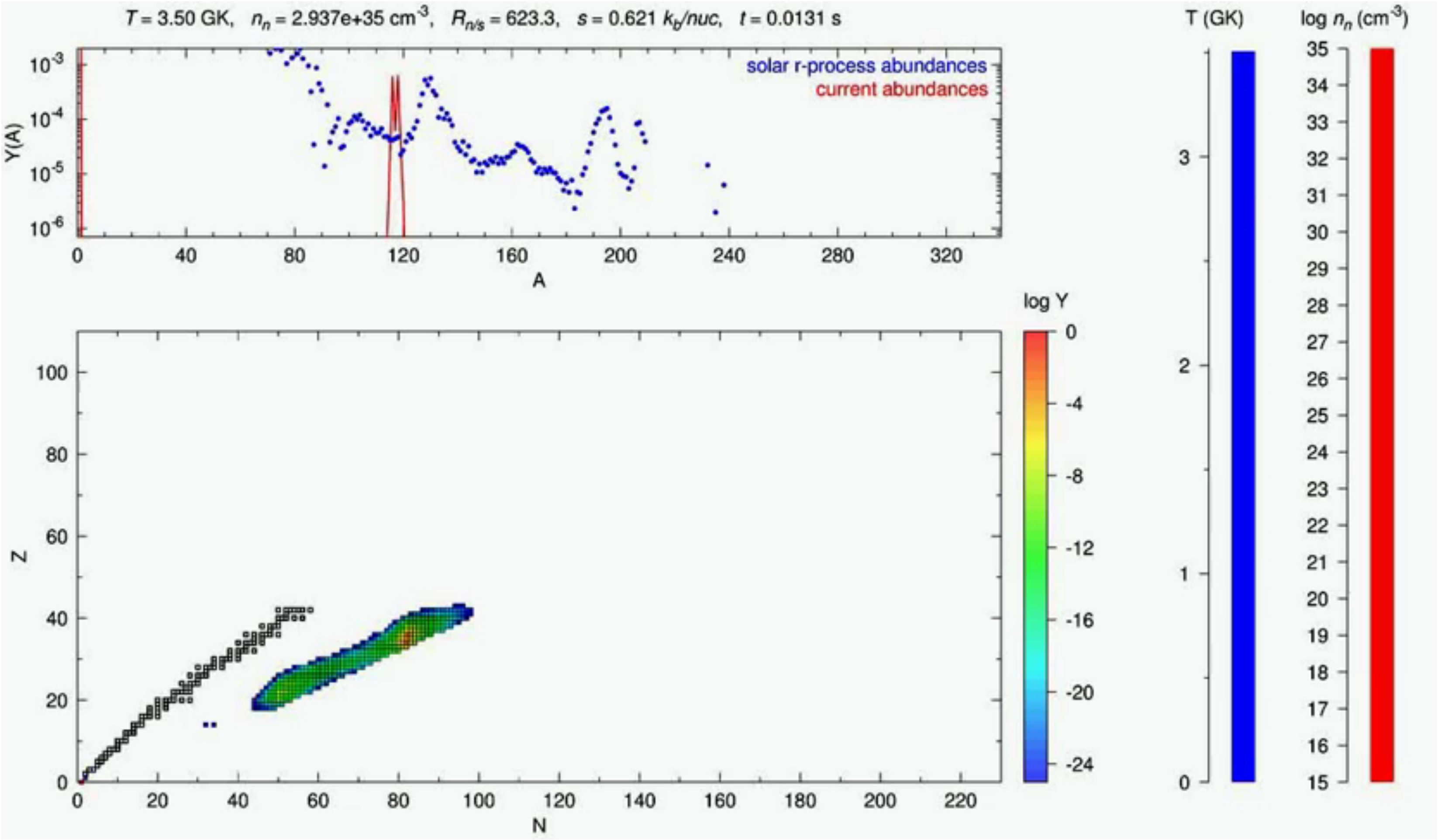
- r-process: the **r**apid neutron capture **process**
- Occurs in supernova and neutron star mergers
- Source of neutrons?
  - Neutron star mergers – obvious
  - Supernova – Neutrino driven wind

$$e^- + p \leftrightarrow n + \nu_e$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e$$

- Key parameter: Neutron to seed ratio (i.e. neutron to proton ratio)
  - Supernova: 4:1?
  - Neutron Stars: 100:1

Ejecta Evolution



Ejecta Evolution

- Decompress pasta to simulate ejecta evolution
- Count the number of protons and neutrons in each cluster after fission

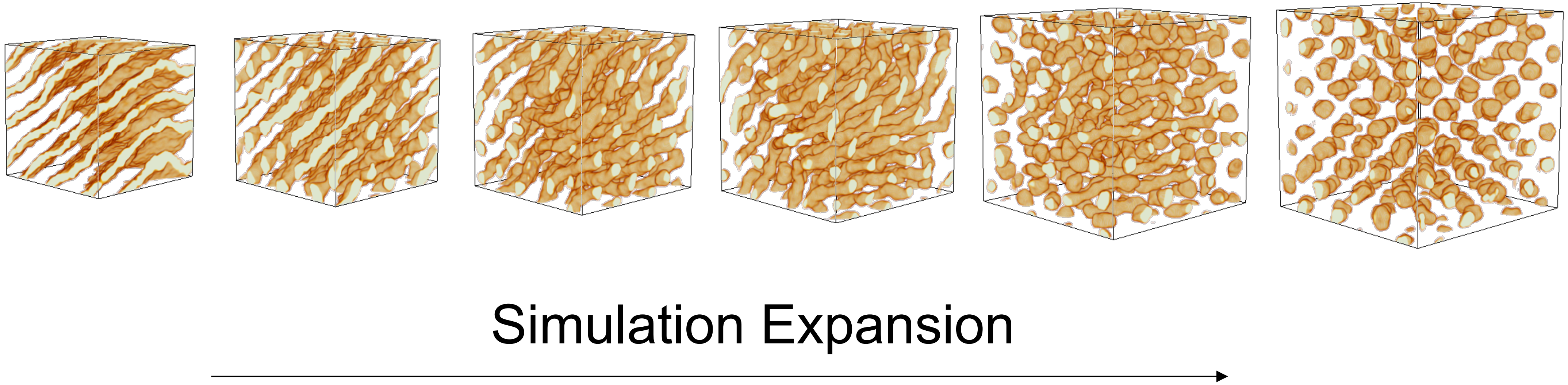
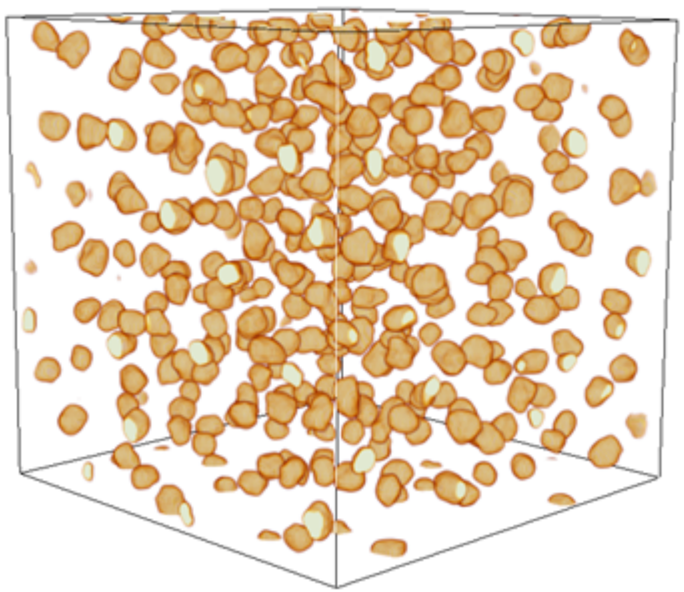
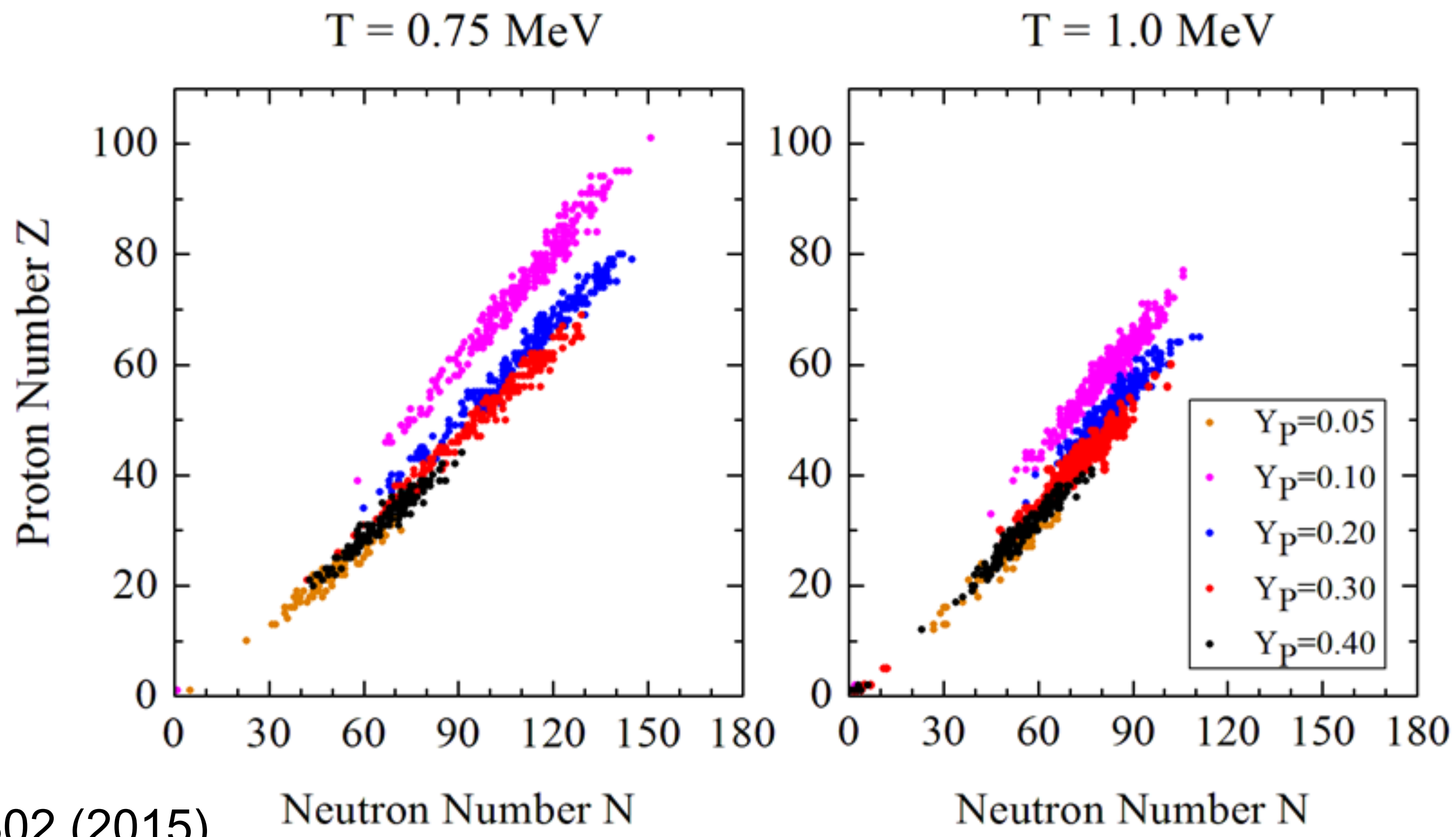


Table of Nuclides

- Pasta gnocchi produce realistic distributions of nuclei

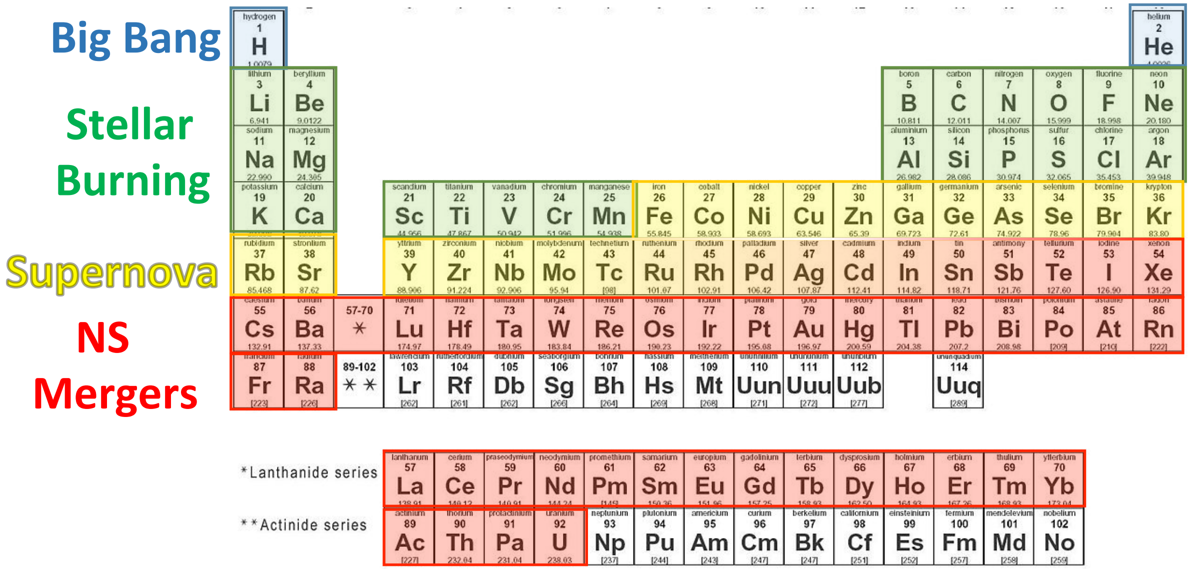




Caplan et al. Phys. Rev. C 91, 065802 (2015)

What ~~fuses~~ the elements heavier than iron?

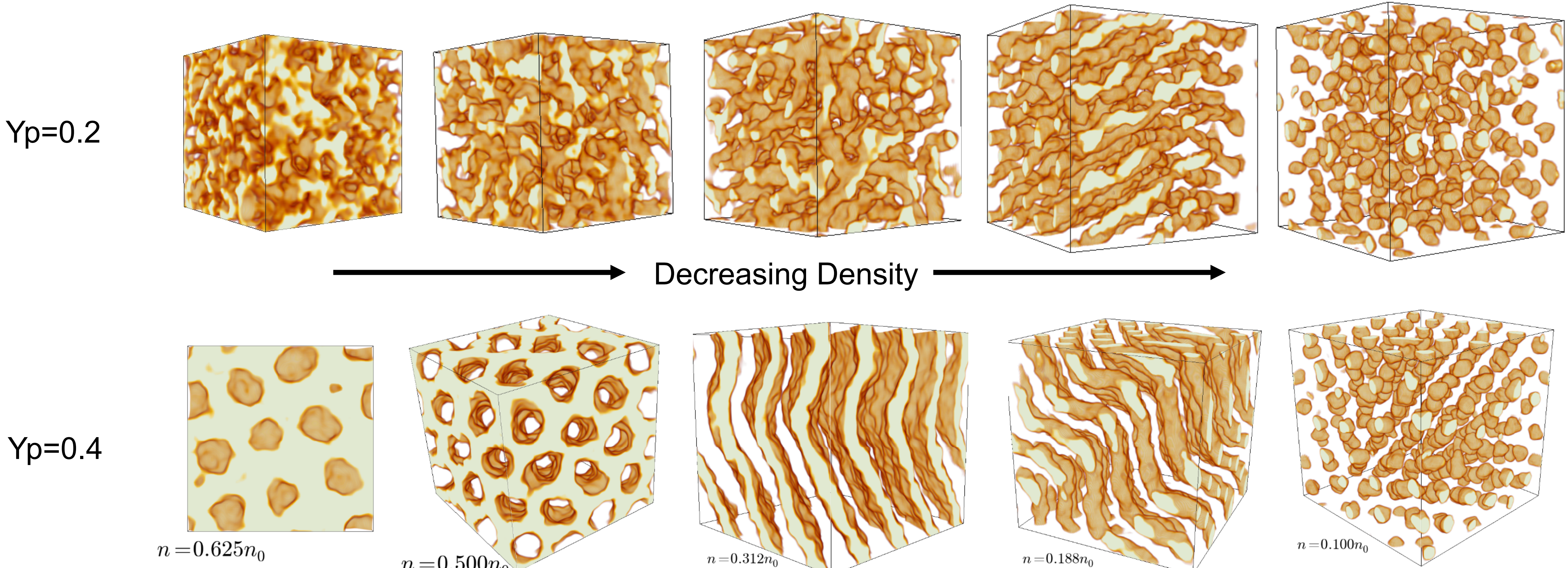
↑ makes



Phase Diagrams

Linear Elasticity

- Simulate pasta with constant temperature and proton fraction
- Observe phase transitions as a function of density



"Thermodynamic" Curvature

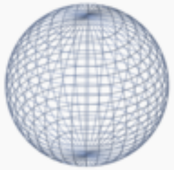
- Use curvature as a thermodynamic quantity
- Discontinuities in curvature indicate phase changes

V	Volume
$A = \int_{\partial K} dA$	Surface Area
$B = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi \, dA$	Mean Breadth
$\chi = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi \, dA$	Euler Characteristic

0

$$\int_M K \, dA + \int_{\partial M} k_g \, ds = 2\pi\chi(M)$$

$$\chi(M) = 2 - 2g$$

Sphere		2
Torus (Product of two circles)		0
Double torus		−2
Triple torus		−4

- Pieces + Cavities - Holes

“Thermodynamic” Curvature

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- Discontinuities in curvature indicate phase changes

V	Volume
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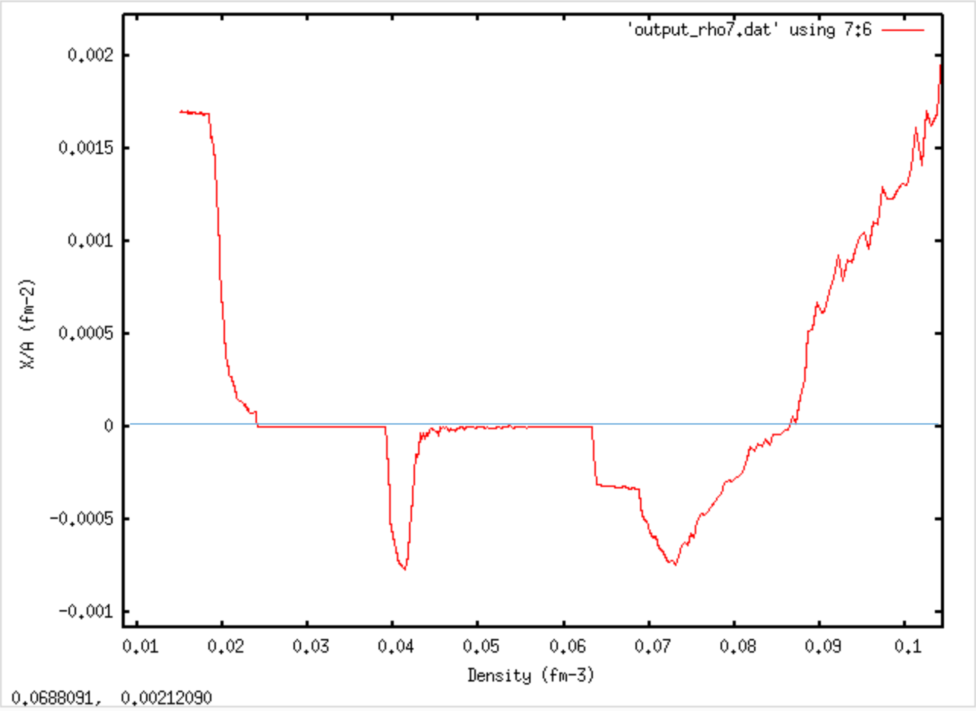
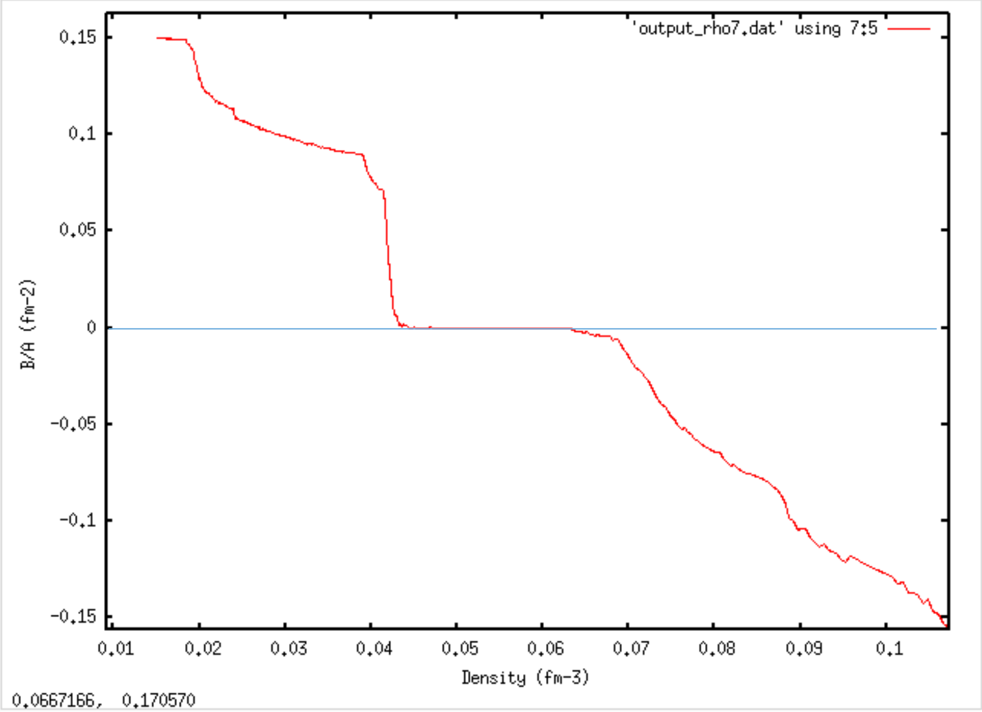
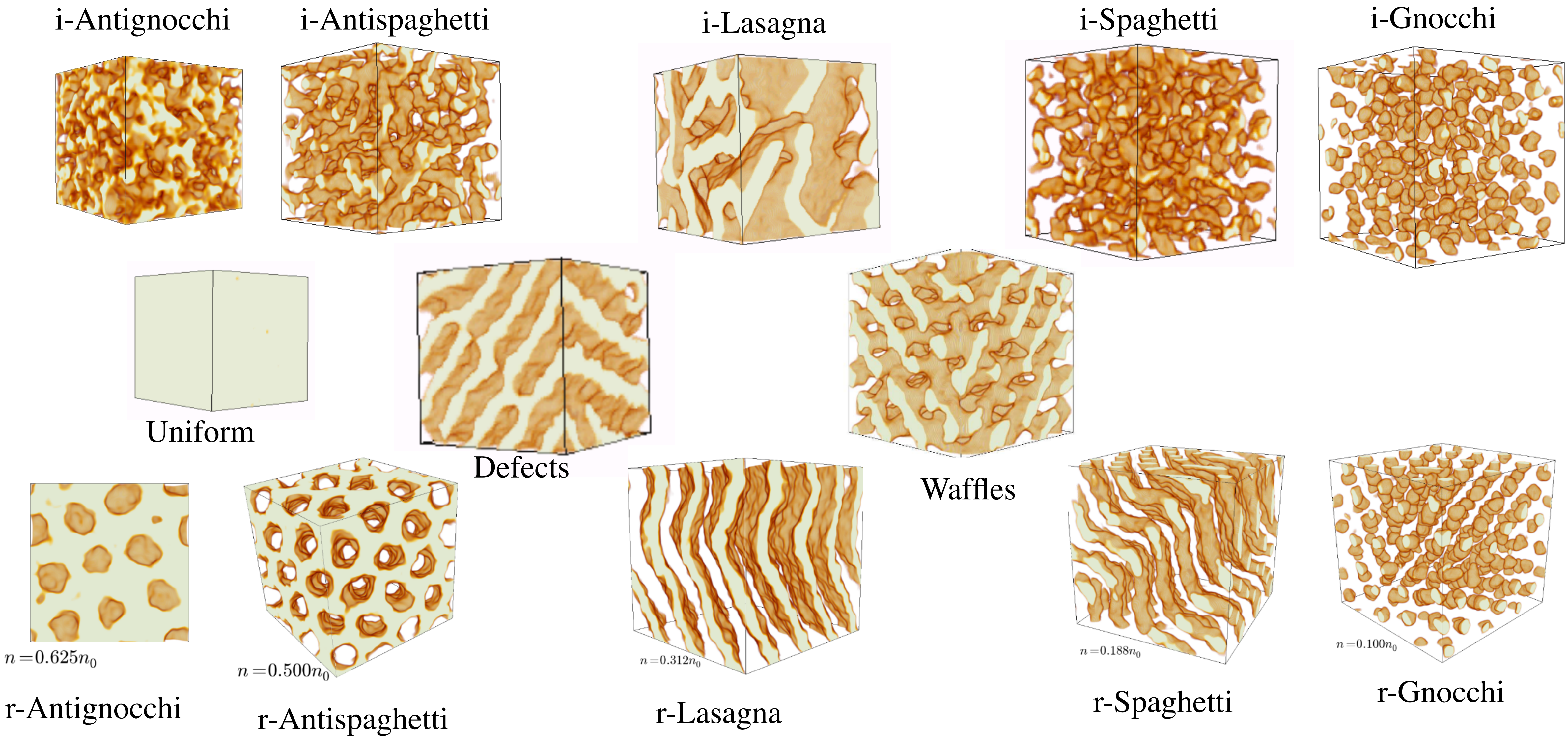
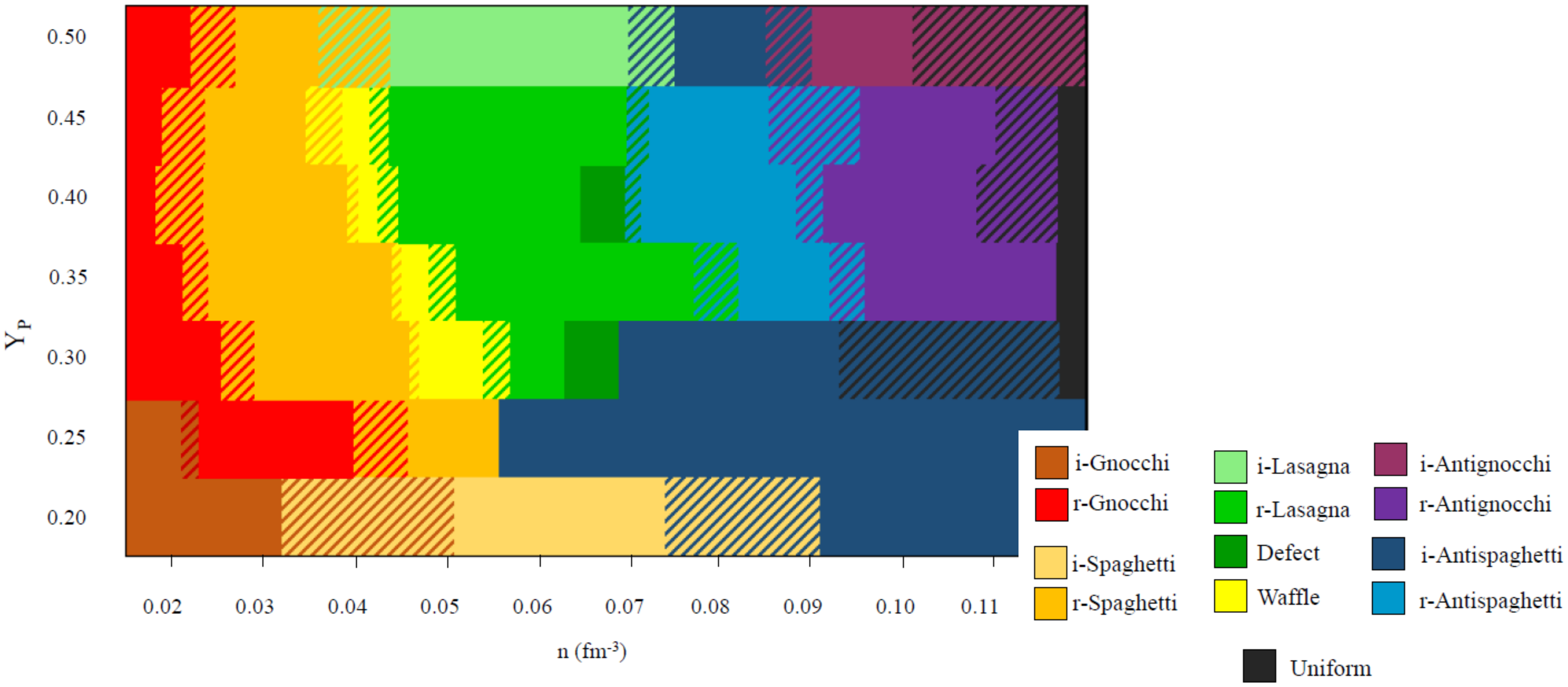




Phases


i-Antignocchi
i-Antispaghetti
i-Lasagna
i-Spaghetti
i-Gnocchi
Uniform
Defects
Waffles
$n = 0.625 n_0$
r-Antignocchi
$n = 0.500 n_0$
r-Antispaghetti
$n = 0.312 n_0$
r-Lasagna
$n = 0.188 n_0$
r-Spaghetti
$n = 0.100 n_0$
r-Gnocchi

Phases (T=1 MeV)



Lepton Scattering

- Why does it matter?
- Lepton scattering from pasta influences a variety of transport coefficients:

- Shear viscosity:

$$\eta = \frac{\pi v_F^2 n_e}{20\alpha^2 \Lambda_{ep}^{\eta}},$$

$$\Lambda_{ep}^{\eta} = \int_0^{2k_F} \frac{dq}{q\epsilon^2(q)} \left(1 - \frac{q^2}{4k_F^2}\right) \left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q)$$

- Electrical conductivity:

$$\sigma = \frac{v_F^2 k_F}{4\pi\alpha\Lambda_{ep}^{\sigma}}$$

$$\Lambda_{ep}^{\kappa} = \Lambda_{ep}^{\sigma} = \int_0^{2k_F} \frac{dq}{q\epsilon^2(q)} \left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q).$$

- Thermal conductivity:

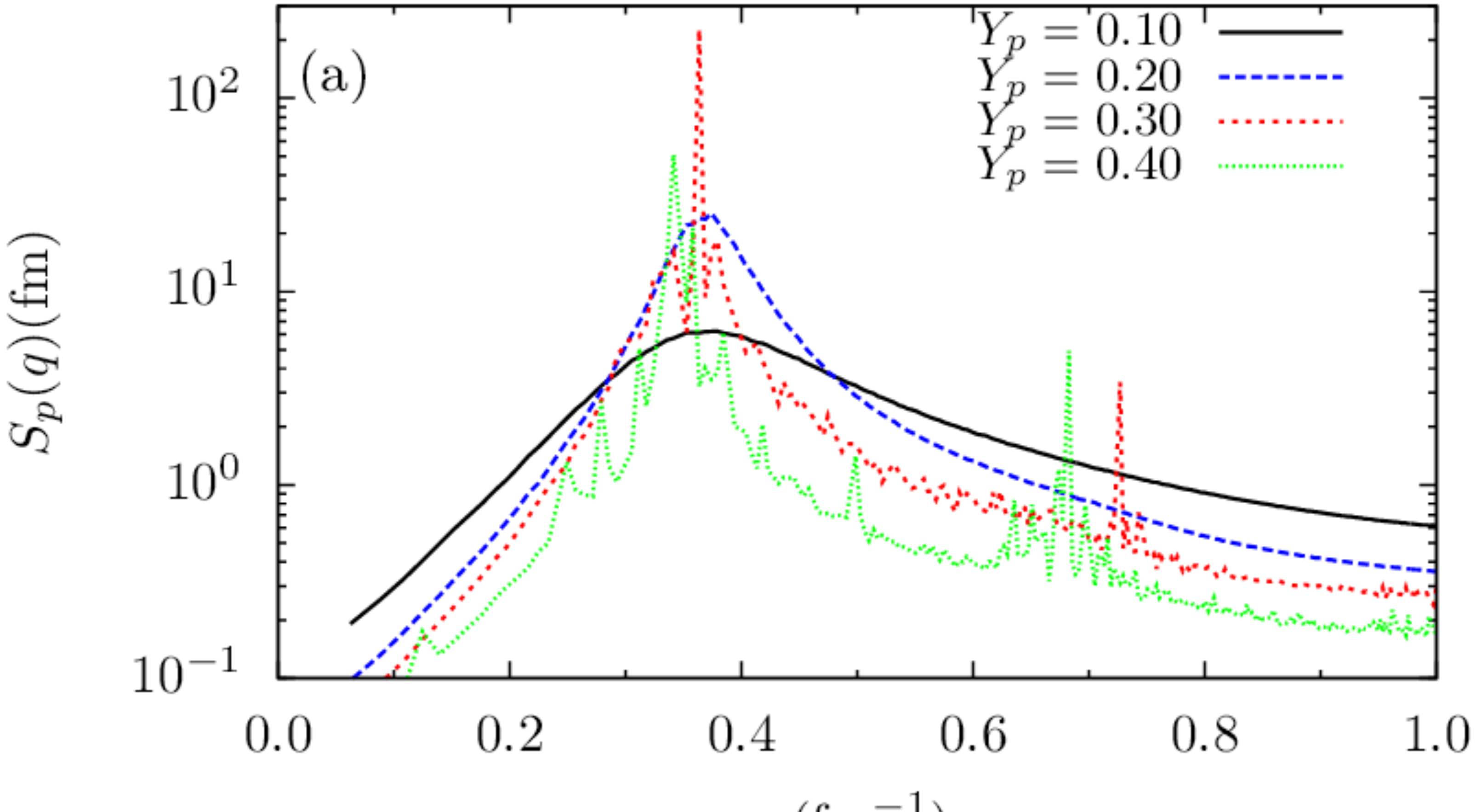
$$\kappa = \frac{\pi v_F^2 k_F k_B^2 T}{12\alpha^2 \Lambda_{ep}^{\kappa}}.$$

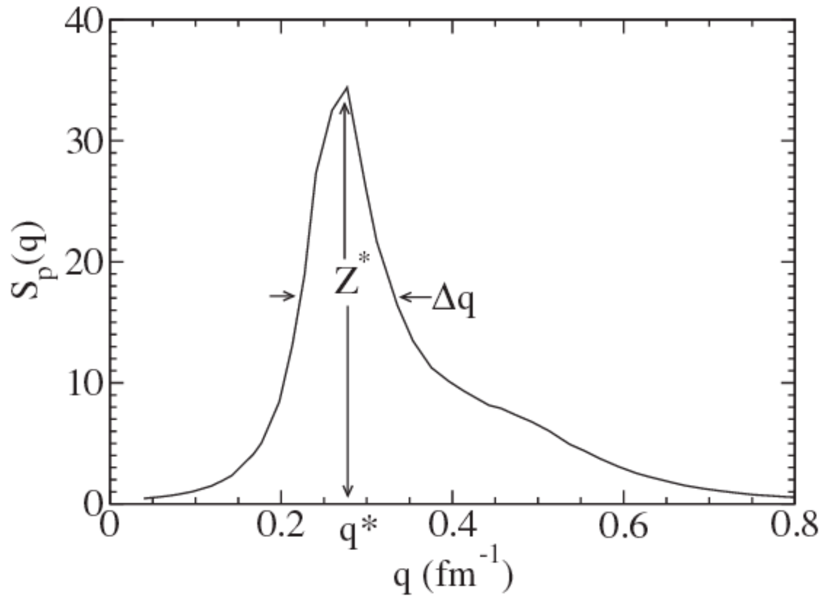
Lepton Scattering

$$S_i(\boldsymbol{q}) = \langle \rho_i^*(\boldsymbol{q},t)\rho_i(\boldsymbol{q},t)\rangle_t - \langle \rho_i^*(\boldsymbol{q},t)\rangle_t \langle \rho_i(\boldsymbol{q},t)\rangle_t$$

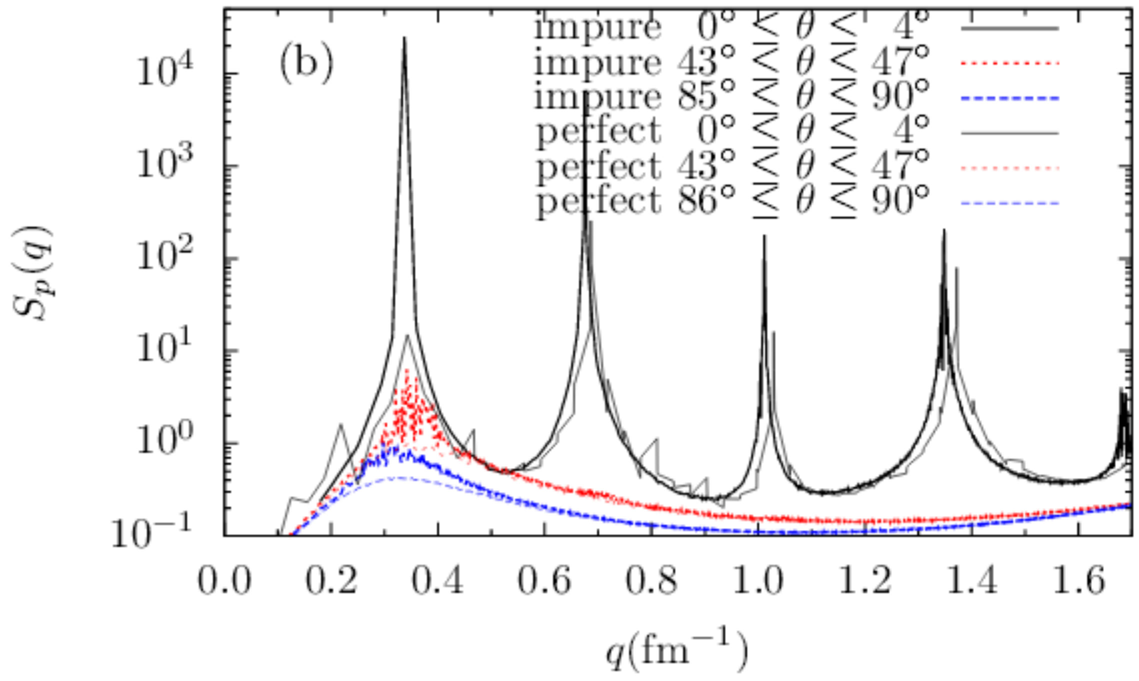
$$\rho_i(\boldsymbol{q},t) = N_i^{-1/2} \sum_{j=1}^{N_i} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_j(t)}$$

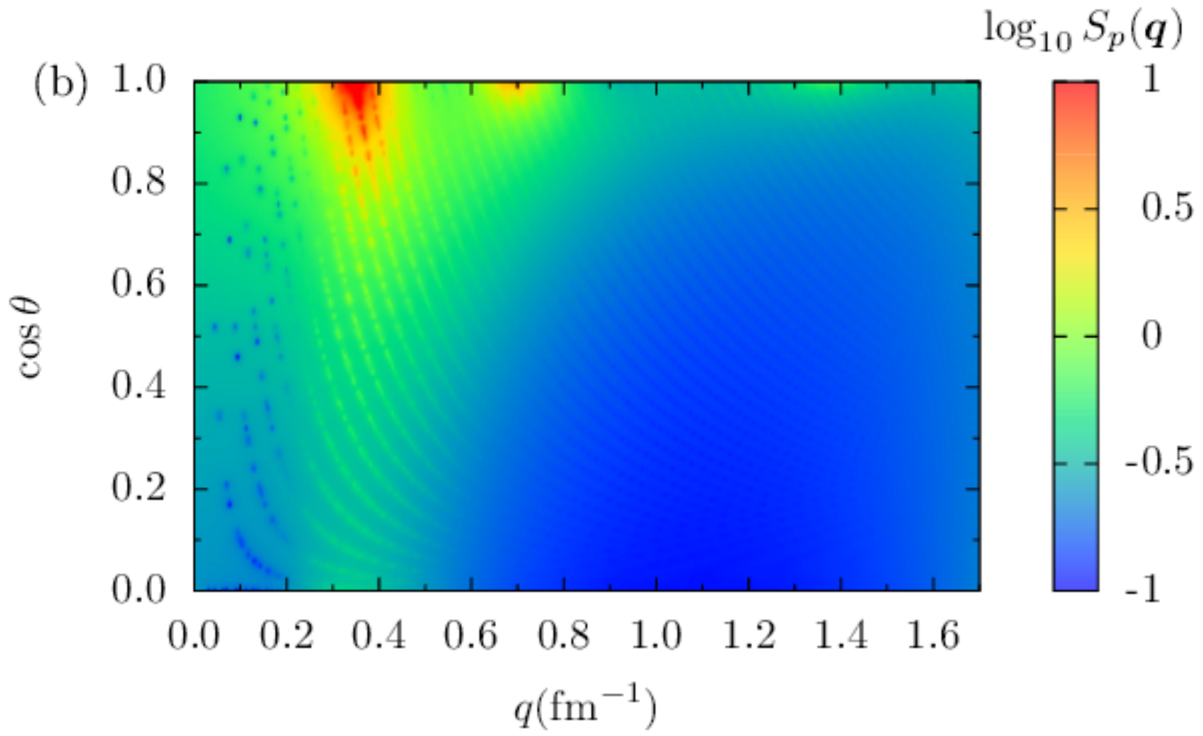
$$\Lambda_{ep} \approx \frac{\Delta q^* Z^*}{q^*}$$





Lepton Scattering





Simulation	$\bar{\eta}(\mathrm{fm}^{-3})$	$\bar{\kappa}(10^3 k_B\,\mathrm{MeV/fm})$	$\bar{Z}^*$
perfect	87.7	6.66	5.5
defects	55.5	4.15	50.2

Self Assembly

- The Helfrich Hamiltonian describes the bending energy, can be found with the principal curvatures, k_1 and k_2, and curvature energies B and $\bar{B}$

Helfrich, Z. Naturforsch. 28 (1973)

$$H_0 = \frac{1}{2}B\int dA(k_1 + k_2)^2 + \bar{B}\int dA(k_1 k_2)^2$$

- Relate the curvature energy to a curvature term in the SEMF

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + a_K A^{1/3} - \delta(A, Z)$$

- Bottom line: minimal surfaces minimize surface energy and the curvature energy settles the tie

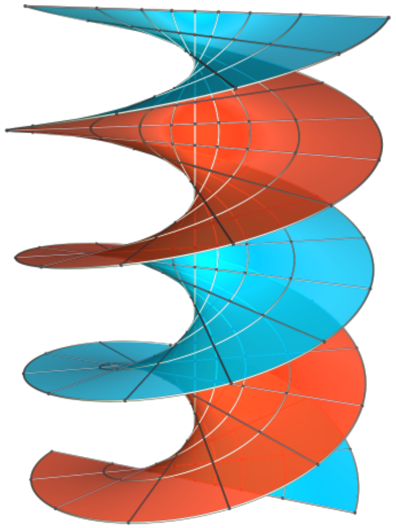
Self Assembly

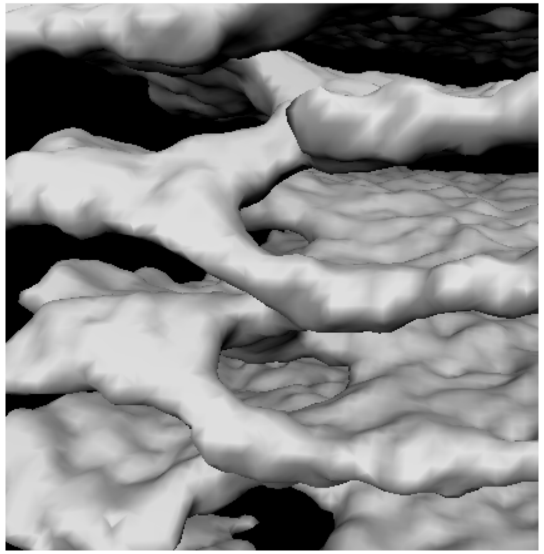
- What minimal surfaces do we see in pasta?

1) $k_1 = k_2 = 0$, Flat plates
2) $k_1 = -k_2$, Hyperbola
3) Other minimal surfaces:

$$H_0 = \frac{1}{2}B\int dA(k_1 + k_2)^2 + \bar{B}\int dA(k_1k_2)^2$$

Helicoids:





Gyroids