On neutrino masses and phenomenological implications: $\mu \to e\gamma$ and μ-e conversion

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Outline

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2. Review
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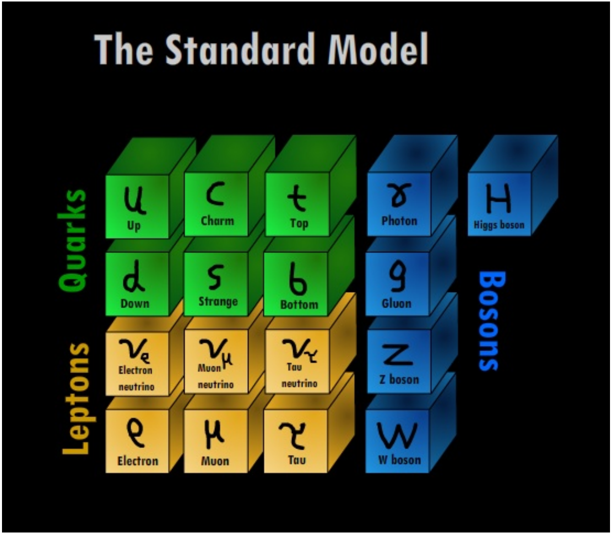
   - Motivations to the EW-ν_R Model
   - Minimal EW-ν_R Model
   - Discrete Symmetry A_4
3. Model of neutrino masses
4. Phenomenological Implications
   - U_{PMNS} vs. V_{CKM}
   - $l_i \to l_j \gamma$
   - μ-e Conversion
5. Conclusion

Introduction

Neutrino in the Standard Model



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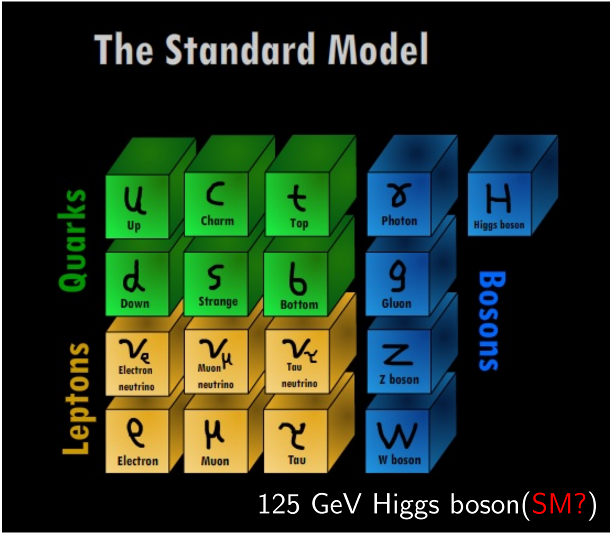


Neutrino in the Standard Model

In the Standard Model (SM)

- neutrinos have exactly zero masses
- there are exactly three neutrinos belonging to three lepton families (e, ν_e), (μ, ν_μ), (τ, ν_τ); lepton number is conserved
- neutrinos and antineutrinos are distinct
- all neutrinos are left-handed, and all antineutrinos are right-handed.

2 problems

Solar neutrino problem

2 problems

Neutrino Oscillation's Evidences

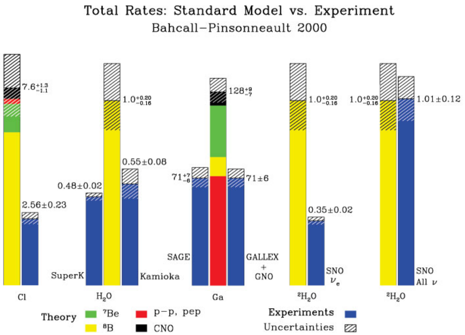
Solar neutrino problem

2 problems

Atmospheric Neutrino Anomaly

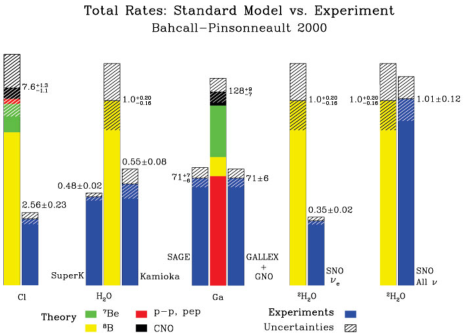
Solar neutrino problem

Solar neutrino problem



- First crack - Ray Davis Cl^{37} solar ν experiments : Φ_{ν_e}(observed) $=$ 1/3 Φ_{ν_e} (SSM)

Solar neutrino problem



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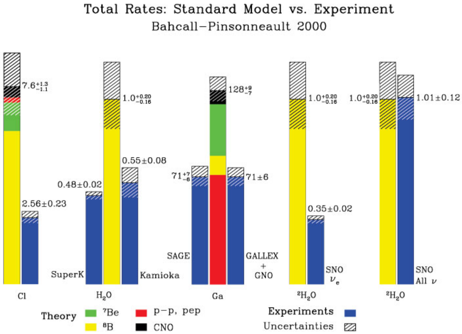

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- GALLEX and SAGE saw about $62(\pm 10)\%$ of SSM prediction

Atmospheric Neutrino Anomaly

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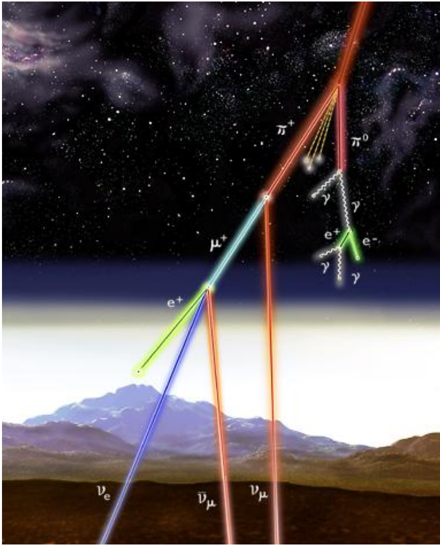
Prediction
$N_{\nu_\mu} : N_{\nu_e} \simeq 2 : 1$

Observation
$N_{\nu_\mu} : N_{\nu_e} = 1.3/1$
by Super-Kamiokande in 1998

$\rightarrow$ discovery of neutrino oscillations



Neutrino Oscillation

Neutrino oscillation arises from a mixture between the flavor and mass eigenstates of neutrinos.

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^{lepton}|\nu_i\rangle$$

where

- $|\nu_\alpha\rangle$ is flavor eigenstate. $\alpha = e,\ \mu,\ \tau$
- $|\nu_i\rangle$ is mass eigenstate. i = 1, 2, 3
- $U_{\alpha i}^{lepton}$ is lepton mixing matrix or **Pontecorvo-Maki-Nakagawa-Sakata** (U_{PMNS}) matrix

Motivation

The discovery of neutrino oscillations

- has revealed many valuable information concerning the mixing matrix U_{PMNS} and the Δm^2 in the neutrino sector.
- first evidence of physics beyond the Standard Model (BSM).

Puzzled questions

- The origin of neutrino masses?
- Why is the mass of neutrino so tiny ($m_\nu < O(eV)$)?
- Can we access experimentally the physics that are responsible for the tininess of the neutrino masses and their mixings?
- Why is the leptonic mixing matrix U_{PMNS} so different from V_{CKM} of the quark sector?

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Motivation

New Physics Search in Charged Lepton Flavor

For instance, $\mu \to e\gamma$, μ-e conversion

- Observing these will remove a hurdle to understand why particles in the same category (family) decay from heavy to lighter, more stable mass states.
- Physicists have searched for these since the 1940s.
- Discovering them is central to understand what physics lies beyond the SM.

Review

Neutrino masses

Dirac mass

Dirac neutrino masses are the neutrino analogues of the SM quark and charged lepton masses. They come from Yukawa coupling to the SM Higgs field $\tilde{\Phi}$

$$\underbrace{g_\nu}_{\text{Yukawa coupling}} \bar{\nu}_L \tilde{\Phi} \nu_R + h.c. \Rightarrow g_\nu \langle \tilde{\Phi} \rangle \bar{\nu}_L \nu_R + h.c. \equiv \underbrace{m_\nu^D}_{\text{Dirac Mass}} \left(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R \right)$$

Neutrinos have a phase of $e^{-i\phi}$ and antineutrinos have a phase of $e^{i\phi}$. Therefore, in the Dirac mass term, these phases cancel out $\rightarrow$ lepton number is conserved

Neutrino masses

Majorana mass of ν_R

$$M_R \ \nu_R^T \sigma_2 \nu_R$$

Majorana mass

In the Majorana mass term, the phase of $\nu_R^T \nu_R$ is not zero $\rightarrow$ lepton number is violated

Seesaw Mechanism

A generic model used to understand the observed neutrino masses ($\sim$ O(eV)), compared to those of quarks and charged leptons which are much much heavier.

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A generic model used to understand the observed neutrino masses ($\sim O(\text{eV})$), compared to those of quarks and charged leptons which are much much heavier.

With $\chi \equiv \sigma_2 \nu_R^*$ and $\nu \equiv \nu_L$ the mass terms can be written as

$$\begin{pmatrix} \nu^T & \chi^T \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & M_R \end{pmatrix}}_{\text{M}} \sigma_2 \begin{pmatrix} \nu \\ \chi \end{pmatrix}$$

Seesaw Mechanism

With the assumption: $m_\nu^D \ll M_R$, diagonalizing the matrix **M** gives eigenvalues

$$m_\nu \approx \frac{\left(m_\nu^D\right)^2}{M_R} \text{ and } M_R$$

Experimental neutrino mass

- Cosmological constraints [1]: $\sum m_\nu < 0.23eV$
- Neutrino oscillation experiments [2]:
  the largest Δm^2 is $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2 \Rightarrow$ the heaviest $m_\nu \gtrsim 4.9 \times 10^{-2}\ eV$.

Cosmology + Oscillation: $4.9 \times 10^{-2}\ eV \lesssim m_\nu^{heaviest} \lesssim 0.23\ eV$

[1] Planck 2015 results
[2] Particle Data Group

Motivations to the EW-ν_R Model

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- ν_R is a singlet under $SU(2)_L \times U(1)_Y$.
- $M_R \sim$ Grand Unified (GUT) mass scale of 10^{16} GeV naturally.

$\rightarrow M_R$ is too large.

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Questions

- Can we make the Seesaw testable?
- Can M_R be of the order of Λ_{EW} (246 GeV)?
- Keeping the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$?
- No new gauge interactions added?

Non-perturbative phenomena

Such important non-perturbative phenomena

- EW phase transition (from $\langle \phi^0 \rangle = 0$ to $\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$)
- related Physics ("sphaleron",...)

can be study by a usual approach: **Lattice regularization**

Can we put the SM gauge theory ($SU(2)_L \times U(1)_Y$) on the lattice?

- Nielsen-Ninomiya no-go theorem: there appear an equal number of right- and left-handed particles of given quantum numbers in a regularized theory with a chiraly invariant action.
- Since the Standard Model is chiral. Left- and right-handed fermions are treated differently by weak interactions, for example, only left-handed doublets coupled to W's. The Nielsen-Ninomiya theorem implies that one cannot put the SM on the lattice.

Anomalies Cancelation

• The SM contains gauge triangle anomalies which breaks gauge invariance. Anomalies cancelation in the SM gives

$$\sum_i Q_i = 0 \tag{1}$$

for each family $\rightarrow$ cancelation between quarks and leptons.

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• Witten anomaly: the theory is trivial unless the number of doublets is even.

- SM has 4 doublets per family (1 lepton and 3 color quark doublets) $\rightarrow$ Witten anomaly free.
- SM with mirror particles: not a chiral gauge theory $\rightarrow$ No Witten anomaly.

Can we solve the problems?

Introducing...

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The Non-sterile Electroweak-scale Right-handed neutrino (EW ν_R) Model

[P. Q. Hung, PLB 649 (2007)]

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- ✓ Can M_R be of the order of Λ_{EW} (246 GeV)?
- ✓ Keeping the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$?
- ✓ No new gauge interactions added?
- ✓ Study non-perturbative phenomena by using lattice regularization?
- ✓ Is carefree toward anomalies?

Minimal EW-ν_R Model [3]

[3] P.Q. Hung, 2007

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What is it?

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Model in which right-handed neutrinos have Majorana masses of the order of Λ_{EW} naturally.

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Gauge group

$SU(3)_C \times SU(2) \times U(1)_Y$

[3] P.Q. Hung, 2007

Model Content

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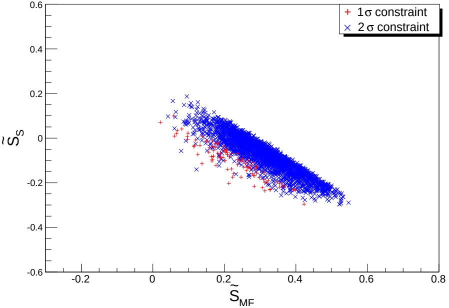
Mirror particles are totally different from the SM particles!

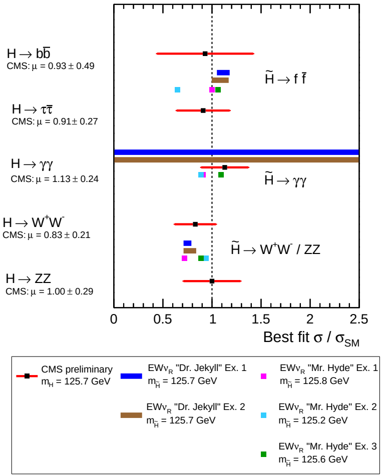
EW precision

V. Hoang, P. Q. Hung and A. S. Kamat, Nucl. Phys. B **877**, 190 (2013) [arXiv:1303.0428 [hep-ph]].



Implications of the 125-GeV SM-like scalar Dr Jekyll (SM-like) & Mr Hyde (very different from SM)

V. Hoang, P. Q. Hung and A. S. Kamat, Nucl. Phys. B **896** (2015) 611-656 [arXiv:1412.0343 [hep-ph]].



Question

What are Higgs sectors for Majorana and Dirac masses?

Majorana mass of ν_R

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$$L_M = g_M \left(l_R^{M,T} \, \sigma_2\right) \, (i \, \tau_2 \, \tilde{\chi}) \, l_R^M \tag{2}$$

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$$
\begin{aligned}
L_M &= g_M \left(l_R^{M,T}\ \sigma_2\right)\ (i\ \tau_2\ \tilde{\chi})\ l_R^M \\
&= g_M\ \nu_R^T\ \sigma_2\ \nu_R\ \chi^0 - \frac{1}{\sqrt{2}}\ \nu_R^T\ \sigma_2\ e_R^M\ \chi^+ + \ldots
\end{aligned}
\tag{2}
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\begin{aligned}
\tilde{\chi} &= (3,\ Y/2 = 1) \\
\tilde{\chi} &= \frac{1}{\sqrt{2}}\ \vec{\tau}.\vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix}
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From (2), the Majorana mass $M_R = g_M v_M$ where $\langle \chi^0 \rangle = v_M \sim \Lambda_{EW}$

ν_R couples to Z-boson and contribute to Γ_Z

$\rightarrow$ Γ_Z's constraint (number of light neutrinos = 3) implies $M_R > M_Z/2$

Dirac mass

Dirac mass

The singlet scalar field ϕ_S couples to fermion bilinear.

$$\begin{aligned} L_S &= g_{Sl}\ \bar{l}_L\ \phi_S\ l_R^M + h.c. \\ &= g_{Sl}\bar{\nu}_L\ \phi_S\ \nu_R + ... + h.c. \end{aligned} \tag{3}$$

$$\phi_S\,(1,\ \ Y/2 = 0)$$

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$$m_\nu \approx \frac{\left(m_\nu^D\right)^2}{M_R} \lesssim 0.23\ eV$$

- $v_S \sim 10^{5-6}$ eV with $g_{Sl} \sim \mathcal{O}(1)$
- $v_S \sim \Lambda_{EW}$ with $g_{Sl} \sim \mathcal{O}(10^{-6})$

Charged fermion mass

We also need a Higgs doublet for charged fermion masses (leptons and quarks)

$$
\begin{aligned}
L_{Y_l} &= g_l \, \bar{l}_L \, \Phi_2 \, e_R + h.c. \qquad (4) \\
L_{Y_q} &= g_q \, \bar{q}_L \, \Phi_2 \, u_R + h.c. \qquad (5)
\end{aligned}
$$

$$
\Phi_2 = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \langle \phi^0 \rangle = \frac{v_2}{\sqrt{2}}
$$

Experimentally

[4] Werner Rodejohann, 2012

Experimentally

• For the quark sector we use the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4]

$$|V_{CKM}| = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

which is really close to a unit matrix.

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which is really close to a unit matrix.

• For the lepton sector, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is used to study the mixings [4]

$$|U_{PMNS}| = \begin{pmatrix} 0.779...0.848 & 0.510...0.604 & 0.122...0.190 \\ 0.183...0.568 & 0.385...0.728 & 0.613...0.794 \\ 0.200...0.576 & 0.408...0.742 & 0.589...0.775 \end{pmatrix}$$

[4] Werner Rodejohann, 2012

Model of neutrino masses

It was conjectured by Cabibbo[5] and Wolfenstein[6] independently that

$$U_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{6}$$

[5] N. Cabibbo, 1978
[6] L. Wolfenstein, 1978

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Experimentally, $U_{PMNS} \simeq U_{CW}$

Is there a symmetry that can give rise to U_{CW}?

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For instance, A_4 Symmetry

[5] N. Cabibbo, 1978

[6] L. Wolfenstein, 1978

A_4 Symmetry

Why A_4?

Why A_4?

With 3 families, we need a group containing a $\underline{3}$ representation.

The simplest one is A_4.

What is A_4?

What is A_4?

- Non-Abelian discrete group
- Four irreducible representations: **Three** 1-dimension representations called $\underline{1}$, $\underline{1}'$, $\underline{1}''$ and **One** 3-dimension representation called $\underline{3}$

A_4 Symmetry

If denoting $\underline{3}$ as (1, 2, 3) then

Multiplication rule[7]

$$\begin{aligned} \underline{3} \times \underline{3} &= \underline{1}(11 + 22 + 33) + \underline{1}'(11 + \omega^2 22 + \omega 33) + \underline{1}''(11 + \omega 22 + \omega^2 33) \\ &+ \underline{3}(23, 31, 12) + \underline{3}(32, 13, 21) \end{aligned}$$

where $\omega = e^{i2\pi/3}$

[7] Ernest Ma, 2007

- In a standard scenario, one usually requires three Higgs doublets to couple to SM charged fermions.
- LHC 125-GeV SM-like Higgs boson put a very very tight constraint on the scalar sector. So it's hard to satisfy those data when 2 or more Higgs doublets are present in the standard scenario.

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- LHC 125-GeV SM-like Higgs boson put a very very tight constraint on the scalar sector. So it's hard to satisfy those data when 2 or more Higgs doublets are present in the standard scenario.

$\Rightarrow$ Our model of neutrino masses: minimal EW ν_R model + 1 Higgs doublet + 2 Higgs triplets $\tilde{\chi},\ \xi$ becomes more relevant.

The form of U_{CW} in our work is contained in ν sector, NOT in charged lepton sector as in some generic models.

Model of neutrino masses

Assignments of the model's content

Field	$(\nu, l)_L$	$(\nu, l^M)_R$	e_R	e_L^M	ϕ_{0S}	$\tilde{\phi}_S$	Φ_2
A_4	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{3}$	$\underline{1}$

Notice: An extension to four Higgs singlet fields $\rightarrow$ No constraints from the LHC!

Neutrino Dirac mass

The Yukawa interactions

$$L_S = \bar{l}_L \left(g_{0S}\phi_{0S} + g_{1S}\tilde{\phi}_S + g_{2S}\tilde{\phi}_S\right) l_R^M + h.c. \tag{7}$$

[8] Ernest Ma, 2007

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$$\underline{3} \otimes (\quad \underline{1} \qquad \underline{3} \qquad \underline{3} \;) \; \underline{3}$$

where g_{1S} and g_{2S} reflect the two different ways that $\tilde{\phi}_S$ couples to the product of $\bar{l}_L$ and l_R^M.

[8] Ernest Ma, 2007

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$$L_S = \bar{l}_L \left(g_{0S}\phi_{0S} + g_{1S}\tilde{\phi}_S + g_{2S}\tilde{\phi}_S\right) l_R^M + h.c. \qquad (7)$$
$$\underline{3} \otimes (\quad \underline{1} \qquad \underline{3} \qquad \underline{3} \;) \; \underline{3}$$

where g_{1S} and g_{2S} reflect the two different ways that $\tilde{\phi}_S$ couples to the product of $\bar{l}_L$ and l_R^M.

Multiplication rule[8]

$$\begin{aligned} \underline{3} \times \underline{3} &= \underline{1}(11+22+33) + \underline{1}'(11+\omega^2 22+\omega 33) + \underline{1}''(11+\omega 22+\omega^2 33) \\ &+ \underline{3}(23,31,12) + \underline{3}(32,13,21) \end{aligned}$$

[8] Ernest Ma, 2007

Neutrino Dirac mass

Neutrino Dirac mass matrix:

$$M_\nu^D = \begin{pmatrix} g_{0S}v_0 & g_{1S}v_3 & g_{2S}v_2 \\ g_{2S}v_3 & g_{0S}v_0 & g_{1S}v_1 \\ g_{1S}v_2 & g_{2S}v_1 & g_{0S}v_0 \end{pmatrix} \tag{8}$$

where $v_0 = \langle \phi_{0S} \rangle$ and $v_i = \langle \phi_{iS} \rangle$ with $\imath = 1, 2, 3$.

Neutrino Dirac mass

If $v_1 = v_2 = v_3 = v \sim O(10^5\ eV)$ [9], M_ν^D can be diagonalized as follows

$$U_{\nu_L}^\dagger M_\nu^D U_{\nu_R} = U_\nu^\dagger M_\nu^D U_\nu = \begin{pmatrix} m_{1D} & 0 & 0 \\ 0 & m_{2D} & 0 \\ 0 & 0 & m_{3D} \end{pmatrix} \tag{9}$$

where $U_\nu = U_{CW}^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$

Notice that $U_{\nu_L} = U_{\nu_R} = U_\nu$.

[9] P.Q. Hung, 2007

Neutrino Dirac mass

The neutrino Dirac masses are

$$m_{1D} = g_{0S}v_0 + g_{1S}v + g_{2S}v \tag{10}$$

$$m_{2D} = g_{0S}v_0 + g_{1S}v\omega^2 + g_{2S}v\omega \tag{11}$$

$$m_{3D} = g_{0S}v_0 + g_{1S}v\omega + g_{2S}v\omega^2 \tag{12}$$

Reality of the masses require that

$$g_{2S} = g_{1S}^* \tag{13}$$

Neutrino Majorana Mass

From the Lagrangian

$$L_M = g_M \,(l_{iR}^{M,T}\,\sigma_2)(i\,\tau_2\,\tilde{\chi})\,l_{jR}^{M} + h.c. \tag{14}$$

Because of the constraints from 125-GeV SM-like boson, the Higgs triplet $\tilde{\chi}$ transforms as $\underline{1}$. Right-handed Majorana mass matrix

$$M_R = \begin{pmatrix} g_M\langle\chi^0\rangle & 0 & 0 \\ 0 & g_M\langle\chi^0\rangle & 0 \\ 0 & 0 & g_M\langle\chi^0\rangle \end{pmatrix} = g_M v_M \mathbb{I} \tag{15}$$

Neutrino Mass Matrix

$$M_\nu = \begin{pmatrix} 0 & M_\nu^D \\ M_\nu^D & M_R \end{pmatrix}$$

The 3×3 see-saw mass matrix for the light neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ becomes

$$m_\nu \sim -M_\nu^D M_R^{-1} M_\nu^{D,T} \tag{16}$$

Charged-lepton mass

[10] P.Q. Hung, 2007

Charged-lepton mass

• Charged leptons can couple to singlet Higgs field which give rise to a mass mixing between charged SM and mirror leptons. However, the mixing is very small so its contribution to the charged-lepton mass matrix can be negligible [10].

[10] P.Q. Hung, 2007

Charged-lepton mass

• Charged leptons can couple to singlet Higgs field which give rise to a mass mixing between charged SM and mirror leptons. However, the mixing is very small so its contribution to the charged-lepton mass matrix can be negligible [10].

• The Yukawa couplings (with Higgs doublet)

$$\begin{aligned} L_{Y_l} &= g_l \, \bar{l}_L \;\; \Phi_2 \;\; e_R + h.c. \\ &= \quad \underline{3} \otimes \underline{1} \otimes \underline{3} \end{aligned} \tag{17}$$

[10]P.Q. Hung, 2007

Charged lepton mass

The charged-lepton mass matrix is

$$\mathcal{M}_l = g_l \frac{v_2}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{18}$$

which gives rise to

$$U_{lL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{19}$$

This is <u>not satisfactory</u> because it causes degenerate charged leptons. We will modify this later.

Phenomenological implications

Why is U_{PMNS} different from V_{CKM}?

Ansätz for U_{LL}.
Toward $M_l M_l^\dagger$

Lepton Flavor Violating (LFV) processes:

- $\mu \to e\gamma$
- μ–e conversion

Why is the U_{PMNS} different from the V_{CKM}?

$$U_{\nu_L} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} ; \; U_{lL} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The PMNS Matrix

$$U_{PMNS} = U_{\nu_L}^{\dagger} \; U_{lL} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{20}$$

which mainly comes from neutrino mixing matrix.

Why is the U_{PMNS} different from the V_{CKM}?

- It has known that $V_{CKM} = U^{\dagger}_{U,L} U_{D,L}$ comes totally from couplings between quarks and Higgs doublet.
- We are showing that the $U_{PMNS} = U^{\dagger}_{\nu_L} U_{lL}$ comes from
  - $U_{lL} \Longleftarrow$ couplings between leptons and Higgs doublet
  - $U_{\nu_L} \Longleftarrow$ couplings between leptons and Higgs singlets

Why is the U_{PMNS} different from the V_{CKM}?

In a nutshell

There are two different sources of PMNS matrix whereas the CKM matrix comes totally from one source.

One expects a natural difference between V_{CKM} and U_{PMNS}.

Ansätz for U_{IL}

Ansätz for U_{lL}

A_4 requires degenerate charged leptons e, μ, $\tau \Rightarrow\ U_{lL} = \mathbb{I}$.

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Ansätz for U_{lL}

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We can use Wolfenstein-like parametrization to construct U_{lL}.

$$U_{lL} \rightarrow U_{lL} = \begin{pmatrix} 1-\frac{\lambda_l^2}{2} & \lambda_l & A_l\lambda_l^3(\rho_l - i\eta_l) \\ -\lambda_l & 1-\frac{\lambda_l^2}{2} & A_l\lambda_l^2 \\ A_l\lambda_l^3(1-\rho_l-i\eta_l) & -A_l\lambda_l^2 & 1 \end{pmatrix} \tag{21}$$

where $A_l,\ \rho_l,\ \eta_l$ are real parameters of O(1).

Ansätz for U_{lL}

$$U_{PMNS} = U_{\nu_L}^{\dagger} U_{lL} =$$

$$\frac{1}{\sqrt{3}}\begin{pmatrix} A_l\lambda_l^3(1-\rho_l-i\eta_l)-\frac{\lambda_l^2}{2}-\lambda_l+1 & -\left(A_l+\frac{1}{2}\right)\lambda_l^2+\lambda_l+1 & A_l\lambda_l^3(\rho_l-i\eta_l)+A_l\lambda_l^2+1 \\ \omega^2A_l\lambda_l^3(1-\rho_l-i\eta_l)-\frac{\lambda_l^2}{2}-\omega\lambda_l+1 & -\left(\omega^2A_l+\frac{\omega}{2}\right)\lambda_l^2+\lambda_l+\omega & A_l\lambda_l^3(\rho_l-i\eta_l)+\omega A_l\lambda_l^2+\omega^2 \\ \omega A_l\lambda_l^3(1-\rho_l-i\eta_l)-\frac{\lambda_l^2}{2}-\omega^2\lambda_l+1 & -\left(\omega A_l+\frac{\omega^2}{2}\right)\lambda_l^2+\lambda_l+\omega^2 & A_l\lambda_l^3(\rho_l+i\eta_l)+\omega^2A_l\lambda_l^2+\omega \end{pmatrix}$$

Combine with the experimental data, we are able to constrain parameters $A_l,\ \lambda_l,\ \rho_l,\ \eta_l$.

Toward $\mathcal{M}_l \mathcal{M}_l{}^\dagger$

Diagonalizing mass matrices $\mathcal{M}_l$ and $\mathcal{M}_l{}^\dagger$ as follows.

$$U_{lL}^\dagger \mathcal{M}_l U_{lR} \quad ; \quad U_{lR}^\dagger \mathcal{M}_l^\dagger U_{lL}$$

Therefore,

$$U_{lL}^\dagger \mathcal{M}_l \mathcal{M}_l{}^\dagger U_{lL} = \begin{pmatrix} m_e{}^2 & 0 & 0 \\ 0 & m_\mu{}^2 & 0 \\ 0 & 0 & m_\tau{}^2 \end{pmatrix}$$

$$\mathcal{M}_l \mathcal{M}_l{}^\dagger = U_{lL} \cdot \begin{pmatrix} m_e{}^2 & 0 & 0 \\ 0 & m_\mu{}^2 & 0 \\ 0 & 0 & m_\tau{}^2 \end{pmatrix} \cdot \; U_{lL}^\dagger$$

Toward $\mathcal{M}_l \mathcal{M}_l^\dagger$

$\Rightarrow$ Up to O(λ_l^2)

$$\begin{pmatrix} (1-\lambda_l^2)\ {m_e}^2 + \lambda_l\ {m_\mu}^2 & \lambda_l({m_\mu}^2 - {m_e}^2) & 0 \\ \lambda_l({m_\mu}^2 - {m_e}^2) & (1-\lambda_l^2)\ {m_\mu}^2 + \lambda_l\ {m_e}^2 & A\lambda_l^2({m_\tau}^2 - {m_\mu}^2) \\ 0 & A_l\lambda_l^2({m_\tau}^2 - {m_\mu}^2) & {m_\tau}^2 \end{pmatrix} \tag{22}$$

A_l, λ_l are extracted from U_{PMNS} and experimental values m_e, m_μ, m_τ.

Wrap up

- The differences between CKM and PMNS matrices come from the fact that U_{PMNS} is constructed by couplings with Higgs singlets and mainly comes from neutrinos.

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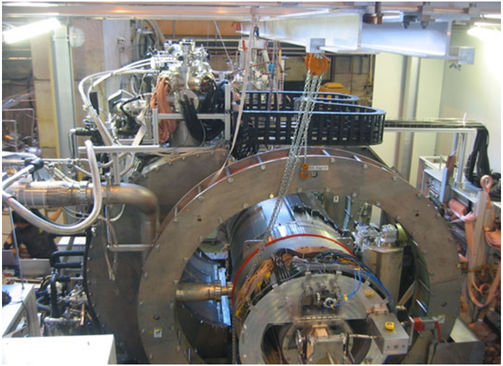
- The differences between CKM and PMNS matrices come from the fact that U_{PMNS} is constructed by couplings with Higgs singlets and mainly comes from neutrinos.
- The simplicity of our approach as compared with previous works is due to the source of the neutrino Dirac masses which comes from the Higgs singlets as opposed to Higgs doublets.

Wrap up

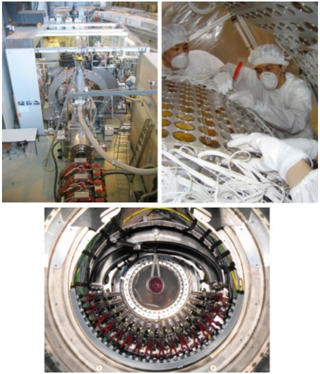
- The differences between CKM and PMNS matrices come from the fact that U_{PMNS} is constructed by couplings with Higgs singlets and mainly comes from neutrinos.
- The simplicity of our approach as compared with previous works is due to the source of the neutrino Dirac masses which comes from the Higgs singlets as opposed to Higgs doublets.
- By slightly breaking A_4 symmetry, we avoided the case of degenerate charged-lepton mass and were able to extract $\mathcal{M}_l \mathcal{M}_l^\dagger$ for the charged-lepton sector (as well as the quark sector).

$$\mu \to e\gamma$$

MEG Experiment



MEG apparatus by Prof. Saoshi Mihara



© MEG collaboration

$$B(\mu^+ \rightarrow e^+ \gamma) < 5.7 \times 10^{-13}$$

Projected Sensitivity = 4.0×10^{-14}

$l_i \to l_j\gamma$

It was argued in model of neutrino masses [11] that the appropriate set of singlet scalars is composed of an A_4-**singlet** ϕ_{0S} and an A_4-**triplet** $\{\phi_{iS}\}$ $(i = 1, 2, 3)$.

The total Yukawa interactions can be written as

$$\mathcal{L}_S = -\bar{l}_L\, U^{\dagger}_{\mathrm{PMNS}}\, \tilde{M}_\phi\, U^{M}_{\mathrm{PMNS}} l^{M}_R - \bar{l}_R\, U'^{\dagger}_{\mathrm{PMNS}}\, \tilde{M}'_\phi\, U'^{M}_{\mathrm{PMNS}} l^{M}_L + \mathrm{H.c.} \quad (23)$$

where

- $\tilde{M}_\phi = U^{\dagger}_{\nu} M_\phi U_\nu$, $\tilde{M}'_\phi = U^{\dagger}_{\nu} M'_\phi U_\nu$ and M'_ϕ is the same as M_ϕ.
- $U_{\mathrm{PMNS}} = U^{\dagger}_{\nu} U^{l}_{L}\,,\; U^{M}_{\mathrm{PMNS}} = U^{\dagger}_{\nu} U^{l^M}_{R}$
- $U'_{\mathrm{PMNS}} = U^{\dagger}_{\nu} U^{l}_{R}\,,\; U'^{M}_{\mathrm{PMNS}} = U^{\dagger}_{\nu} U^{l^M}_{L}$

[11] P.Q. Hung, T. Le, JHEP 1509, 001 (2015)

$l_i \to l_j\gamma$

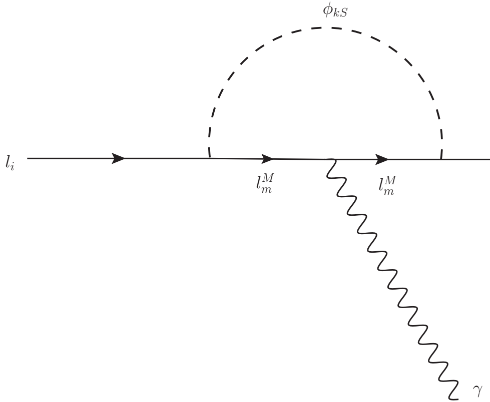


Figure : One-loop induced Feynman diagram for $l_i \to l_j\gamma$ in EW-scale ν_R model.

The relevant Yukawa couplings between the leptons, mirror leptons and the A_4 singlet and triplet scalars can be deduced by recasting the Lagrangian.

$$\mathcal{L}_S = -\sum_{k=0}^{3}\sum_{i,m=1}^{3}\left(\bar{l}_{Li}\,\mathcal{U}_{im}^{L\,k}\, l_{Rm}^{M} + \bar{l}_{Ri}\,\mathcal{U}_{im}^{R\,k}\, l_{Lm}^{M}\right)\phi_{kS} + \mathrm{H.c.} \quad (24)$$

where

$$\mathcal{U}_{im}^{L\,k} \equiv \left(U_{\mathrm{PMNS}}^{\dagger}\cdot M^{k}\cdot U_{\mathrm{PMNS}}^{l^M}\right)_{im}, \quad (25)$$

$$= \sum_{j,n=1}^{3}\left(U_{\mathrm{PMNS}}^{\dagger}\right)_{ij} M_{jn}^{k}\left(U_{\mathrm{PMNS}}^{M}\right)_{nm}, \quad (26)$$

and

$$\mathcal{U}_{im}^{R\,k} \equiv \left(U_{\mathrm{PMNS}}^{\prime\,\dagger}\cdot M^{\prime\,k}\cdot U_{\mathrm{PMNS}}^{\prime\,l^M}\right)_{im}, \quad (27)$$

$$= \sum_{j,n=1}^{3}\left(U_{\mathrm{PMNS}}^{\prime\,\dagger}\right)_{ij} M_{jn}^{\prime\,k}\left(U_{\mathrm{PMNS}}^{\prime\,M}\right)_{nm}. \quad (28)$$

For the process $l_i^-(p) \to l_j^-(p') + \gamma(q)$

- The amplitude

$$\mathcal{M}\left(l_i^- \to l_j^- \gamma\right) = \epsilon_\mu^*(q)\bar{u}_j(p')\left\{i\sigma^{\mu\nu}q_\nu\left[C_L^{ij}P_L + C_R^{ij}P_R\right]\right\}u_i(p) \quad , \tag{29}$$

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- The partial width

$$\Gamma\left(l_i \to l_j\gamma\right) = \frac{1}{16\pi}m_{l_i}^3\left(1 - \frac{m_{l_j}^2}{m_{l_i}^2}\right)^3\left(|C_L^{ij}|^2 + |C_R^{ij}|^2\right) \quad . \tag{30}$$

Coefficients

$$C_L^{ij} = +\frac{e}{16\pi^2}\sum_{k=0}^{3}\sum_{m=1}^{3}\left\{\frac{1}{m^2_{l_m^M}}\left[m_i\mathcal{U}_{jm}^{R\,k}\left(\mathcal{U}_{im}^{R\,k}\right)^* + m_j\mathcal{U}_{jm}^{L\,k}\left(\mathcal{U}_{im}^{L\,k}\right)^*\right]\mathcal{I}\left(\frac{m^2_{\phi_{kS}}}{m^2_{l_m^M}}\right) + \frac{1}{m_{l_m^M}}\mathcal{U}_{jm}^{R\,k}\left(\mathcal{U}_{im}^{L\,k}\right)^*\mathcal{J}\left(\frac{m^2_{\phi_{kS}}}{m^2_{l_m^M}}\right)\right\}\ , \tag{31}$$

$$C_R^{ij} = +\frac{e}{16\pi^2}\sum_{k=0}^{3}\sum_{m=1}^{3}\left\{\frac{1}{m^2_{l_m^M}}\left[m_i\mathcal{U}_{jm}^{L\,k}\left(\mathcal{U}_{im}^{L\,k}\right)^* + m_j\mathcal{U}_{jm}^{R\,k}\left(\mathcal{U}_{im}^{R\,k}\right)^*\right]\mathcal{I}\left(\frac{m^2_{\phi_{kS}}}{m^2_{l_m^M}}\right) + \frac{1}{m_{l_m^M}}\mathcal{U}_{jm}^{L\,k}\left(\mathcal{U}_{im}^{R\,k}\right)^*\mathcal{J}\left(\frac{m^2_{\phi_{kS}}}{m^2_{l_m^M}}\right)\right\}\ . \tag{32}$$

Considering $m_{l_m^M} \gg m_{i,j}$ and setting $m_{i,j} \to 0$.

$$
\begin{aligned}
\mathcal{I}(r) &= \frac{1}{12(1-r)^4}\left[-6r^2\log r + r(2r^2+3r-6)+1\right] \quad , \qquad (33)\\
\mathcal{J}(r) &= \frac{1}{2(1-r)^3}\left[-2r^2\log r + r(3r-4)+1\right] \quad . \qquad (34)
\end{aligned}
$$

Numerical Analysis

The branching ratio $\mathrm{B}(\mu \to e\gamma)$ is given by

$$\mathrm{B}(\mu \to e\gamma) = \tau_\mu \cdot \Gamma\left(l_i \to l_j\gamma\right) \tag{35}$$

where τ_μ is the lifetime of the muon

$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6}\ \mathrm{s} \ . \tag{36}$$

Numerical Analysis

- For the masses of the singlet scalars ϕ_{kS}
  $m_{\phi_{0S}} : m_{\phi_{1S}} : m_{\phi_{2S}} : m_{\phi_{3S}} = M_S : 2M_S : 3M_S : 4M_S$ with $M_S = 10$ MeV.
- For the masses of the mirror lepton l_m^M
  $m_{l_m^M} = M_{\rm mirror} + \delta_m$ with $\delta_1 = 0$, $\delta_2 = 10$ GeV, $\delta_3 = 20$ GeV and $100 \; GeV \leq M_{\rm mirror} \leq 800 GeV$
- Scenario 1 $U^M_{\rm PMNS} = U'_{\rm PMNS} = U'^M_{\rm PMNS} = U^\dagger_{CW}$
- Scenario 2 $U^M_{\rm PMNS} = U'_{\rm PMNS} = U'^M_{\rm PMNS} = U_{\rm PMNS}$

Some examples

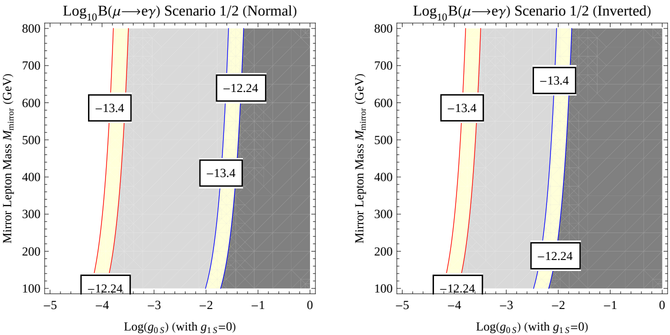

Figure : Contour plots of $\mathrm{Log}_{10}B(\mu \to e\gamma)$ on the $(\mathrm{Log}_{10}(g_{0S}), M_{\mathrm{mirror}})$ plane for normal (left panel) and inverted (right panel) hierarchy in scenarios 1 (red curves) and 2 (blue curves) with $g_{0S} = g'_{0S}$ and $g_{1S} = g'_{1S} = 0$.

Numerical Analysis

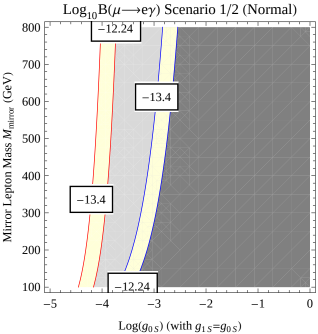

Figure : Same as previous figure with $g_{0S} = g'_{0S} = g_{1S} = g'_{1S}$ instead.

Wrap up

In our analysis, we are showing that constraints from $\mu \to e\gamma$ imply Yukawa couplings $< 10^{-3}$.

- The small couplings now brings the singlet VEV up to $\mathcal{O}(100\text{ MeV})$ or even $\mathcal{O}(1\text{ GeV})$. There does not appear to be much of a hierarchy problem in EW-ν_R model.

Wrap up

In our analysis, we are showing that constraints from $\mu \rightarrow e\gamma$ imply Yukawa couplings $< 10^{-3}$.

- The small couplings now brings the singlet VEV up to $\mathcal{O}(100 \text{ MeV})$ or even $\mathcal{O}(1 \text{ GeV})$. There does not appear to be much of a hierarchy problem in EW-ν_R model.

- Due to small couplings, searching for mirror particles of this model at the LHC would be quite interesting since they might decay outside the beam pipe and inside silicon vertex detectors.

  Search for mirror quarks at the LHC
  S. Chakdar, K. Ghosh, V. Hoang, P. Q. Hung and S. Nandi,
  Phys. Rev. D **93**, No. 3, 035007 (2016),
  DOI:10.1103/PhysRevD.93.035007, [arXiv:1508.07318 [hep-ph]].

μ-e conversion

Photo by Reidar Hahn, Fermilab

SINDRUM II @ PSI

SINDRUM II: $B(\mu^- + Au \rightarrow e^- + Au) < 7 \times 10^{-13}$

SINDRUM II: $B(\mu^- + Ti \rightarrow e^- + Ti) < 6.1 \times 10^{-13}$

Projected Sensitivity for
$B(\mu^- + Al \rightarrow e^- + Al)$

Mu2e: 6×10^{-17}

COMET:3×10^{-17}

Effective Lagrangian for μ-e Conversion

$$\begin{aligned}\mathcal{L}_{\rm eff} = & - \frac{1}{\Lambda^2}\Bigg[\left(C_{DR} m_\mu \bar{e}\sigma^{\alpha\beta}P_L\mu + C_{DL} m_\mu \bar{e}\sigma^{\alpha\beta}P_R\mu\right)F_{\alpha\beta} \\ & + \sum_{q=u,d,s}\left(C_{VR}^{(q)}\bar{e}\gamma^\alpha P_R\mu + C_{VL}^{(q)}\bar{e}\gamma^\alpha P_L\mu\right)\bar{q}\gamma_\alpha q \\ & + \sum_{q=u,d,s} m_\mu m_q G_F\left(C_{SR}^{(q)}\bar{e}P_R\mu + C_{SL}^{(q)}\bar{e}P_L\mu\right)\bar{q}q \\ & + m_\mu\left(C_{GQR}G_F\bar{e}P_L\mu + C_{GQL}G_F\bar{e}P_R\mu\right)\frac{\beta_L}{2g_s^3}G^{a\alpha\beta}G^a_{\alpha\beta} + \mathrm{H.c.}\Bigg].\end{aligned} \tag{37}$$

where $C_{D(L,R)}$, $C^{(q)}_{V(L,R)}$, $C^{(q)}_{S(L,R)}$ and $C_{GQ(L,R)}$ are dimensionless coupling constants depending on specific LFV model.

μ-e Conversion

Conversion rate (general formula) [12]

$$\begin{aligned}\Gamma_{\rm conv} \;=\; \frac{m_\mu^5}{4\Lambda^4}\Bigg(&\bigg|C_{DR}D + 4\tilde{C}^{(p)}_{VR}V^{(p)} + 4\tilde{C}^{(n)}_{VR}V^{(n)} + \\ &+4G_F m_\mu\left(m_p\tilde{C}^{(p)}_{SR}S^{(p)} + m_n\tilde{C}^{(n)}_{SR}S^{(n)}\right)\bigg|^2 \\ &+\bigg|C_{DL}D + 4\tilde{C}^{(p)}_{VL}V^{(p)} + 4\tilde{C}^{(n)}_{VL}V^{(n)} + \\ &+4G_F m_\mu\left(m_p\tilde{C}^{(p)}_{SL}S^{(p)} + m_n\tilde{C}^{(n)}_{SL}S^{(n)}\right)\bigg|^2\Bigg) \;. \qquad (38)\end{aligned}$$

where D, V, S are overlap integrals of the relativistic wave functions of μ and e in the electric field of nucleus.

[12] R. Kitano, M. Koike, Y. Okada (2007)

μ-e Conversion

Contributions to the conversion rate

- Photonic contributions
  $\mu^-(p) \to e^-(p')\gamma^*(q)$ with an off-shell photon.
- Four-fermion coupling constants from
  - γ exchange
  - Z exchange
  - box diagrams
  - scalar Higgs exchange

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  - box diagrams
  - scalar Higgs exchange

μ-e Conversion

The formula for the conversion rate (from γ contributions ONLY)

$$\Gamma_{\rm conv} \simeq \frac{m_\mu^5}{4\Lambda^4}\left(\left|C_{DR}D + 4\tilde{C}_{VR}^{(p)}V^{(p)} + 4\tilde{C}_{VR}^{(n)}V^{(n)}\right|^2 + \left|C_{DL}D + 4\tilde{C}_{VL}^{(p)}V^{(p)} + 4\tilde{C}_{VL}^{(n)}V^{(n)}\right|^2\right) . \qquad (39)$$

Question

Is there any relation between μ-e conversion and $\mu \to e\gamma$?

μ-e Conversion

The formula for the conversion rate (from γ contributions ONLY)

$$\Gamma_{\rm conv} \simeq \frac{m_\mu^5}{4\Lambda^4}\left(\left|C_{DR}D + 4\tilde{C}_{VR}^{(p)}V^{(p)} + 4\tilde{C}_{VR}^{(n)}V^{(n)}\right|^2 + \left|C_{DL}D + 4\tilde{C}_{VL}^{(p)}V^{(p)} + 4\tilde{C}_{VL}^{(n)}V^{(n)}\right|^2\right). \tag{39}$$

Question

Is there any relation between μ-e conversion and $\mu \to e\gamma$?

The relation is

$$\Gamma^{\gamma *}_{conv}(q^2 \to 0) \approx \pi D^2 \Gamma^{\gamma} \tag{40}$$

So in terms of the branching ratio, we have

$$B_{\mu N \to eN} = \frac{\Gamma^{\gamma *}_{conv}}{\Gamma_{capt}} = \pi D^2 \frac{\Gamma_{\mu}}{\Gamma_{capt}} B_{\mu \to e\gamma} \tag{41}$$

μ-e Conversion

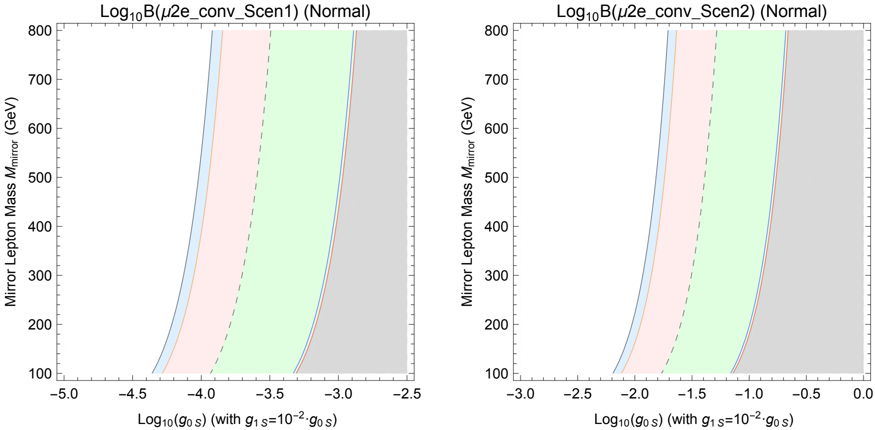

Figure : Contour plots of $\mathrm{Log}_{10}B(\mu \to$ *e conversion*$)$ on the $(\mathrm{Log}_{10}(g_{0S}), M_{\mathrm{mirror}})$ plane for normal hierarchy in scenario 1 (left panel) and scenario 2 (right panel) with $g_{0S} = g'_{0S}$ and $g_{1S} = g'_{1S} = 10^{-2} g_{0S}$.

μ-e Conversion

Let's zoom in

μ-e Conversion

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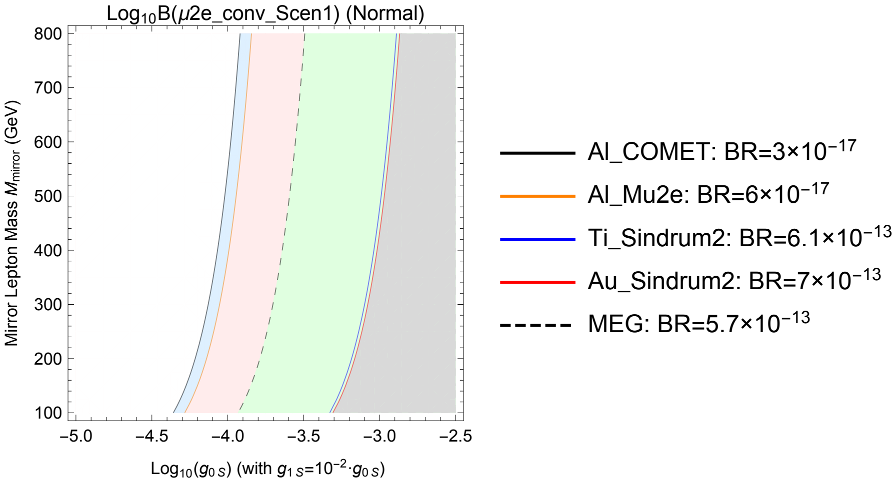

μ-e Conversion

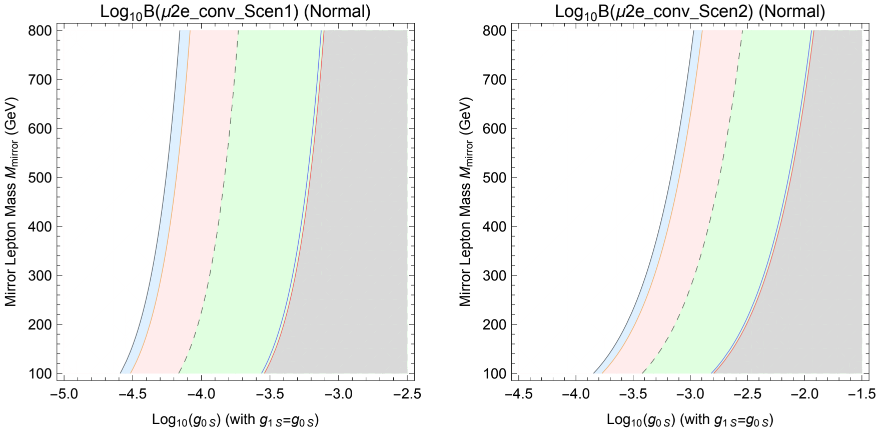

Figure : Contour plots of $\mathrm{Log}_{10}B(\mu \to e\ \textit{conversion})$ on the $(\mathrm{Log}_{10}(g_{0S}), M_{\rm mirror})$ plane for normal hierarchy in scenario 1 (left panel) and scenario 2 (right panel) with $g_{0S} = g'_{0S} = g_{1S} = g'_{1S}$.

Wrap up

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- The current limit from $\mu \to e\gamma$ excludes almost half of the searched region for the branching ratio of μ-e conversion. Therefore, it may help to narrow down future searches for μ-e conversion.
- **μ-e Conversion in Mirror Fermion Model with Electroweak Scale Right-handed Neutrinos**,
  P.Q. Hung, T. Le, V.Q. Tran and T.C. Yuan (paper in preparation).

On-going project

We are working on a **Quarks Project** using the similar *ansätz* that we have made for leptons.

Stay tuned. We have more things coming up.

Conclusion

- We present a model of neutrino masses in the framework of the Electroweak scale Right-handed neutrinos (EW-ν_R) model, which is constructed with a horizontal A_4 symmetry. Such a model has several interesting phenomenological implications.
- We not only obtain the experimentally desired form of the PMNS matrix but also provide an explanation of why U_{PMNS} is very different from V_{CKM}. By making a simple *ansätz* we extract $\mathcal{M}_l \mathcal{M}_l{}^\dagger$ for the charged lepton sector. A similar *ansätz* is proposed for the quark sector.
- The one-loop induced lepton flavor violating radiative decays $l_i \to l_j \gamma$ and μ-e conversion in an extended mirror model might be related to each other under a good approximation that we have established.
- Implications concerning the possible detection of mirror leptons at the LHC and the ILC as well as future searches for μ-e conversion at Fermilab and J-PARC COMET are also discussed.


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Appendix

1. Characters of A_4 reperesentations

A_4	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_3	2	1	1	1	-1
C_4	3	1	ω	ω^2	0
$C_{4'}$	3	1	ω^2	ω	0

where $\omega = e^{i2\pi/3}$ which is the cube root of unity.

Appendix

$$(1)\quad 0.779 < \frac{1}{\sqrt{3}}|A\lambda^3(1-\rho-i\eta) - \frac{\lambda^2}{2} - \lambda + 1| < 0.848$$

$$(2)\quad 0.510 < \frac{1}{\sqrt{3}}|-\left(A+\frac{1}{2}\right)\lambda^2 + \lambda + 1| < 0.604$$

$$(3)\quad 0.122 < \frac{1}{\sqrt{3}}|A\lambda^3(\rho-i\eta) + A\lambda^2 + 1| < 0.190$$

$$(4)\quad 0.183 < \frac{1}{\sqrt{3}}|\omega^2 A\lambda^3(1-\rho-i\eta) - \frac{\lambda^2}{2} - \omega\lambda + 1| < 0.568$$

$$(5)\quad 0.385 < \frac{1}{\sqrt{3}}|-\left(\omega^2 A+\frac{\omega}{2}\right)\lambda^2 + \lambda + \omega| < 0.728$$

$$(6)\quad 0.613 < \frac{1}{\sqrt{3}}|A\lambda^3(\rho-i\eta) + \omega A\lambda^2 + \omega^2| < 0.794$$

$$(7)\quad 0.200 < \frac{1}{\sqrt{3}}|\omega A\lambda^3(1-\rho-i\eta) - \frac{\lambda^2}{2} - \omega^2\lambda + 1| < 0.576$$

$$(8)\quad 0.408 < \frac{1}{\sqrt{3}}|-\left(\omega A+\frac{\omega^2}{2}\right)\lambda^2 + \lambda + \omega^2| < 0.742$$

$$(9)\quad 0.589 < \frac{1}{\sqrt{3}}|A\lambda^3(\rho-i\eta) + \omega^2 A\lambda^2 + \omega| < 0.775$$

$$-4.8517 < A < -4.4580, \quad -0.2404 < \lambda < -0.1882,$$
$$-5.6339 < \rho < -5.5712, \quad -4.7160 < \eta < 4.8912$$

Appendix

3. Sample numerical results
Taking upper limit values of $A = -4.4580$, $\lambda = -0.1882$, $\rho = -5.5712$ and $\eta = 4.8912$

$$U_l = \begin{pmatrix} 0.9823 & -0.1882 & -0.1656 - 0.1454i \\ 0.1882 & 0.9823 & -0.1579 \\ 0.1953 - 0.1454i & 0.1579 & 1 \end{pmatrix}$$

$$U_l {U_l}^\dagger = \begin{pmatrix} 1.0489 & 0.0261 + 0.0230i & -0.0035 - 0.0026i \\ 0.0261 - 0.0230i & 1.0253 & 0.0340 + 0.0274i \\ -0.0035 + 0.0026i & 0.0340 - 0.0274i & 1.0842 \end{pmatrix}$$

$$\simeq \mathbb{I}$$

Appendix

Using the above numerical U_l and putting in the values of $m_e = 0.51 \times 10^{-3}$ GeV, $m_\mu = 0.1057$ GeV and $m_\tau = 1.7768$ GeV we get

$$\mathcal{M}_l \mathcal{M}_l{}^\dagger \simeq \begin{pmatrix} 0.1537 & 0.0805 + 0.0725i & -0.5231 - 0.4590i \\ 0.0805 - 0.0725i & 0.0895 & -0.4968 \\ -0.5231 + 0.4590i & -0.4968 & 3.1573 \end{pmatrix}$$

Appendix

4. Possible signature of EW ν_R model

The fact

1. ν_R interacts with the W and Z (part of a doublet)
2. Both ν_R and e_R^M interact with ν_L and e_L through the singlet scalar field ϕ_S

Since $m_{\phi_S} \sim O(10^5\ eV)$, it's possible

$$\begin{aligned} \nu_R &\rightarrow \nu_L + \phi_S \\ e_R^M &\rightarrow e_L + \phi_S \end{aligned}$$

If $m_{\nu_R} \lesssim m_{e_R^M}$:

$$\begin{aligned} e_M^R \rightarrow \quad & \nu_R + e_L + \bar{\nu}_L \\ & \nu_R \rightarrow \nu_L + \phi_S \end{aligned}$$

Possible signature of EW ν_R model

The heaviest ν_R could be pair produced

$$
\begin{aligned}
q + \bar{q} &\rightarrow Z \rightarrow \nu_R + \nu_R \\
\nu_R &\rightarrow e_R^M + W^*(W) \\
e_R^M &\rightarrow e_L + \phi_S
\end{aligned}
$$

at a 'displaced' vertex.
If ν_R is Majorana

$$e_R^{M,-} + W^+ + e_R^{M,-} + W^+ \rightarrow e_L + e_L + W^+ + W^+ + 2\phi_S$$

same-sign dilepton event which is distinctively different from the Dirac case!

Appendix

5. How to stop neutrinos?

Q: If one uses a wall of lead, how thick should it be to stop a beam of neutrinos?

A: Typical low energy (MeV) cross section $\sigma \approx 10^{-47}\ m^2$.
Mean free path for neutrinos going through e.g. lead:

- Number density of nucleons in Pb: $n = \dfrac{11400\ kg/m^3}{1.76 \times 10^{-27}\ kg}$
- Number of interaction per meter:
  $\sigma \times n = 10^{-47}\ m^2 \times \dfrac{11400\ kg/m^3}{1.76 \times 10^{-27}}\ kg$
- Mean free path: $\lambda = \dfrac{1}{\sigma \times n} \approx 1.5 \times 10^{17}\ m \approx$ 1.6 *light years*