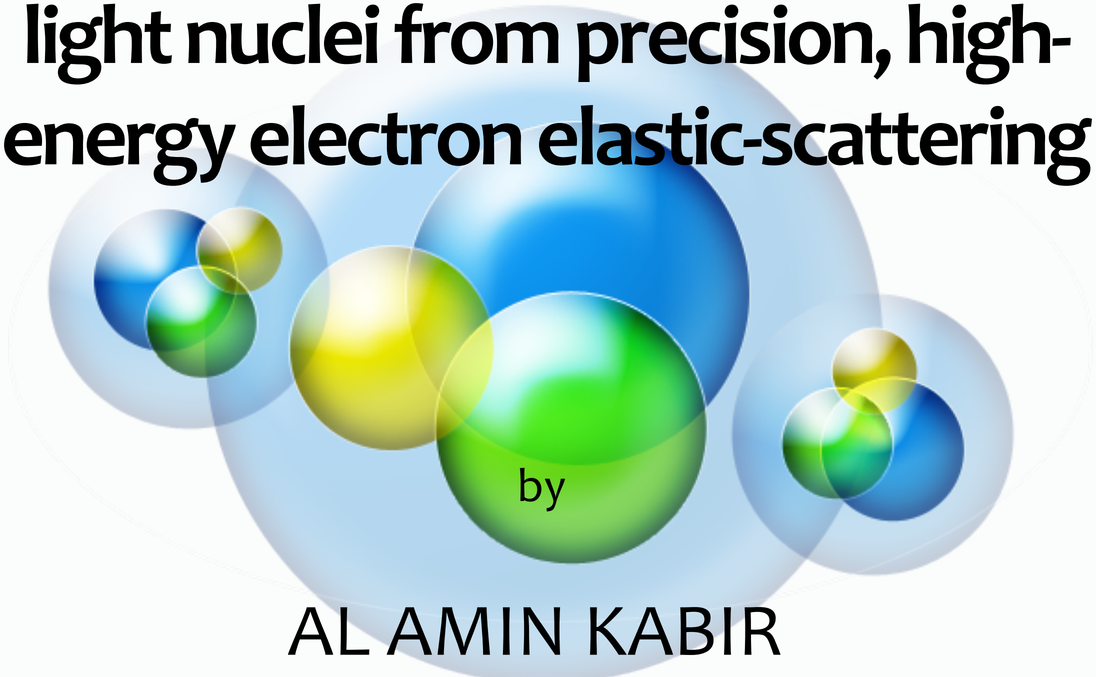
Determination of the charge radii of light nuclei from precision, high-energy electron elastic-scattering



by

AL AMIN KABIR

KENT STATE UNIVERSITY

With JLab Hall A Colaboration

UVA Talk 01/26/2016

What is a rms charge radius and how is it measured?

- There is no definite nuclear boundary, so we end up measuring a range of radii from which the mean can be determined.

- Typically determined by analyzing electron elastic scattering measurements. Almost everything we know about nuclear and atomic physics has been discovered by scattering experiments. (e.g., Rutherford's discovery of nucleus, discovery of quarks etc.).

- Can be determined by measuring the effect of of finite nuclear size on the energy levels of atomic electrons.

- Also can be determined from studying the transition energies in muonic atoms.

Why Charge Radii are Important

F. Buchinger et.al., Phys. Rev. C 49, 1402 (1994)

The charge radius of a nuclus is one of the most fundamental measurements in nuclear physics:

- **It plays a key role in studying the characteristics of a nucleus and testing theoretical models of nuclei.**
- **It provides direct information of the Coulomb energy of nuclei; therefore, it is important for nuclear mass formulae.**
- **It can serve to impose strong constraints on the saturation properties of nuclear forces .**
- **The study of nuclear charge radii is critically important for atomic physics in precision spectroscopy studies of atoms.**
- **Connects atomic and subatomic physics.**

How well do we know the charge radii of nuclei?

- It is very well known for carbon(^{12}C), and other heavy nuclei.
- Thought to be very well known for hydrogen(^{1}H) until recently.
  - Large discrepency found between the lamb shift measurement of muonic hydrogen (~0.8408+/-0.0004 fm) and previous measurements from electron scattering experiments and spectroscopic measurements from atomic hydrogen (~0.875+/- 0.006 fm).
- There is no reliable electron scattering results for boron(^{10}B) and lithium(^{6}Li).

The LEDEX Experiment and our Motivation

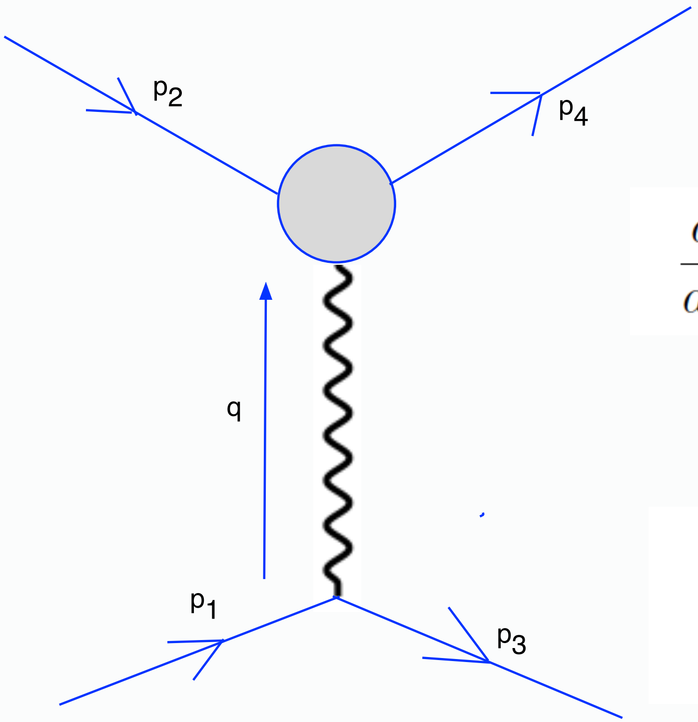
- **Low Energy Deuteron Experiment (LEDEX) was proposed to resolve discrepencies between the existing world data sets for describing the structure function of the deuteron.**

- **Liquid hydrogen and carbon were added to cross-check the experimental procedure and to allow the deuteron form factor to be determined relative to these cross sections at every point.**

- **Renewed attention due to the discrepencies found in the charge radius determinations of the proton.**

- **Motivated from the new ab-intitio theoretical calculations for the charge radii of Lithium and Boron.**

Feynman Diagrams and Cross-Section Formulations

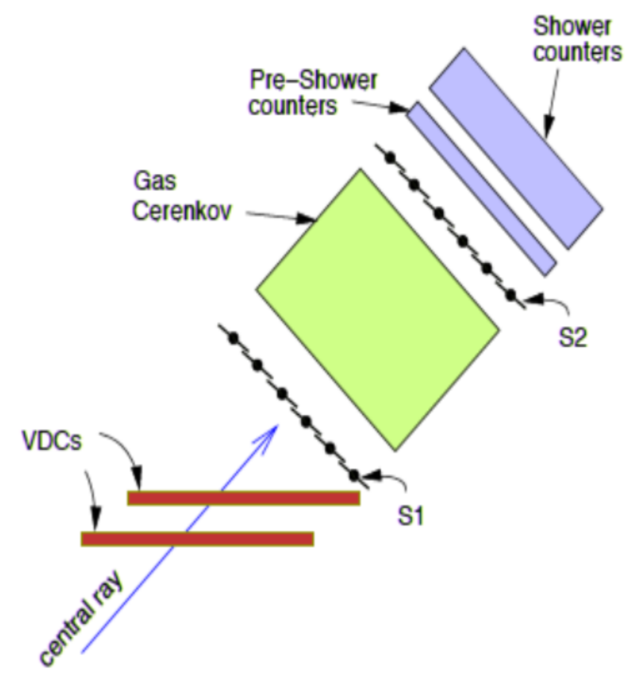


$$\frac{d\sigma}{d\Omega'} = \left(\frac{d\sigma}{d\Omega'}\right)_{Mott}\left[2\tau G_M^2 tan^2(\Theta/2) + \frac{G_E^2 + \tau G_M^2}{1+\tau}\right].$$

$$\frac{d\sigma}{d\Omega'}\Big|_{point} = \frac{d\sigma}{d\Omega'}\Big|_{Mott} = \frac{(Z\alpha)^2 E^2}{4k^2 sin^4(\Theta/2)}\frac{E'}{E}cos^2\frac{\Theta}{2}.$$

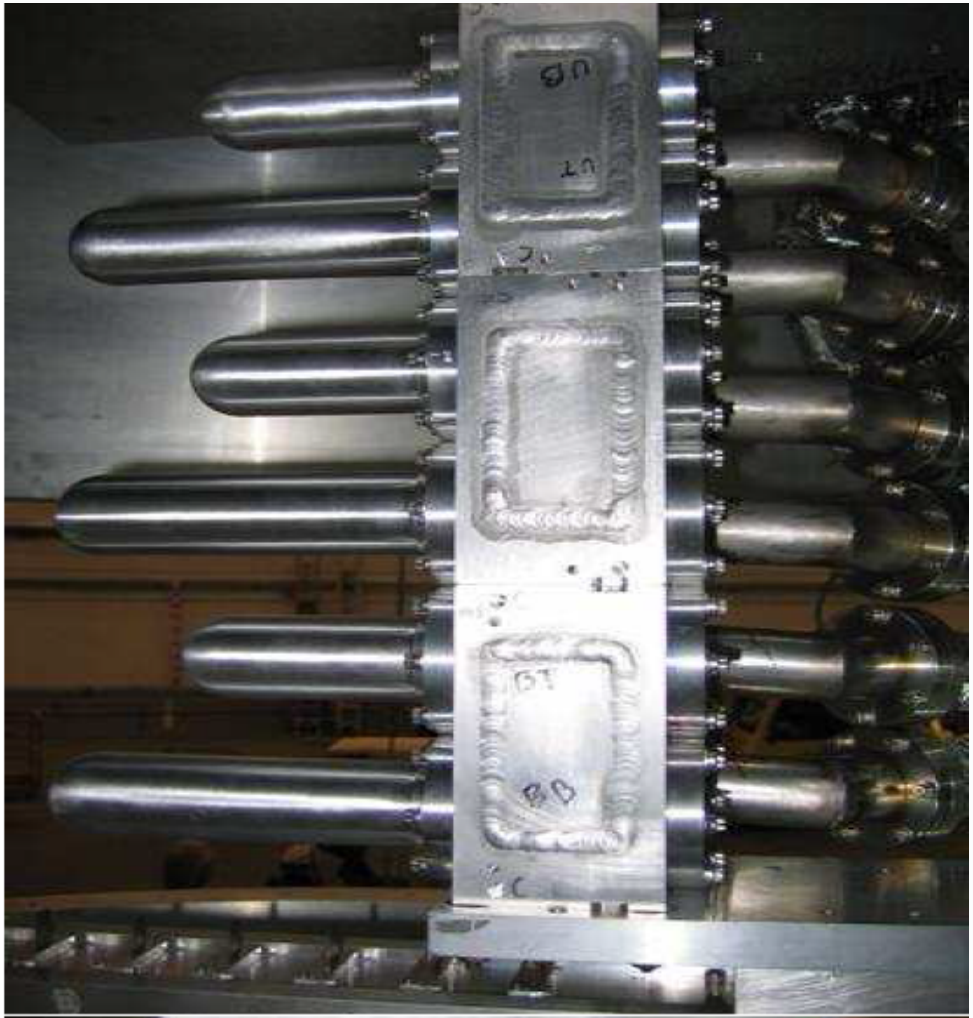
Hall A Setup

LHRS
RHRS
Target
Beam Line

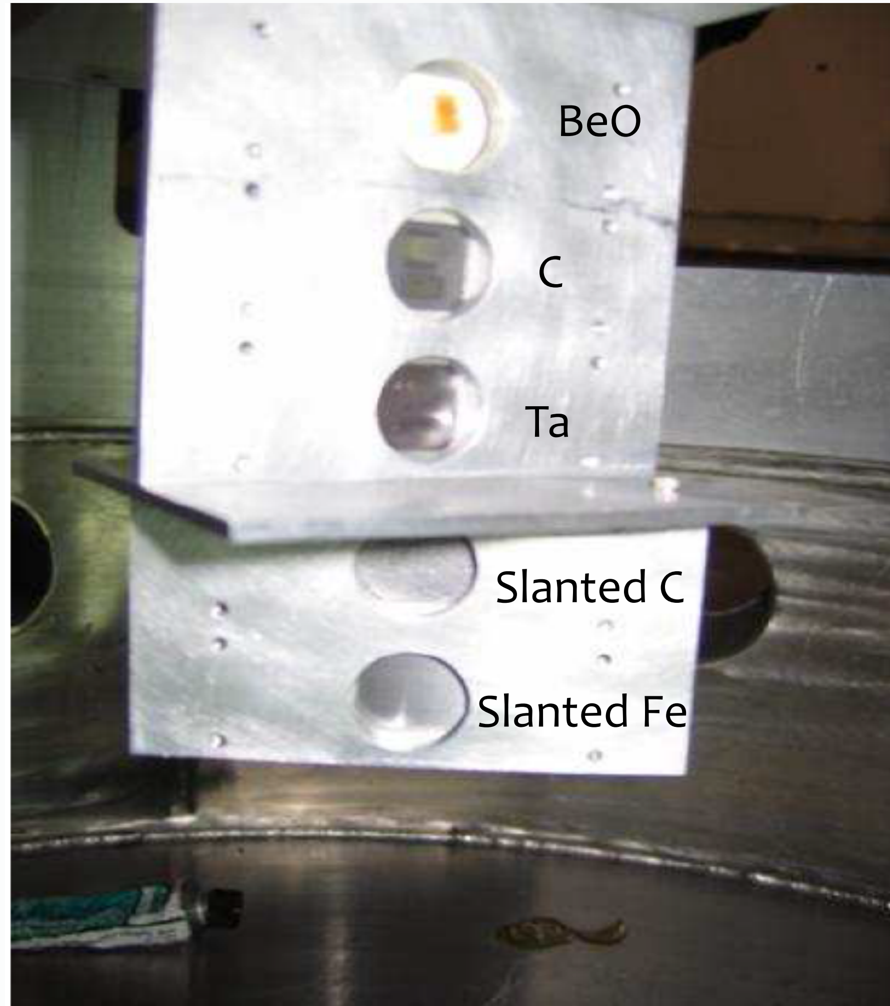


Shower counters
Pre-Shower counters
Gas Cerenkov
S2
VDCs
S1
central ray

LEDEX Target system



Liquid Target Ladder



Solid Targets

Extraction of the Experimental Cross-Section

- The working formula for extracting the cross section is:

  $d\sigma / d\Omega = (1/(L_t \times t \times \Delta\Omega \times \Pi_i \; \varepsilon_i)) \times \text{Counts} \times R$

- Obtain net counts after substracting the back-ground and dummy from a calibrated run and apply necessary cuts (e.g. acceptance, particle Identification etc.).

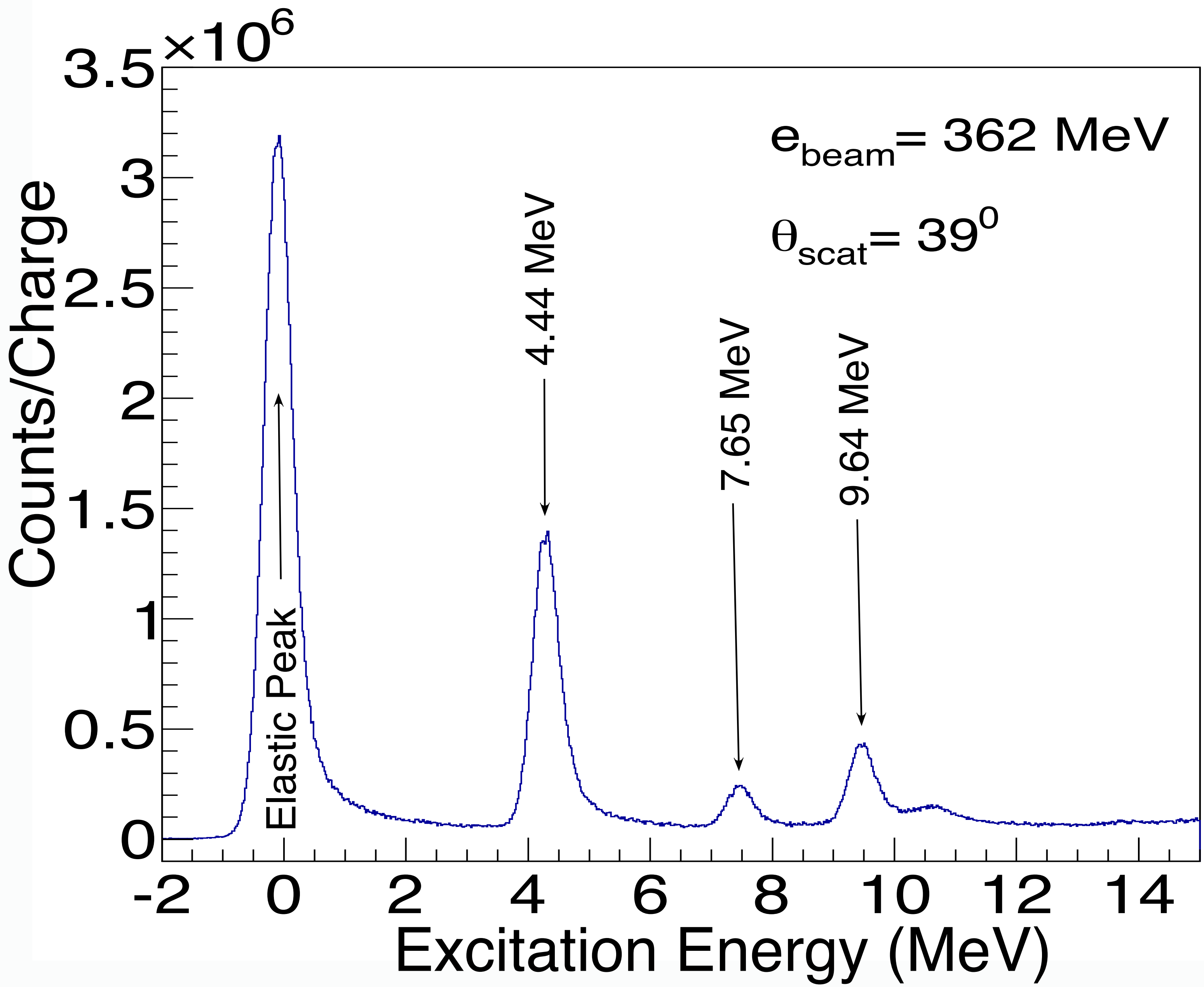
- Obtain the wire chamber Efficiencies (e.g. triggering, tracking etc.)

- Use MCEEP to find the radiative correction factor.

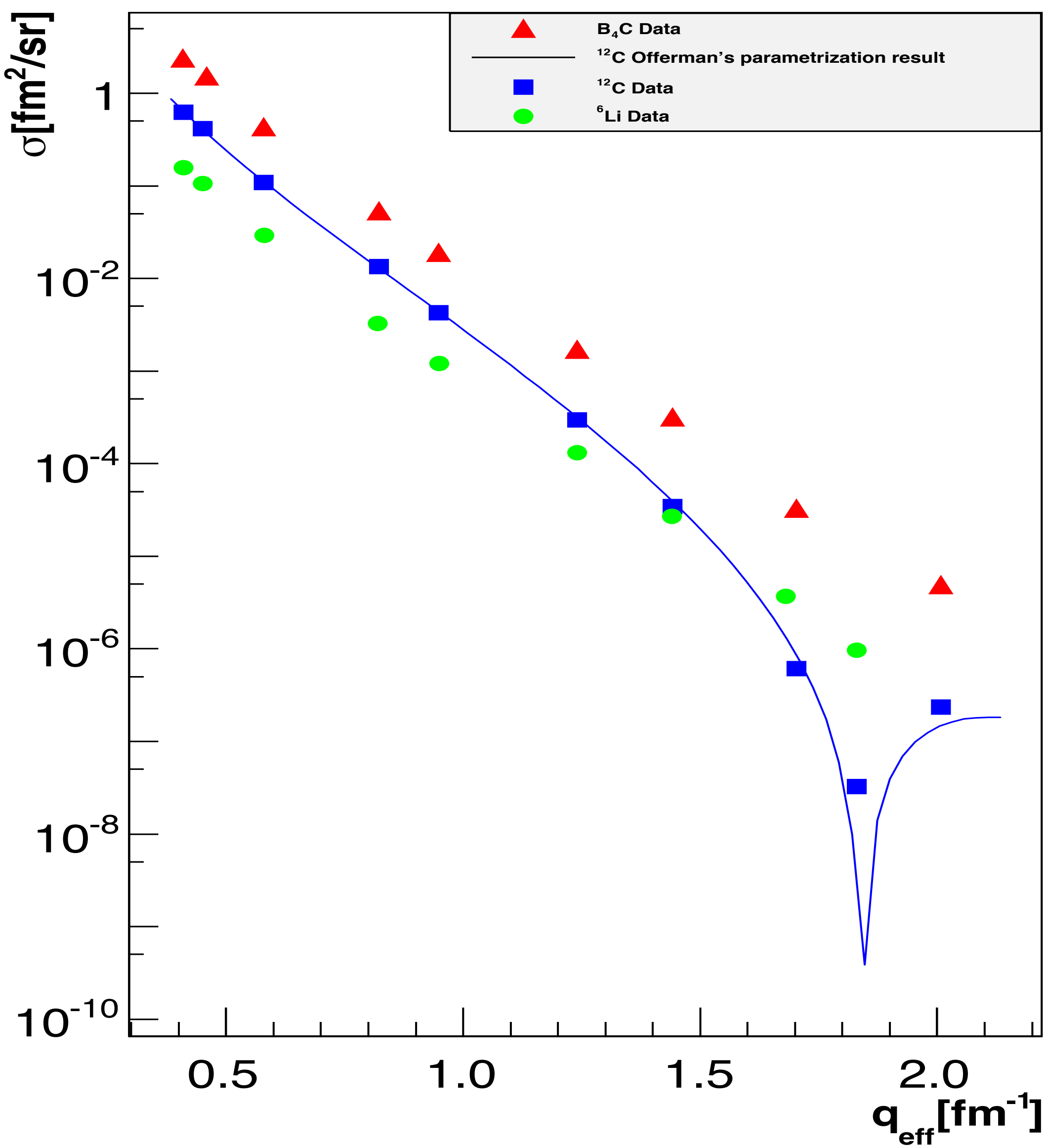
- Divide the yield by mceep phase space and efficiencies and multiply by radiative correction factor to find the cross –section for hydrogen, deuteruim and carbon.

- For lithium and boron, there is no simulation method, and hence, we found their cross-sections relative to carbon.

Example of a calibrated Carbon Run

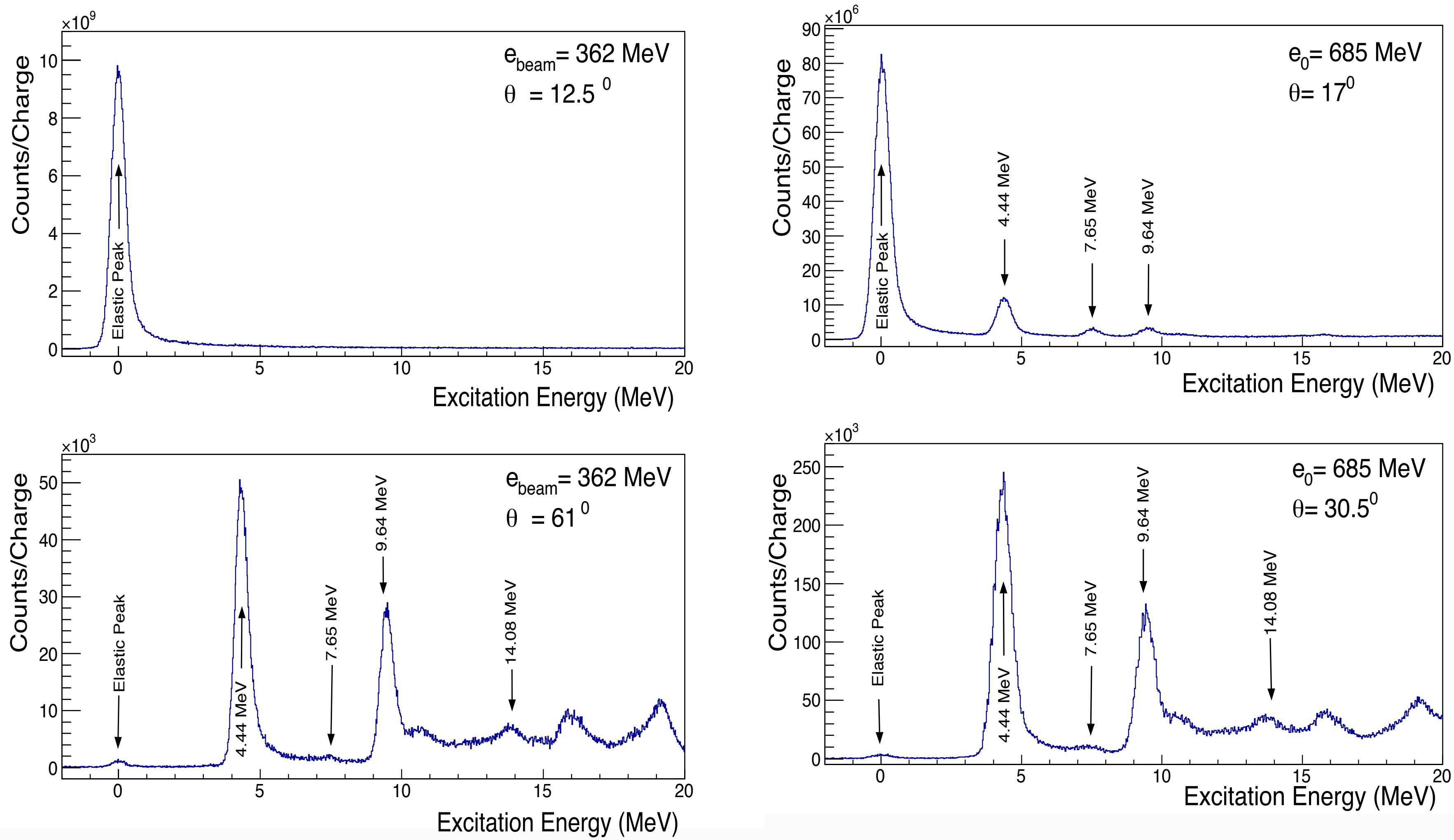


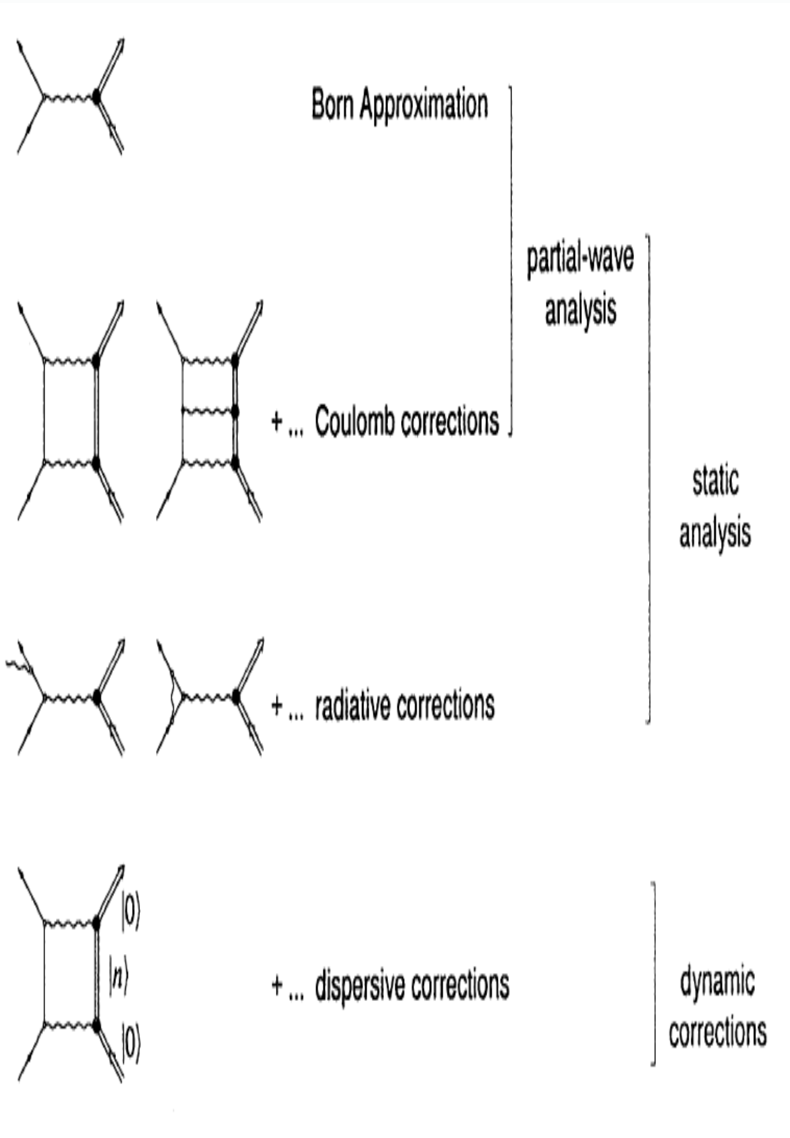
Cross-section results for boron carbide and lithium with respect to carbon

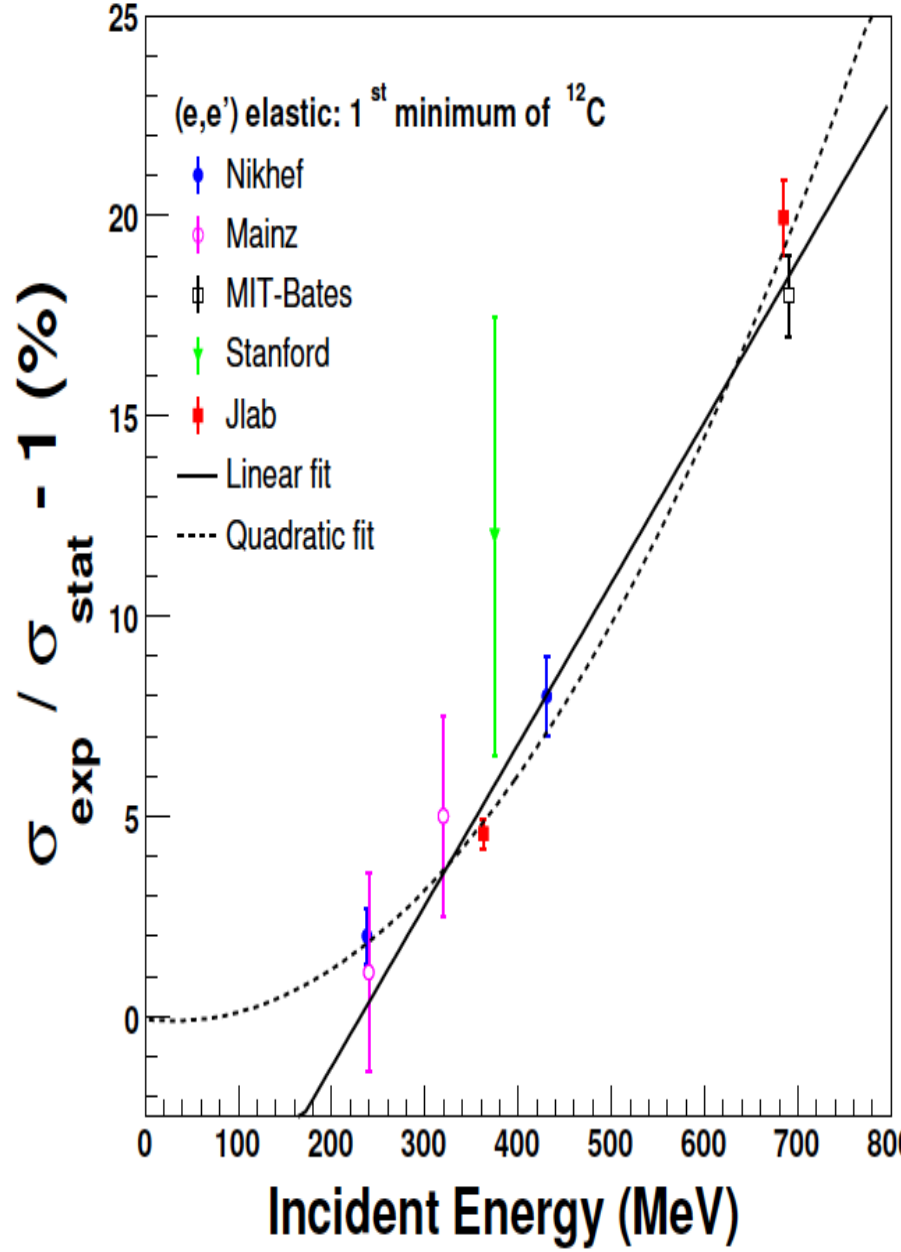


Behaviour at Diffraction Miniama

P. Gueye et.al., in preparation




Born Approximation
partial-wave analysis
+ ... Coulomb corrections
static analysis
+ ... radiative corrections
|0⟩
|n⟩
|0⟩
+ ... dispersive corrections
dynamic corrections


(e,e') elastic: 1st minimum of 12C
Nikhef
Mainz
MIT-Bates
Stanford
Jlab
Linear fit
Quadratic fit
σexp / σstat - 1 (%)
Incident Energy (MeV)
0
5
10
15
20
25
0
100
200
300
400
500
600
700
800

Uncertainties

- **Statistcal ~** $\frac{1}{\sqrt{Counts}}$
- **Systematic:**

Table 4.3: Systematic Uncertainties

Quantity	Normalization [%]	Random[%]
Beam Current	0.5	—
Solid Angle	1.0	—
Composition	0.05	—
Target thickness	0.6	—
Efficiency	—	1.0
Radiation correction	1.0	—
Background subtraction	—-	1.0
Overall	1.53	1.41

Form Factor and its Parametrization

- For spin zero nucleus the form factor is all electric and for an unpolarized beam it is related to cross-section as

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{Mott} \left|F(q)\right|^2$$

- In the kinematics of this experiment the charge form factor is dominant and magnetic and multipole contributions are negligible except for boron .

- The charge form factor can be approximated by a fourier transformation of charge distribution

$$F(q) = \frac{1}{Z}\int_0^\infty \rho(r)\frac{\sin(qr)}{qr}d^3r$$

Form Factor and its Parametrization

The charge distribution can be written as a combination of a complete set of zeroth order Bessel functions of the first kind :

$$\rho(r) = \begin{cases} \sum_{n=1}^{\infty} a_n \dfrac{\sin(\frac{n\pi r}{R})}{\frac{n\pi r}{R}}, r \le R \\ 0, r > R \end{cases}$$

By choosing a cutoff radius beyond which charge distribution would be zero, we get

$$F(q) = \frac{1}{Z}\int_0^R \sum_{n=1}^{\infty} a_n \frac{\sin(\frac{n\pi r}{R})}{(\frac{n\pi r}{R})} \frac{\sin(q.r)}{qr} 4\pi r^2 dr$$

$$= \frac{8}{3q}[a_1 \frac{8\pi\sin(8q)}{\pi^2 - 64q^2} - a_2 \frac{8\pi\sin(8q)}{4\pi^2 - 64q^2} + a_3 \frac{8\pi\sin(8q)}{9\pi^2 - 64q^2} - a_4 \frac{8\pi\sin(8q)}{16\pi^2 - 64q^2} + ...]$$

Thus, if we know the values of F(q) around the roots of the Bessel function, the coefficients can be found by solving a set of linear equations.

Charge Radius

- **Charge radii have been extracted using mathematica by evaluating the integral:**

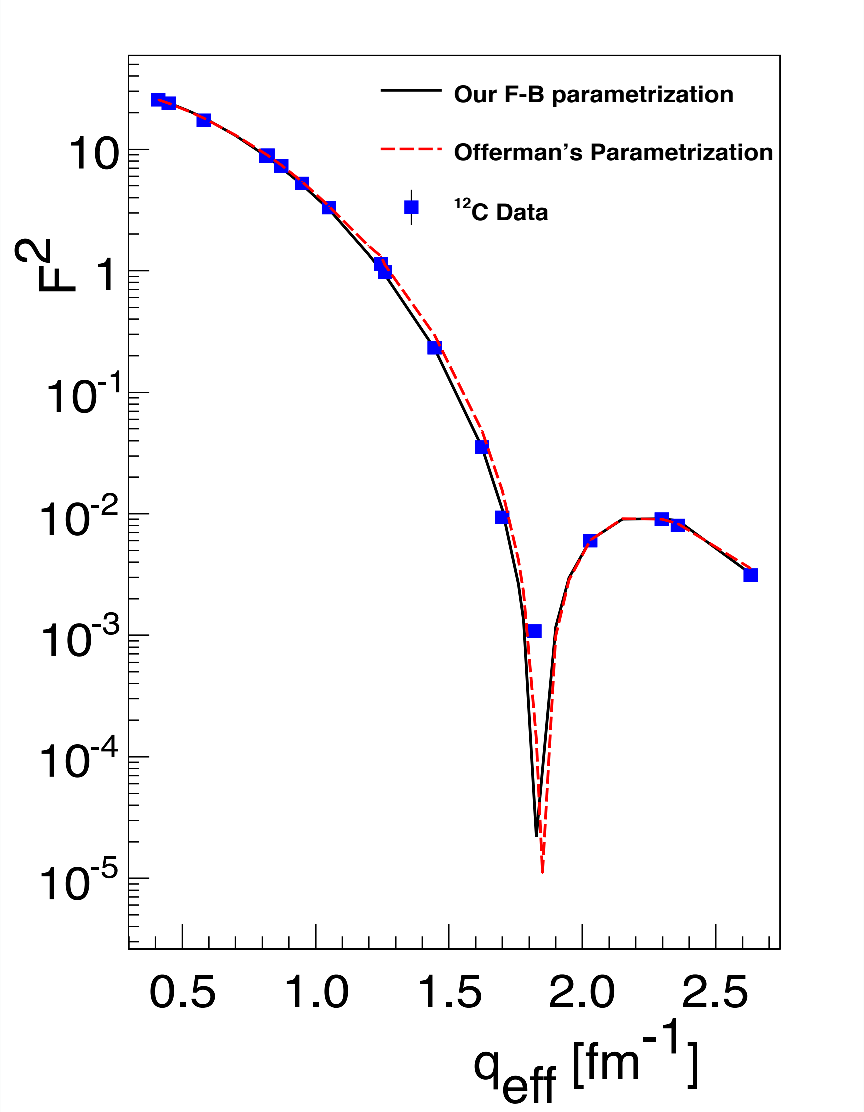
$$\left\langle r^2 \right\rangle_{rms} = \frac{\int_0^{R_{cut}} r^2 \rho(r) d^3 r}{\int_0^{R_{cut}} \rho(r) d^3 r}$$

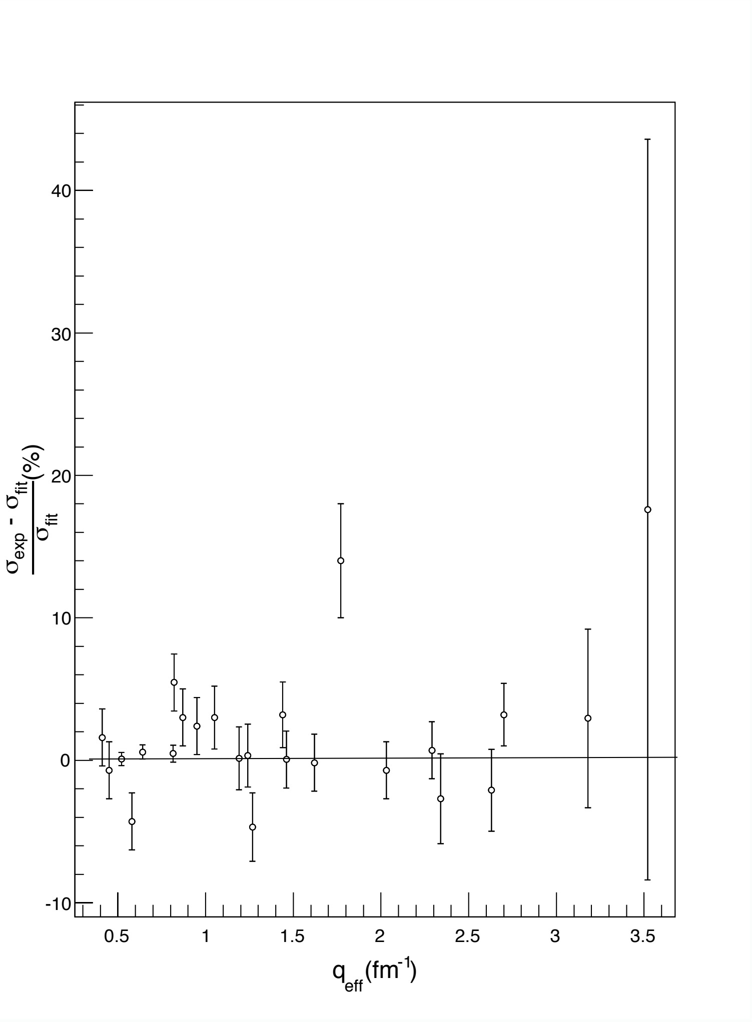
Where,

$$\int_0^{R_{cut}} \rho(r) d^3 r = Z$$

- **The error in the charge radii are found through a Monte Carlo simulation Technique.**

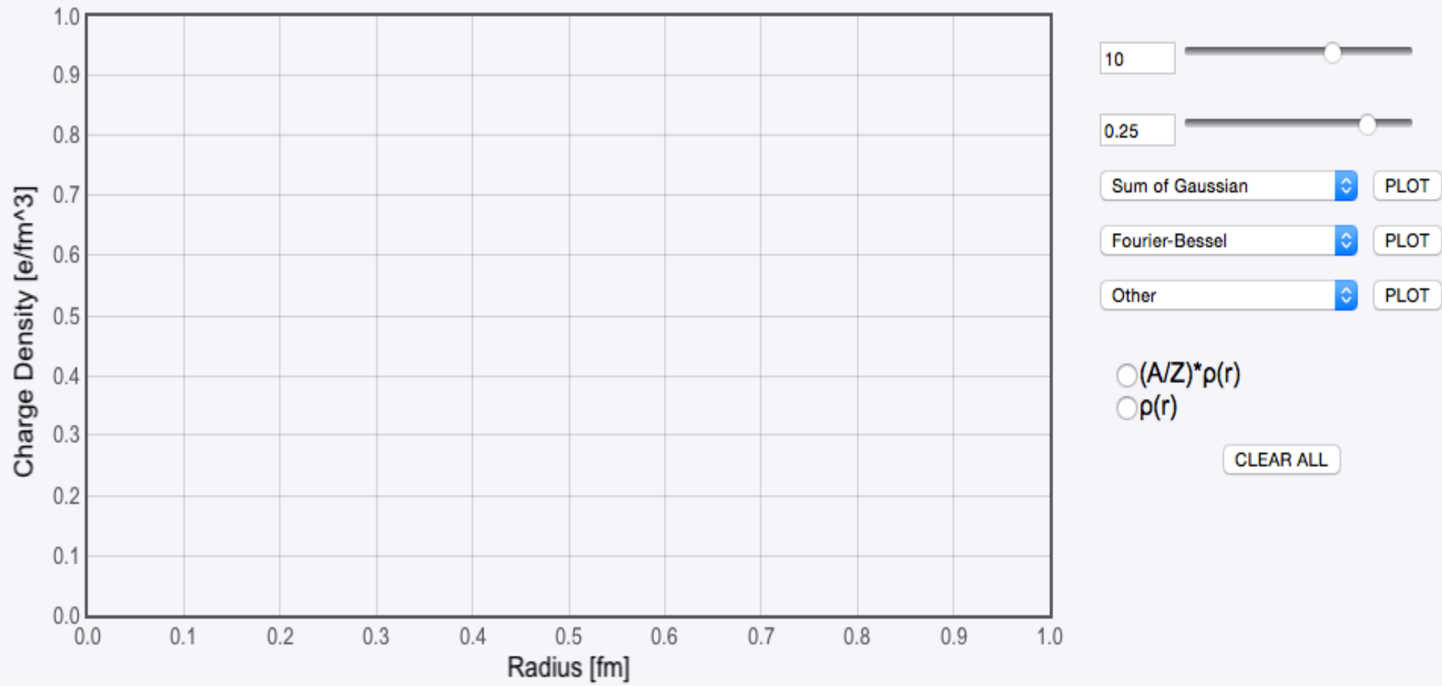
F-B Parametrization Result for Carbon





Home Plot Download Charge Radii About

Nuclear Charge Density Archive - Plots



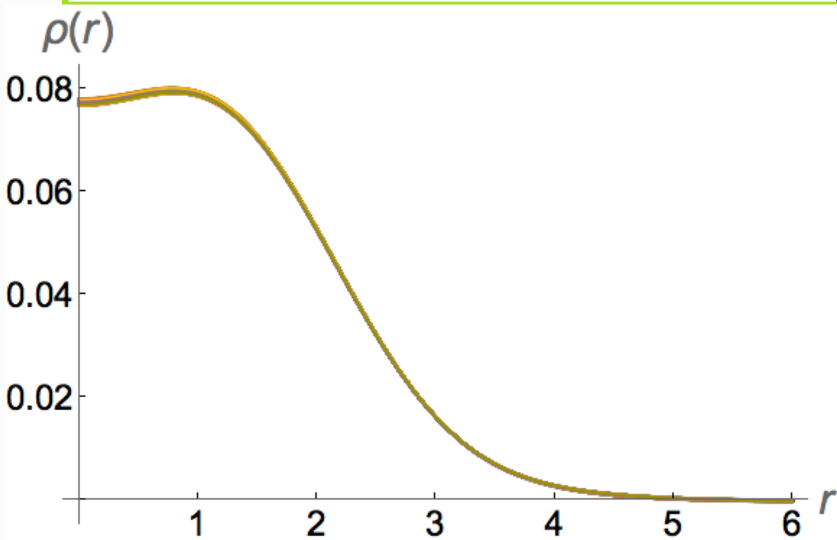
Select a nuclei from the drop-down box on the right. Then, select a normalization type using the radio buttons that appear (a radio button must be selected). Then click 'PLOT' to add the nuclei to the plot. To remove a plot, simply click the 'REMOVE' button next to the nuclei you would like to remove from the table on the left. To updat the axes, use the sliders on the right side, then click 'PLOT' to update. To save an image of the graph, grab a screenshot of the page'

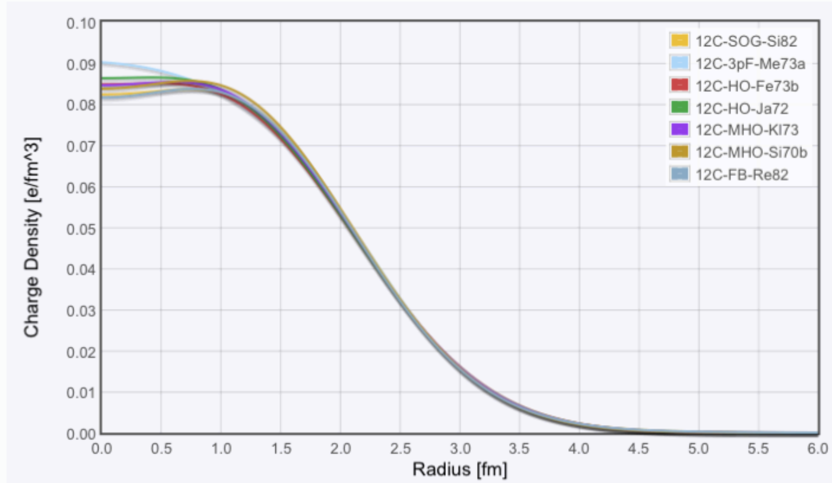
Note that 'non-normalized' (ρ(r)) graphs do not have their plots scaled, whereas 'normalized' ((A/Z)*ρ(r)) graphs are scaled by a factor of (A/Z).

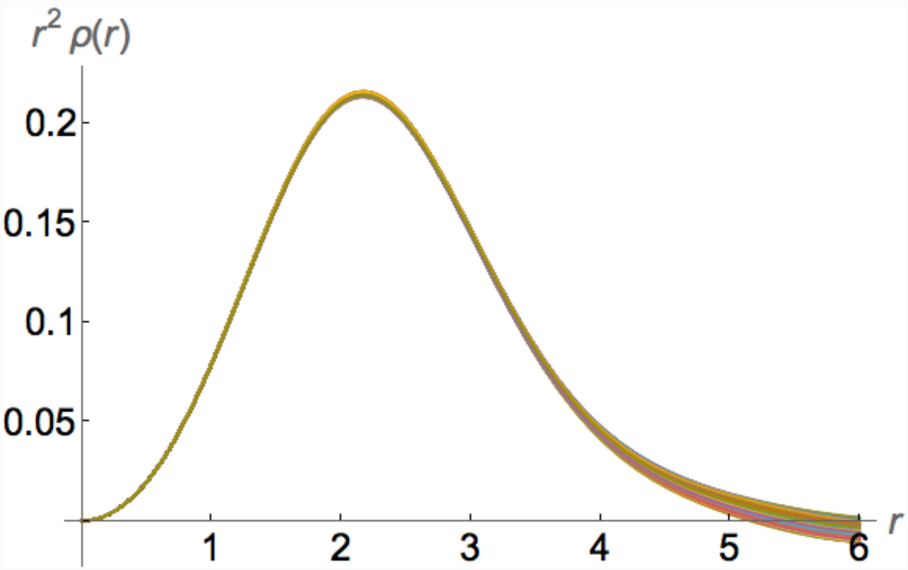
Information about specific plot models can be found on the About page.

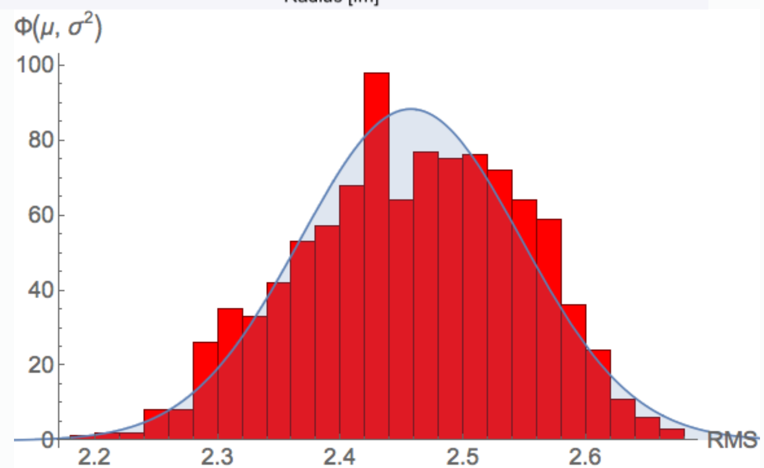
http://faculty.virginia.edu/ncd/plot.html

Charge Density Distribution of Carbon and its Charge Radius





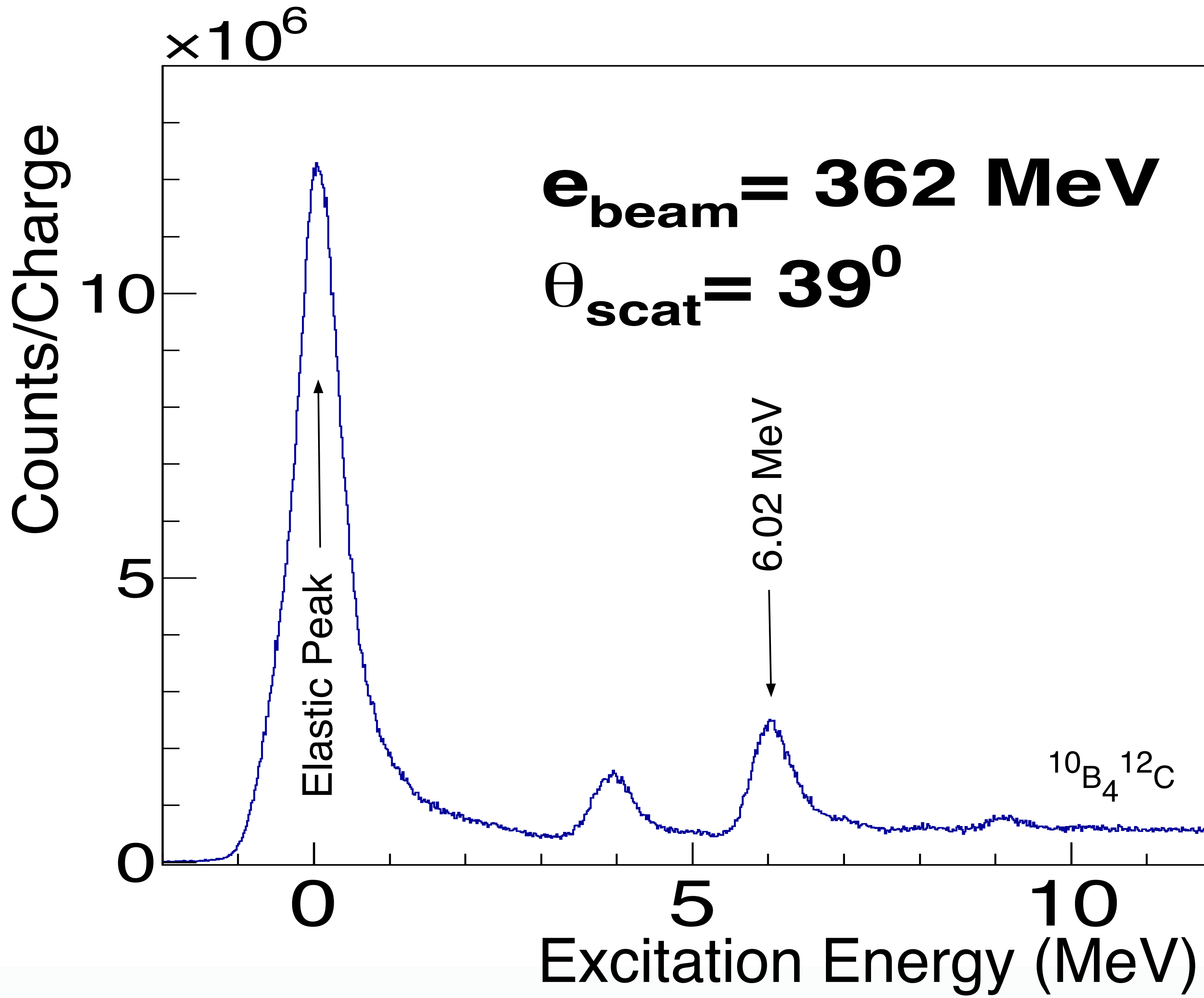




Our Result, r_{rms} =2.45 ± 0.09 fm

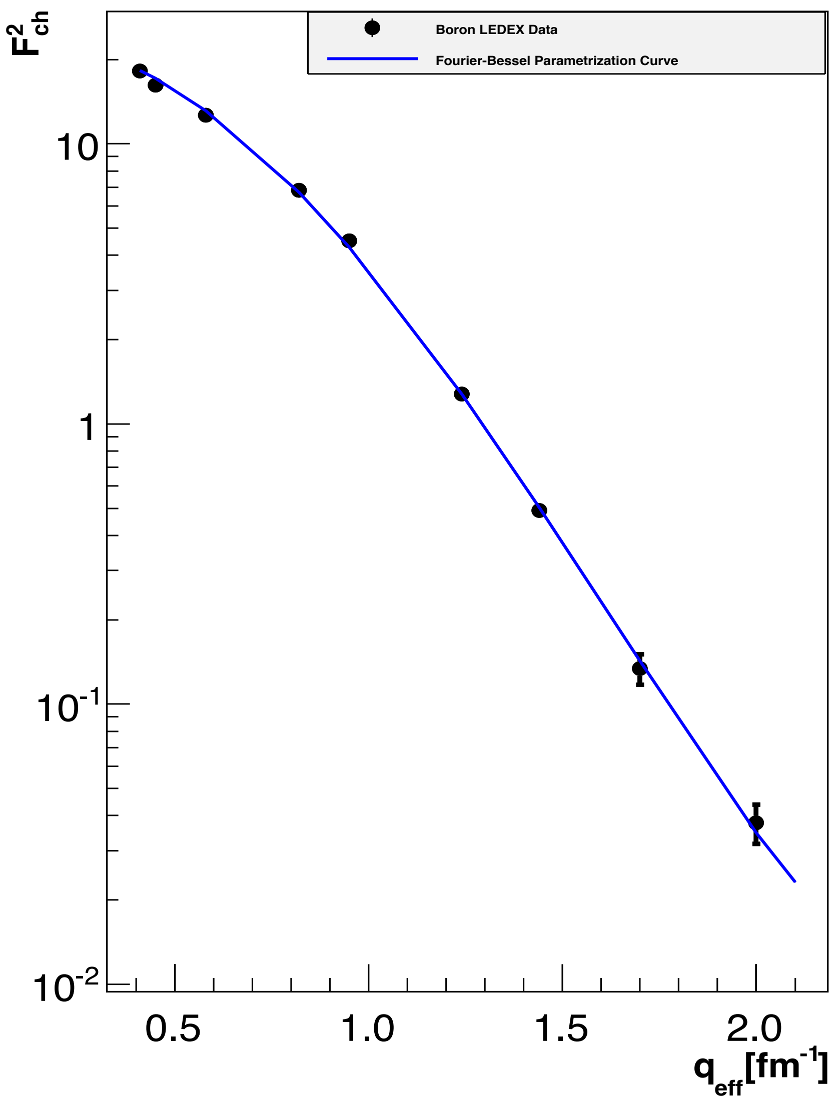
GFMC Result, r_{rms}=2.46 fm

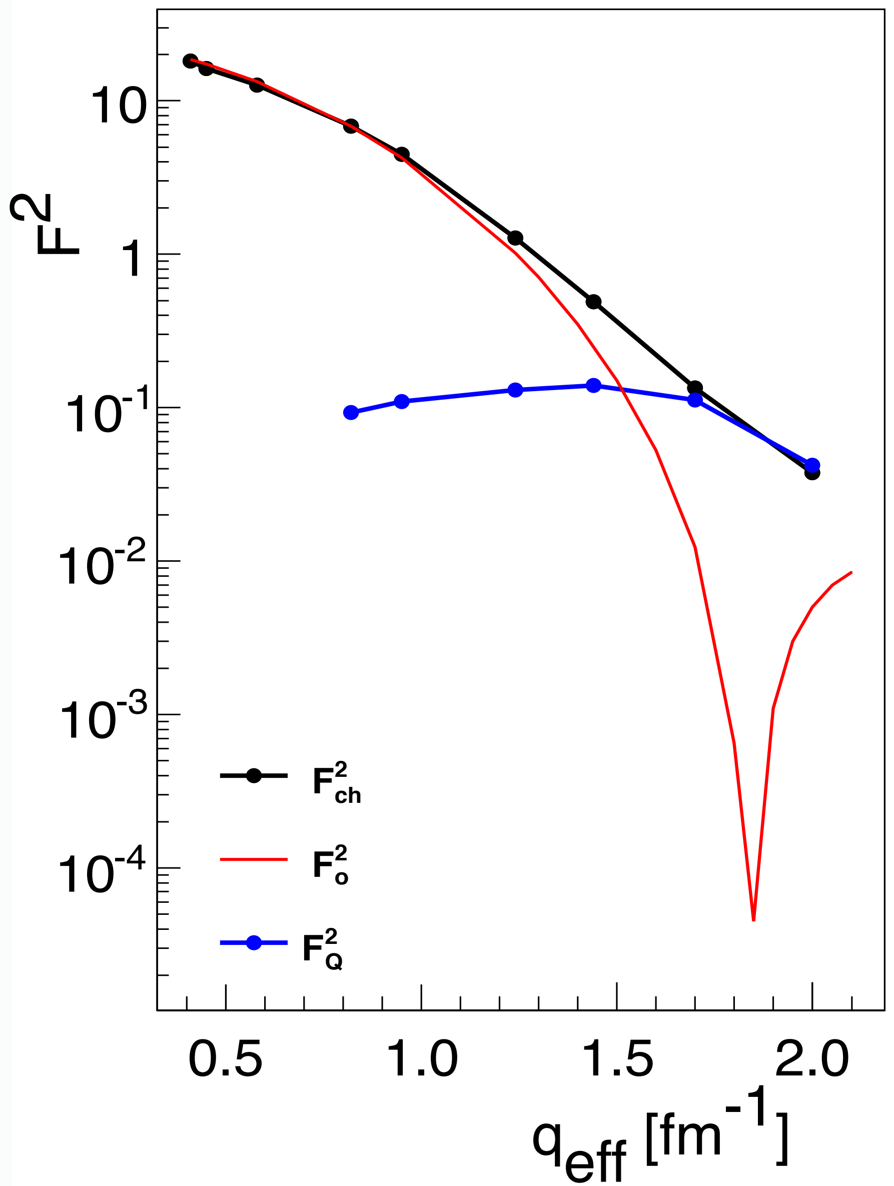
Excitation Spectra for Boron-Carbide



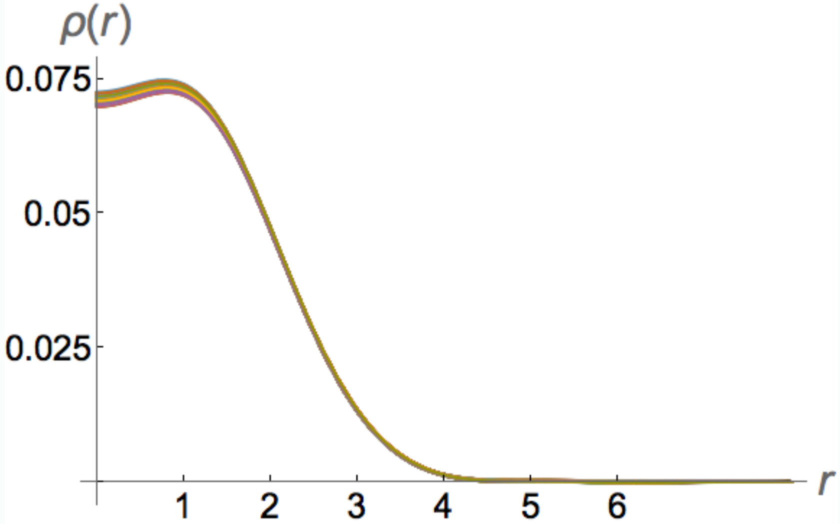
F-B Parametrization Results for Boron

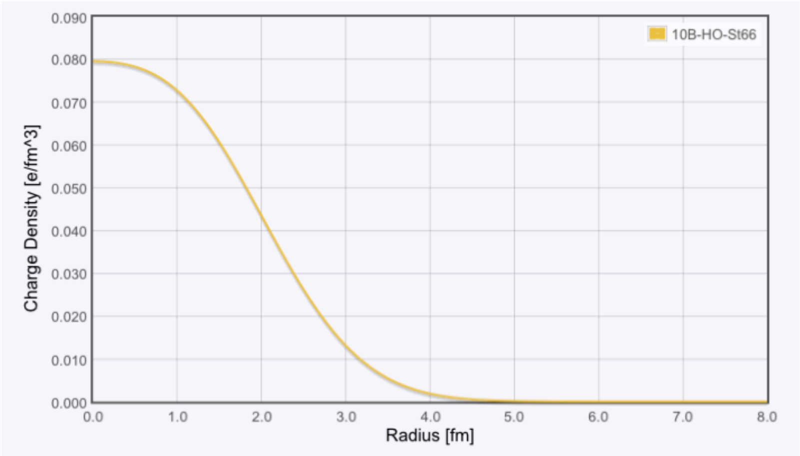
A.A Kabir et. al., in preparation

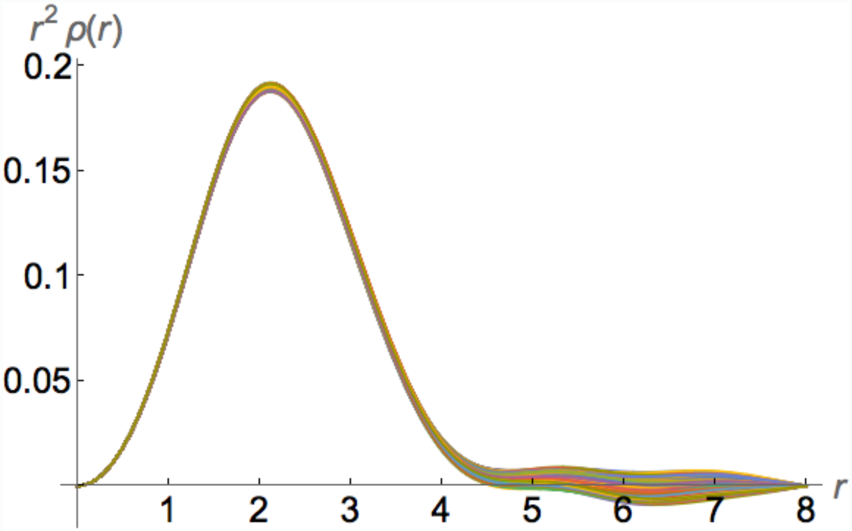


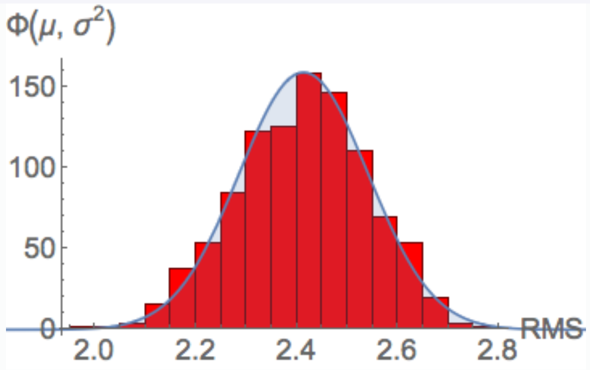


Charge Density and Charge Radius for Boron





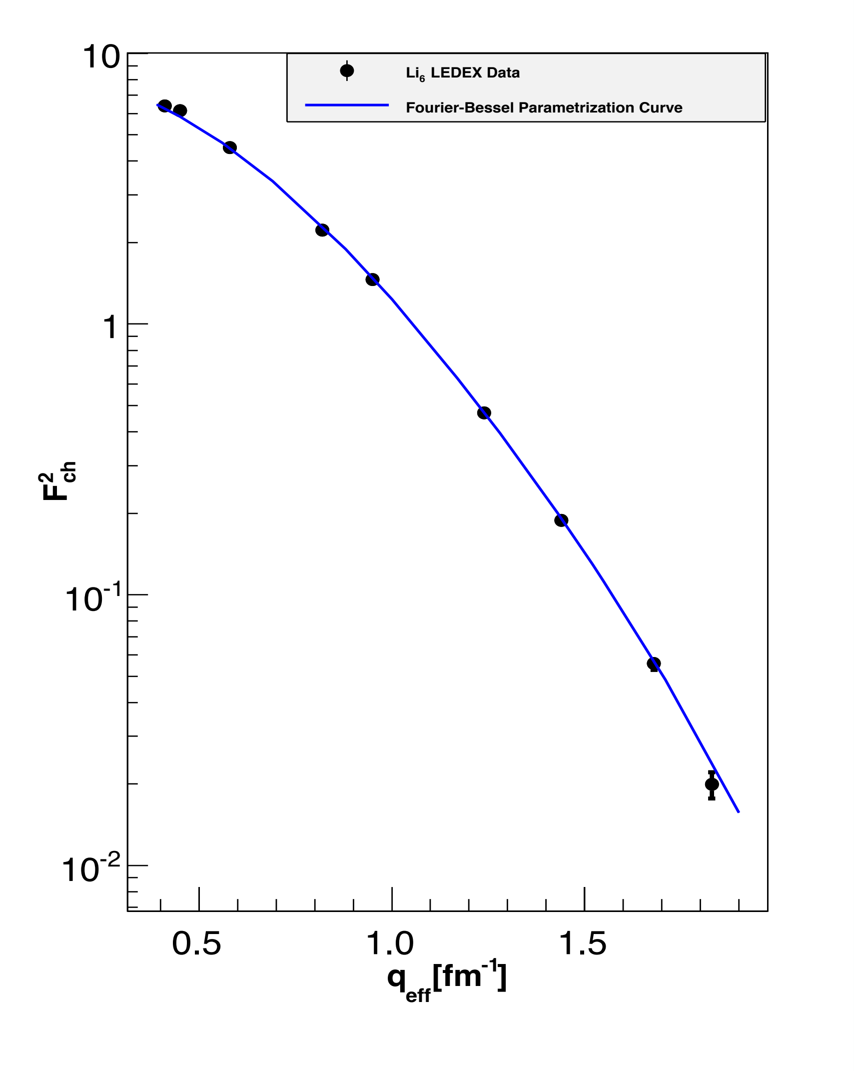


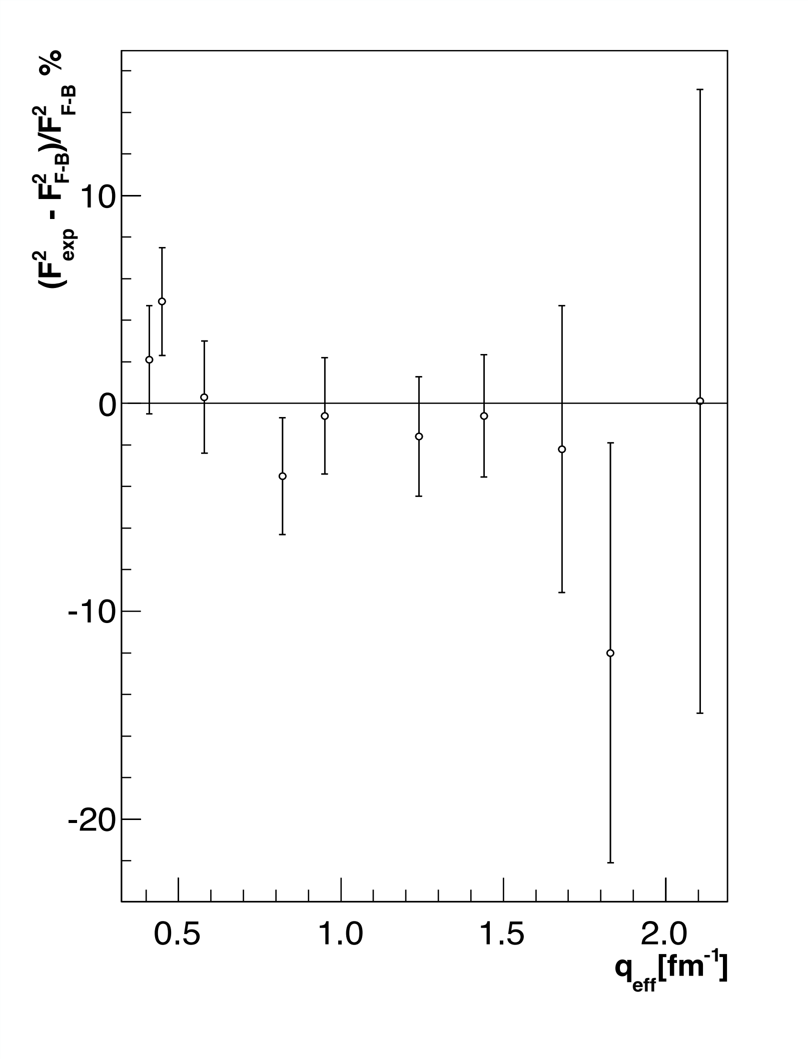


Our result, r_{rms} =2.40 ± 0.122 fm

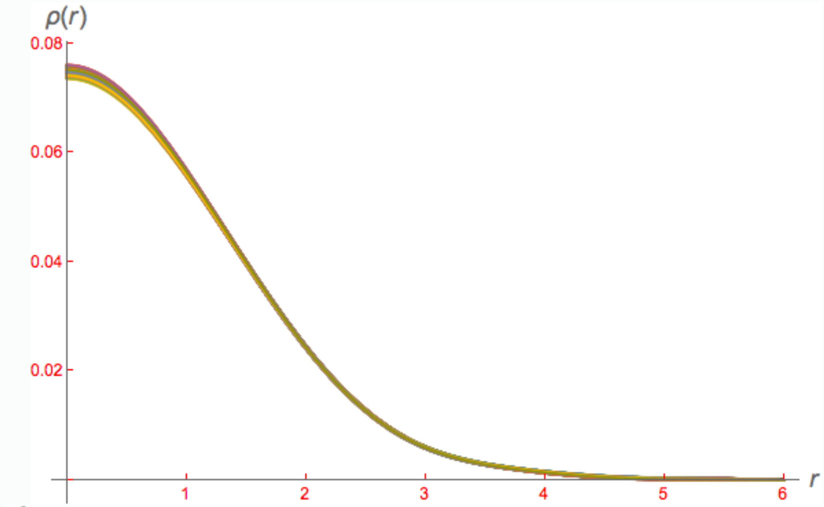
GFMC Result, r_{rms} =2.43 fm

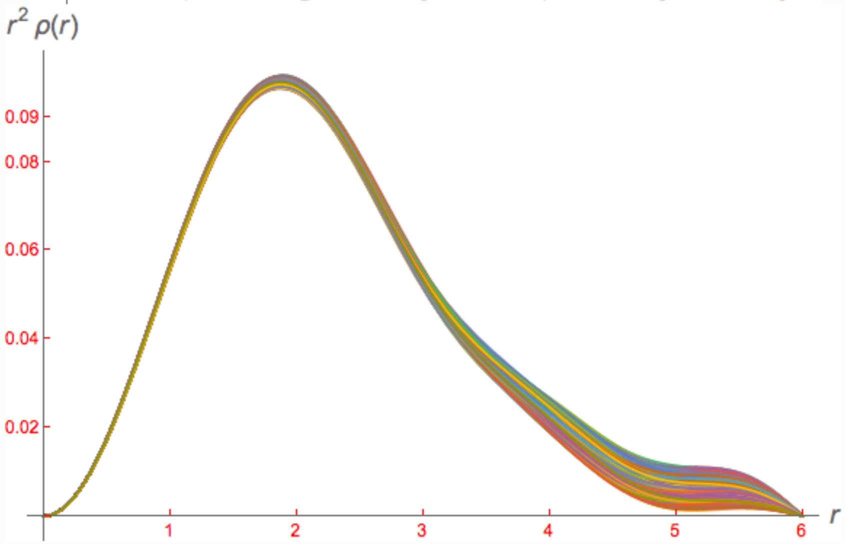
F-B Parametrization Results for Lithium

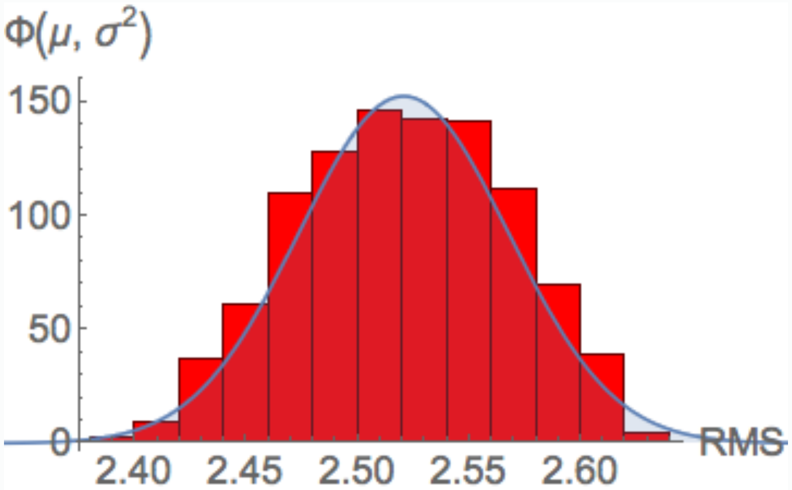




Charge Density and Charge Radius for Lithium







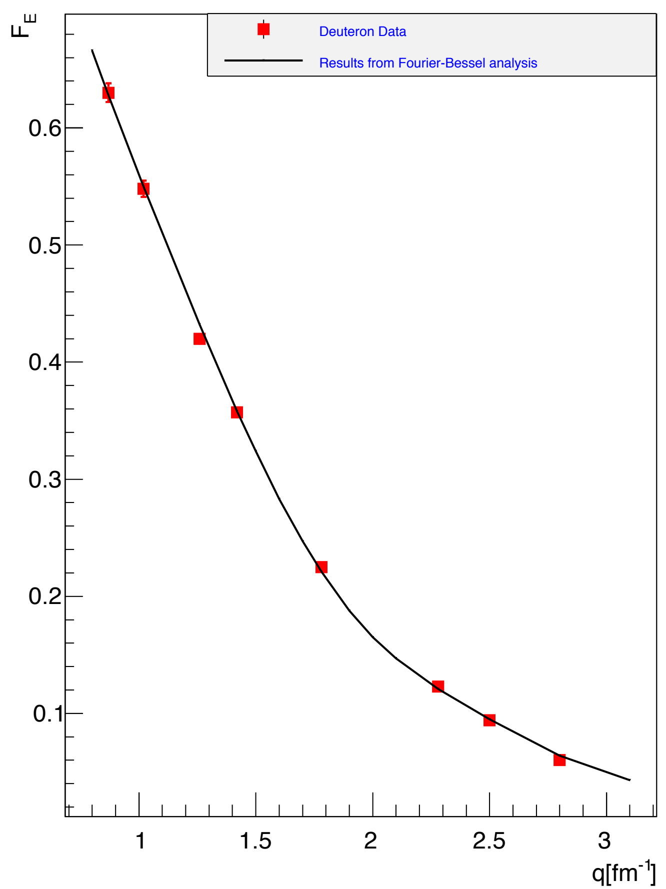
Our F-B Parametrization Result

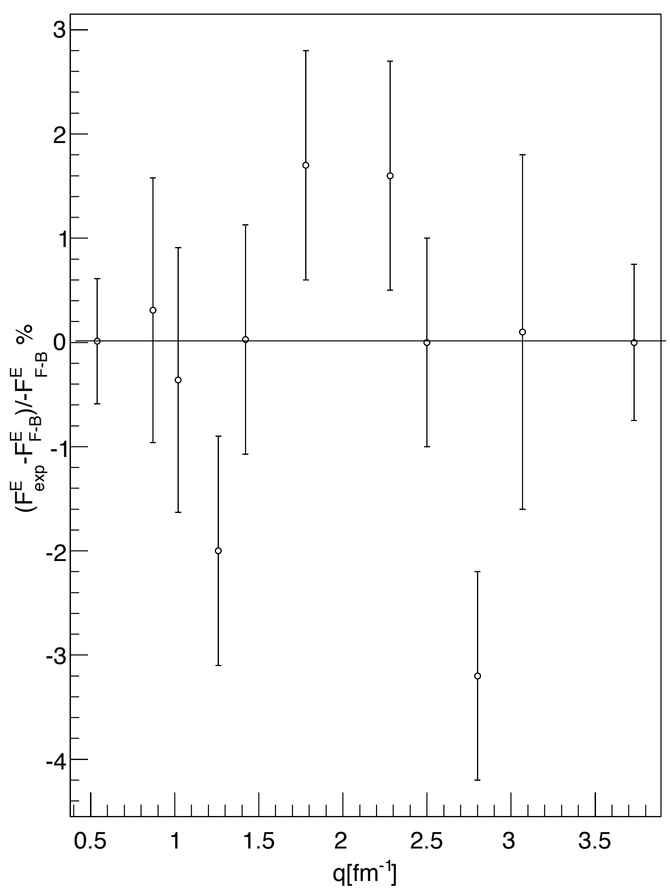
r_{rms} =2.52 ± 0.05 fm

GFMC Result

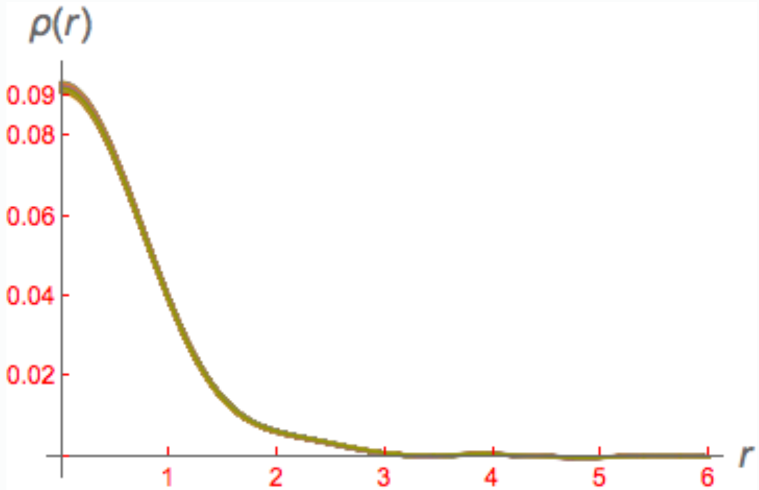
r_{rms} =2.53 fm

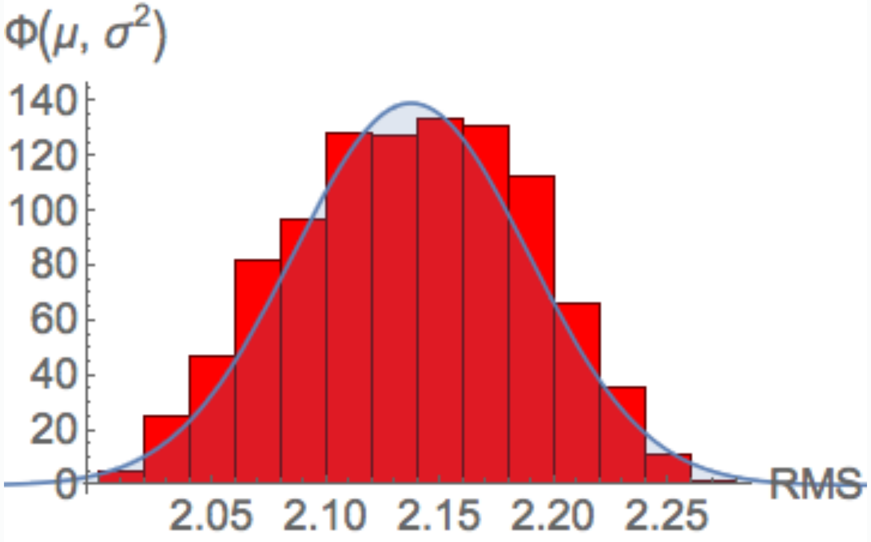
F-B Parametrization Results for Deuterium

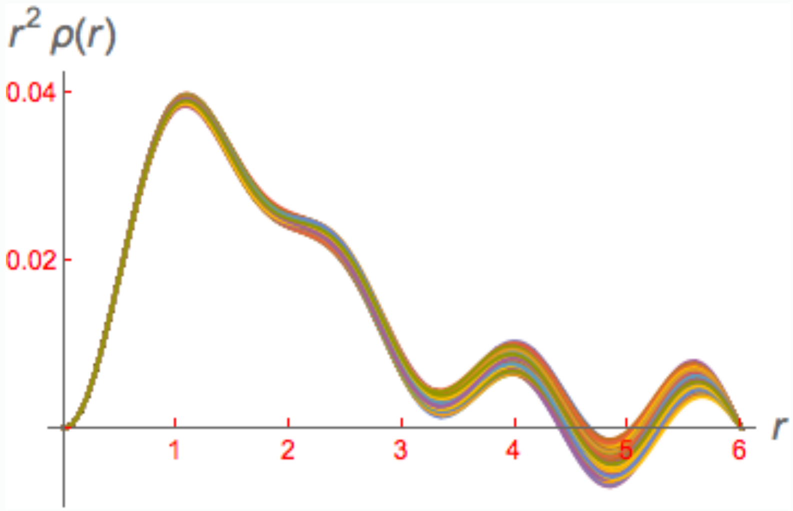




Charge Density and Charge Radius for Deuterium



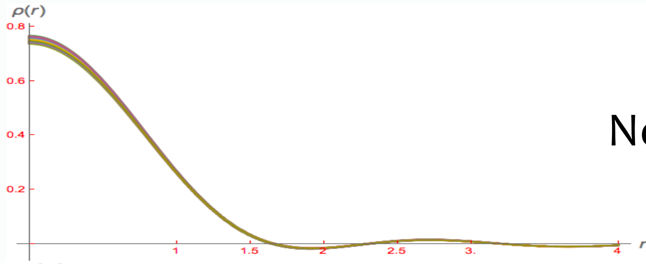




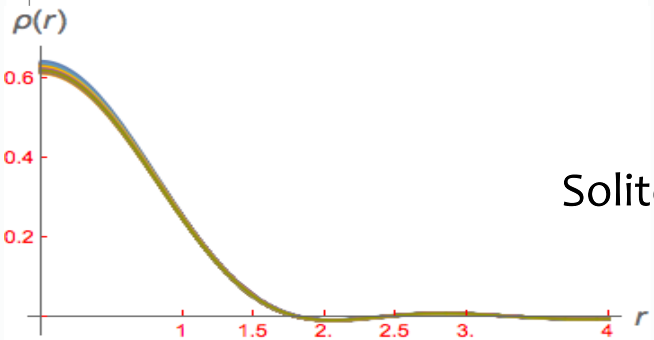
Our F-B Parametrization result:
r_{rms} =2.136 ± 0.051 fm

GFMC Result
r_{rms} = 2.146 fm

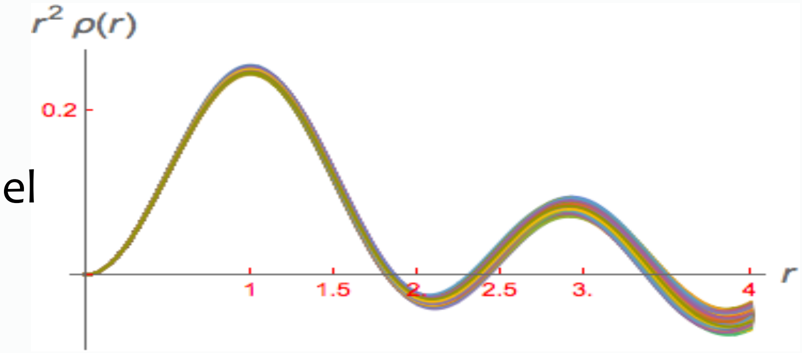
F-B Parametrization result for Hydrogen

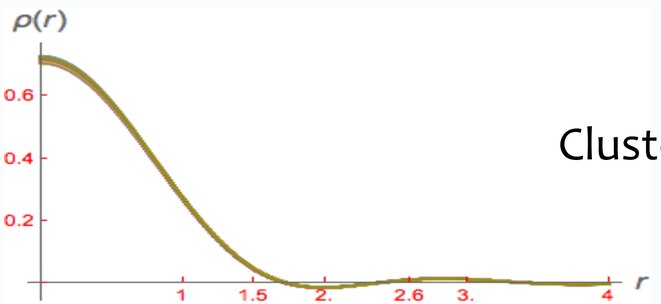


No recoil

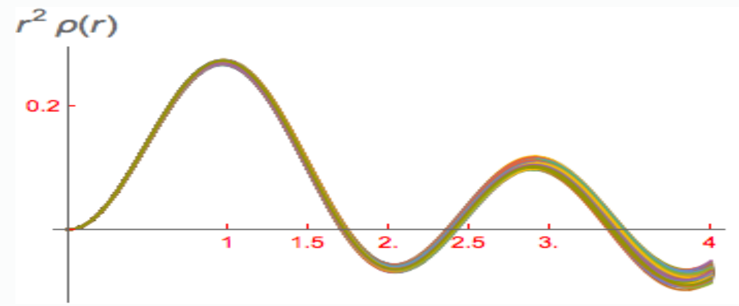


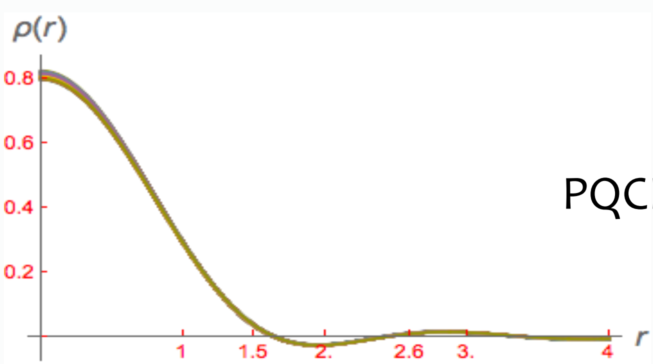
Soliton Model





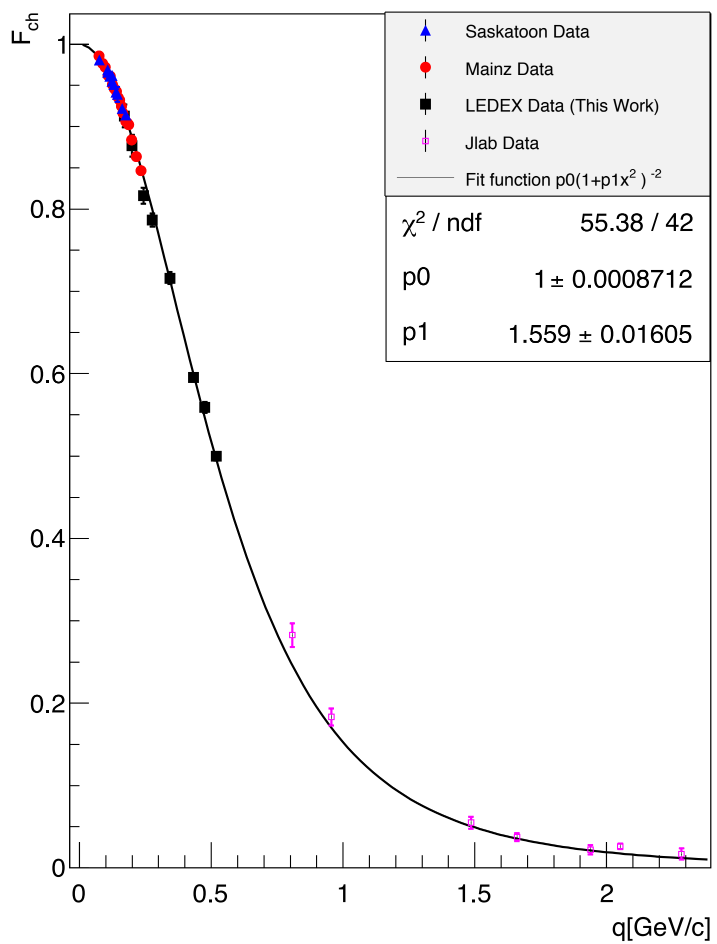
Cluster Model

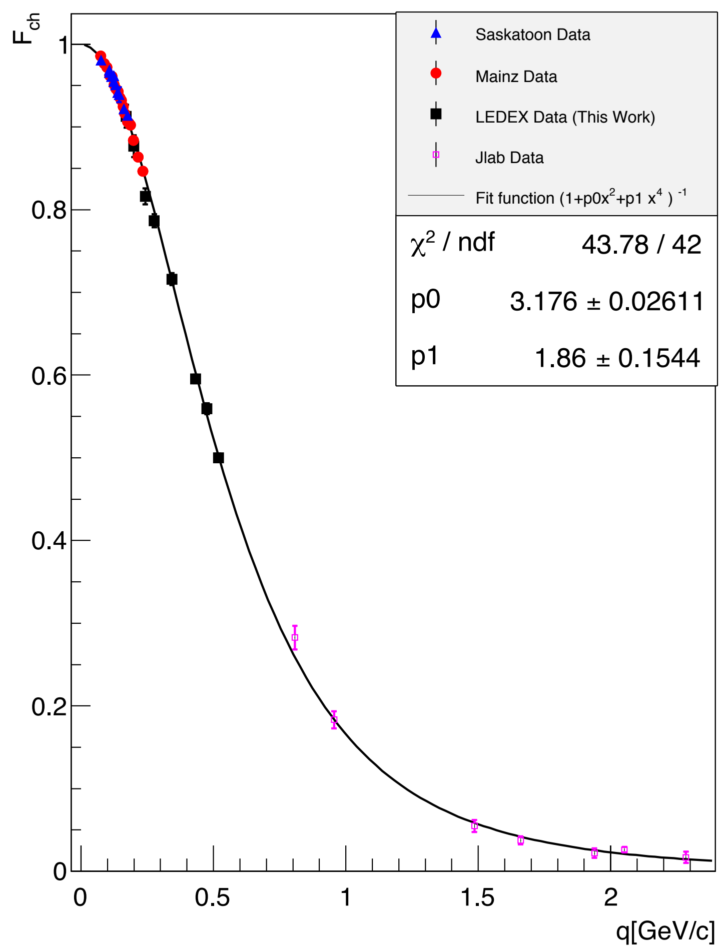




PQCD Model

Hydrogen Charge Radius Result from World Data Set





Breakdown of F-B parametrization technique

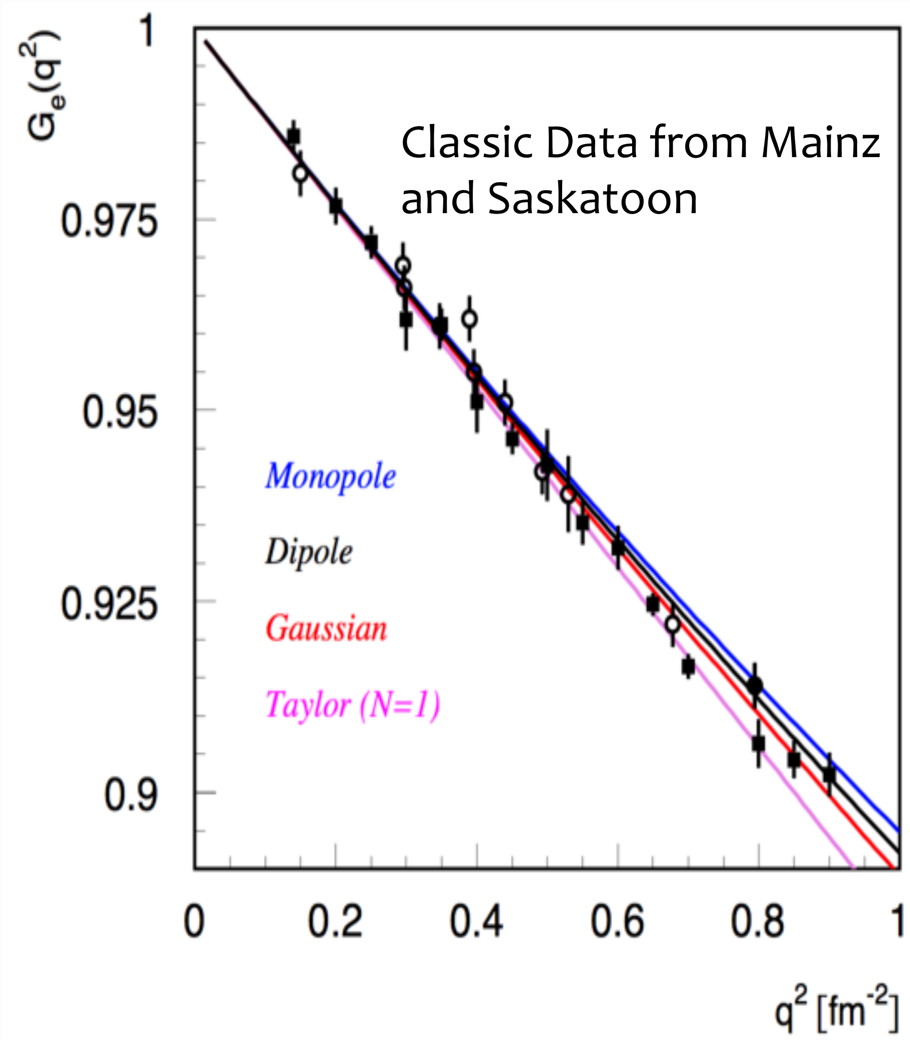
- F-B Parametrization technique does not work work well for very light nuclei.
- Needs unrealistically large number of Fourier-Bessel coefficients.
- Solution is to go with the usual technique of finding the slope of the form factor at q=0, as

$$\begin{aligned}
F(q) &= \frac{1}{Z}\int_0^R \rho(r)\frac{1}{qr}[qr - \frac{(qr)^3}{3!} + \frac{(qr)^5}{5!}]d^3r \\
&= \frac{1}{Z}\int_0^R \rho(r)d^3r - \frac{q^2}{6Z}\int_0^R \rho(r)r^2 d^3r + \ldots \\
&= 1 - \frac{q^2}{6} < r_{rms}^2 > + \ldots \\
\therefore < r_{rms}^2 > &= -6\frac{dF(q)}{dq^2}|_{q\to 0}
\end{aligned}$$

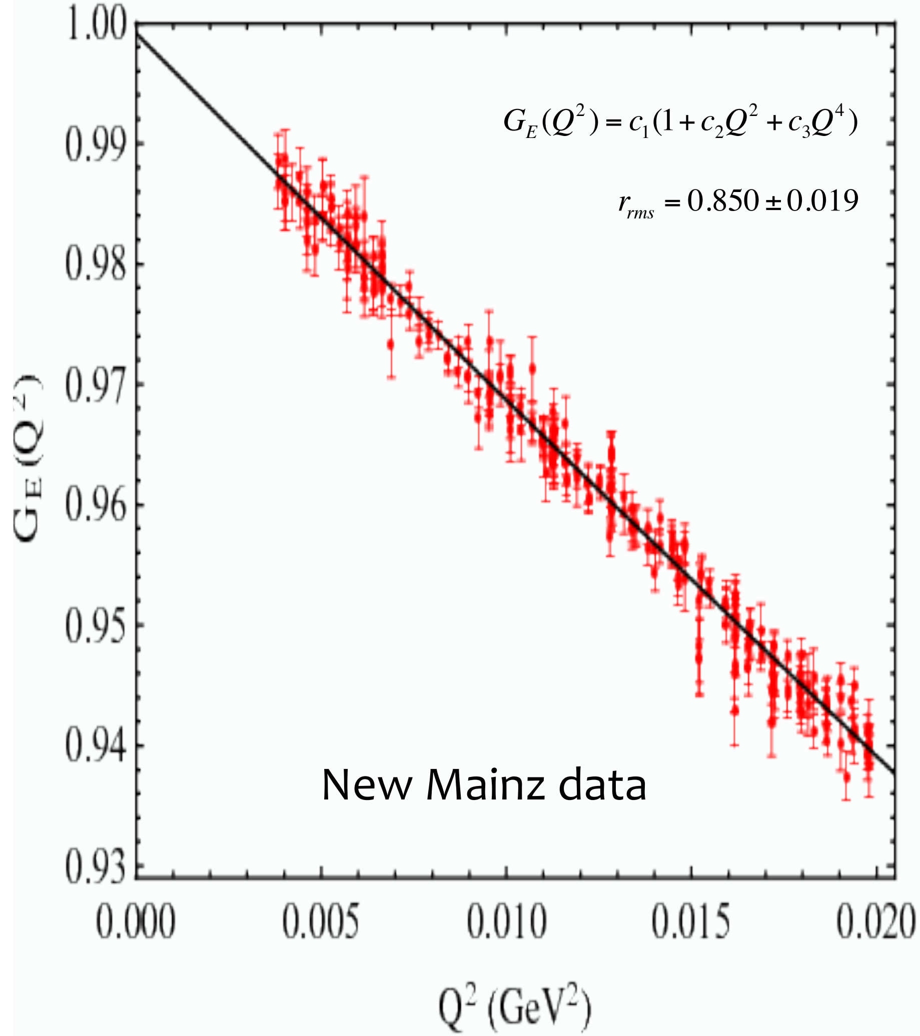
- But this introduces new problems. The fit function needs to be extrapolated in the region where we do not have experimental data and different fit functions produce different results.
- Next, we focus on the lowest momentum transfer data available where the definition of rms radius as a slope of form factor is more appropriate.

^{1}H charge radius result from the lowest momentum transfer Measurements

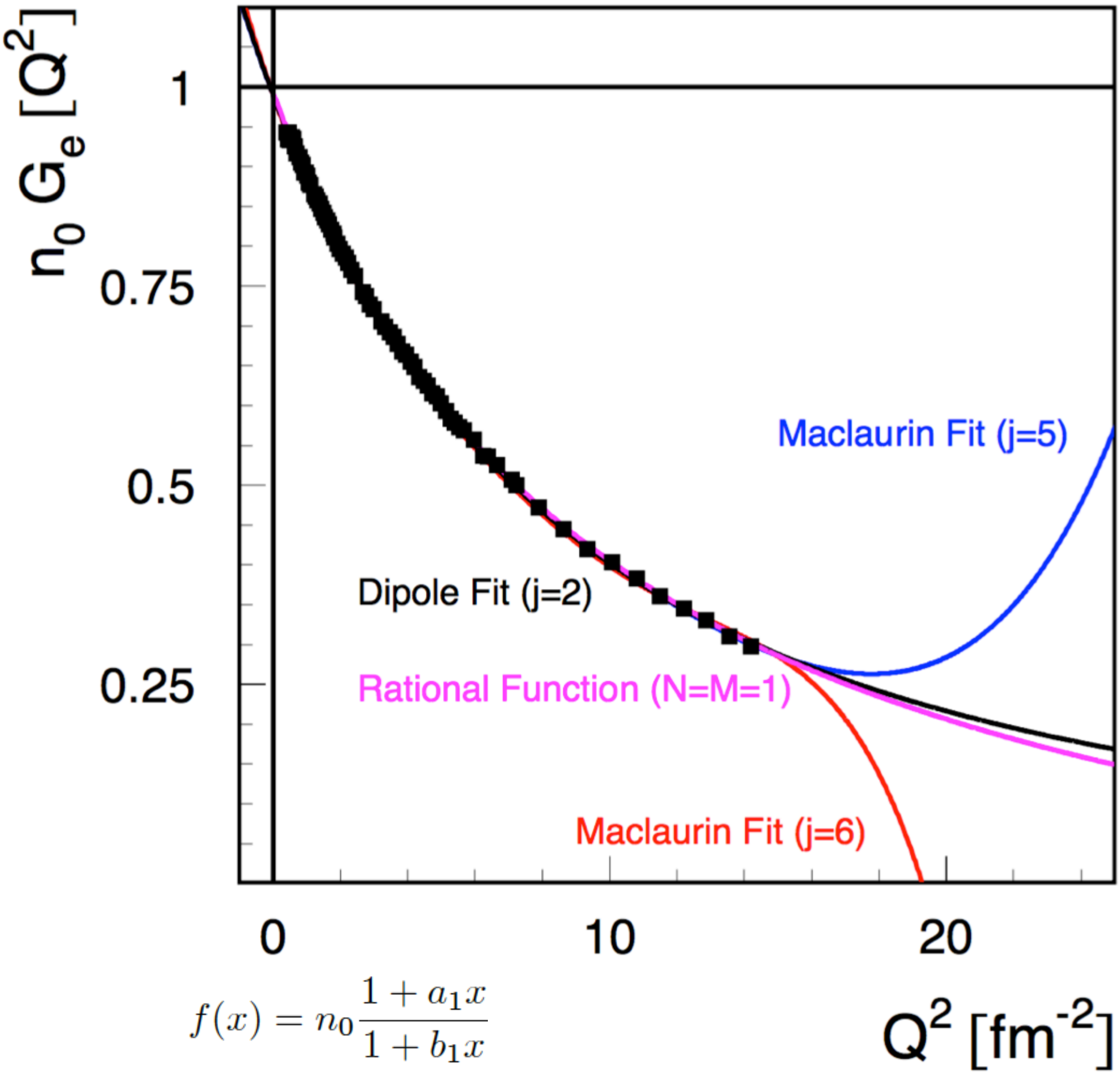
D.W. Higinbotham et.al.,arXiv:1510.01293



K. Griffioen et. al., arXiv: 1509.0667



1(GeV/c)^2=25.684 fm $^{-2}$



$$f(x) = n_0 \frac{1 + a_1 x}{1 + b_1 x}$$

Conclusions

- Elastic scattering data for several light nuclei have been analyze and Form factors are parametrized with Fourier Bessel analysis and charge densities are found.

- The charge radii were determined by integrating over the normalized charge density distribution function and are compared with theoretical calculations.

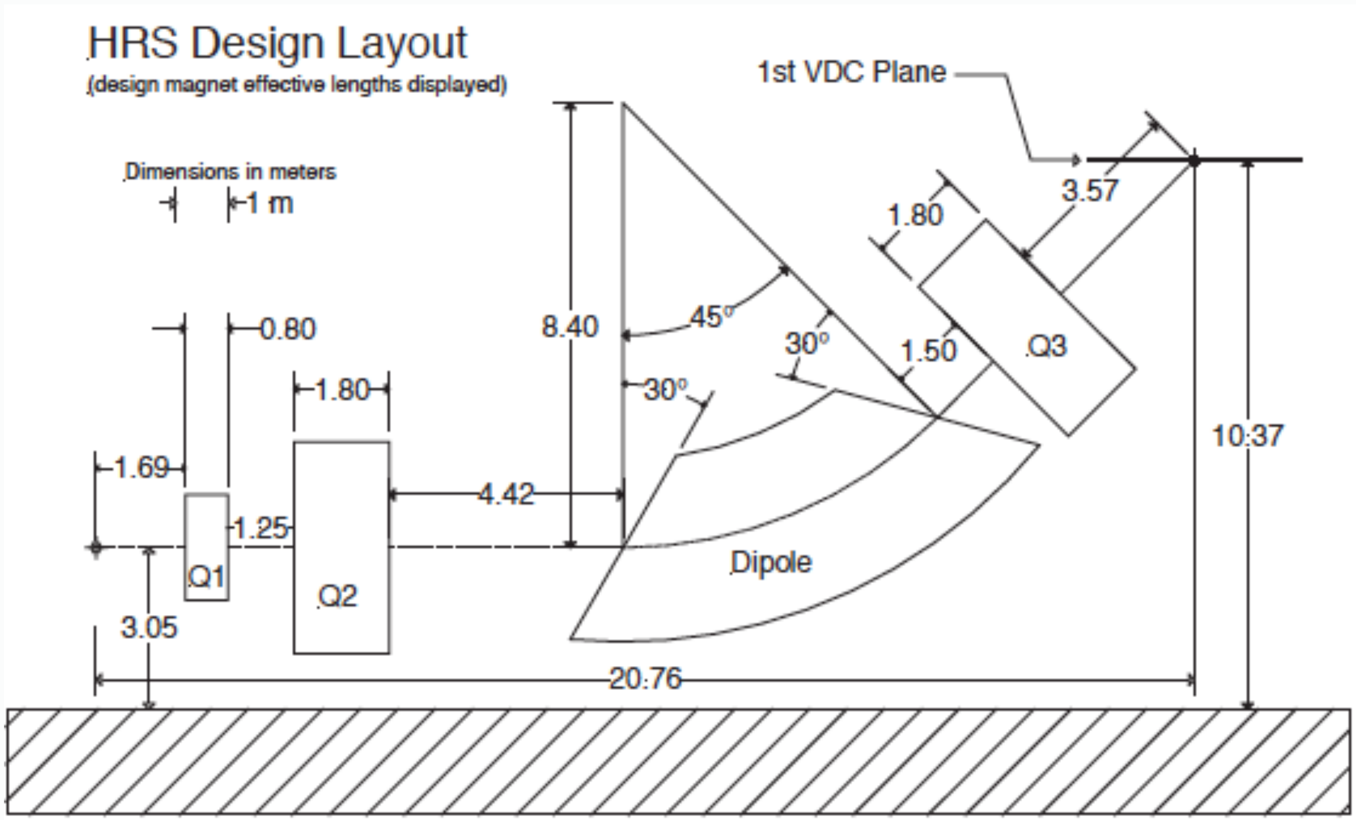
- Results for ^{12}C are consistent with previous measurements and new results are obtained for ^{10}B and ^{6}Li. The results for carbon, boron, lithium and deuterium are found to be in excellent agreement with the latest theoretical calculations.

- Ambiguities in determining the charge radius for the proton from elastic scattering experiments are discussed. Lowest momentum transfer measurement with model independent fit function yield a charge radius result for hydrogen closer to the muonic hydrogen result.

Beamline

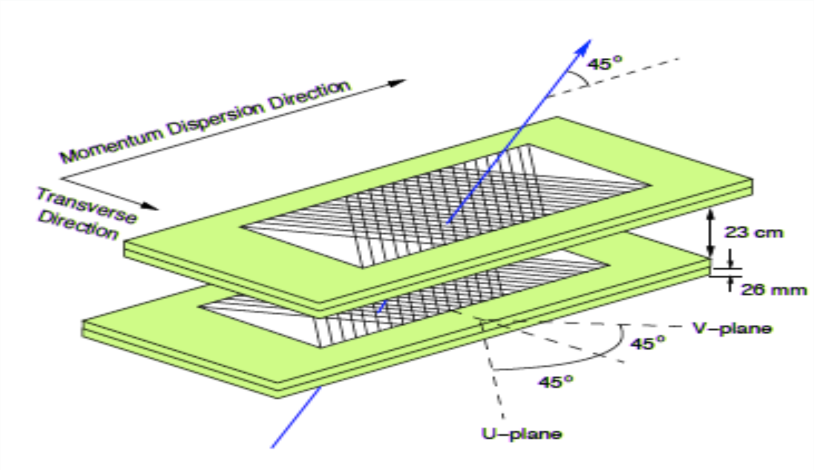
- Beam Energy Measurement
  - The Tiefenbach Method
  - eP Method
- Beam Charge Monitor
  - Unser Monitor
  - Silver Calorimeter
- Beam Positon Monitor

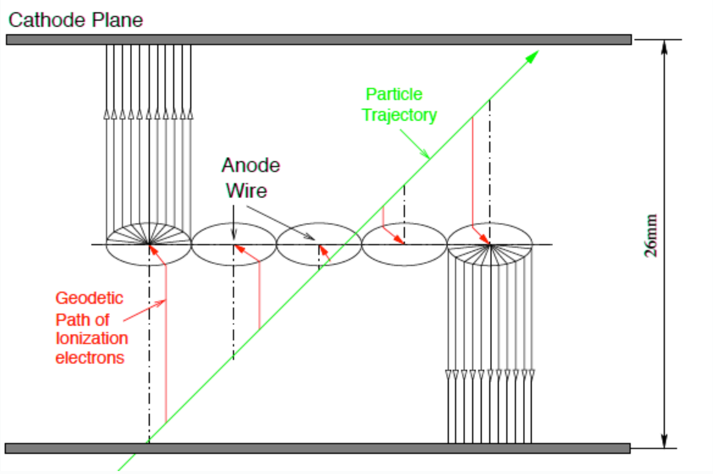
HRS Spectrometer



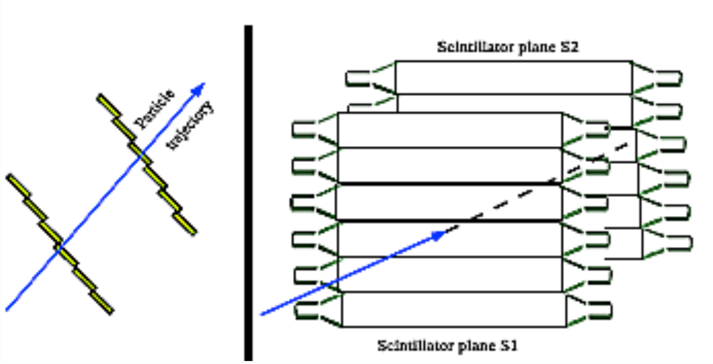
Tracking and Triggering

The VDC

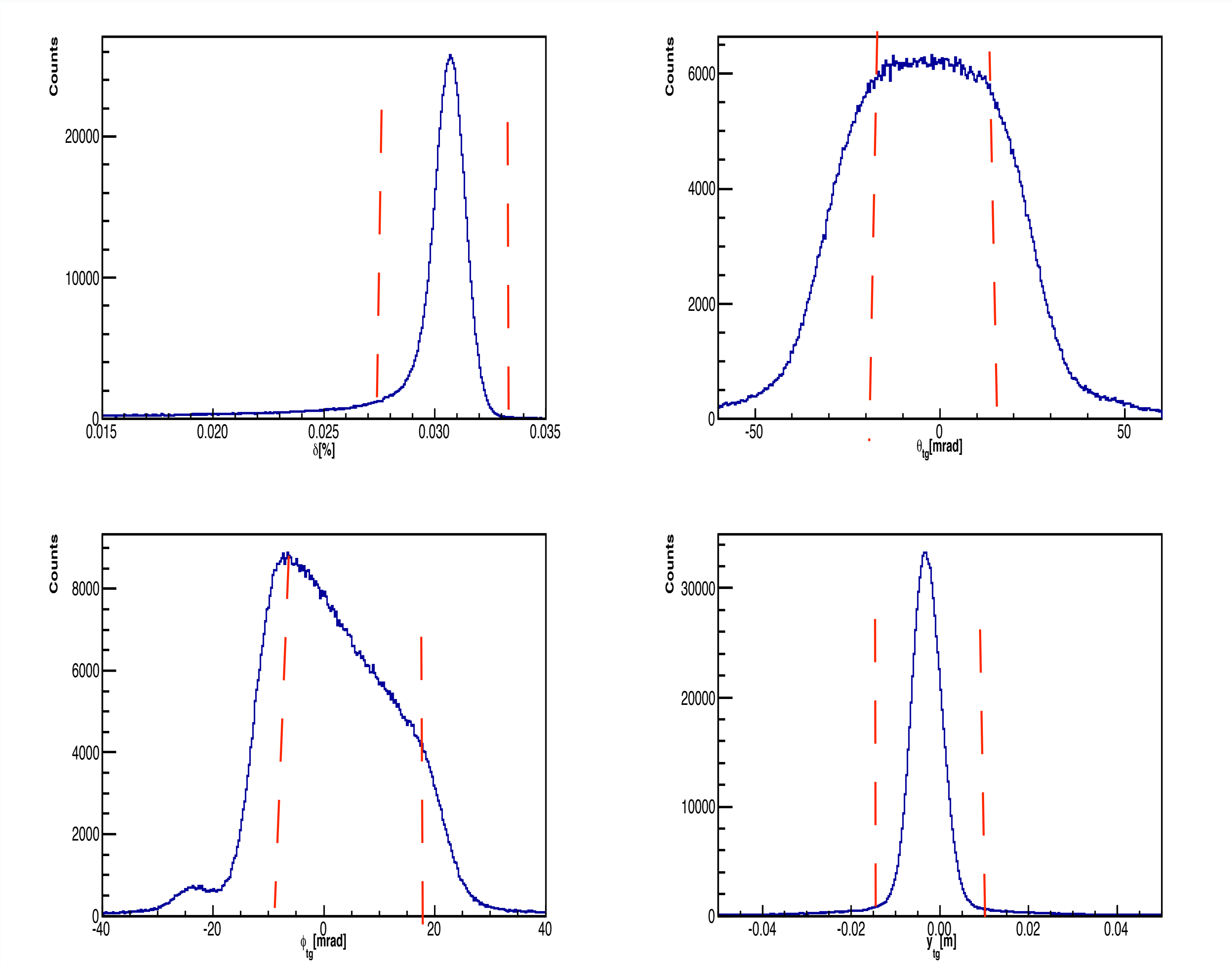


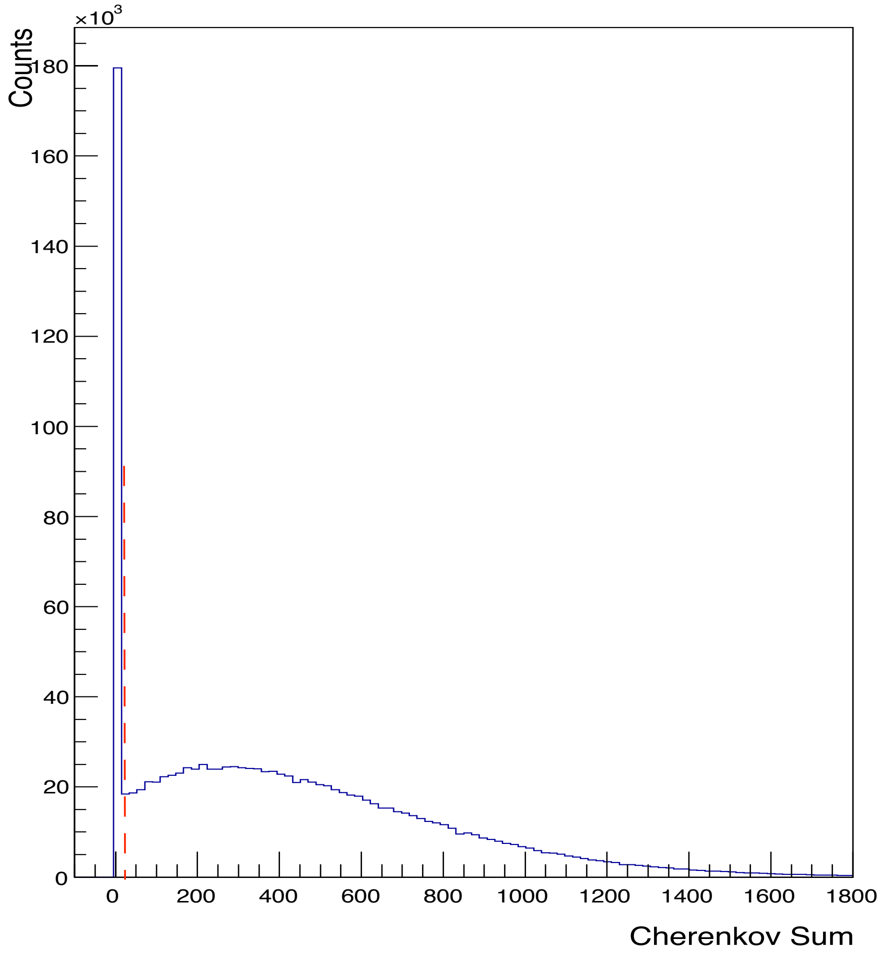


The Scintillator

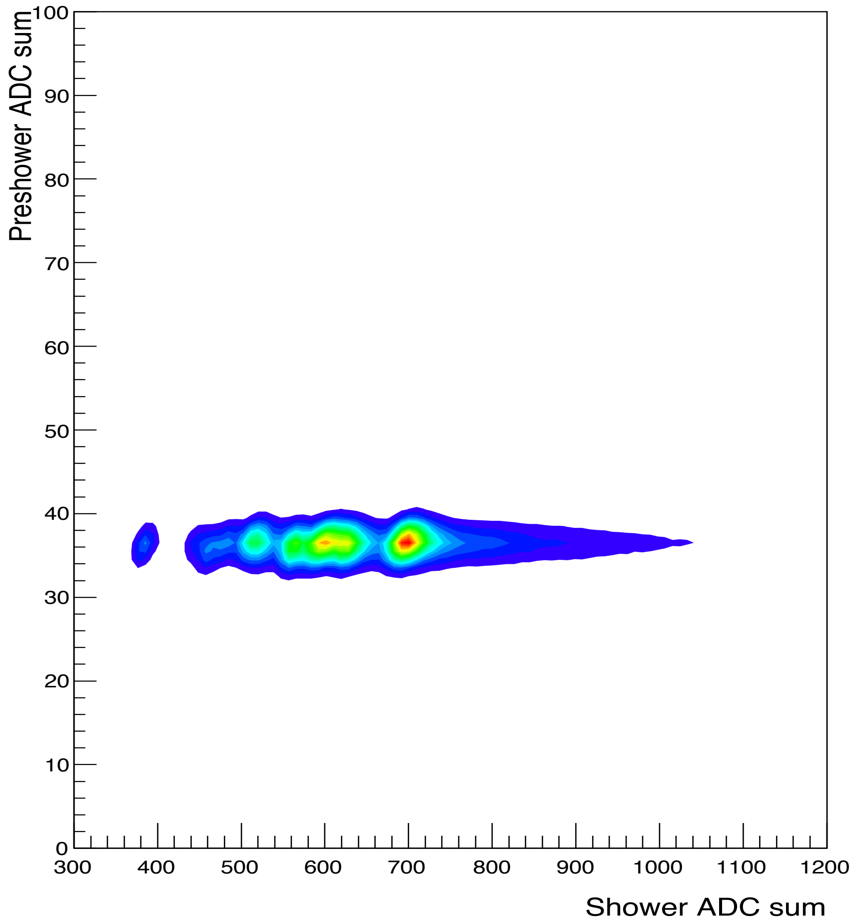


Geometric and Particle Identification Cuts



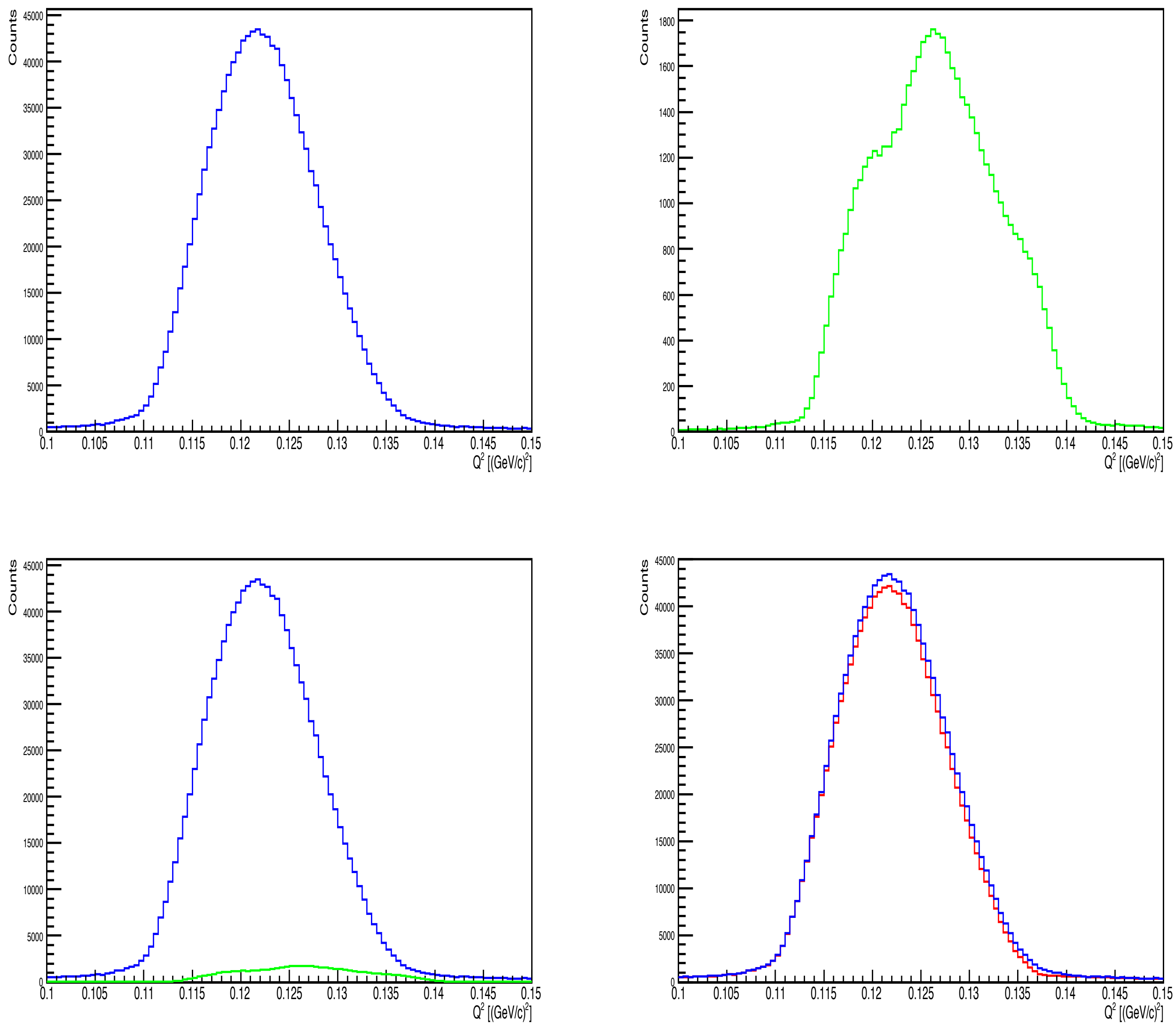


×10^3
Counts
180
160
140
120
100
80
60
40
20
0
0
200
400
600
800
1000
1200
1400
1600
1800
Cherenkov Sum



Preshower ADC sum
100
90
80
70
60
50
40
30
20
10
0
300
400
500
600
700
800
900
1000
1100
1200
Shower ADC sum

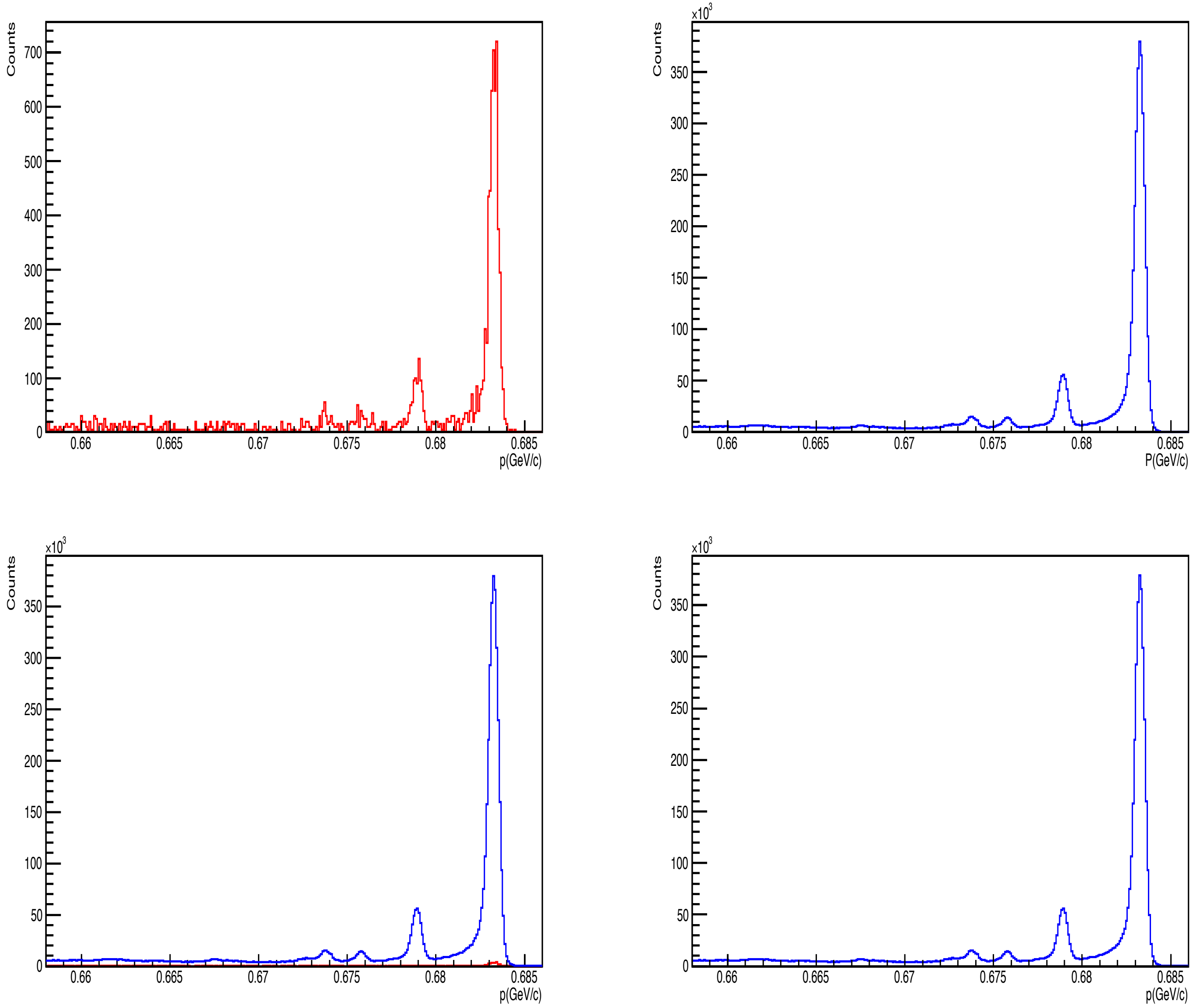
Dummy Subtraction



Efficiencies

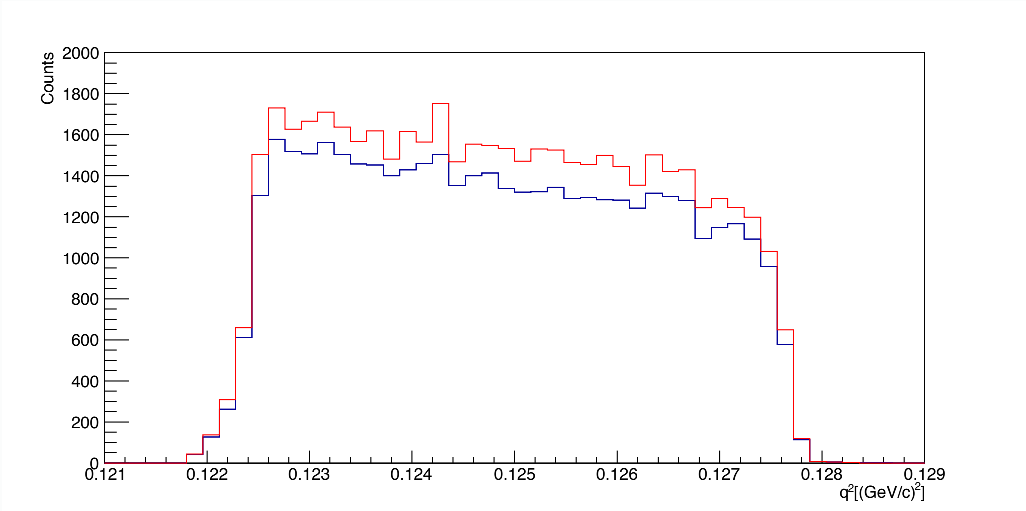
- **Triggering efficiency:** $$\varepsilon_{trig} = \frac{T_3}{T_3 + T_4}$$
- **Tracking efficiency:** $$\varepsilon_{track} = \frac{N_{cut}}{N}$$
- **DAQ Efficiency:** $$\varepsilon_{DE} = \exp(-R\tau)$$
- **Computer Dead time:** $$\varepsilon_{dc} = \frac{PS.N}{N_{trig}}$$

Background Subtraction

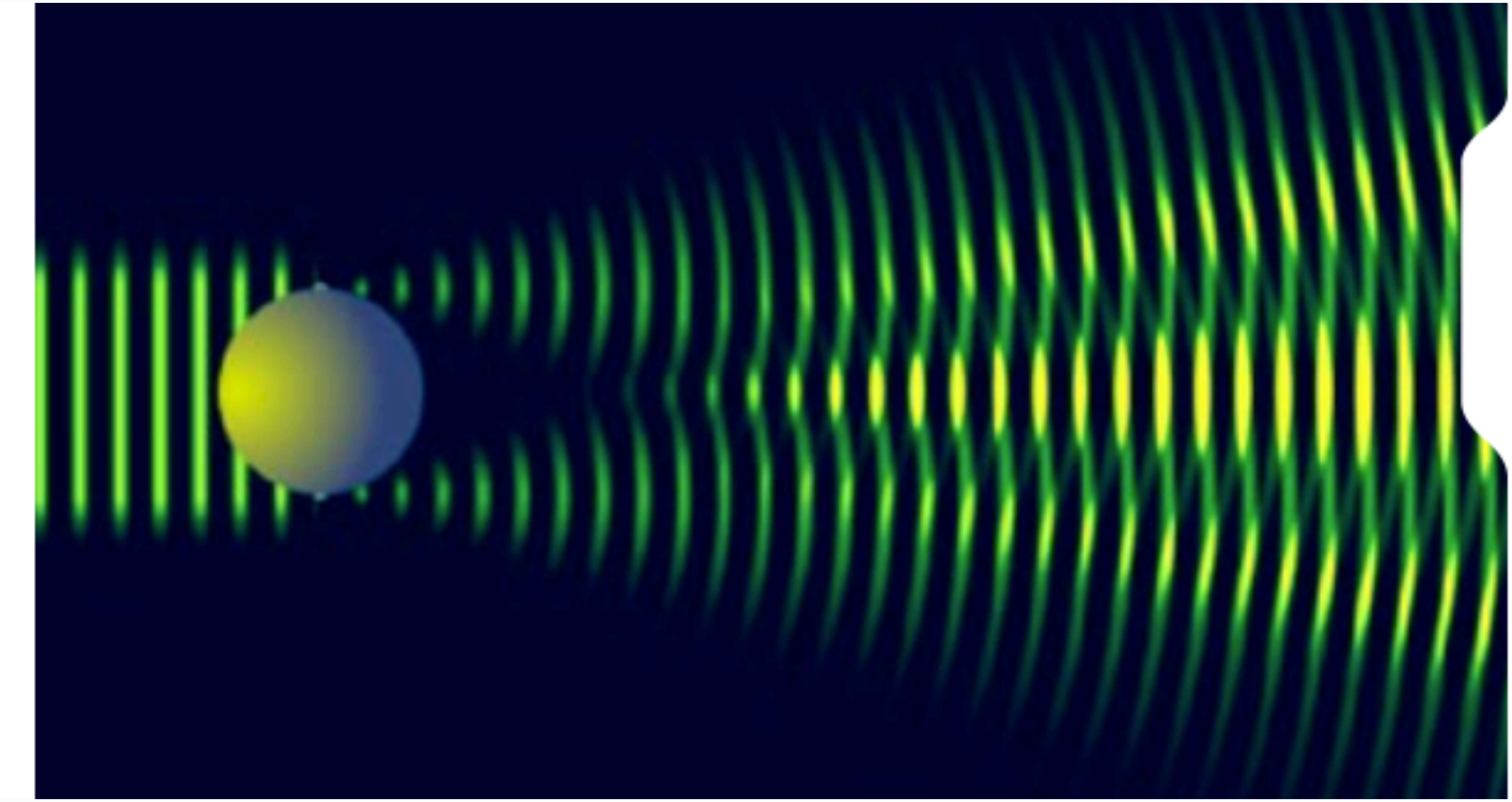


Radiation Correction

- Internal Radiation
  - Real
    - Soft (< 0.35 MeV)
    - Hard (> 0.35 MeV)
  - Virtual
- External Radiation
  - Atomic Collision
  - Straggling
  - External Bremsstrahlung
  - Multiple scattering



Scattering of light from a Sphere



Why Electron Scattering ?

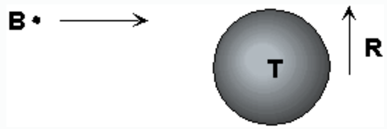
- **Positives:**
  - **Simple Reaction Mechanism.**
  - **Cross-section is calculable**
  - **Interaction strength is small ($\alpha = \frac{1}{137}$): involves small perturbation.**
  - **Very accurately known; gives confidence in predicting the result.**
- **Negatives**
  - **Needs High Intensity beam, thicker targets, large solid angle detectors.**
  - **Radiative effects need to be corrected for.**

Energy of the electron beam

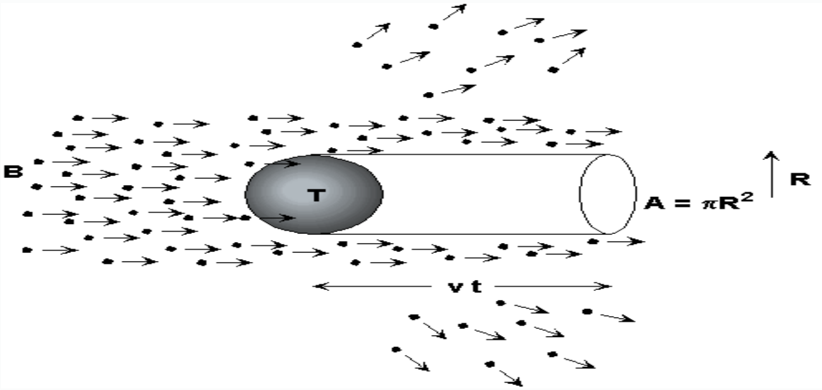
- Electron's Debroglie wavelength (h/mv) is long compared the size of nucleus: nucleus is seen as a point.
- Wavelength is comparable to the size of the nucleus: can resolve the finite size.
- Wavelength much shorter than nuclear size: resolve the nuclear internal structure.

Scattering Cross-Section

In general , the cross-section is the effective area of the collision region.



The cross-section for this diagram is, $\sigma = \pi R^2$.

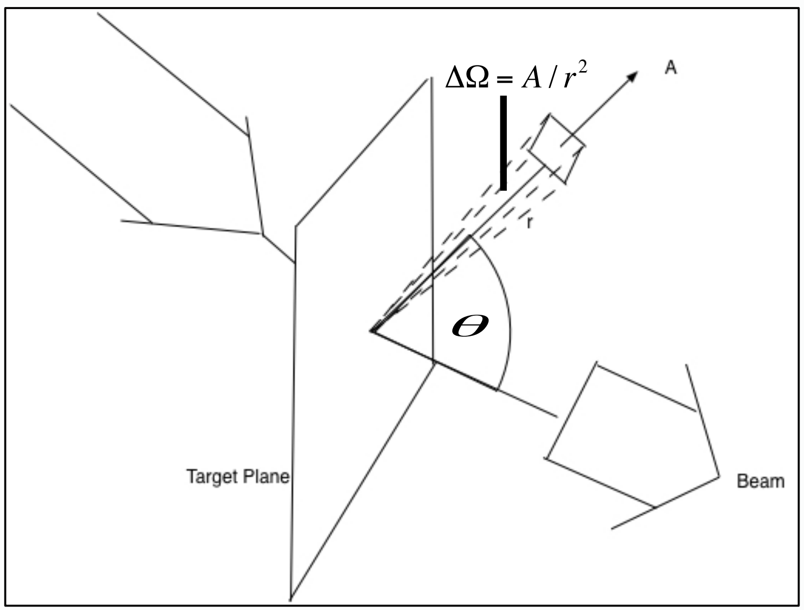


When a beam of particles hits the target, the no. of particles colliding in a time t with the target, will be a cylindrical region equal to

$$N = n_a . v_a . t . \sigma \quad \Rightarrow \quad \dot{N} = \Phi_a . \sigma$$

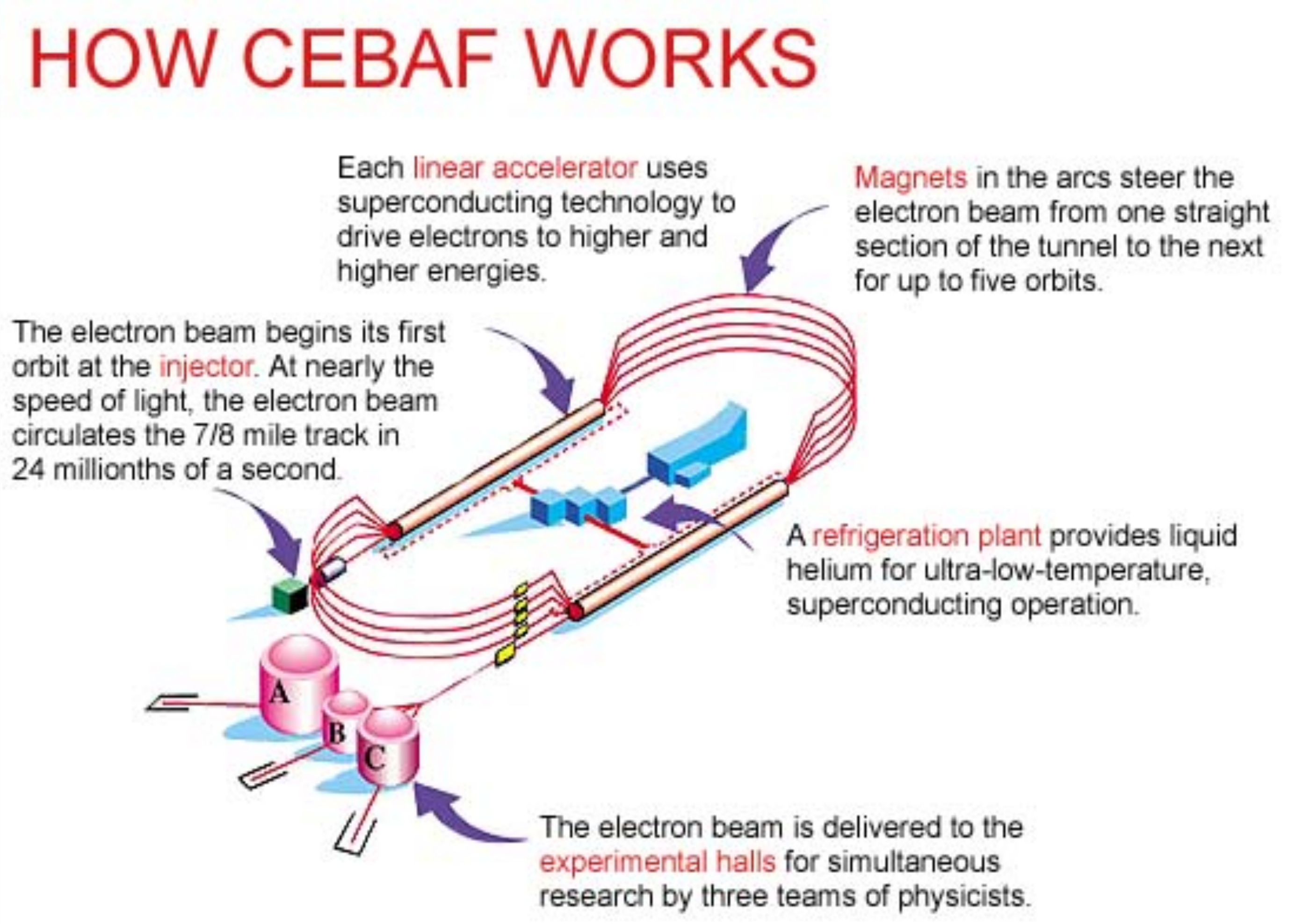
Differential Scattering Cross-section

In reality, only a fraction of all reactions are detected. A detector of area 'A' is placed at a distance r and at an angle θ with respect to the beam direction, covering a solid angle ΔΩ.



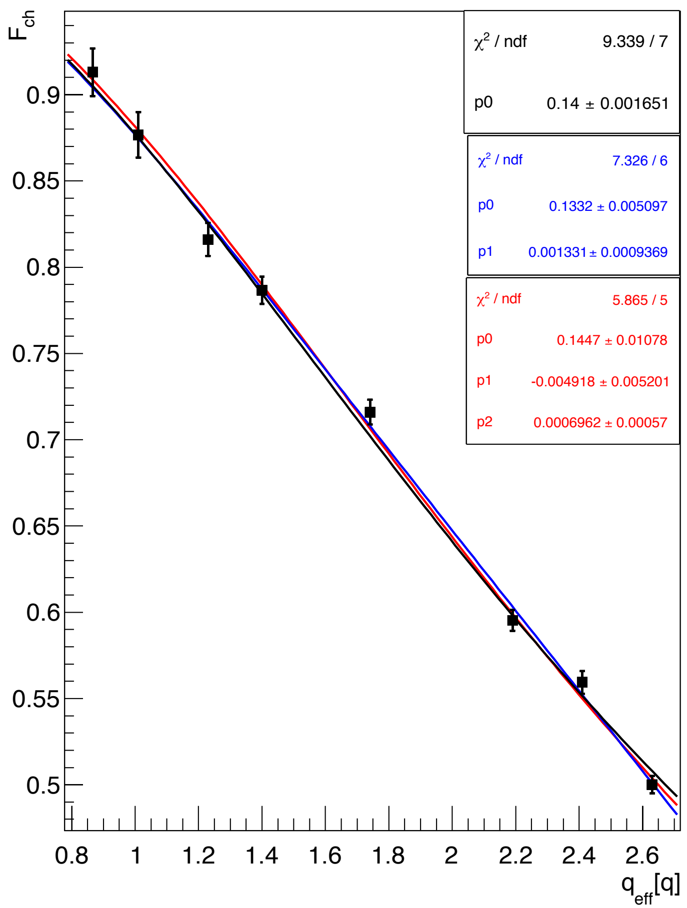
$$\frac{d\sigma(E,\theta)}{d\Omega} = \frac{\dot{N}(E,\theta,\Delta\Omega)}{\Phi . N_t . \Delta\Omega}$$

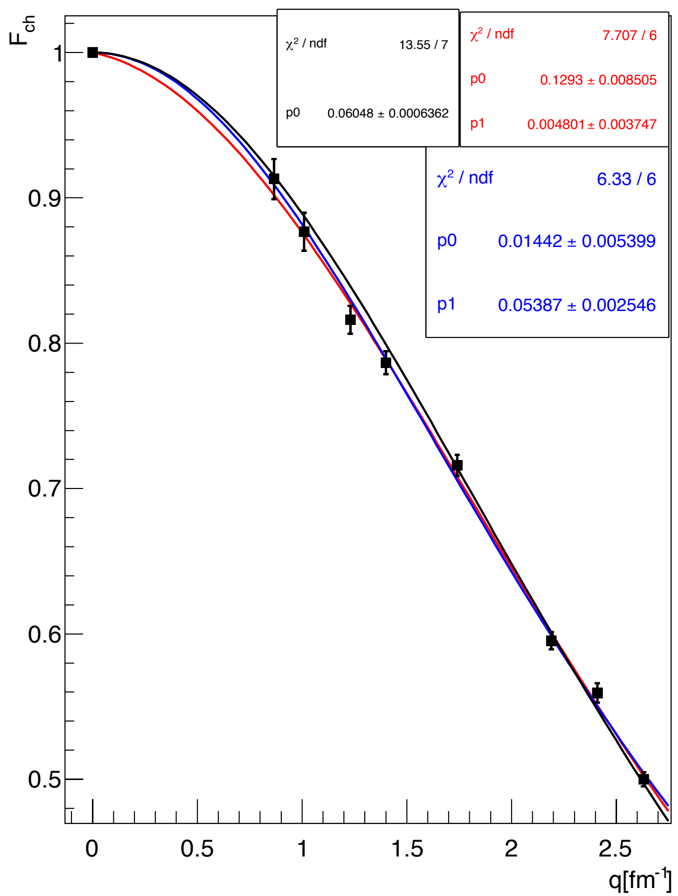
CEBAF Accelerator



Some Other Parametrization Models for Hydrogen

Fit function, $F_{ch} = \dfrac{1}{(1 + p_0 q^2 + p_1 q^4 + \ldots)^{\pm n}}$





Results for hydrogen charge radius

Fit-Function(Model)	Data Source	$r_{rms}(fm)$
$\frac{1}{(1+p0q^2)^2}$	$LEDEX+Saskatoon+Mainz+JLab$	0.849 ± 0.004
$\frac{1}{1+p0q^2+p1q^4}$	$LEDEX+Saskatoon+Mainz+JLab$	0.861 ± 0.003
F-B (Soliton)	$LEDEX$	0.912 ± 0.0147
F-B (Cluster)	$LEDEX$	0.893 ± 0.0142
F-B (PQCD)	$LEDEX$	0.879 ± 0.0147
$\frac{1}{1+p0q^2+p1q^4}$	$LEDEX$	$0.894 \pm 0.02 \pm 0.004$
$\frac{p0}{1+p1\tau^2}$	$LEDEX$	0.912 ± 0.006
$\frac{p0}{1+p1\tau^2+p2\tau^4}$	$LEDEX$	0.8938 ± 0.0178
$\frac{1}{(1+p0q^2)^2}$	$LEDEX$	0.853 ± 0.007
$\frac{p0}{1+11.9\tau^2+p1\tau^4}$	$LEDEX$	$0.877 \pm ...$
$\frac{p0}{1+11.45\tau^2+p1\tau^4}$	$LEDEX$	$0.86 \pm ...$
$\frac{p0}{1+11.0\tau^2+p1\tau^4}$	$LEDEX$	$0.842 \pm ...$