Generalized Parton Distributions in Exclusive Electroproduction and Time-like Compton Scattering: From Cross Sections and Asymmetries to Recoil Polarization

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Collaborators

GPDs, Extension to Chiral Odd Sector

S. Liuti, O. Gonzalez Hernandez

- GRG, O. Gonzalez-Hernandez, S.Liuti, PRD91, 114013 (2015)
- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438
- Ahmad, GRG, Liuti, PRD79, 054014, (2009)
- Gonzalez, GRG, Liuti PRD84, 034007 (2011)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- Gonzalez Hernandez, Liuti, GRG, Kathuria, PRC88, 065206 (2013)

Abstract:

GPDs underpin many exclusive electroproduction processes - Deeply Virtual Compton Scattering, Deeply Virtual Meson Production - as well as Deeply Virtual Time-like Compton Scattering. One way GPDs have been parameterized is in terms of covariant spectator models. It will be shown how the determination of individual GPDs and the confirmation of various models rests on measurements of azimuthal asymmetries, target and beam polarizations, and recoil baryon polarization.

(Acronyms: DVCS, DVMP, DVπ, TCS)

Abstract:

GPDs underpin many exclusive electroproduction processes -
Deeply Virtual ... Virtual Meson
Production - a ... ne-like Compton
Scattering. One ... erized is in terms
of covariant sp ... shown how the
determination ... confirmation of
various mode ... s of azimuthal
asymmetries, ta ... and recoil baryon
polarization.

(Acronyms: DVCS, DVMP, DVπ, TCS)

Composition of Hadrons - nucleons
Momentum Distribution & Spin or Helicity or Transversity

- Spin for NR systems, e.g. Spin ½:

  $S_z\,|\pm\frac{1}{2}\rangle = \pm\frac{1}{2}\,|\pm\frac{1}{2}\rangle$ Goudsmit & Uhlenbeck (1925)

- Helicity for Relativistic scattering:

  $S\cdot p\,|p,\pm\frac{1}{2}\rangle = \pm\frac{1}{2}\,|p,\pm\frac{1}{2}\rangle$ Jacob & Wick (1959)

- Transversity for Single Spin Asymmetries in Relativistic 2-body or inclusive scattering with ***n*** normal to the plane:

  $\boldsymbol{S\cdot n}\,|\pm\frac{1}{2}\rangle_T = \pm\frac{1}{2}\,|\pm\frac{1}{2}\rangle_T$

  with $|\pm\frac{1}{2}\rangle_T = (|+\frac{1}{2}\rangle \pm i\,|-\frac{1}{2}\rangle)/\sqrt{2}$ GRG & Moravcsik (1976)

Composition of Hadrons - nucleons
Momentum Distribution & Spin or Helicity or Transversity

- Spin for NR systems, e.g. Spin ½:

  $S_z\, |\pm½ \rangle = \pm½ |\pm½ \rangle$ NR 3-quark models $S_{q1} \otimes S_{q2} \otimes S_{q3} \otimes L$
- Helicity for Relativistic scattering & QCD asymp. free:

  $S\cdot p\, |p,\pm½ \rangle = \pm½ |p,\pm½ \rangle$ pdf's $g_1(x)$ & axial charge
- Transversity for Single Spin Asymmetries in Relativistic 2-body or inclusive scattering -- ***n*** normal to the plane:

  $\boldsymbol{S\cdot n}\, |\pm½ \rangle_T = \pm½ |\pm½ \rangle_T$

  with $|\pm½ \rangle_T = (|+½ \rangle \pm i|-½ \rangle)/\surd 2$ pdf's $h_1(x)$

  & "tensor charge"

Spin in scattering

- "Spinful" processes:
  - Compton Scattering γ+N→γ'+N´ 2x2x2x2 ***P***→ 8 ***PT***→6 complex amplitudes
  - DVCS γ^*+N→γ'+N´ 3x2x2x2 ***P***→ 12 amps
    - (e^-→$e^{-\prime}$+γ^* beam can be polarized increasing to 20 amps but reduced by QED factorization & helicity conservation)
  - Pseudoscalar Photoproduction γ+N→π or η +N´ ***P***→4 amps
  - DVπP γ^*+N→π or η +N´ →6 amps
  - TCS γ+N→γ^*+N´ & γ^*→ $\ell^+ \ell^-$ ***P***→ 12 amps

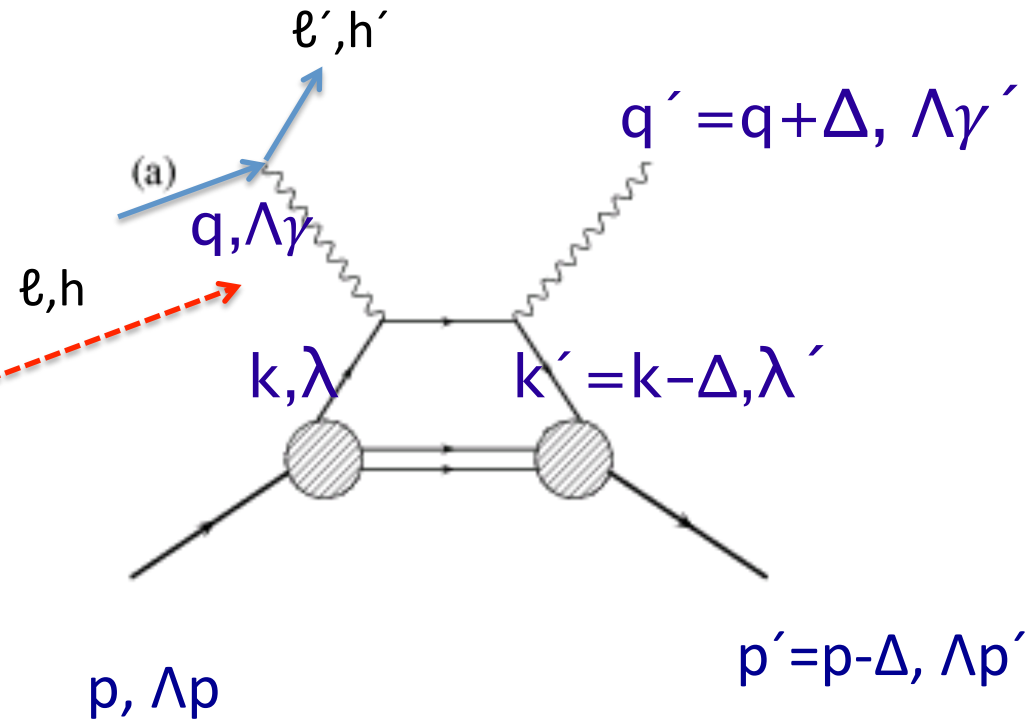
  - ***How are these spin dependent amps determined?***

    All depend on Spin of Nucleon - a composite object

Assigning helicities

DVCS or DVMP

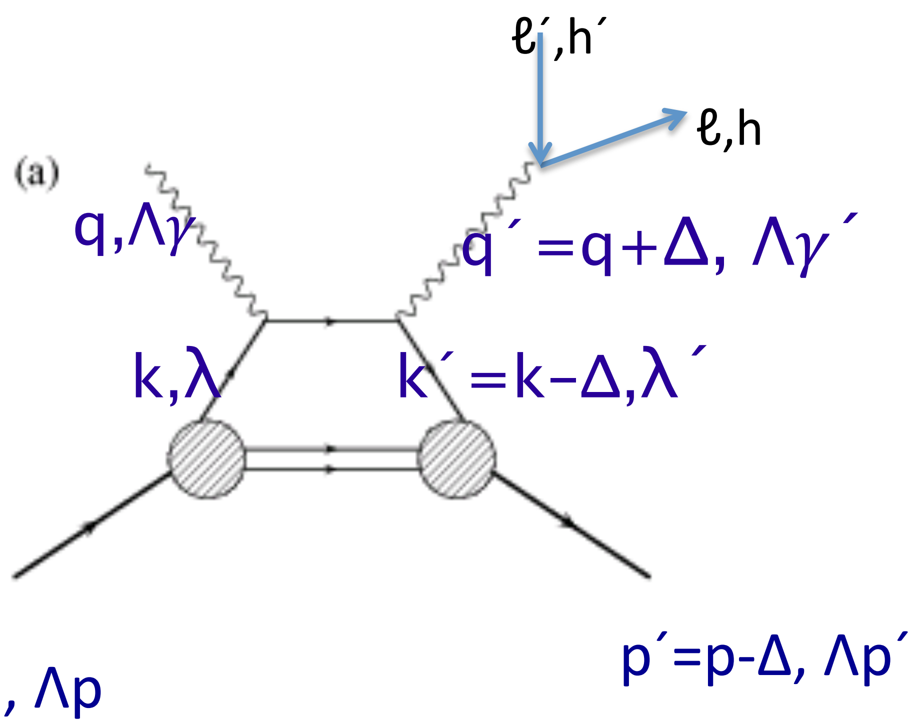
$$T^{h,h',\Lambda p,\Lambda p',\Lambda\gamma'} = \delta_{h,h'} \sum_{\Lambda\gamma} A_h^{\Lambda\gamma} f_{\Lambda\gamma,\Lambda p;\Lambda\gamma',\Lambda p'}/Q^2$$



One γ^* exchange
→ Order α_{QED}

Time-like CS

$$T^{\Lambda\gamma,\Lambda p,\Lambda p',h,h'} = \delta_{h,-h'} \sum_{\Lambda\gamma'} \tilde{A}_h^{\Lambda\gamma'} f_{\Lambda\gamma,\Lambda p;\Lambda\gamma',\Lambda p'} / q'^2$$



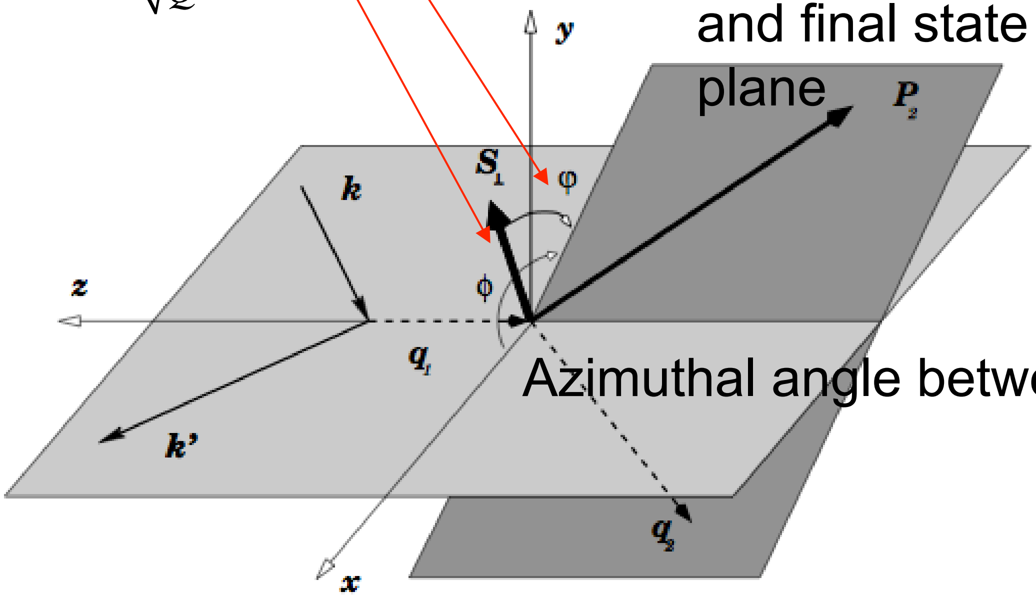
DVCS Phase angles (or TCS viewed backwards)

$$\frac{d\sigma}{dx_{\mathrm{B}}\,dy\,d|\Delta^2|\,d\phi\,d\varphi} = \frac{\alpha^3 x_{\mathrm{B}} y}{16\,\pi^2\,Q^2\sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2$$

Amplitude

$$y = \frac{(P_1 q)}{(P_1 k_1)} \qquad \varepsilon = 2x_B \frac{M}{\sqrt{Q^2}}$$

Angle between transverse spin and final state plane



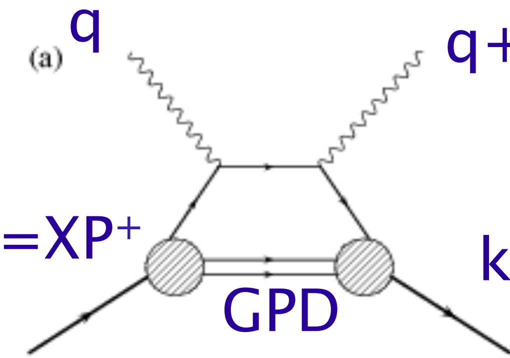
Azimuthal angle between planes

(diagram from Belitsky, Kirchner, Muller, 2002)

DVCS & DVMP $\gamma^*(Q^2)+P \rightarrow (\gamma \text{ or meson})+P'$
partonic picture – leading twist

Light cone variables
$P^+ = (p+p')/\sqrt{2}$

$\left\{ \begin{array}{l} k_T \\ k^+ = XP^+ \end{array} \right.$



$\left. \begin{array}{l} k'_T = k_T - \Delta \\ k'^+ = (X-\zeta)P^+ \end{array} \right\}$

$\left. \begin{array}{l} P'^+ = (1-\zeta)P^+ \\ P'_T = -\Delta \end{array} \right\}$

$\zeta \rightarrow 0$ Large s
Regge-like

Factorized
"handbag"
picture

Quark-spectator
quark+diquark

$X > \zeta$ DGLAP $\quad \Delta_T \text{ -> } b_T$ transverse spatial

$X < \zeta$ ERBL $\quad x = (X-\zeta/2)/(1-\zeta/2);\ \xi = \zeta/(2-\zeta)$

see *Ahmad*, *GG*, Liuti, PRD79, 054014, (2009) for first chiral odd GPD parameterization *Gonzalez*, *GG*, Liuti PRD84, 034007 (2011); PRD91, 114013 (2015)

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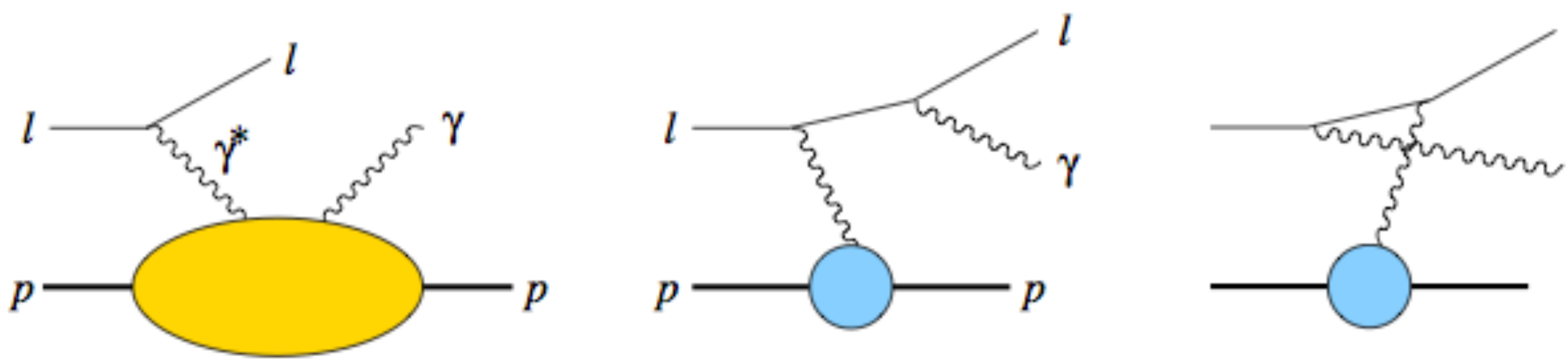
Constructing observable quantities

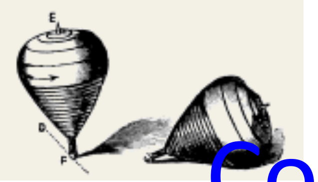
- Unpolarized cross section
- $d\sigma \propto \sum_{\text{all helicities}} (T^{h,h',\Lambda p,\Lambda p',\Lambda\gamma'})^* \, T^{h,h',\Lambda p,\Lambda p',\Lambda\gamma'}$

for DVCS (& TCS) the Bethe-Heitler process also contributes

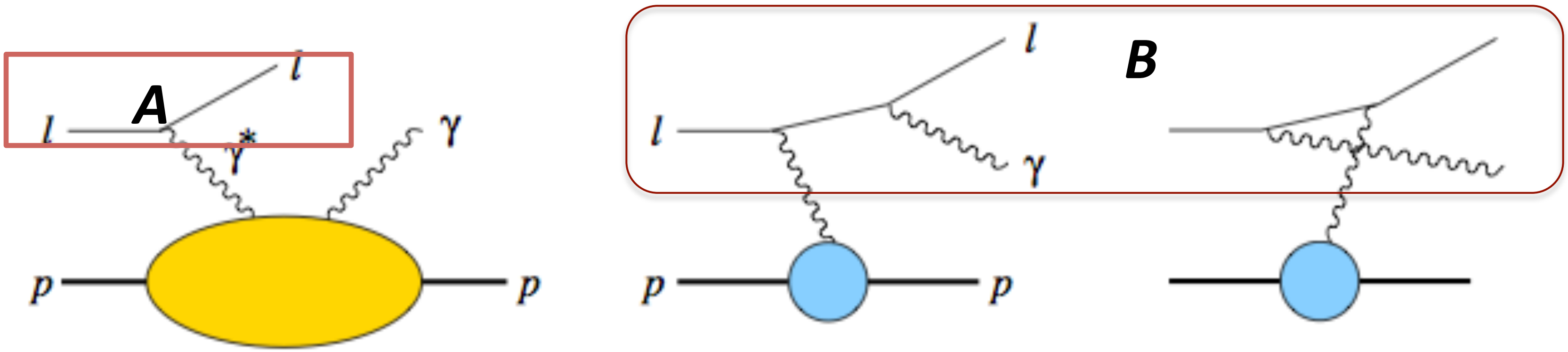
figure from Diehl & Sapeta EPJC41



$$T^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p') = T^{h,\Lambda,\Lambda',\Lambda'_\gamma}_{DVCS}(k,p,k',q',p') + T^{h,\Lambda,\Lambda',\Lambda'_\gamma}_{BH}(k,p,k',q',p')$$



Constructing observable quantities



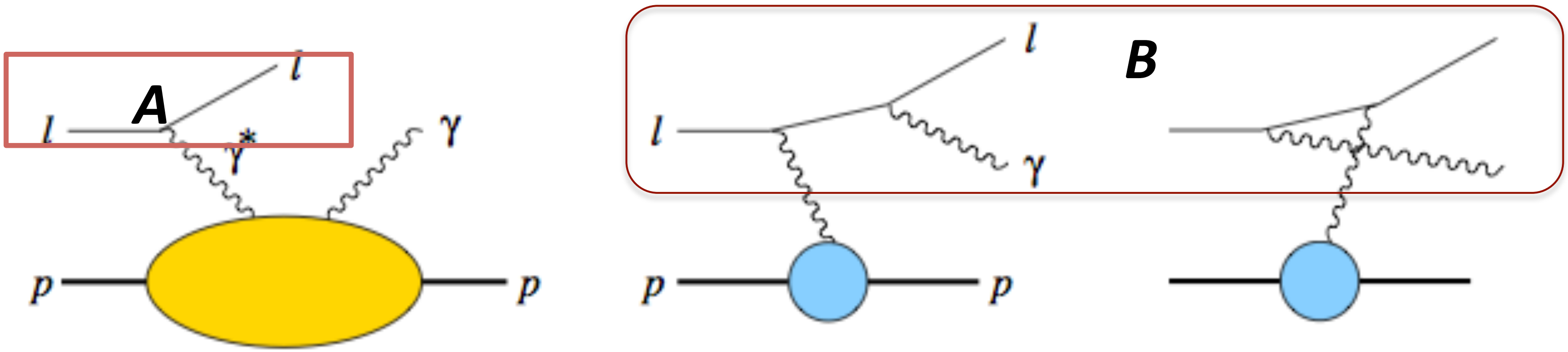
$$T^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p') = T_{DVCS}^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p') + T_{BH}^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p')$$

QED factorizes

$$T_{DVCS}^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p') = \sum_{\Lambda^*_\gamma} A_h^{\Lambda^*_\gamma}(k,k',q)\frac{1}{Q^2} f_{\Lambda,\Lambda'}^{\Lambda^*_\gamma,\Lambda'_\gamma}(q,p,q',p')$$

$$T_{BH}^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p') = \sum_{\tilde{\Lambda}_\gamma} B_{h,\Lambda'_\gamma}^{\tilde{\Lambda}_\gamma}(k,k',q',\Delta)\frac{1}{\Delta^2} J_{\tilde{\Lambda}_\gamma,\Lambda;\Lambda'}(\Delta,p,p')$$

Constructing observable quantities



Phases are important!

$$T_{DVCS}^{h,\Lambda,\Lambda',\Lambda'_\gamma}(k,p,k',q',p') = \sum_{\Lambda^*_\gamma} A_h^{\Lambda^*_\gamma}(k,k',q)\frac{1}{Q^2} f_{\Lambda,\Lambda'}^{\Lambda^*_\gamma,\Lambda'_\gamma}(q,p,q',p')$$

$$= A_h^{+1} e^{+i(1-\Lambda-\Lambda'_\gamma+\Lambda')\phi} f_{\Lambda\Lambda'}^{+1\Lambda'_\gamma,} + A_h^{-1} e^{-i(-1-\Lambda-\Lambda'_\gamma+\Lambda')\phi}\, \tilde{f}_{\Lambda,\Lambda'}^{-1\Lambda'_\gamma} + \gamma A_h^3 e^{-i(-\Lambda-\Lambda'_\gamma+\Lambda')\phi}\, \tilde{f}_{\Lambda;,\Lambda'}^{0\Lambda'_\gamma}$$

DVCS unpolarized cross section

$$\frac{d\sigma}{d\Phi} = \Gamma \sum_{\Lambda,\Lambda',\Lambda'_\gamma} \Big\{ 2 \mid f_{+1,\Lambda;\Lambda'_\gamma,\Lambda'} \mid^2 + 2\epsilon \mid f_{0,\Lambda;\Lambda'_\gamma,\Lambda'} \mid^2$$
$$+\ 4\sqrt{\epsilon(1+\epsilon)} \left[\cos\phi\, \mathcal{R}e(f^*_{+1,\Lambda;\Lambda'_\gamma,\Lambda'} f_{0,\Lambda;\Lambda'_\gamma,\Lambda'}) \right]$$
$$-\ 4\epsilon \left[\cos 2\phi\, \mathcal{R}e(f^*_{+1,\Lambda;\Lambda'_\gamma,\Lambda'} f_{-1,\Lambda;\Lambda'_\gamma,\Lambda'}) \right] \Big\}$$

For polarized lepton beam

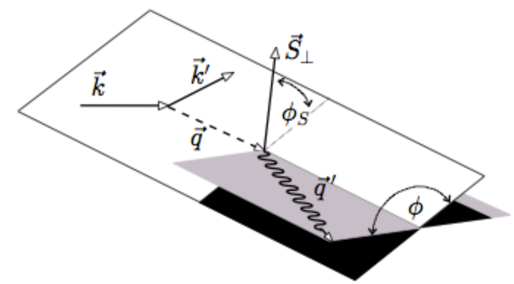
$$\frac{d\sigma}{d\Phi}\Big|_{\text{beam}} = \mathcal{P}_l \Gamma \sum_{\Lambda,\Lambda',\Lambda'_\gamma} \Big\{ -4\sqrt{\epsilon(1-\epsilon)}\, \sin\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,\Lambda'} f_{0,\Lambda;\Lambda'_\gamma,\Lambda'}) \Big\}$$

For DVπ set $\Lambda\gamma'$=0

Cross section with φ modulations & beam/target polarized

$$
\begin{aligned}
\frac{d^4\sigma}{dx_{Bj}\,dy\,d\phi\,dt} = \Gamma \Bigg\{ \Bigg[& F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)} \cos\phi F_{UU}^{\cos\phi} + \boxed{h \sqrt{2\epsilon(1-\epsilon)}\, \sin\phi F_{LU}^{\sin\phi}} \Bigg] \\
+ & \; S_{||} \Bigg[\boxed{\sqrt{2\epsilon(\epsilon+1)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi}} + h \boxed{\left(\sqrt{1-\epsilon^2}\, F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\, \cos\phi\, F_{LL}^{\cos\phi} \right)} \Bigg] \\
+ & \; S_{\perp} \Bigg[\sin(\phi-\phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \epsilon \left(\sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right) \\
+ & \; \sqrt{2\epsilon(1+\epsilon)} \left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \Bigg] \\
+ & \; S_{\perp} h \Bigg[\sqrt{1-\epsilon^2} \cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \left(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right) \Bigg] \Bigg\}
\end{aligned}
$$

$$
A_{LU} = \sqrt{\epsilon(1-\epsilon)}\, \frac{F_{LU}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}}
$$



$$
A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)} \sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}}
$$

$$
F_{UU,T} = \frac{d\sigma_T}{dt}, \quad F_{UU,L} = \frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi} = \frac{d\sigma_{LT}}{dt},
$$
$$
F_{UU}^{\cos 2\phi} = \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi} = \frac{d\sigma_{LT'}}{dt}
$$

$$
A_{LL} = \frac{N^{\rightarrow}_{s_z=+} - N^{\rightarrow}_{s_z=-} + N^{\leftarrow}_{s_z=+} - N^{\leftarrow}_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2}\; F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\, \cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon F_{UU,L}}
$$

from Diehl & Sapeta; Bacchetta, *et al*. SIDIS Review; …

Asymmetries & helicity amps for DVπ

structure functions for the unpolarized beam and single transversely polarized target,

$$
\begin{aligned}
F_{UT,T}^{\sin(\phi-\phi_S)} &= \Im m\, F_{11}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda' *} f_{10}^{-\Lambda'} = \Im m \left[f_{10}^{++*} f_{10}^{-+} + f_{10}^{+-*} f_{10}^{--} \right] \\
F_{UT,L}^{\sin(\phi-\phi_S)} &= \Im m\, F_{00}^{+-} = \Im m \sum_{\Lambda'} f_{00}^{+\Lambda' *} f_{00}^{-\Lambda'} = \Im m \left[f_{00}^{++*} f_{00}^{-+} + f_{00}^{+-*} f_{00}^{--} \right] \\
F_{UT}^{\sin(\phi+\phi_S)} &= \Im m\, F_{1-1}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda' *} f_{-10}^{-\Lambda'} = \Im m \left[-f_{10}^{++*} f_{10}^{+-} + f_{10}^{+-*} f_{10}^{++} \right] \\
F_{UT}^{\sin(3\phi+\phi_S)} &= \Im m\, F_{1-1}^{-+} = \Im m \sum_{\Lambda'} f_{10}^{-\Lambda' *} f_{-10}^{+\Lambda'} = \Im m \left[f_{10}^{-+*} f_{10}^{--} - f_{10}^{--*} f_{10}^{-+} \right] \\
F_{UT}^{\sin\phi_S} &= \Im m\, F_{10}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda' *} f_{00}^{-\Lambda'} = \Im m \left[f_{10}^{++*} f_{00}^{-+} + f_{10}^{+-*} f_{00}^{--} \right] \\
F_{UT}^{\sin(2\phi-\phi_S)} &= \Im m\, F_{10}^{-+} = \Im m \sum_{\Lambda'} f_{10}^{-\Lambda' *} f_{00}^{+\Lambda'} = \Im m \left[f_{10}^{-+*} f_{00}^{++} + f_{10}^{--*} f_{00}^{+-} \right],
\end{aligned}
$$

and three for the longitudinally polarized lepton and transversely polarized target,

$$
\begin{aligned}
F_{LT}^{\cos(\phi-\phi_S)} &= \Re e\, F_{11}^{+-} = \Re e \sum_{\Lambda'} f_{10}^{+\Lambda' *} f_{10}^{-\Lambda'} = \Re e \left[f_{10}^{++*} f_{10}^{-+} + f_{10}^{+-*} f_{10}^{--} \right] \\
F_{LT}^{\cos\phi_S} &= \Re e\, F_{10}^{+-} = \Re e \sum_{\Lambda'} f_{10}^{+\Lambda' *} f_{00}^{-\Lambda'} = \Re e \left[f_{10}^{++*} f_{00}^{+-} + f_{10}^{+-*} f_{00}^{--} \right] \\
F_{LT}^{\cos(2\phi-\phi_S)} &= \Re e\, F_{10}^{-+} = \Re e \sum_{\Lambda'} f_{10}^{-\Lambda' *} f_{00}^{+\Lambda'} = \Re e \left[f_{10}^{-+*} f_{00}^{++} + f_{10}^{--*} f_{00}^{+-} \right].
\end{aligned}
$$

How do GPDs enter in helicty amps?
GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\boxed{H^q}\gamma^+ + \boxed{E^q}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\boxed{\tilde{H}^q}\gamma^+\gamma_5 + \boxed{\tilde{E}^q}\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

Chiral even GPDs
Liuti, et al. → "flexible parameterization"
How to measure and/or parameterize them?
-> Ji sum rule

$$\left\langle J_q^x \right\rangle = \tfrac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\boxed{H_T^q}\,i\sigma^{+i} + \boxed{\tilde{H}_T^q}\frac{P^+\Delta^i-\Delta^+P^i}{m^2}\right.$$
$$\left. + \boxed{E_T^q}\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \boxed{\tilde{E}_T^q}\frac{\gamma^+P^i-P^+\gamma^i}{m}\right]u(p,\lambda)$$

Chiral odd GPDs
-> transversity
How to measure and/or parameterize them?

Normalizing GPDs – Chiral even

Form factor, Forward limit

$$\int_0^1 H_q(x,\xi,t)\,dx = F_1^q(t), \quad H_q(x,0,0) = q(x)$$

Integrates to charge

$$\int_0^1 E_q(x,\xi,t)\,dx = F_2^q(t)$$

→Anomalous magnetic moments

$$\int_0^1 \tilde{H}_q(x,\xi,t)\,dx = g_A^q(t), \quad \tilde{H}_q(x,0,0) = \Delta q(x) = q^{\rightarrow}_{\Rightarrow}(x) - q^{\leftarrow}_{\Rightarrow}(x)$$

Integrates to axial charge

$$\int_0^1 \tilde{E}_q(x,\xi,t)\,dx = g_P^q(t)$$

Pseudoscalar form factor

How do we normalize chiral-odd GPDs?

Some Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_1(x)$$

Transversity

Integrates to tensor charge δ_q

Form Factors

$$\int H_T^q(x,\xi,t)\,dx = \delta q(t)$$

$$\int \bar{E}_T^q(x,\xi,t)\,dx = \int \left(2\tilde{H}_T^q + E_T^q\right) dx = \kappa_T^q(t)$$

"transverse moment" κ_T^q

$$\int \tilde{E}_T(x,\xi,t)\,dx = 0$$

No direct interpretation of E_T .

Factoring hard from soft including helicities

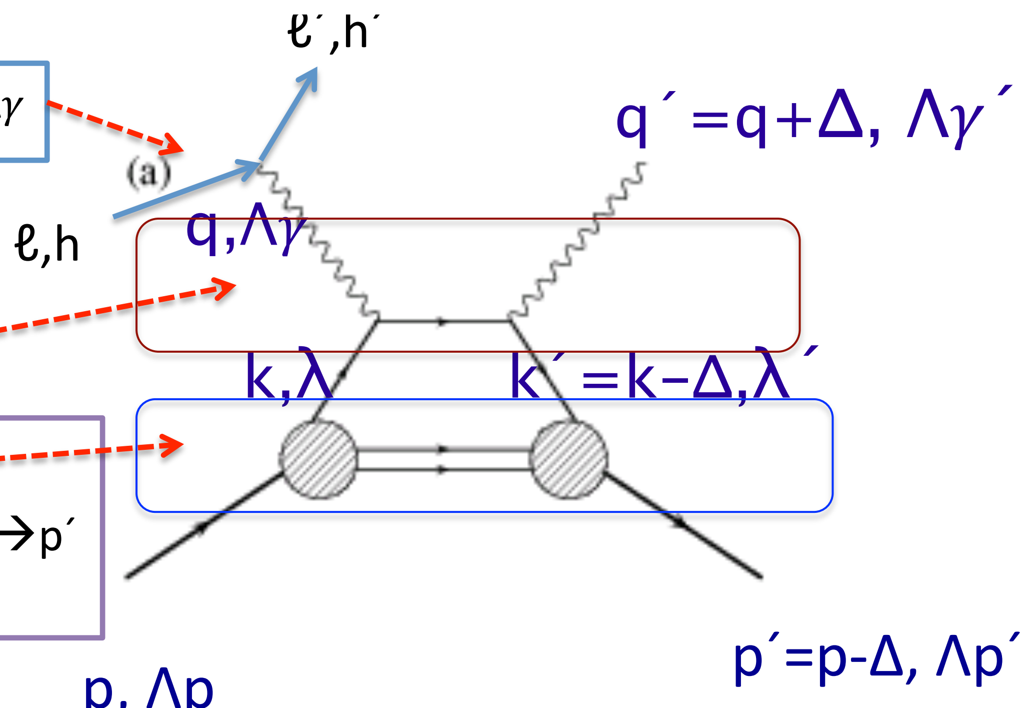
$$T^{h,h',\Lambda p,\Lambda p',\Lambda\gamma'} = \delta_{h,h'} \sum_{\Lambda\gamma} A_h^{\Lambda\gamma} f_{\Lambda\gamma,\Lambda p;\Lambda\gamma',\Lambda p'}/Q^2$$

$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda'_\gamma}(X,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(X,\zeta,t),$$

$\mathbf{A_h^{\Lambda\gamma}}$

Hard amp γ^* + quark
→γ + quark′ g

Soft amp $A_{\lambda p}$ . . .
p→ quark(k) : quark′(k′)→p′
~ ***GPDs***



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$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\boxed{H^q}\gamma^+ + \boxed{E^q}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\boxed{\tilde{H}^q}\gamma^+\gamma_5 + \boxed{\tilde{E}^q}\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

To obtain helicity amps expand quark field operators in creation and annihilation operators in the helicity basis

$\boldsymbol{A}_{\lambda p',\lambda';\Lambda p,\lambda}$

Helicity amps for quark GPDs

$$
\begin{aligned}
A_{++,++} &= \sqrt{1-\xi^2}\left(\frac{H^q+\tilde{H}^q}{2} - \frac{\xi^2}{1-\xi^2}\,\frac{E^q+\tilde{E}^q}{2}\right) \\
A_{-+,-+} &= \sqrt{1-\xi^2}\left(\frac{H^q-\tilde{H}^q}{2} - \frac{\xi^2}{1-\xi^2}\,\frac{E^q-\tilde{E}^q}{2}\right) \\
A_{++,-+} &= -\epsilon\,\frac{\sqrt{t_0-t}}{2m}\,\frac{E^q-\xi\tilde{E}^q}{2}, \\
A_{-+,++} &= \epsilon\,\frac{\sqrt{t_0-t}}{2m}\,\frac{E^q+\xi\tilde{E}^q}{2},
\end{aligned}
$$

Chiral even GPDs
Liuti, et al. → “flexible parameterization”

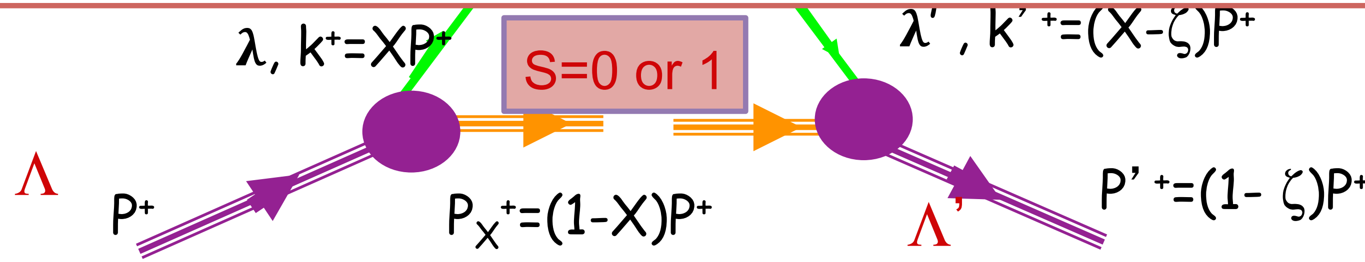
$$
\begin{aligned}
A_{++,+-} &= \epsilon\,\frac{\sqrt{t_0-t}}{2m}\left(\tilde{H}_T^q + (1-\xi)\,\frac{E_T^q+\tilde{E}_T^q}{2}\right), \\
A_{-+,--} &= \epsilon\,\frac{\sqrt{t_0-t}}{2m}\left(\tilde{H}_T^q + (1+\xi)\,\frac{E_T^q-\tilde{E}_T^q}{2}\right), \\
A_{++,--} &= \sqrt{1-\xi^2}\left(H_T^q + \frac{t_0-t}{4m^2}\,\tilde{H}_T^q - \frac{\xi^2}{1-\xi^2}\,E_T^q + \frac{\xi}{1-\xi^2}\,\tilde{E}_T^q\right) \\
A_{-+,+-} &= -\sqrt{1-\xi^2}\,\frac{t_0-t}{4m^2}\,\tilde{H}_T^q
\end{aligned}
$$

Chiral odd GPDs

The GPD Model – Reggeized Diquarks

The Model – first for Chiral Even – Reggeized Diquarks

Procedure to construct GPDs & observables
Spectator diquark model & Reggeization



Product of diquark l.c.w.f.'s $\rightarrow A_{\Lambda\,\lambda;\,\Lambda'\lambda'}$

$\downarrow$

$A_{\Lambda\lambda;\Lambda'\lambda}$ → chiral even GPDs

$\downarrow$

$g \otimes A \rightarrow$ exclusive process helicity amps

$\downarrow$

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

$\downarrow$

vertex parity$\rightarrow A_{\Lambda\lambda;-\Lambda'-\lambda'}$ → chiral odd GPDs→pdf's, …

Recursive fit

GRG, Gonzalez Hernandez, Liuti, PRD84 (2011)

Advantage: control over the number of parameters to be fitted at different stages so that it can be optimized

Functional form:

From DIS

$$\boxed{q(x,Q_o^2) = A_q\, x^{-\alpha_q}(1-x)^{\beta_q} F(x,c_q,d_q,\ldots)}$$

to DVCS, DVMP

$$H_q(x,\xi,t;Q_o^2) = N_q\, x^{-\left[\alpha_q+\alpha'_q(1-x)^p t\right]} G^{a_1 a_2 a_3 ..}(x,\xi,t)$$

$$a_1 = m_q,\ a_2 = M_X^q,\ a_3 = M_\Lambda^q, \ldots$$

pdf's
Form Factors
dσ / dΩ
Asymmetries

"Flexible" parameterization based on the Reggeized quark-diquark model.

Sea quarks and gluon parametrization, work in progress

Details of Regge-diquark model

$$A^{(0)}_{\Lambda'\lambda',\Lambda\lambda} = \int d^2k_\perp \phi^*_{\Lambda'\lambda'}(k',P')\phi_{\Lambda\lambda}(k,P),$$

Scalar diquark → helicity "amps"

$$\phi_{\Lambda,\lambda}(k,P) = \Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2-m^2}$$

l.c.w.f. 's

$$\phi^*_{\Lambda'\lambda'}(k',P') = \Gamma(k')\frac{\overline{U}(P',\Lambda')u(k',\lambda')}{k'^2-m^2}$$

$$\Gamma = g_s\frac{k^2-m_q^2}{(k^2-M_\Lambda^{q\,2})^2}$$

$$\begin{aligned}
\phi^*_{++}(k,P) &= \mathcal{A}(m+MX) \\
\phi^*_{+-}(k,P) &= \mathcal{A}(k_1+ik_2), \\
\phi_{--}(k,P) &= \phi_{++}(k,P) \\
\phi_{-+}(k,P) &= -\phi^*_{+-}(k,P).
\end{aligned}$$

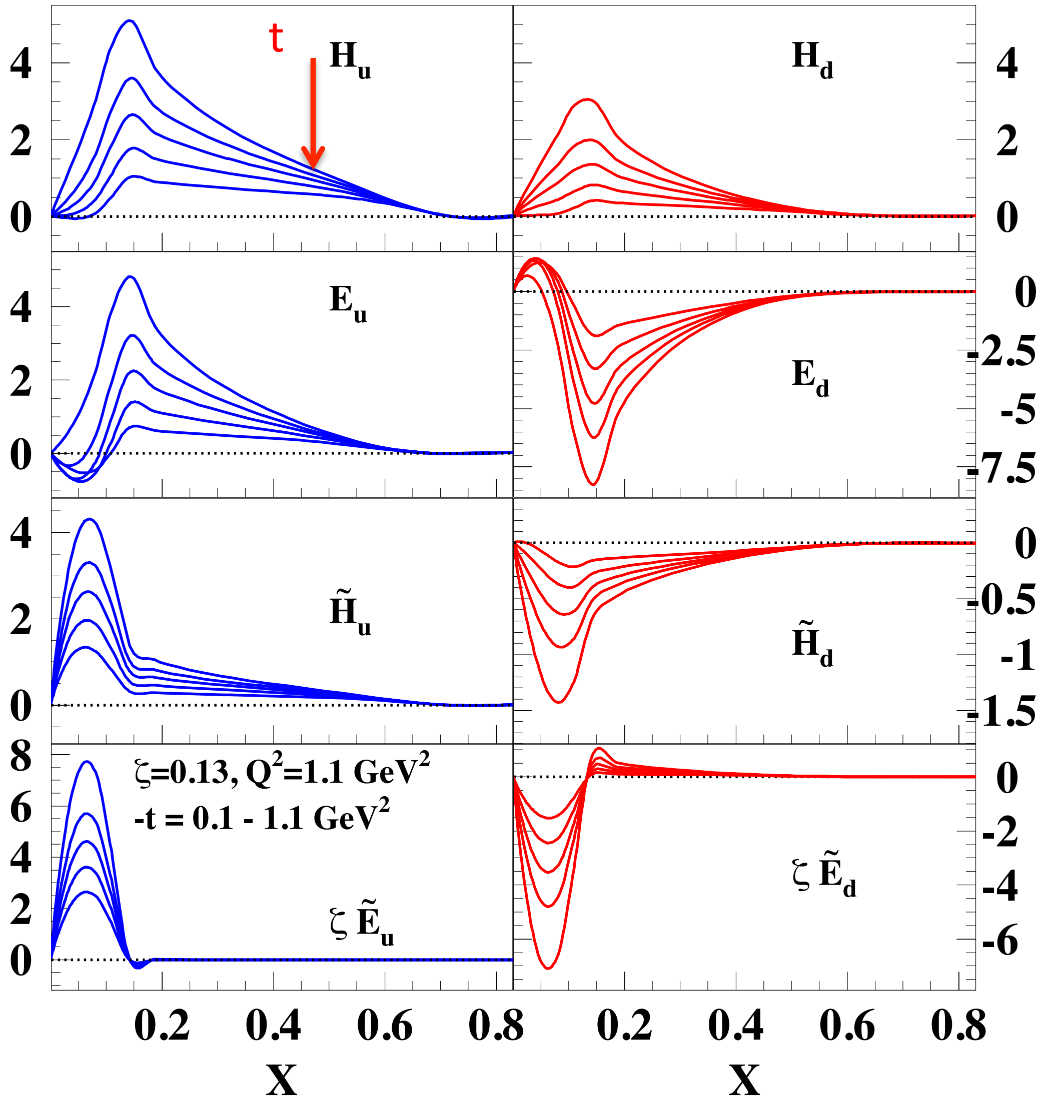
Add axial vector diquark → helicity amps

Chiral odd GPDs = helicity amps

$$\begin{aligned}
\tau\left[2\widetilde{H}_T(X,0,t)+E_T(X,0,t)\right] &= A_{++,+-}+A_{-+,--} \\
&= A^{T_Y}_{++,++}-A^{T_Y}_{+-,+-}+A^{T_Y}_{-+,-+}-A^{T_Y}_{--,--} \\
H_T(X,0,t) &= A_{++,--}+A_{-+,+-} \\
&= A^{T_X}_{++,++}-A^{T_X}_{+-,+-}-A^{T_X}_{-+,-+}+A^{T_X}_{--,--} \\
\tau^2\widetilde{H}_T(X,0,t) &= -A_{-+,+-} \\
&= A^{T_Y}_{++,++}-A^{T_Y}_{+-,+-}-A^{T_X}_{-+,-+}+A^{T_X}_{--,--} \\
\widetilde{E}_T(X,0,t) &= A_{++,+-}-A_{-+,--}=0
\end{aligned}$$

diquark → ***A*** helicity "amps"
→ GPDs → convolute with
hard amp → CFFs with
helicity → ***DVCS f's***
combine with lepton vertex
→ e+p→e+γ(or ρ,ω, π,η .. .)+p
amps → observables

Chiral even GPDs



From GPDs with evolution to Compton Form Factors

CFFs to helicity amps

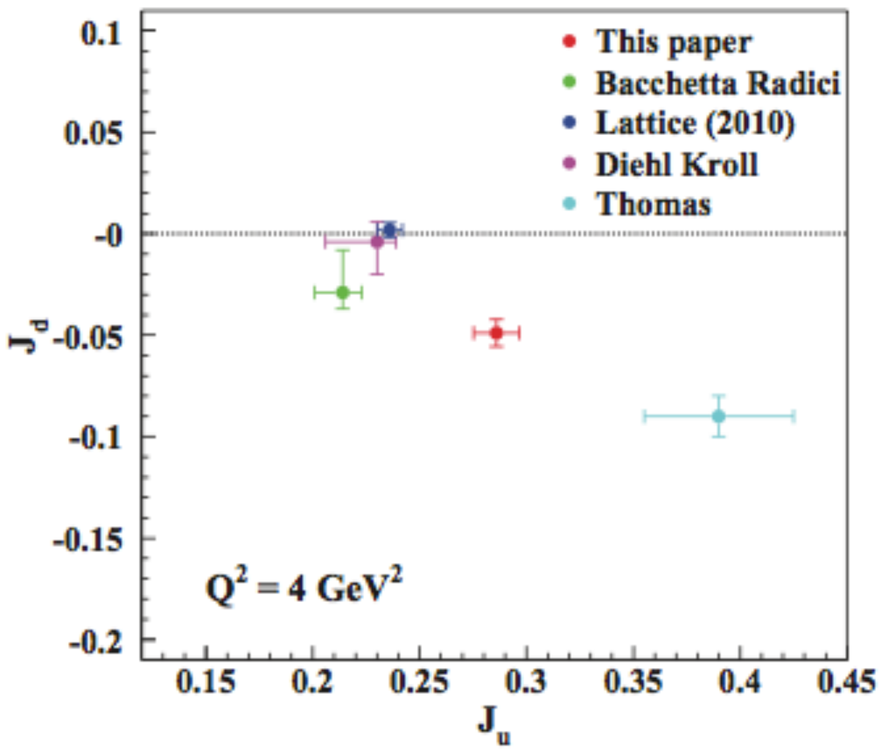
helicity amps to observables

<-> parameters

Valence quark angular momenta – from "flexible" chiral even model applied to EM form factors, pdf's & some cross section & asymmetry data

Gonzalez Hernandez, Liuti, GRG, Kathuria

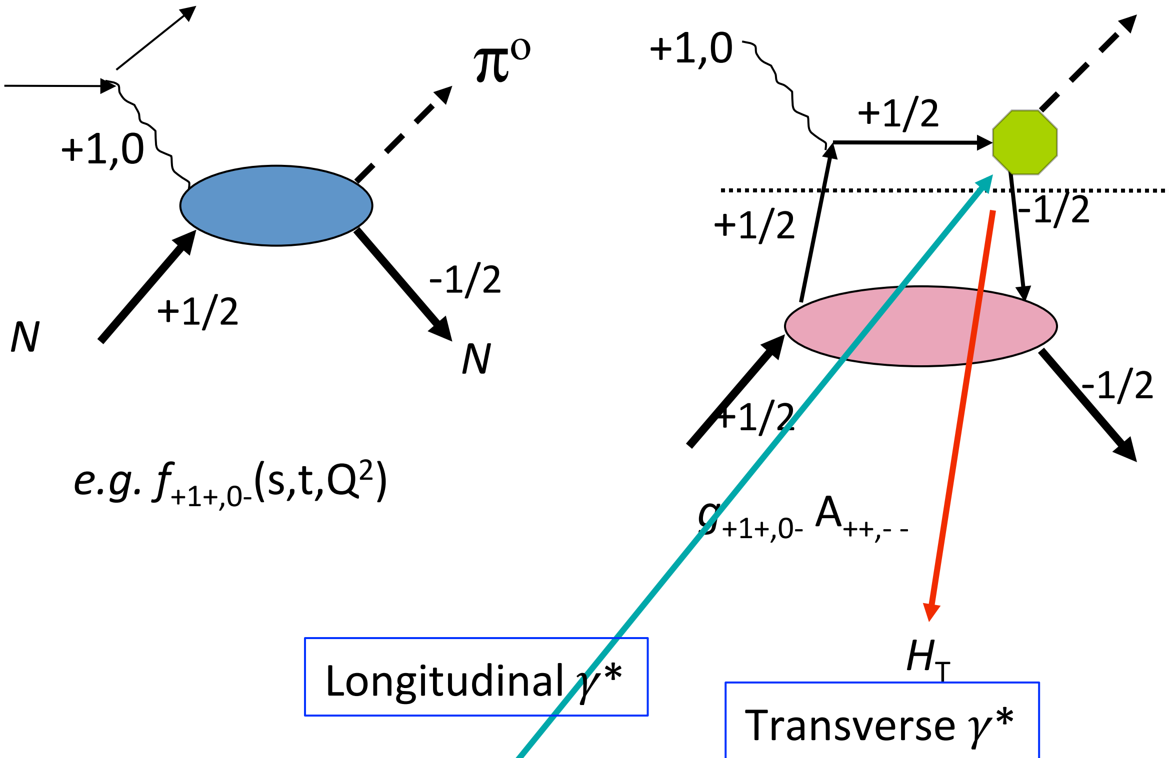
PHYSICAL REVIEW C **88**, 065206 (2013)



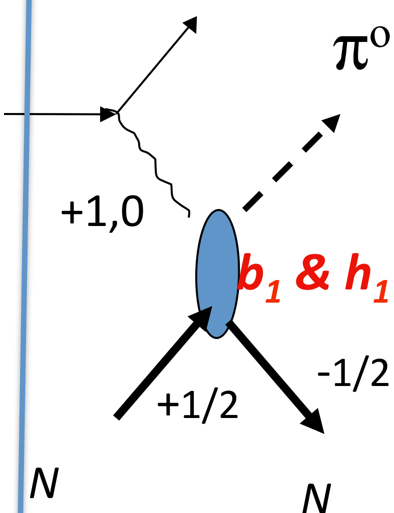
Improved precision based on EM Form Factor measurements
G. D. Cates, et al,, Phys. Rev. Lett. **106**, 252003 (2011).

How to single out **chiral odd GPDs**?

Exclusive Lepto-production of π^o or η, η' to measure **chiral odd GPDs & Transversity**



$$\mathcal{P} = K f_\pi \left\{ \gamma_5 \, \not{q}' \phi_\pi(\tau) + \gamma_5 \mu_\pi \phi_\pi^{(3)}(\tau) \right\}$$



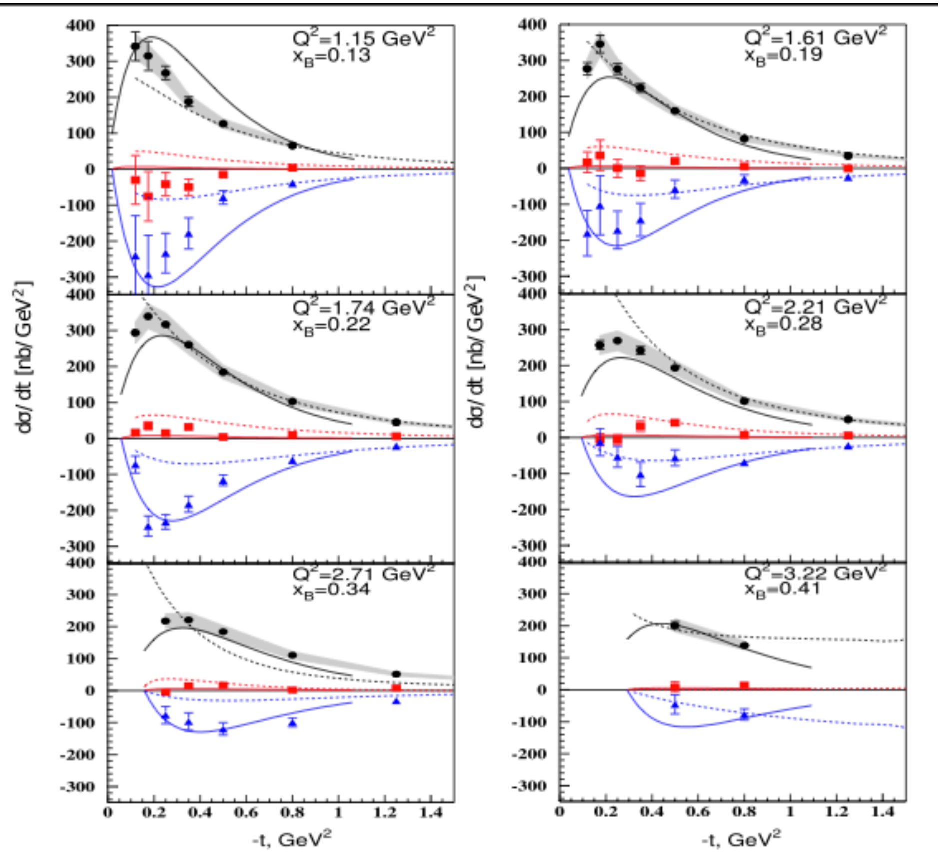
t-channel J^{PC} quantum numbers enhance chiral odd

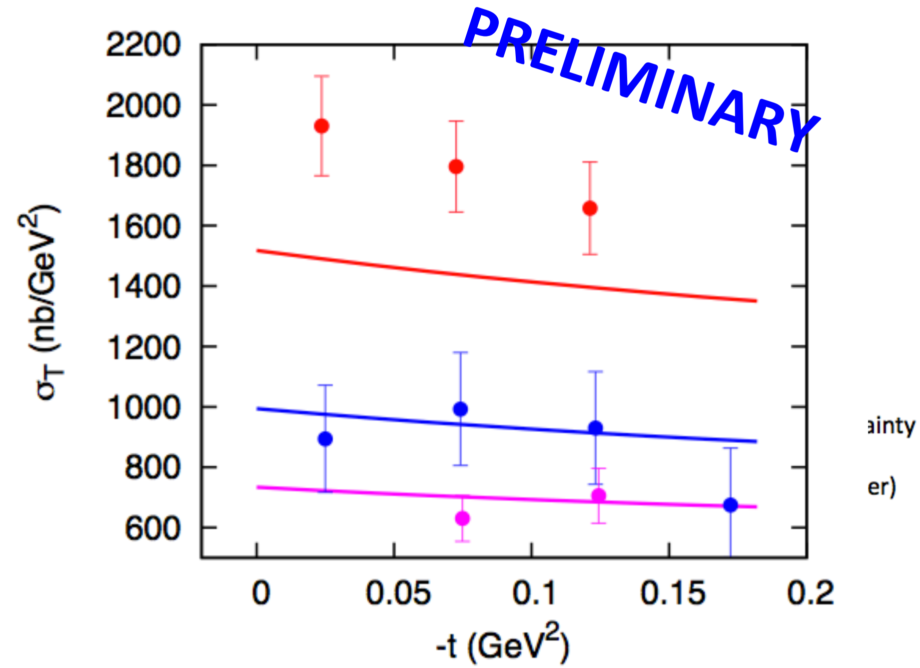
30

Why consider chiral-odd GPDs? Why go beyond leading twist?

$\pi 0$ electroproduction **data dictate** necessity of transverse photons

CLAS; Hall A separated cross sections ; Asymmetries distinguish models





courtesy F. Sabatie, Hall A @ CIPANP

Dashed curve: GGL . . Solid curve: G&K

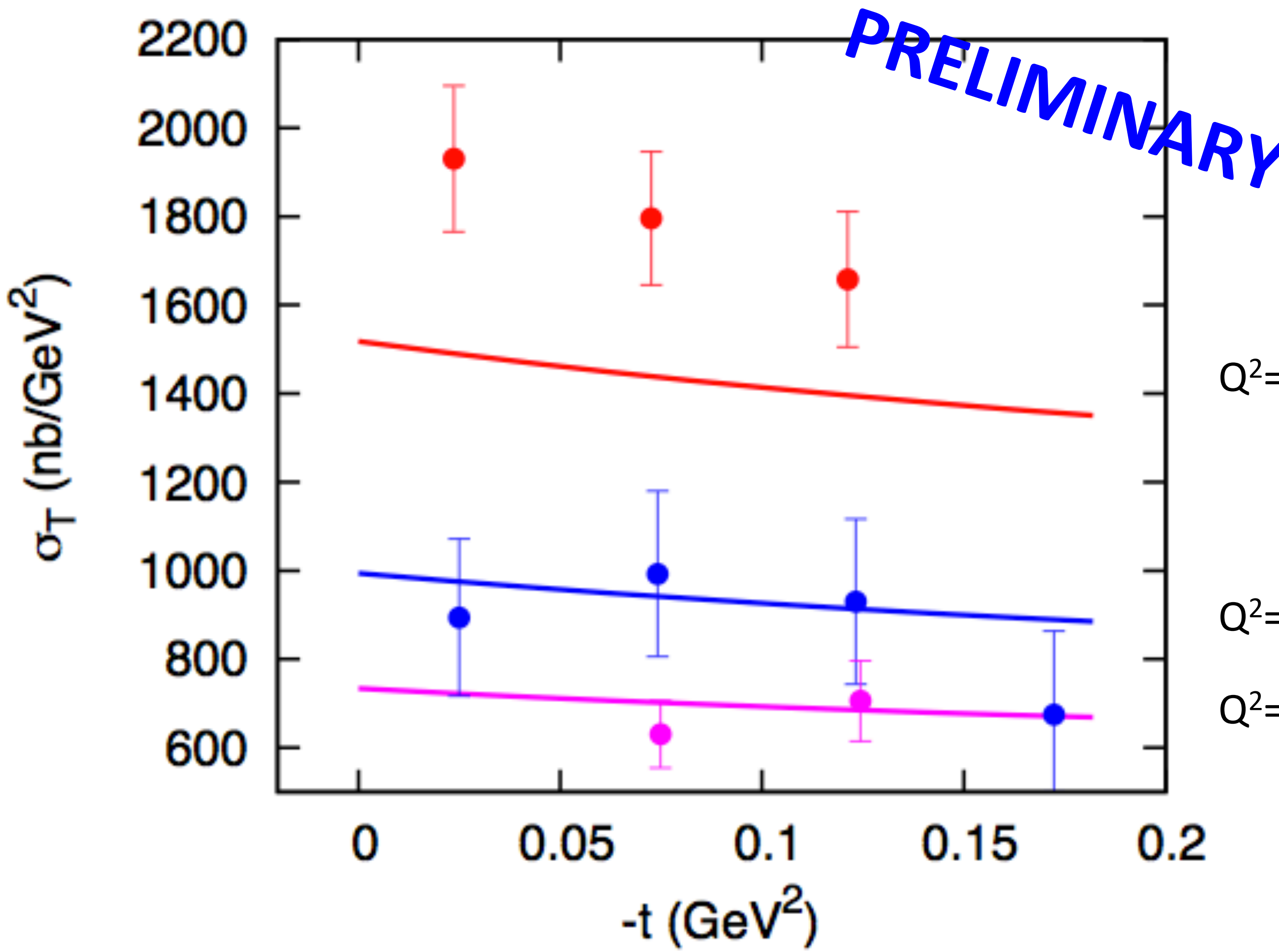
CLAS: Bedlinskiy, et al., PRL 109, 112001 (2012)

G. R. Goldstein, J. O. Gonzalez Hernandez, and S. Liuti, Phys. Rev. D 84, 034007 (2011).

See also S. V. Goloskokov and P. Kroll, Eur. Phys. J. A 47, 112 (2011).

Hall A data x_B=0.36

courtesy F. Sabatie & M. Defurne

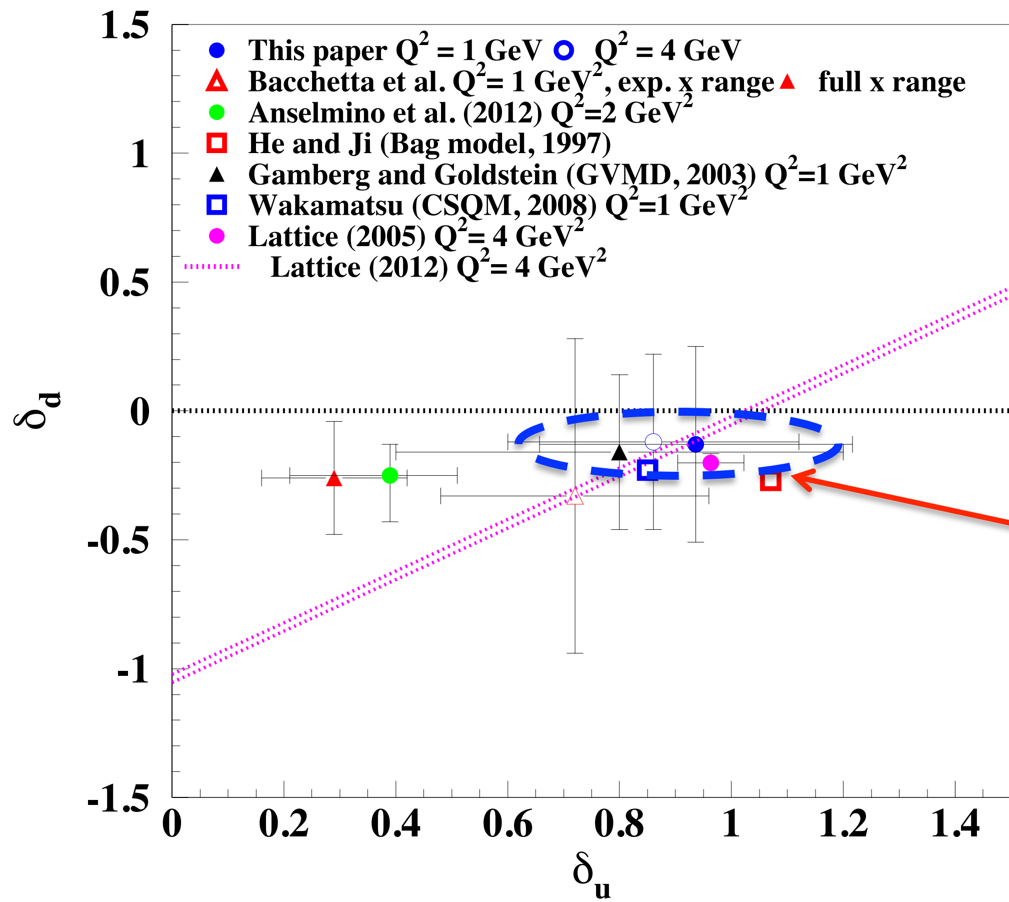


Q^2=1.5 GeV^2

Q^2=1.75 GeV^2

Q^2=2.0 GeV^2

We look at Chiral odd GPDs - why? → $H_T(x,\xi,t)$→ $h_1(x)$ **Transversity** → **tensor charges** δ_q to get complete picture of spin decomposition



GG, Gonzalez, Liuti, PRD91, 114013 (2015))

Observables expressed in bilinears of helicity amps – 6 amps for π^0

Compton Form Factors

Couplings $g_{\pi}^{V\ \&/\mathrm{or}\ A}(Q^2)$

f_1

$$f_{10}^{++} = g_{\pi}^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1+\xi)\left(\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_{\pi}^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1}{2-\zeta}E_T + \frac{1}{2-\zeta}\widetilde{\mathcal{E}}_T\right],$$

f_2

$$f_{10}^{+-} = \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2}\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]$$

$$= \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\zeta^2/4}{1-\zeta}\mathcal{E}_T + \frac{\zeta/2}{1-\zeta}\widetilde{E}_T\right]$$

f_3

$$f_{10}^{-+} = -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

$$= -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

f_4

$$f_{10}^{--} = g_{\pi}^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1-\xi)\left(\mathcal{E}_T - \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_{\pi}^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1-\zeta}{2-\zeta}\mathcal{E}_T + \frac{1-\zeta}{2-\zeta}\widetilde{\mathcal{E}}_T\right]$$

Also Chiral Even CFFs

f_5

$$f_{00}^{+-} = g_{\pi}^{A,odd}(Q)\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

f_6

$$f_{00}^{++} = -g_{\pi}^{A,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\xi\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

Selecting transversity

$$f_{10}^{++} \propto \Delta\left(2\widetilde{\mathcal{H}}_T + (1+\xi)\mathcal{E}_T - (1+\xi)\widetilde{\mathcal{E}}_T\right)$$

$$f_{10}^{+-} \propto \boxed{\mathcal{H}_T +} \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T \boxed{-} \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T$$

$$f_{10}^{-+} \propto \Delta^2\,\widetilde{\mathcal{H}}_T$$

$$f_{10}^{--} \propto \Delta\left(2\widetilde{\mathcal{H}}_T + (1 \boxed{-} \xi)\mathcal{E}_T + (1 \boxed{-} \xi)\widetilde{\mathcal{E}}_T\right),$$

$$\boxed{2\tilde{H}_T + E_T \equiv \bar{E}_T}$$

Compare also f_{long}^{odd} & with chiral even f_{long}^{even}

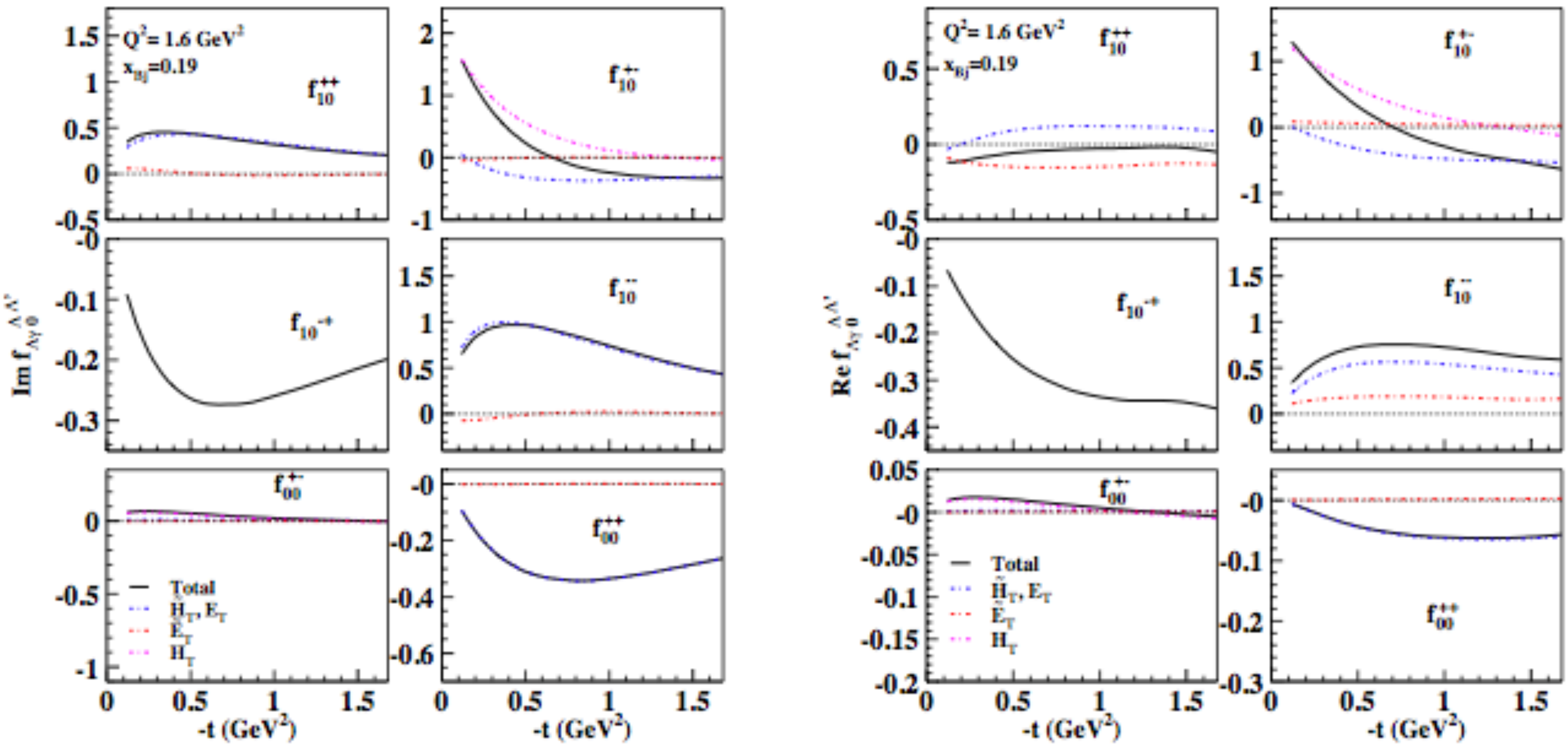
$$f_{00}^{+-} = g_\pi^{A,odd}(Q)\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]\frac{\sqrt{t_0-t}}{2M}$$

$$f_{00}^{++} = -g_\pi^{A,odd}(Q)\left(\frac{\sqrt{t_0-t}}{2M}\right)^2\left[\xi\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right].$$

$$f_{00}^{+-,\,even} = \frac{\zeta}{\sqrt{1-\zeta}}\frac{1}{1-\zeta/2}\frac{\sqrt{t_o-t}}{2M}\widetilde{\mathcal{E}},$$

$$f_{00}^{++,\,even} = \frac{\sqrt{1-\zeta}}{1-\zeta/2}\widetilde{\mathcal{H}} + \frac{-\zeta^2/4}{(1-\zeta/2)\sqrt{1-\zeta}}\widetilde{\mathcal{E}},$$

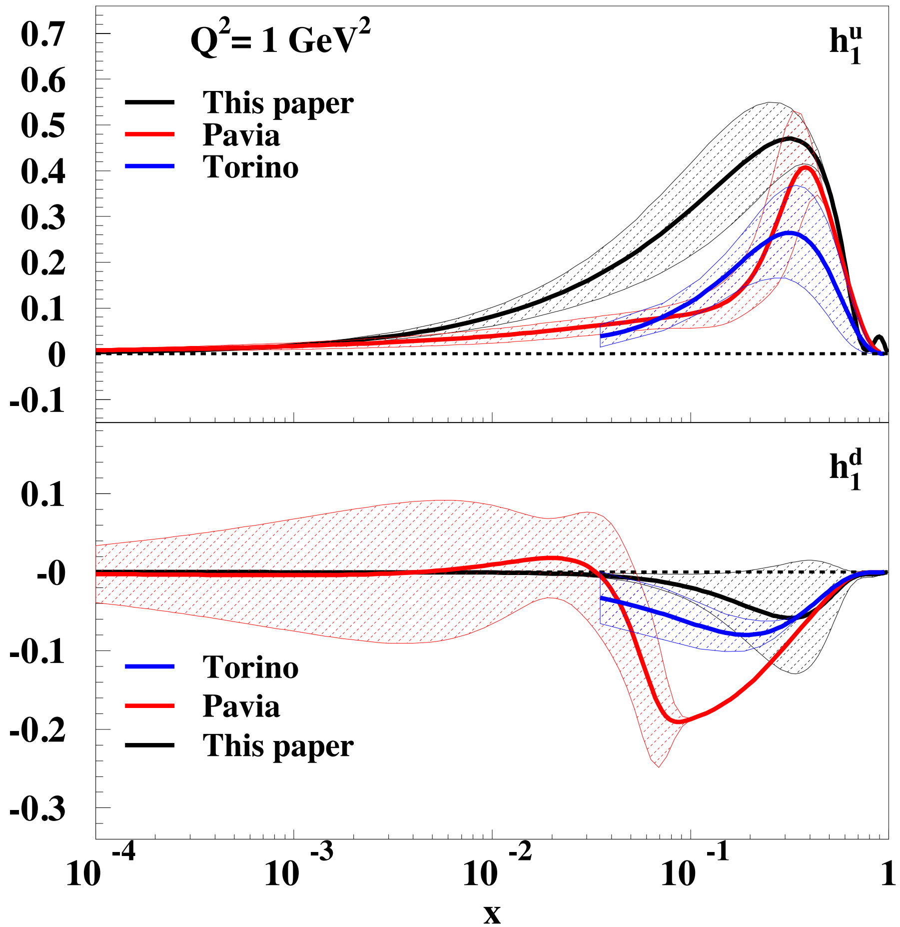
6 helicity amps for π^0 after Compton Form Factors evaluated



$f_{10}^{+-} \propto \Delta^0$ dominates & $\supset H_T$ Transversity

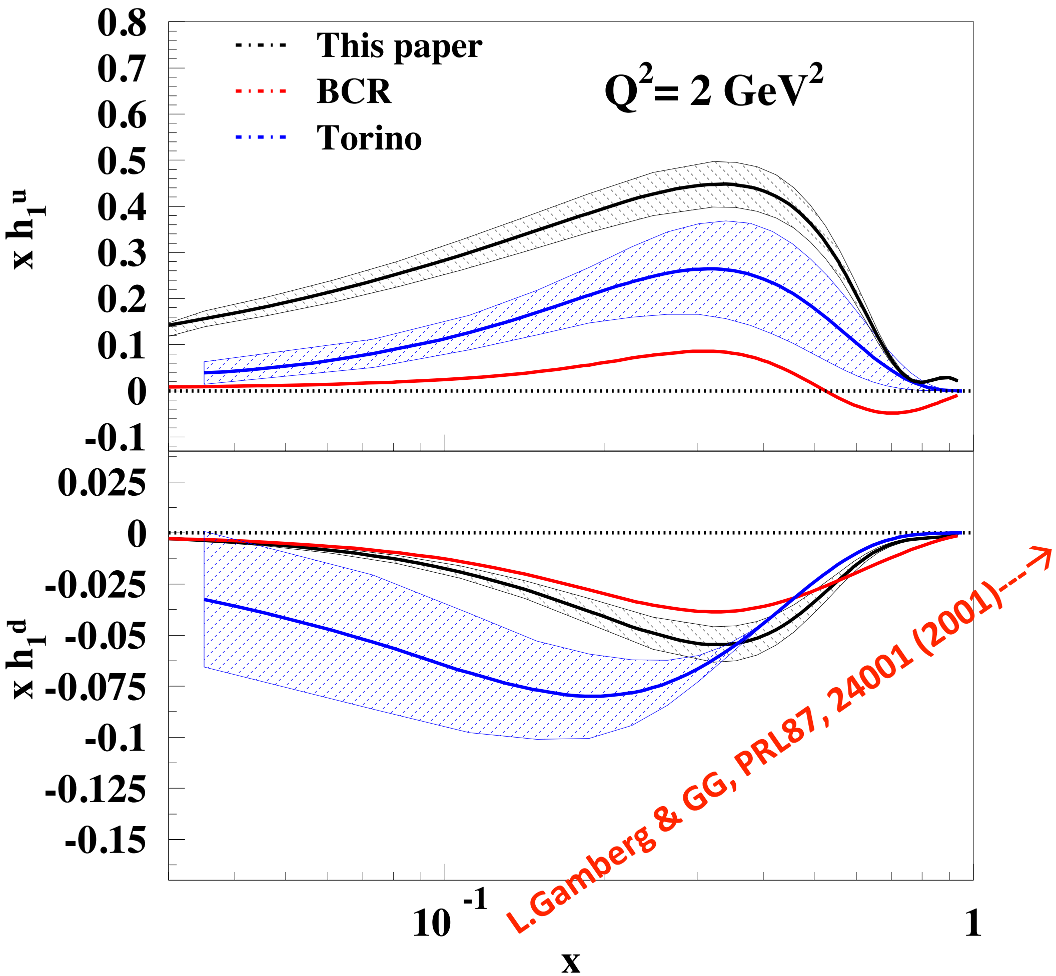
f_{10}^{++} & $f_{10}^{--} \propto \Delta^1$ & $\supset 2\tilde{H}_T + E_T$

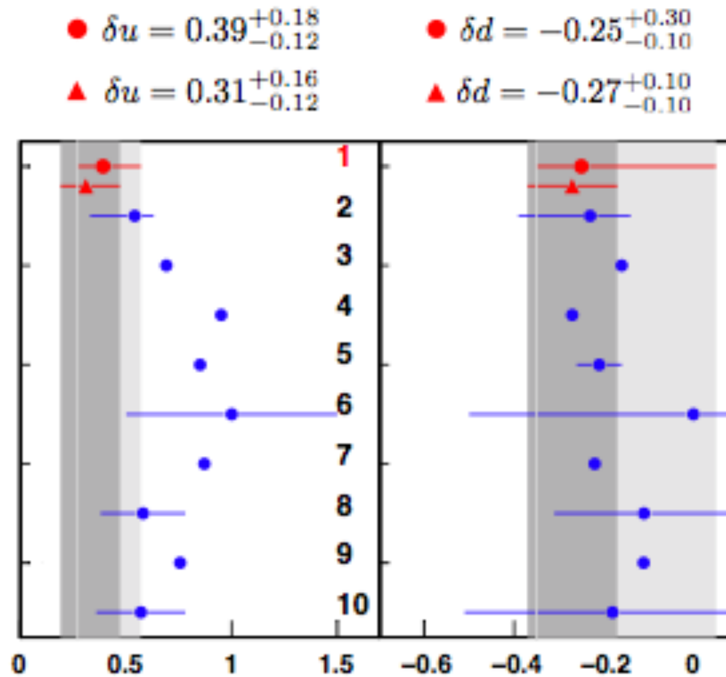
Chiral odd GPDs →
Transversity → pdf's: $h_1^q(x,Q^2)$



GG, Gonzalez, Liuti, arXiv:1311.0483 [hep-ph] 1401.0438 PRD91, 114013 (2015)

Extraction of tensor charge-- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438 PRD91, 114013 (2015)



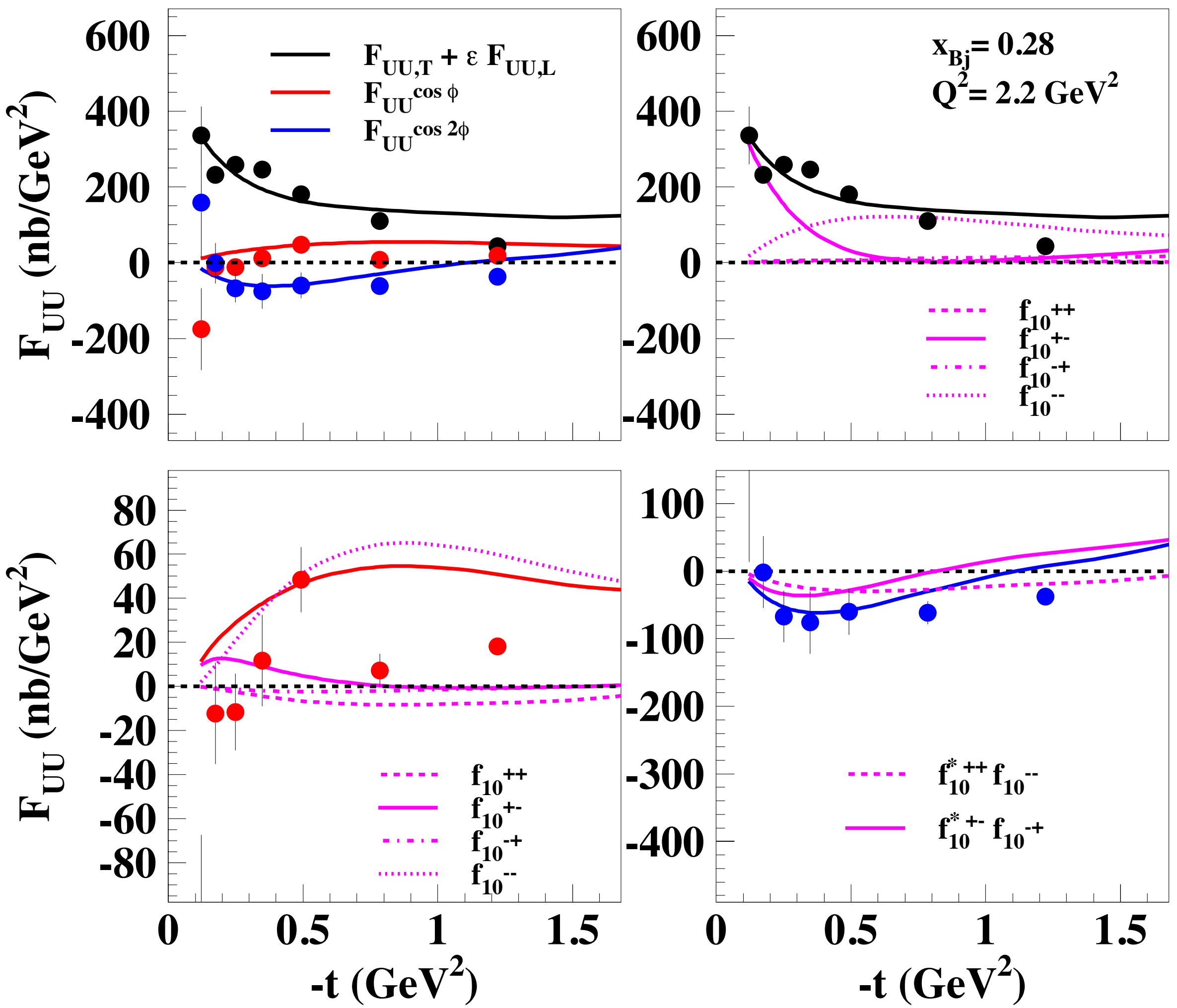


Anselmino, Boglione, et al., Phys.Rev. D87 (2013) 094019
$\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

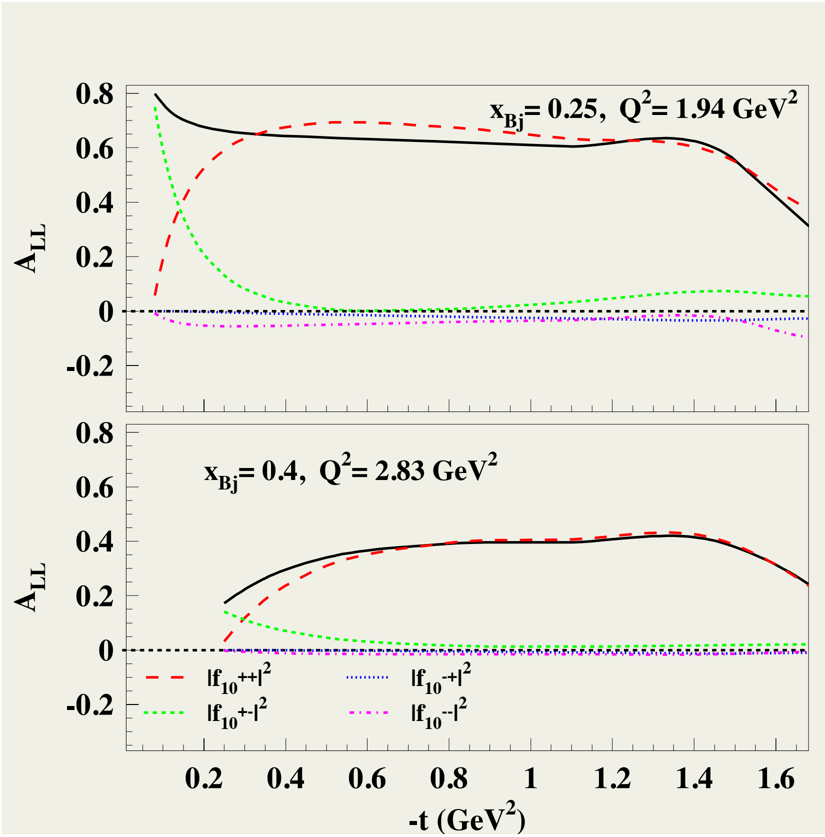
From our Reggeized form
$\delta u \approx 1.2$ $\delta d \approx -0.08$
Closer to QCD sum rule values

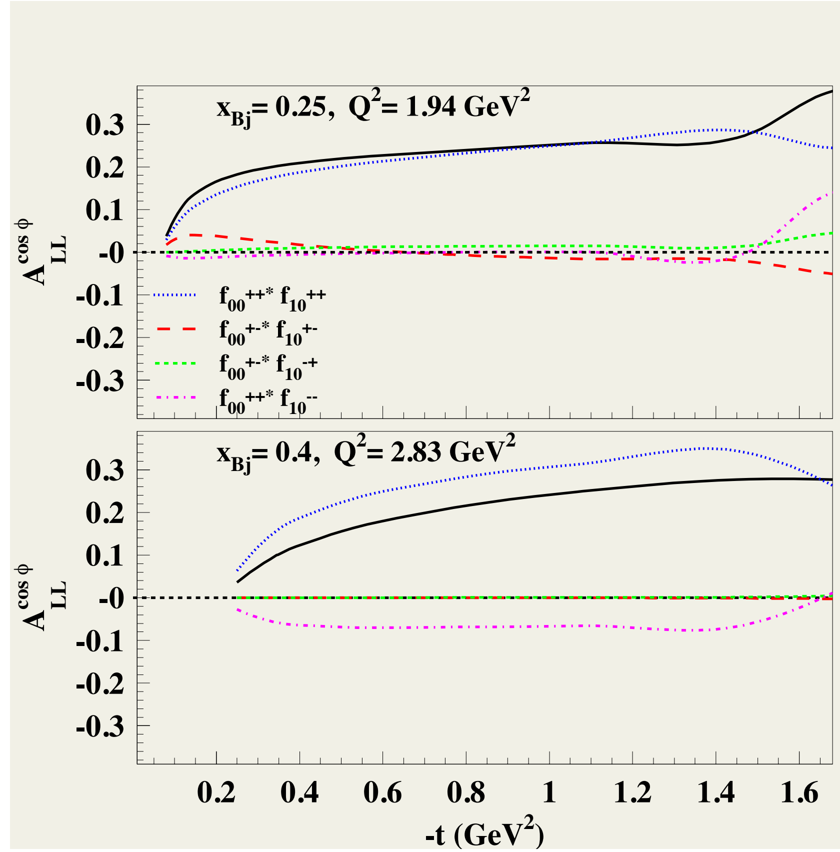
How well do the parameters fixed with DVCS data reproduce π° electroproduction data?

Hall B data, Bedlinskii, et al. PRL 109, 112001 (2012)



Longitudinally polarized beam and target

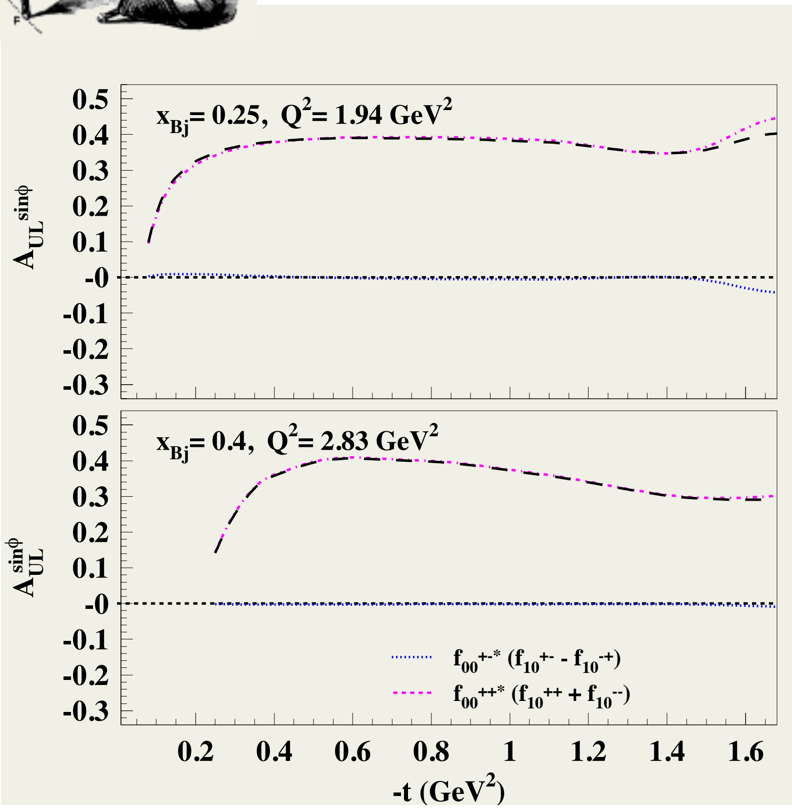


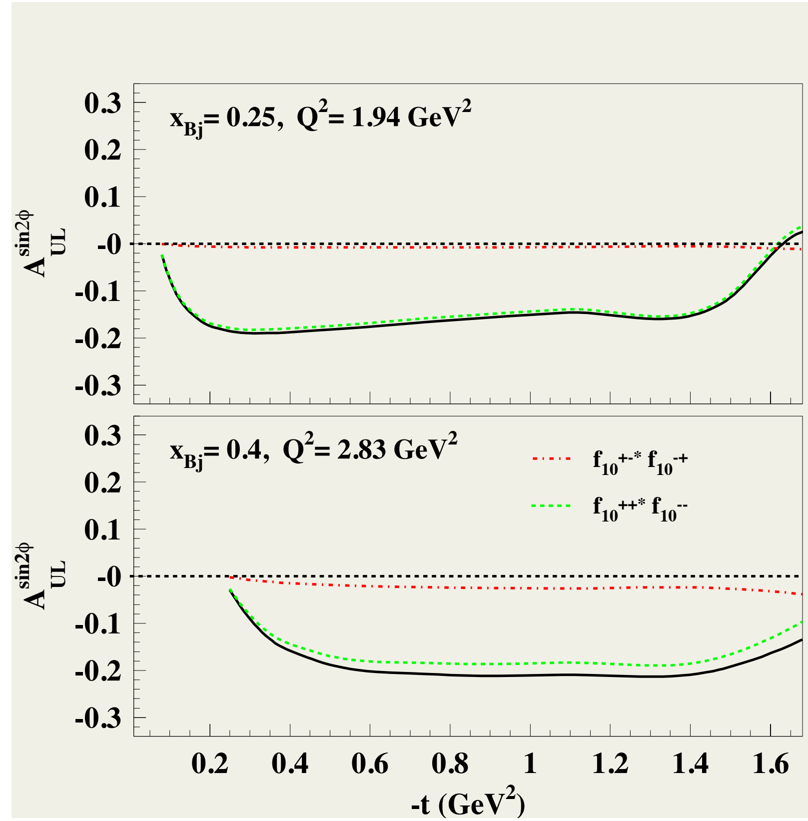


Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

Longitudinally polarized target



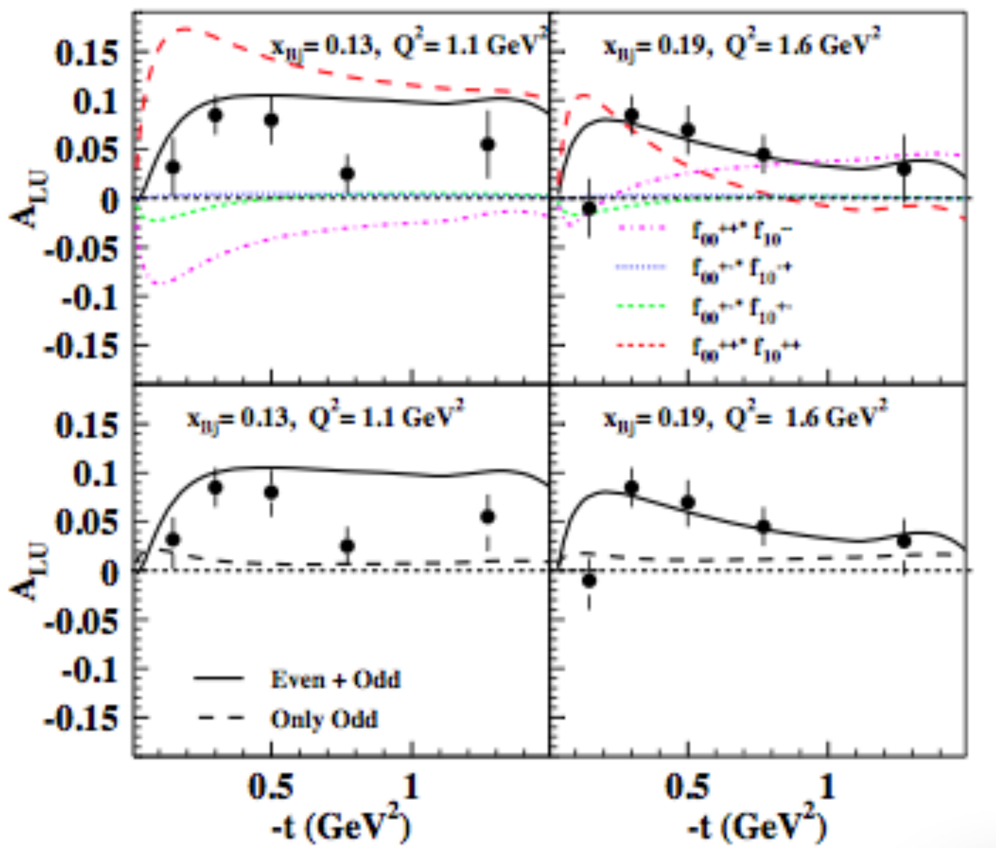


Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

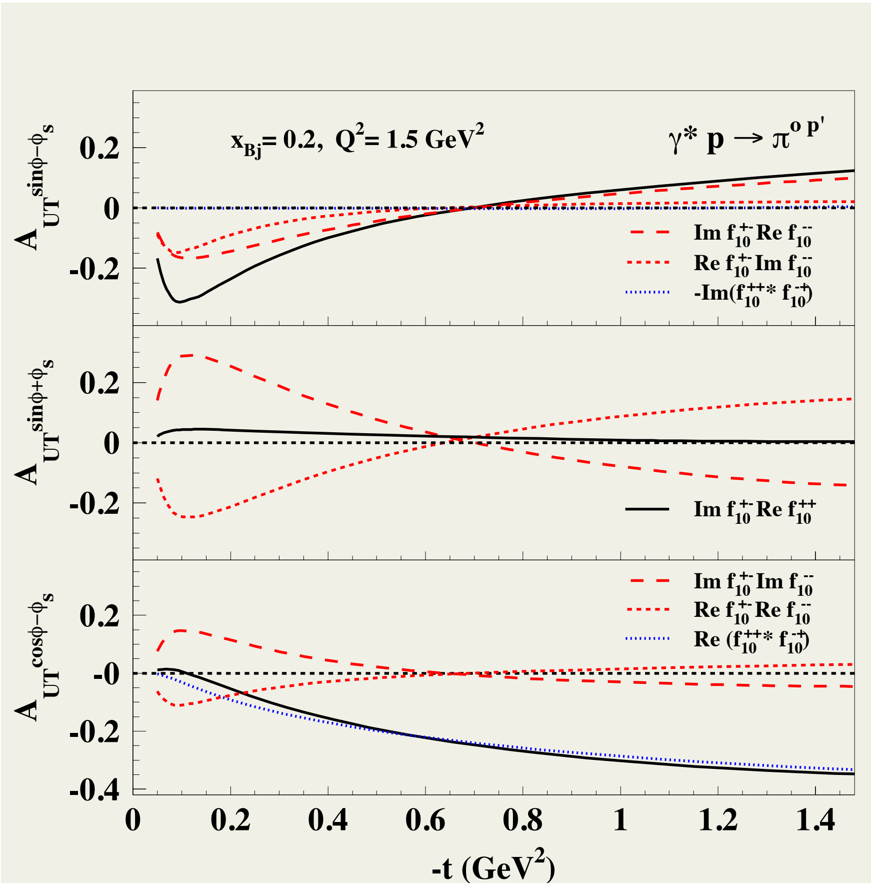
f_{10}^{+-} , f_{10}^{++}, f_{10}^{--} , f_{10}^{-+}
$\propto \Delta^0$, Δ^1 , Δ^1 , Δ^2 resp.

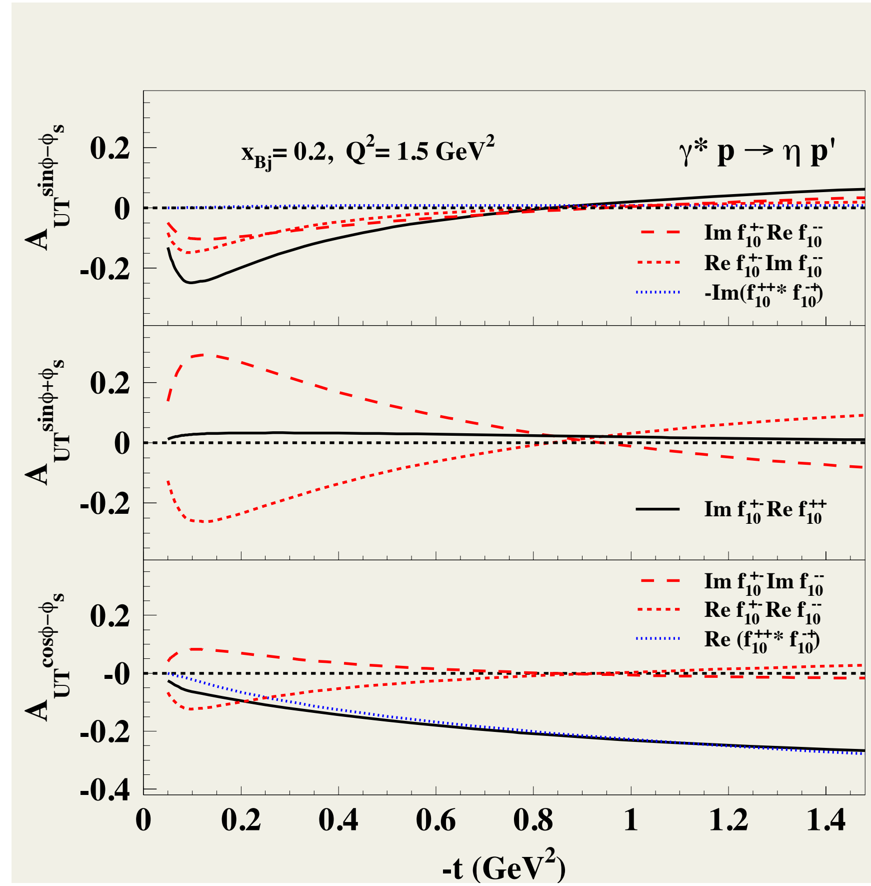
Beam spin asymmetry
shows importance of $\tilde{H}$ chiral even (CLAS data –DeMasi, et al.)



f_{10}^{+-} , f_{10}^{++},f_{10}^{--} , f_{10}^{-+}
$\propto \Delta^0$, Δ^1 , Δ^1 , Δ^2 resp.

Transverse target

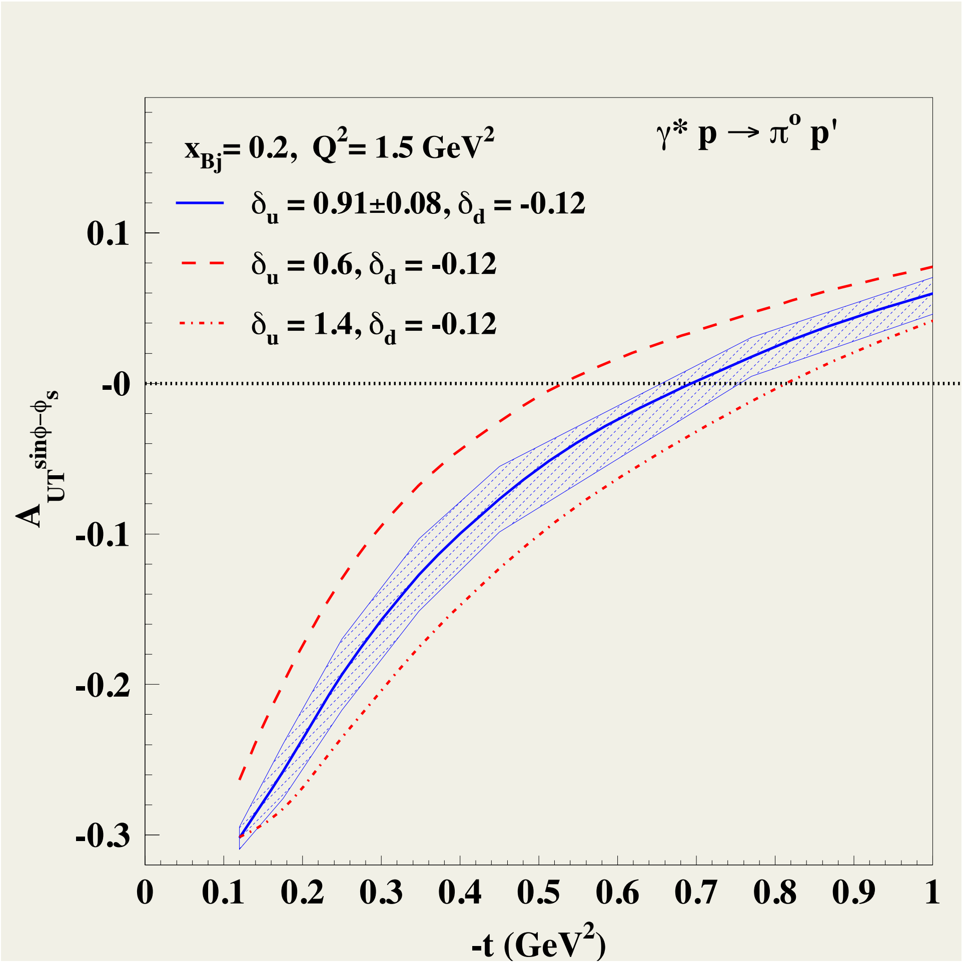




Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++},f_{10}^{--}

Asymmetry sensitive to tensor charge



$$A_{UT}^{\sin(\phi+\phi_S)} = -\frac{\epsilon}{2}\frac{F_{UT}^{\sin(\phi+\phi_S)}}{F_{UU,T}+\epsilon F_{UU,L}} = -\epsilon\,\frac{\Re e f_{10}^{+-}\,\Im m f_{10}^{++} - \Re e f_{10}^{++}\,\Im m f_{10}^{+-}}{d\sigma/dt}$$

Recoil nucleon or baryon polarization

(e.g. e+p→e'+K$^+$+Λ↑)

- General form for DVCS or DVπ (set Λ_γ'=0 in formula)

$f_{\Lambda_{\gamma^*},\Lambda;\Lambda'_\gamma,\Lambda'}$ or $f^{\Lambda^*_\gamma,\Lambda'_\gamma}_{\Lambda,\Lambda'}$

longitudinal

$$\frac{d\sigma_{+,+}}{d\Phi} - \frac{d\sigma_{-,-}}{d\Phi} = \Gamma(2(1-\epsilon)) \sum_{h,\Lambda,\Lambda'_\gamma} \frac{1}{\gamma}\sqrt{\epsilon(1+\epsilon)} \Big[\sin\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} - f^*_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}}) - \epsilon \sin 2\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) \Big]$$

transverse in hadron plane

$$\frac{d\sigma_{+,-}}{d\Phi} + \frac{d\sigma_{-,+}}{d\Phi} = \Gamma \sum_{h,\Lambda,\Lambda'_\gamma} \Big\{ \frac{2}{\gamma}\sqrt{\epsilon(1+\epsilon)} \Big[2\sin\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} + f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) \Big] - 2\epsilon \Big[\cos 2\phi\, \mathcal{R}e(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} + f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) + \sin 2\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} + f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) \Big] \Big\}$$

transverse ⊥ hadron plane

$$-i\left(\frac{d\sigma_{+,-}}{d\Phi} - \frac{d\sigma_{-,+}}{d\Phi}\right) = \Gamma \sum_{h,\Lambda,\Lambda'_\gamma} \Big\{ -2\mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}}) - 2\epsilon(f^*_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f^*_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}}) - 4\sqrt{\epsilon(1+\epsilon)}\cos\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} - f^*_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) - 2\epsilon \Big[-\cos 2\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} - f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) + \sin 2\phi\, \mathcal{R}e(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} - f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) \Big] \Big\}$$

Recoil nucleon or baryon polarization

(e.g. $e+p \rightarrow e' + K^+ + \Lambda\uparrow$)

- General form for DVCS or DVπ (set $\Lambda_\gamma'=0$ in formula)

longitudinal

$$
\begin{aligned}
\frac{d\sigma_{+,+}}{d\Phi} - \frac{d\sigma_{-,-}}{d\Phi} &= \Gamma(2(1-\epsilon)) \sum_{h,\Lambda,\Lambda'_\gamma} \frac{1}{\gamma}\sqrt{\epsilon(1+\epsilon)} \Big[\sin\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} - f^*_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}}) \\
&\quad -\epsilon\, \sin 2\phi\, \boxed{\mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}})}\Big]
\end{aligned}
$$

Leading terms for π

transverse

in hadron plane

$$
\begin{aligned}
\frac{d\sigma_{+,-}}{d\Phi} + \frac{d\sigma_{-,+}}{d\Phi} &= \Gamma \sum_{h,\Lambda,\Lambda'_\gamma} \Big\{\frac{2}{\gamma}\sqrt{\epsilon(1+\epsilon)} \Big[2\sin\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} + f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}})\Big] \\
&\quad -2\epsilon\Big[\cos 2\phi\, \boxed{\mathcal{R}e(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} + f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}})} \\
&\quad + \sin 2\phi\, \boxed{\mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} + f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}})}\Big]\Big\}
\end{aligned}
$$

transverse

$\perp$ hadron plane

$$
\begin{aligned}
-i\left(\frac{d\sigma_{+,-}}{d\Phi} - \frac{d\sigma_{-,+}}{d\Phi}\right) &= \Gamma \sum_{h,\Lambda,\Lambda'_\gamma} \Big\{-2\mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}}) - 2\epsilon(f^*_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f^*_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}}) \\
&\quad - 4\sqrt{\epsilon(1+\epsilon)}\cos\phi\, \mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{0,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} - f^*_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{0,\Lambda;\Lambda'_\gamma,+\frac{1}{2}}) \\
&\quad -2\epsilon\Big[-\cos 2\phi\, \boxed{\mathcal{I}m(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} - f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}})} \\
&\quad + \sin 2\phi\, \boxed{\mathcal{R}e(f^*_{+1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}} f_{-1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} - f_{+1,\Lambda;\Lambda'_\gamma,-\frac{1}{2}} f^*_{-1,\Lambda;\Lambda'_\gamma,+\frac{1}{2}})}\Big]\Big\}
\end{aligned}
$$

Recoil Polarization & DVPseudoscalar

- *cos*2φ & *sin*2φ terms involve transverse virtual photons => probe chiral odd GPDs ($\sigma_T >> \sigma_L$ as previous π^0 & η data show!)
- Long*: Im* $\{f_{10}^{++} * f_{10}^{--} - f_{10}^{-+} * f_{10}^{+-}\} \propto \Delta^2 = t_0 - t$
- Trans: *Im* $\{f_{10}^{++} * f_{10}^{-+} \pm f_{10}^{+-} * f_{10}^{--}\} \supset Im\{H_T^* (2\tilde{H}_T + E_T)\}$
- c.f. $A_{UT}^{\sin(\varphi - \varphi s)}$

f_{10}^{+-} , f_{10}^{++},f_{10}^{--} , f_{10}^{-+}
$\propto \Delta^0$, Δ^1 , Δ^1 , Δ^2 resp.

Observables expressed in bilinears of helicity amps – 6 amps for π^0

Compton Form Factors

Couplings $g_\pi^{V\ \&/\mathrm{or}\ A}(Q^2)$

f_1

$$f_{10}^{++} = g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1+\xi)\left(\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1}{2-\zeta}E_T + \frac{1}{2-\zeta}\widetilde{\mathcal{E}}_T\right],$$

f_2

$$f_{10}^{+-} = \frac{g_\pi^{V,odd}(Q)+g_\pi^{A,odd}(Q)}{2}\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]$$

$$= \frac{g_\pi^{V,odd}(Q)+g_\pi^{A,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\zeta^2/4}{1-\zeta}\mathcal{E}_T + \frac{\zeta/2}{1-\zeta}\widetilde{E}_T\right]$$

f_3

$$f_{10}^{-+} = -\frac{g_\pi^{A,odd}(Q)-g_\pi^{V,odd}(Q)}{2}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

$$= -\frac{g_\pi^{A,odd}(Q)-g_\pi^{V,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

f_4

$$f_{10}^{--} = g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1-\xi)\left(\mathcal{E}_T - \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1-\zeta}{2-\zeta}\mathcal{E}_T + \frac{1-\zeta}{2-\zeta}\widetilde{\mathcal{E}}_T\right]$$

Also Chiral Even CFFs

f_5

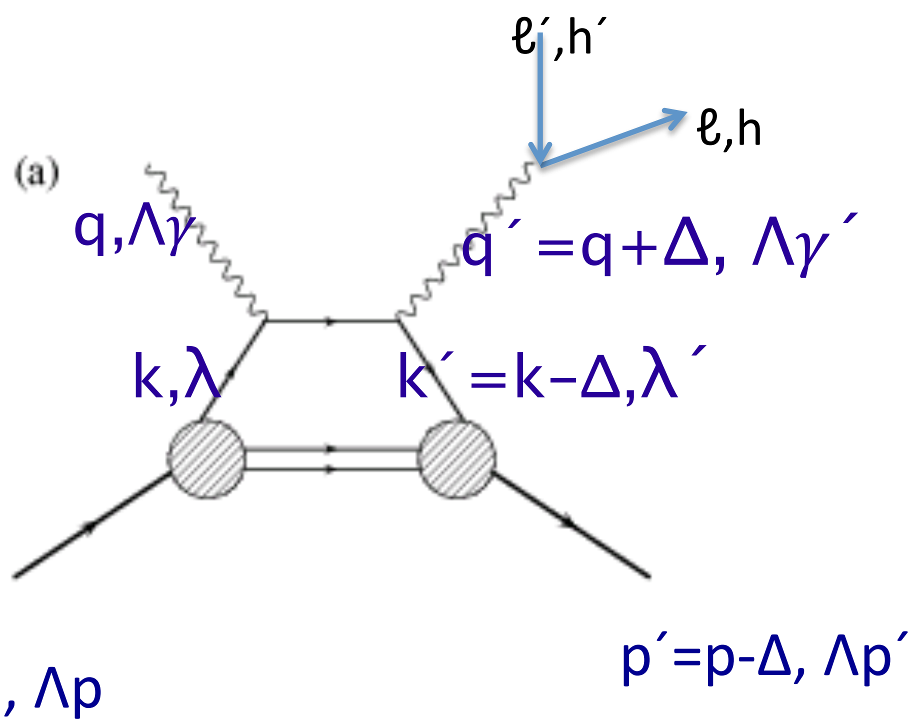
$$f_{00}^{+-} = g_\pi^{A,odd}(Q)\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

f_6

$$f_{00}^{++} = -g_\pi^{A,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\xi\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

Time-like CS

$$T^{\Lambda\gamma,\Lambda p,\Lambda p',h,h'} = \delta_{h,-h'} \sum_{\Lambda\gamma'} \tilde{A}_h^{\Lambda\gamma'} f_{\Lambda\gamma,\Lambda p;\Lambda\gamma',\Lambda p'}/q'^2$$



TCS observables

- BH Interference selects GPDs linearly
- photon-nucleon helicity amps accessed in TCS are related to DVCS – kinematics ξ, t with Q'^2 large vs. ξ, t with $|Q^2|$ large
- Azimuthal modulations involve $\gamma' \to \ell^+ \ell^-$ with $q' = k_+ + k_-$
- Leptons form plane w.r.t. (γ, p, p´) plane & q´ direction. Orientation of lepton plane is φ' .

TCS amps: $f_{\Lambda\gamma,\Lambda p;\Lambda\gamma',\Lambda p'}$ with $\gamma' \rightarrow \ell^+ \ell^-$ at leading order in t/Q'^2 (or Δ/Q'), $\Lambda\gamma'=\Lambda\gamma$

$$f_{+1,+;+1,+} = \sqrt{1-\xi^2}\left(\mathcal{H} + \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}(\mathcal{E} + \tilde{\mathcal{E}})\right)$$

$$f_{+1,-;+1,-} = \sqrt{1-\xi^2}\left(\mathcal{H} - \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}(\mathcal{E} - \tilde{\mathcal{E}})\right)$$

$$f_{+1,+;+1,-} = \frac{\sqrt{t_0 - t}}{2m}(\mathcal{E} - \xi\tilde{\mathcal{E}}) \,,$$

$$f_{+1,-;+1,+} = -\frac{\sqrt{t_0 - t}}{2m}(\mathcal{E} + \xi\tilde{\mathcal{E}}) \,.$$

Same form as DVCS amps – different kinematics

At $x=\pm\xi$ $\mathcal{H}(-\xi,\xi,t) = \mathcal{H}^*(\xi,\xi,t)$. & ± for other CFFs (from Wilson coeffs)

=> TCS:$f_{\Lambda\gamma,\Lambda p;\Lambda\gamma',\Lambda p'}$ = *DVCS:*$f^*_{-\Lambda\gamma,\Lambda p;-\Lambda\gamma',\Lambda p'}$ (leading twist & $-Q^2=Q'^2$)

How to get chiral odd GPDs for $\ell^+ \ell^-$?

Consider $\pi^- + p \rightarrow \ell^+ \ell^- + n$ vs. $\ell^- + p \rightarrow \ell^- + \pi^+ + n$

Summary

- Flexible parameterization for chiral even from form factors, pdfs & DVCS **R✖Dq** Many predictions
- Extended **R✖Dq** to chiral odd sector
- DVMP – π^0, η many dσ ‘s & Asymmetries measure ***Transversity***
- **Compared to new Hall A data – showed agreement within error bands.**
- **Recoil baryon polarization π^0,η → combinations of Chiral Odd leading GPDs**
- **TCS – similar to DVCS & different access to GPDs**