Matrix product states for the fractional quantum Hall effect

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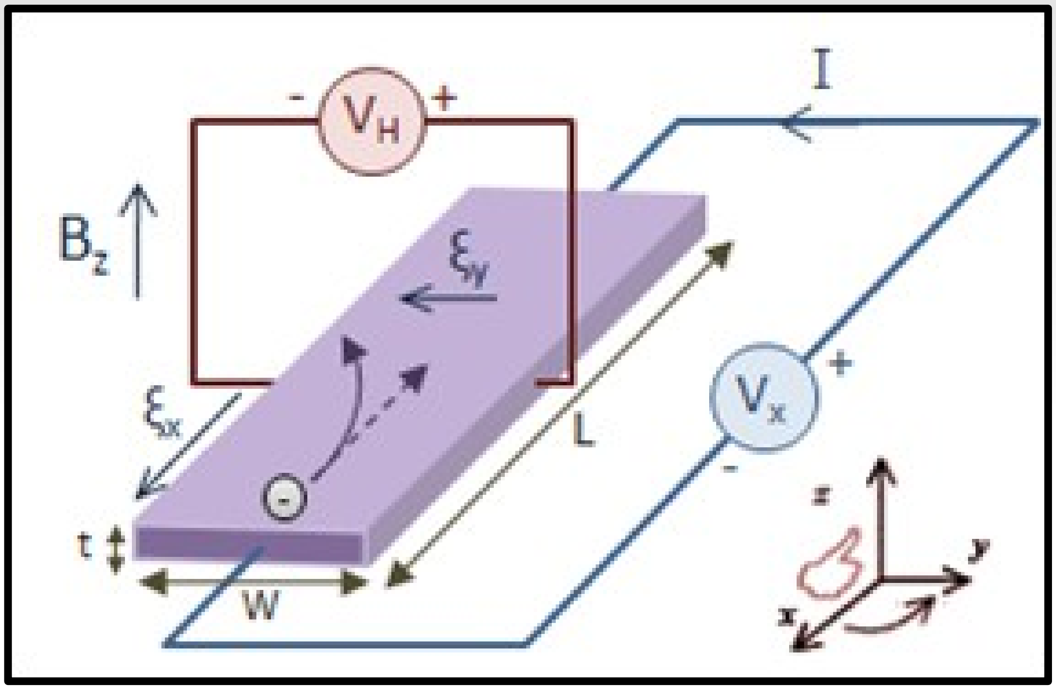
Michael Zaletel, RM, PRB **86**, 245305 [arXiv:1208.4862]
Michael Zaletel, RM, Frank Pollmann PRL **110**, 236801 [arXiv:1211.3733]

Outline

- Why study quantum Hall?
- Quantum entanglement
  - Topological phases
  - Matrix product states (MPS)
- 1. MPS for quantum Hall model wavefunctions
- 2. Modeling physical systems with DMRG (density matrix renormalization group)
- 3. Extracting topological content from ground states

Quantum Hall

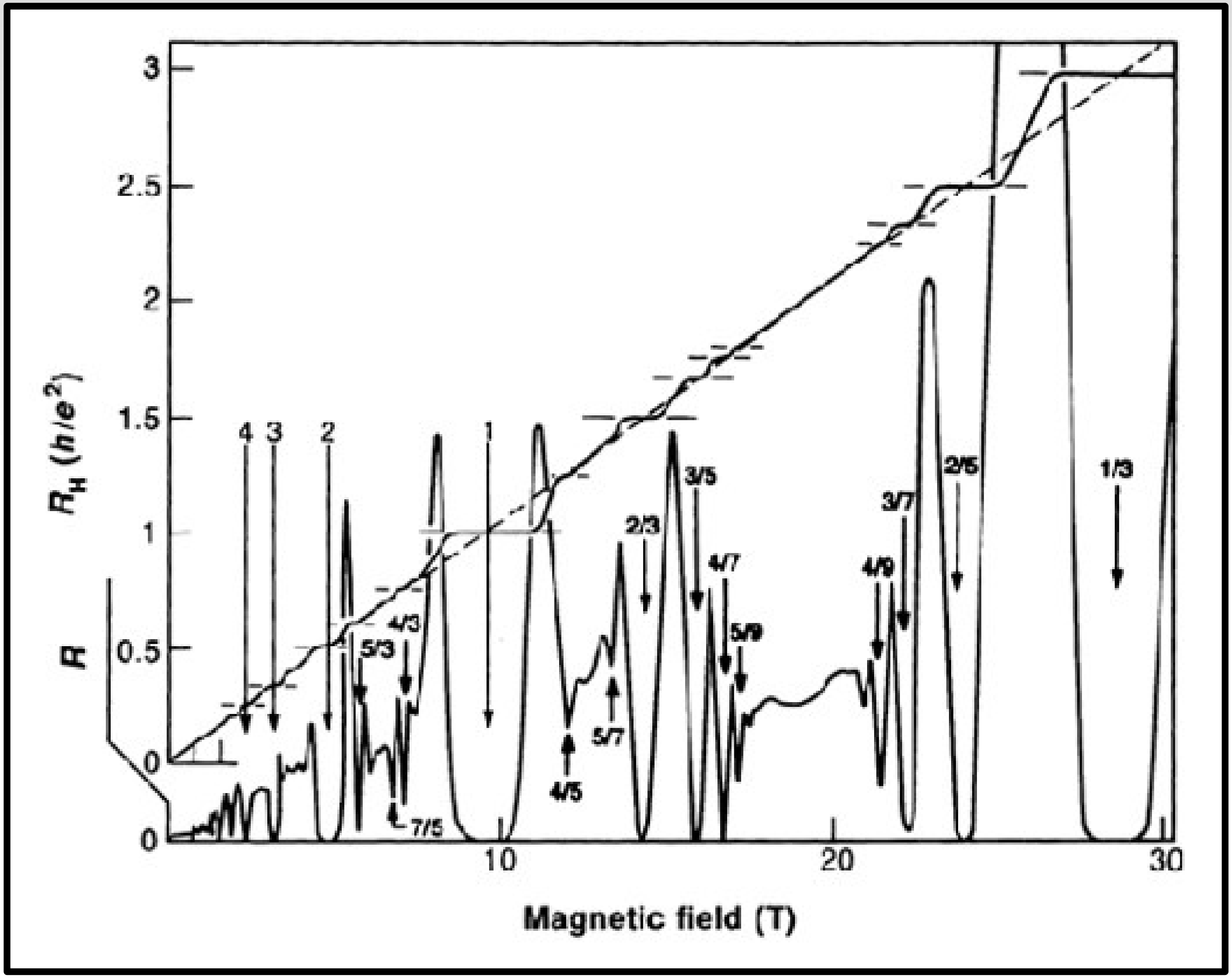


2-dimensional electron gas

Hall resistance $R_{\mathrm{H}} = R_{xy} = V_{\mathrm{H}}/I$

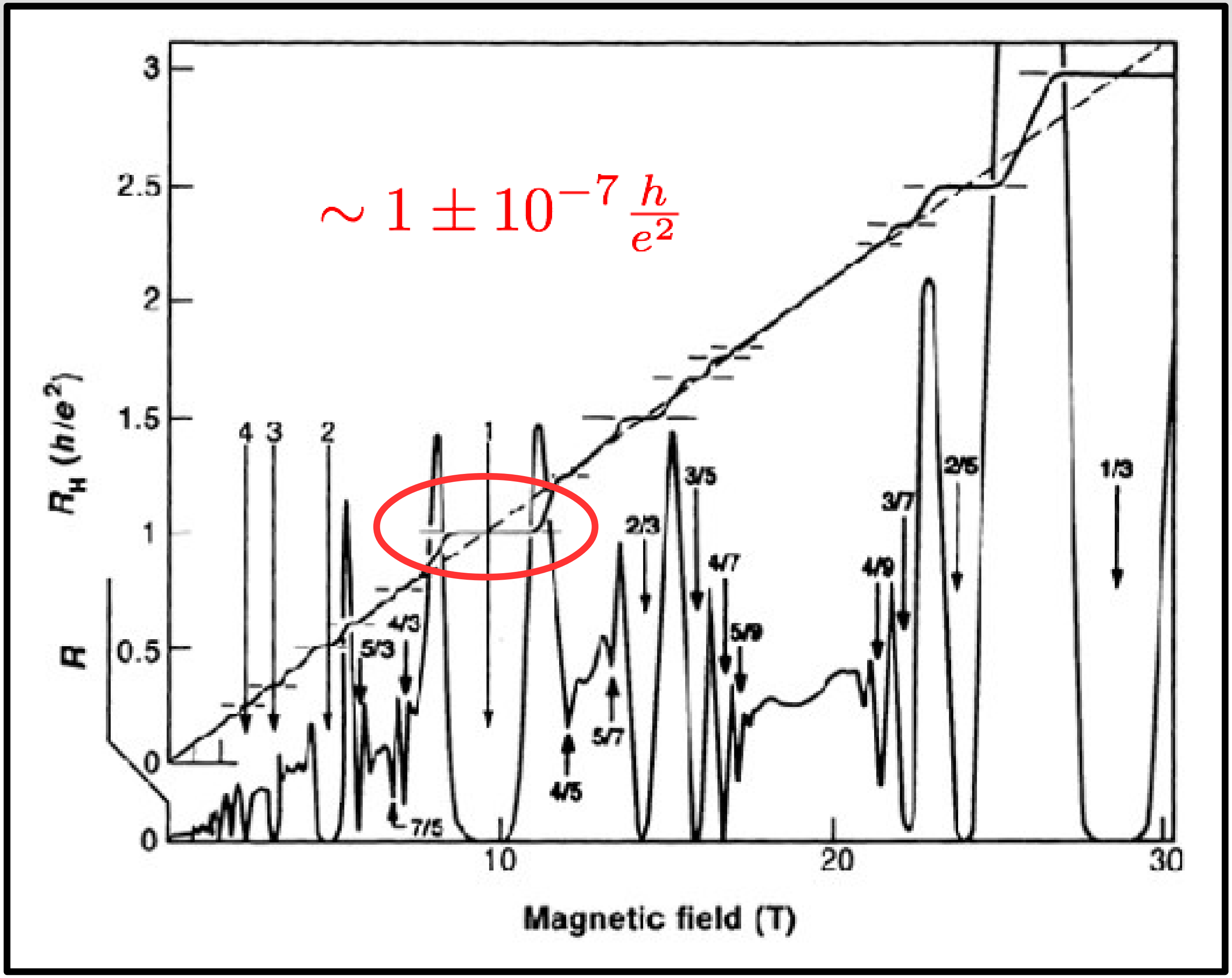
Longitudinal resistance $R_{xx} = V_x/I$

Fractional Quantum Hall



H. L. Stormer, Physica B177, 401 (1992).

Fractional Quantum Hall



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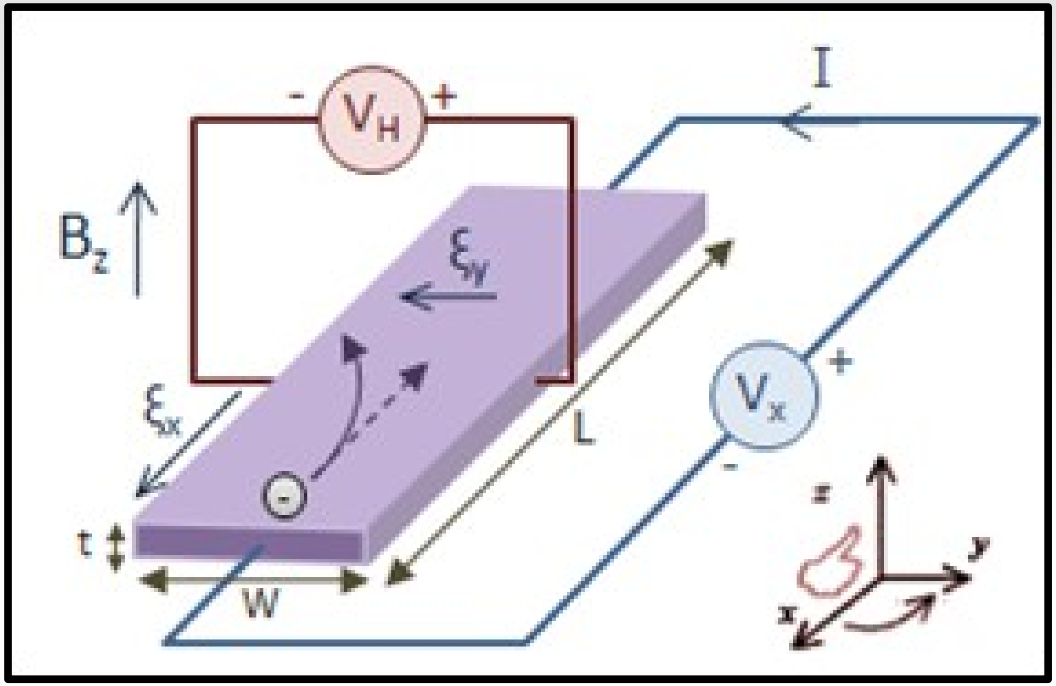
Quantum Hall



Independent from

- Geometry
- Material
- Impurities

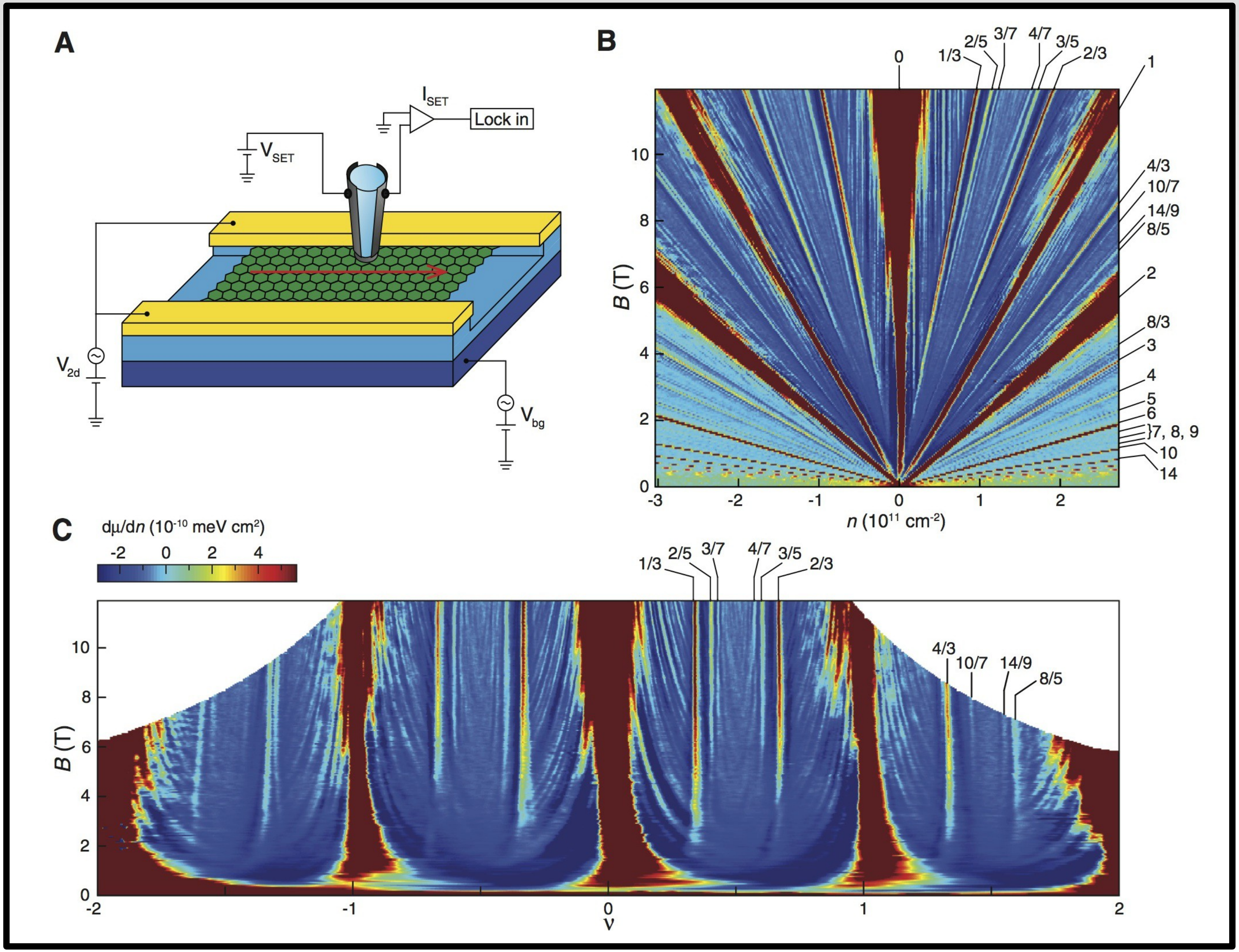
Signs of fundamental physics!

Why is 2D special?

$$\sigma = R^{-1} L^{2-D}$$

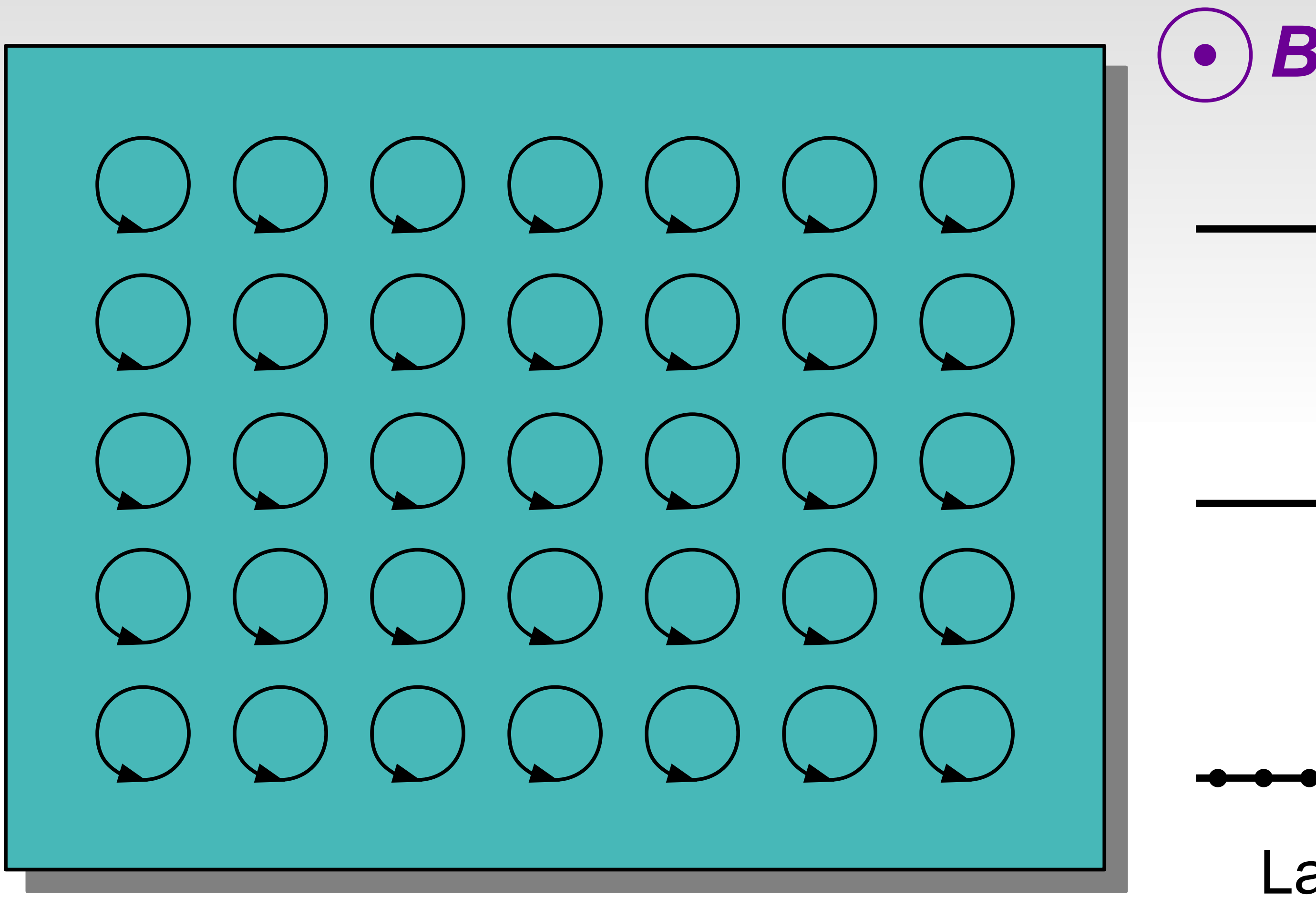
σ has units $\dfrac{e^2}{h}$

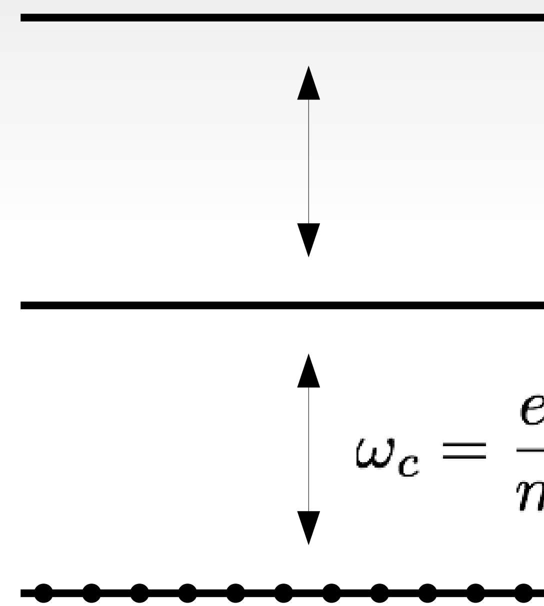
FQH in Graphene



Feldman, Krauss, Smet, Yacoby, Science 337, 1196 (2012)

Integer Quantum Hall



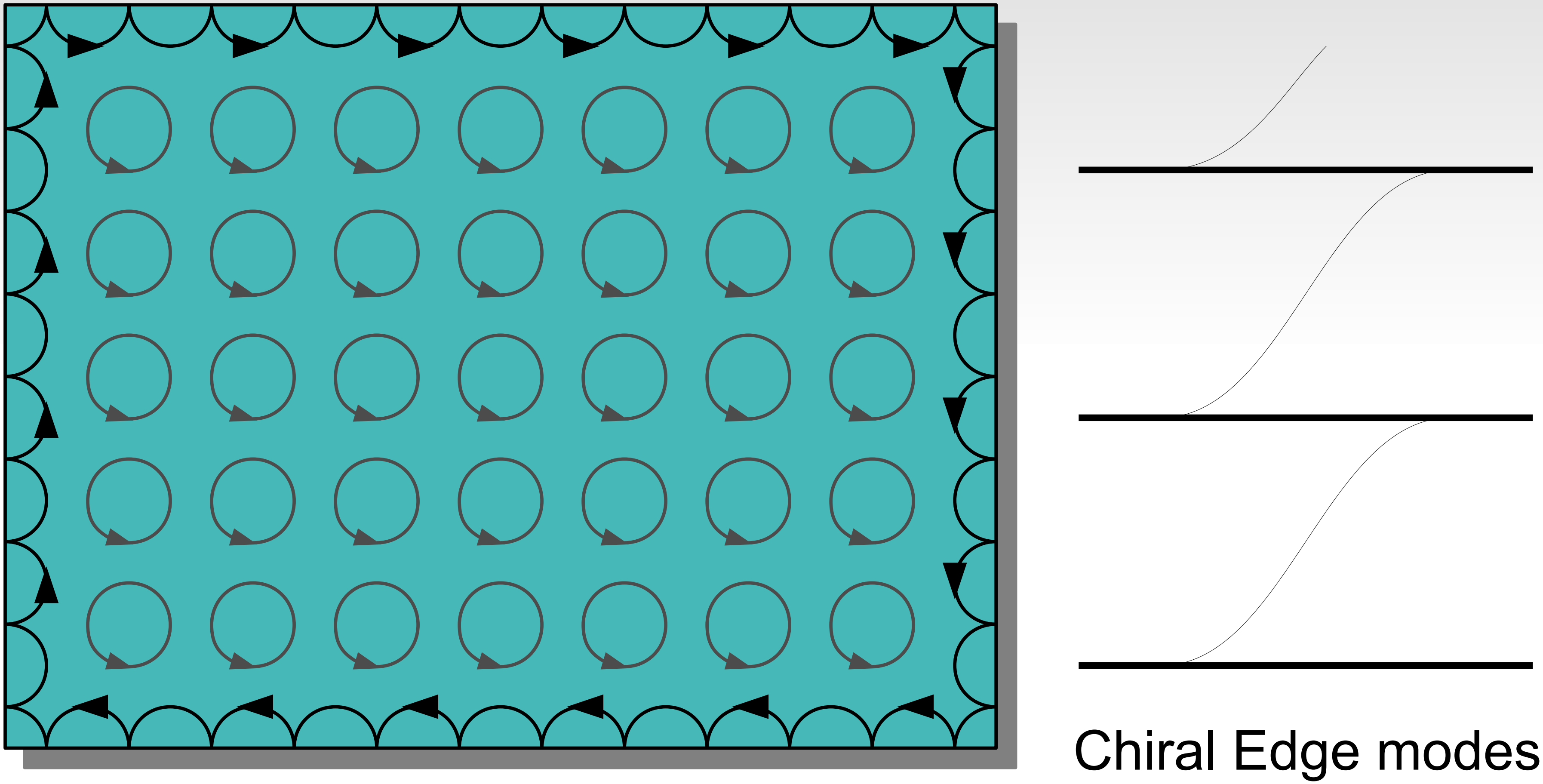


$$\omega_c = \frac{eB}{m^*}$$

Landau levels

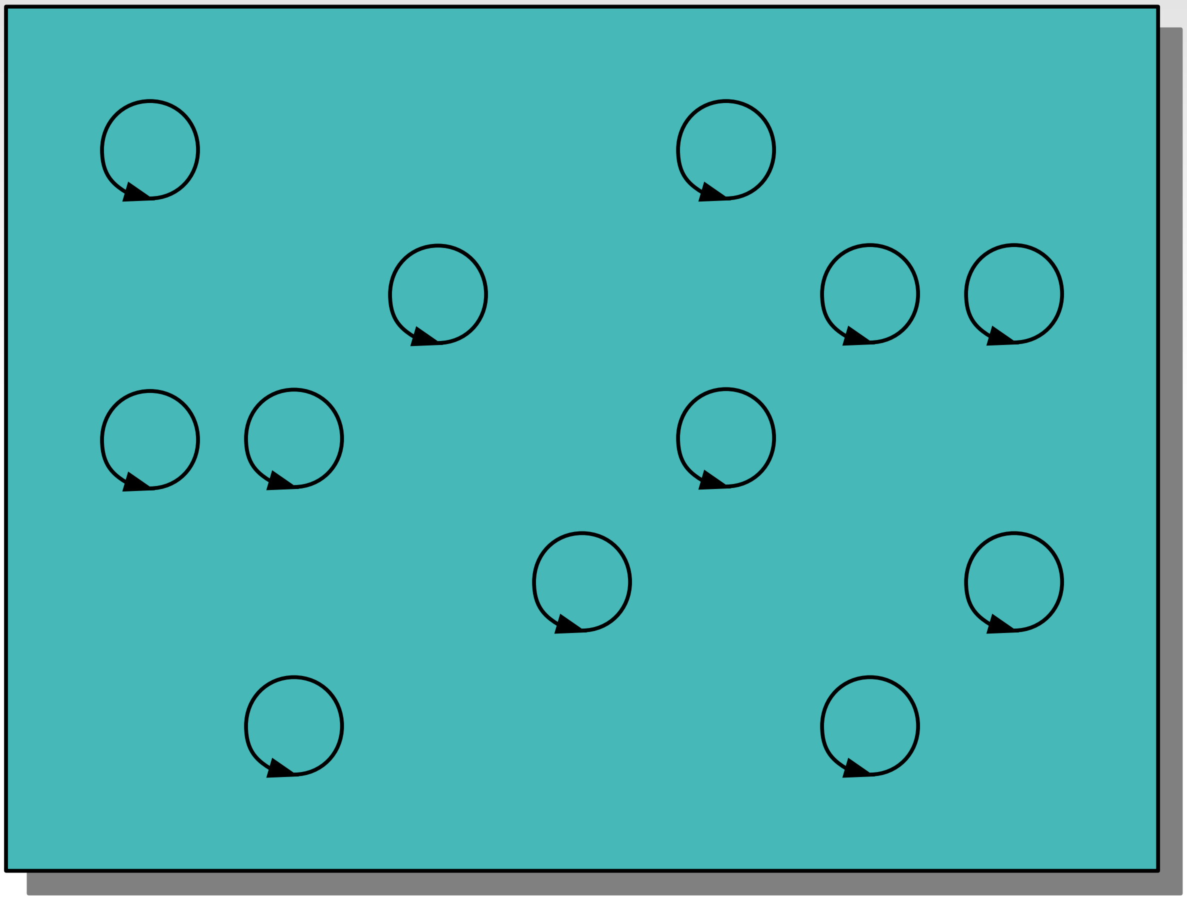
Magnetic length: $\ell_B = \sqrt{\frac{h}{eB}}$

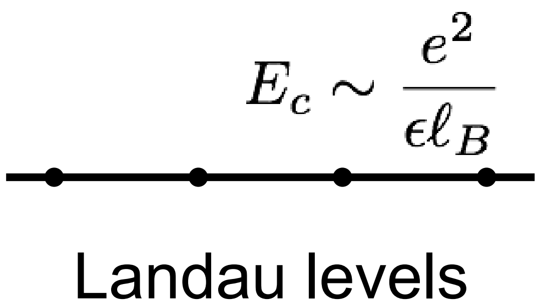
Integer Quantum Hall



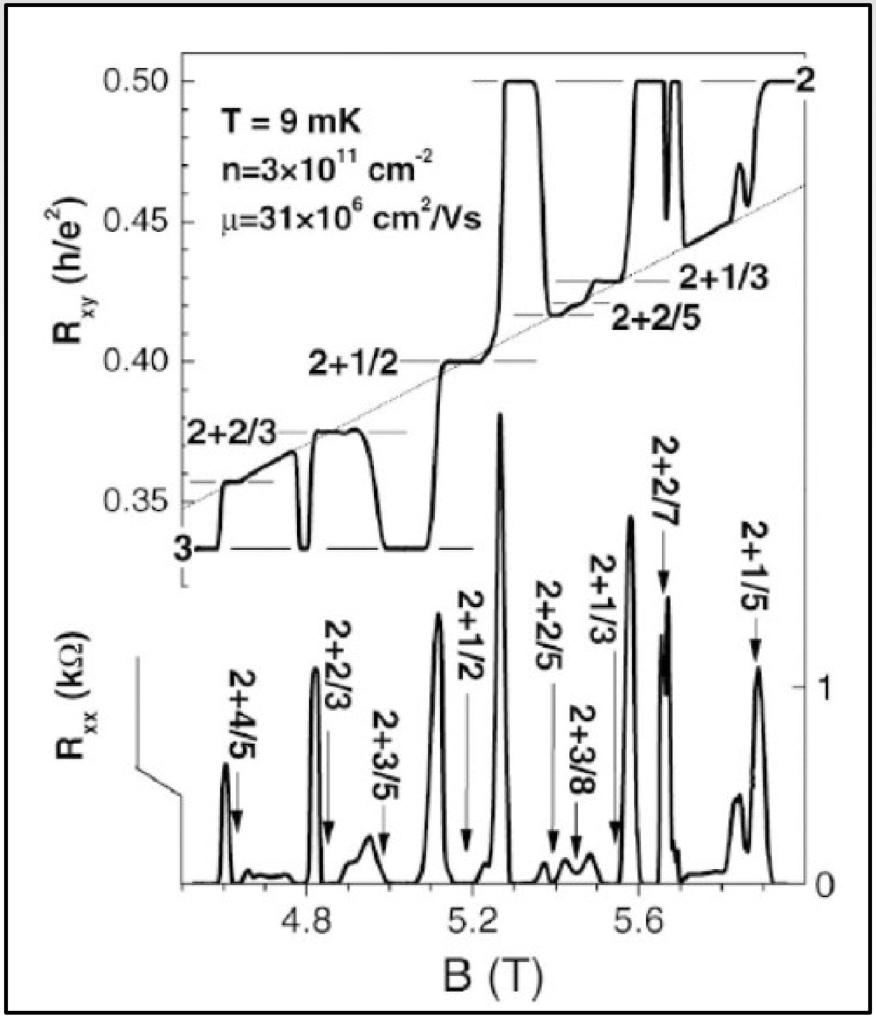
Chiral Edge modes

Fractional Quantum Hall





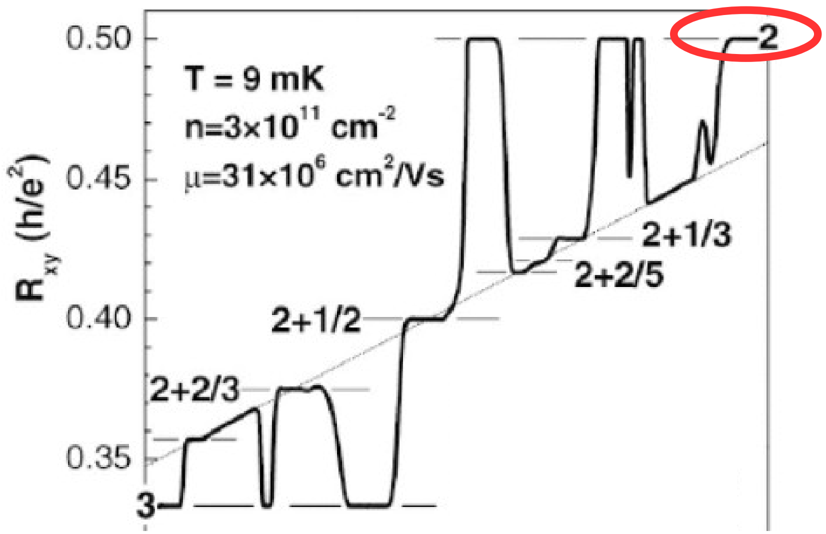
Fractional Quantum Hall



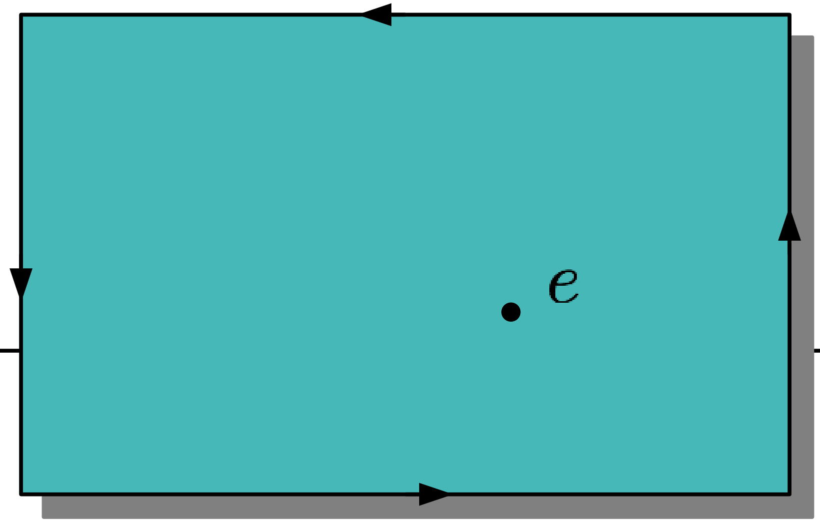
Xia et al., PRL 93, 176809

- Strongly interacting systems
- Fractionally filled Landau levels
- Emergent fractional charges
- Edge modes
- Exotic braiding statistics
- Quantum computer?

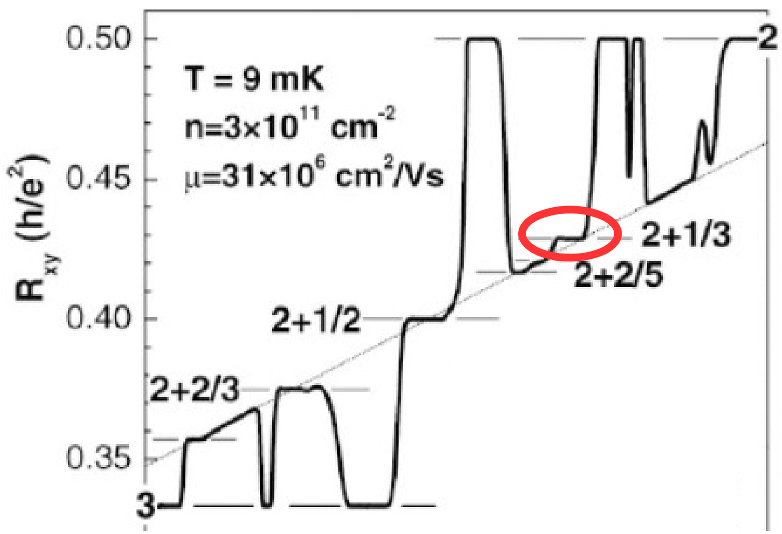
Fractional Quantum Hall



Integer quantum Hall $\nu = 2$



Fractional Quantum Hall



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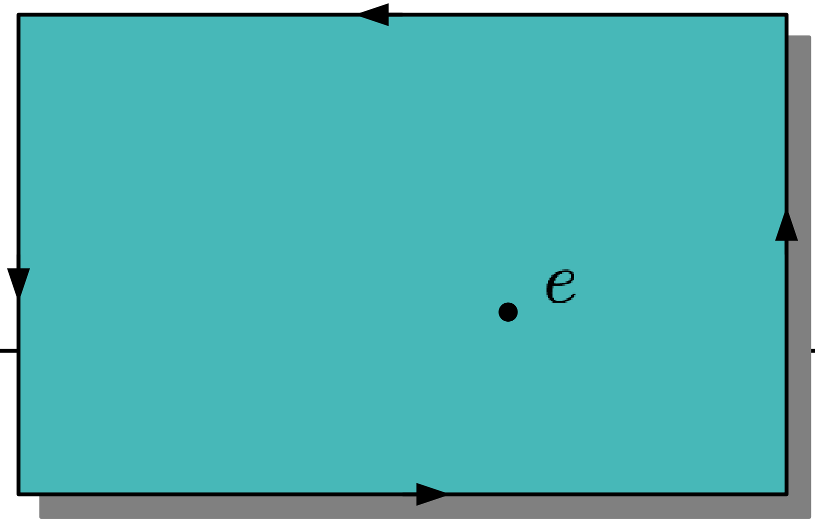
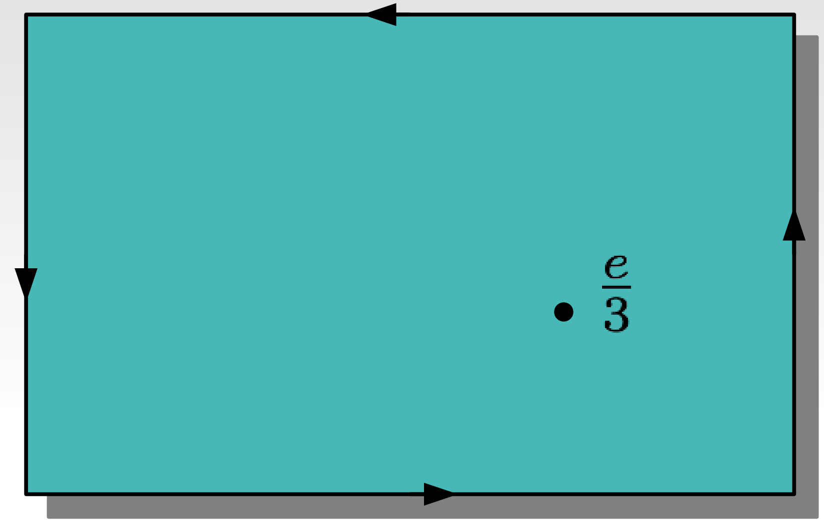
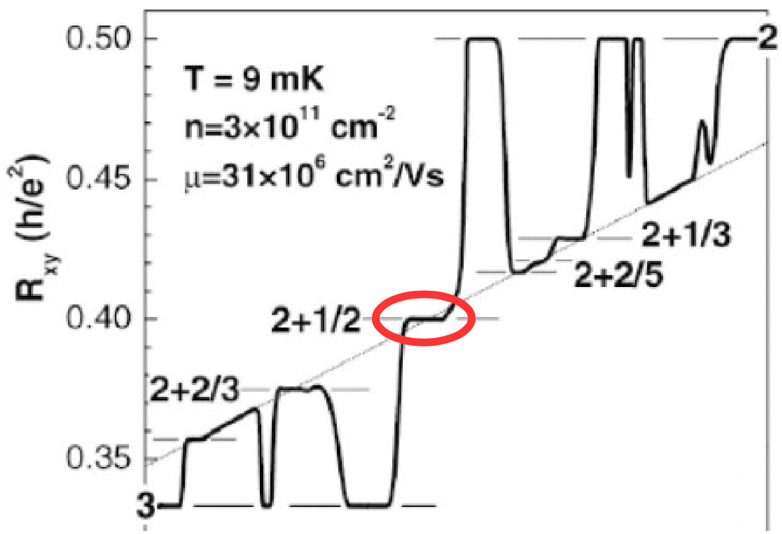


Laughlin $\nu = 2 + 1/3$



Fractional Quantum Hall



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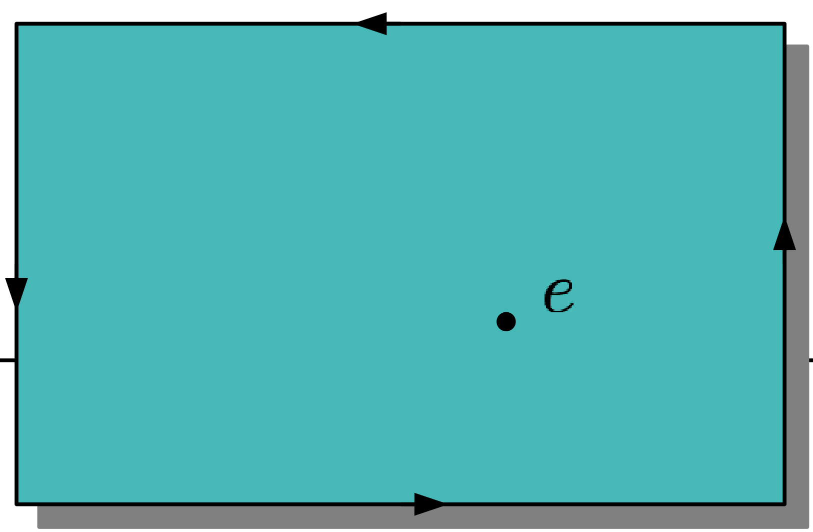

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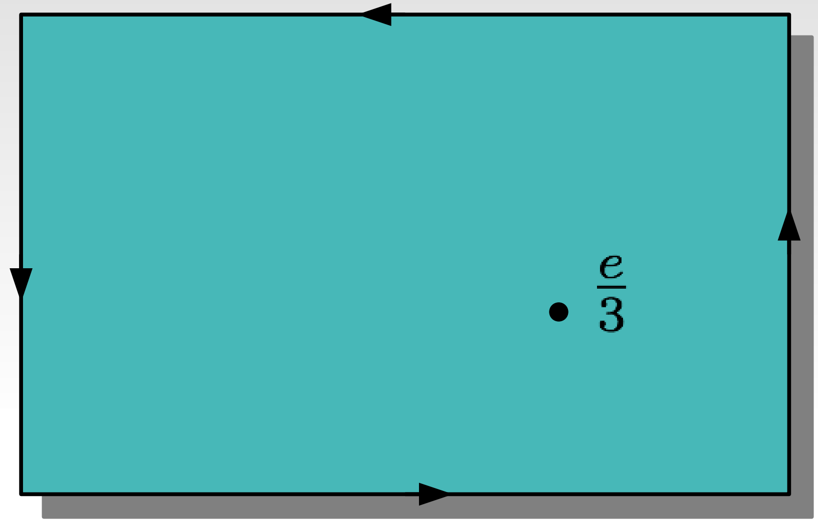
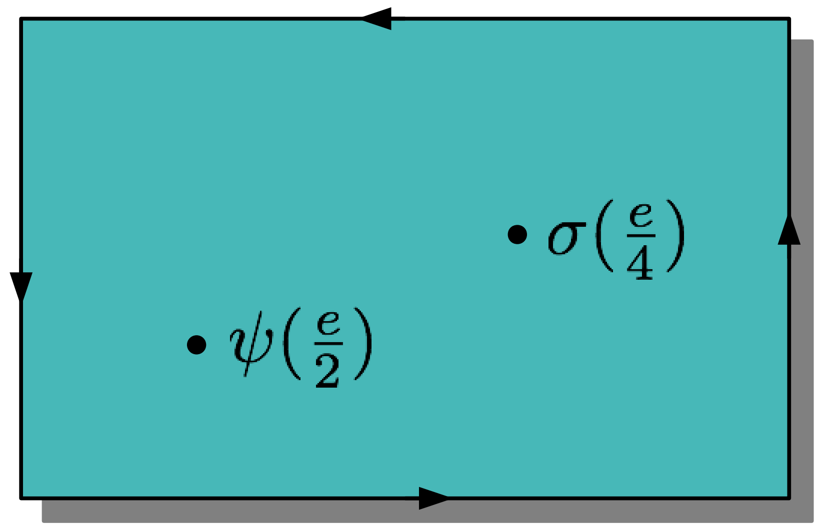
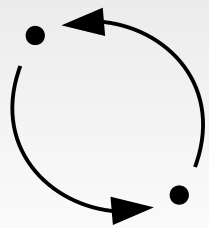


Moore-Read? $\nu = 2 + 1/2$



Braiding Statistics

Bosons



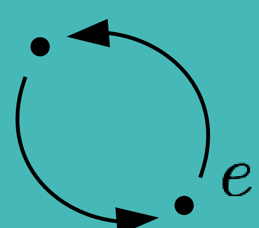
$$e^{i\theta} = +1$$

Laughlin $\nu = 2 + 1/3$

$\frac{e}{3}$

Integer quantum Hall $\nu = 2$

Fermions



$$e^{i\theta} = -1$$

Moore-Read? $\nu = 2 + 1/2$

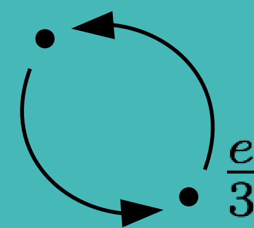
$\sigma\left(\frac{e}{4}\right)$

$\psi\left(\frac{e}{2}\right)$

Braiding Statistics

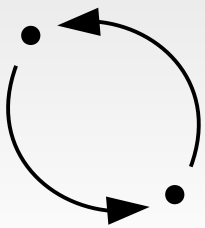
Laughlin $\nu = 2 + 1/3$

Abelian Anyons



$$e^{i\theta} = e^{i\pi/3}$$

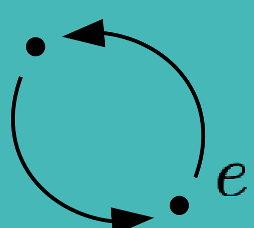
Bosons



$$e^{i\theta} = +1$$

Integer quantum Hall $\nu = 2$

Fermions



$$e^{i\theta} = -1$$

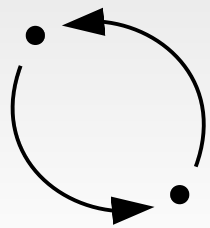
Moore-Read? $\nu = 2 + 1/2$

• $\sigma(\frac{e}{4})$

• $\psi(\frac{e}{2})$

Braiding Statistics

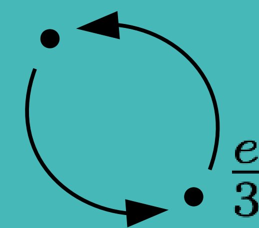
Bosons



$$e^{i\theta} = +1$$

Laughlin $\nu = 2 + 1/3$

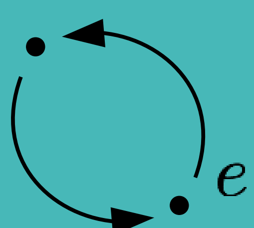
Abelian Anyons



$$e^{i\theta} = e^{i\pi/3}$$

Integer quantum Hall $\nu = 2$

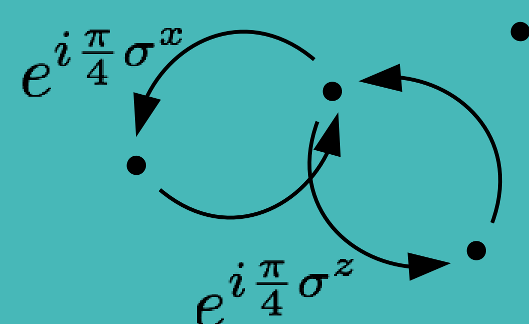
Fermions



$$e^{i\theta} = -1$$

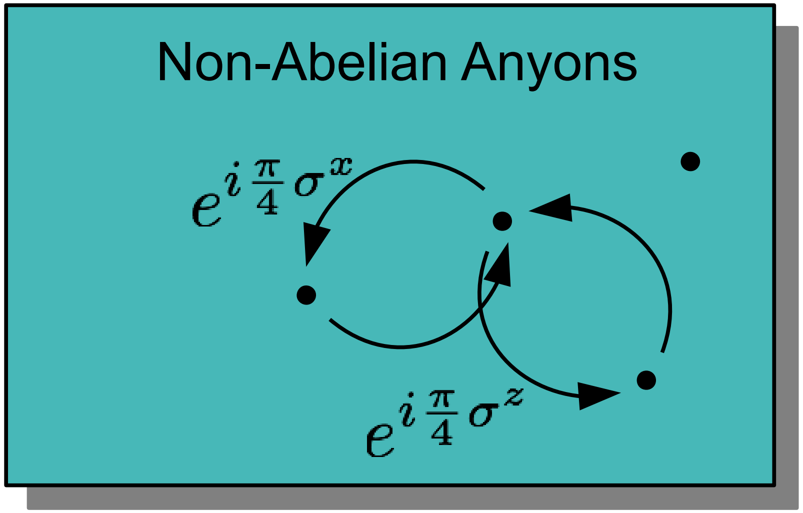
Moore-Read? $\nu = 2 + 1/2$

Non-Abelian Anyons

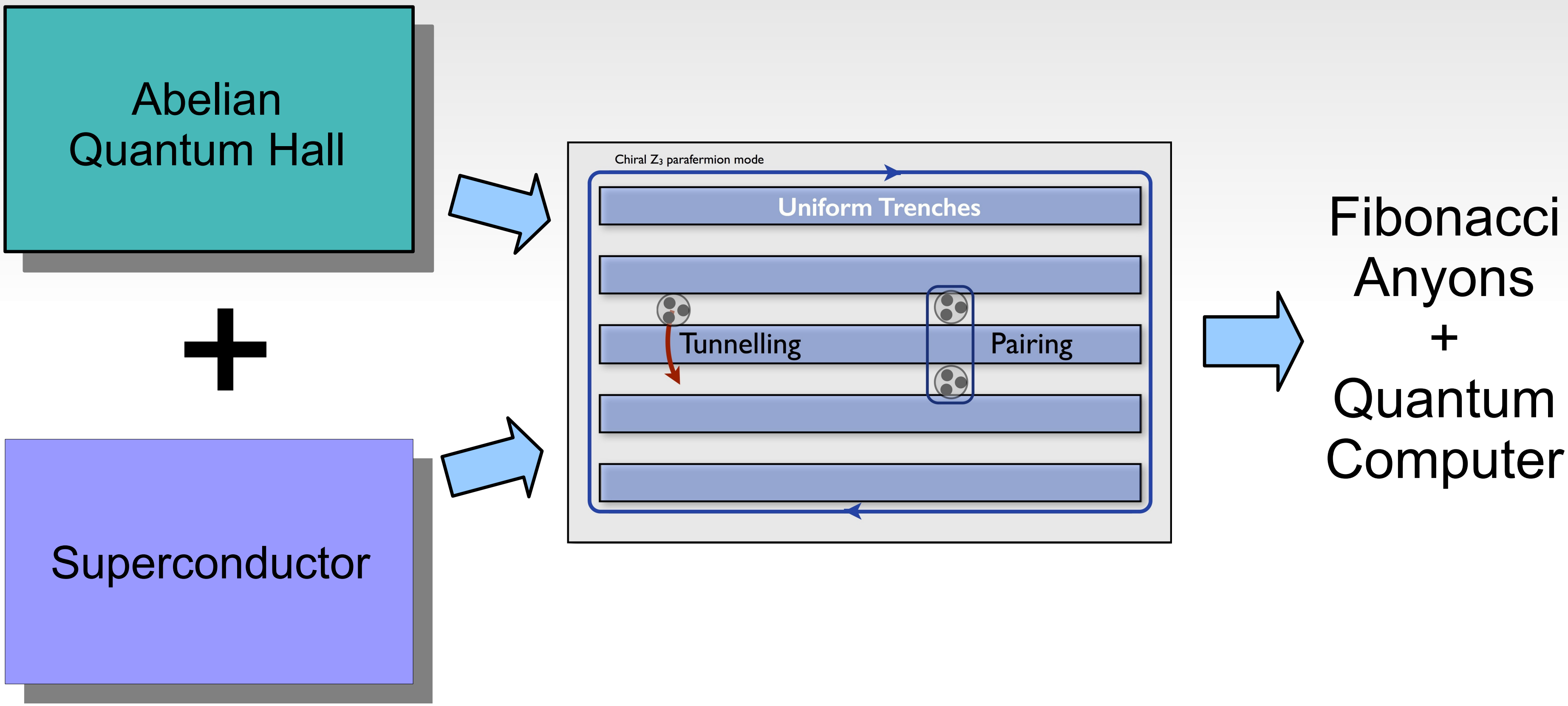


Topological Order

- Excitation degeneracy $\propto (\sqrt{2})^{\#\text{ of anyons}}$
  $\rightarrow$ Quantum dimension of the anyon $= \sqrt{2}$
  [Moore, Read 1990; Nayak, Wilczek 1996]
- Non-commuting operations from braiding
- Fault-tolerant topological quantum computer
  [Kitaev 2003]
  - Qubits stored non-locally
  - Operators from braiding



Universal Topological Quantum Computing?



RM, D. Clarke, J. Alicea, N. Lindner, P. Fendley, C. Nayak, Y. Oreg, A. Stern, E. Berg, K. Shtengel, M.P.A. Fisher, arXiv:1307.4403. (to appear in **PRX**)

Fractional Quantum Hall

- Archetypal 'topological phases'
- Strongly interacting systems
- Emergent fractional charges
- Exotic Abelian/non-Abelian braiding statistics
- Excitation spectrum degeneracy
- Candidate platform for topological quantum computing

How to study these phases?

Model Wavefunction

vs.

Numerical Study

How to study these phases?

Model Wavefunction

vs.

Numerical Study

Ferromagnet

$$|\psi\rangle = |\cdots \uparrow\uparrow\uparrow\uparrow\uparrow \cdots\rangle$$

Excitation: $|\cdots \uparrow\uparrow\downarrow\uparrow\uparrow \cdots\rangle$

Model wavefunctions

- Simplest construction for a phase
- Ground state of (fine-tuned) a parent Hamiltonian
- Captures the essential topological aspects
  - Anyon content (charge, spin, braid statistics)
- Clean edge spectrum
- Analytical structure
  - Conformal field theory, topological quantum field theory

Example: Laughlin wavefunction

$$\psi(z_1, \ldots, z_N) \propto \prod_{a<b} (z_a - z_b)^3 \cdot e^{-\frac{1}{4}\sum_a |z_a|^2}$$

Complex coordinate: $z = x + iy$

Numerical Study

Compute

- Energetics
- Non-topological excitations
- Edge reconstruction

Numerical methods

- Exact diagonalization limited by the number of electrons (~24)

Numerical Study

Compute

- Energetics
- Non-topological excitations
- Edge reconstruction

Numerical methods

- Exact diagonalization limited by the number of electrons (~24)
- Use matrix product states (MPSs) representation and the density-matrix-renormalization-group (DMRG) methods!

MPSs stores the entanglement within a wavefunction

Exact Diag. vs MPS/DMRG

	Maximum size via exact diagonalization	Maximum size via MPS/DMRG
One-Dimensions		
Spin 1 chain (Heisenberg)	~ 24	∞
Spin ½ chain (Critical Ising)	~ 35	$> 10^4$
Two-Dimensions		
Quantum Hall (1/3 filling)	$20\ell_B \times 20\ell_B$	$35\ell_B \times \infty$
Quantum Hall (5/2 filling)	$10\ell_B \times 10\ell_B$	$22\ell_B \times \infty$

Entanglement is only sensitive to local fluctuations

Extract Topological Data

$|\psi\rangle \Rightarrow$ Fractional charges, braiding properties

What are (numerical) signatures of a topological phase?

Extract Topological Data

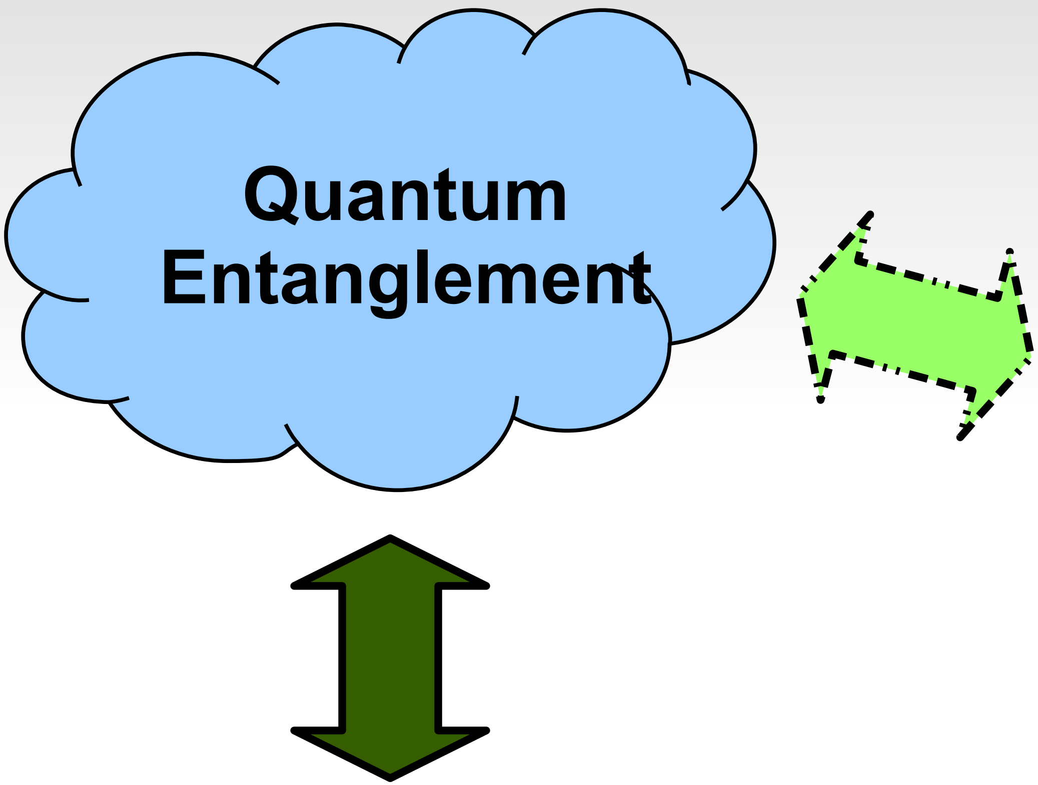
$|\psi\rangle \Longrightarrow$ Fractional charges, braiding properties

What are (numerical) signatures of a topological phase?

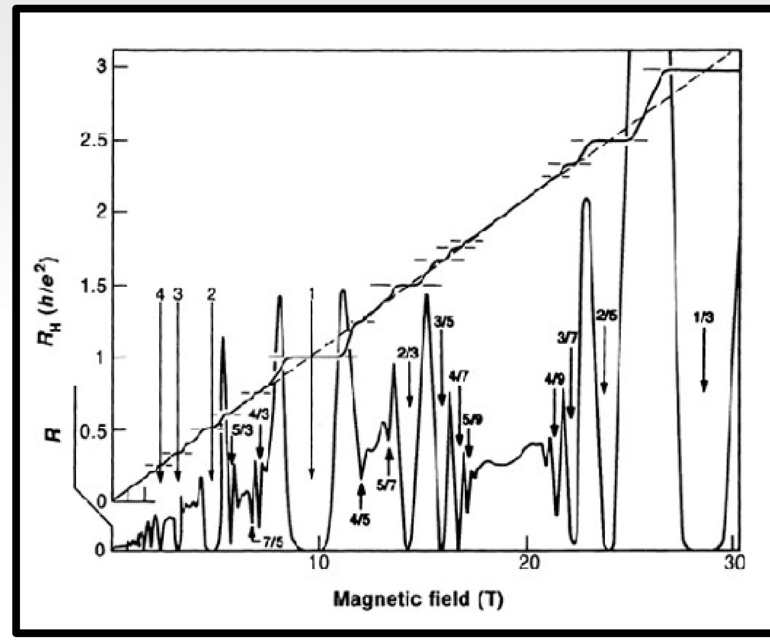
Via entanglement of the ground state wavefunction

- Quantum dimensions
  [Kitaev, Preskill 2006; Levin, Wen 2006]
- Edge spectrum
  [Li, Haldane 2008]
- Anyon spin, braiding statistics, chiral central charge
  [Zhang, Grover, Turner, Oshikawa, Vishwanath 2012;
  Zaletel, RM, Pollmann 2013; Tu, Zhang, Qi 2013]

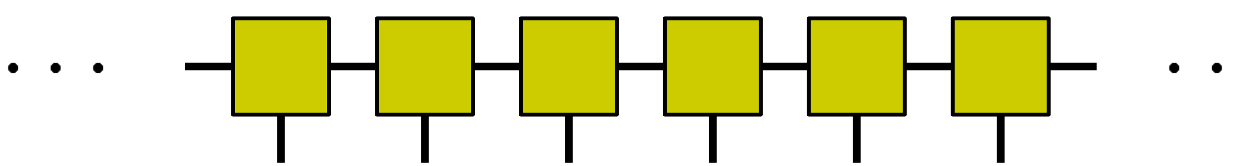
FQH, Entanglement, MPS



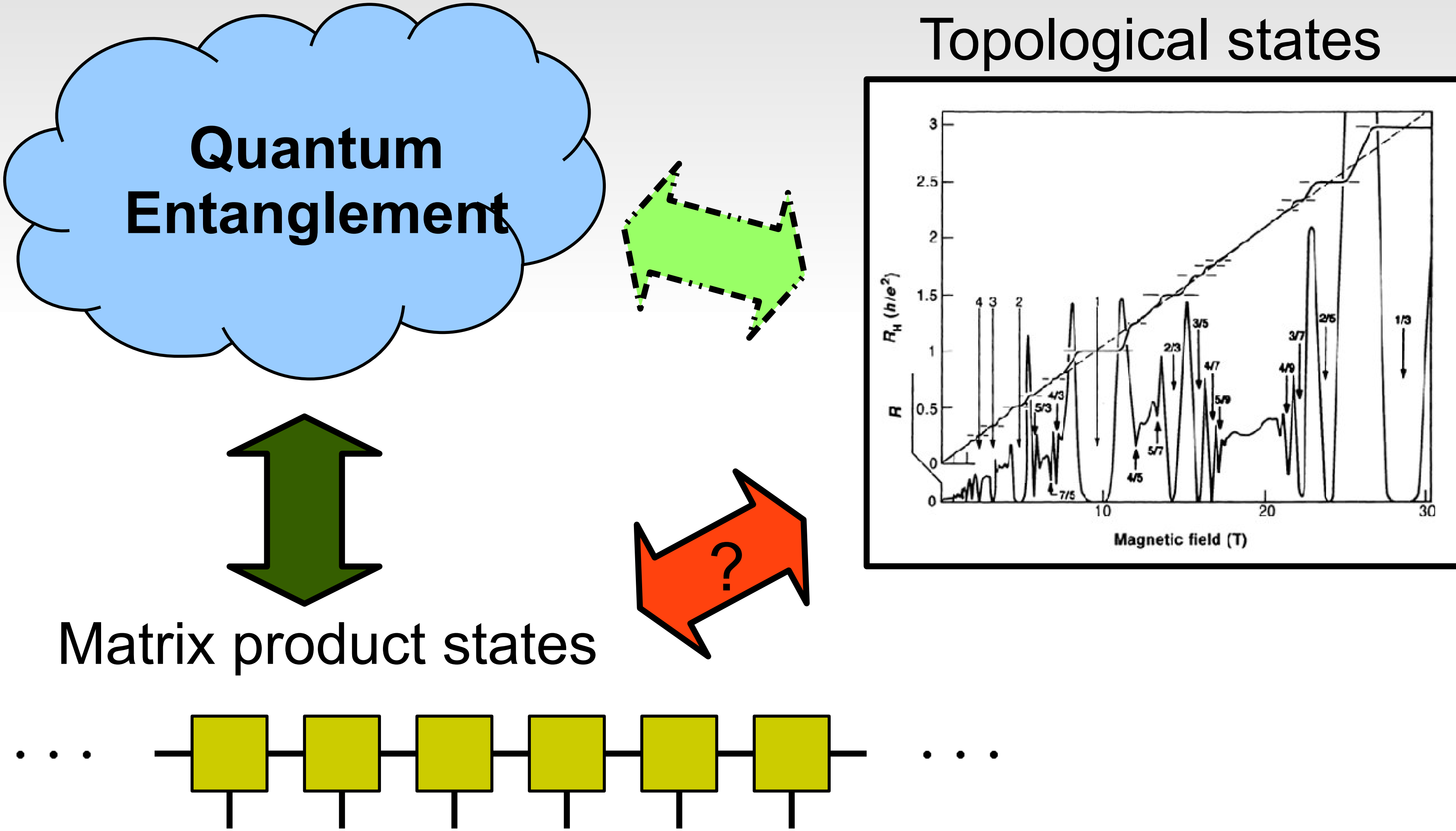
Topological states



Matrix product states



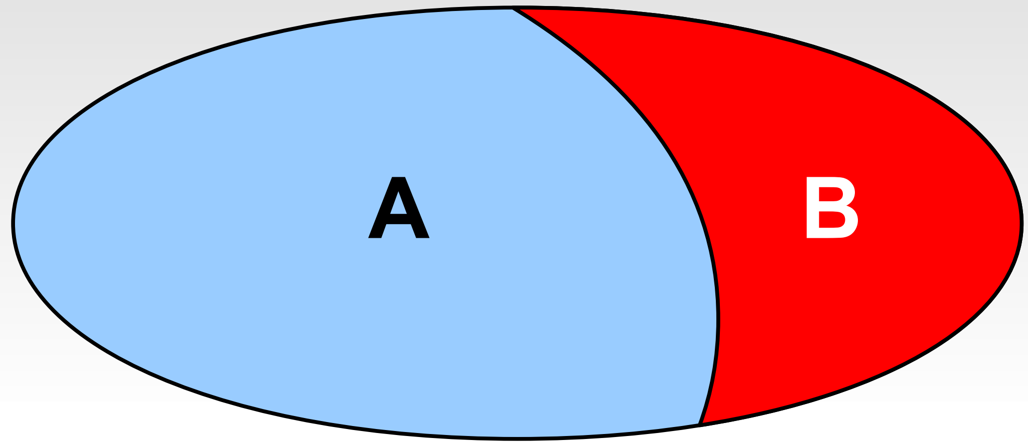
FQH, Entanglement, MPS



Entanglement

Bipartition

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$



$$|\Psi\rangle = \sum_{\alpha} \lambda_\alpha \, |\varphi_\alpha\rangle^A \otimes |\phi_\alpha\rangle^B$$

$|\Psi\rangle \in \mathcal{H}$

$\lambda_\alpha > 0$

$|\varphi_\alpha\rangle \in \mathcal{H}_A$

$|\phi_\beta\rangle \in \mathcal{H}_B$

Entanglement

Schmidt decomposition

$$|\Psi\rangle = \sum_{\alpha} \lambda_{\alpha} |\varphi_{\alpha}\rangle^{A} \otimes |\phi_{\alpha}\rangle^{B}$$

Entanglement spectrum

$$\lambda_{\alpha} = e^{-\frac{1}{2}\tilde{E}_{\alpha}}$$

Entanglement entropy

$$S = -\sum_{\alpha} \lambda_{\alpha}^{2} \log(\lambda_{\alpha}^{2})$$

Entanglement

Examples

- Product state $S = 0$

$$|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle$$

Entanglement

Examples

- Product state

$$|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle$$

$$S = 0$$

- Spin singlet

$$S = \log 2$$

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}} |\uparrow\rangle \otimes |\downarrow\rangle + \frac{1}{\sqrt{2}} |\downarrow\rangle \otimes (-|\uparrow\rangle)$$

Entanglement

Examples

- Product state $S = 0$

$$|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle$$

- Spin singlet $S = \log 2$

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}|\uparrow\rangle \otimes |\downarrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \otimes (-|\uparrow\rangle)$$

- Sum of all combinations

$$\frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Entanglement

Examples

- Product state $S = 0$

$$|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle$$

- Spin singlet $S = \log 2$

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}|\uparrow\rangle \otimes |\downarrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \otimes (-|\uparrow\rangle)$$

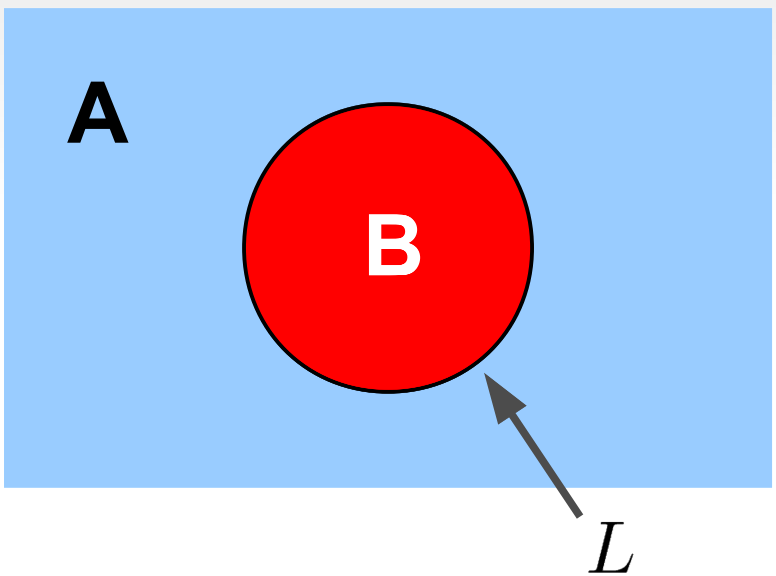
- Sum of all combinations $S = 0$

$$\frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$= \left[\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right] \otimes \left[\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right]$$

Entanglement Entropy

'Area Law': $S = aL - \gamma + \ldots$



Topological entanglement entropy

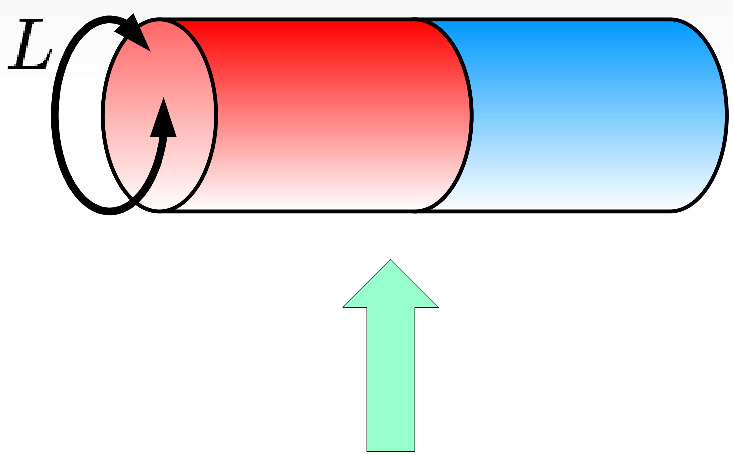
$$2\gamma = \log \sum_{\text{qp types } a} d_a^2$$

(e.g. $d = 1$ per Abelian qp, $d = \sqrt{2}$ per Majorana species)

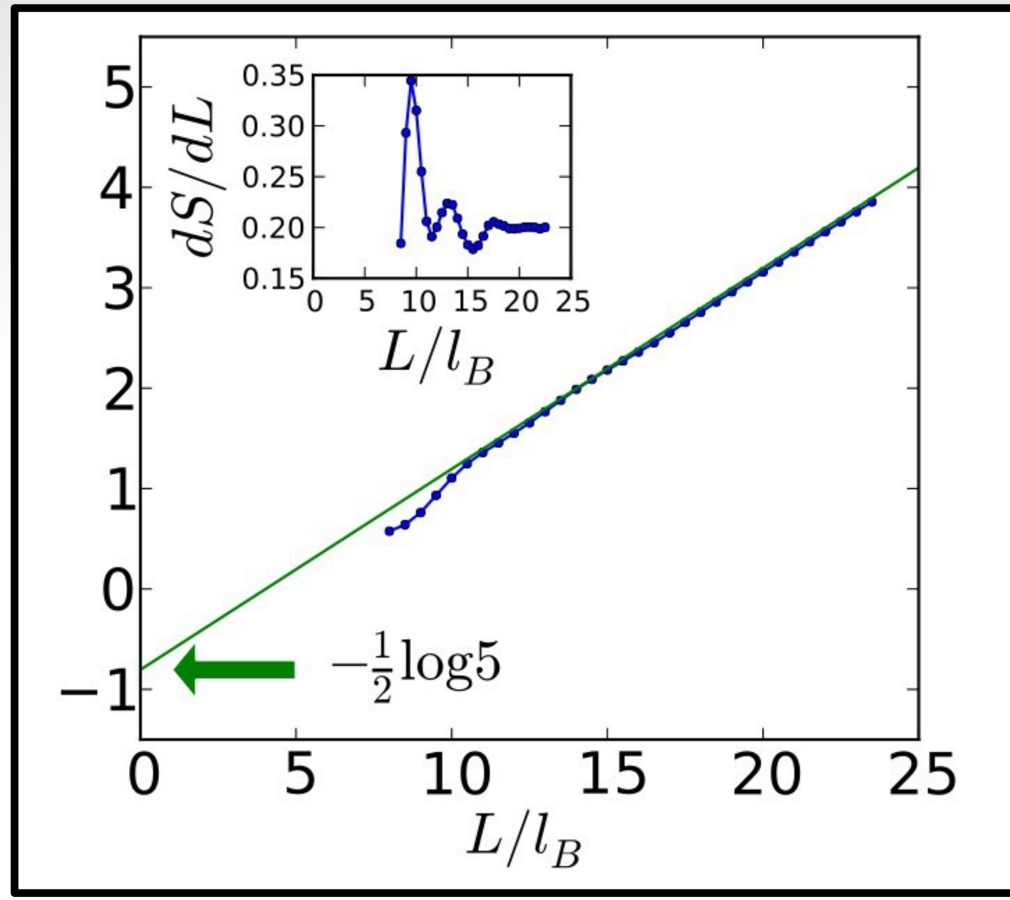
Kitaev, Preskill, PRL 96, 110404 (2006)
Levin, Wen, PRL 96, 110405 (2006)

Entanglement Entropy

$$S = aL - \gamma + \ldots$$



$$\nu = 2/5$$



Michael Zaletel, RM, Frank Pollmann 2013

Entanglement Spectrum

Spectrum: $\tilde{E}_\alpha = -\log \lambda_\alpha^2$

λ_α^2 are the eigenvalues of the density matrix ρ

ρ commutes with the symmetries of the system

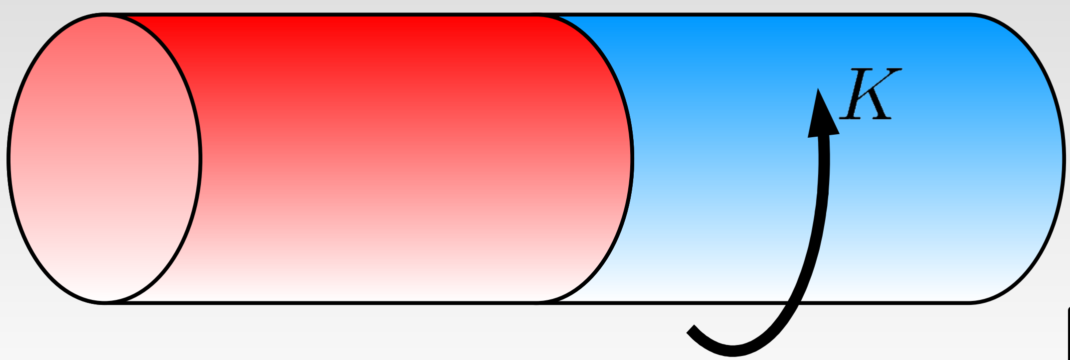
$\Rightarrow$ Each entanglement energy / Schmidt value has definitive charge

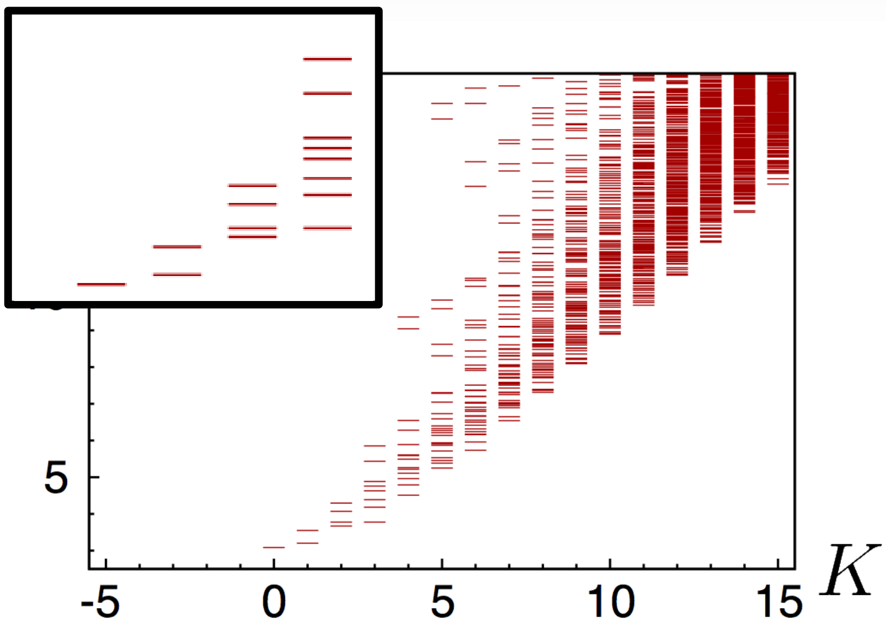
Examples: charge, momentum, spin,... etc.

The entanglement spectrum of the ground state encodes universal topological characteristics!

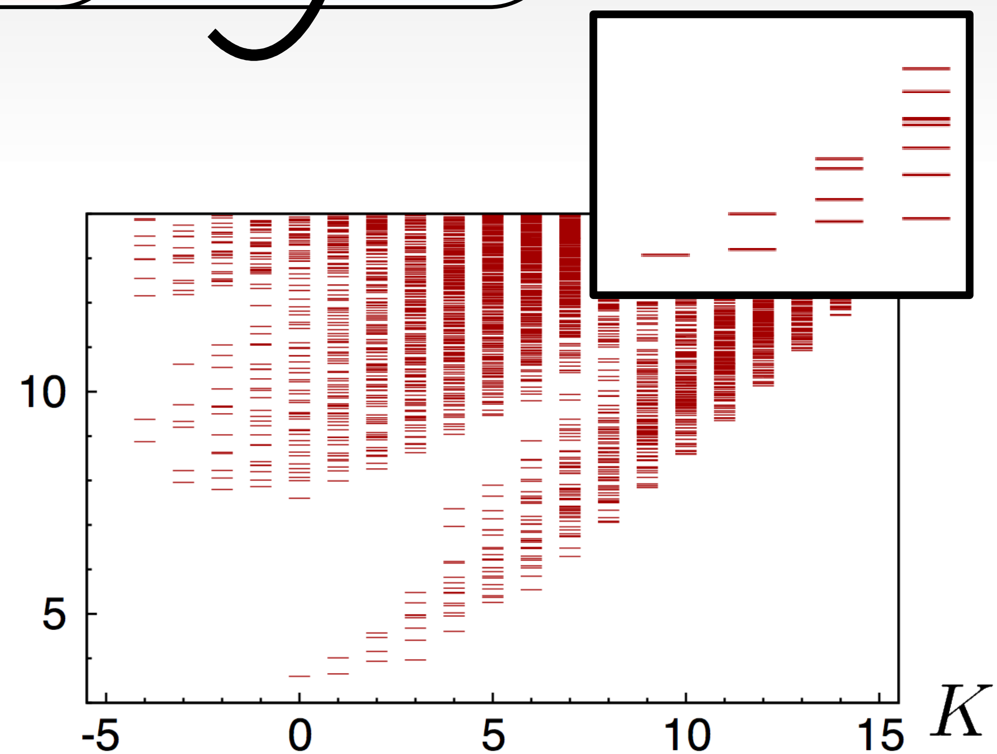
Li, Haldane, PRL 101, 010504 (2008)

Quantum Hall Entanglement





Model wavefunction

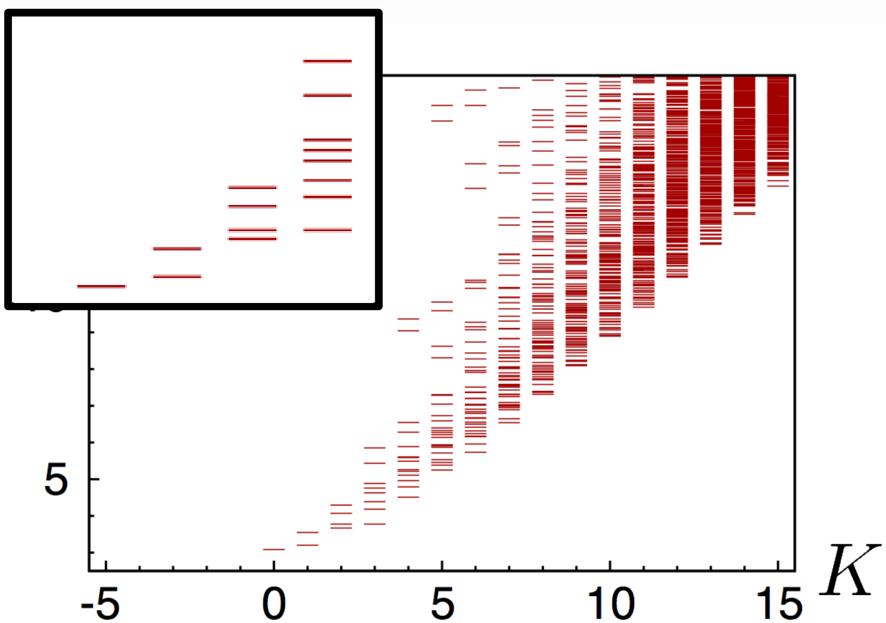


Wavefunction from Hamiltonian

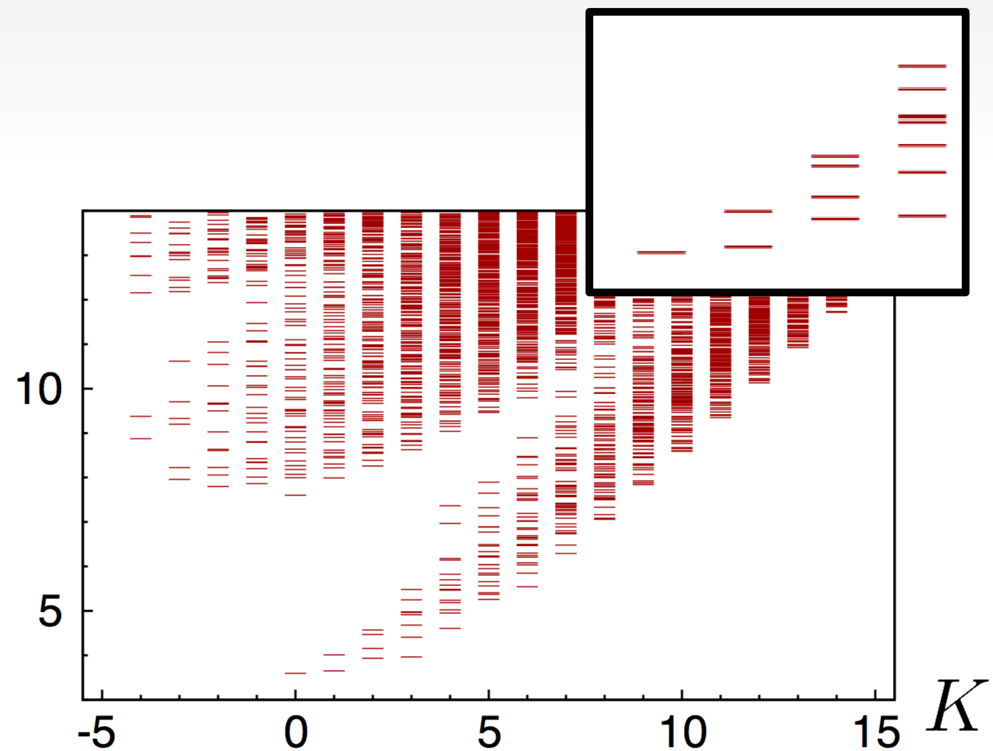
Quantum Hall Entanglement

The low entanglement energy states are in one-to-one correspondence with the (physical) edge theory of the quantum Hall phase!

Counting: 1, 2, 4, 8, 14, ...



Model wavefunction



Wavefunction from Hamiltonian

Entanglement

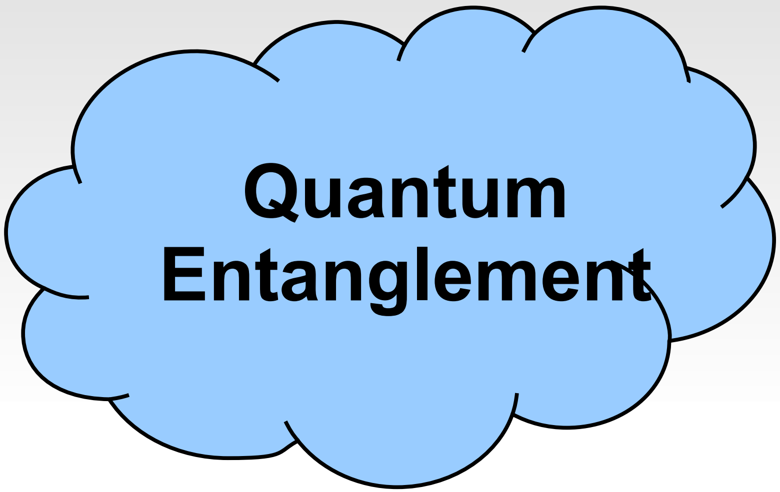
Extract topological nature from ground state wavefunction(s)

- Topological entanglement entropy
- Entanglement spectrum

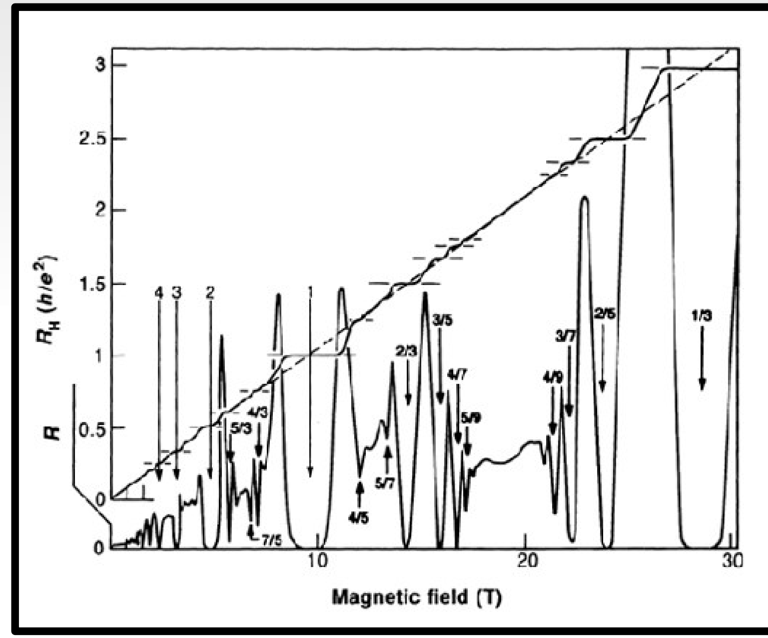
Use entanglement to efficiently represent many-body wavefunctions

- Matrix Product States

FQH, Entanglement, MPS



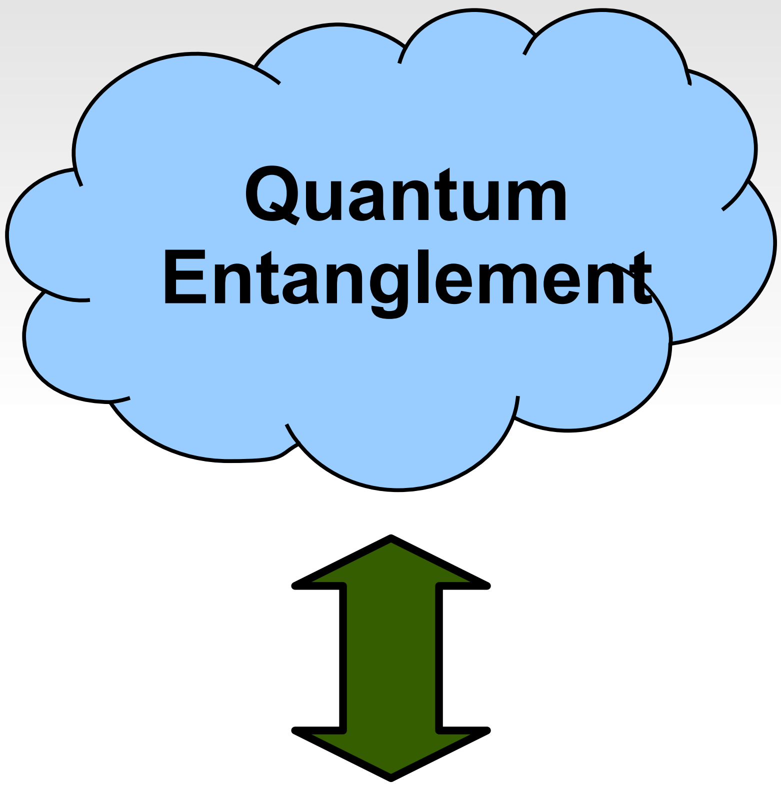
Topological states



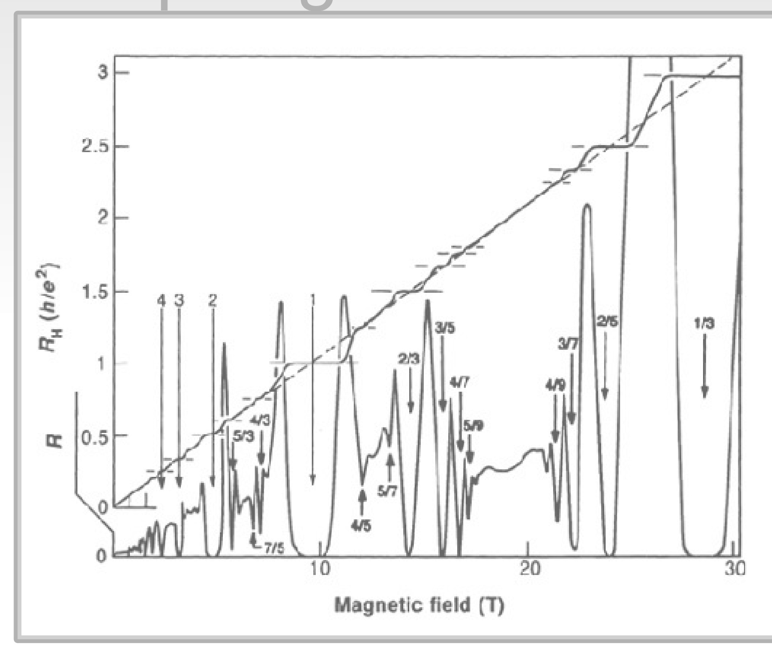
- Topological entanglement entropy
- Entanglement spectrum / Edge correspondence

A. Kitaev, J. Preskill, PRL 96, 110404 (2006)
M. Levin, X.-G. Wen, PRL 96, 110405 (2006)
H. Li, F. D. M. Haldane, Phys. Rev. Lett. 101, 010504 (2008)

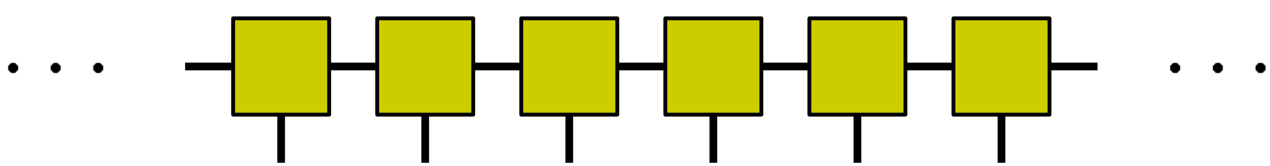
FQH, Entanglement, MPS



Topological states



Matrix product states



Wavefunction Representation

Exact diagonalization:

⇨ Hilbert space dimension $\approx d^N$

Example: Spin 2 chain (in 1D) $(d = 2S + 1 = 5)$

$$S = 2, N = 25 \longrightarrow 10^{14} \text{ numbers!}$$

Wavefunction Representation

Exact diagonalization:

Hilbert space dimension $\approx d^N$

Example: Spin 2 chain (in 1D) $(d = 2S + 1 = 5)$

$$S = 2, N = 25 \longrightarrow 10^{14} \text{ numbers!}$$

MPS representation

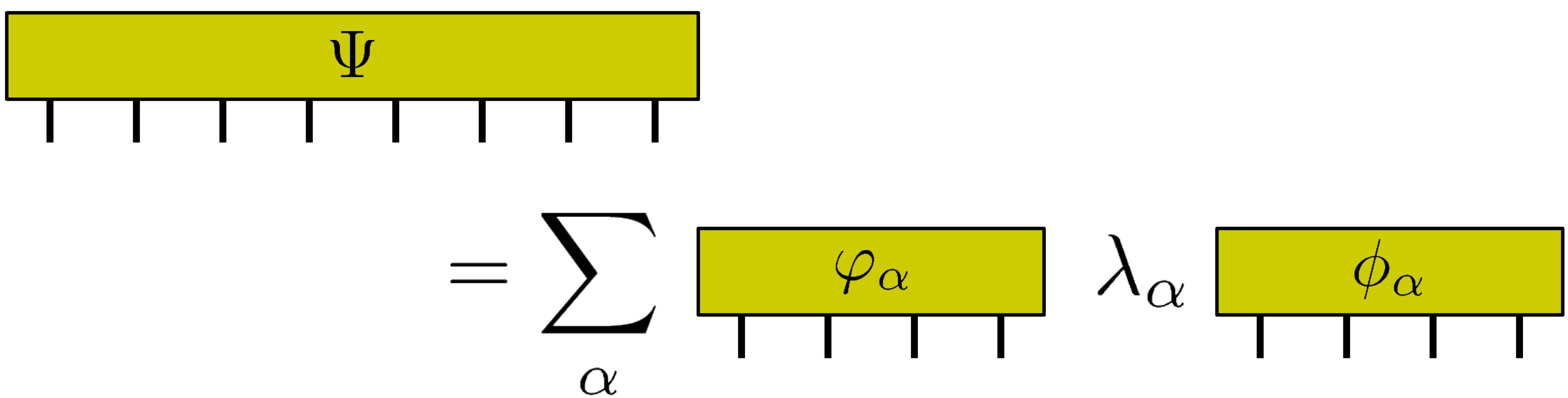
Scales with entanglement $\approx \mathcal{O}(1)^{\text{Entanglement}}$

$$S = 2 \longrightarrow 10^{6} \text{ numbers!}$$

Schmidt Decomposition

Schmidt decomposition

$$|\Psi\rangle = \sum_{\alpha} \lambda_{\alpha} \, |\varphi_{\alpha}\rangle^{A} \otimes |\phi_{\alpha}\rangle^{B}$$

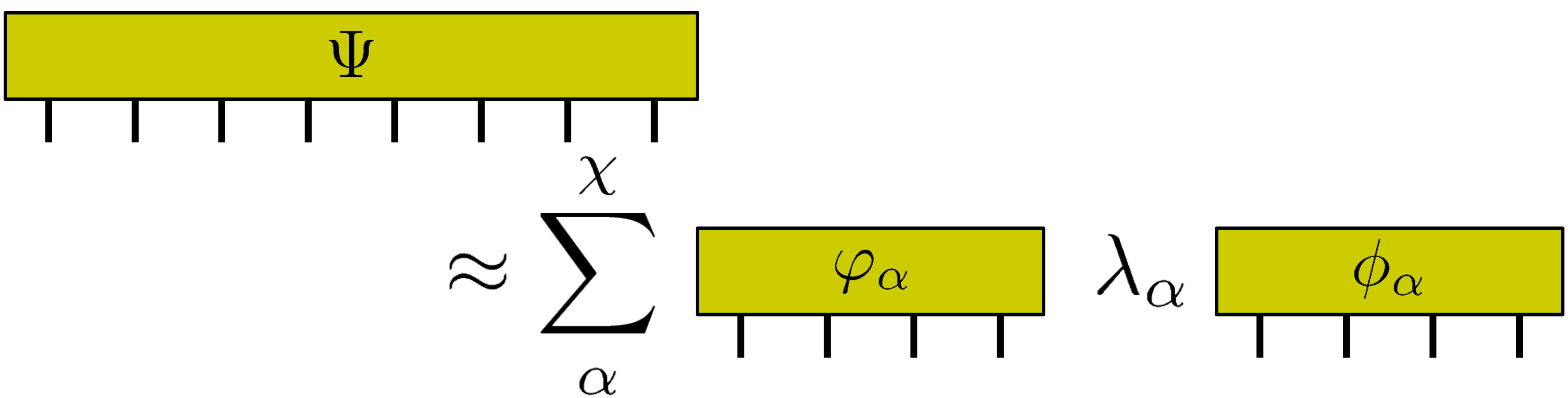


Schmidt Decomposition

Schmidt decomposition

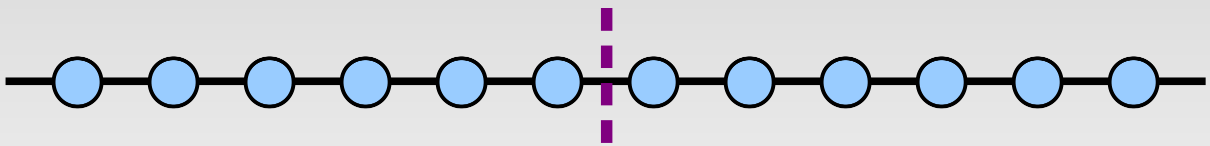
$$|\Psi\rangle = \sum_{\alpha} \lambda_\alpha \, |\varphi_\alpha\rangle^{A} \otimes |\phi_\alpha\rangle^{B}$$

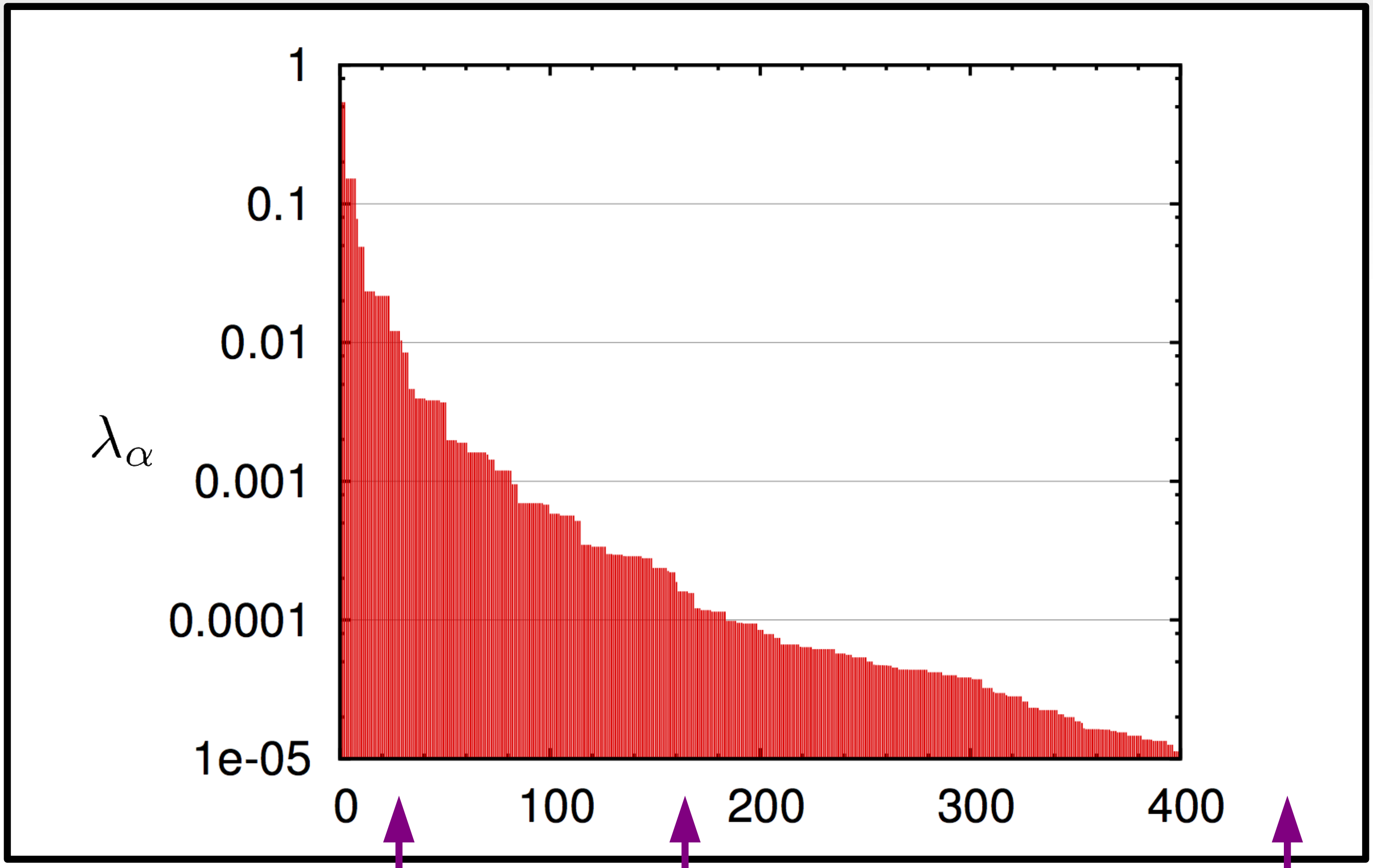
$$\approx \sum_{\alpha=1}^{\chi} \lambda_\alpha \, |\varphi_\alpha\rangle^{A} \otimes |\phi_\alpha\rangle^{B}$$



Schmidt Decomposition

Spin-2 Heisenberg





$\epsilon < 0.001$ $\epsilon < 10^{-6}$ $\epsilon < 10^{-8}$

Schmidt Decomposition

Schmidt decomposition

$$|\Psi\rangle \approx \sum_{\alpha}^{\chi} \lambda_\alpha \, |\varphi_\alpha\rangle^A \otimes |\phi_\alpha\rangle^B$$

$$d^N \Longrightarrow \chi + \chi d^{N/2} + \chi d^{N/2}$$

Schmidt Decomposition

Schmidt decomposition

$$|\Psi\rangle \approx \sum_{\alpha}^{\chi} \lambda_\alpha \, |\varphi_\alpha\rangle^A \otimes |\phi_\alpha\rangle^B$$

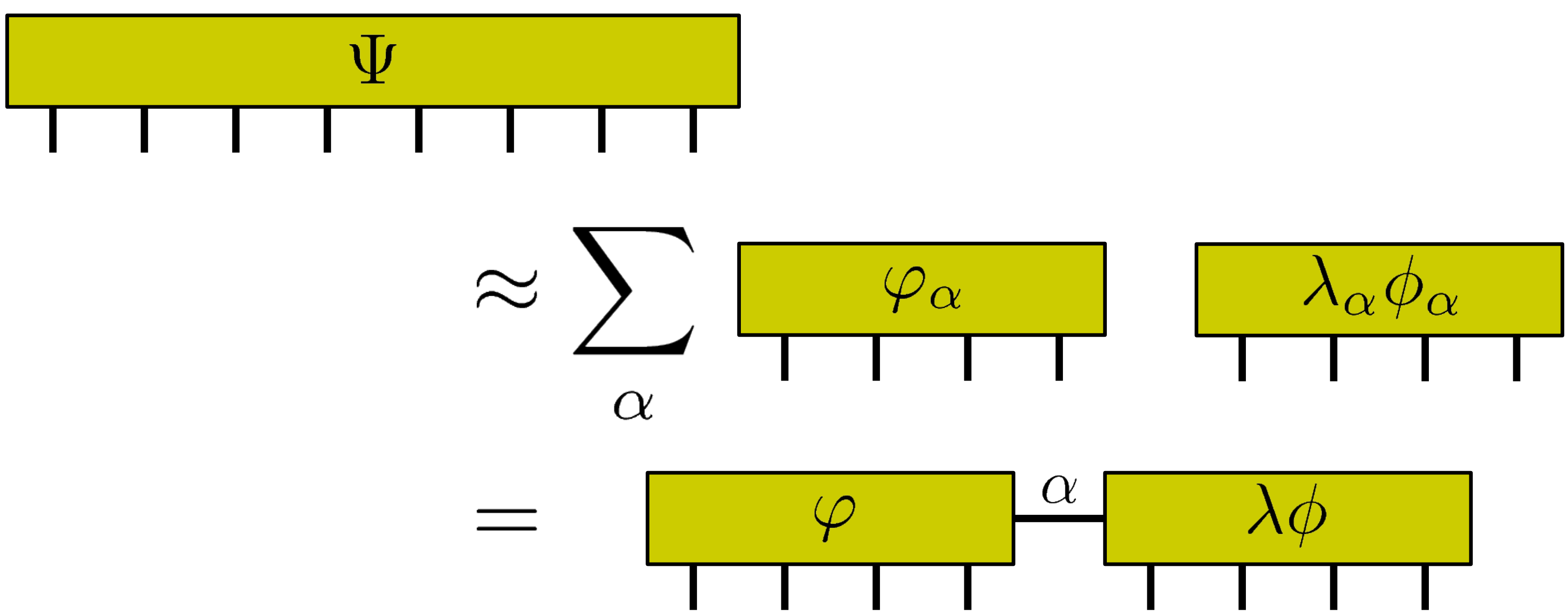
$$d^N \Longrightarrow \chi + \chi d^{N/2} + \chi d^{N/2}$$

What if we repeat this process?

'Vector Product States'

$$|\Psi\rangle \approx \sum_{\alpha}^{\chi} \lambda_\alpha \, |\varphi_\alpha\rangle^A \otimes |\phi_\alpha\rangle^B$$

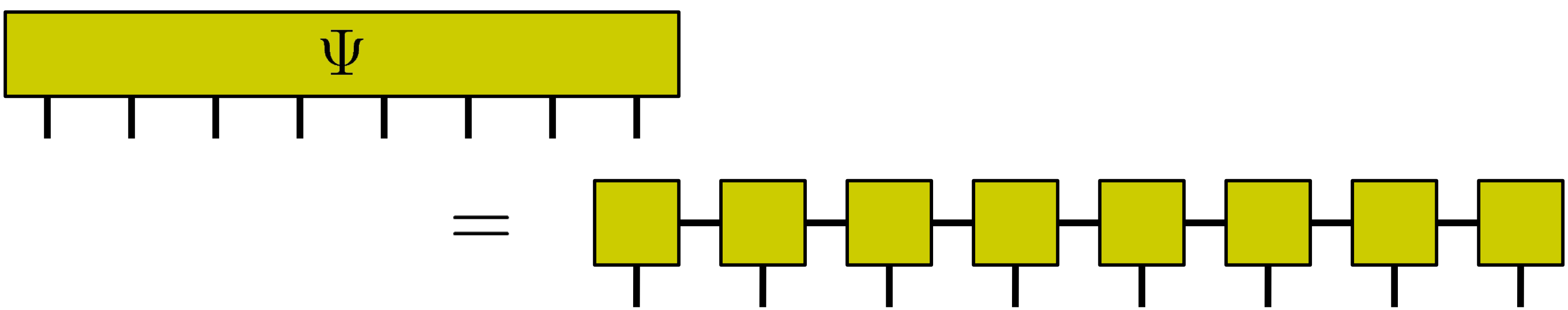
$$= |\vec{\varphi}\rangle^A \cdot |\vec{\lambda\phi}\rangle^B$$



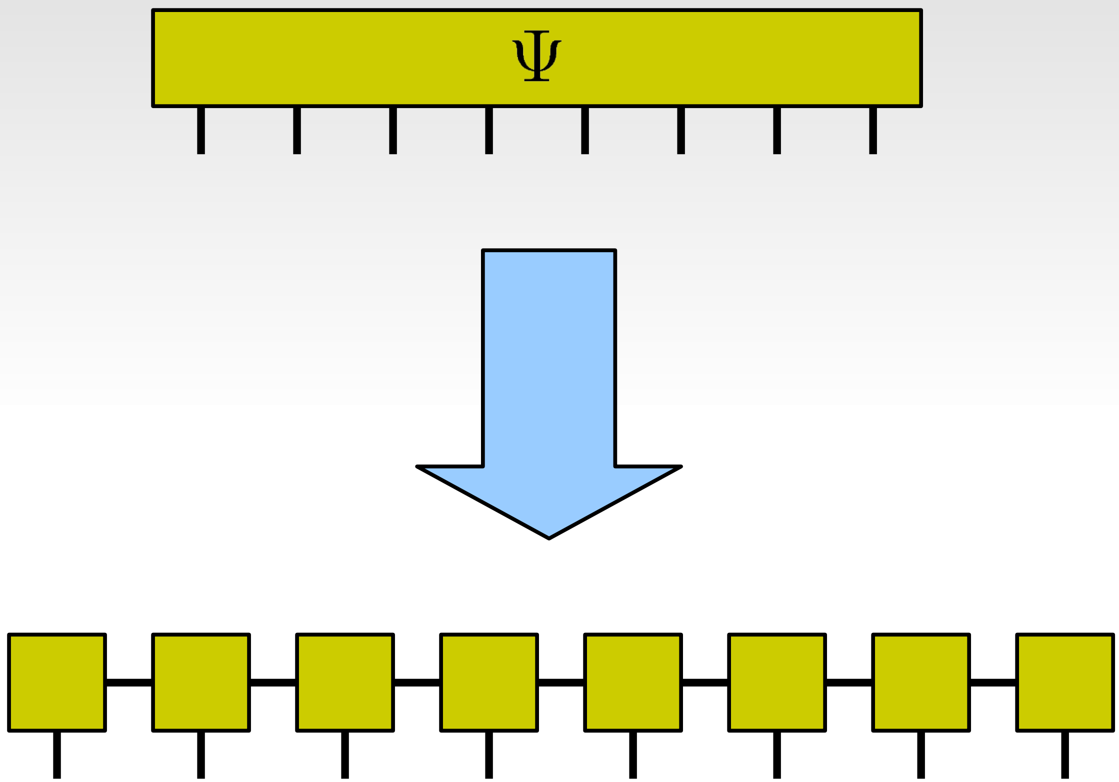
Matrix Product States

$$|\Psi\rangle = |\vec{\phi}\rangle^{[1]} \cdot |\vec{\vec{\lambda}}\phi\rangle^{[2]} \cdot |\vec{\vec{\lambda}}\phi\rangle^{[3]} \cdots |\vec{\vec{\lambda}}\phi\rangle^{[7]} \cdot |\lambda\vec{\phi}\rangle^{[8]}$$

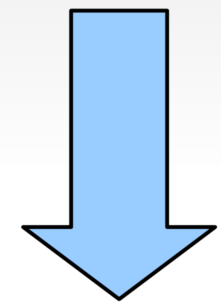
Matrix of wavefunctions



Matrix Product States



$$d^N$$



$$N \cdot (\chi^2 d)$$

$$\sum_{\{s\}} \left[B^{[1]}_{s_1} B^{[2]}_{s_2} \cdots B^{[N]}_{s_N} \right] |s_1 s_2 \cdots s_N\rangle$$

Matrix Product States

MPS ansatz

$$|\Psi\rangle = \sum_{\{s\}} \left[B^{[1]}_{s_1} B^{[2]}_{s_2} \cdots B^{[N]}_{s_N}\right] |s_1 s_2 \cdots s_N\rangle$$

At each site, there are d matrices, $B^{[n]}_1, \ldots, B^{[n]}_d$

Infinite MPS ansatz

$$|\Psi\rangle = \sum_{\{s\}} \left[\cdots B^{[-1]}_{s_{-1}} B^{[0]}_{s_0} B^{[1]}_{s_1} \cdots\right] |\cdots s_{-1} s_0 s_1 \cdots\rangle$$

MPS examples

Product state: $|\mathrm{AF}\rangle = \cdots |\uparrow\rangle\,|\downarrow\rangle\,|\uparrow\rangle\,|\downarrow\rangle\,|\uparrow\rangle \cdots$

Matrix product state:

$$|\mathrm{AKLT}\rangle \propto \cdots \begin{pmatrix} |0\rangle & \sqrt{2}|\uparrow\rangle \\ \sqrt{2}|\downarrow\rangle & -|0\rangle \end{pmatrix} \begin{pmatrix} |0\rangle & \sqrt{2}|\uparrow\rangle \\ \sqrt{2}|\downarrow\rangle & -|0\rangle \end{pmatrix} \begin{pmatrix} |0\rangle & \sqrt{2}|\uparrow\rangle \\ \sqrt{2}|\downarrow\rangle & -|0\rangle \end{pmatrix} \cdots$$

General (translationally invariant) MPS form

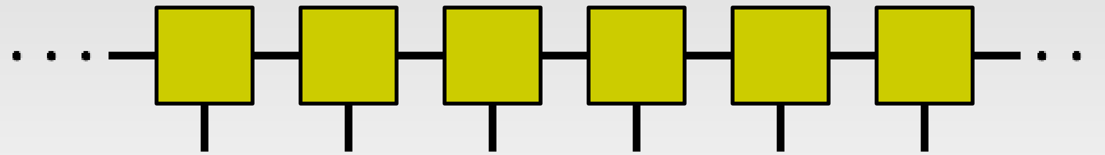
$$|\Psi\rangle = \cdots BBBB \cdots , \qquad B = \sum_s B_s |s\rangle$$

$\chi \times \chi$

Equivalent representation

$$\langle \cdots s_{-1} s_0 s_1 \cdots |\Psi\rangle = \cdots B_{s_{-1}} B_{s_0} B_{s_1} \cdots$$

Matrix Product States



- An efficient way to store a wavefunction

$$d^N \longrightarrow N\chi^2 d$$

  - 1D: Entanglement independent of system size

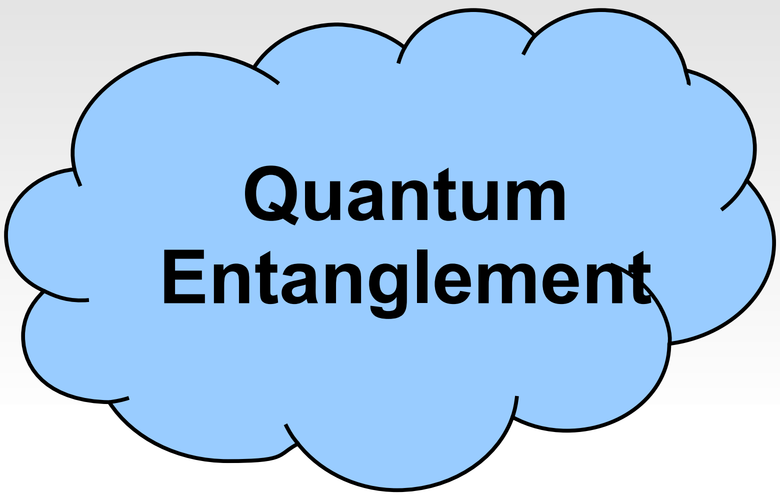
$$\chi \sim \mathrm{const}$$

  - 2D: Entanglement grows linearly as length

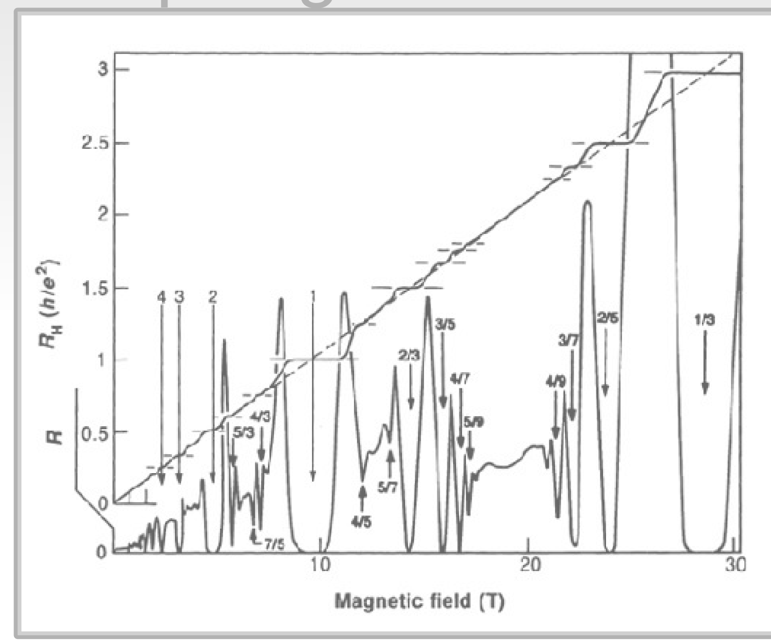
$$\chi \sim b^L \sim b^{\sqrt{N}}$$

- Easy to extract the entanglement spectrum

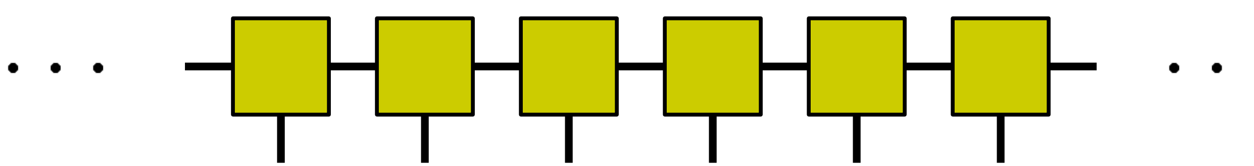
FQH, Entanglement, MPS



Topological states



Matrix product states



Outline

- Why study quantum Hall?
- Quantum entanglement
  - Topological phases
  - Matrix product states (MPS)
- 1. MPS for quantum Hall model wavefunctions
- 2. Modeling physical systems with density matrix renormalization group (DMRG)
- 3. Extracting topological content from ground states

MPS and DMRG

1 Model wavefunctions

→ Matrix product states wavefunctions

2 Computing ground states

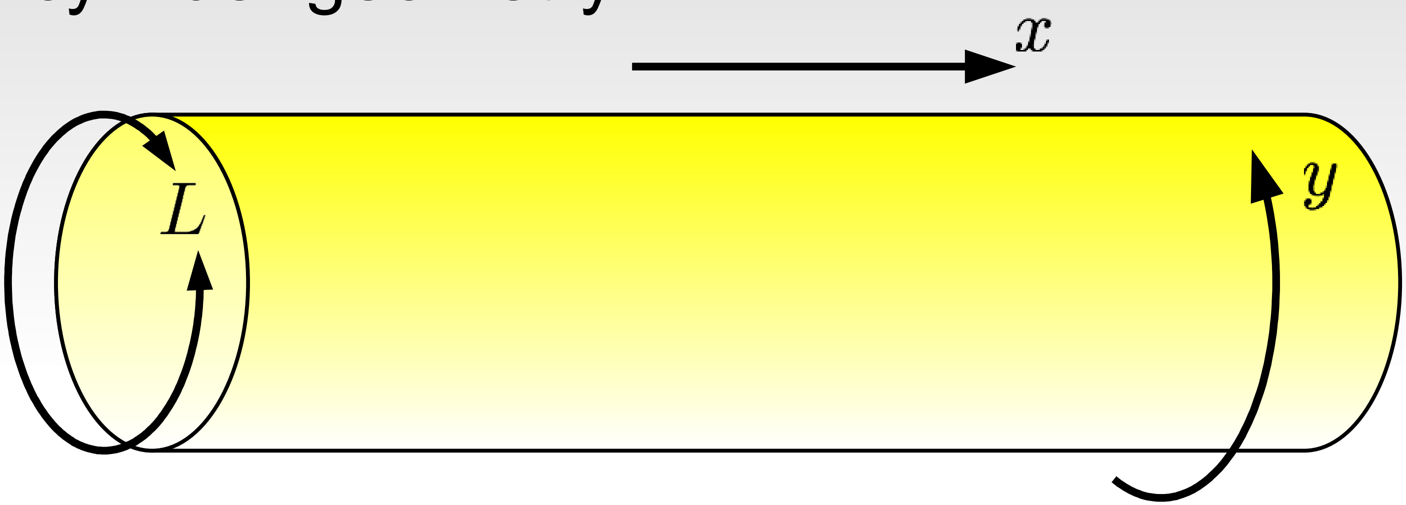
Quantum Hall Hamiltonian → DMRG → Matrix product states wavefunctions

3 Extracting topological information

Matrix product states wavefunctions → Braiding statistics

Quantum Hall on a cylinder

Infinite cylinder geometry



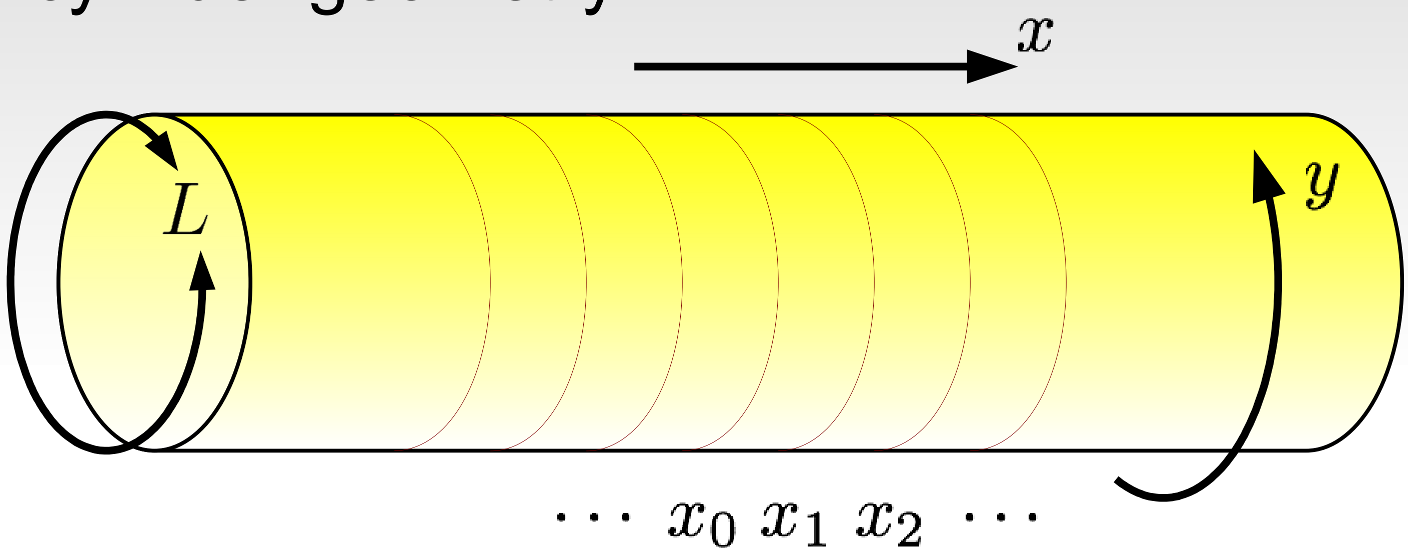
Landau gauge

$$\mathbf{A}(x,y) = \ell_B^{-2}(0,x)$$

$\Rightarrow$ $k_y = -i\partial_y$ is a good quantum number

Map to 1D chain

Infinite cylinder geometry



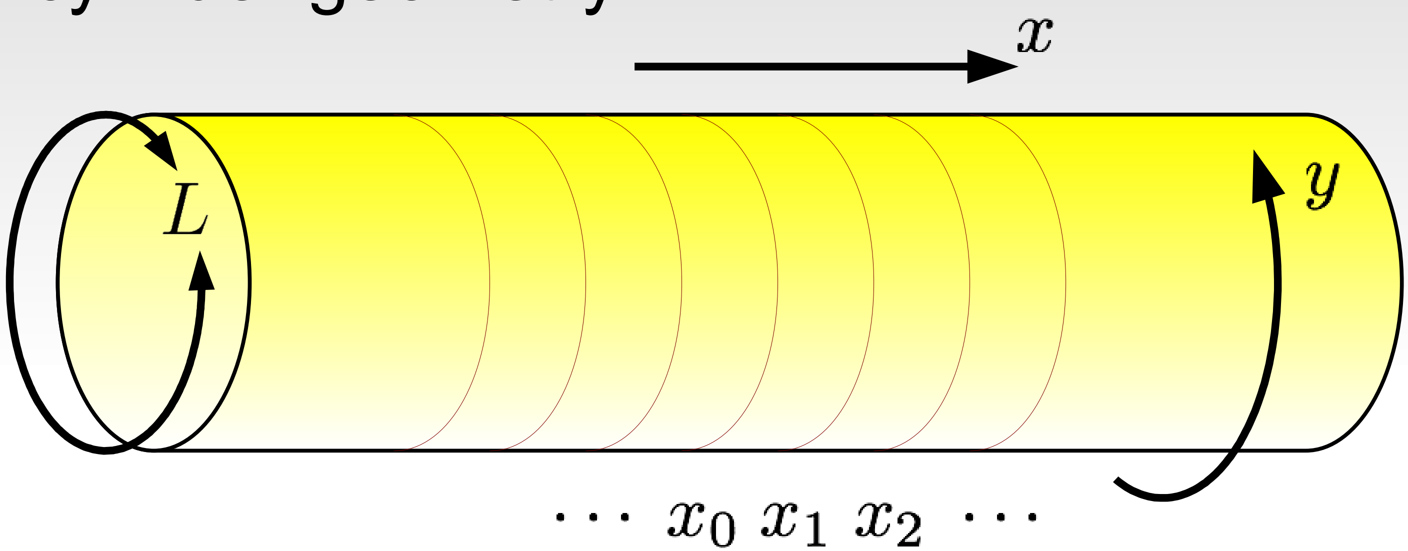
Landau gauge

$$\varphi_n = \frac{1}{\sqrt{L\ell_B \pi^{1/2}}} e^{ik_n y} e^{-\frac{1}{2\ell_B^2}(x-x_n)^2}$$

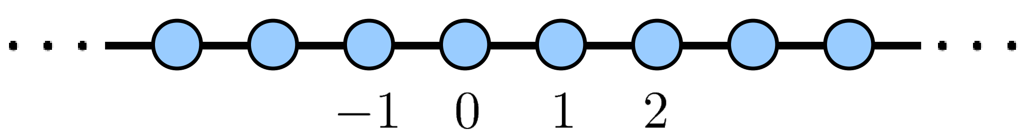
$$k_n = n\frac{2\pi}{L}, \quad x_n = \ell_B^2 k_n$$

Map to 1D chain

Infinite cylinder geometry



Chain of Landau orbitals



Occupation basis: $|\Psi\rangle = \sum_{\{m\}} c(\{m\}) | \cdots m_{-1} m_0 m_1 m_2 \cdots \rangle$

Conformal Field Theory & Quantum Hall

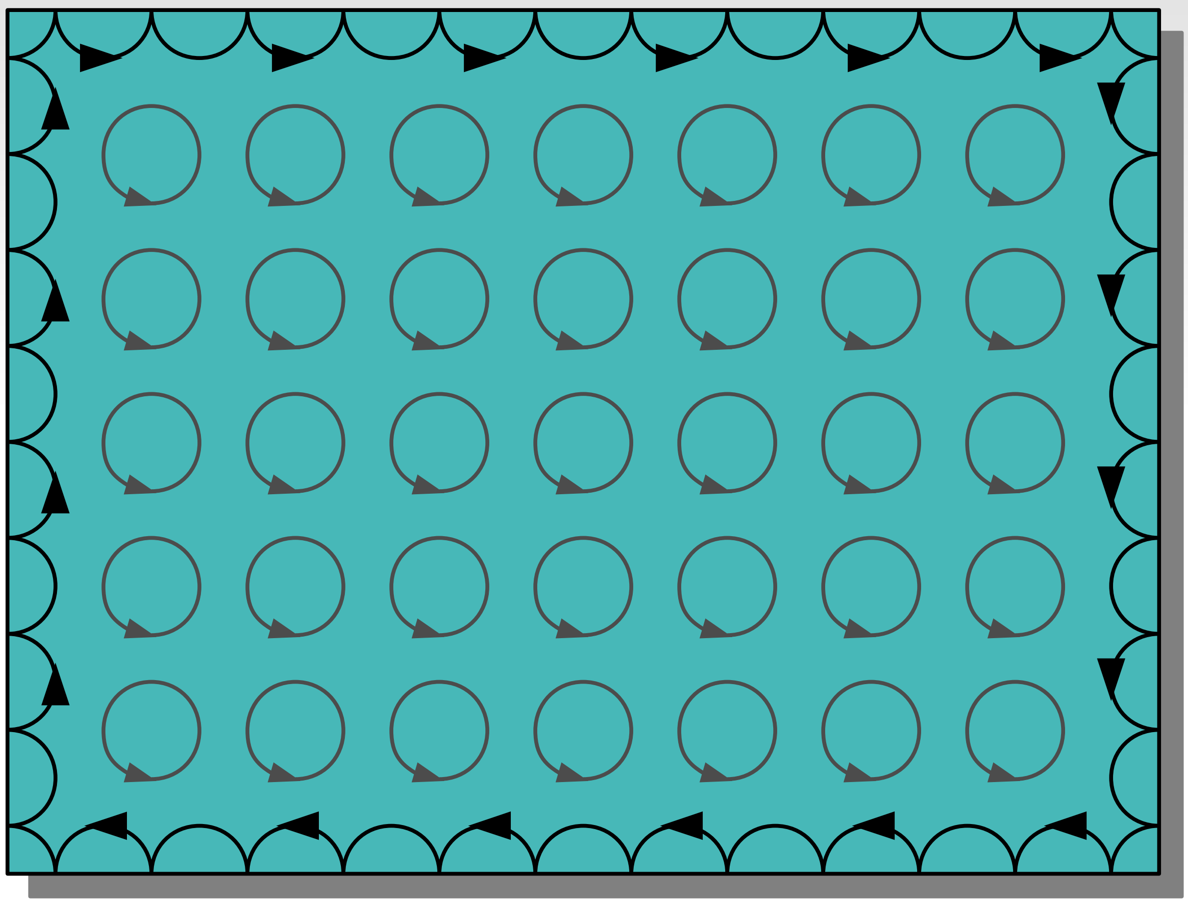
The following are all described by a chiral CFT:

- Edge excitations (1+1D CFT)
- Model wavefunction (2D CFT conformal blocks) [Moore, Read]
- Quasiparticle content (CFT primary fields) [Moore, Read]
- Entanglement spectrum [Li, Haldane]

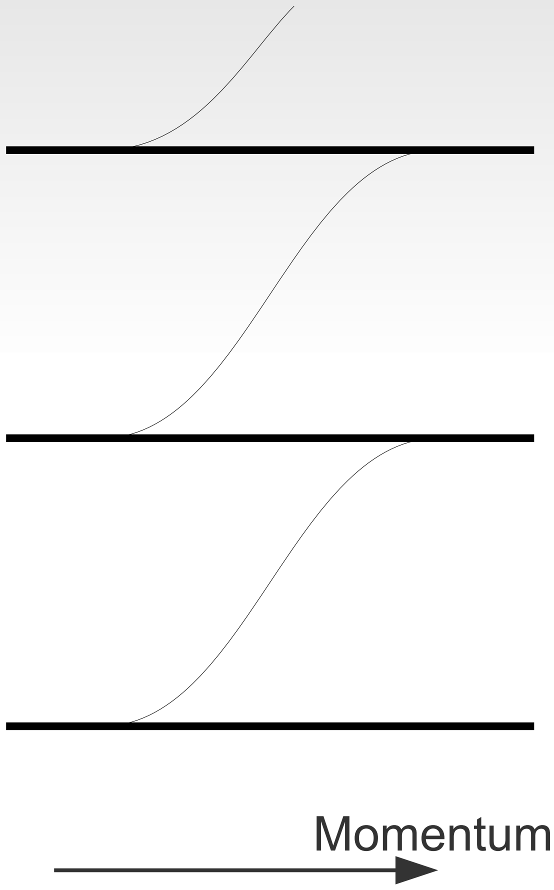
Examples:

- Laughlin: compact U(1) boson
- Hierarchy states: multiple U(1) bosons
- Moore-Read: U(1) boson + Ising/Majorana
- Read-Rezayi: U(1) boson + parafermion

Quantum Hall Edge

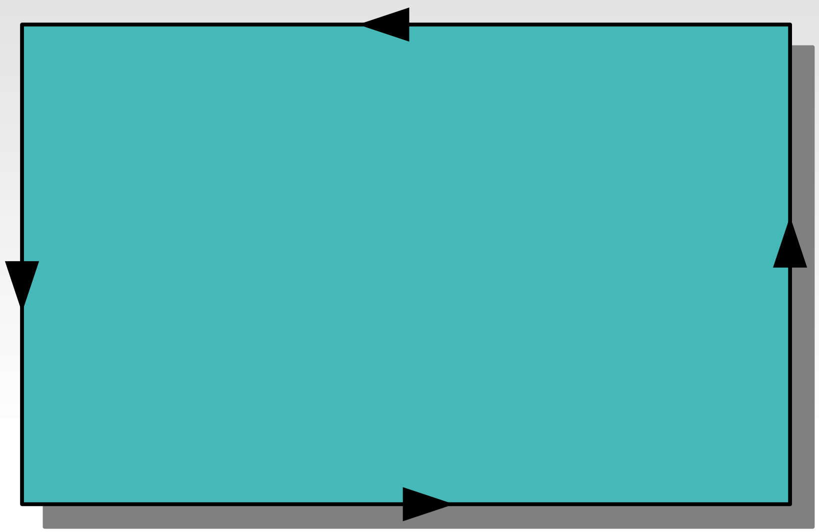


Chiral Edge modes

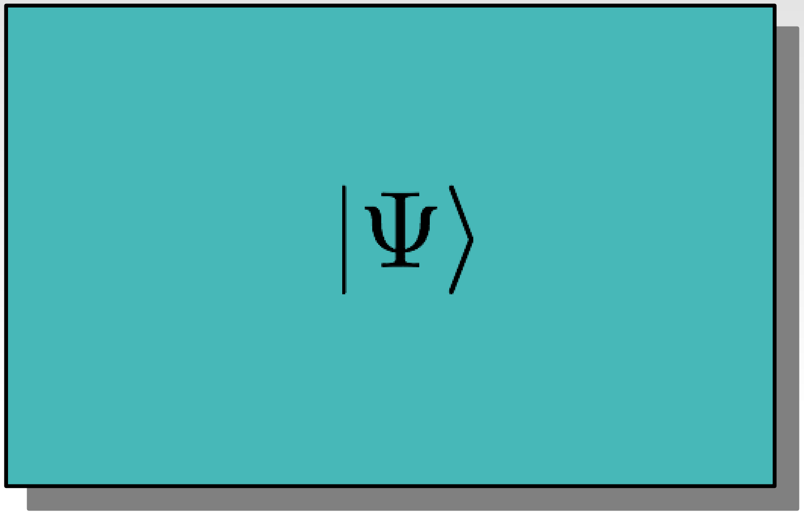


CFT & Quantum Hall

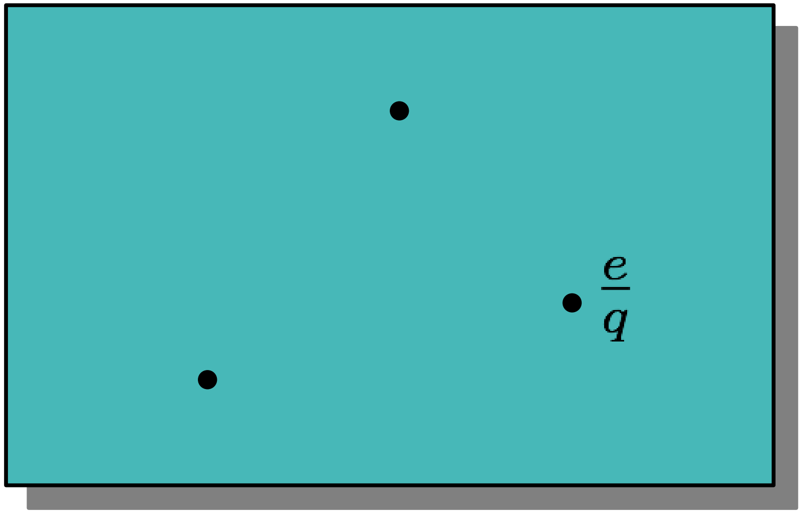
Edge



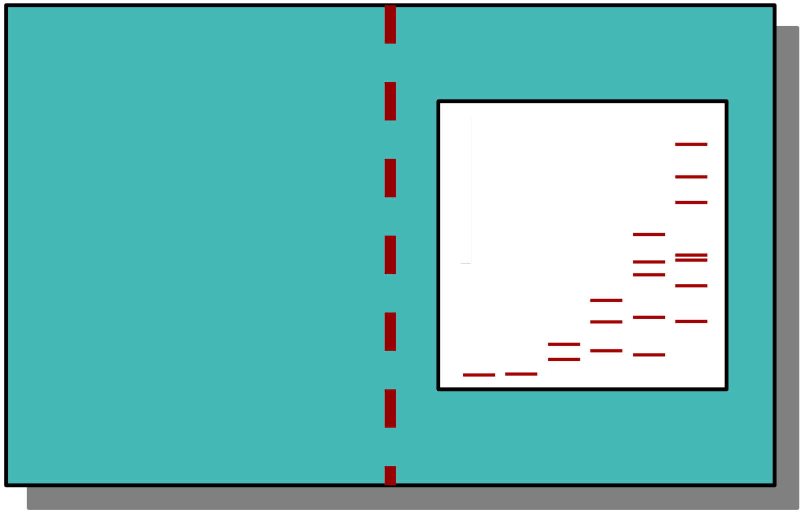
Bulk Wavefunction



Quasiparticles



Entanglement Spectrum



Edge Structure

Laughlin states $\Longleftrightarrow$ Chiral free boson

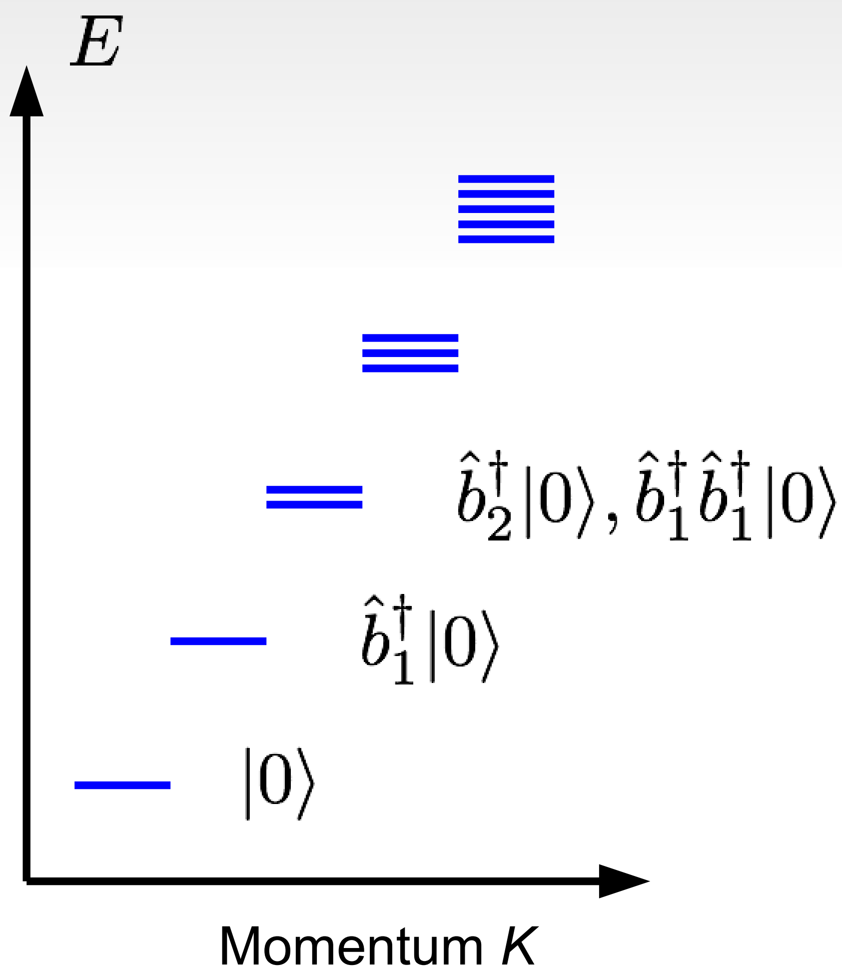
Action

$$S = \frac{1}{2}\int dx\, dt\, \partial_x\phi(\partial_t\phi - \partial_x\phi)$$

Hamiltonian

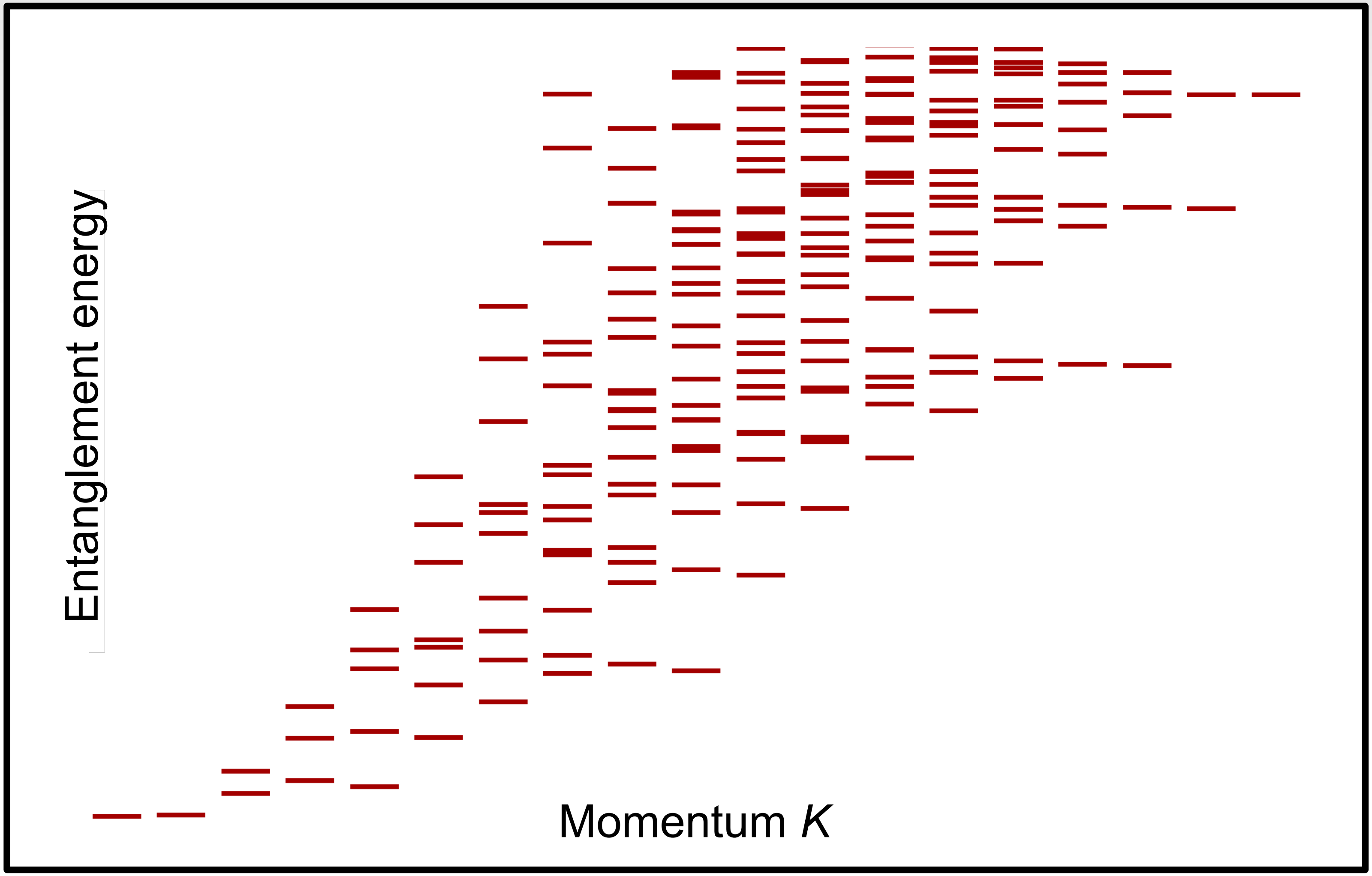
$$\hat{H} = \sum_{n\geq 1} n\hat{b}_n^\dagger \hat{b}_n + \begin{pmatrix}\text{Charging}\\ \text{energy}\end{pmatrix}$$

Neutral excitations



Laughlin wavefunction

Entanglement spectrum



Model wavefunction

Model wavefunctions written in terms of a correlation function

Example: Laughlin wavefunction $\nu = 1/q$

$$\Psi(z_a) \propto \prod_{a<b} (z_a - z_b)^q \cdot e^{-\frac{1}{4}\sum_a |z_a|^2}$$

$$\propto \left\langle e^{i\sqrt{q}\phi(z_1)} e^{i\sqrt{q}\phi(z_2)} \dots e^{i\sqrt{q}\phi(z_N)} \; e^{\frac{-i}{\sqrt{q}} \int \frac{d^2 z}{2\pi\ell_B^2} \phi(z)} \right\rangle_\phi$$

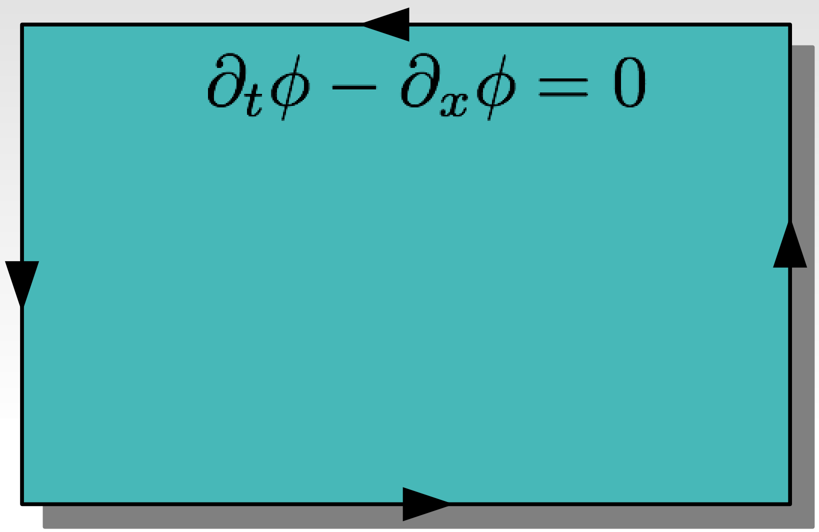
chiral U(1) boson ϕ

$$\langle \phi(z)\phi(z') \rangle = -\log(z - z')$$ (2D Coulomb potential)

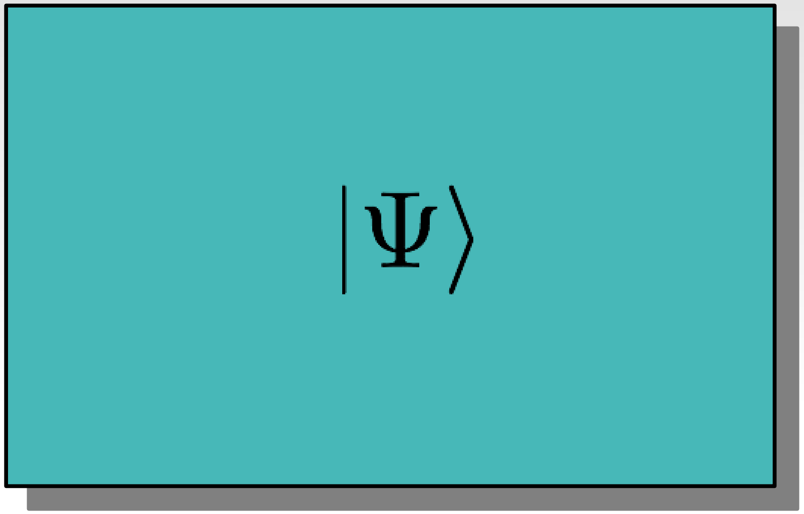
$$e^{i\sqrt{q}\phi(z)} e^{i\sqrt{q}\phi(z')} \sim \exp\langle -q\phi(z)\phi(z') \rangle \sim (z - z')^q$$

CFT & Quantum Hall

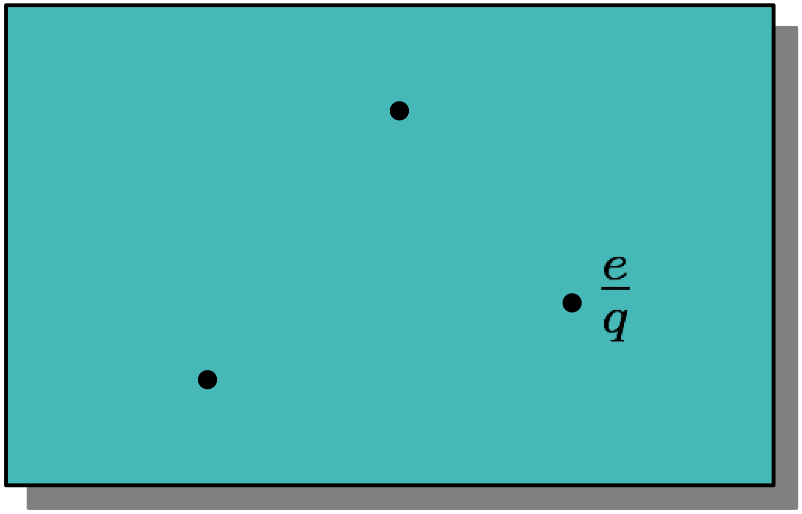
Edge



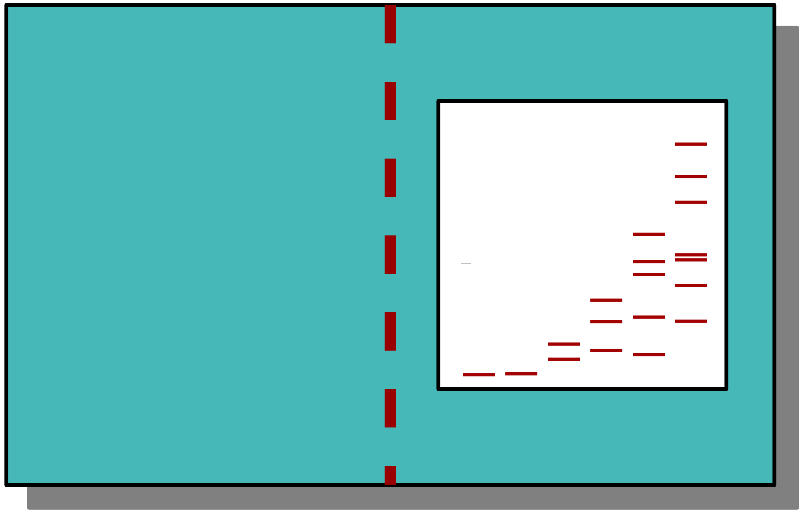
Bulk Wavefunction



Quasiparticles

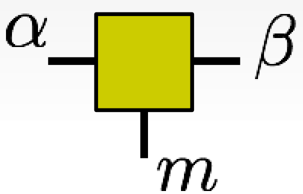


Entanglement Spectrum



Exact quantum Hall MPS

Convert a model wavefunctions constructed from CFT and rewrite it as an MPS in the orbital basis.



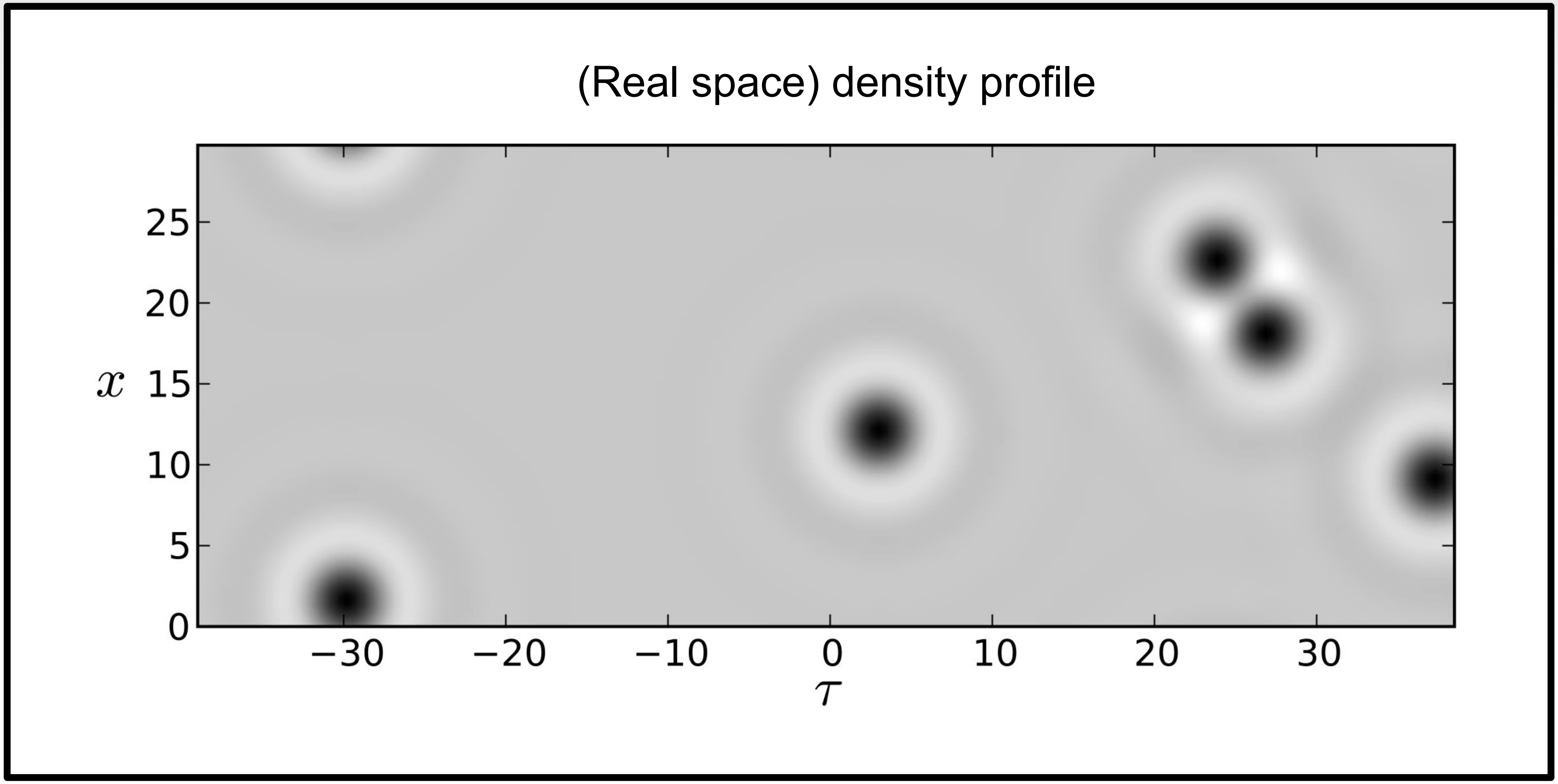
- MPS written in terms of tensors $B_{m;\alpha\beta}$
- m_n is the orbital filling at orbital n
- α and β label states of the auxilary CFT
- The B_m's are operators in the CFT

$$B_{m;\alpha\beta} = \left\langle \alpha \left| \frac{\left(\hat{\mathcal{V}}_0\right)^m}{\sqrt{m!}} e^{-i\sqrt{\nu}\phi_0 - (\delta\tau)H} \right| \beta \right\rangle$$

Michael Zaletel, RM, PRB 86, 245305

Exact quantum Hall MPS

- Add quasiparticles



Michael Zaletel, RM, PRB 86, 245305

Model Wavefunctions MPSs

- Formally relate bulk to entanglement
- Proof of principle: It is possible to apply 1D techniques to 2D
- Elucidate the role of charge and momentum in quantum Hall
- Reveal the structure of quasiparticle excitations
- Insights on how not to get 'stuck' with DMRG

Outline

- Why study quantum Hall?
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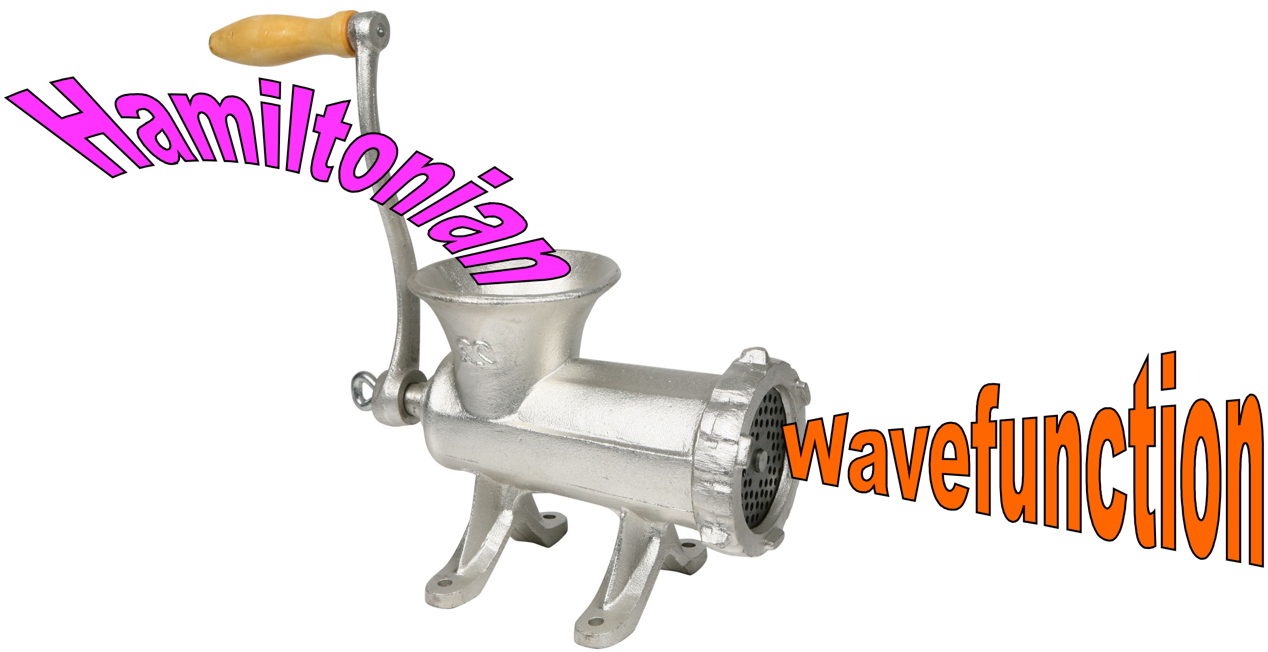
DMRG

The density matrix renormalization group (DMRG) variationally optimizes the MPS by locally minimizing

$$\langle \Psi | H | \Psi \rangle$$

Control parameter: χ



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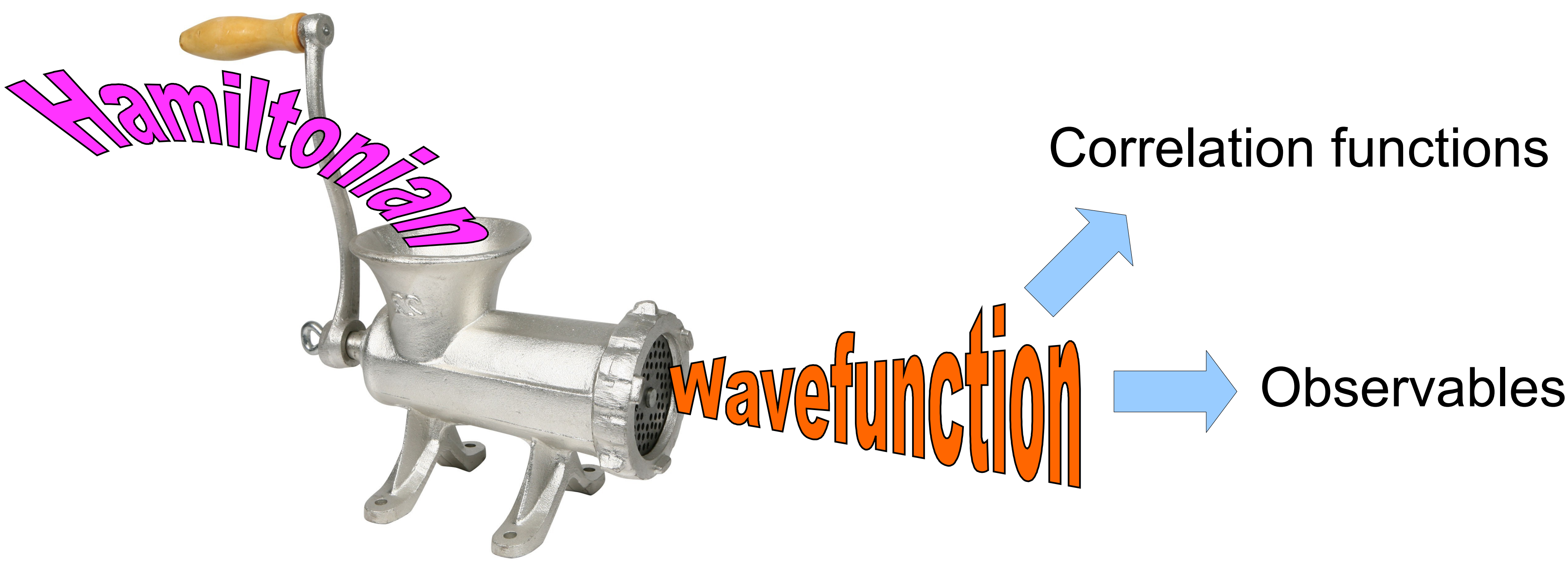
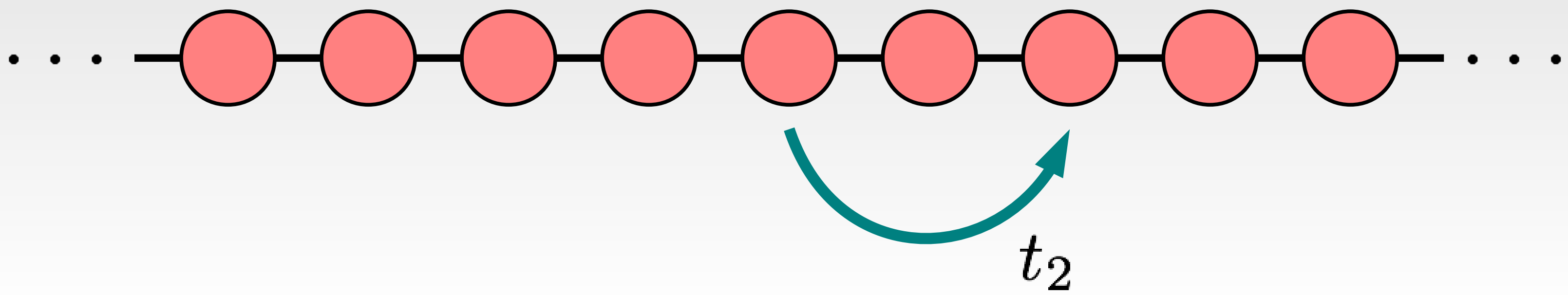


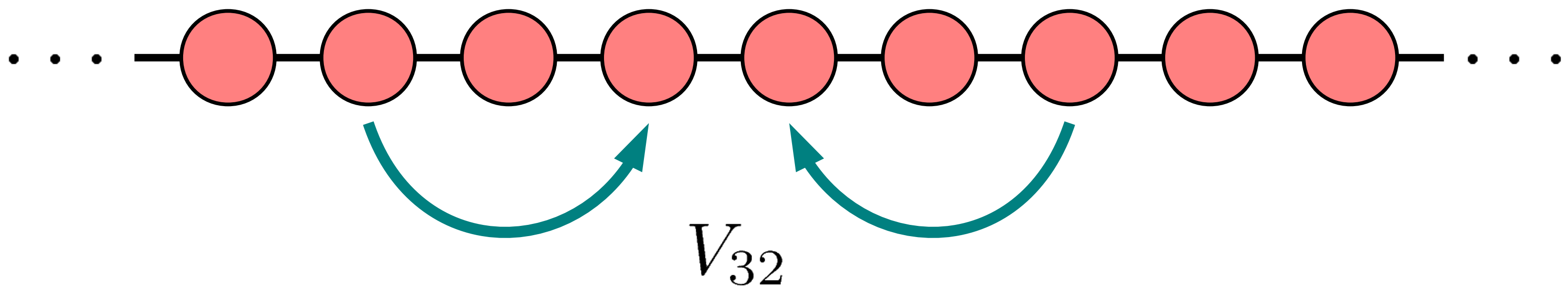
Interaction Hamiltonian

Trap / Confinement potential



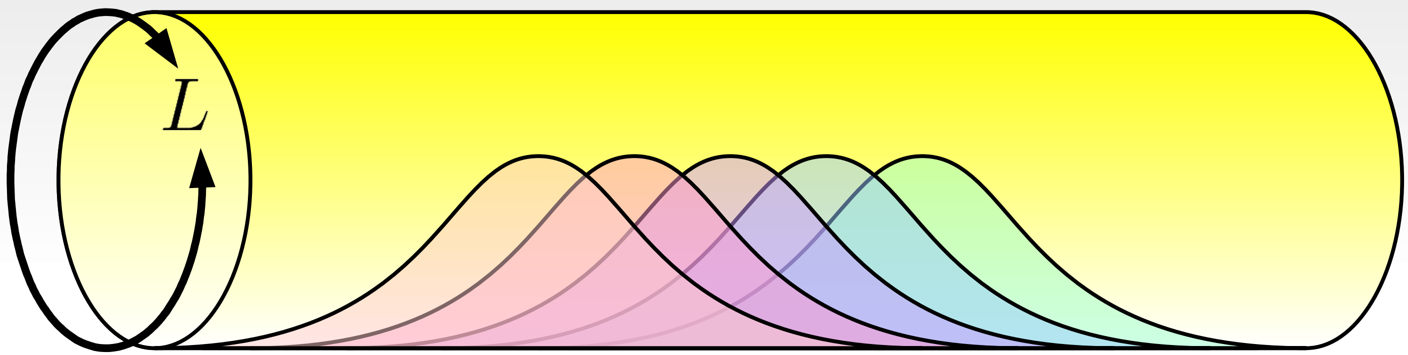
Density-density interaction

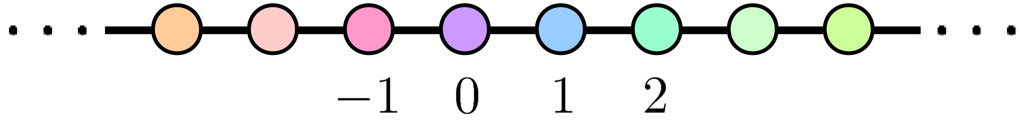
$$H = \sum_j \sum_{k,m} V_{km} \psi^\dagger_{j+k} \psi^\dagger_{j+m} \psi_{j+k+m} \psi_j$$



DMRG

Infinite cylinder geometry

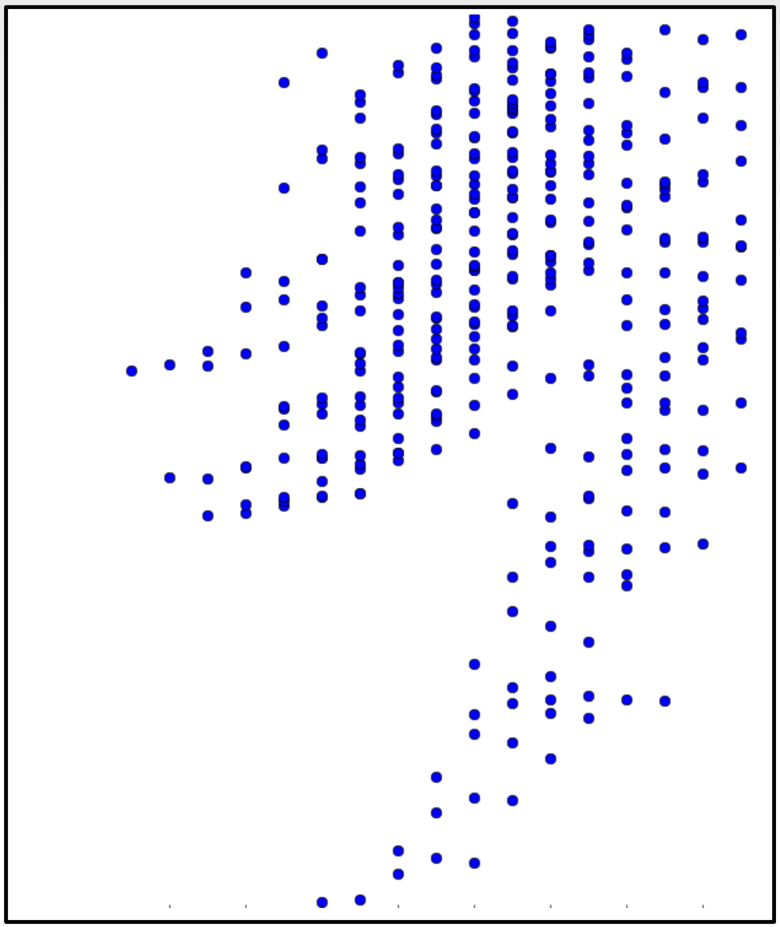




Wavefunction overlap $\mathcal{O}(L/\ell_B)$ orbitals

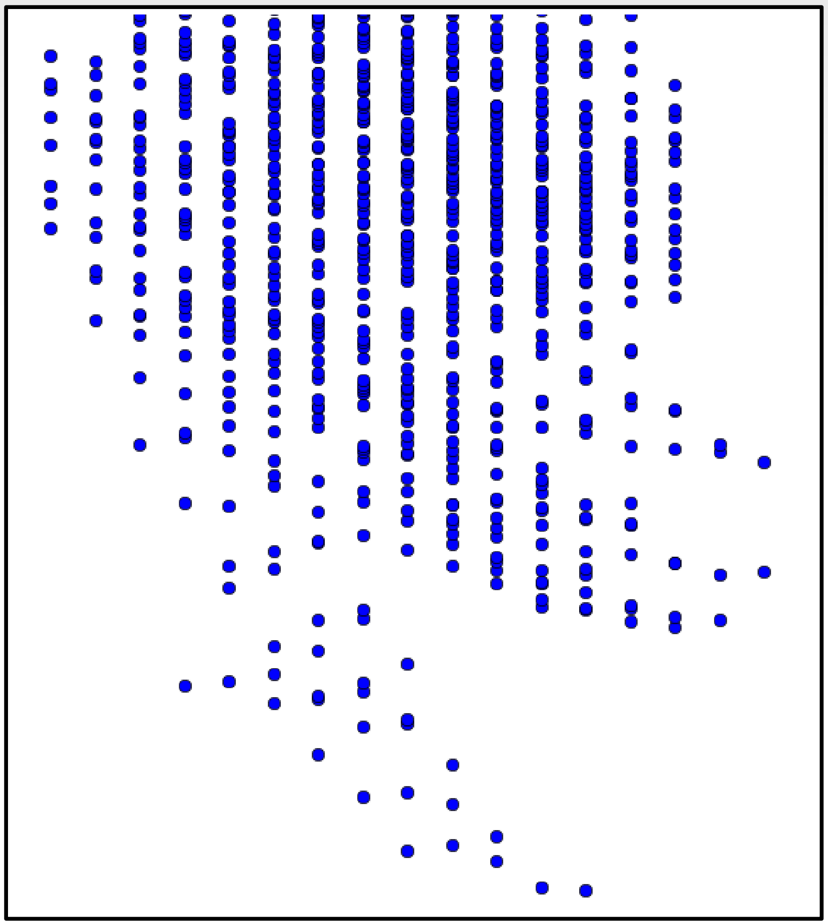
Sample Entanglement Data

1/3 filling with Coulomb repulsion



Momentum

2+2/3 filling with Coulomb repulsion
(spin polarized)



Momentum

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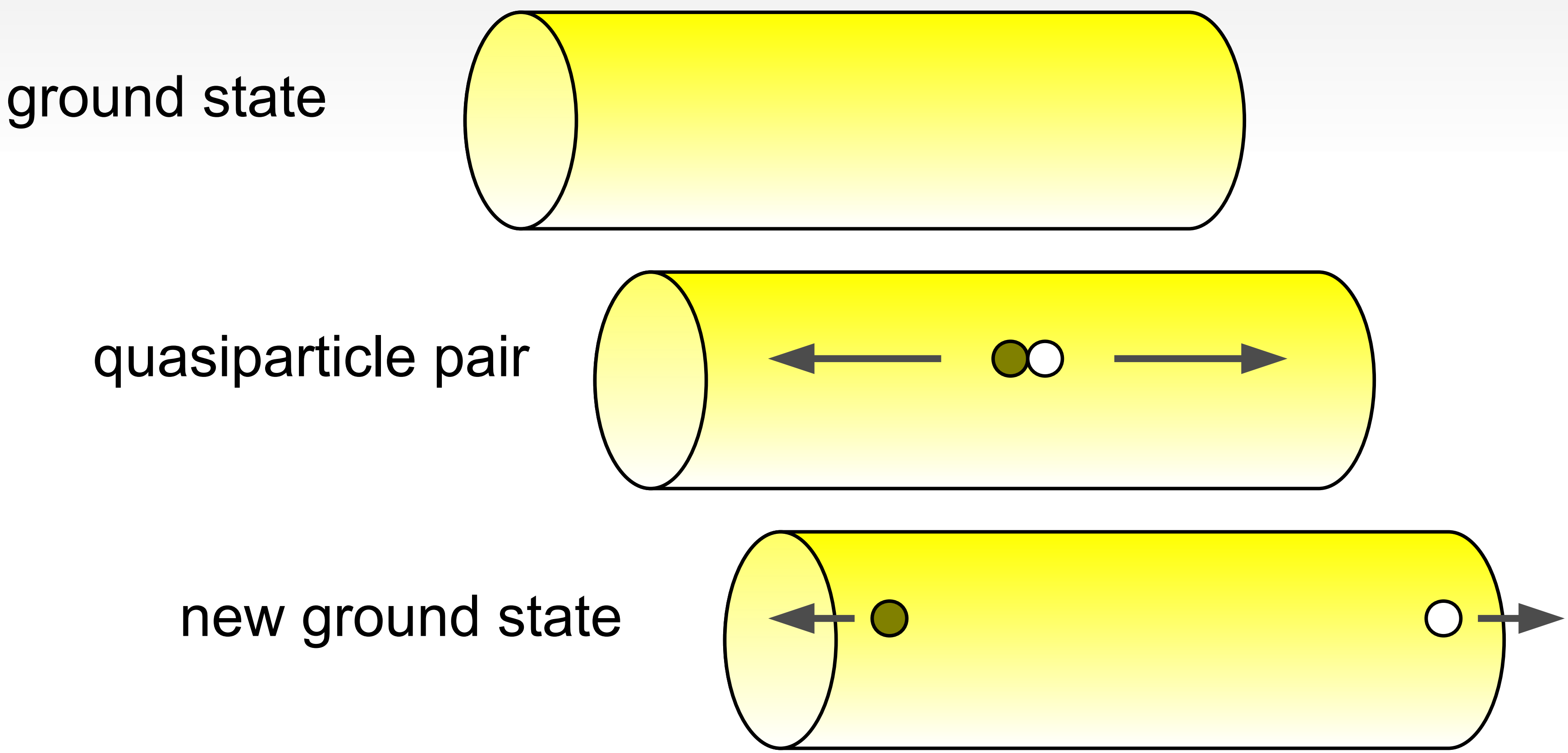
Topological Characteristics

Given a wavefunction, how can we study its phase? (What phase is it in?)

- What are the quasi-particles?
- What are their charges?
- What are their braiding statistics?

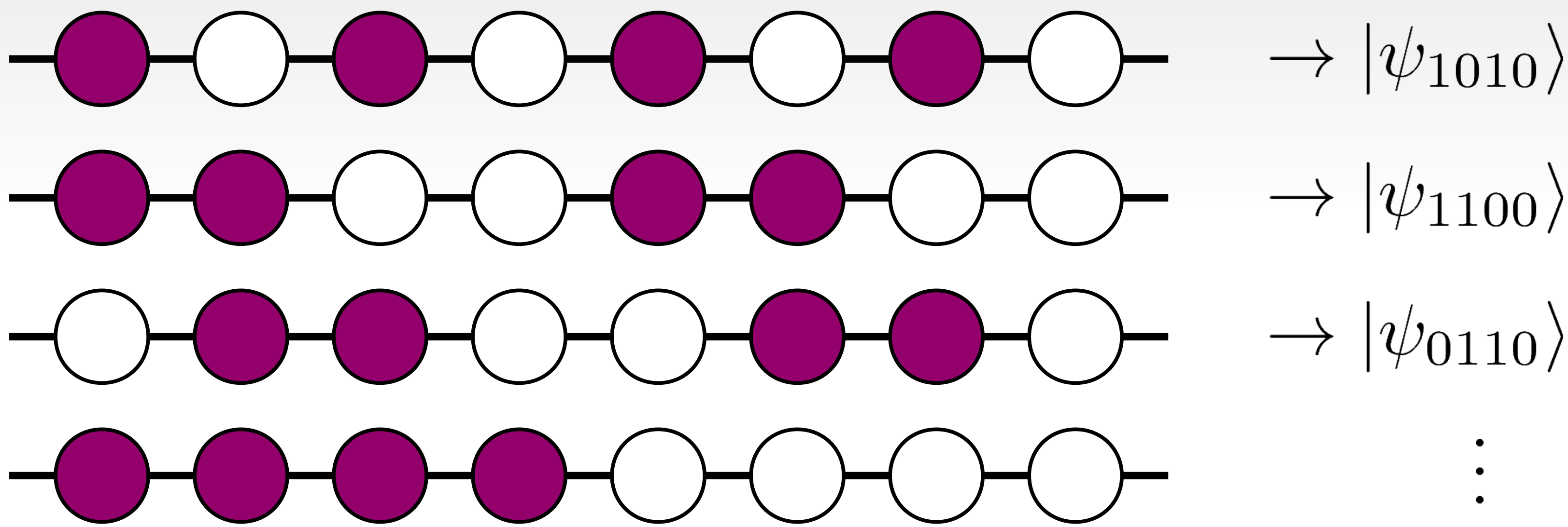
Quasiparticles

There is a one-to-one correspondence between quasiparticles and ground states on a cylinder.



Ground State Degeneracy

By seeding the DMRG with different initial configurations, we can get different possible ground states.

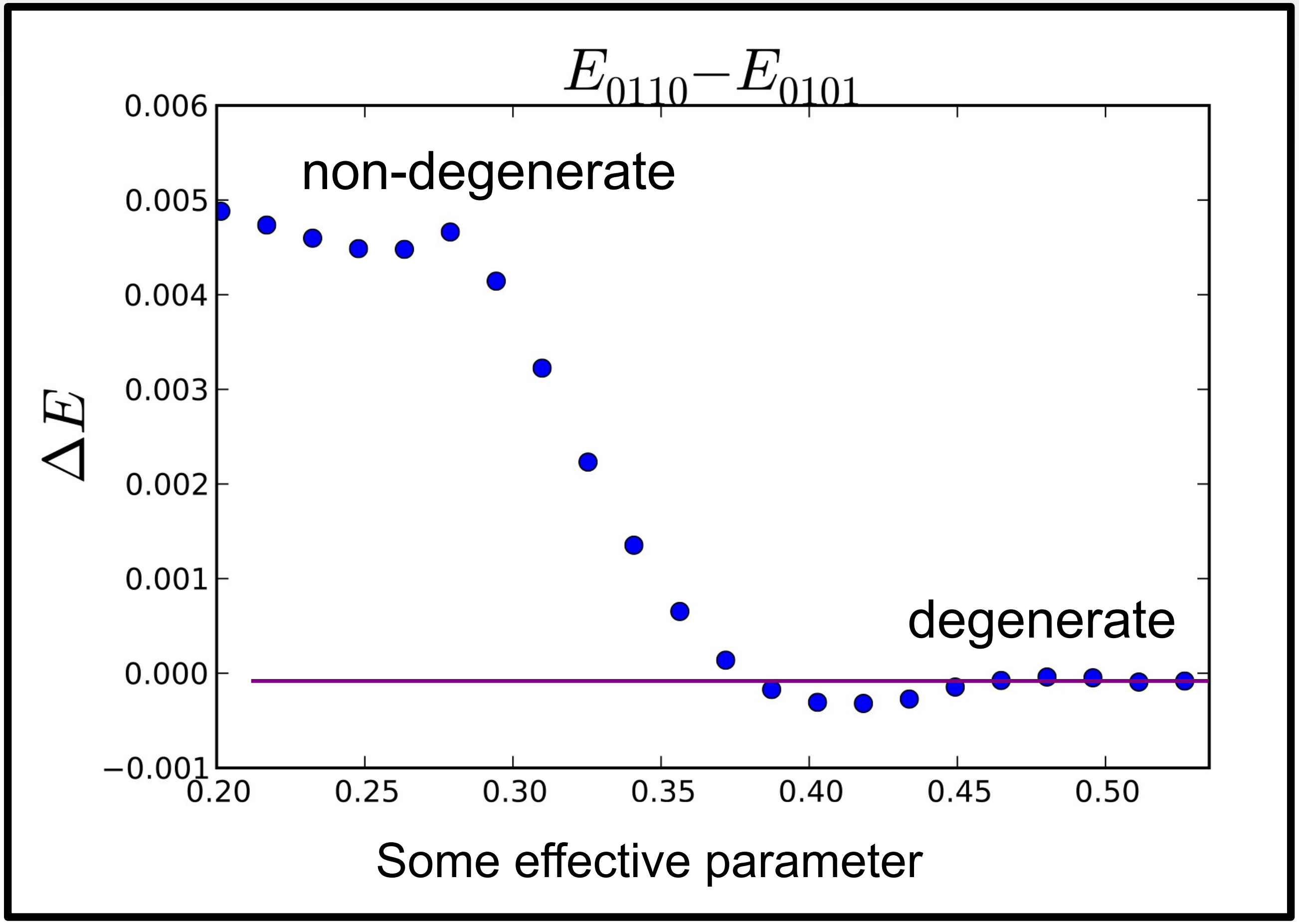


After DMRG, we can check which states are identical, or by checking the energy which are not ground states.

Ground State Degeneracy

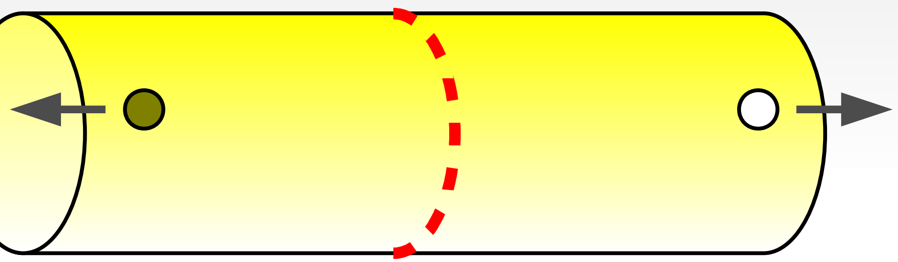
Check overlap $\langle \Psi_{0110} | \Psi_{1010} \rangle = 0$

Energy difference



Entanglement Spectra

There is a one-to-one correspondence between quasiparticles and ground states on a cylinder.



quasiparticle type ⟺ entanglement spectrum

Entanglement Spectra

Topological properties from the ground states' entanglement spectra

- Quantum dimension [Kitaev, Preskill; Levin, Wen]
- 'Completeness' of the quasiparticle set [Cincio, Vidal]
- Edge spectrum [Li, Haldane]

Entanglement Spectra

Topological properties from the ground states' entanglement spectra

- Quantum dimension [Kitaev, Preskill; Levin, Wen]
- 'Completeness' of the quasiparticle set [Cincio, Vidal]
- Edge spectrum [Li, Haldane]

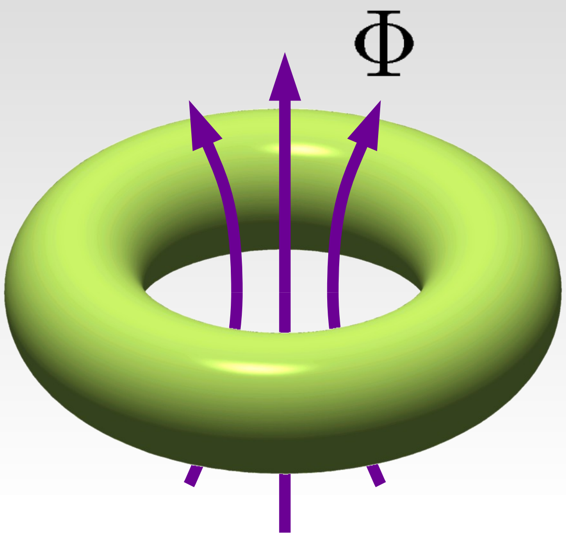
From quantum Hall DMRG [Zaletel, RM, Pollmann]

- Quasiparticle fractional charge
- Topological spin, chiral central charge
- Hall viscosity
- Braiding statistics (S matrix) ?

Entanglement Spectra

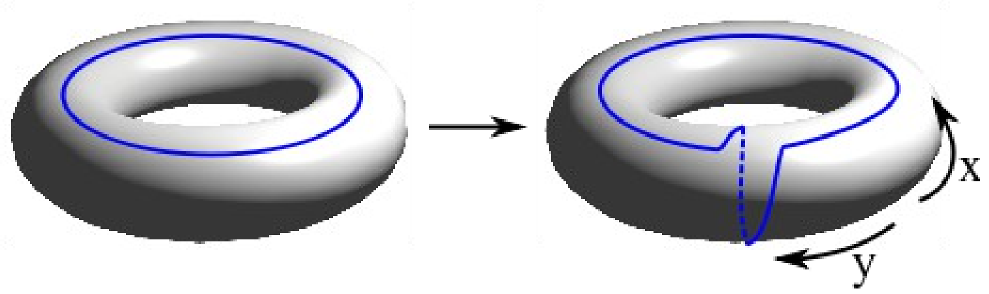
Charge polarization

- Quasiparticle charges
- Flux/quasiparticle insertion



Momentum polarization

- Topological spins
- Chiral central charge
- Hall viscosity

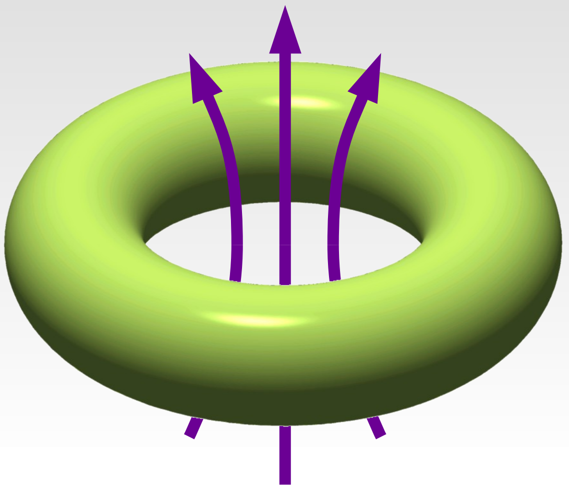


Charge From Entanglement

Berry phase

$$\theta_B = \int_t \langle \Psi(t) | -i\partial_t | \Psi(t) \rangle$$

Closed loop $|\Psi(t_i)\rangle = |\Psi(t_f)\rangle$

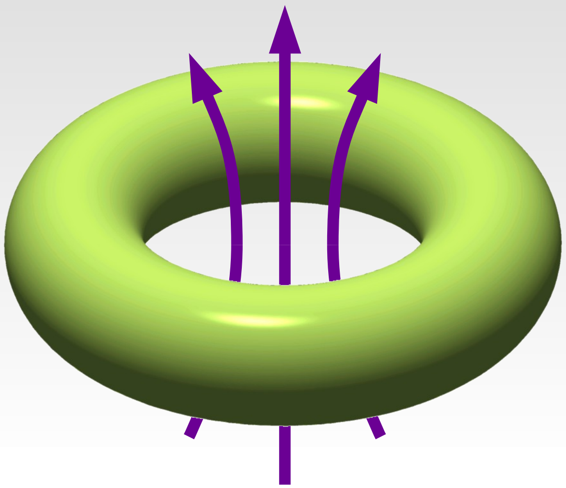


$$\Phi(t) : 0 \to \frac{h}{e}$$

Charge From Entanglement

Berry phase

$$\theta_B = \int_t \langle \Psi(t) | -i\partial_t | \Psi(t) \rangle$$



$$\Phi(t) : 0 \rightarrow \frac{h}{e}$$

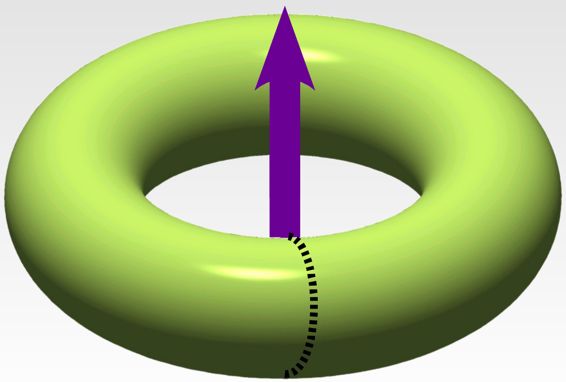
Entanglement spectrum

Anyon charge

Charge From Entanglement

Berry phase

$$\theta_B = \int_t \langle \Psi(t) | -i\partial_t | \Psi(t) \rangle$$



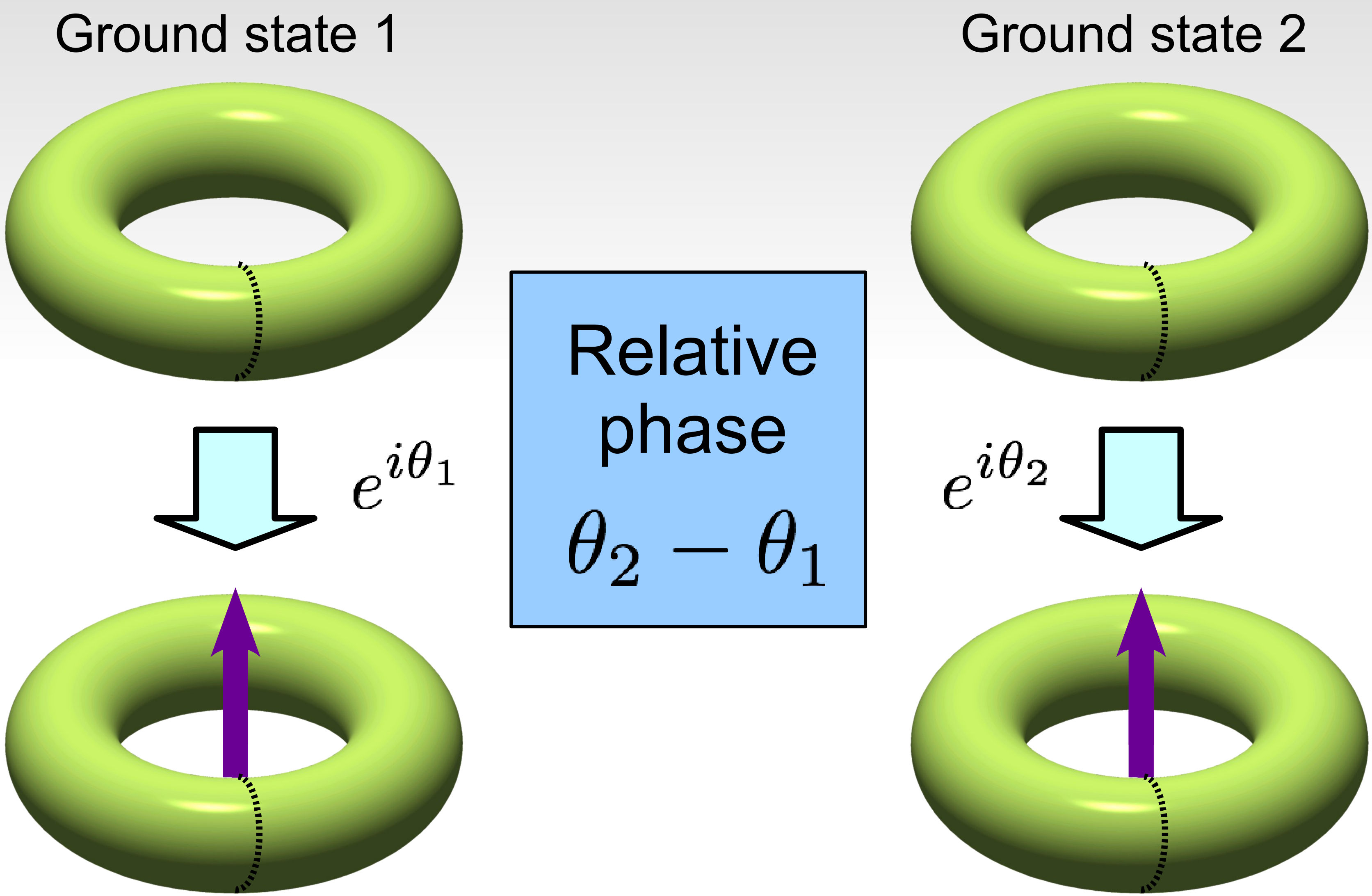
Berry phase from entanglement

$$\theta_B \;=\; 2\pi \sum_\alpha e^{-\tilde{E}_\alpha} Q_\alpha \;=\; 2\pi \langle Q \rangle$$

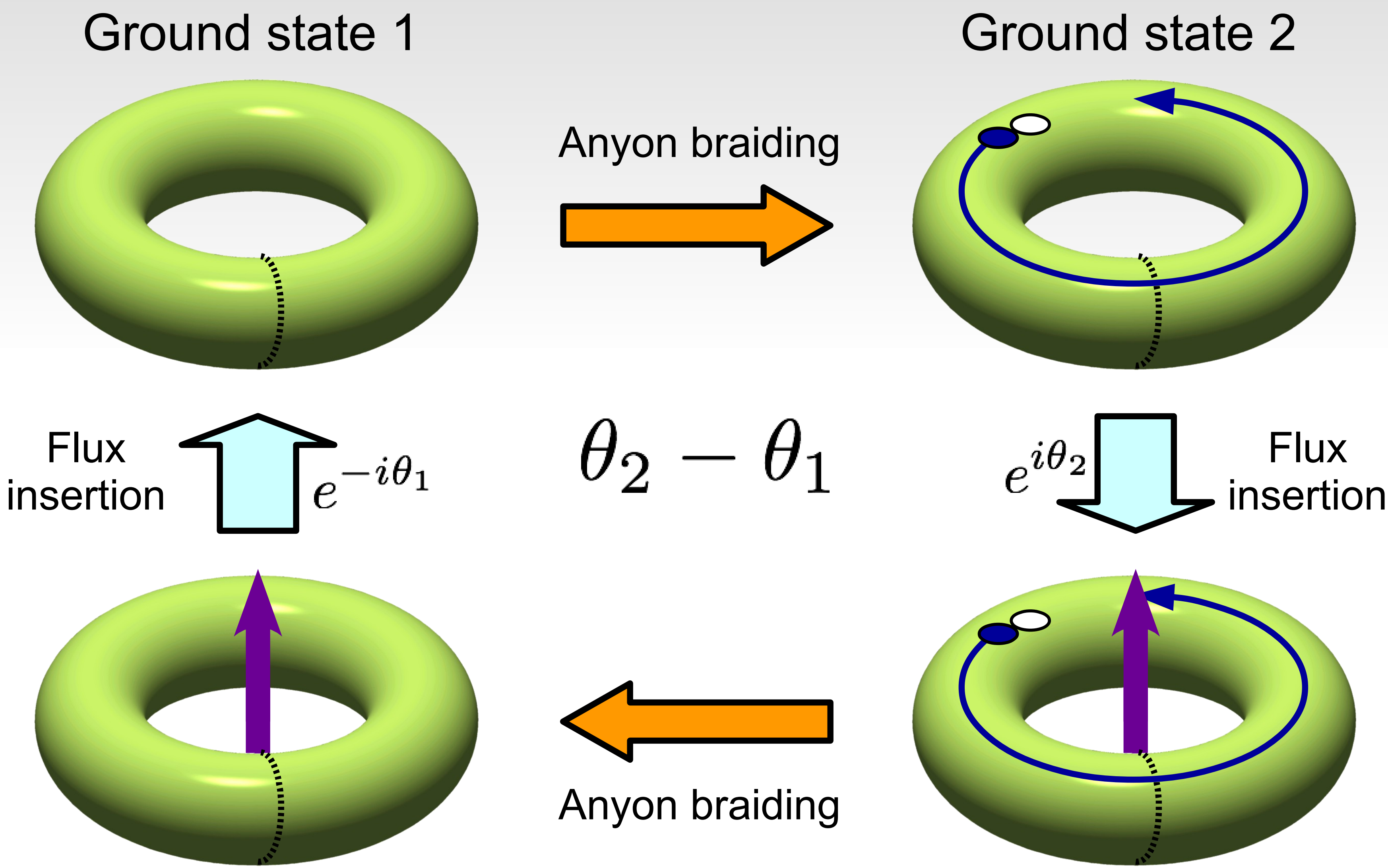
Charge of schmidt vectors

Entanglement energies

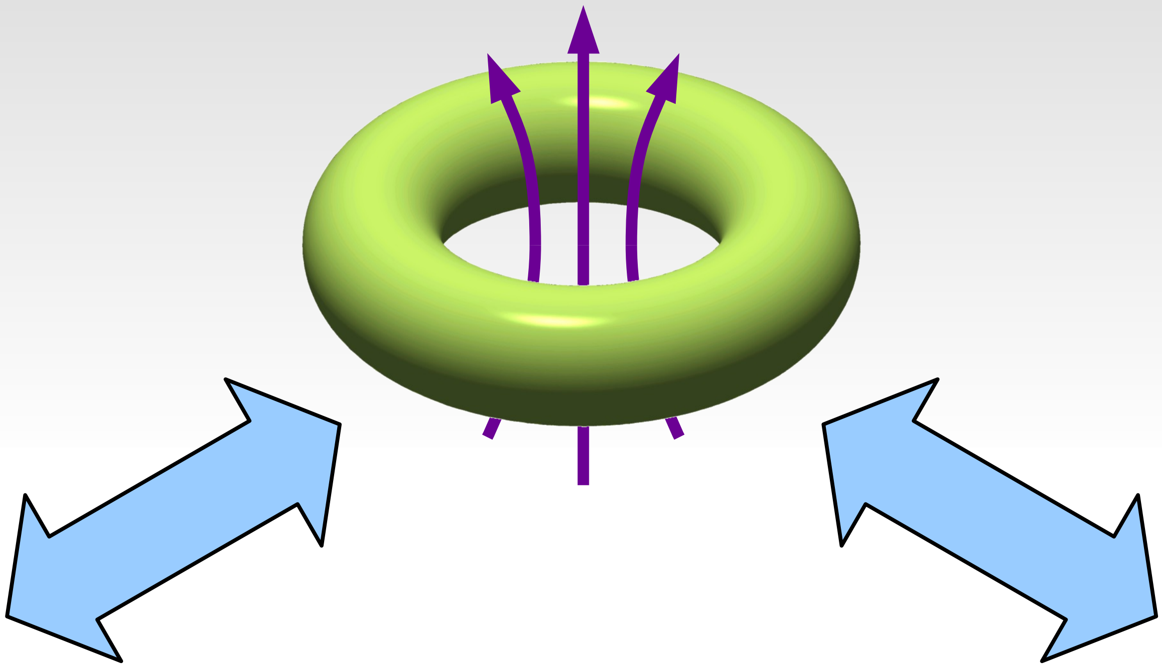
Charge From Entanglement



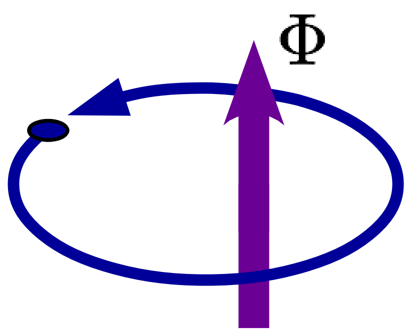
Charge From Entanglement



Charge From Entanglement



Anyon charge

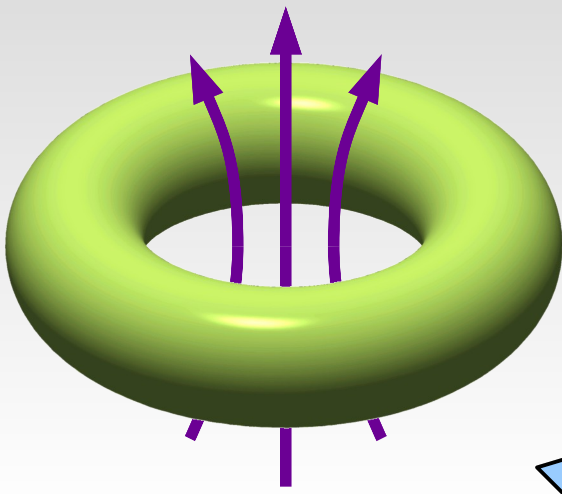


Entanglement spectrum

$$\sum_{\alpha} e^{-\tilde{E}_{\alpha}} Q_{\alpha}$$

Michael Zaletel, RM, Frank Pollmann 2013

Charge From Entanglement



Response to electric field

$$E = -\frac{\partial}{\partial t} A$$

[Resta 1992;
King-Smith, Vanderbilt 1993]

Electric polarization

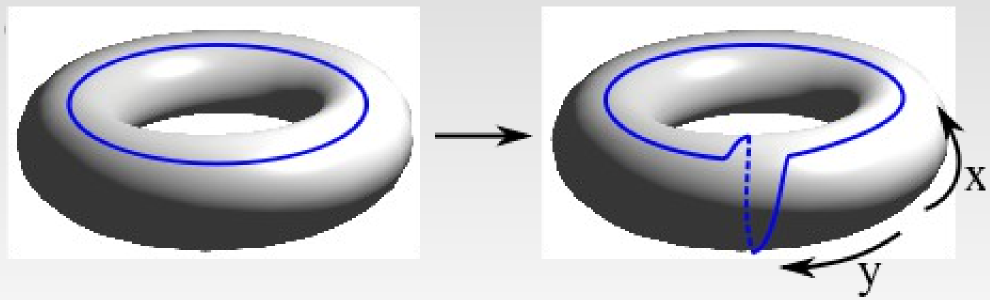
[Turner, Zhang, RM,
Vishwanath 2012]

Quasiparticle
Charges

[Zaletel, RM, Pollmann]

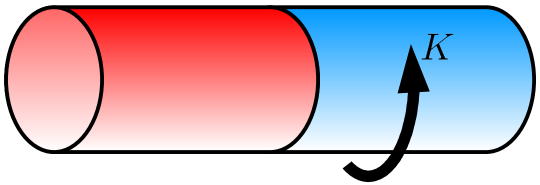
Quantum fluctuations
of charge / Entanglement

Torus (Dehn) Twist

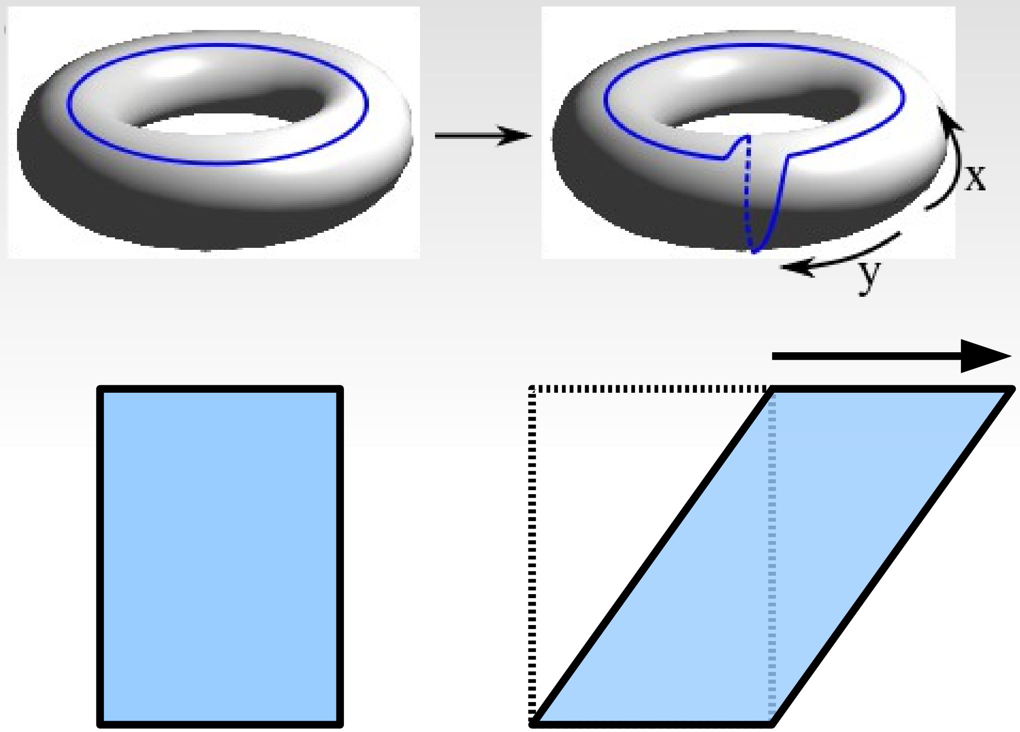


Electric Charge ⟷ Magnetic Flux

Momentum ⟷ Translation

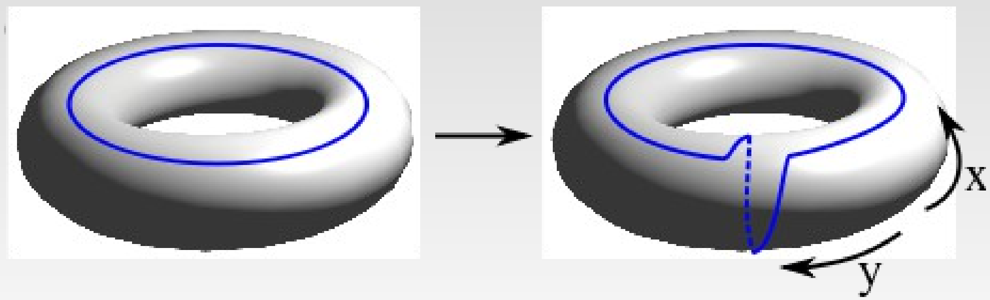


Torus (Dehn) Twist



- 'Momentum polarization'
- Bulk response: Hall viscosity
- Topological response: 'modular T-transformation'

Torus (Dehn) Twist



Berry phase

$$U_T(a) = \exp\left[2\pi i\left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar}L^2 + \cdots\right)\right]$$

Topological spin

Hall viscosity

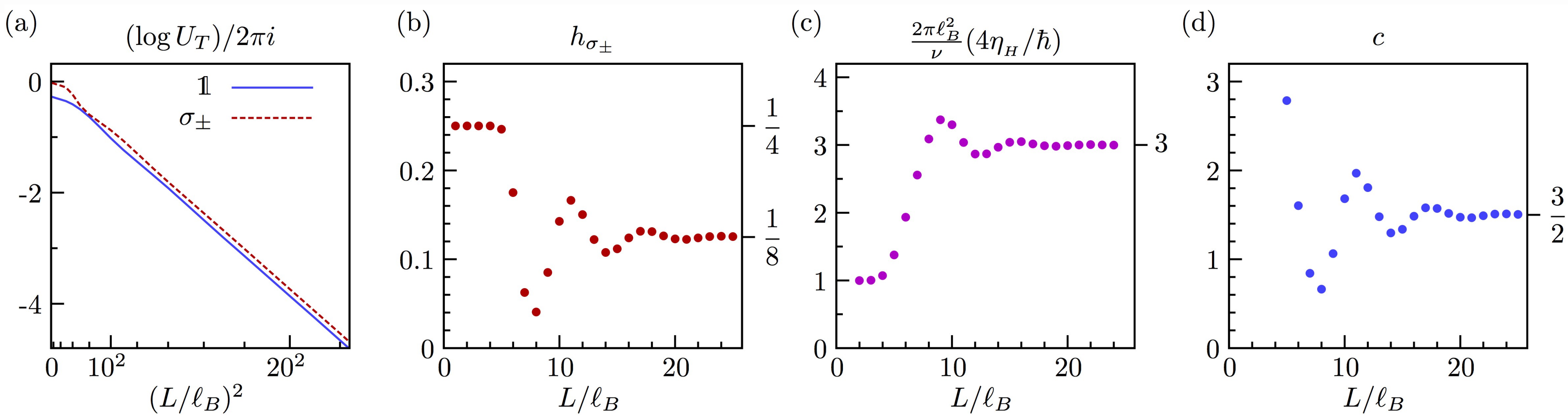
Chiral central charge

Michael Zaletel, RM, Frank Pollmann 2013

Torus (Dehn) Twist

Extract spin, central charge, Hall viscosity

$$U_T(a) = \exp\left[2\pi i\left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar}L^2 + \cdots\right)\right]$$

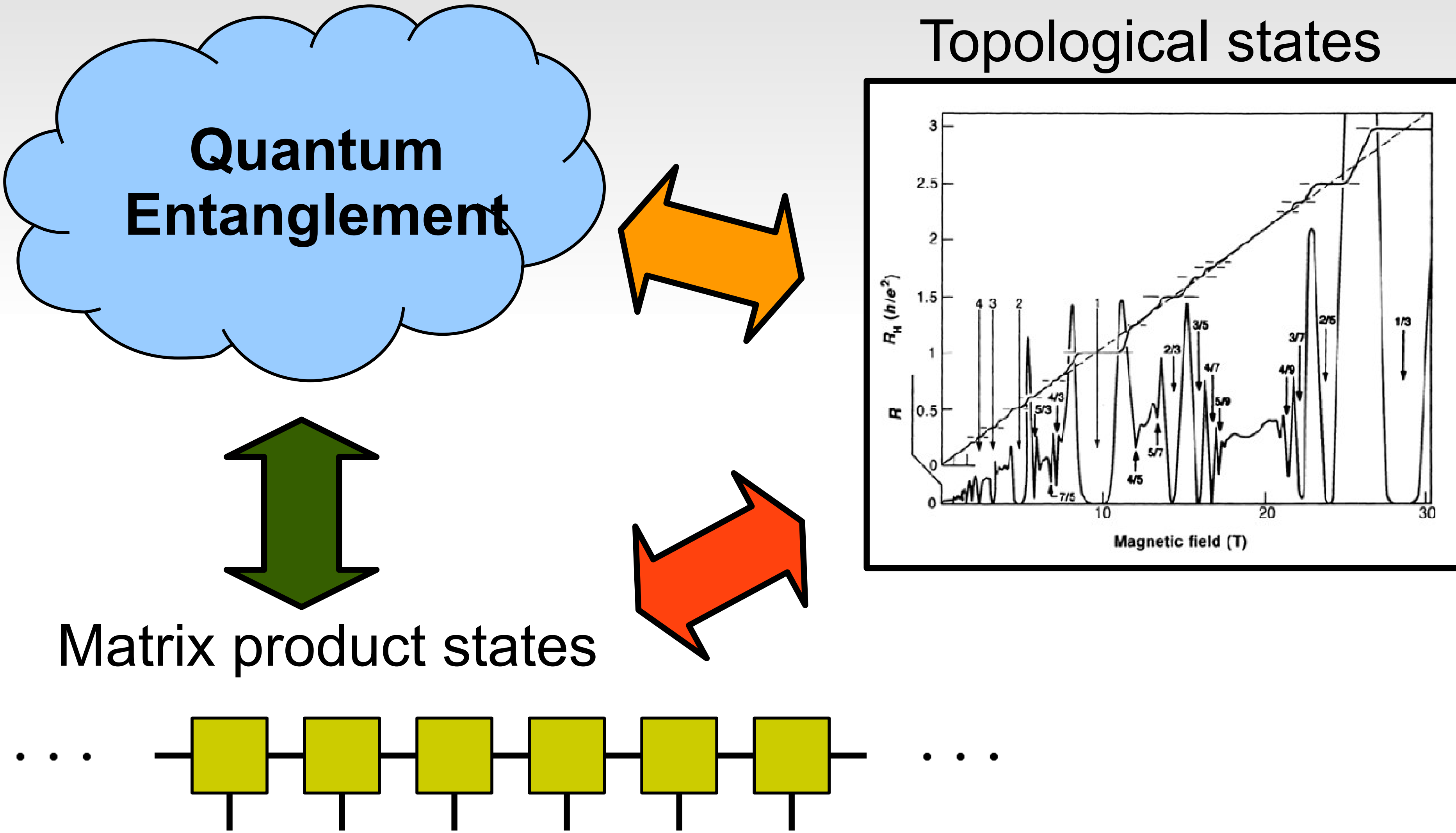


Raw data | Toplogical spin | Hall viscosity | Chiral central charge

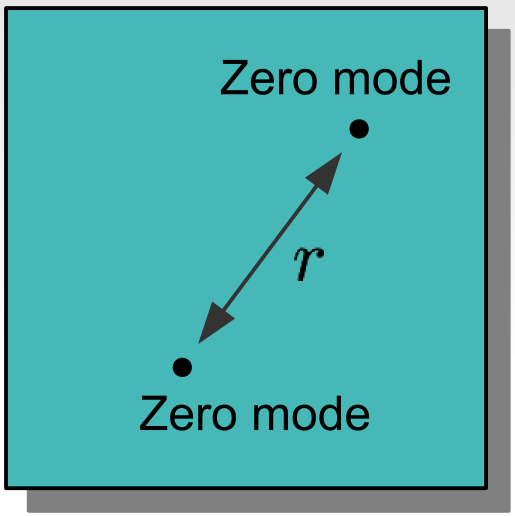
Summary

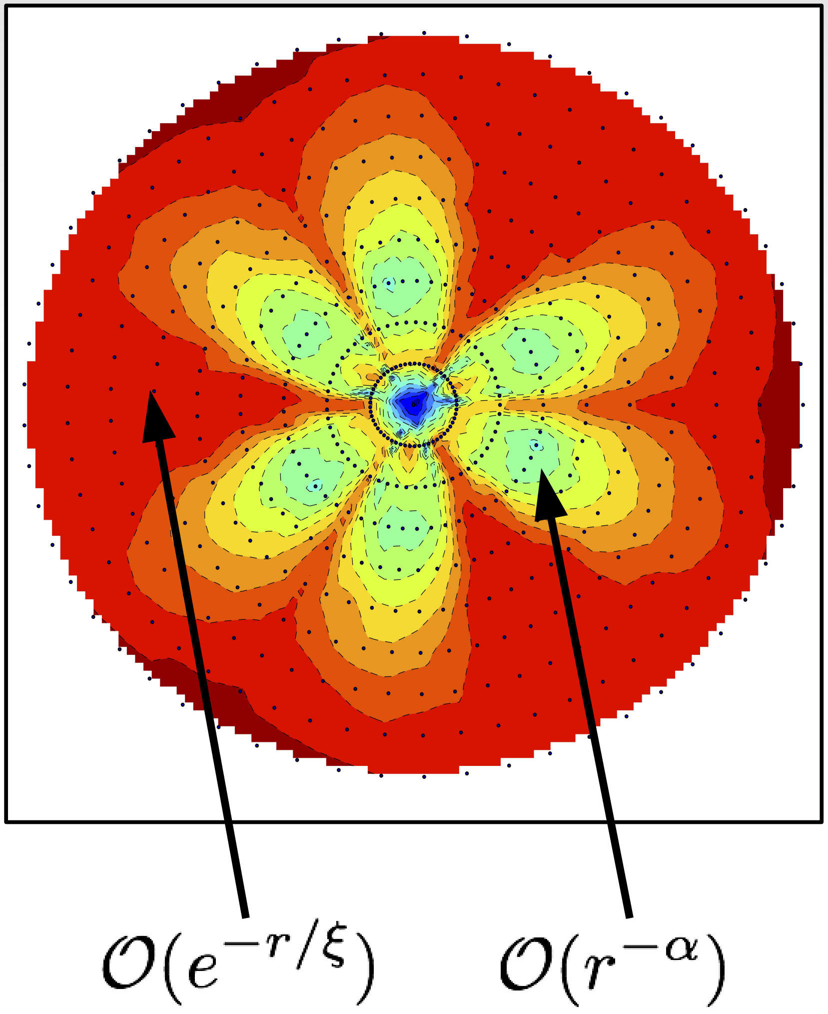
- Why study quantum Hall?
- Quantum entanglement
  - Topological phases
  - Matrix product states (MPS)
- 1. MPS for quantum Hall model wavefunctions
- 2. Modeling physical systems with density matrix renormalization group (DMRG)
- 3. Extracting topological content from ground states

FQH, Entanglement, MPS



Parafermion Zero Modes



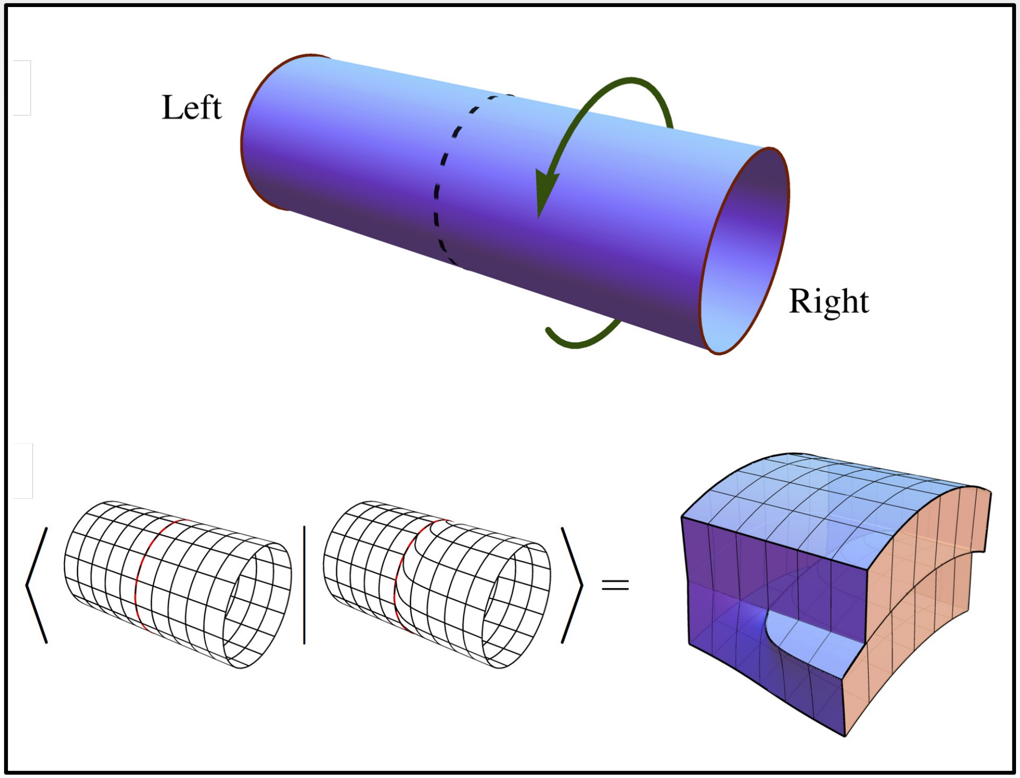


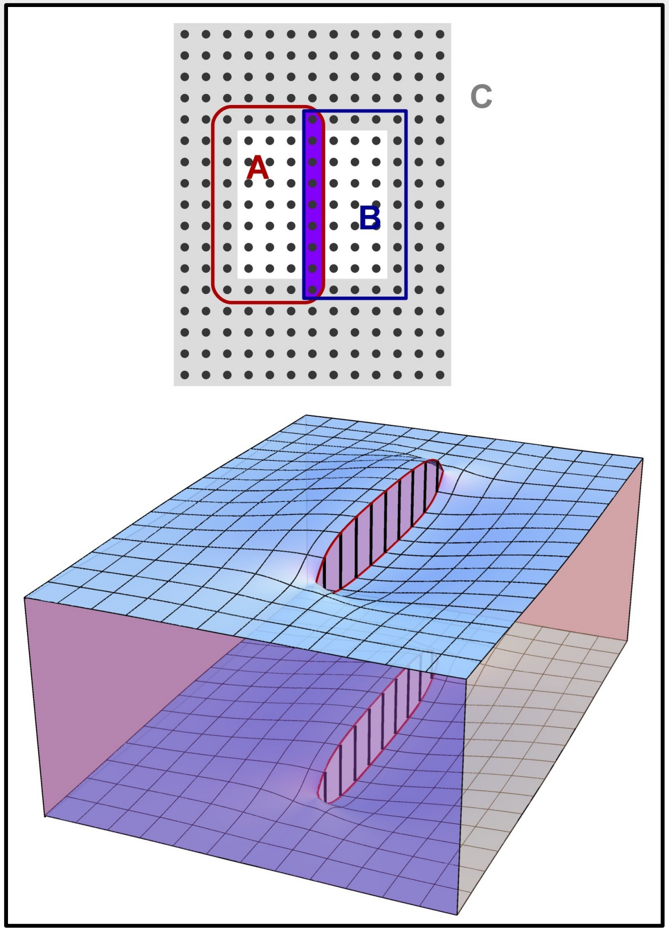
Exponential or algebraic energy separation in the excited spectrum?

Adam Jermyn, RM, Jason Alicea

Gravitational Response

Momentum polarization ⟷ Spacetime deformation

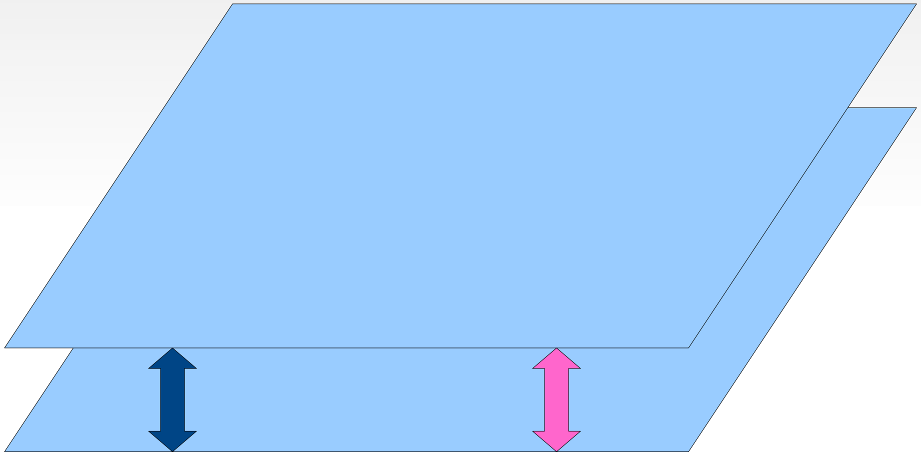




RM, Michael Zaletel, XiaoLiang Qi

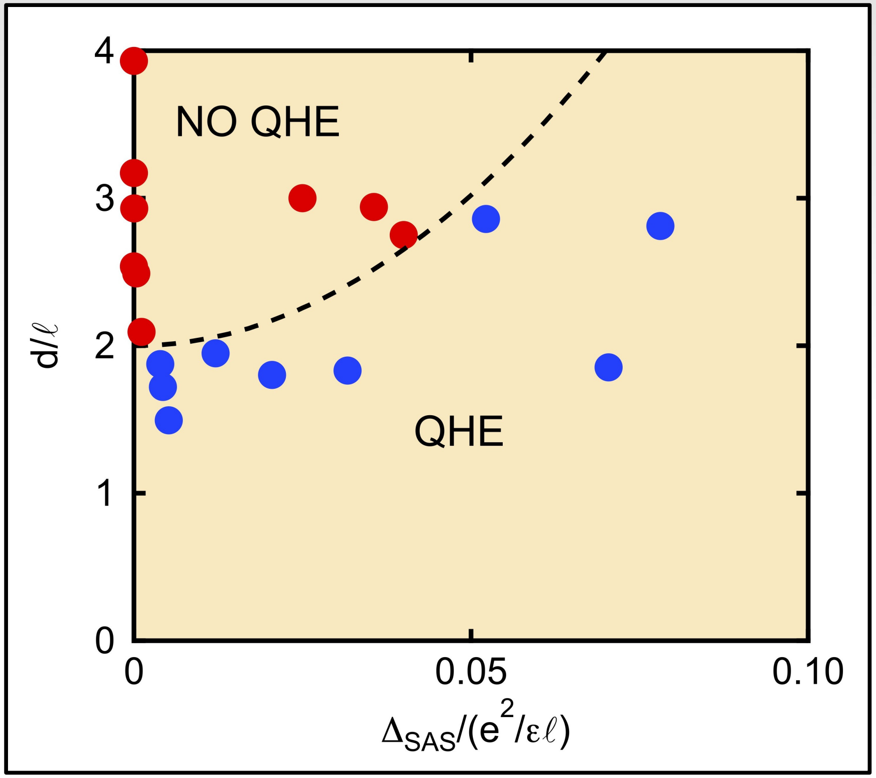
Quantum Hall Bilayer

$$\nu = \frac{1}{2} + \frac{1}{2}$$



Electron tunneling

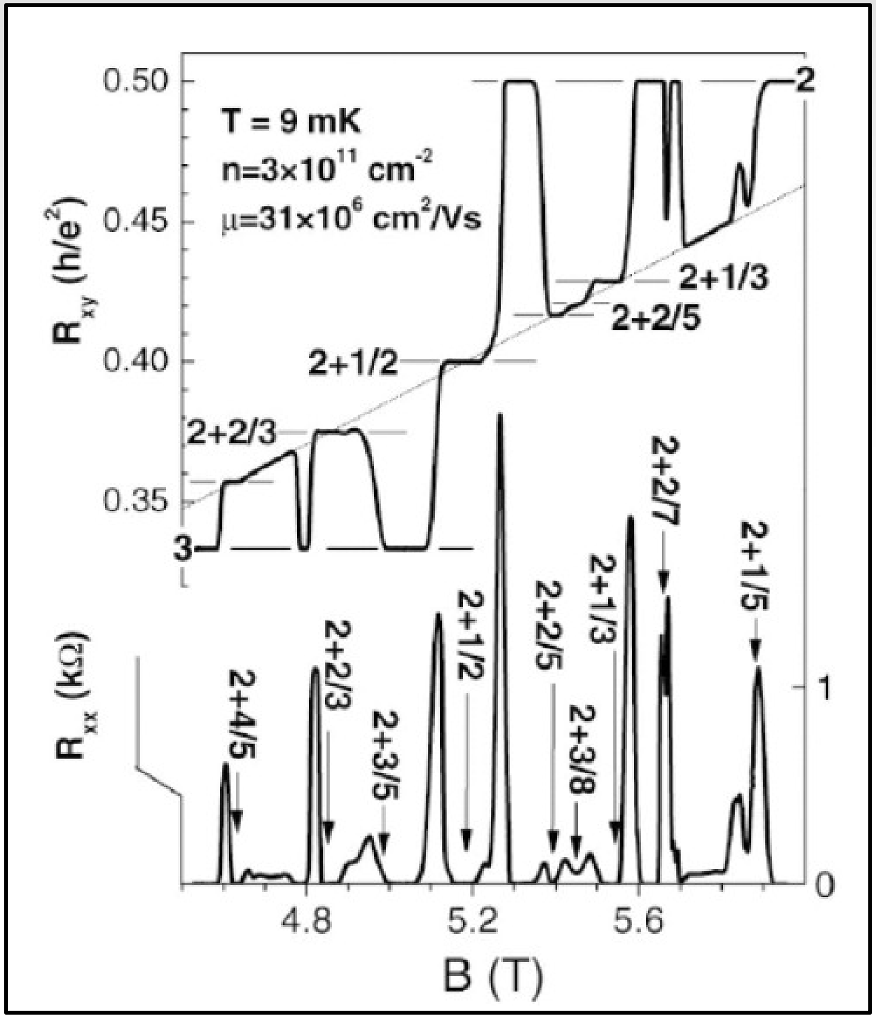
Electron Repulsion



Murphy, Eisenstein, Boebinger, Pfeiffer, West, PRL 72, 728

Can we see this effect for different Landau levels?

Fractional Quantum Hall



Xia et al., PRL 93, 176809

- What are these mysterious phases?
- Does the 5/2 or 12/5 state support non-Abelian anyons?
- Can we use these states for quantum computing?
- Can we do 'ab-initio' for strongly correlated states?