

16 June 2014, Virginia



# Accessing the frequency resource of broadband bi-photons

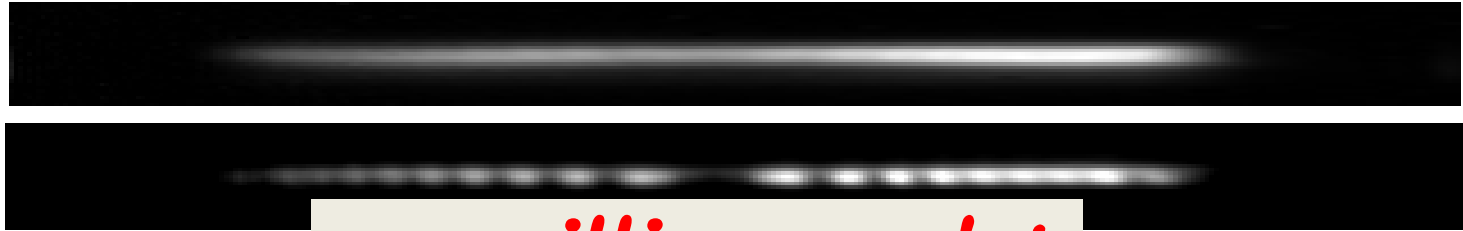
Avi Pe'er

Rafi Vered, Yaakov Shaked, Michael Rosenbluh

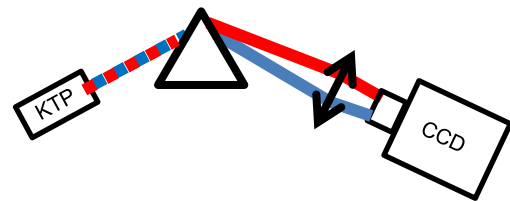
Physics Dept. and BINA center for nanotechnology, Bar Ilan University

\$\$ □ ISF, EU-IRG,  
Kahn Foundation

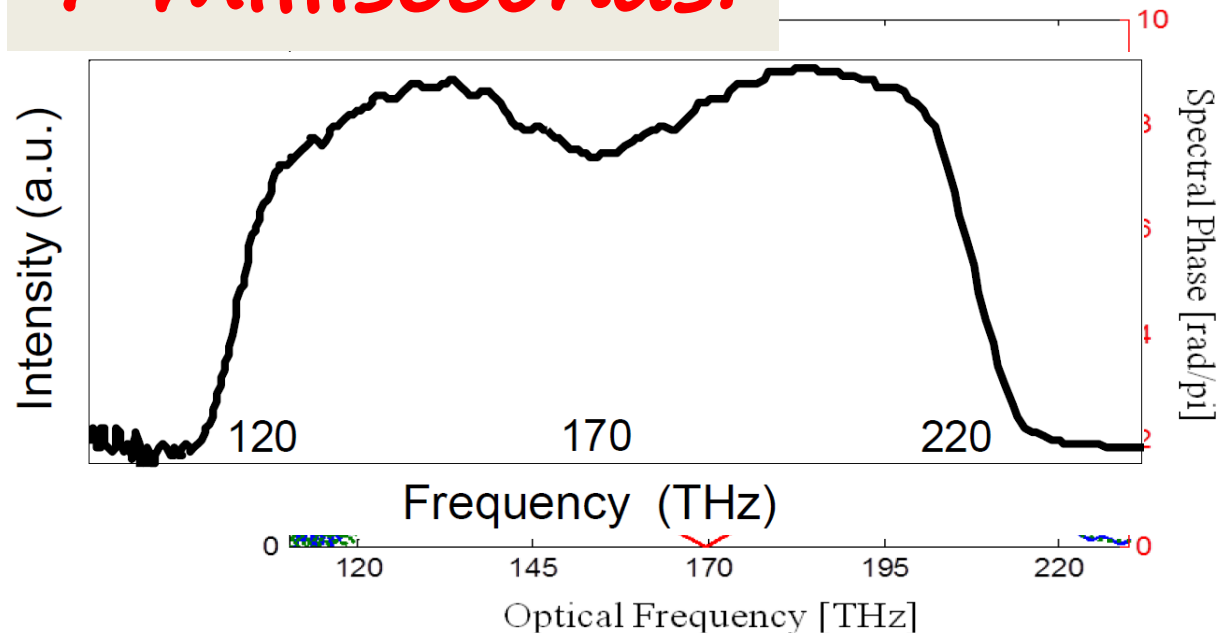
# Broadband Bi-photons Full Quantum Wave Function



7 milliseconds!

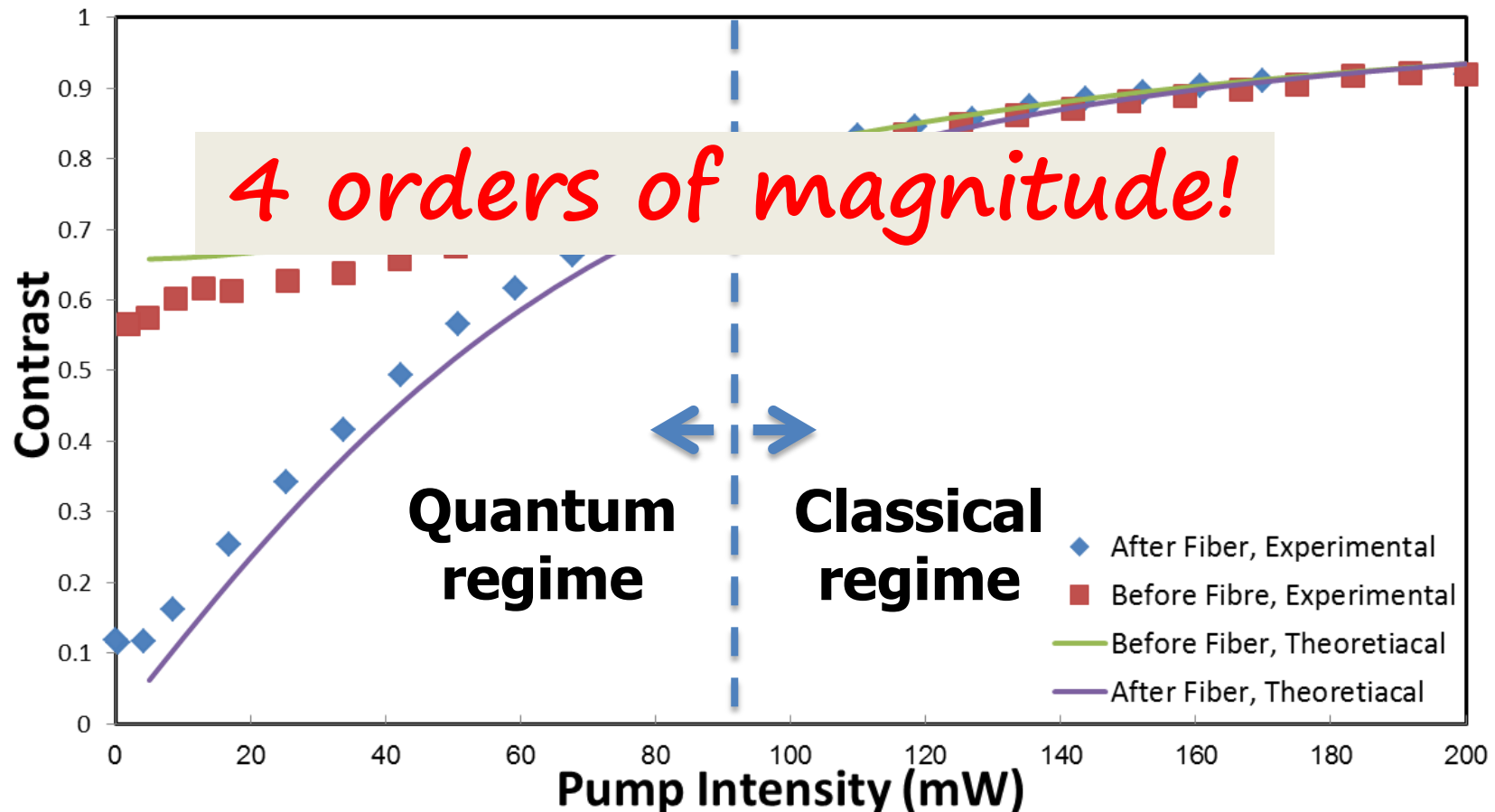


- Amplitude
- Phase
- Purity
- Sum frequency Correlation



# Classical to Quantum Transition

The contrast as a function of the attenuation.



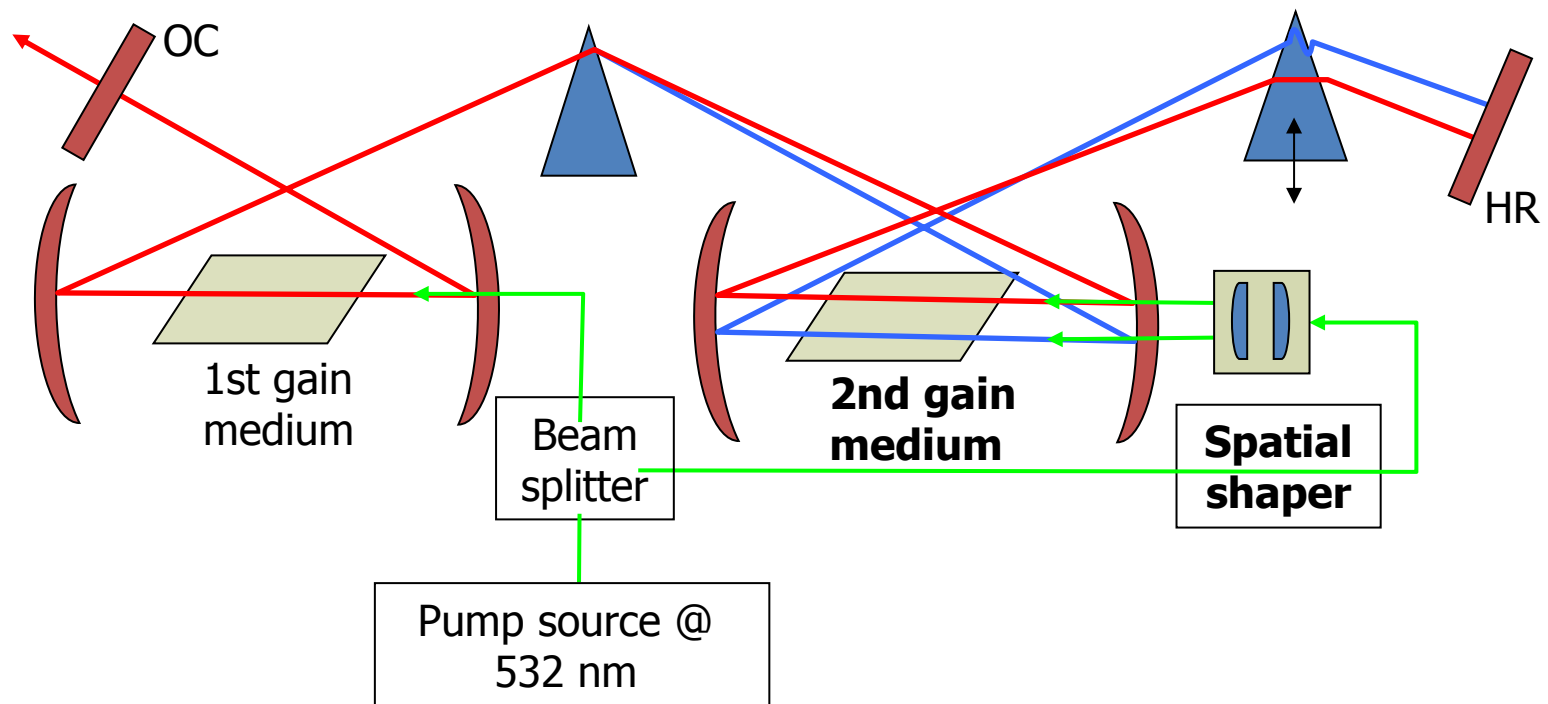
# Outline

- Time-energy entangled photons are great !
- Why no one uses them ? (How to measure ?)
- Efficient measurement with a quantum bi-photon interference
- Fringe contrast as a nonclassical witness
- The classical-to-quantum transition
- New effects - FWM with imaginary gain
- (if time permits) An efficient source of high-power broadband two-mode squeezing (OPO)
- Conclusions

# Group Overview

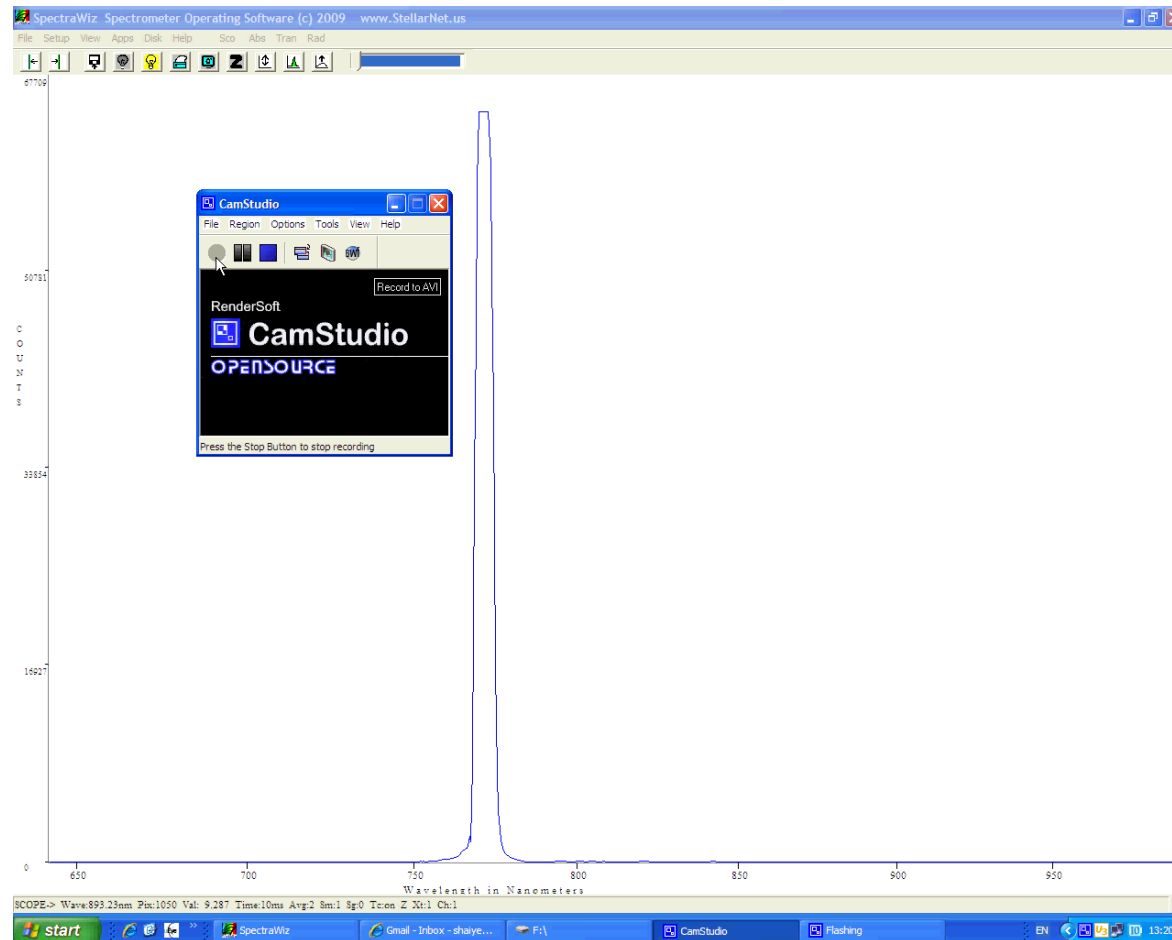
- Frequency comb sources
  - Control of mode-locking physics
  - New configurations of Kerr-lens mode locked lasers
- Precision measurements of ultrafast dynamics in molecules
- Ultra-broadband, time-energy correlated light – Sources and applications
  - Bi-photons (low power) and coherent squeezed light (high power)

# Controlling the comb spectrum by Gain shaping



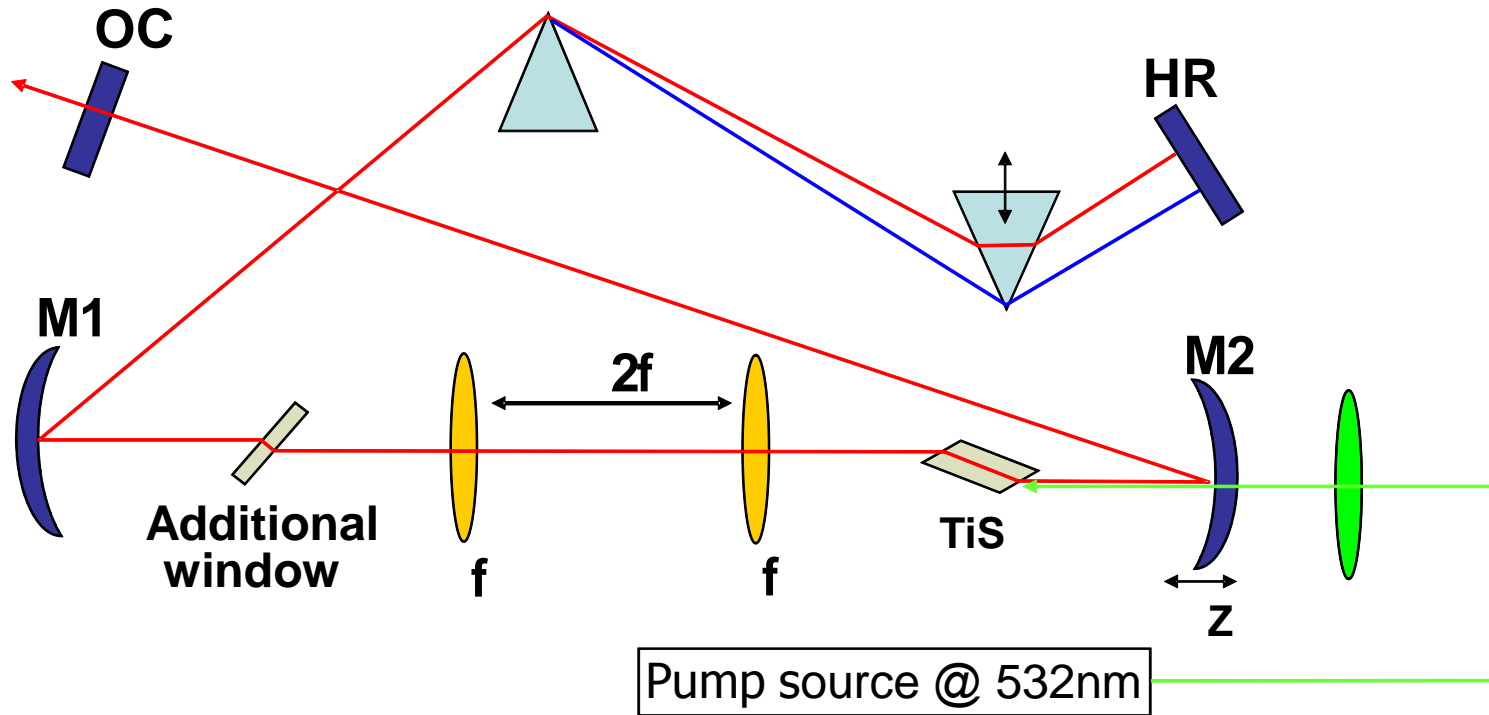
**Spatial** shape of pump = **Spectral** shape of gain

# Manipulating the Mode-locking spectrum in real time



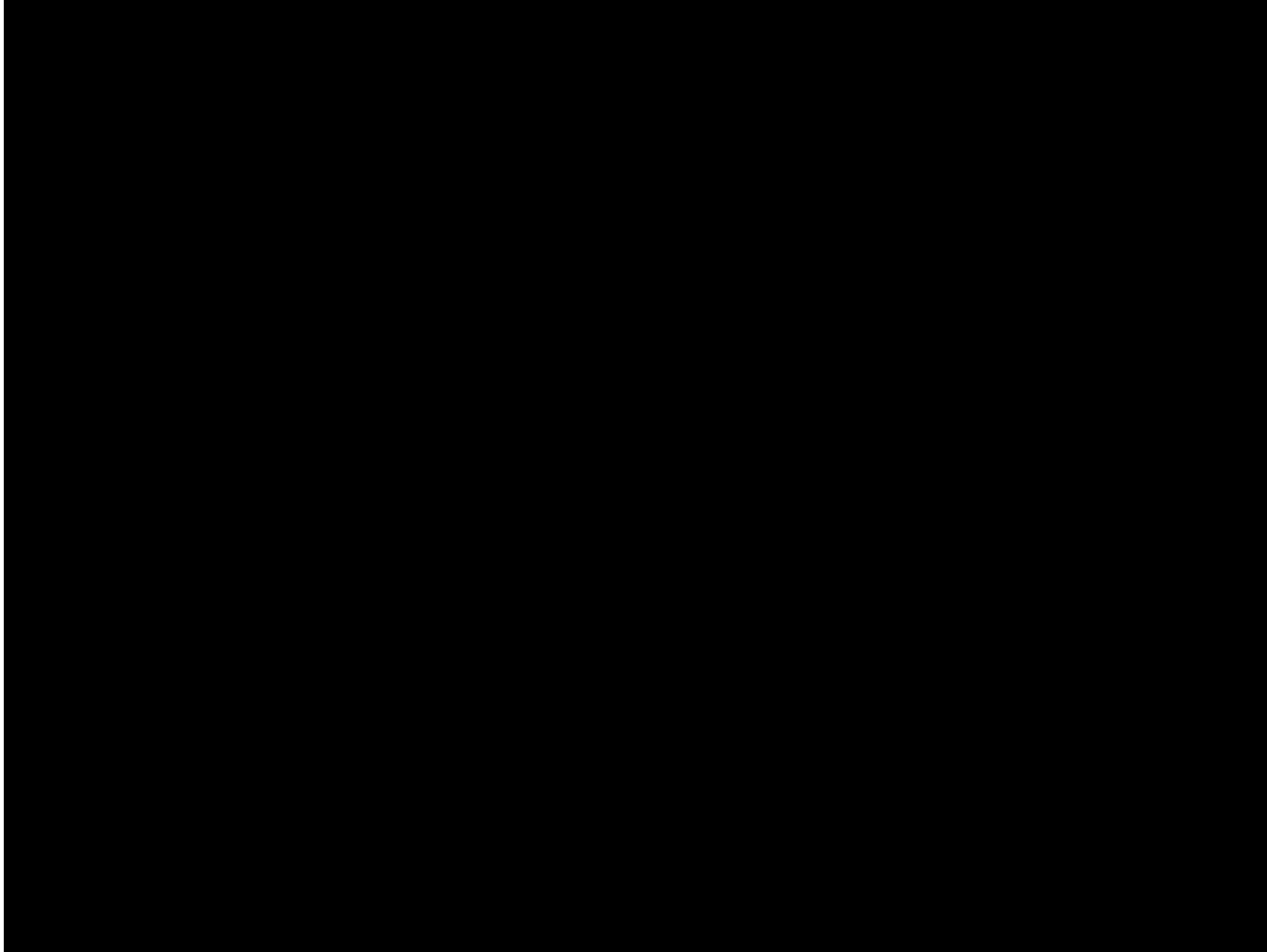
*Optics Express* **20**, 9991-9998 (2012),  
**"Intra-cavity gain shaping of mode-locked Ti:Sapphire laser oscillations"**

# Mode locking below the CW threshold





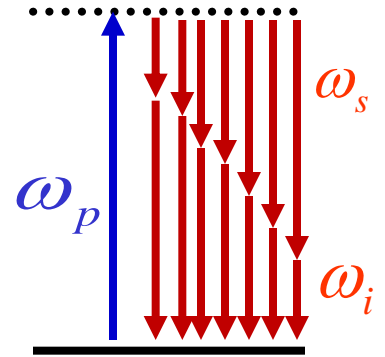
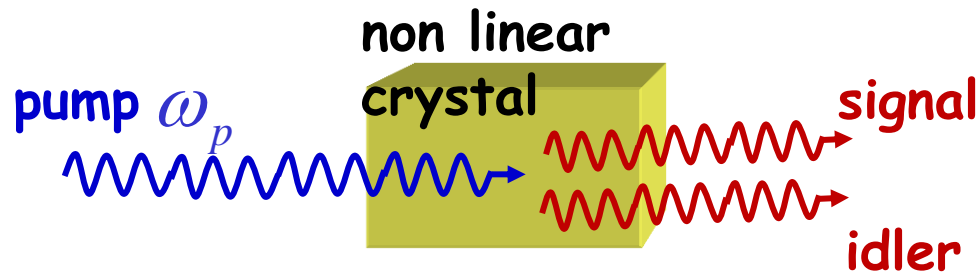
# Mode locking below the CW threshold



[Opt. Express \*\*21\*\*, 19040–19046 \(2013\),](#)

**"Mode locking with enhanced nonlinearity - a detailed study"**

# Time-Energy Entangled Photons



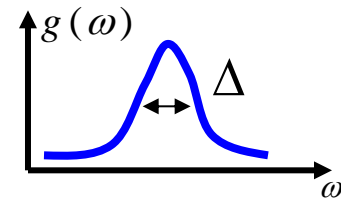
$$\omega_s + \omega_i = \omega_p$$

$$\omega_s - \omega_i = ?$$

**The two-photon state (monochromatic pump)**

$$|\psi\rangle = |0\rangle + \varepsilon \int d\omega g(\omega) |1_{\omega_0-\omega}, 1_{\omega_0+\omega}\rangle$$

**Entanglement**



# Time-Energy Correlation

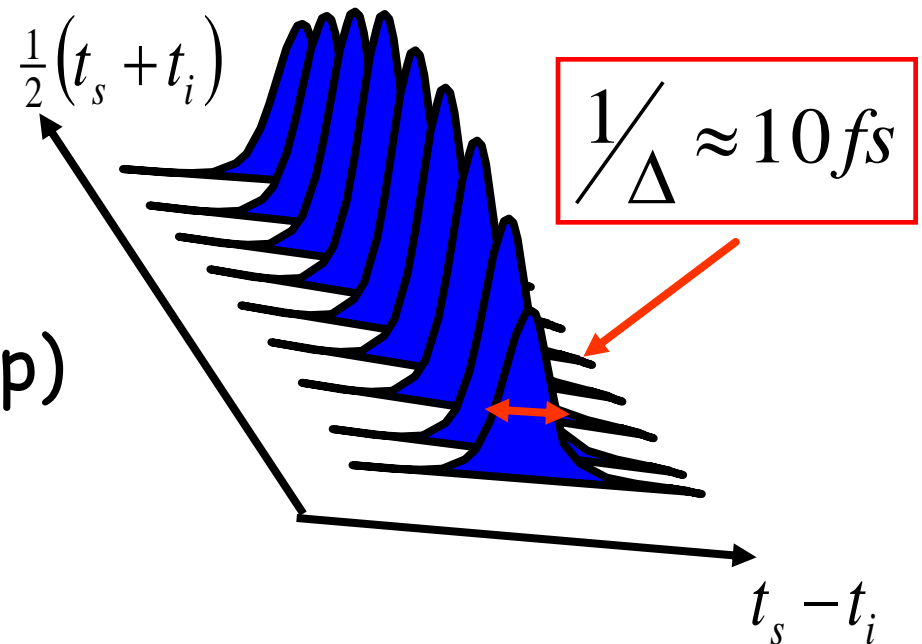
$$|\psi\rangle = (1 - \varepsilon)|0\rangle + \varepsilon \int d\omega g(\omega) |1_{\omega_{p/2}-\omega}, 1_{\omega_{p/2}+\omega}\rangle$$

uncertainty  
relation

$$\rightarrow t_s + t_i = ? \quad t_s - t_i \approx 1/\Delta$$

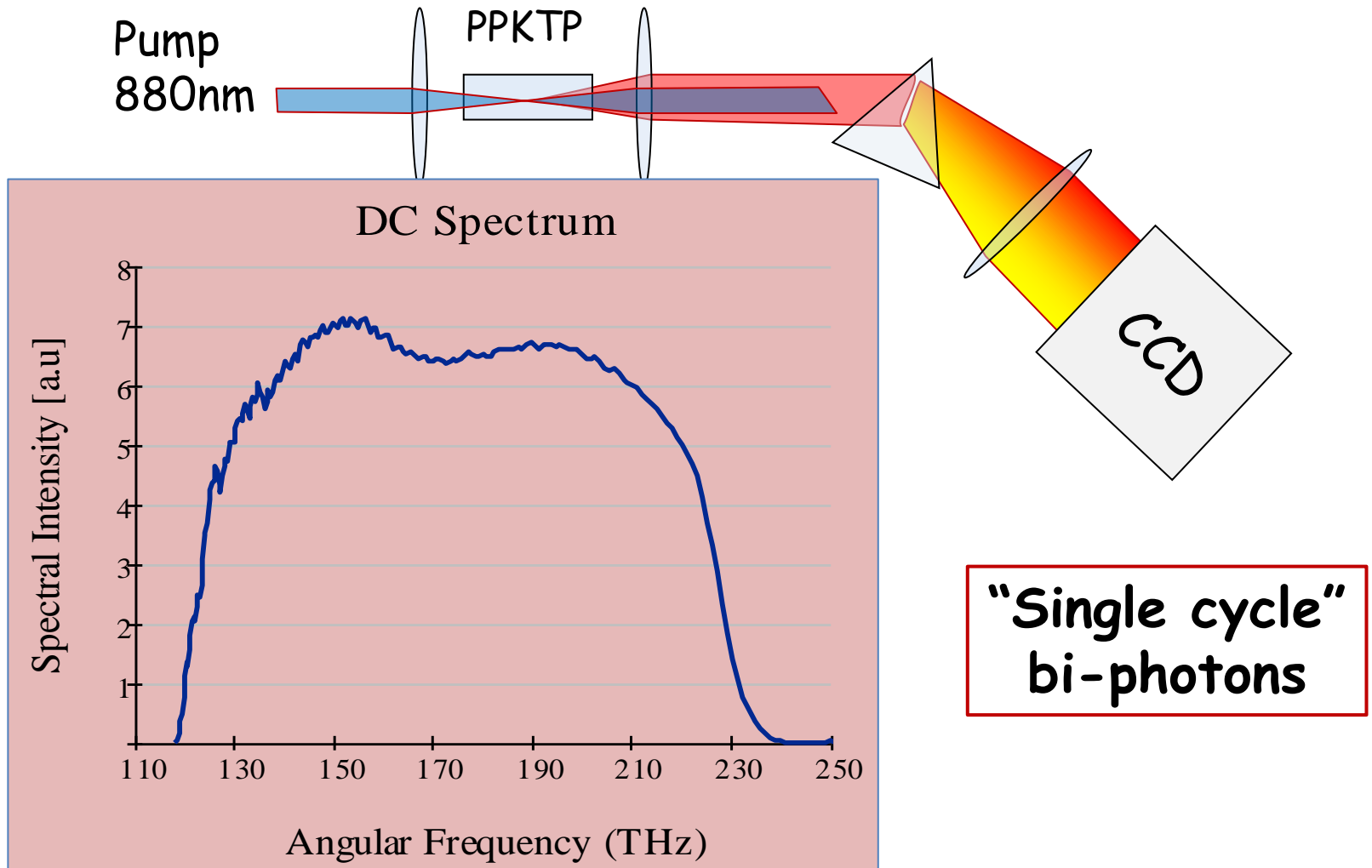
the two-photon  
wave function  
(monochromatic pump)

$$\Psi(t_s, t_i) \propto G(t_s - t_i)$$



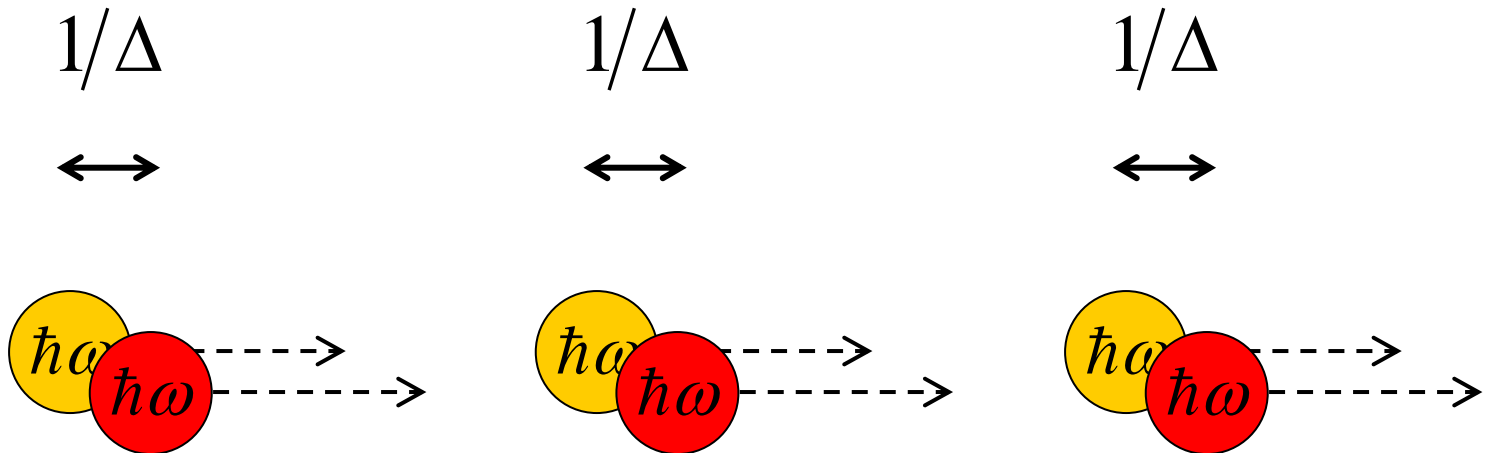
# Ultra-Broadband bi-Photons

**Zero Dispersion !**



# Why ultra-broad photon pairs ?

Because there are so many of them !



$$\Phi_{\max} \approx \Delta \approx 10^{14} \text{ pairs} / s \approx 12 \mu W$$

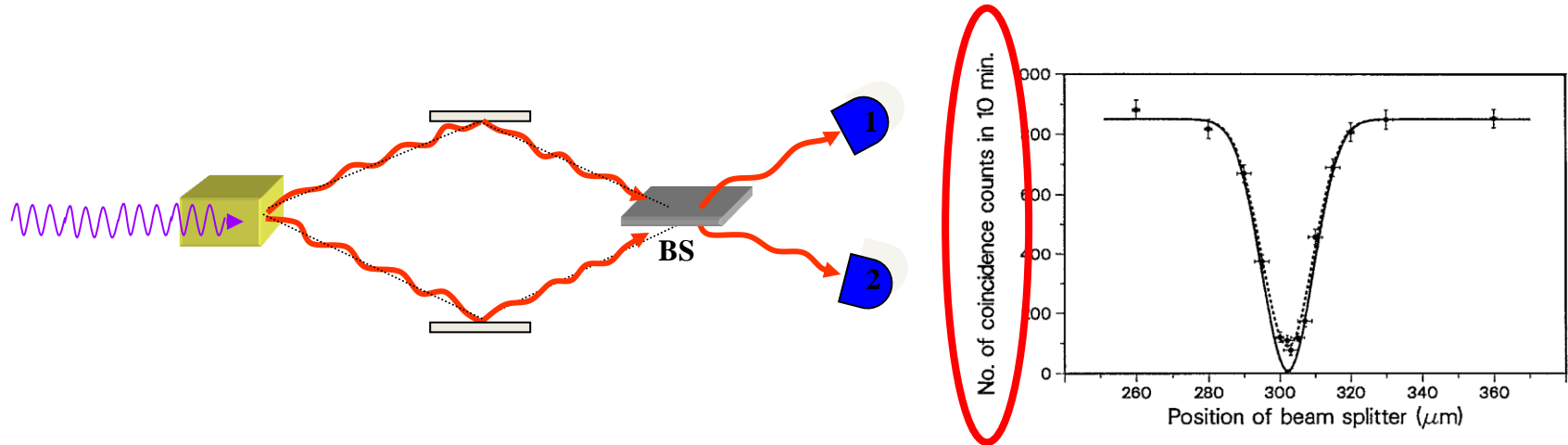
# Why not?

## Direct detection does not work:

- Slow detectors cannot observe the sharp time-correlation
- Slow detectors = energy distinguishability

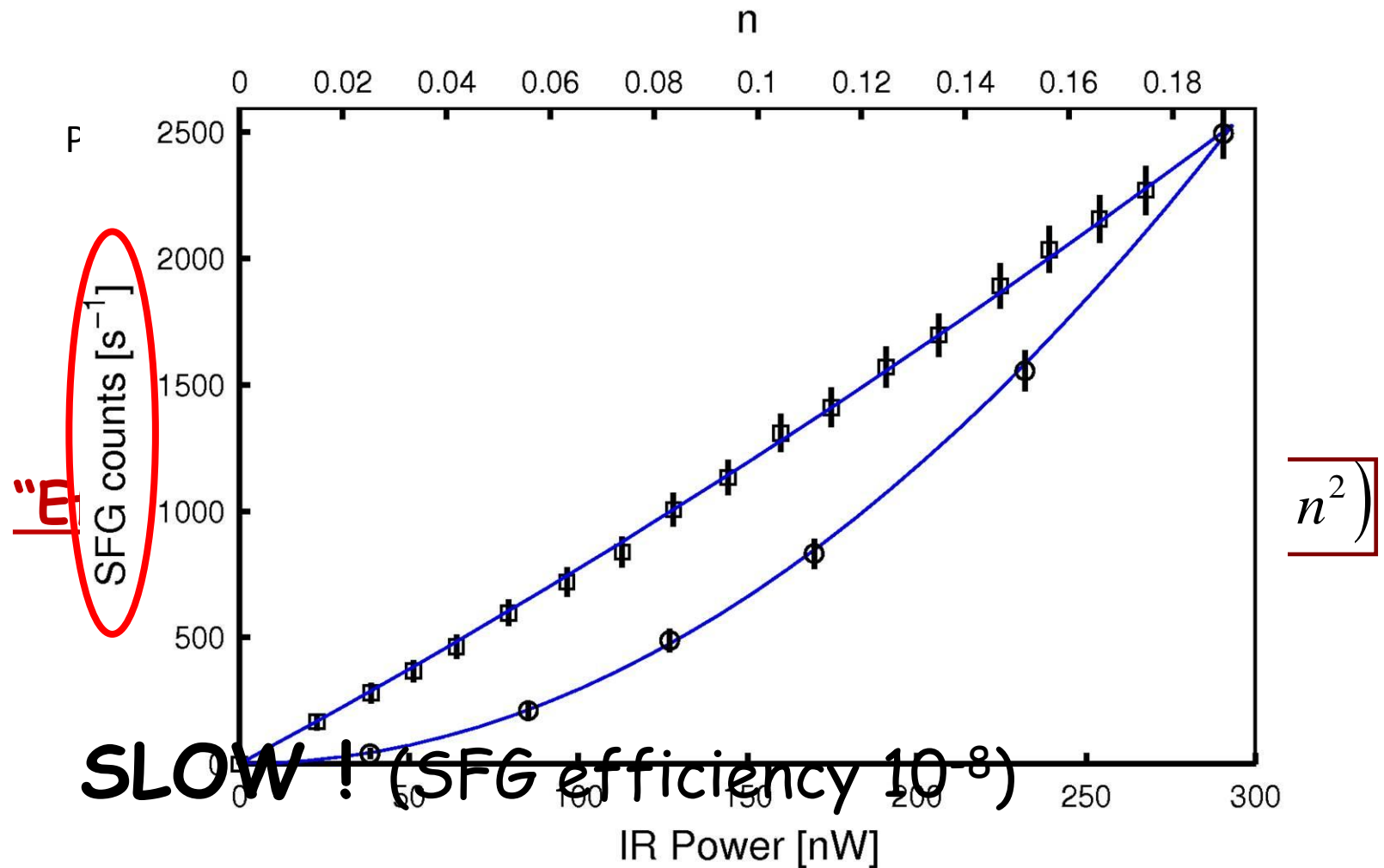
Need some other scheme

# Measuring bi-photons - HOM



**SLOW !** (coincidence limit  $<10^6$  ph/s)

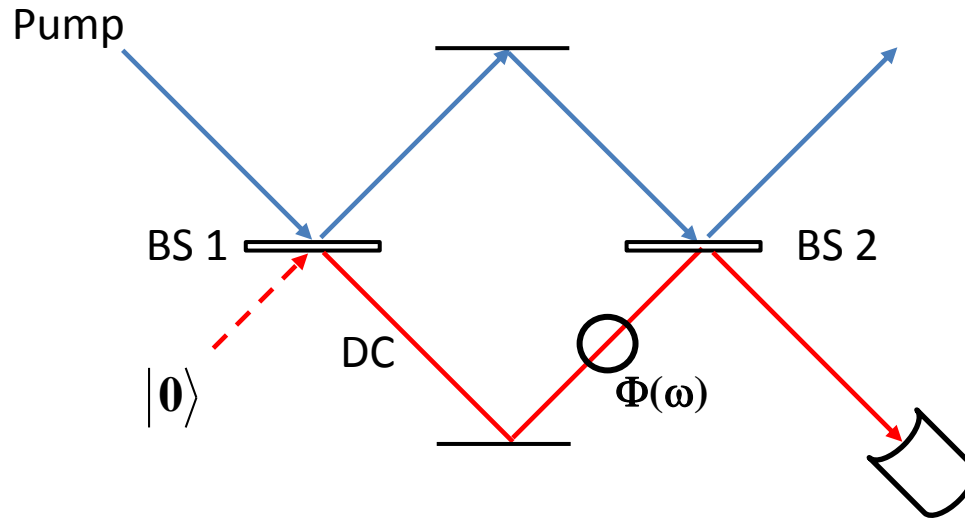
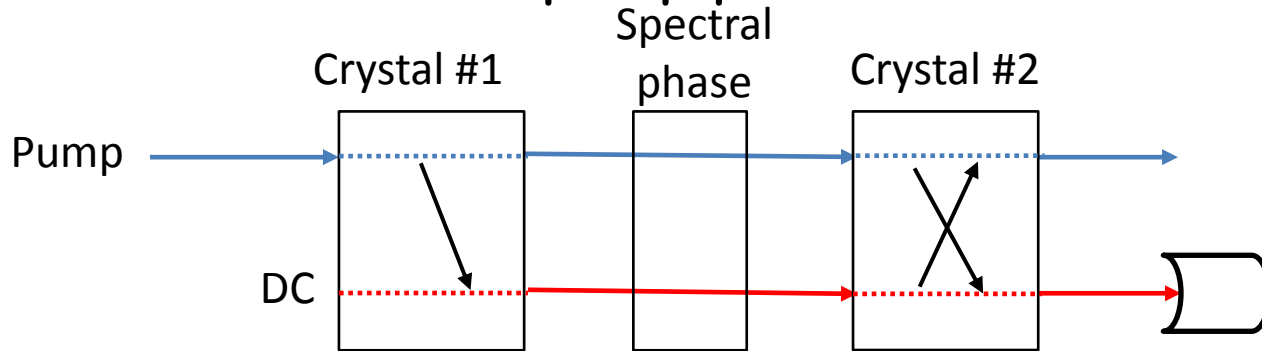
# Measuring bi-photons - SFG





# Quantum two-photon interference

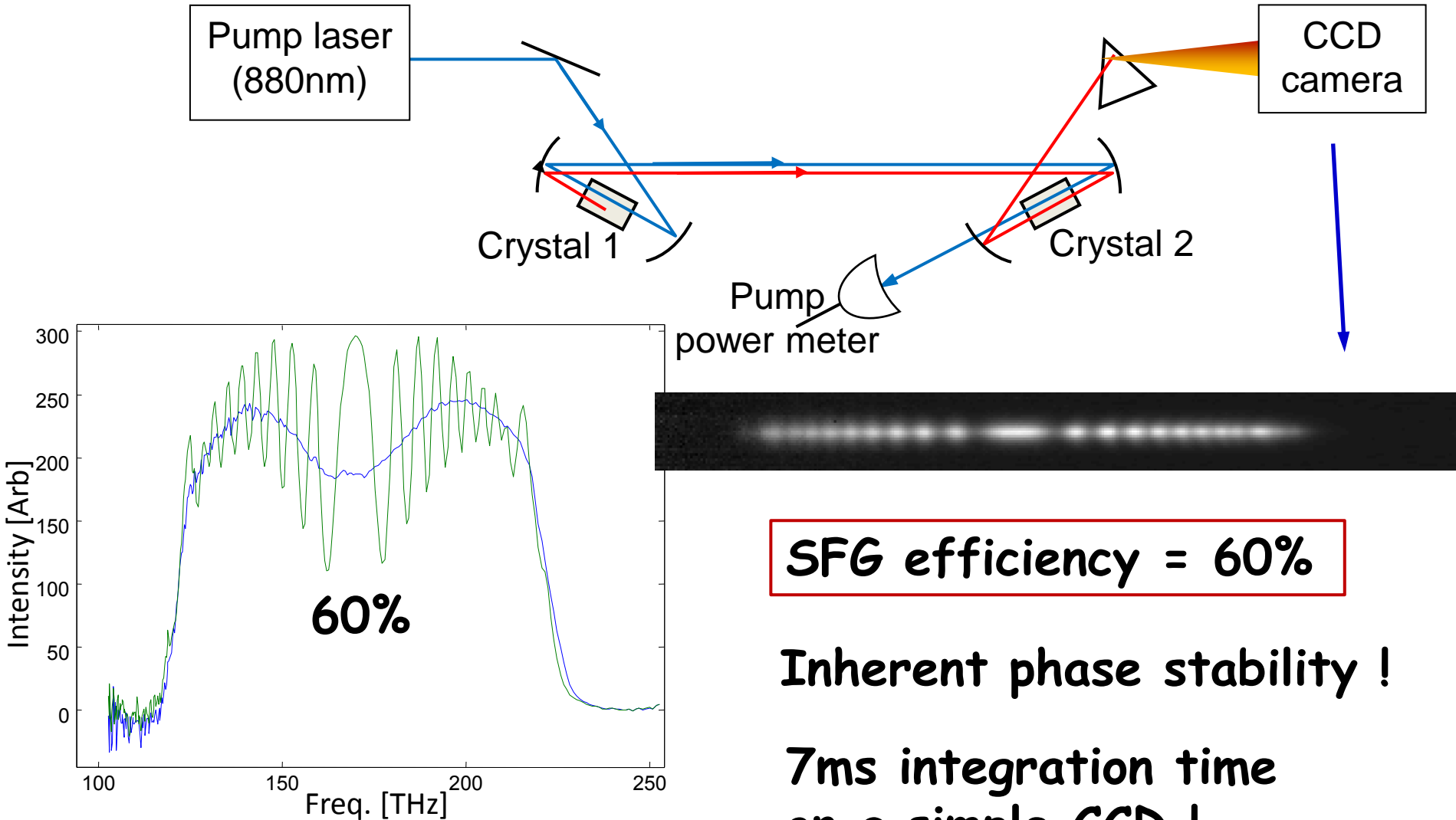
What if we let the pump pass ?



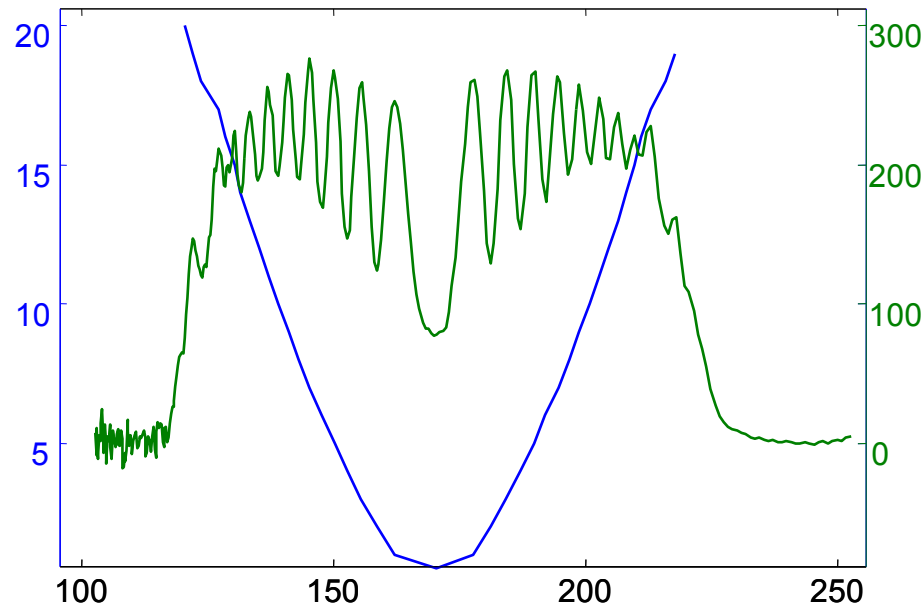
"Frustrated Two-Photon Creation via Interference", T. J. Herzog, J. G. Rarity, H. Weinfurt & A. Zeilinger, *Phys. Rev. Lett.* **72**, 629-632 (1993).

**Detection of bi-photons by attempting to annihilate them**

# Quantum two-photon interference

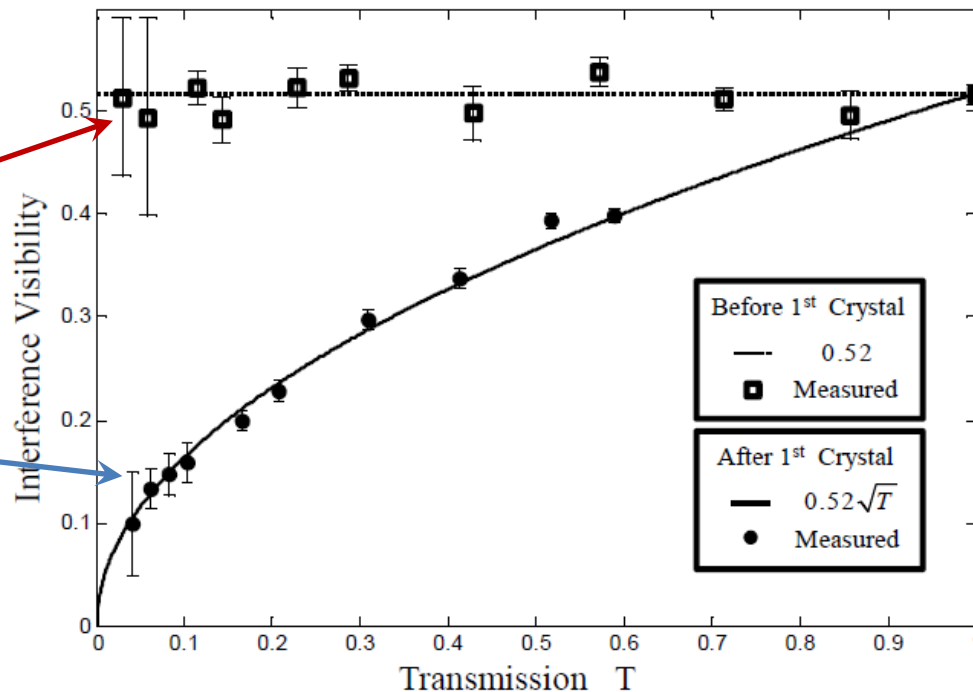
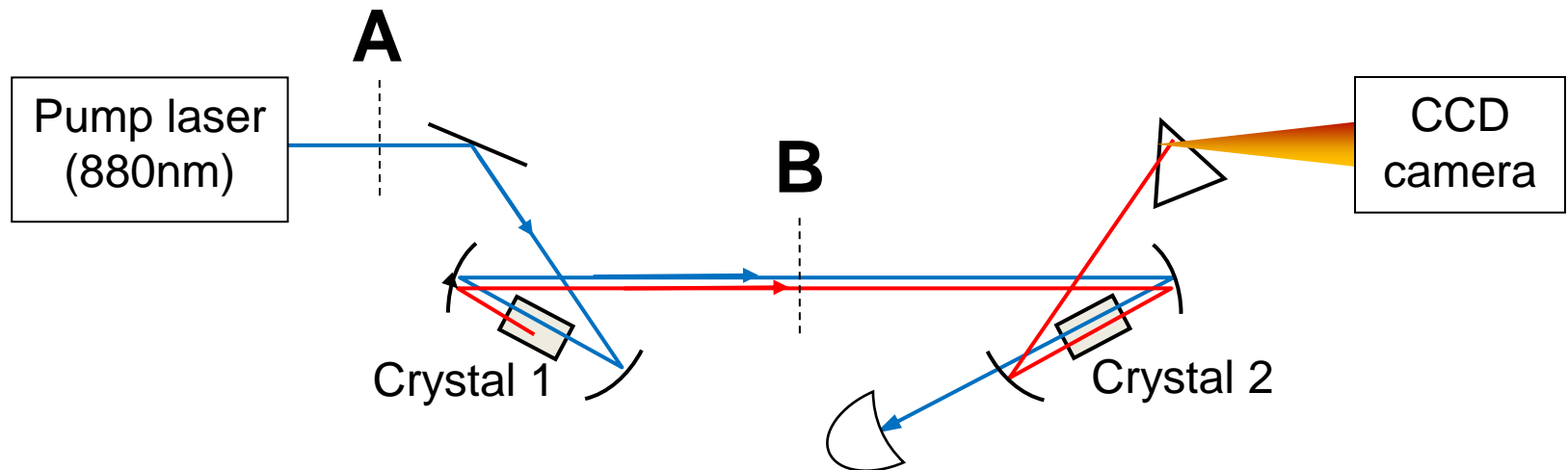


# Reconstruct the spectral phase



- Phase mismatch
- Dispersion from the dielectric mirrors

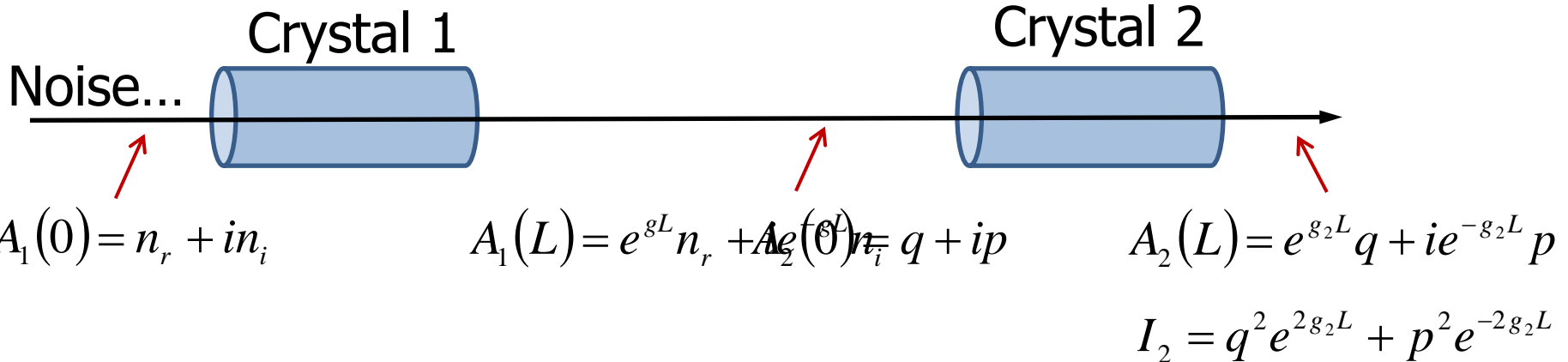
# What is non-classical ?



**Measure of the  
two-photon purity**

**SPEED !  
Full flux detected**

# Classical model (simplified)



$$V_{classical} = \frac{I_2^{\max} - I_2^{\min}}{I_2^{\max} + I_2^{\min}} = \frac{(q^2 e^{2g_1 L} + p^2 e^{-2g_1 L}) - (q^2 e^{-2g_1 L} - p^2 e^{2g_1 L})}{(q^2 e^{2g_1 L} + p^2 e^{-2g_1 L}) + (q^2 e^{-2g_1 L} - p^2 e^{2g_1 L})} = \frac{(q^2 - p^2)}{(q^2 + p^2)} \tanh(2g_1 L)$$

$$\frac{(q^2 - p^2)}{(q^2 + p^2)} = \tanh(2g_1 L)$$

$$V_{classical} = \tanh(2g_1 L) \tanh(2g_2 L)$$

$$V_{classical}^A \approx (2g_1 L)^2 \propto I_P$$

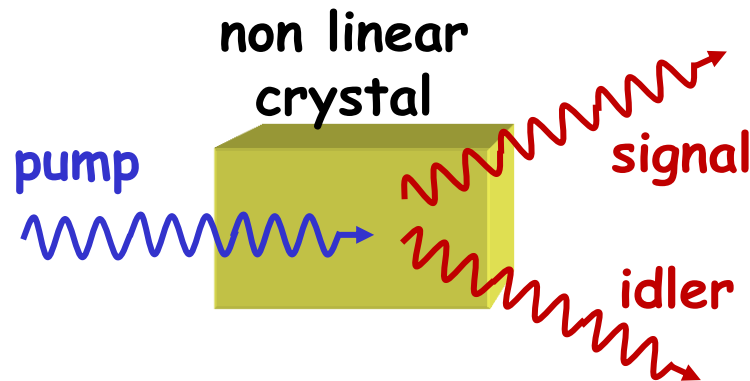
$$V_{classical}^B \approx (2g_1 L)(2g_2 L) \propto \sqrt{I_P}$$

# Conclusions (so far)

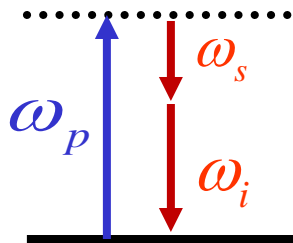
1. Bandwidth allows **ultra-high flux** of collinear **"single cycle"** bi-photons
2. The pumped crystal acts as a bi-photon detector with **near unity efficiency**.
3. **No coincidence detection** ! Standard intensity detection at the bi-photons rate
4. Comparing single photon loss with pair-wise loss verifies non-classical behavior.
5. **Speedup** !  $\times 10^4$  demonstrated,  $\times 10^8$  feasible

# Now to FWM...

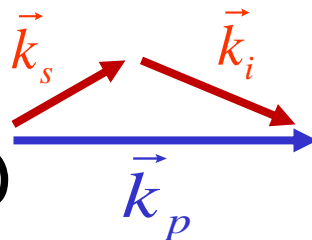
## Down conversion



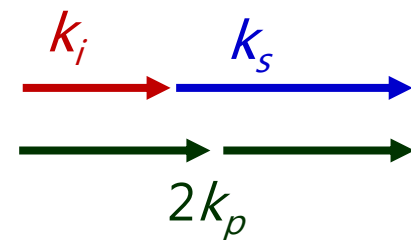
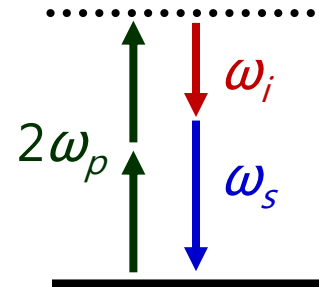
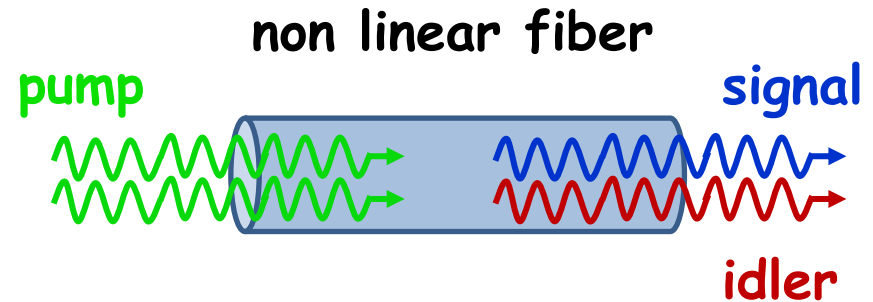
energy conservation



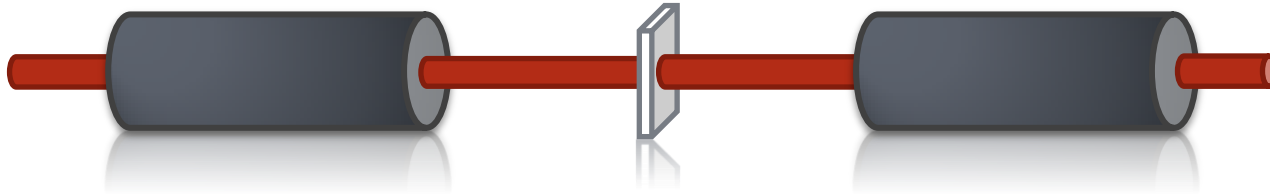
momentum conservation  
(phase matching)



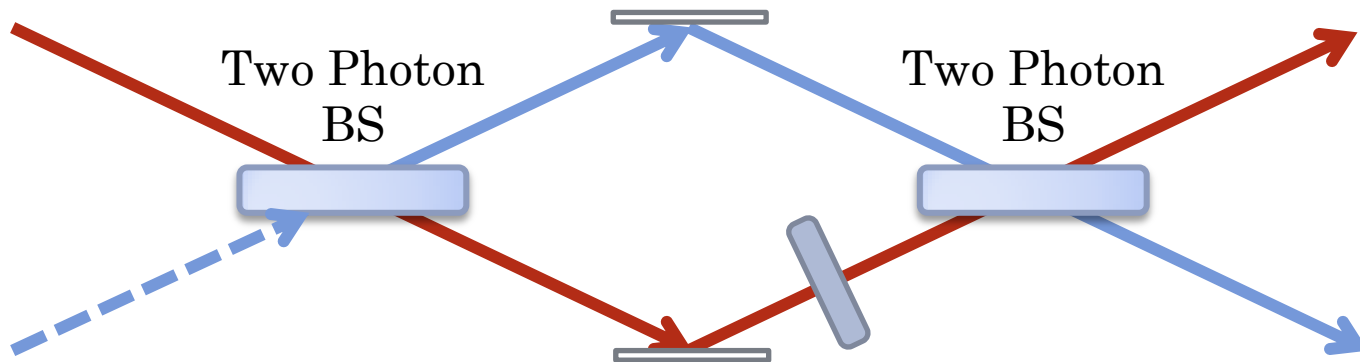
## Four Waves Mixing



# FWM concept



An equivalent Mach-Zehnder interferometer **for Bi-photons**:





# FWM and TWM – Differences...

## Down conversion

$$\frac{\partial}{\partial z} A_s = -i\chi A_p A_i^* e^{-i\Delta k \cdot z}$$

$$\frac{\partial}{\partial z} A_i = -i\chi A_p A_s^* e^{-i\Delta k \cdot z}$$

## Four Waves Mixing

$$\frac{\partial}{\partial z} A_s = -i\gamma \left( 2|A_p|^2 A_s + A_p^2 A_i^* e^{-i\Delta k \cdot z} \right)$$

$$\frac{\partial}{\partial z} A_i = -i\gamma \left( 2|A_p|^2 A_i + A_p^2 A_s^* e^{-i\Delta k \cdot z} \right)$$

## Rescale equations

$$\frac{\partial}{\partial z} B_s = -i\gamma A_{p,0}^2 B_i^* e^{-i\left(\Delta k - 2\gamma|A_p|^2\right) \cdot z}$$

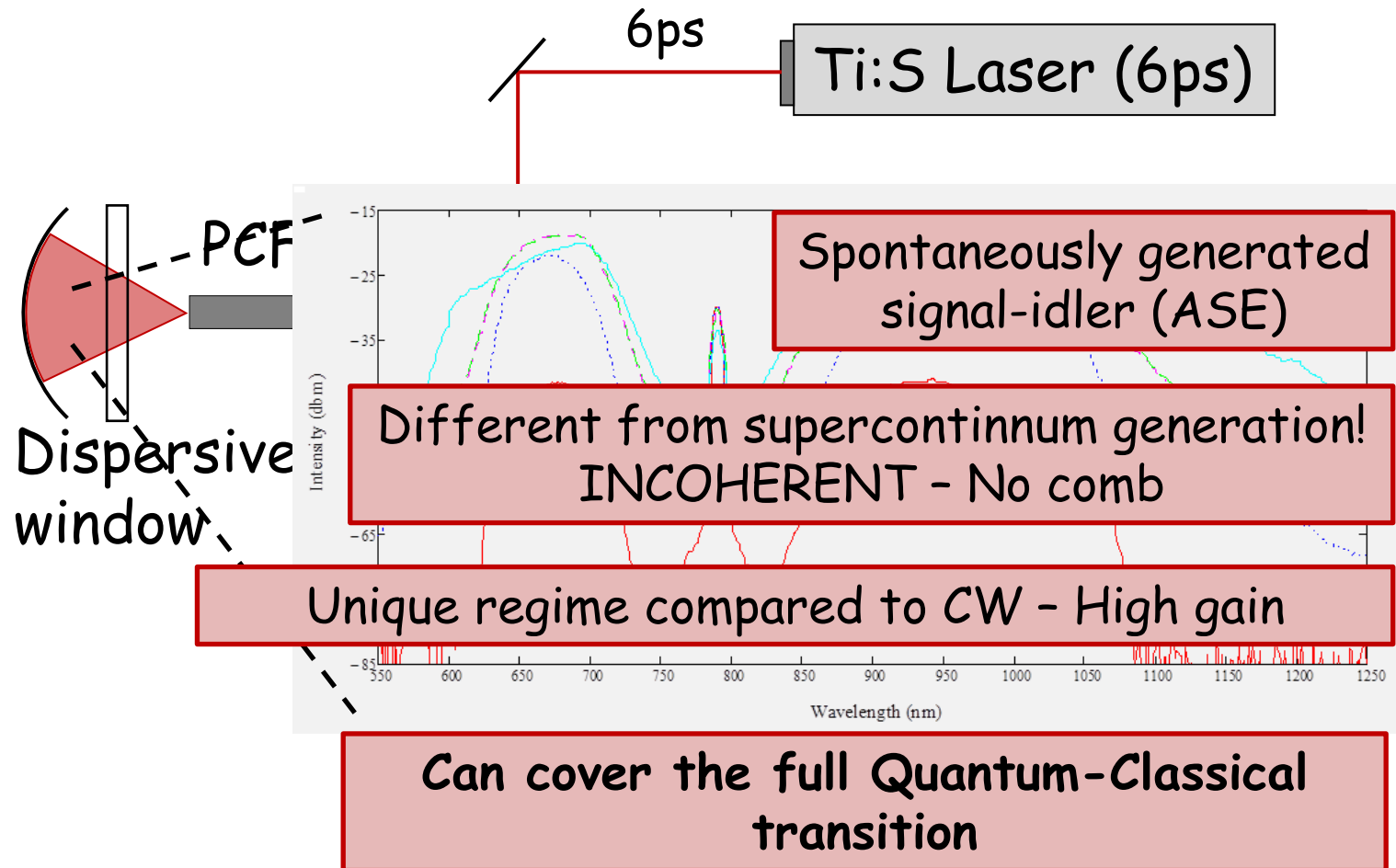
$$\frac{\partial}{\partial z} B_i = -i\gamma A_{p,0}^2 B_s^* e^{-i\left(\Delta k - 2\gamma|A_p|^2\right) \cdot z}$$

$$B_{s,i} = A_{s,i} e^{-2i\gamma|A_p|^2 z}$$

## Generalized phase mismatch

$$\Delta\kappa = \Delta k - 2\gamma|A_p|^2$$

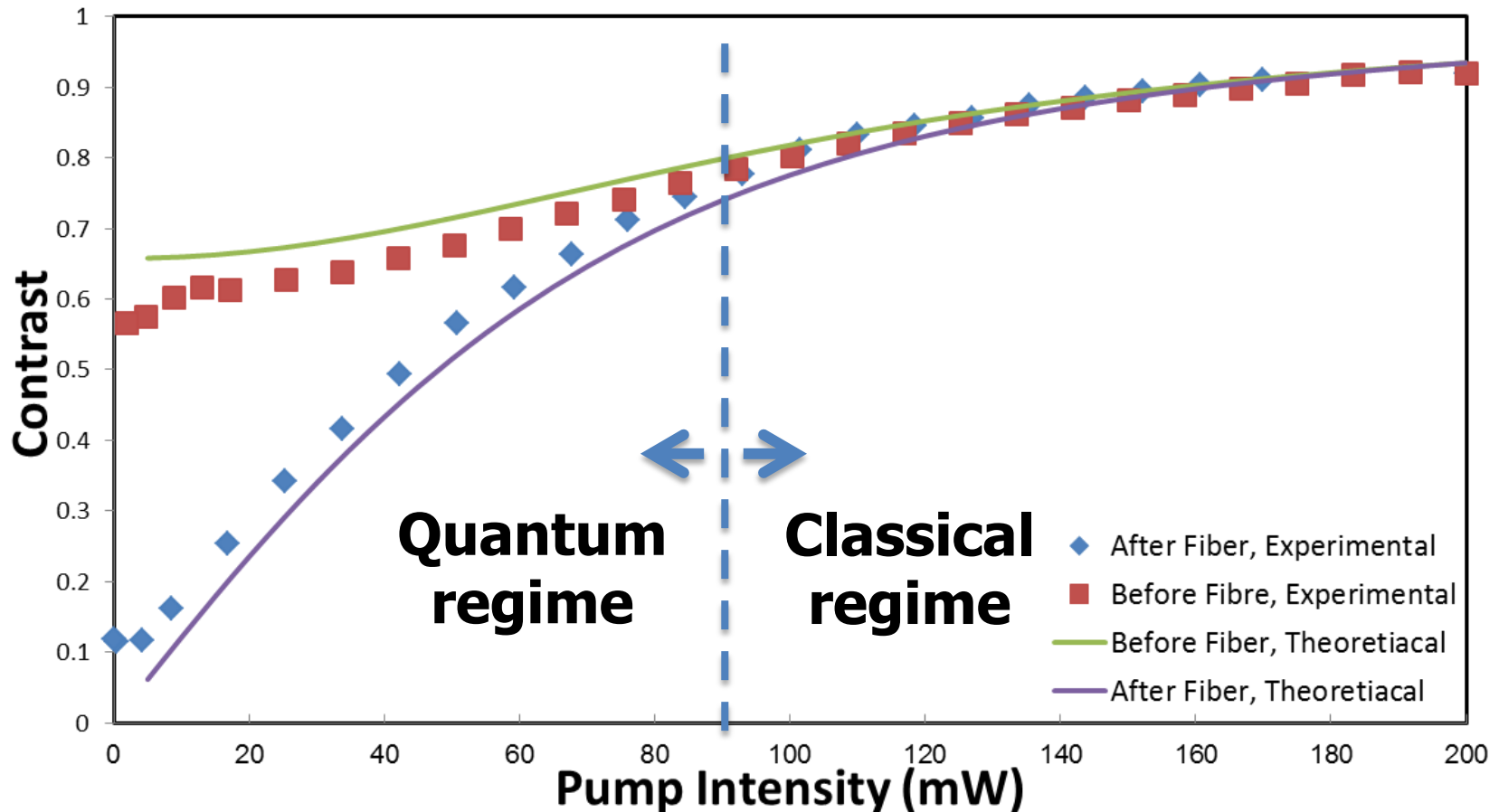
# The Experiment



Rafi Z. Vered, Michael Rosenbluh, and Avi Pe'er,  
***"Two-photon correlation of broadband-amplified spontaneous four-wave mixing"***, Phys. Rev. A **86**, 043837 (2012)

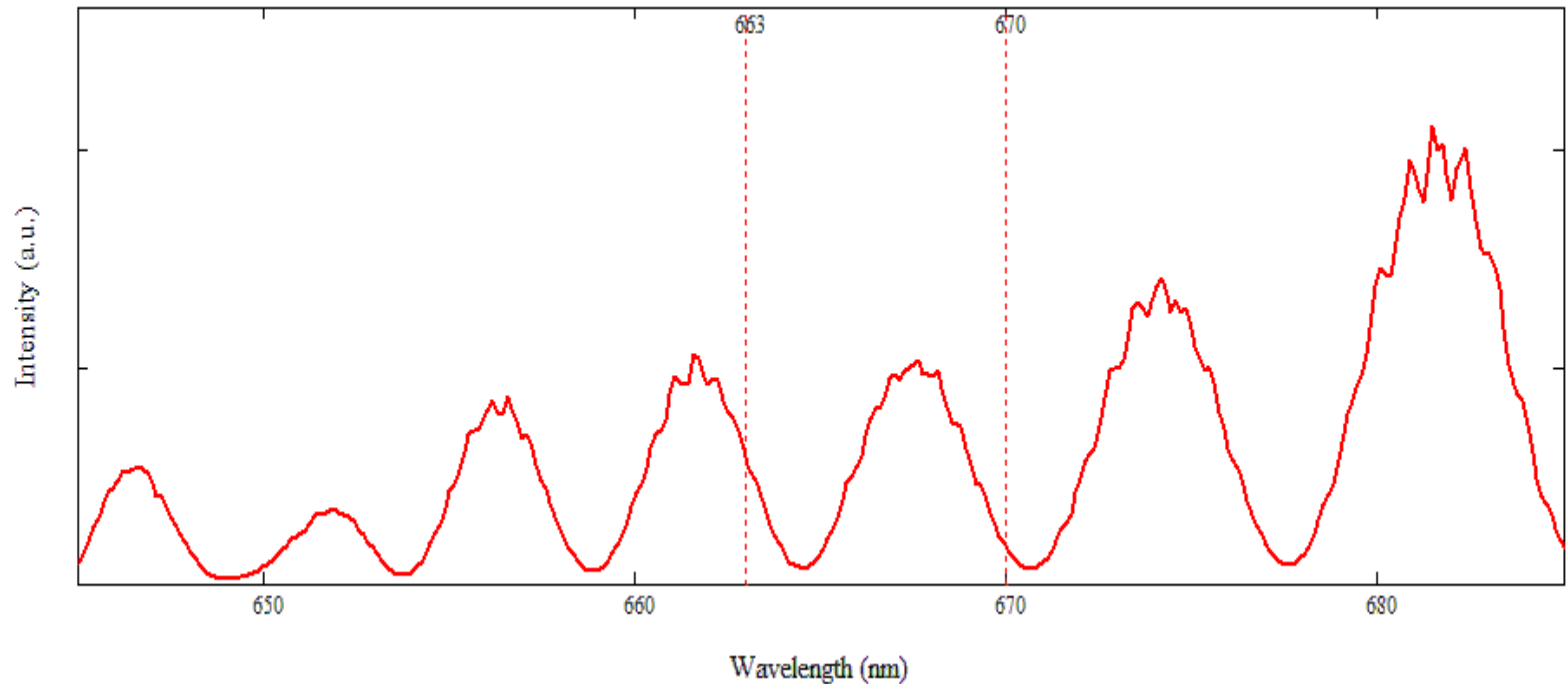
# Classical-to-quantum transition

Near zero dispersion - 784nm ( $\Delta k \approx 0$ , real gain)



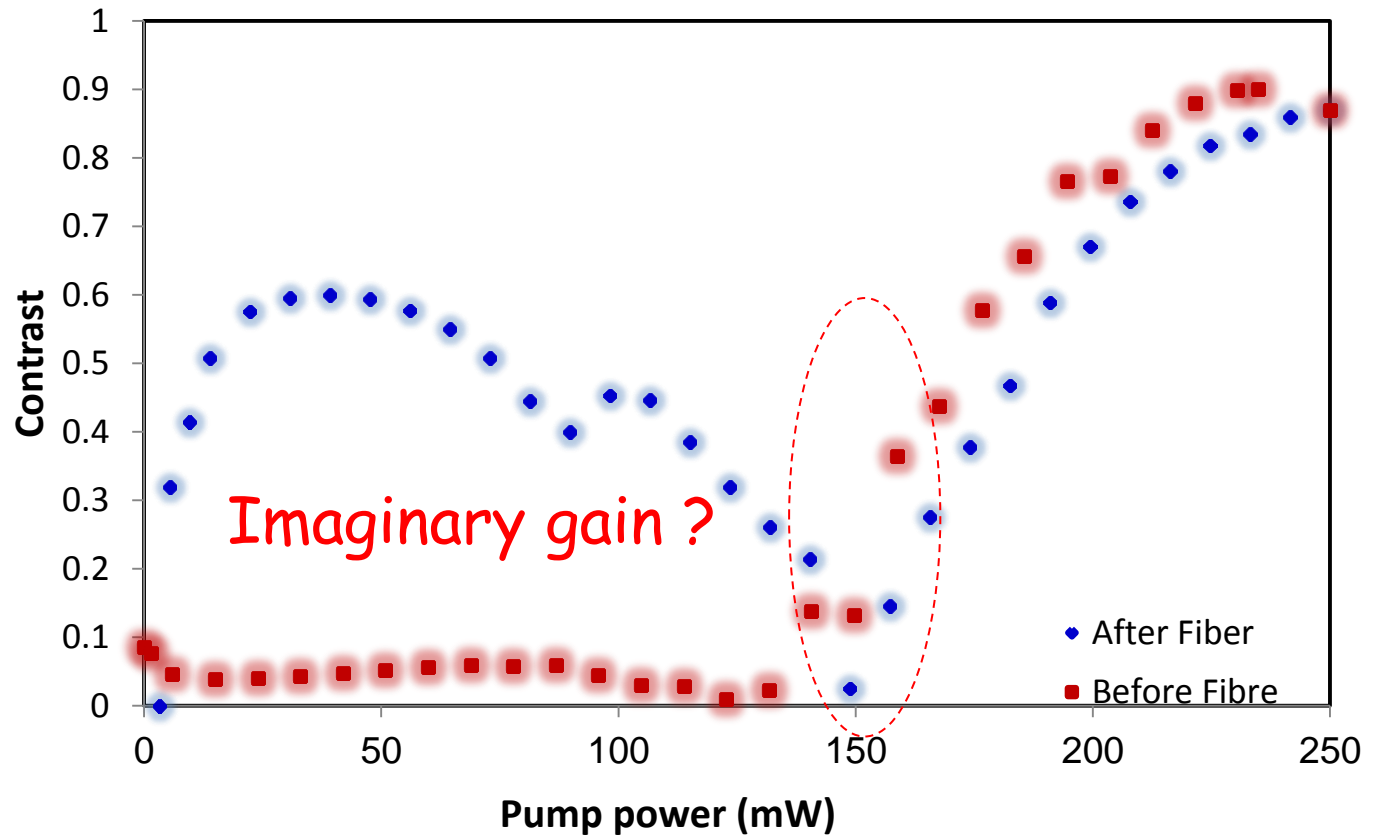
# FWM – nearby pump wavelength

Shift pump - 787nm ( $\Delta k < 0$ , threshold for gain)



**Phase shift with intensity**

# FWM – nearby pump wavelength



Squeezing ?

# FWM Gain Solution

## Signal/idler solution

$$B_{s,i} = b_{s,i}^{\pm} e^{\pm g \cdot z} e^{-i \frac{\Delta q}{2} z}$$

$$I_{s,i} \propto I_p z^2 \left( \frac{\text{Sinh}[gz]}{gz} \right)^2$$

$$g = \sqrt{\gamma^2 |A_p|^4 - \frac{\Delta q^2}{4}}$$

Similar to 3-waves,  
but...

Generalized phase  
mismatch

$$\Delta q = \Delta k - 2\gamma |A_p|^2$$

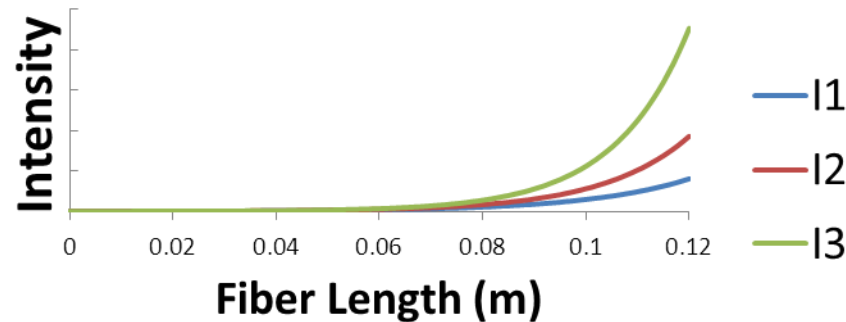
**Gain can become imaginary !**

**Correlation ?**

# Imaginary gain ?

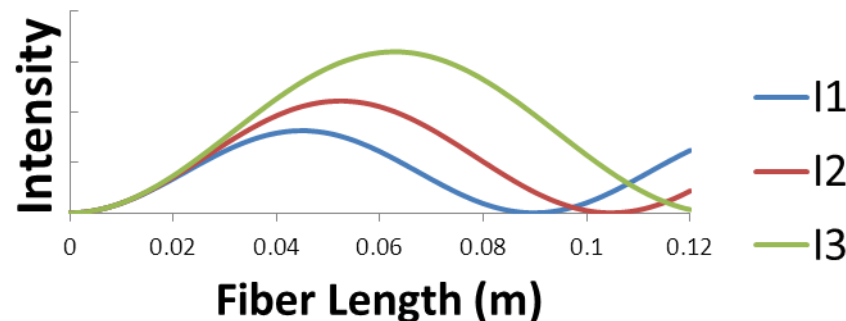
For real gain:

$$I_{FWM} \propto \frac{I_p^2}{g^2} \sinh^2(gl)$$



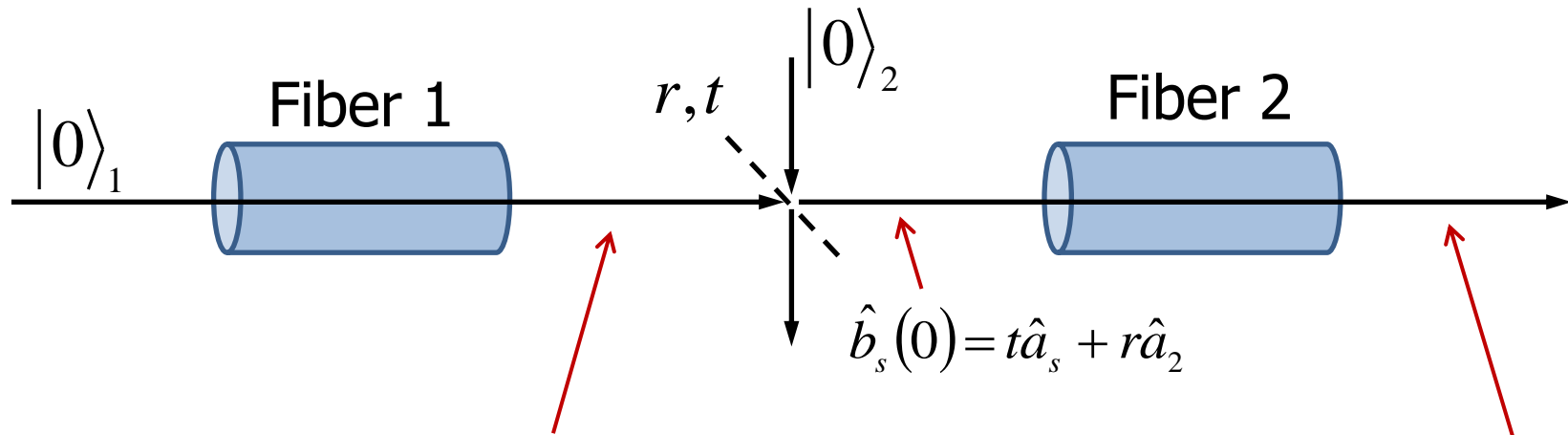
For imaginary gain:

$$I_{FWM} \propto l^2 I_p^2 \text{sinc}^2(|g|l)$$



$$I_3 > I_2 > I_1$$

# Quantum model (full)



$$\hat{a}_s(L) = \left[ \left( \cosh(g_1 L) + i \frac{\Delta\kappa}{2g_1} \sinh(g_1 L) \right) \hat{a}_s(0) + i \frac{\gamma |A_P|^2}{g_1} \sinh(g_1 L) \hat{a}_i^+(0) \right]$$

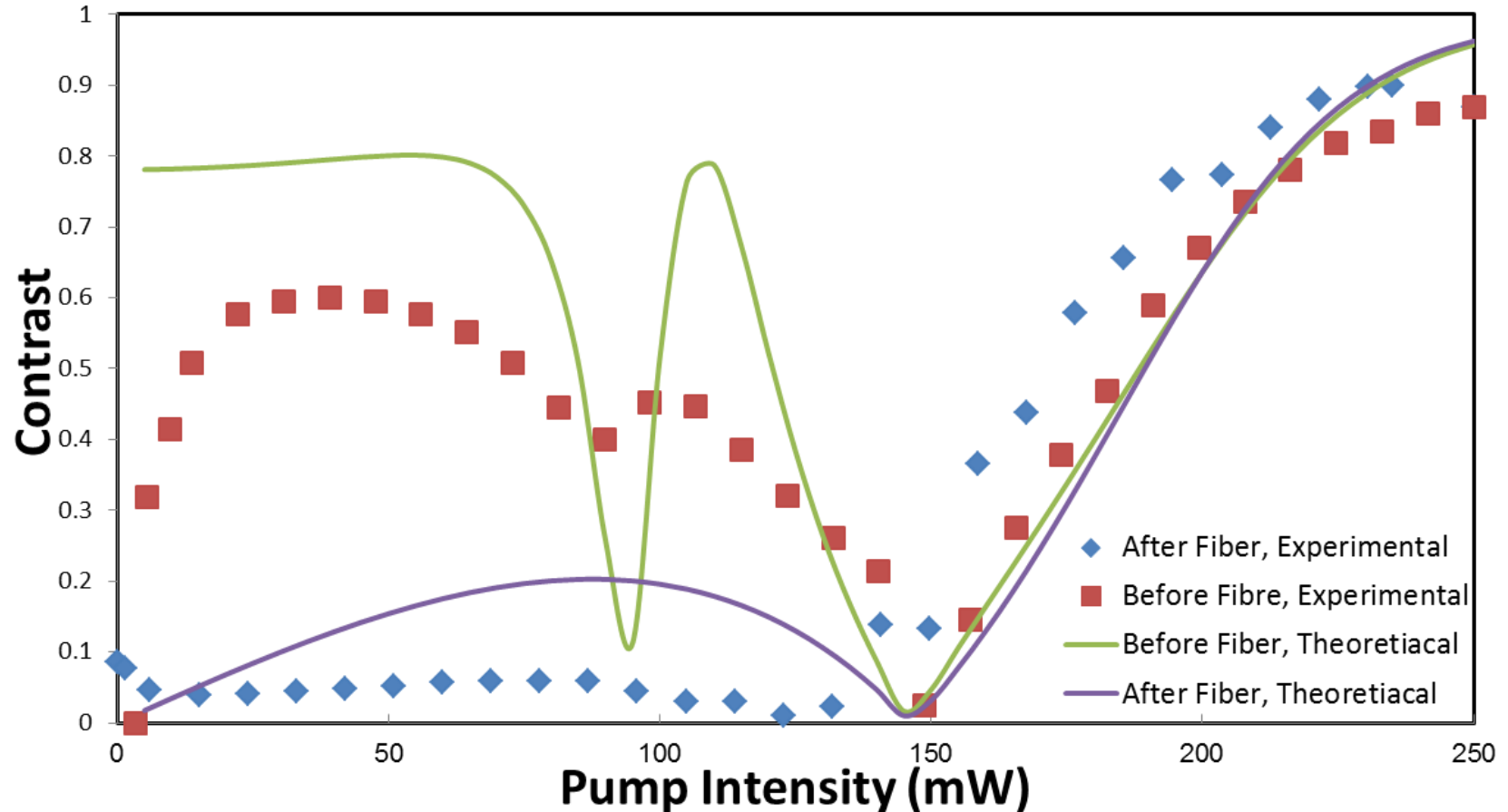
$$\hat{b}_s(L) = \left[ \left( \cosh(g_2 L) + i \frac{\Delta\kappa}{2g_2} \sinh(g_2 L) \right) \hat{b}_s(0) + i \frac{\gamma |A_P|^2 e^{2i\varphi_P}}{g_2} \sinh(g_2 L) \hat{b}_i^+(0) \right]$$

$$N_s(\varphi_P) = {}_{1,2} \langle 0 | \hat{b}_s^+ \hat{b}_s | 0 \rangle_{1,2}$$

CW treatment  
(no time dependence)



# Theory vs experiment



Contrast reduction ?

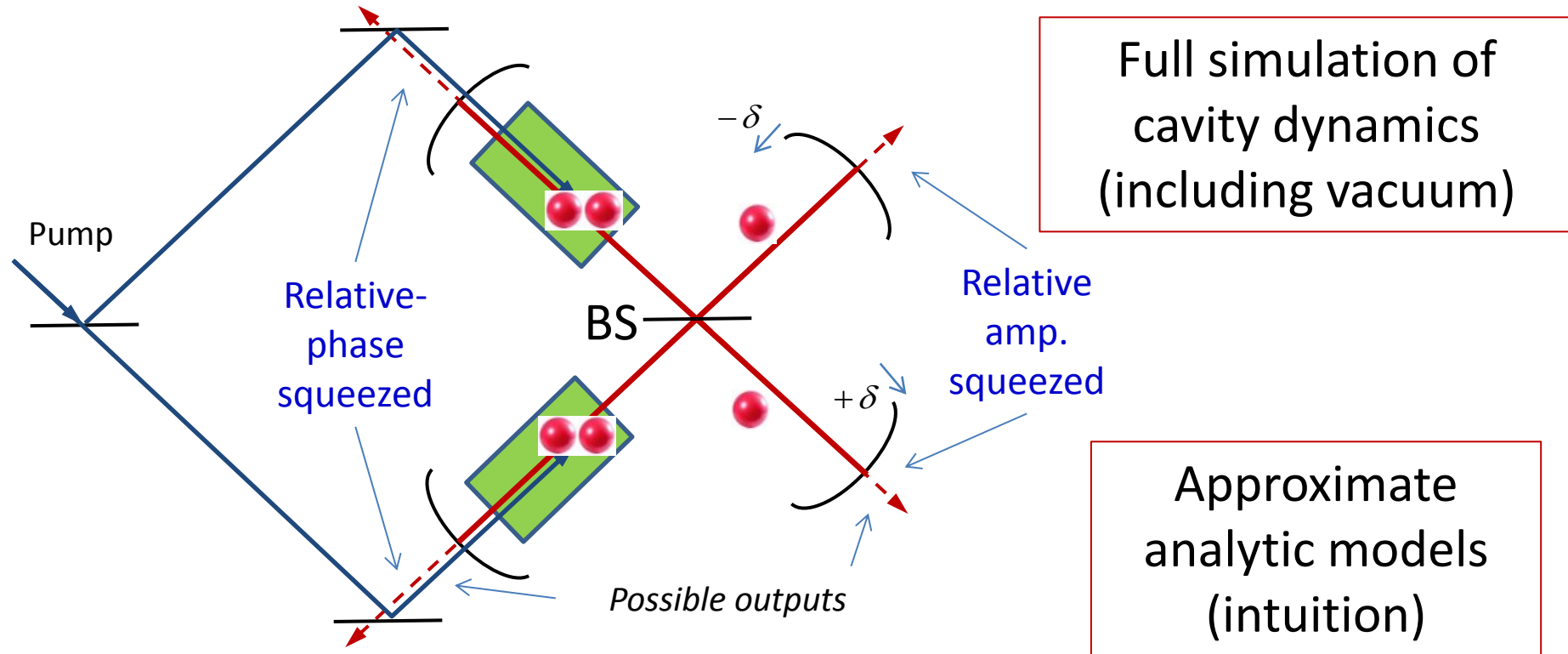
Pulse effects,      Spectrometer resolution

# Conclusions (II)

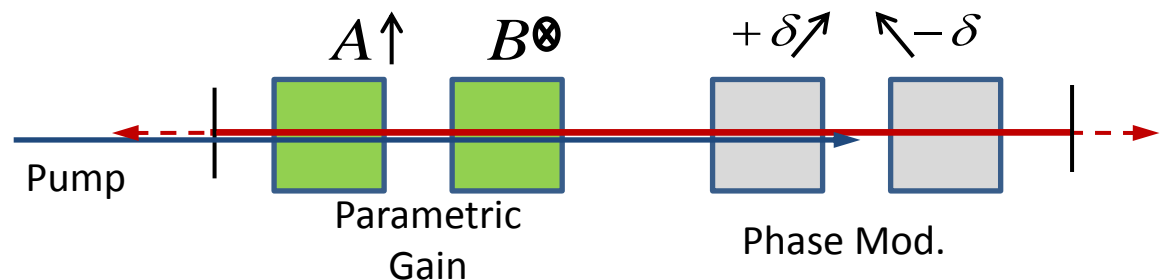
- Observation of the **entire classical-quantum transition** with FWM in fiber
- Span 4 orders of magnitude (and more...)
- **Bi-photon generation with imaginary gain**
- Can this be used to measure broadband (two-mode) squeezing ?

# A high-power efficient source (OPO) ?

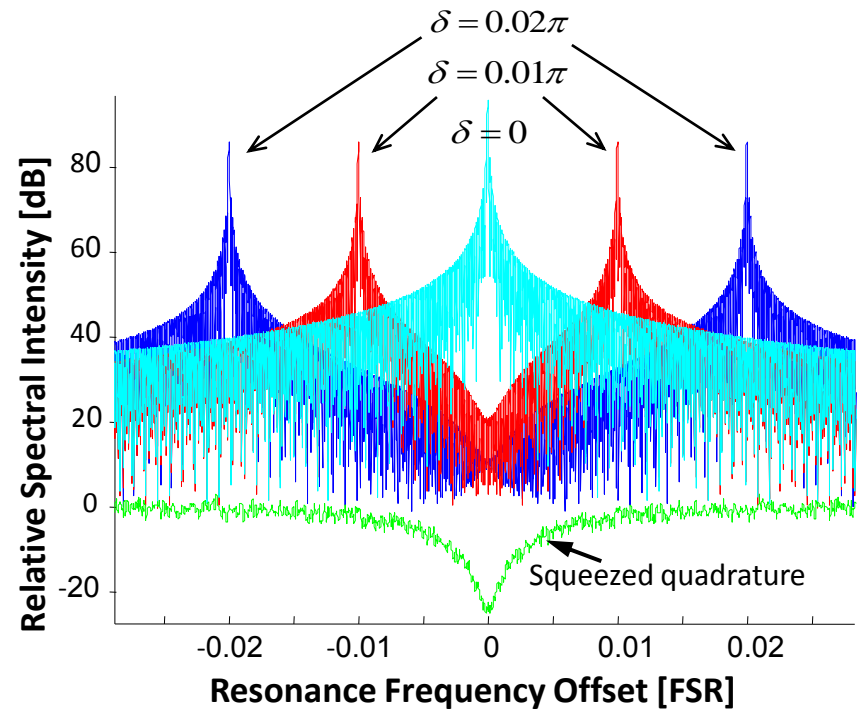
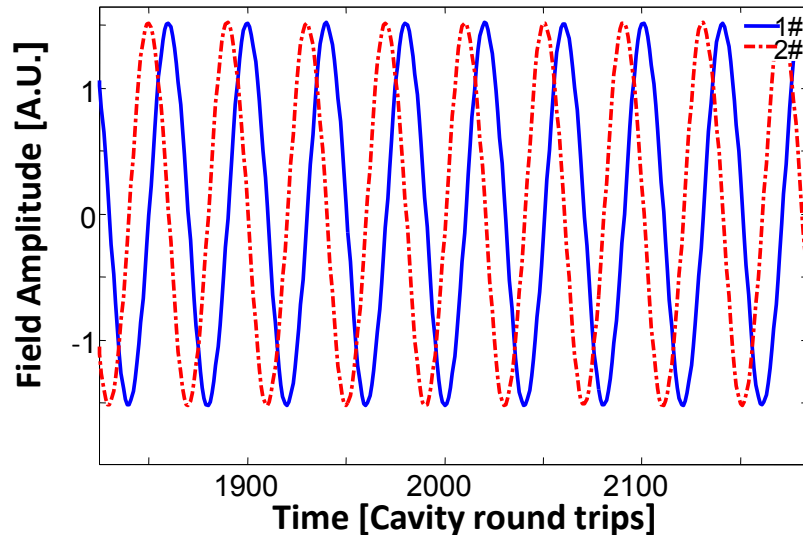
What if we introduce a (HOM) interferometer into the Source ?



**A simple realization:**



# Coupled narrowband oscillation



Quantum Beating at  $\omega_{beat} = 2\delta \cdot f_{rep}$

**Two-mode squeezing with arbitrary, tuned separation !**

**Direct electronic detection / stabilization**

# Coupled OPOs – Narrowband theory

Exact dynamical equation (including vacuum)

$$\tau \frac{d}{dt} A = \left[ -\frac{T^2}{2} A + (\kappa l A_p - \frac{1}{2} \kappa^2 l^2 A^2) A^* \right] \cos \delta + \left[ \left( 1 - \frac{T^2}{2} \right) B + (\kappa l A_p - \frac{1}{2} \kappa^2 l^2 B^2) B^* \right] \sin \delta + T n^A(t)$$

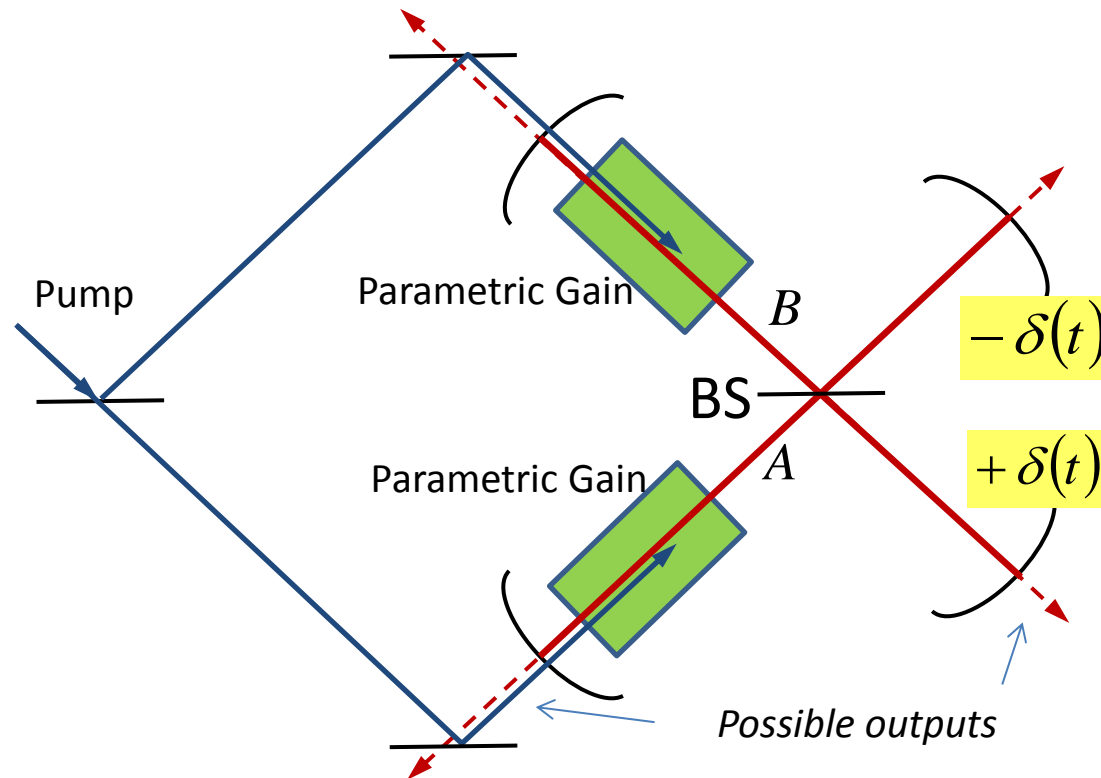
Approximate analytic model

$$\tau \frac{d}{dt} \begin{bmatrix} A \\ B \end{bmatrix} \approx \begin{bmatrix} 0 & \delta \\ \delta & 0 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

**Quantum beats**

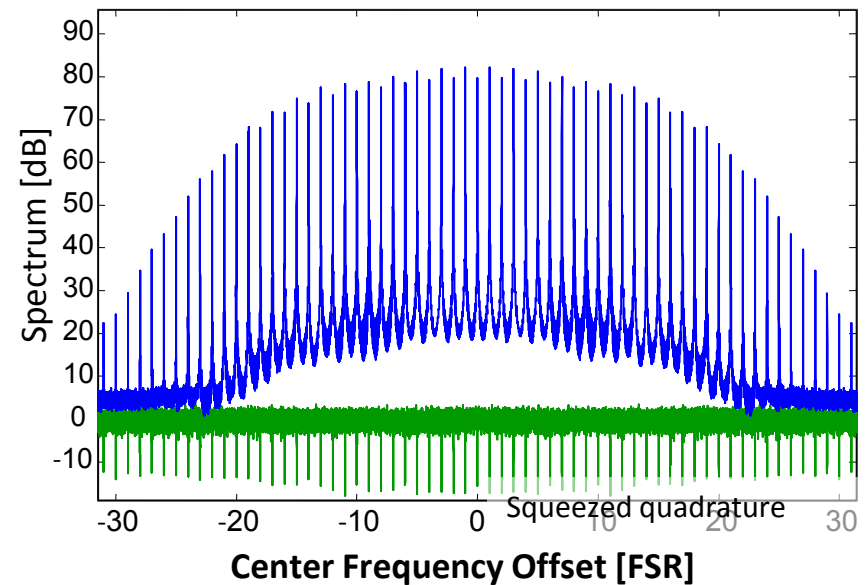
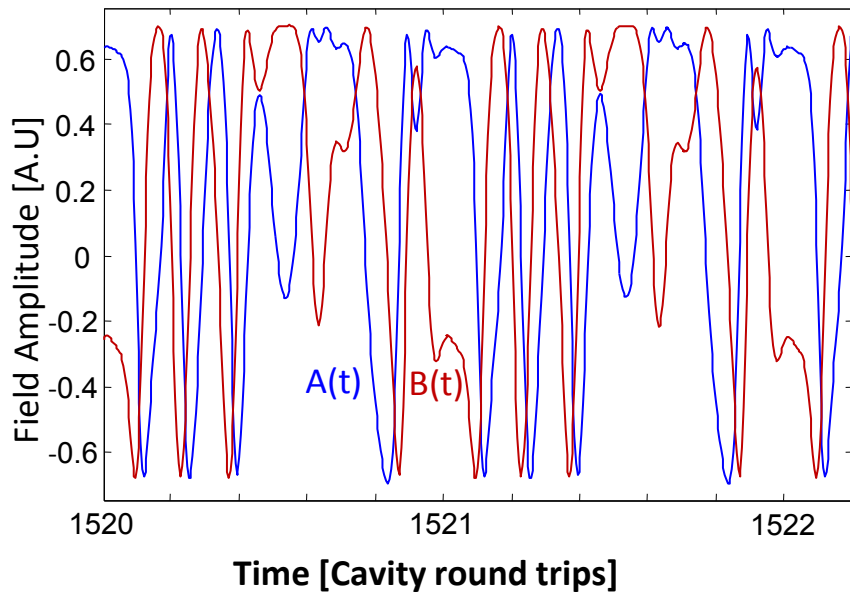
# What if... (II)

What if we modulate the coupling phase at the repetition rate of the cavity ?



# Pairwise mode locking

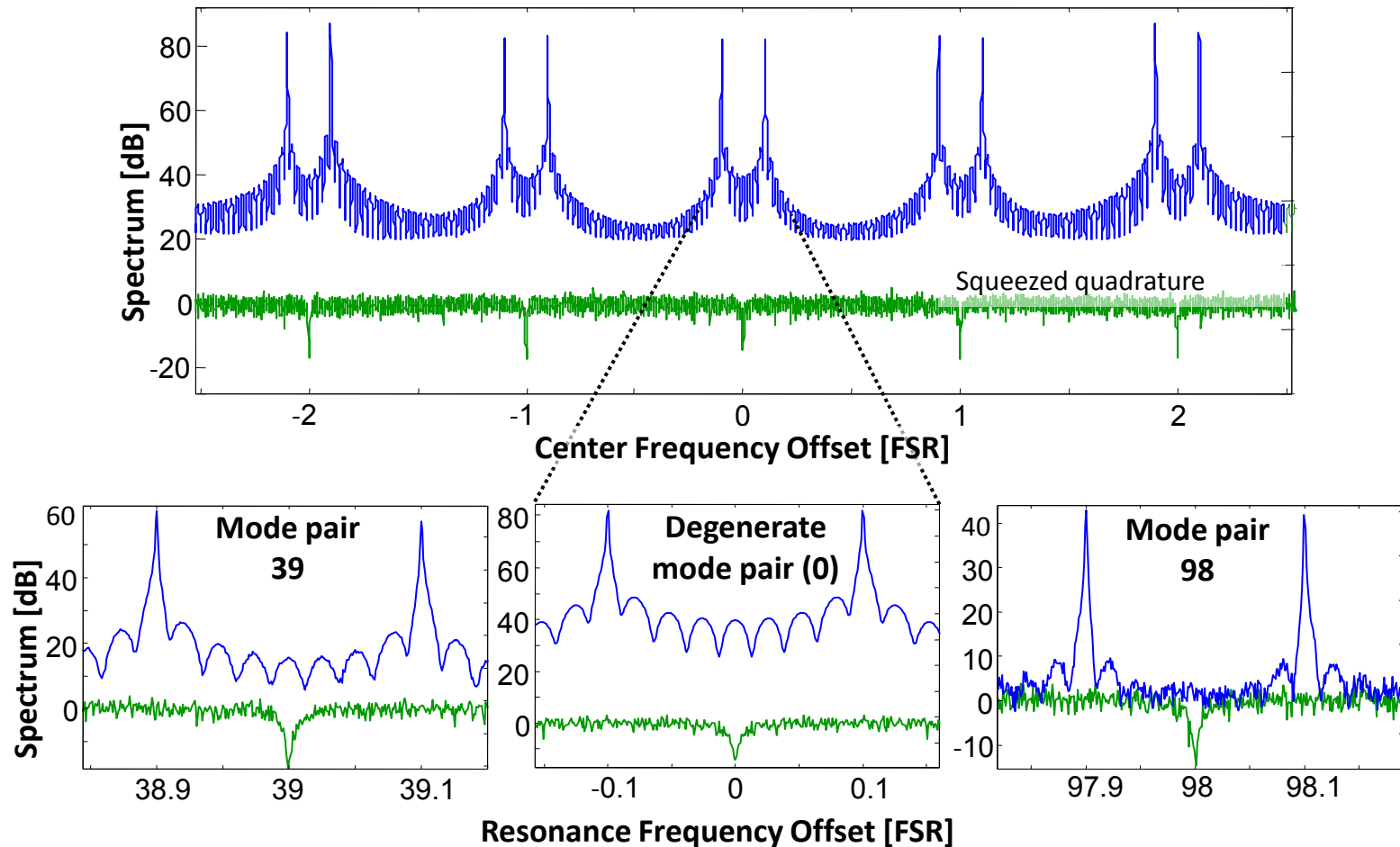
**Energy spread between modes - Pairwise mode locking**



**A two-photon analog for active mode locking in lasers**

**Quantum Frequency Comb!**

# Quantum two-photon comb



**A Coherent Link between all pairs !**



# What is it good for... ?

**The comb**



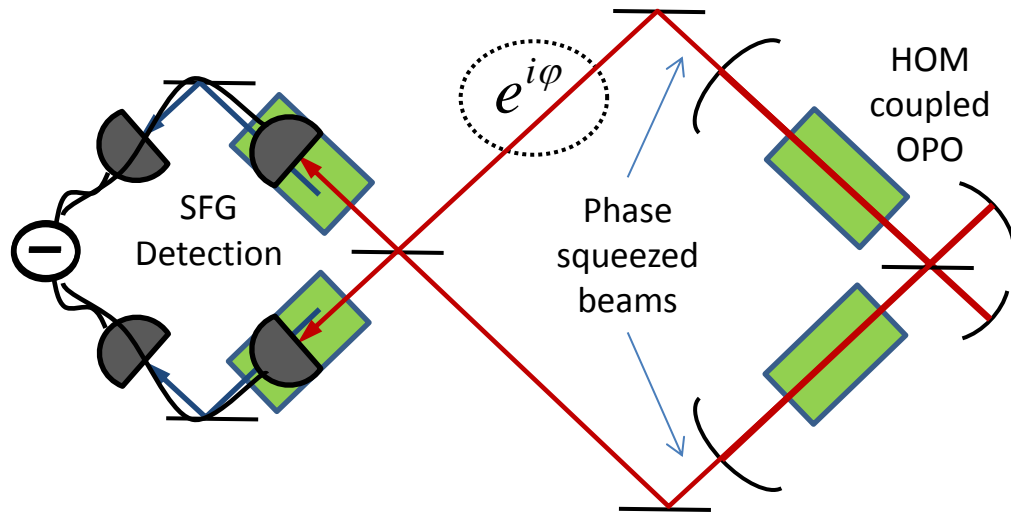
**Coherence transfer across the spectrum**  
(freq. counting)

**Same possibilities with broadband  
two-photon coherence and squeezing**

- **Precision phase measurement – sub shot-noise**
- Atoms as nonlinear mixers (Kimble 1997)
- Modification of atomic natural lifetime in broadband squeezed light (Gardiner 1987)
- Classical applications (spread-spectrum optical communication...)
- What else... ?

# Precision phase measurement

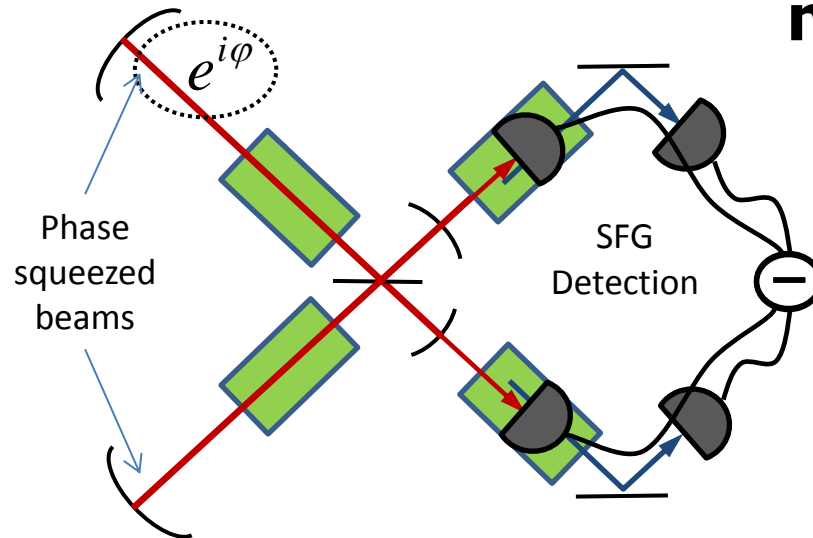
## Mach-Zehnder



Quantum beat –  
**high freq. detection**

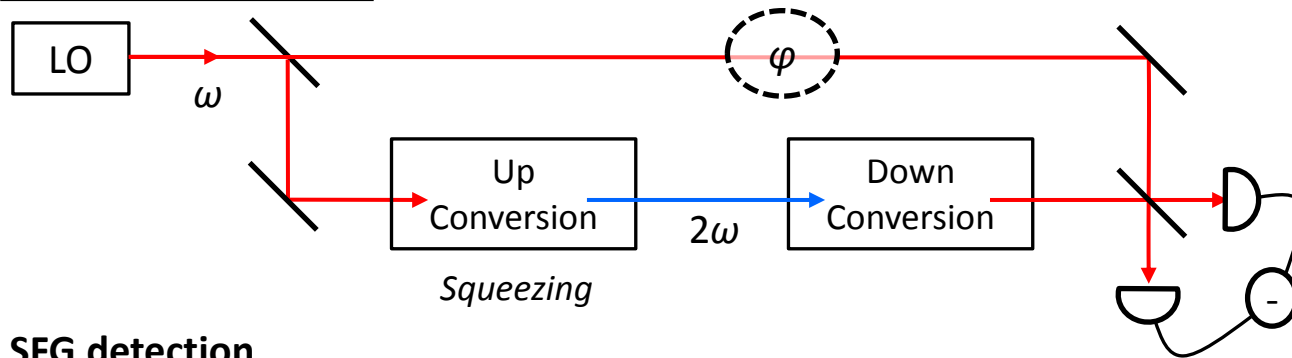
Two Squeezed beams -  
**no local oscillator**

## Michelson

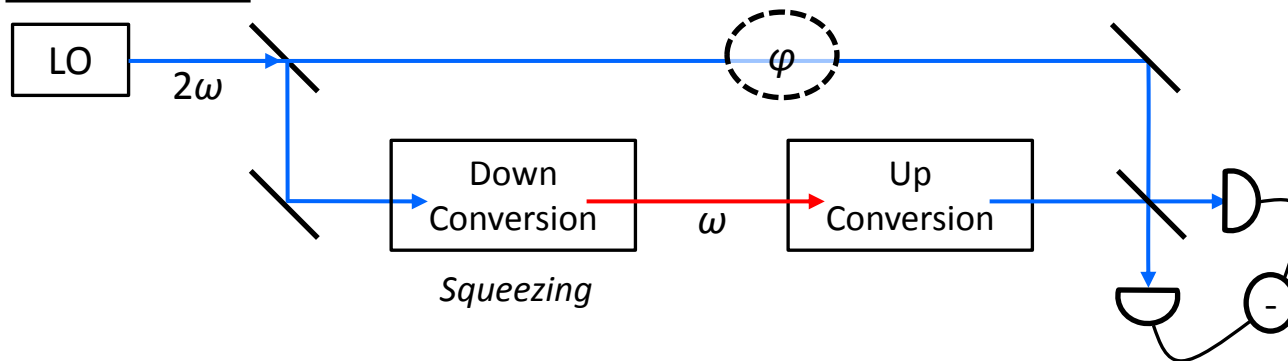


# SFG detection of Squeezing at high-power

## Homodyne detection



## SFG detection



$$V_2^0 = 1 + \frac{4v^2(N + M - v^2/8)}{(1 + 3v^2/4)^2}$$

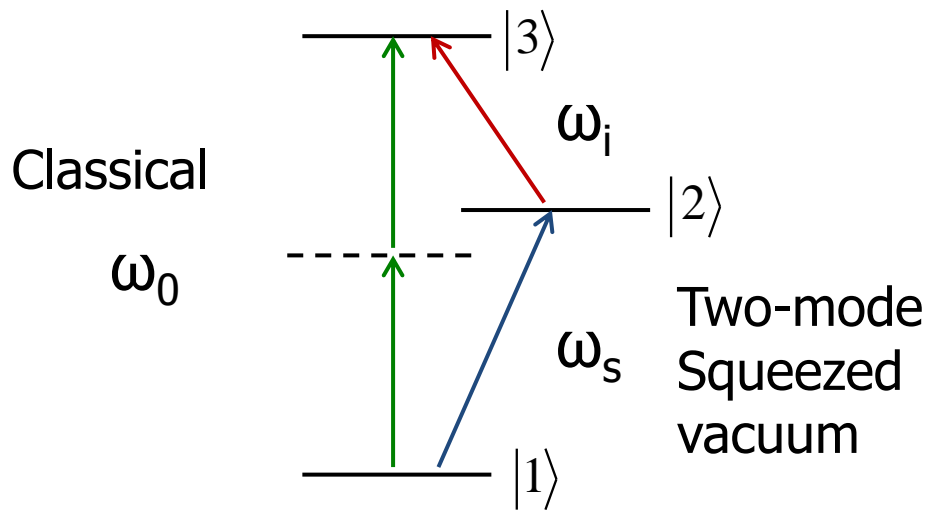
$$V_2^{\pi/2} = 1 + \frac{4v^2(N - M + v^2/8)}{(1 + v^2/4)^2}$$

**Fully quantum analysis**

[\*Phys. Rev. A.\* \*\*88\*\*, 043808 \(2013\)](#)

# Atoms as nonlinear mixers

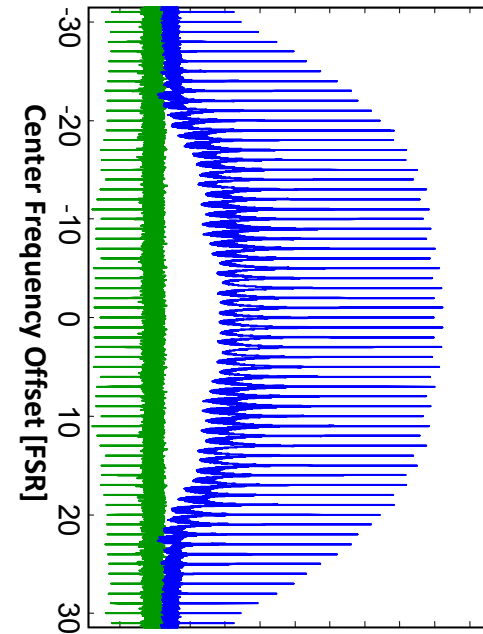
PRA 55, 1605 (1997)



Quantum interference in  
two-photon absorption

**Two pairs** coherently linked

**With quantum comb**



**All pairs squeezed and  
coherently linked !**

# Conclusions (last)

- **HOM coupled OPOs** - A source for above threshold, high-power broadband two-mode squeezed light
- **Pairwise mode-locking** - two-photon analog of active mode-locking
- A pairwise coherent link across a broad spectrum - **Another kind of Quantum comb**
- It will be useful for something
- Experiments on the way...

# Theory – Coupled OPOs - Broadband

Exact dynamical equation (including mismatch):

$$\tau \frac{d}{dt} A_t(\omega) = \left\{ -\frac{T^2}{2} A_t(\omega) + \left[ \kappa l A_p - \frac{1}{2} \kappa^2 l^2 \left( 1 - \frac{1}{3} i \Delta \kappa l \right) \sum_{\omega} A_t(\omega) A_t(-\omega) \right] A_t^*(-\omega) \right\} \cos \delta \\ + \left\{ \left( 1 - \frac{T^2}{2} \right) B_t(\omega) + \left[ \kappa l A_p - \frac{1}{2} \kappa^2 l^2 \left( 1 - \frac{1}{3} i \Delta \kappa l \right) \sum_{\omega} B_t(\omega) B_t(-\omega) \right] B_t^*(-\omega) \right\} \sin \delta + T n_t^A(\omega)$$

Mode locking analog (classical theory):

$$\tau \frac{d}{dt} A(\omega) = G_A A(\omega) + \frac{\delta_{AC}}{2} [B(\omega + \omega_r) + B(\omega - \omega_r)]$$

$$\tau^2 \frac{d^2}{dt^2} \langle |A(\omega)|^2 \rangle = [4G_A^2 - \delta_{AC}^2] \langle |A(\omega)|^2 \rangle + \frac{\delta_{AC}^2}{2} [\langle |B(\omega + \omega_r)|^2 \rangle + \langle |B(\omega - \omega_r)|^2 \rangle]$$

$$4G_A^2 \langle |A(\omega)|^2 \rangle - \delta_{AC}^2 [\langle |A(\omega)|^2 \rangle - \langle |B(\omega)|^2 \rangle] + \frac{\delta_{AC}^2}{2} \omega_r^2 \frac{d^2}{d\omega^2} \langle |B(\omega)|^2 \rangle = 0$$

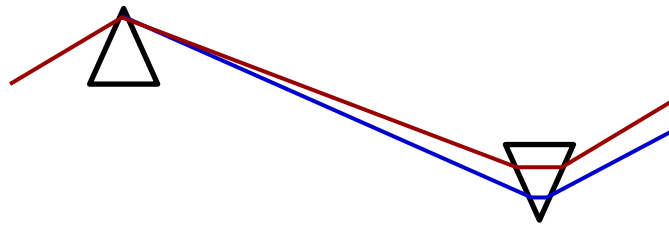
$$\frac{\delta_{AC}^2}{2} \omega_r^2 \frac{d^2}{d\omega^2} \langle |A(\omega)|^2 \rangle = -4G^2 \langle |A(\omega)|^2 \rangle$$

$$\text{Gaussian spectrum } \langle |A(\omega)|^2 \rangle \sim e^{-\omega^2 / \Delta^2} \quad \Delta^2 = \delta_{AC} \omega_r \mu, \quad G_0^2 = \delta_{AC} \omega_r / 4\mu$$

# Dispersion compensation

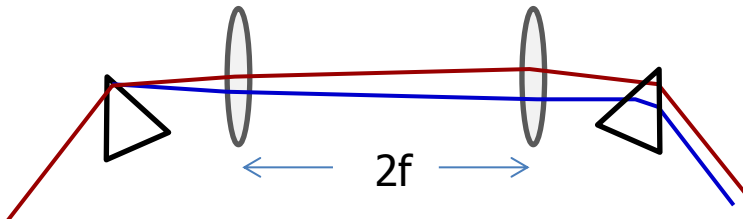
**Spectral symmetry** ➡ **Only even orders important**

Standard



Cannot handle 4<sup>th</sup> order  
(need negative distance)

Novel



Space – material  
interchanged

Telescope = negative distance