# Quantum computing with hypercubes of light

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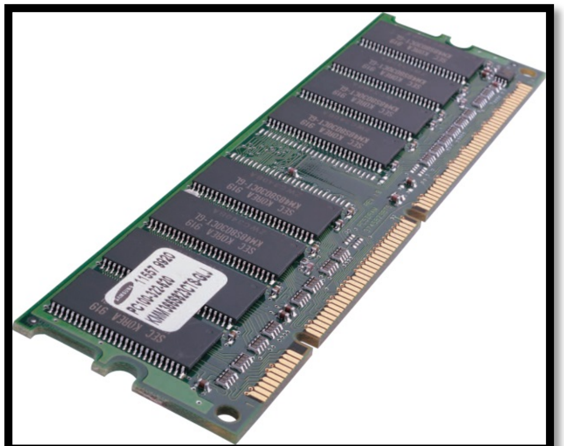
2/21/2014

# Classical & Quantum Computers

**Register** **Processor**

**input** →



→

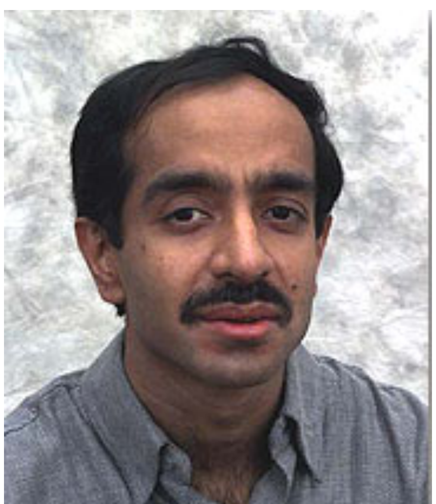

→ **output**

| Computer Type | Register | Processor |
|---|---|---|
| Classical Computer | bits | Classical gates |
| Quantum Computer | qubits | Quantum gates:<br>• Circuit-based<br>• Measurement-based |
| | continuous variables | |

# Why quantum computers?

- **R.Feynman**, 1982:
  simulating quantum physics

- **P.Shor**, 1994:
  integer factoring: $O((\mathrm{Log}\ N)^{3})$



- **L.Grover**,1996:
  unsorted database searching: $O(N^{1/2})$

- **Test** whether the quantum mechanical principles still hold in HUGE entangled systems

# Qubits: the quantum bits

- Qubits: any two-level systems:

$$\{ \, |0\rangle, |1\rangle \, \}$$

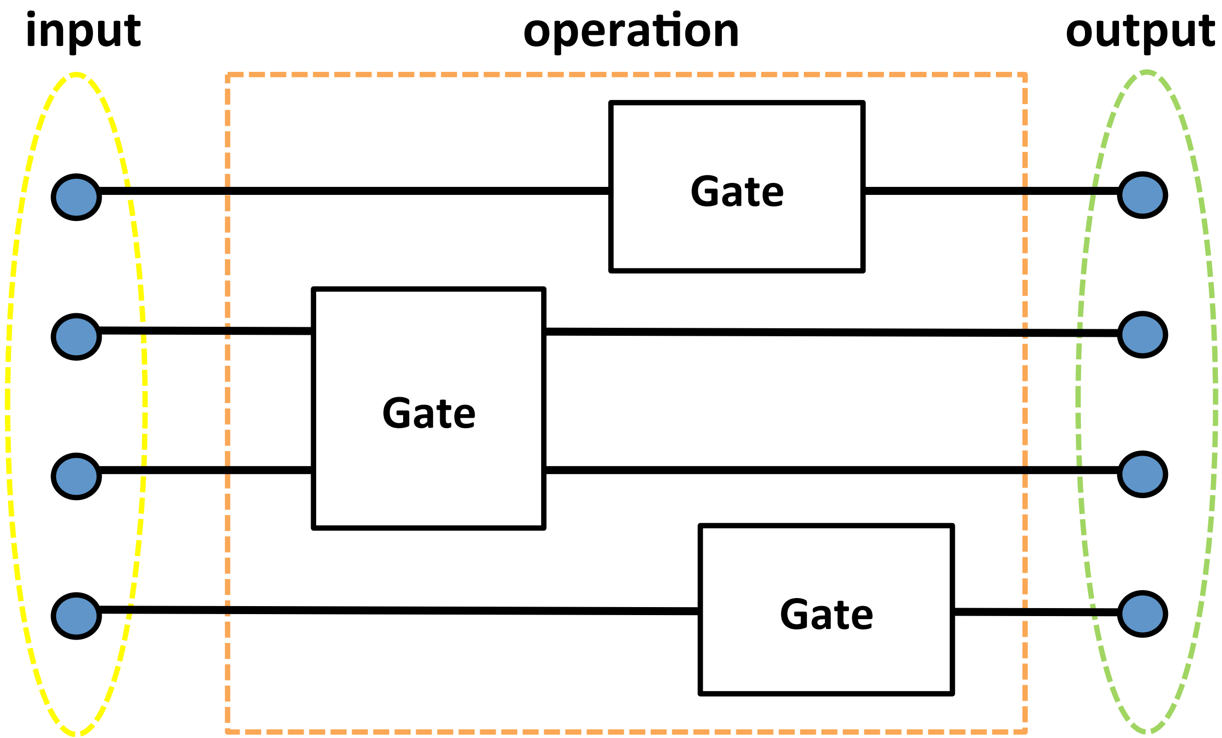
- Superposition:

$$\alpha|0\rangle + \beta|1\rangle$$

- Entanglement:

$$1/\sqrt{2} \ \ (|00\rangle\downarrow 12 + |11\rangle\downarrow 12 \,)$$

# Circuit-based computers




Any N-qubit gate can always be decomposed to combinations of *single*- and *two*-qubit gates.

# Quantum gates

Hadamard gate:

$$|0\rangle \;-\boxed{H}-\; \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle \quad ; \quad |1\rangle \;-\boxed{H}-\; \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$

Pauli Z:

$$|0\rangle \;-\boxed{Z}-\; |0\rangle \quad ; \quad |1\rangle \;-\boxed{Z}-\; -|1\rangle$$

Pauli X:

$$|0\rangle \;-\boxed{X}-\; |1\rangle \quad ; \quad |1\rangle \;-\boxed{X}-\; |0\rangle$$

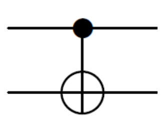
Rotation around Z:

$$|\pm\rangle \;-\boxed{R_Z(\theta)}-\; \frac{1}{\sqrt{2}}(e^{-i\frac{\theta}{2}}|0\rangle \pm e^{i\frac{\theta}{2}}|1\rangle)$$

Controled-NOT(CNOT):

$|k\rangle_1$

$|l\rangle_2$

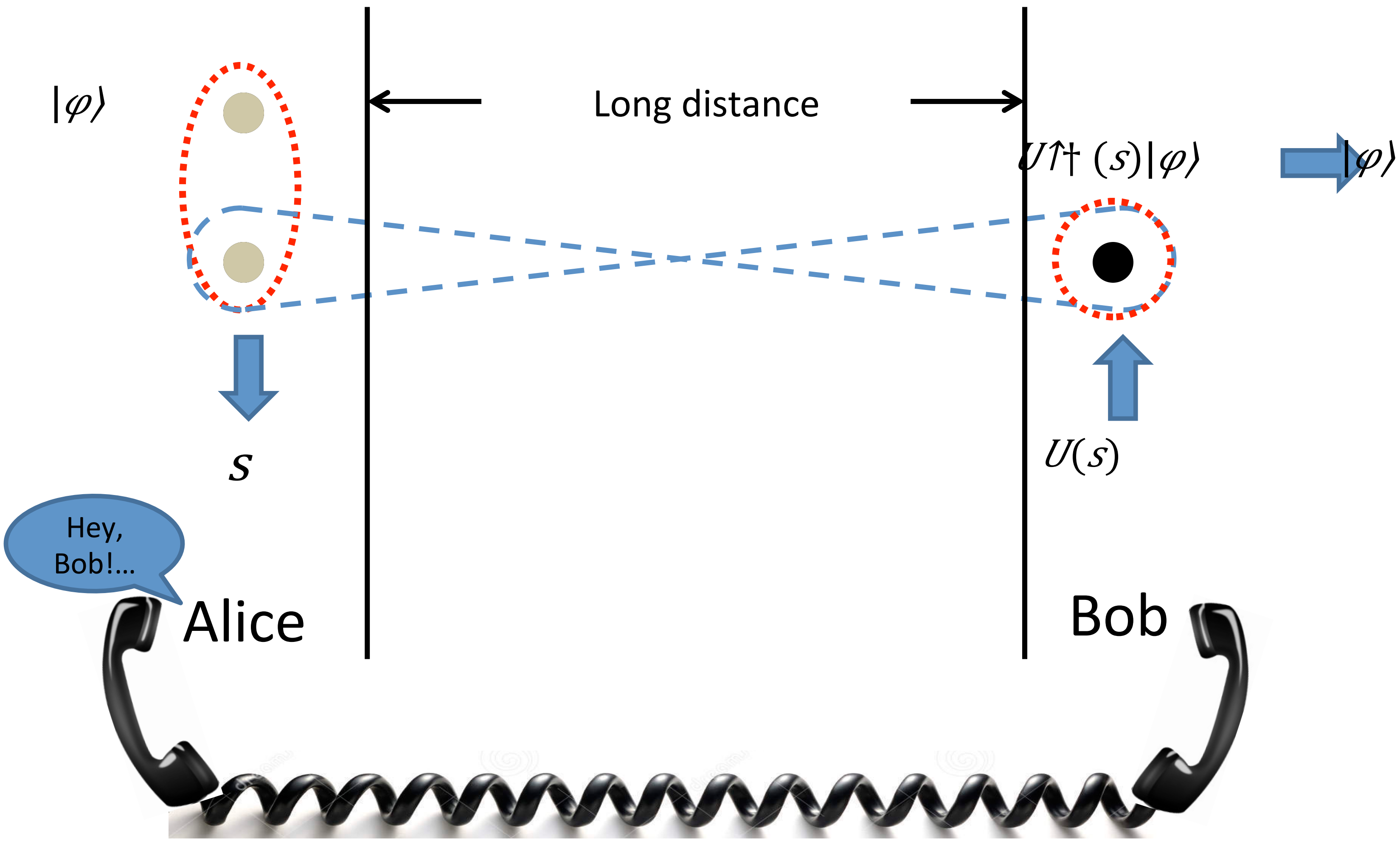


$|k\rangle_1$

$|k \bigoplus l\rangle_1$

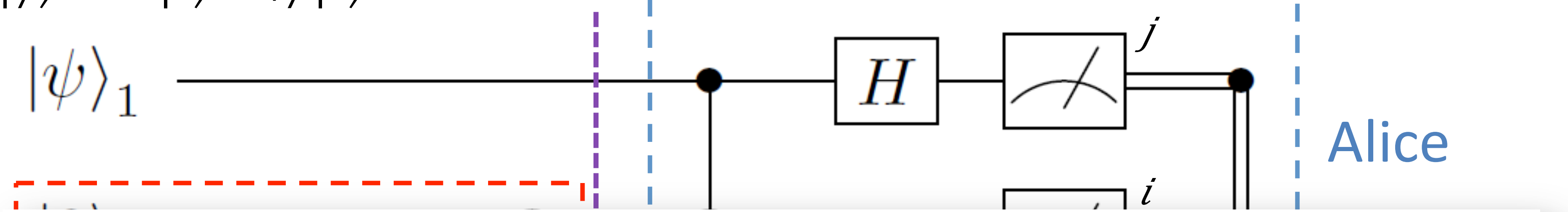
A universal set of gates: $\{H, R_Z(\frac{\pi}{2}), R_Z(\frac{\pi}{4}), \mathrm{CNOT}\}$
This set is able to approximate any unitaries to arbitrary accuracy.

# Quantum teleportation



# The quantum circuit for teleportation is:

$|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$




# PHYSICAL REVIEW LETTERS


VOLUME 70 | 29 MARCH 1993 | NUMBER 13

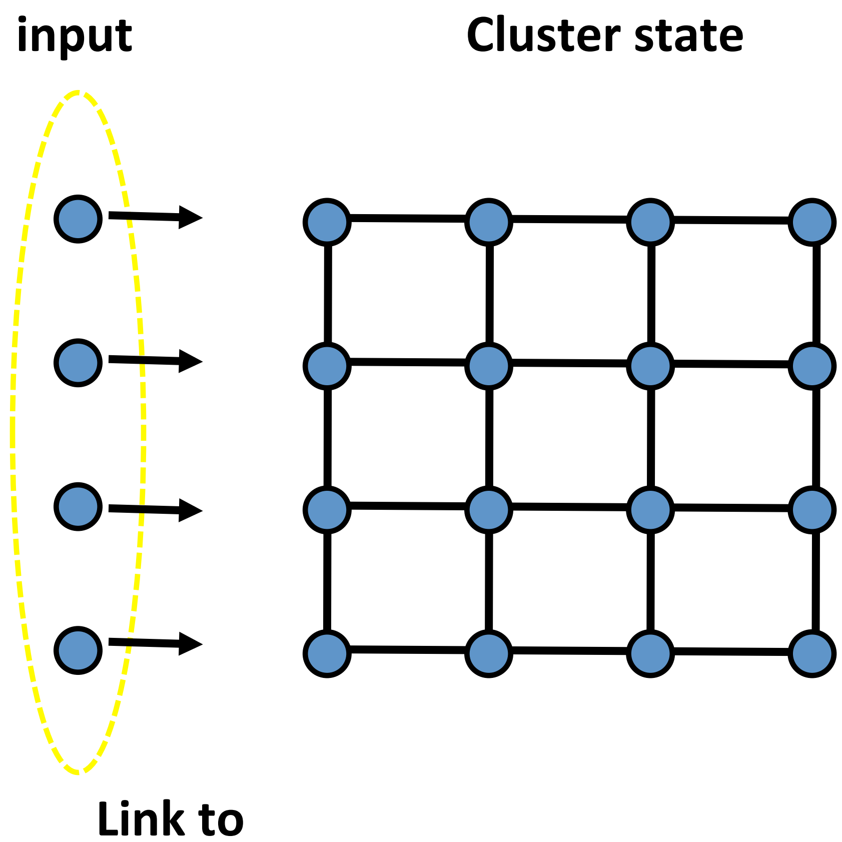

$= \sum_{i,j\in\{0,1\}} |\beta_{ij}\rangle_{12} Z_3$

**Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels**

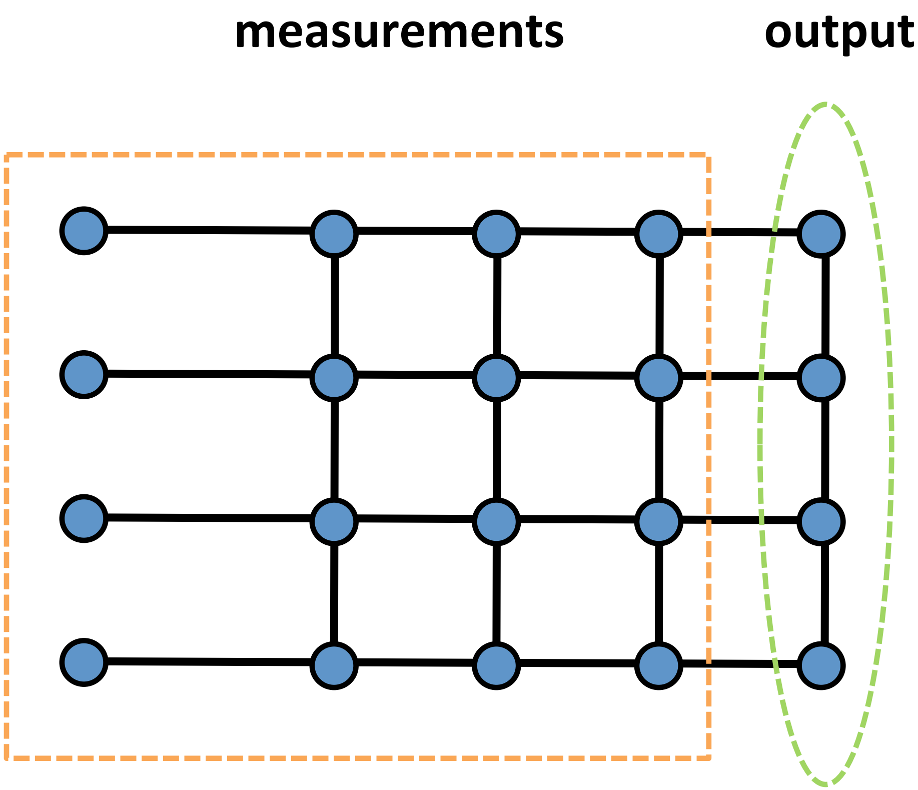

Charles H. Bennett,[1] Gilles Brassard,[2] Claude Crépeau,[2],[3] Richard Jozsa,[2] Asher Peres,[4] and William K. Wootters[5]


| | | |
|---|---|---|
| $\lvert\beta_{00}\rangle_{12} = \lvert 00\rangle_{12} + \lvert 11\rangle_{12}$ | $\alpha\lvert 0\rangle_3 + \beta\lvert 1\rangle_3$ | Do nothing |
| $\lvert\beta_{10}\rangle_{12} = \lvert 00\rangle_{12} - \lvert 11\rangle_{12}$ | $\alpha\lvert 0\rangle_3 - \beta\lvert 1\rangle_3$ | $Z_3$ |
| $\lvert\beta_{01}\rangle_{12} = \lvert 01\rangle_{12} + \lvert 10\rangle_{12}$ | $\alpha\lvert 1\rangle_3 + \beta\lvert 0\rangle_3$ | $X_3$ |

# Measurement-based quantum computers

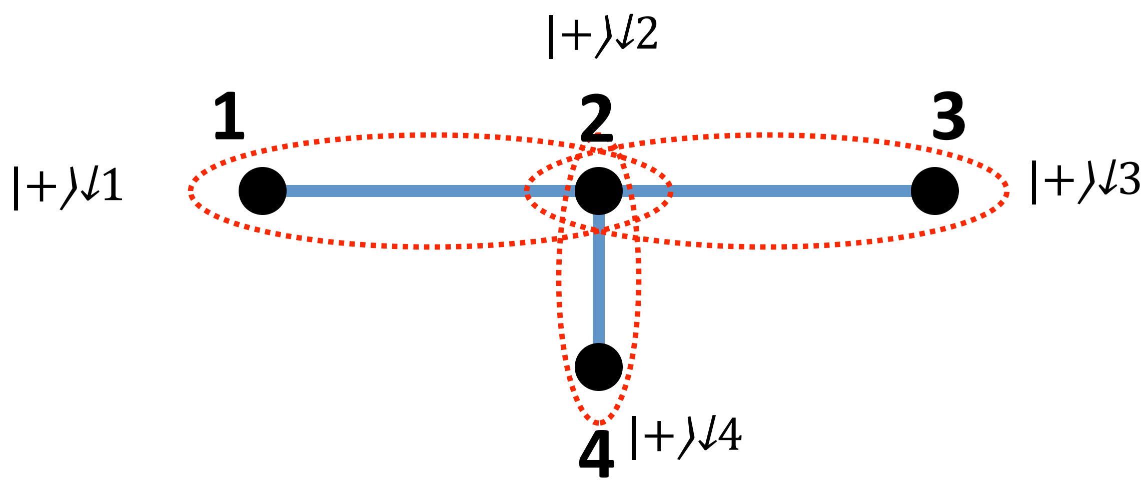


# Measurement-based quantum computers



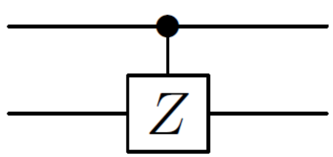

Also known as: One-way quantum computer

# Cluster states




Controled-Z($C_Z$):

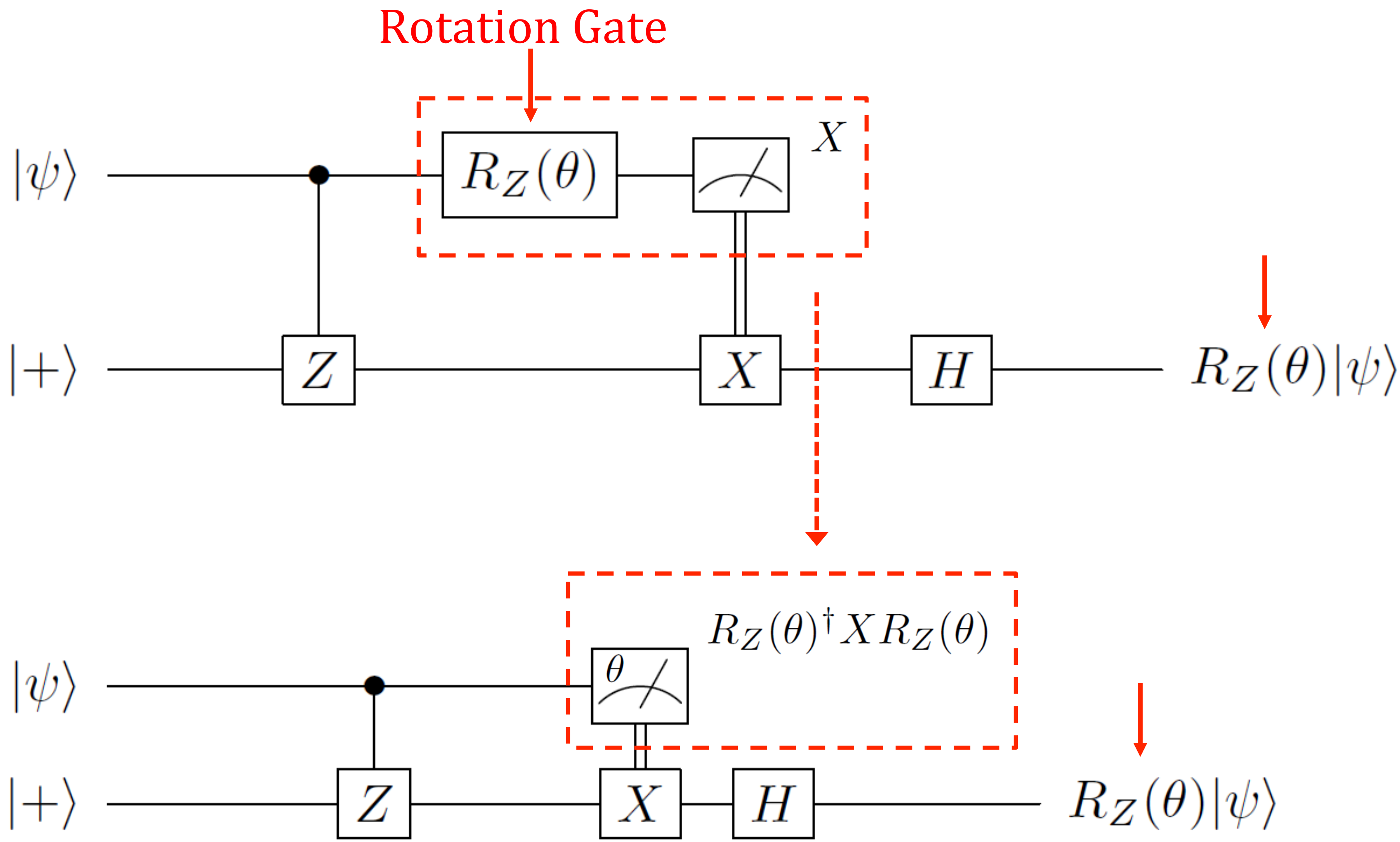
$|k\rangle_1 |l\rangle_2$



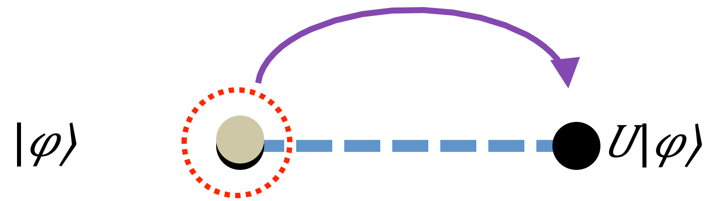

$(-1)^{kl}|k\rangle_1 |l\rangle_2$

$|\varphi\rangle = |+0++\rangle - |-1--\rangle$

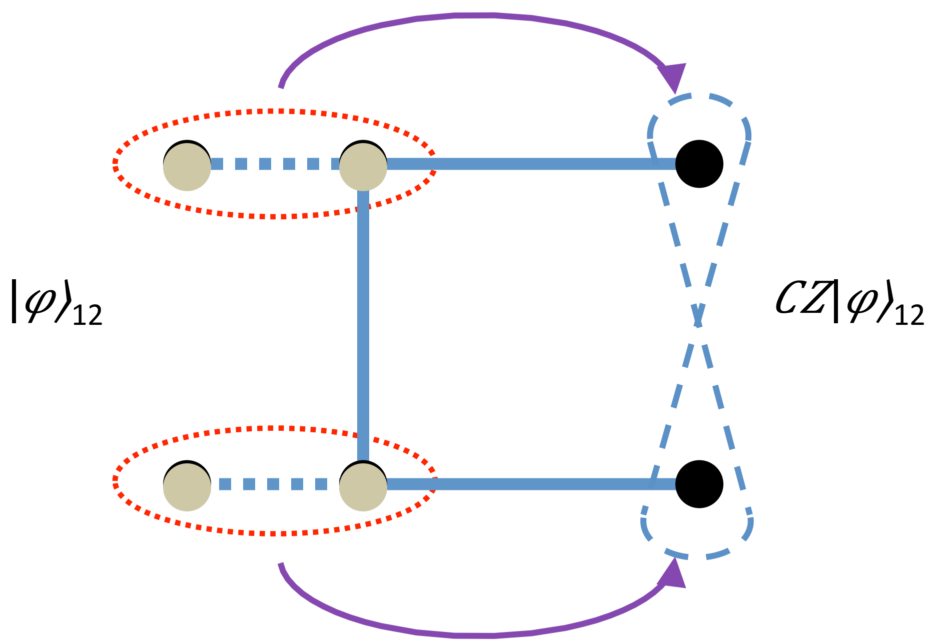
- Any gates can be applied by using a cluster state and measurements!


Rotation Gate
$|\psi\rangle$
$R_Z(\theta)$
$X$
$|+\rangle$
$Z$
$X$
$H$
$R_Z(\theta)|\psi\rangle$
$R_Z(\theta)^\dagger X R_Z(\theta)$
$|\psi\rangle$
$\theta$
$|+\rangle$
$Z$
$X$
$H$
$R_Z(\theta)|\psi\rangle$

# Apply two-qubit gates by cluster state

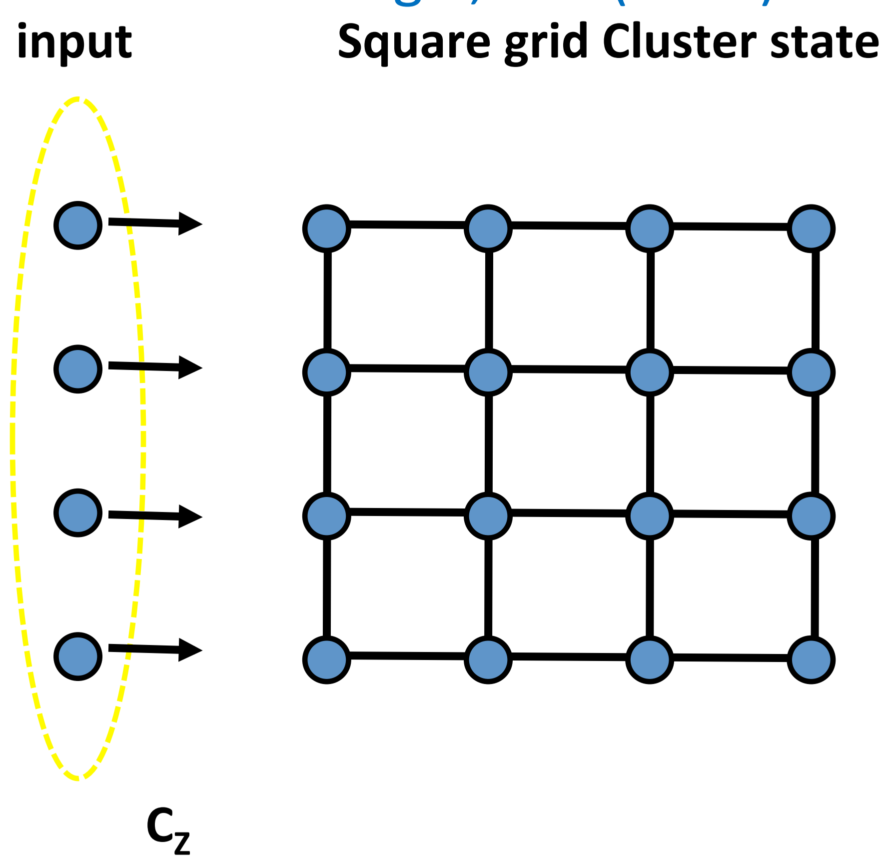



Universal! What kind of cluster state is sufficient?



Universal QC is achieved with a *square grid* cluster state:

R.Raussendorf and H.J.Briegel, PRL (2001)



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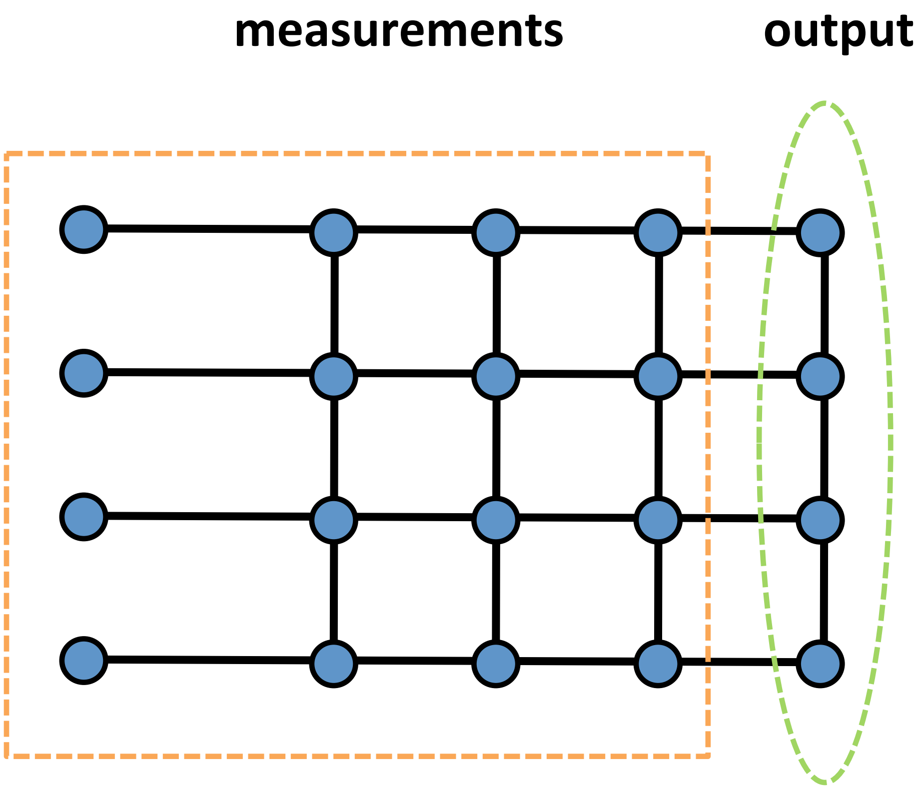
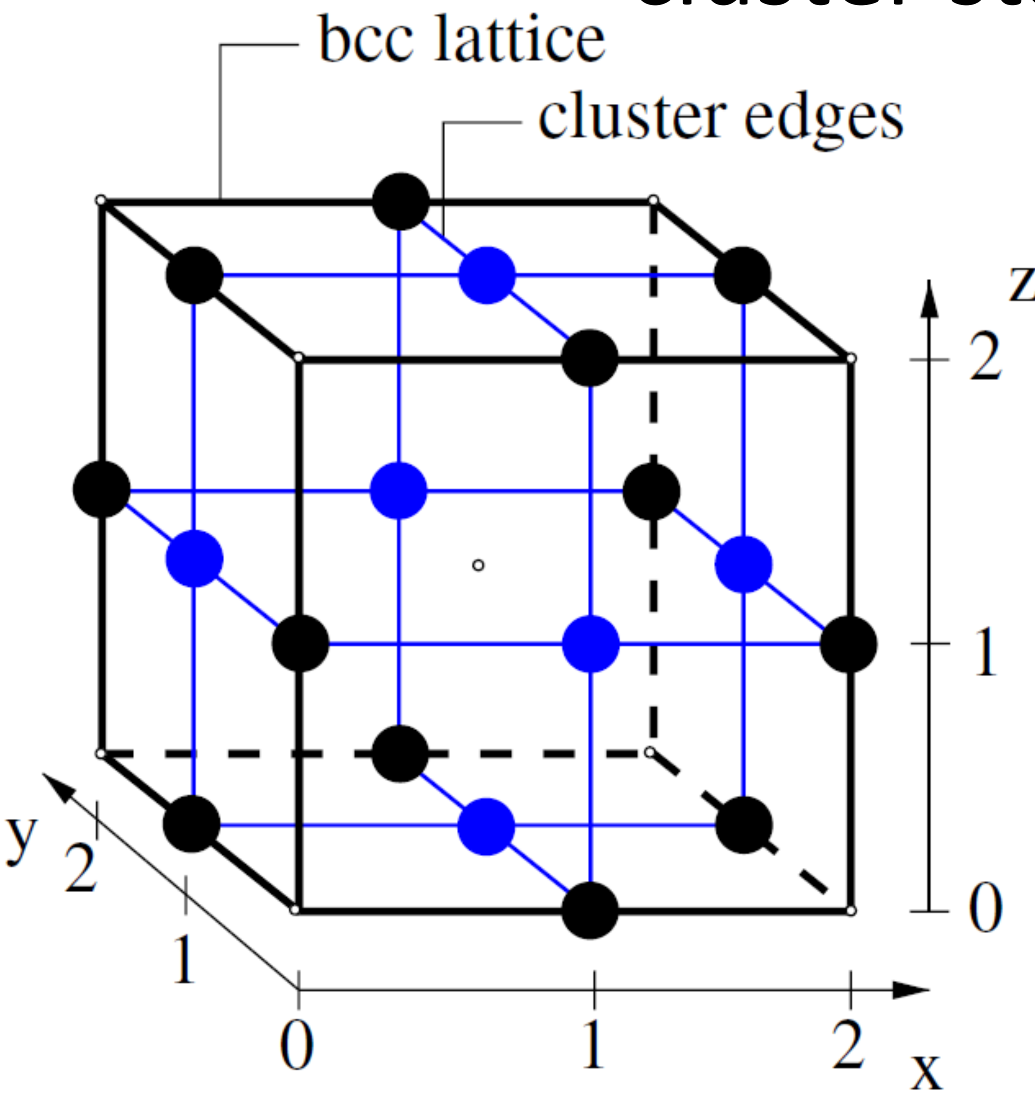


# Quantum error correction using cluster states




- Measurements: *Cubic lattice Cluster State* into 3-D *Kitaev surface code state*

- Topological quantum error correction: R.Raussendorf *et al*, Ann. Phys.(N.Y.) (2006)

# Progress of optical qubit cluster states

- Realization of Shor's algorithm(factorizing 15):

  C.Lu *et al*., PRL (2007)

  B.P.Lanyon *et al*., PRL (2007)

- Blind quantum computing:

  S.Barz *et al*., Science (2012)

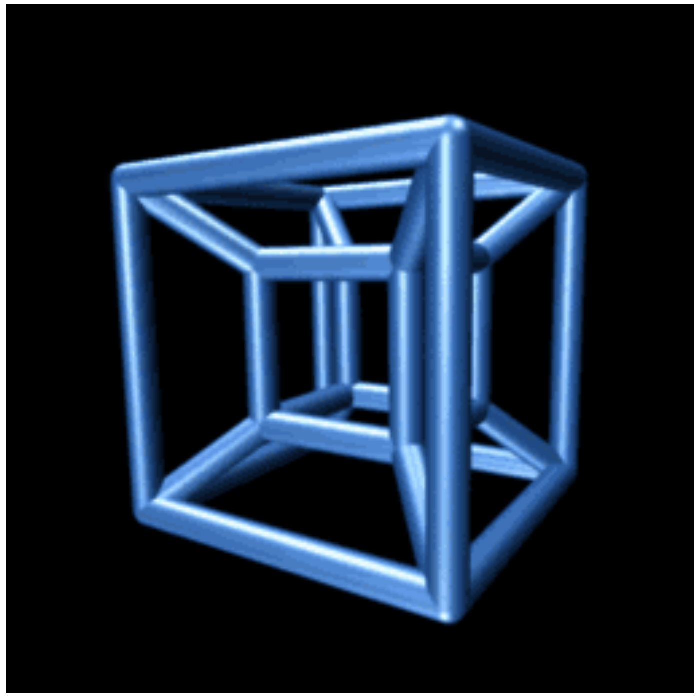
- Topological quantum computing:

  Y.Han *et al*.,PRL (2007)

- Topological error correction:

  X.Yao *et al*., Nature (2012)

# Hypercubic lattice cluster states



Tesseract (4-D hypercube)
Figure from http://en.wikipedia.org/wiki/Tesseract

- We know how to make them in our lab!
- But with *continuous variables* rather than qubits.

# Continuous variables

- Qubits: discrete variables
- Continuous variables: continuous systems, such as electromagnetic field quadratures
  - Amplitude: $q = 1/\sqrt{2}\ (a + a^\dagger)$
  - Phase: $p = 1/i\sqrt{2}\ (a - a^\dagger)$

Harmonic oscillator: $\{x, p\} \leftrightarrow \{q, p\}$

$a$ & $a^\dagger$: field annihilation & creation operators

$[q, p] = i\hbar$

| | Discrete Variables(DV) | Continuous Variables(CV) |
|---|---|---|
| Basis | { } | { , } |
| General states | | |
| Conjugate basis | | |
| Bipartite maximally Entangled states | | |
| Single-qubit (mode) gates | Pauli<br>Pauli $X$ | WH<br>WH $X(\xi)=e^{-i\xi p}$ |
| Two-qubit (mode) gate | | |

# EPR state & two-mode squeezed state

- $$|EPR\rangle_{12} = \int_{-\infty}^{+\infty} |q\rangle_1 |q\rangle_2 \, dq = \int_{-\infty}^{+\infty} |p\rangle_1 |-p\rangle_2 \, dp = \sum_{n=0}^{\infty} |n\rangle_1 |n\rangle_2$$

  Nullifiers:
  $$(q_1 - q_2)|EPR\rangle_{12} = 0|EPR\rangle_{12}$$
  $$(p_1 + p_2)|EPR\rangle_{12} = 0|EPR\rangle_{12}$$

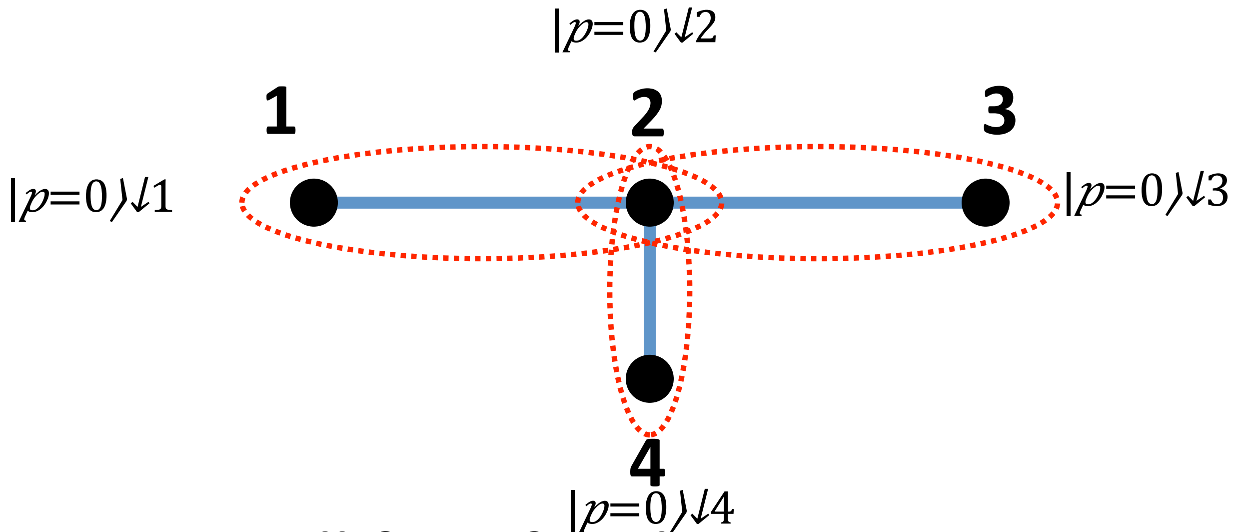
  $$\Delta(q_1 - q_2)^2 = \Delta(p_1 + p_2)^2 = 0$$

- $$|\Psi(r)\rangle_{12} = 1/\cosh^2 r \sum_{n=0}^{\infty} \tanh^n r \, |n\rangle_1 |n\rangle_2 \xrightarrow{r\to\infty} |EPR\rangle_{12}$$

  Approximate nullifiers:
  $$(q_1 - q_2)|\Psi(r)\rangle_{12} = e^{-r}|\Psi(r)\rangle_{12} \xrightarrow{r\to\infty} 0|EPR\rangle_{12}$$
  $$(p_1 + p_2)|\Psi(r)\rangle_{12} = e^{-r}|\Psi(r)\rangle_{12} \xrightarrow{r\to\infty} 0|EPR\rangle_{12}$$

  $$\Delta(q_1 - q_2)^2 = \Delta(p_1 + p_2)^2 = e^{-2r} \xrightarrow{r\to\infty} 0$$

# CV Cluster states




Nullifiers for cluster states:

$$p_i - \sum_{j \in \aleph(i)} q_j \;\;,\;\; i \in [1,N]$$

$$\{p_1 - q_2 \,,\; p_2 - q_1 - q_3 - q_4 \,,\; p_3 - q_2 \,,\; p_4 - q_2 \}$$

# Progress of optical CV cluster states

- CV Teleportation:

  A.Furusawa *et al*., Science (1998)

- CV error threshold for quantum error correction:

  N.C.Menicucci, arXiv: 1310.7596

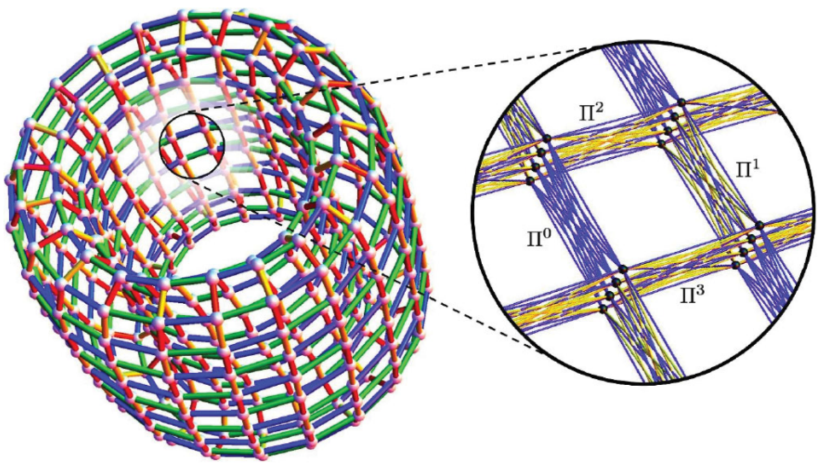
- Detecting topological entropy:

  T.F.Demarie *et al*., arXiv: 1305.0409

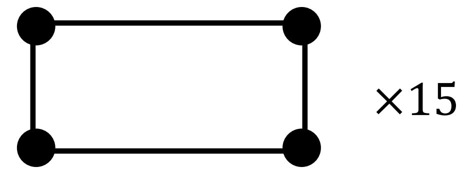
- Scalable CV cluster states:

  *See next slide*

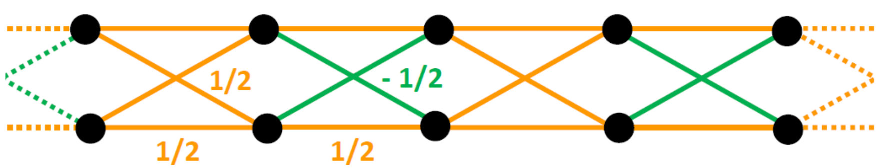
# Scalable CV cluster states



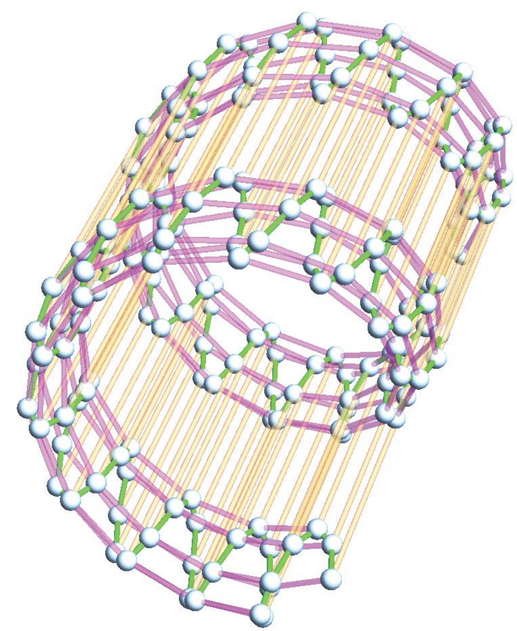

N.C.Menicucci, S.T.Flammia, O.Pfister. PRL 2008




M.Pysher, Y.Miwa, R.Shahrokhshahi, R.Bloomer, O.Pfister, PRL 2011

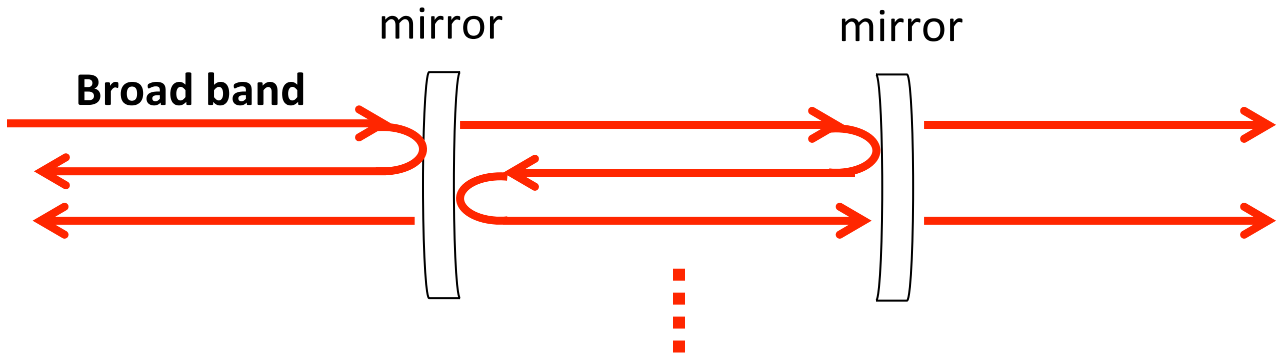


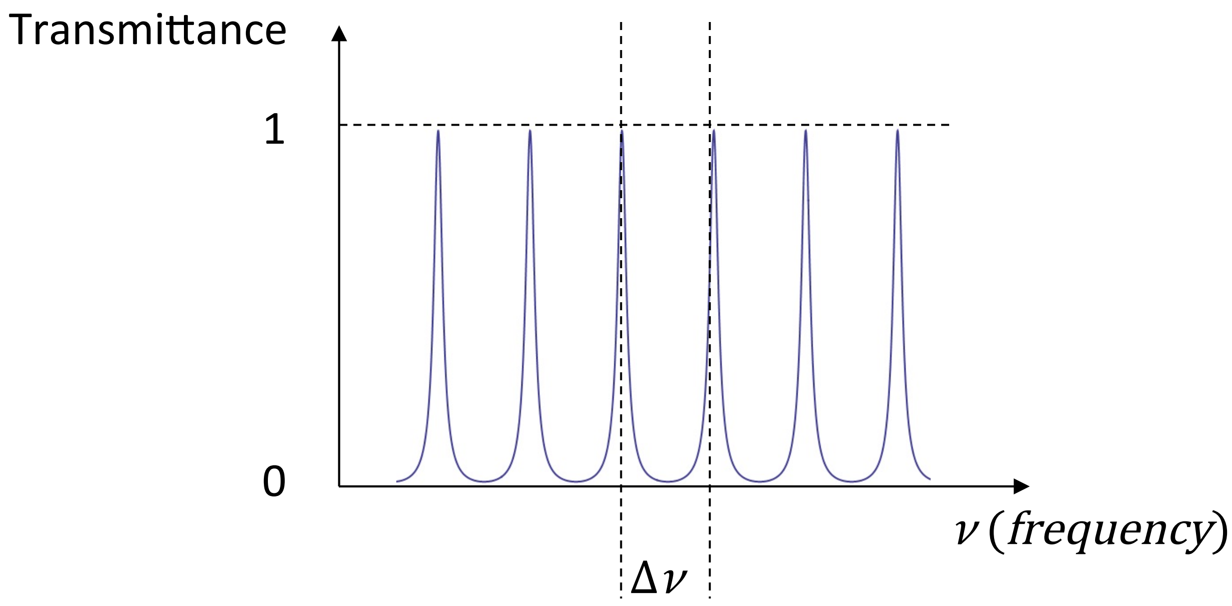

- N.C.Menicucci, PRL 2010
- M.Chen, N.C.Menicucci, O.Pfister, arXiv:1311.2957, 2013
- S.Yokoyama et al., Nat. Photonics 2013



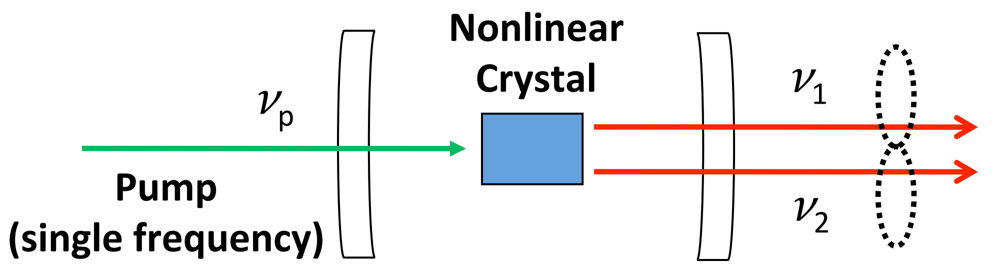
P.Wang, M.Chen, N.C.Menicucci,and O.Pfister, arXiv:1309.4105, 2013

# Optical cavity



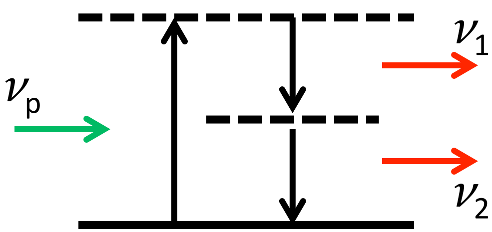



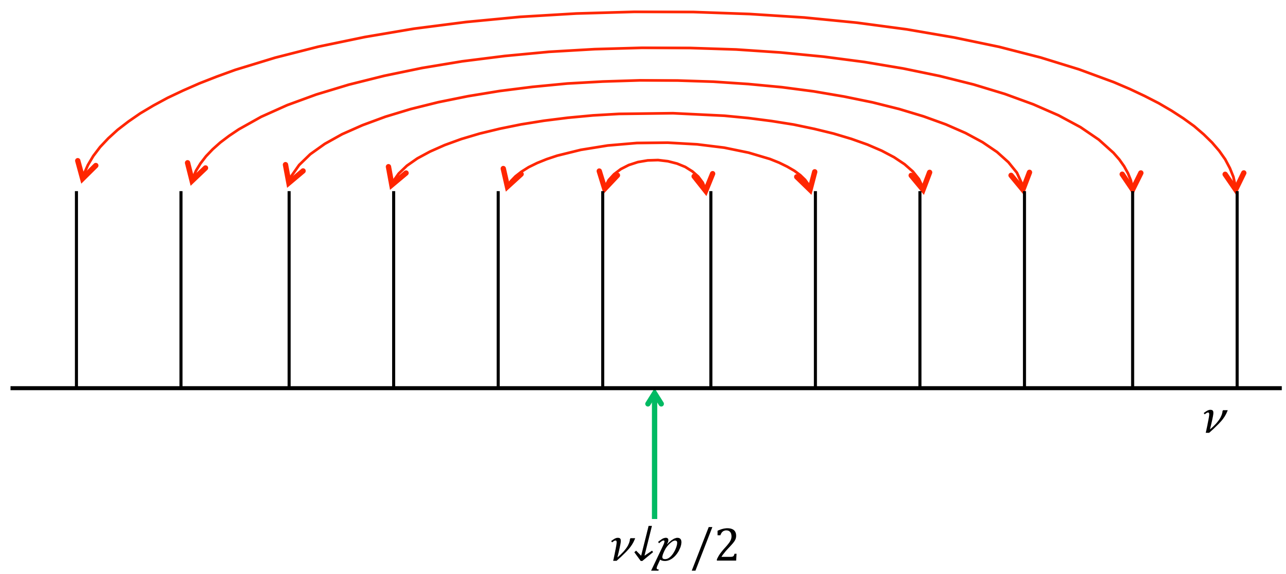
# Optical parametric oscillator(OPO)

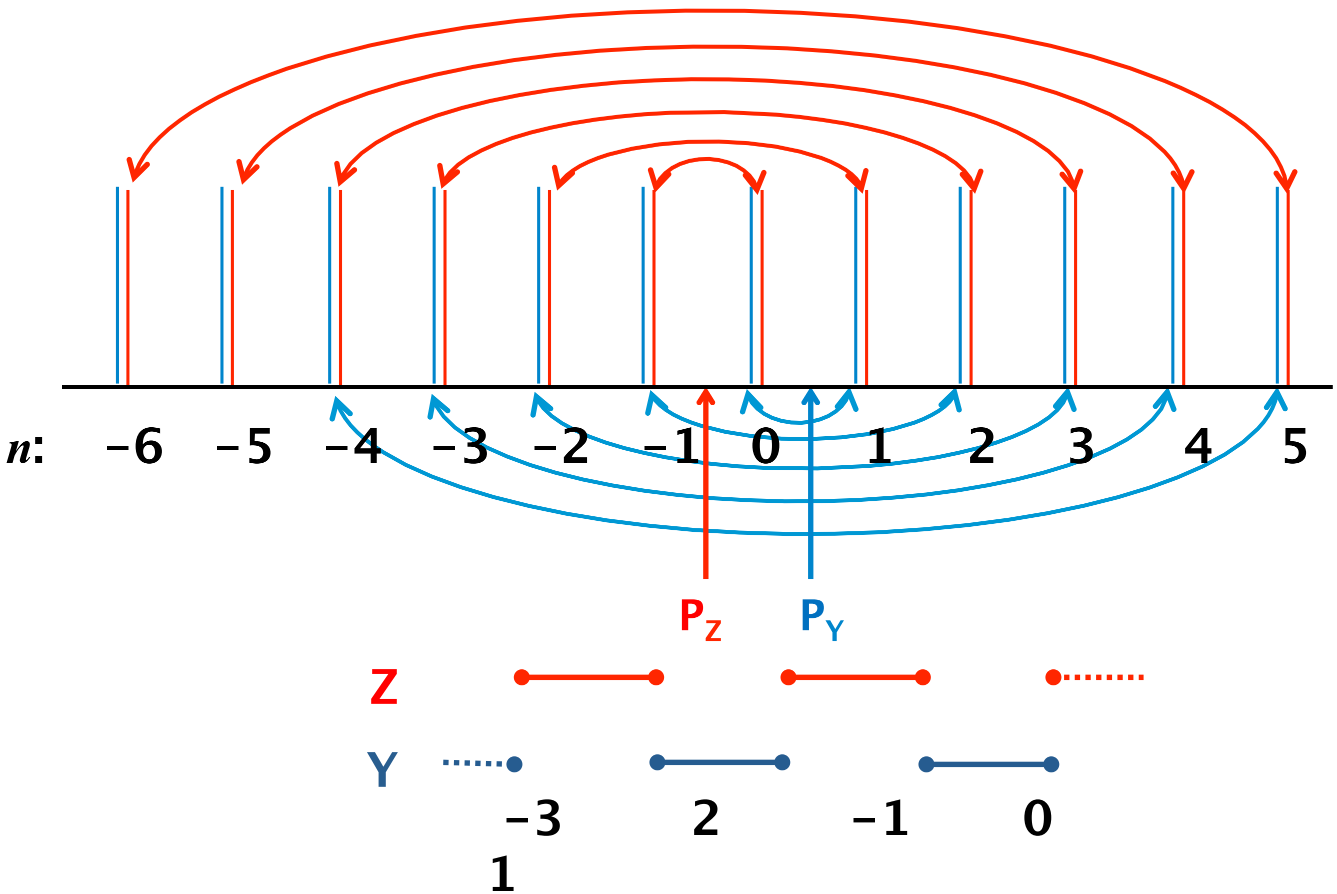



TMSS(or approx. EPR pairs)

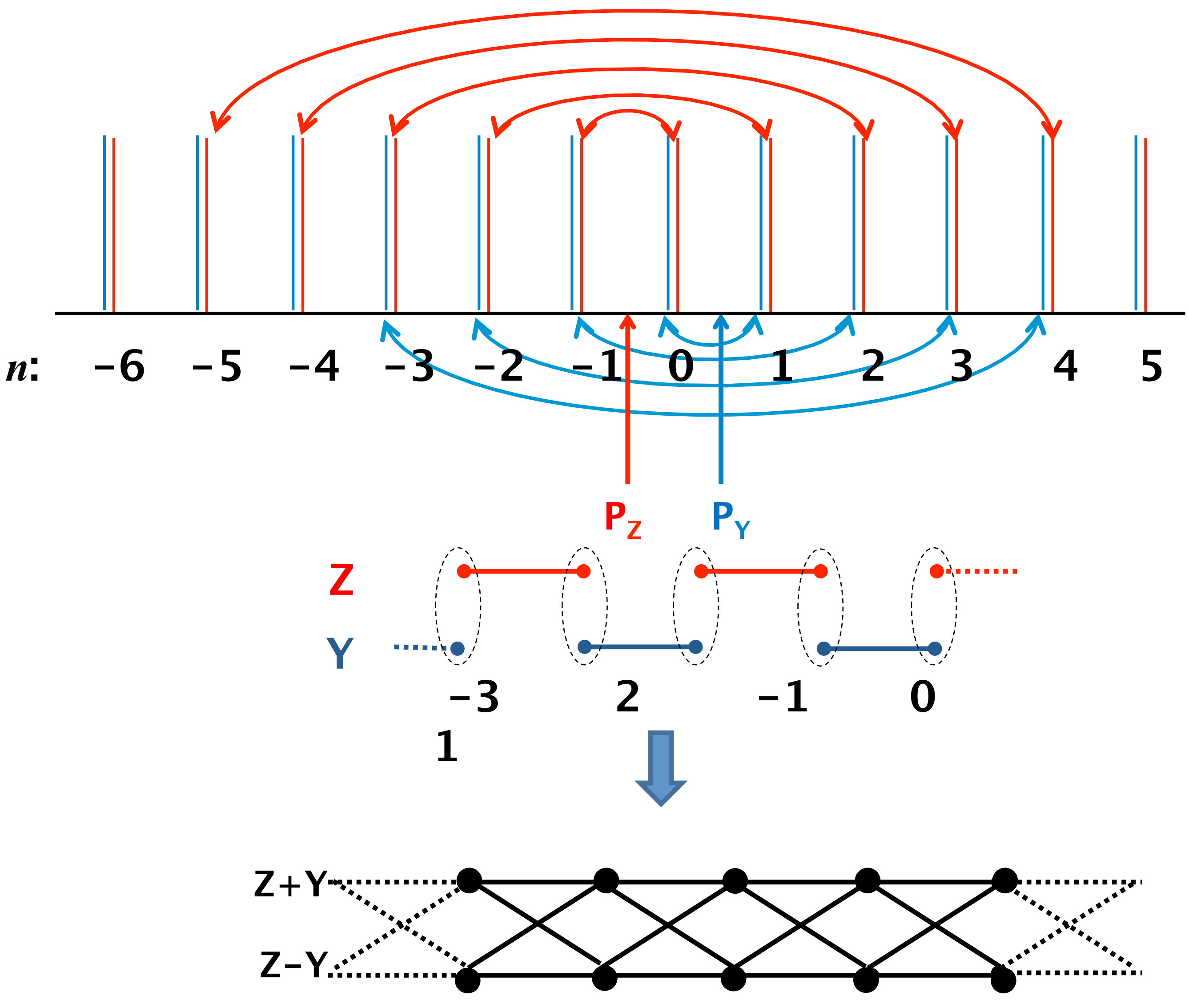
*Energy conservation:*
$\nu_1 + \nu_2 = \nu_p$



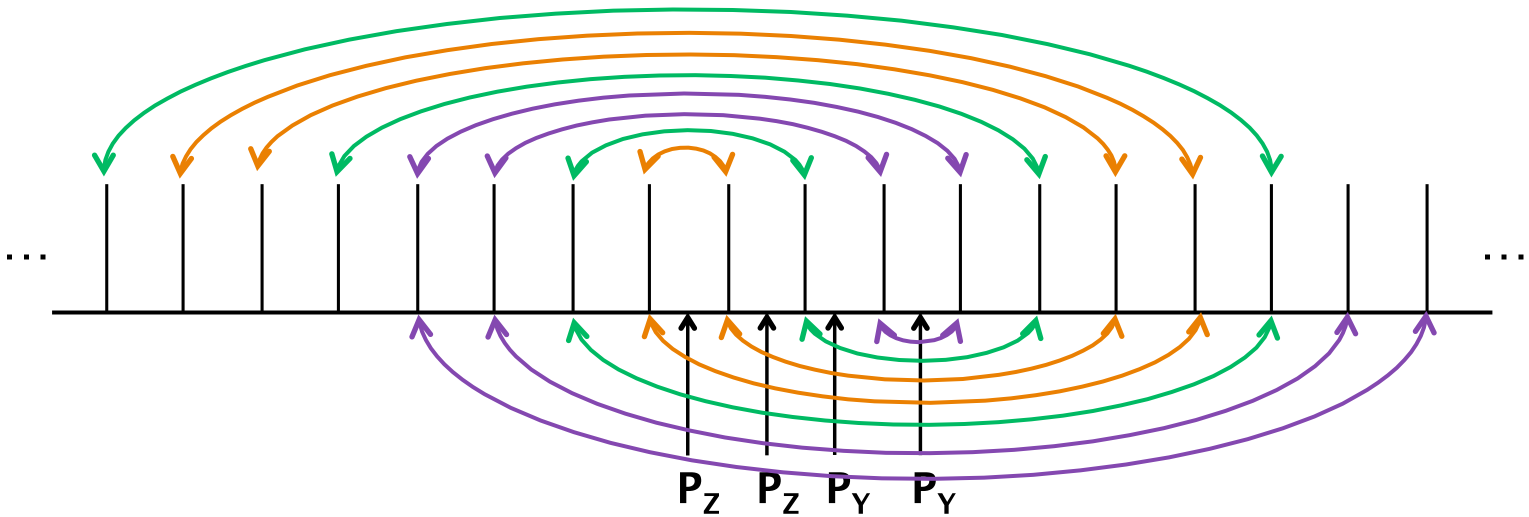


n:
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5
P_Z
P_Y
Z
Y
−3 2 −1 0
1

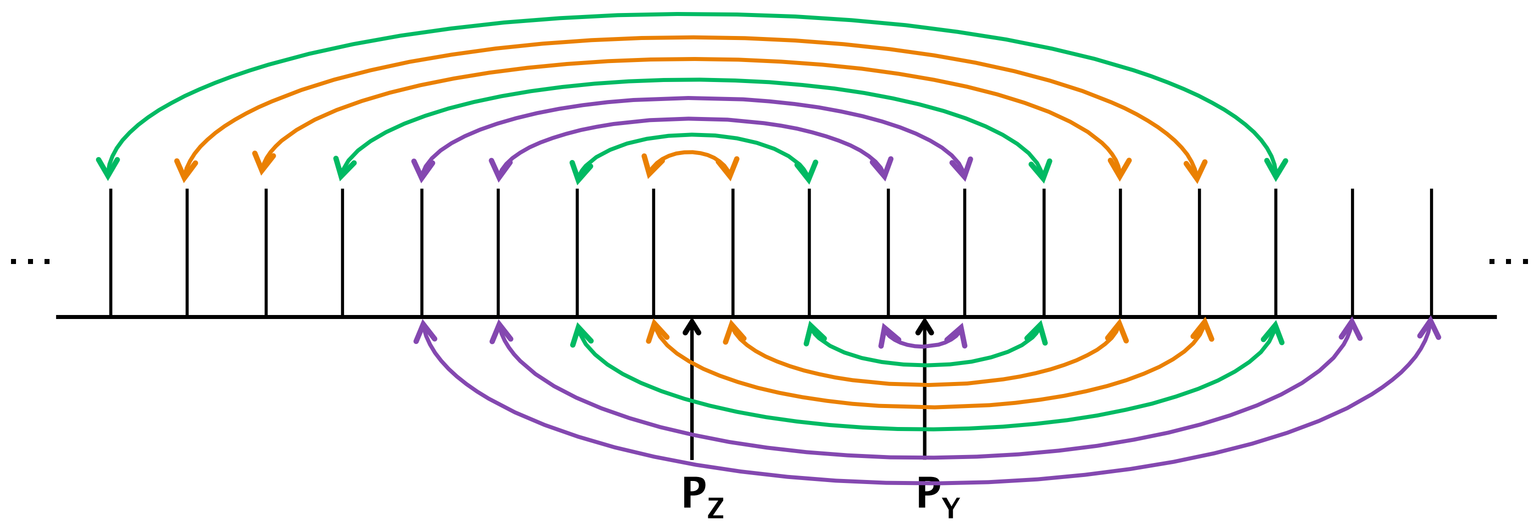



"Dual-rail quantum wire cluster state"

# Multiple copies

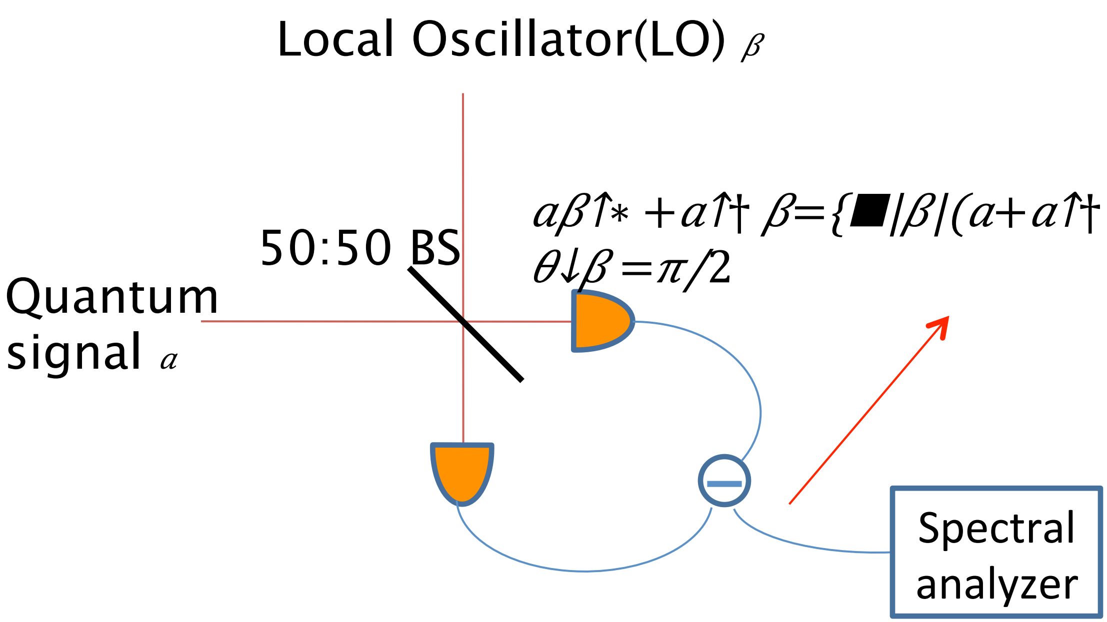


# Multiple copies




- **Three independent dual-rail quantum wires!**
- **The number of quantum wires is proportional to the pump spacing**

# State verification



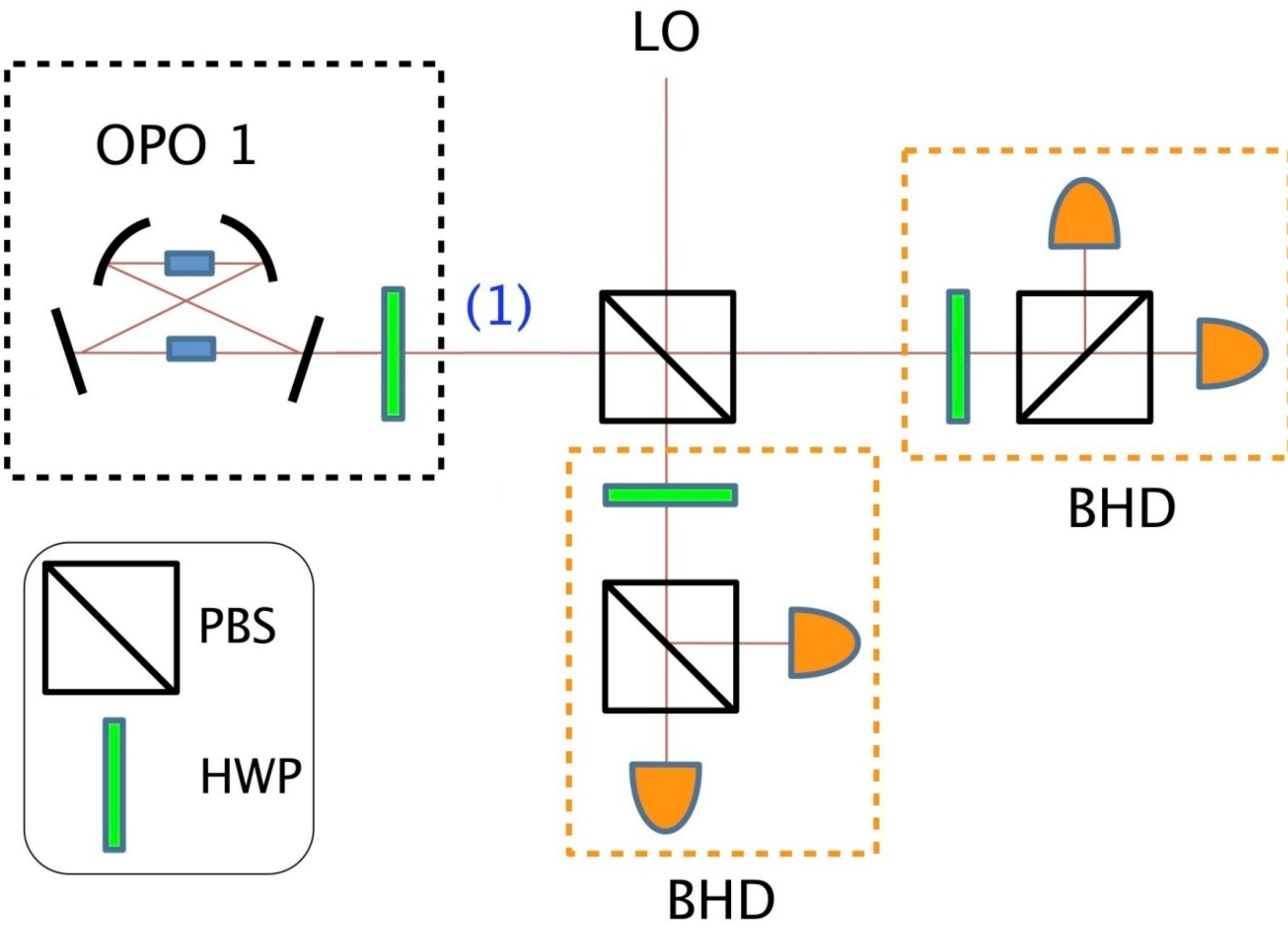

$$a\beta^{*} + a^{\dagger}\beta = \begin{cases} |\beta|(a+a^{\dagger}) = \sqrt{2}\,|\beta|q\ , & \theta_{\beta} = 0 \\ i|\beta|(a-a^{\dagger}) = \surd & \theta_{\beta} = \pi/2 \end{cases}$$

Variance of quadrature combinations

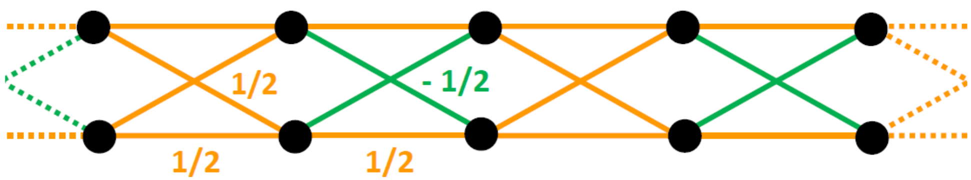
Verify nullifiers

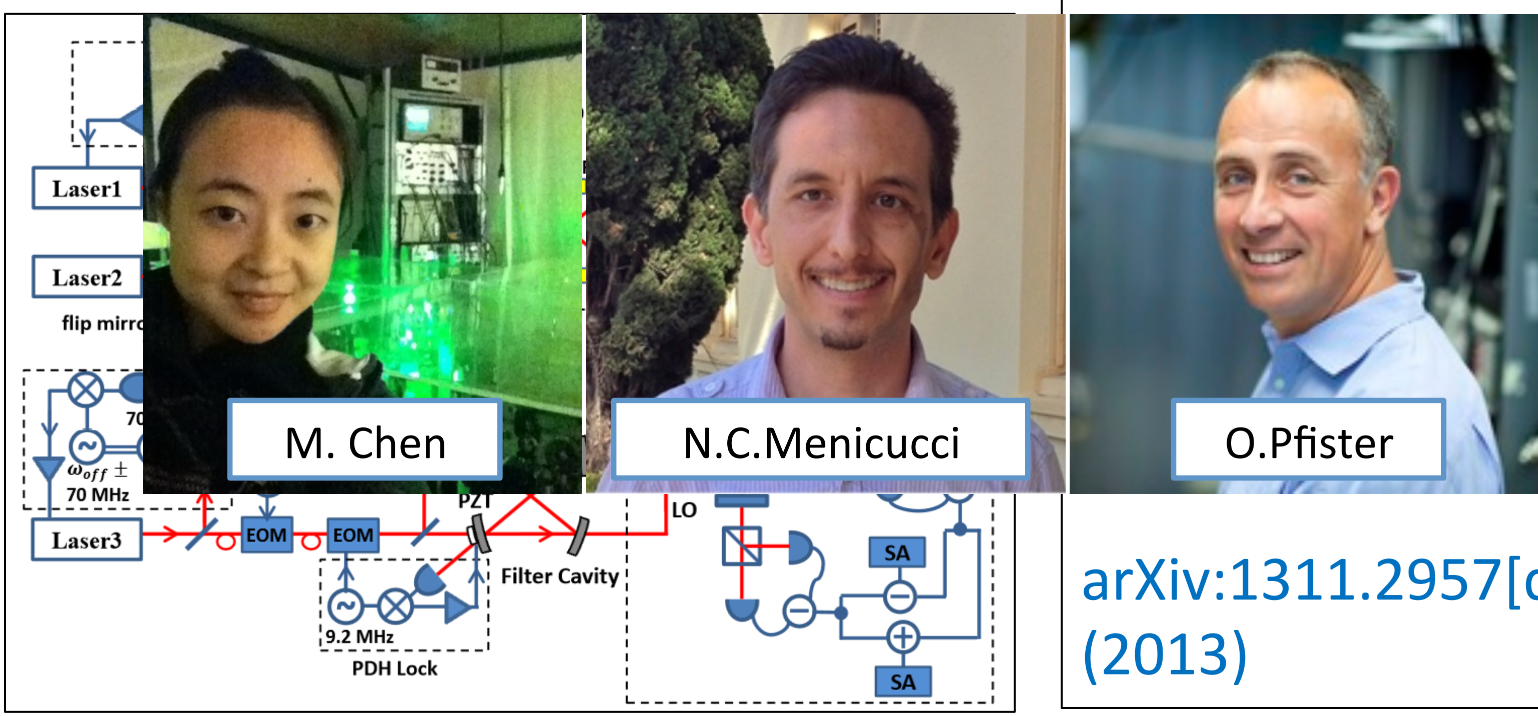
Balanced homodyne detection

# Dual-rail quantum wire cluster state: experimental setup



# Dual-rail quantum wire cluster state: experimental realization




M.Chen,
N.C.Menicucci,
and O.Pfister,

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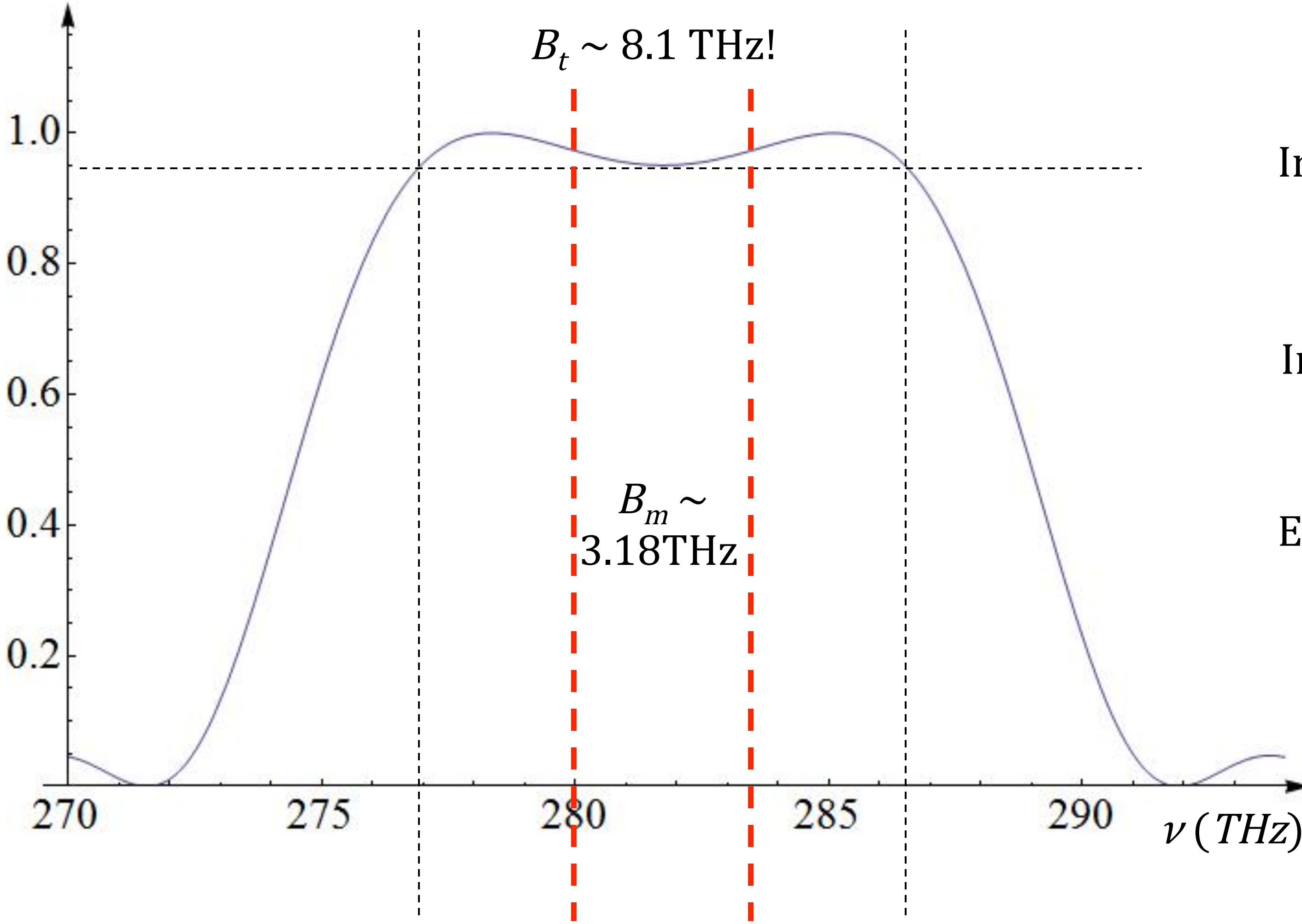
requency

arXiv:1311.2957[quant-ph]
(2013)

# EPR pair generation bandwidth




$\Delta\nu$=0.95 GHz

In theory:

$2B_t/\Delta\nu \sim 17000$

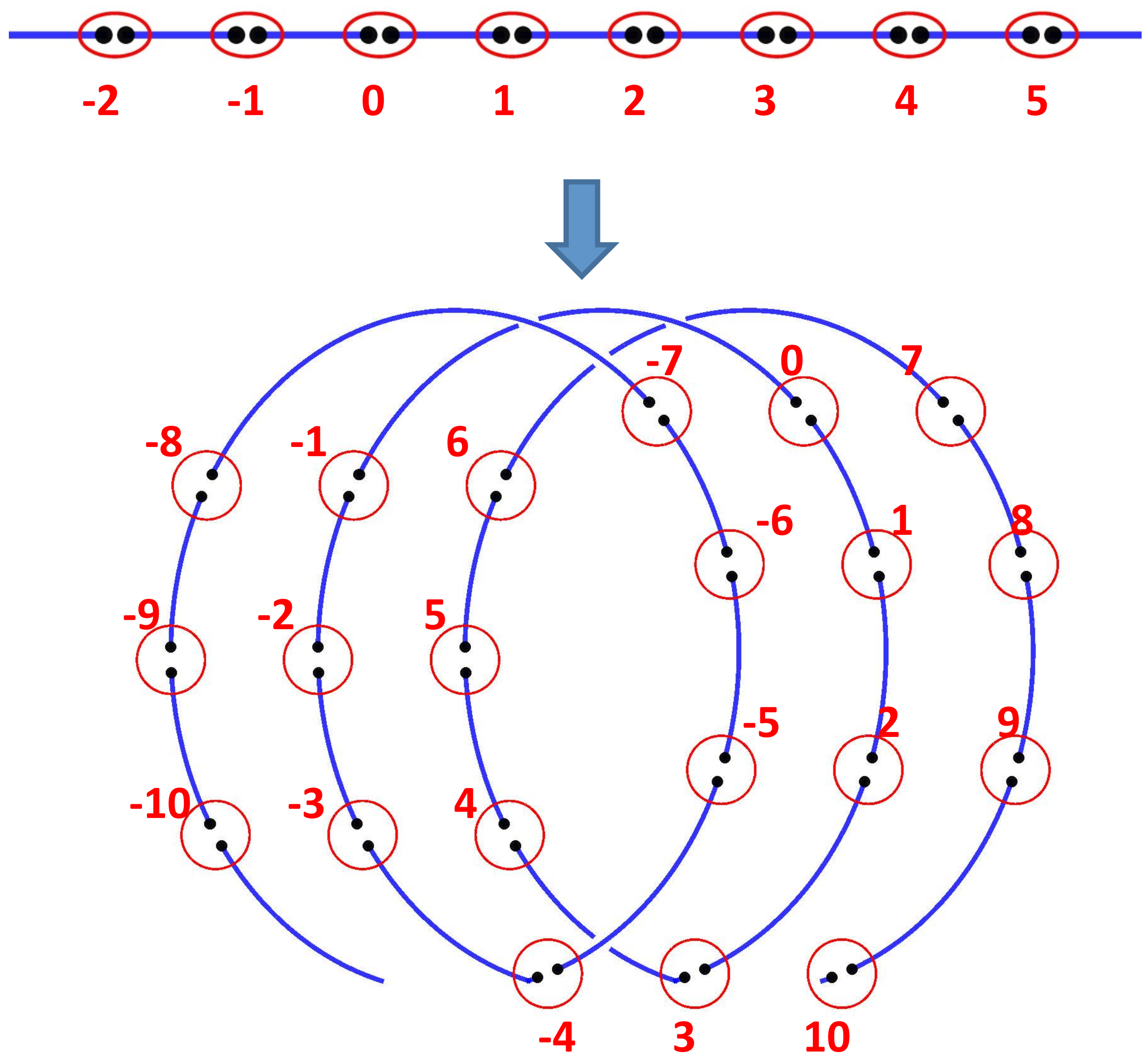
In M.Chen *et al.*, 2013:

$\sim 60$

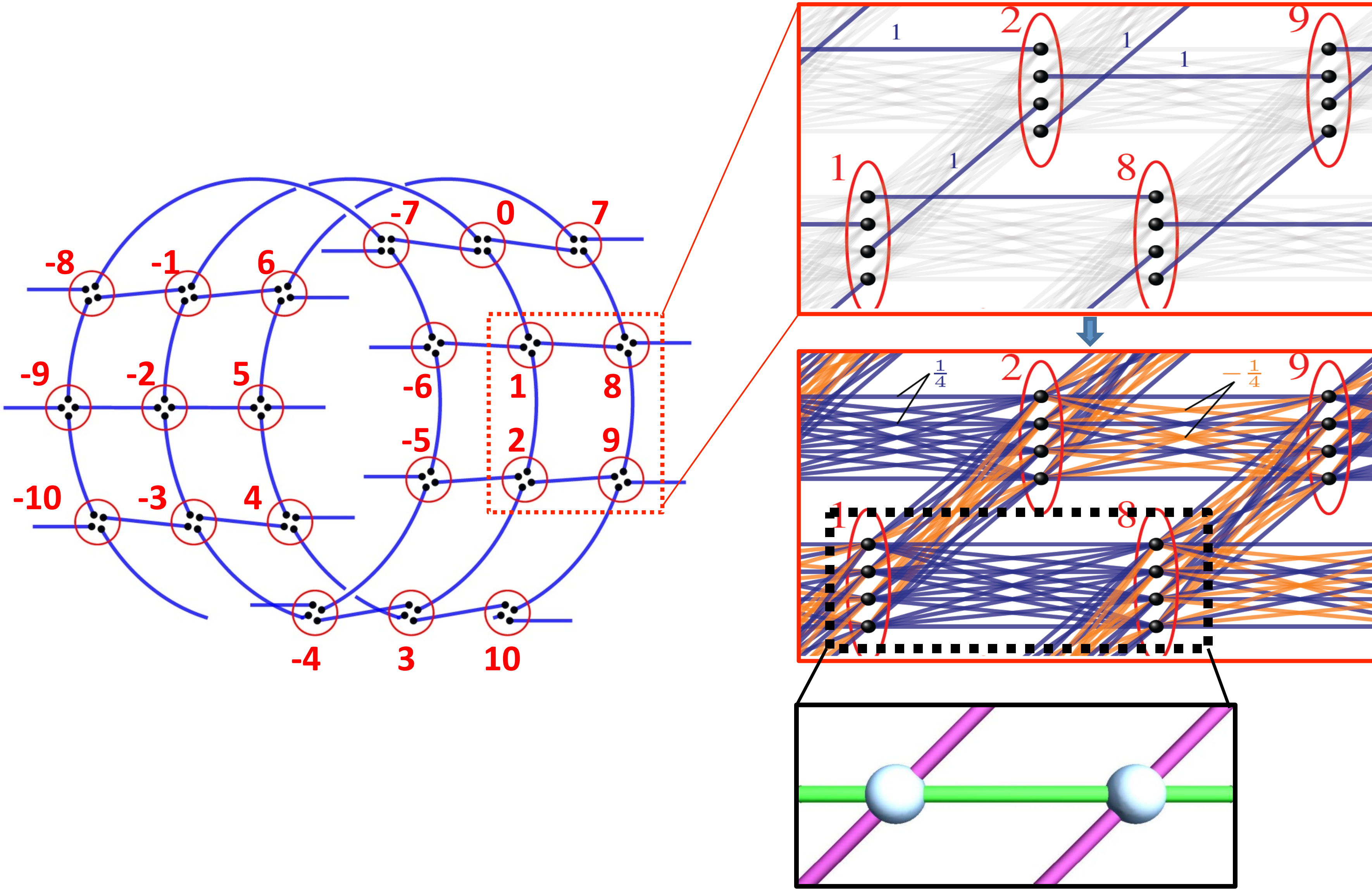
Experiment(expected):

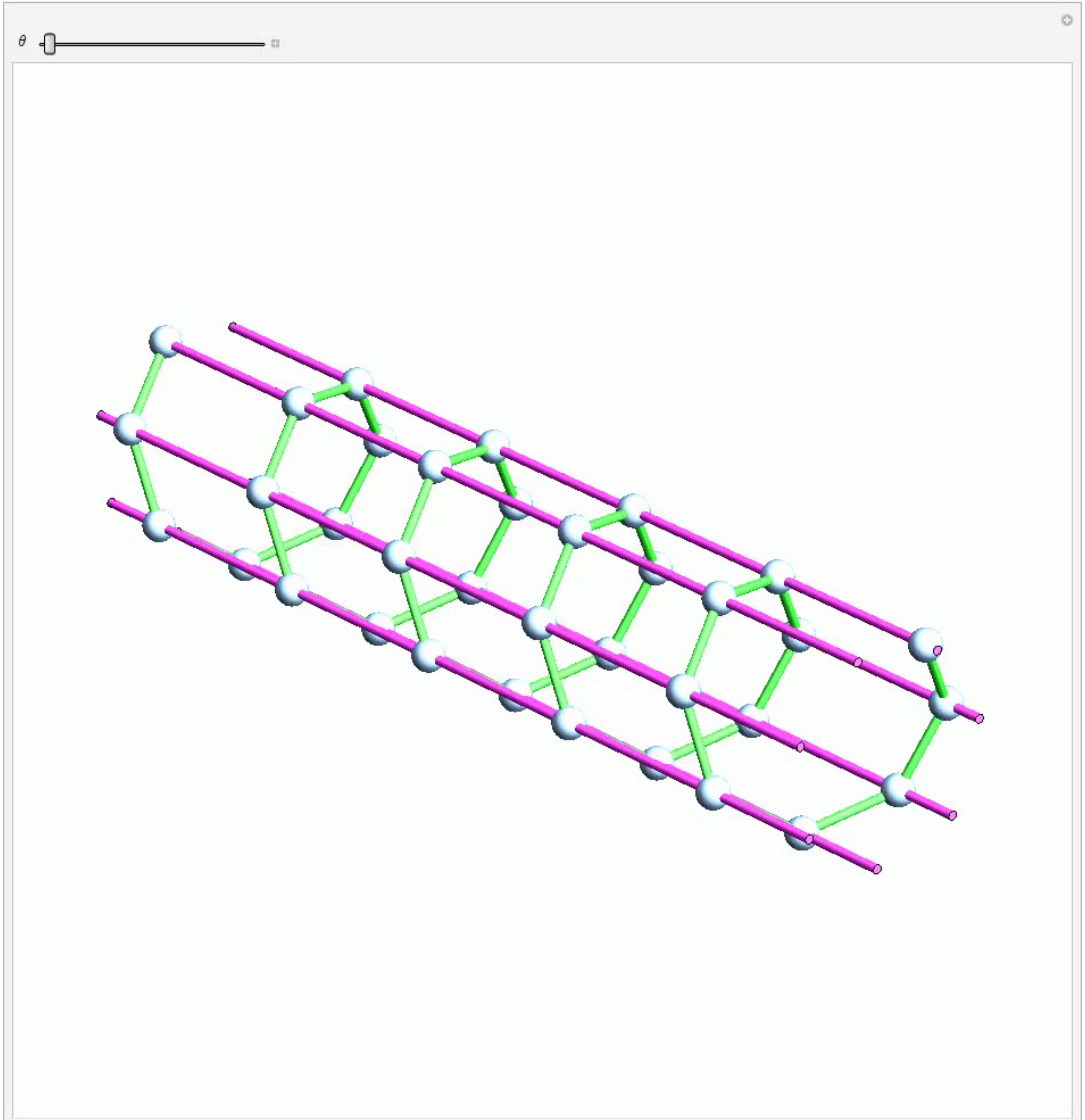
$2B_m/\Delta\nu \sim 6700$

# Square lattice cluster state

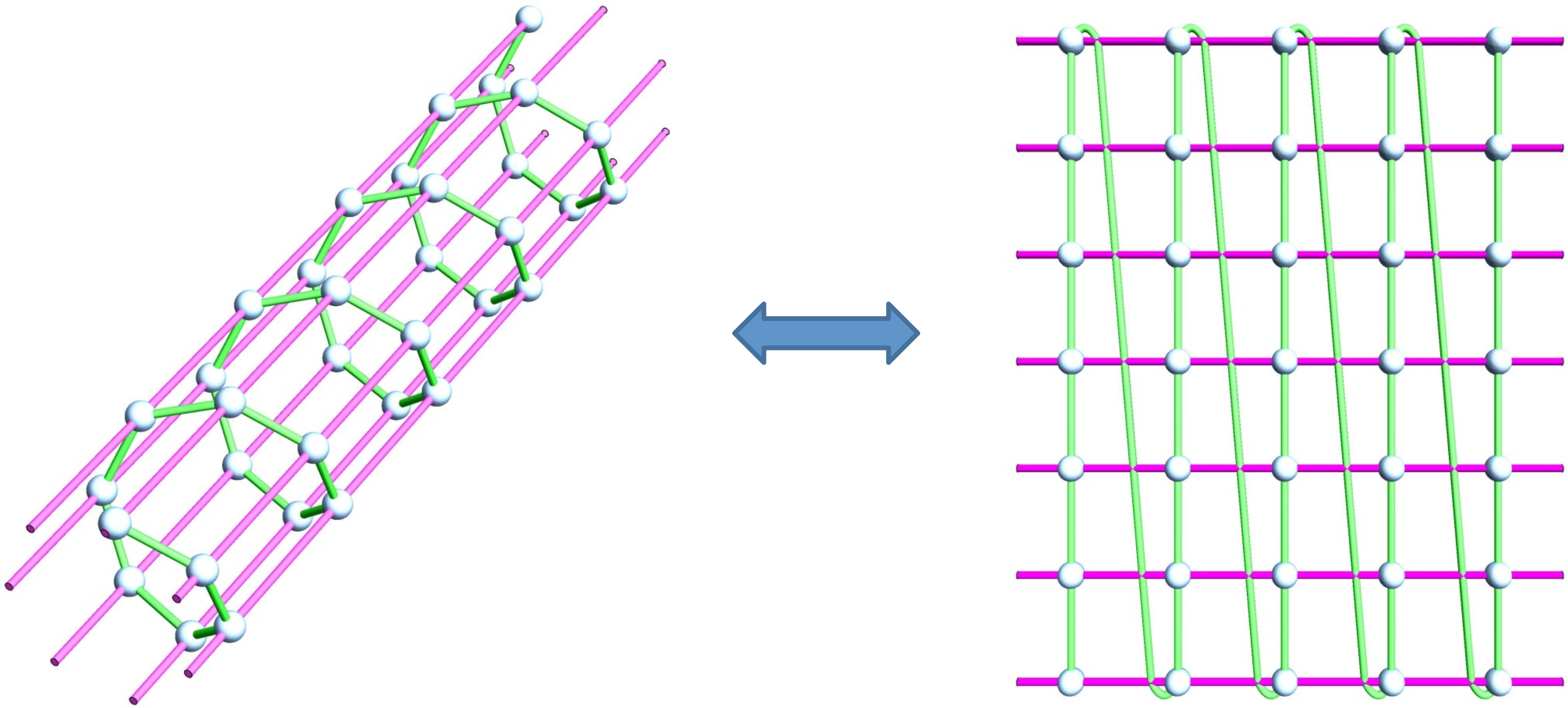


# Square lattice cluster state

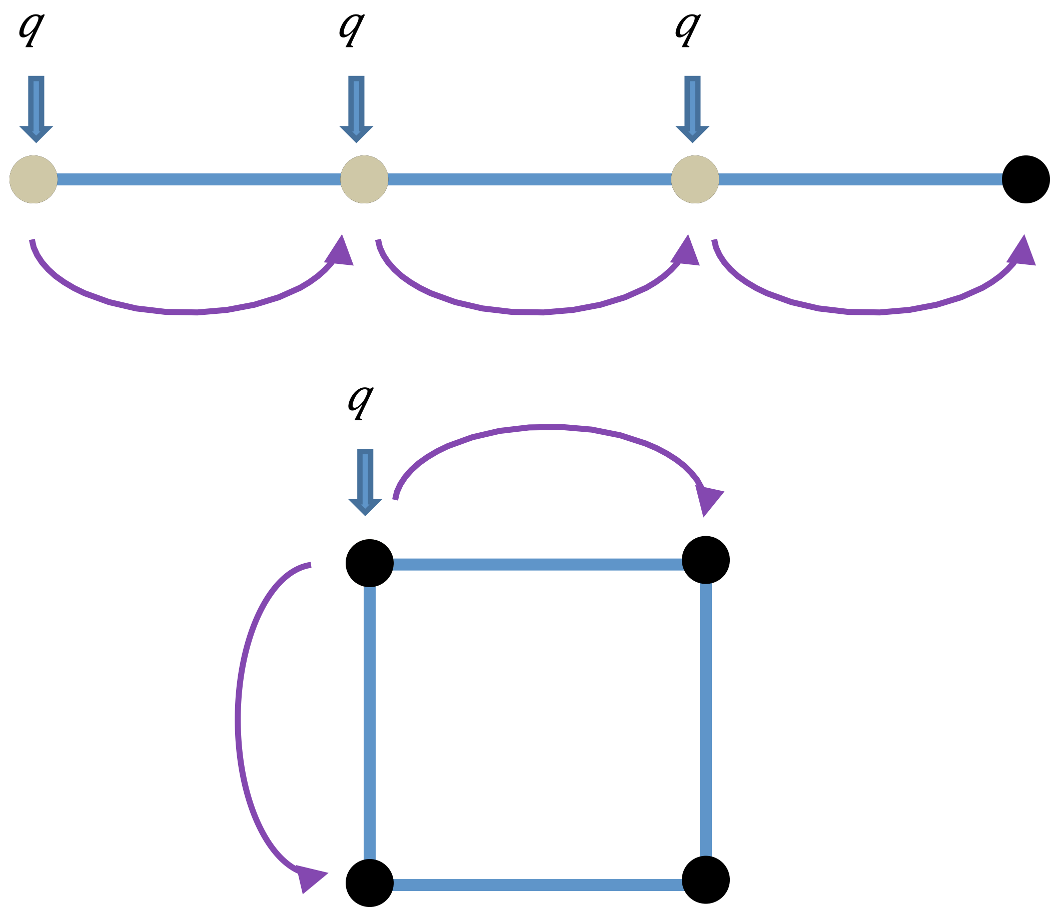


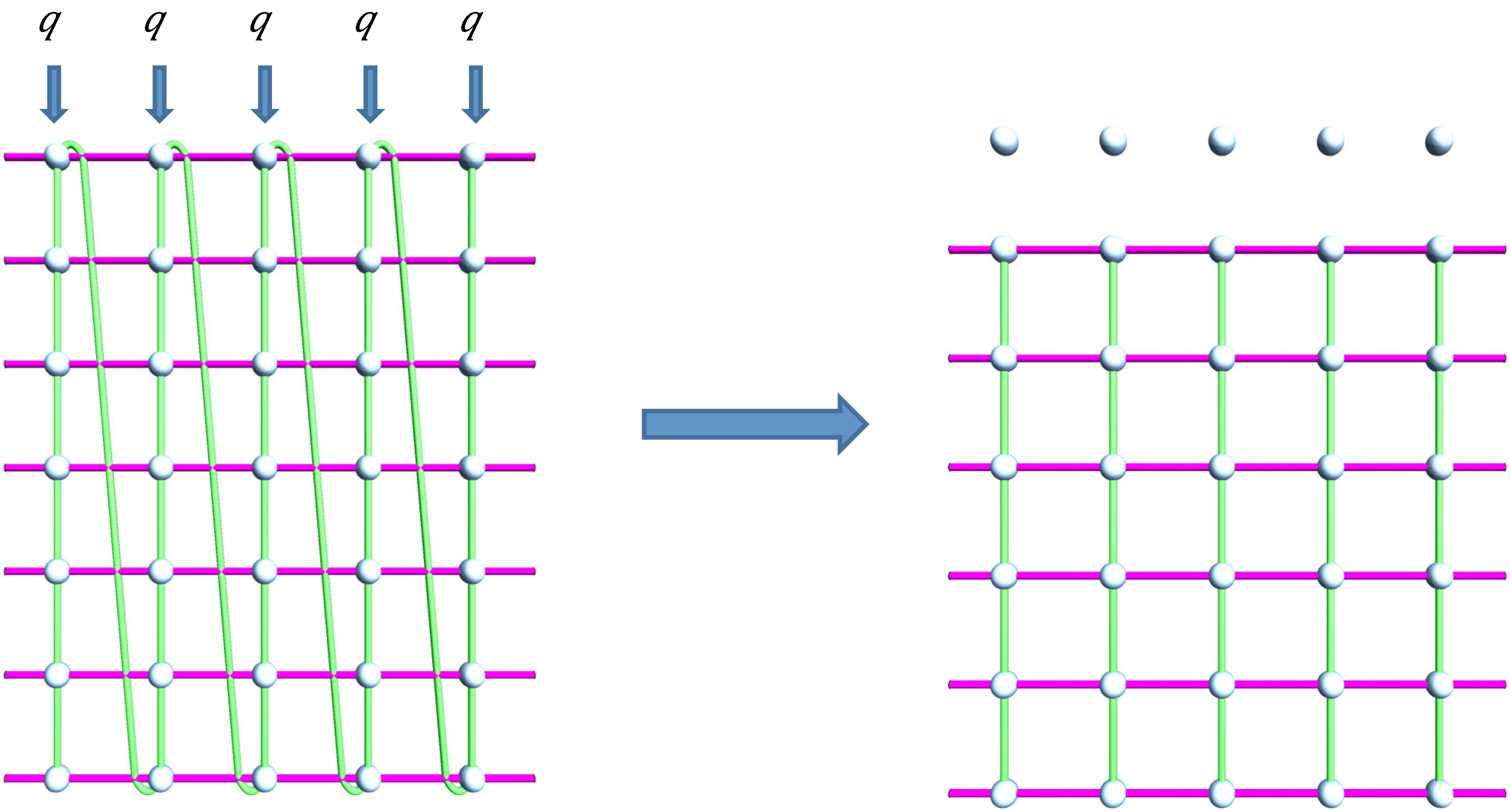

θ

# Square lattice cluster state

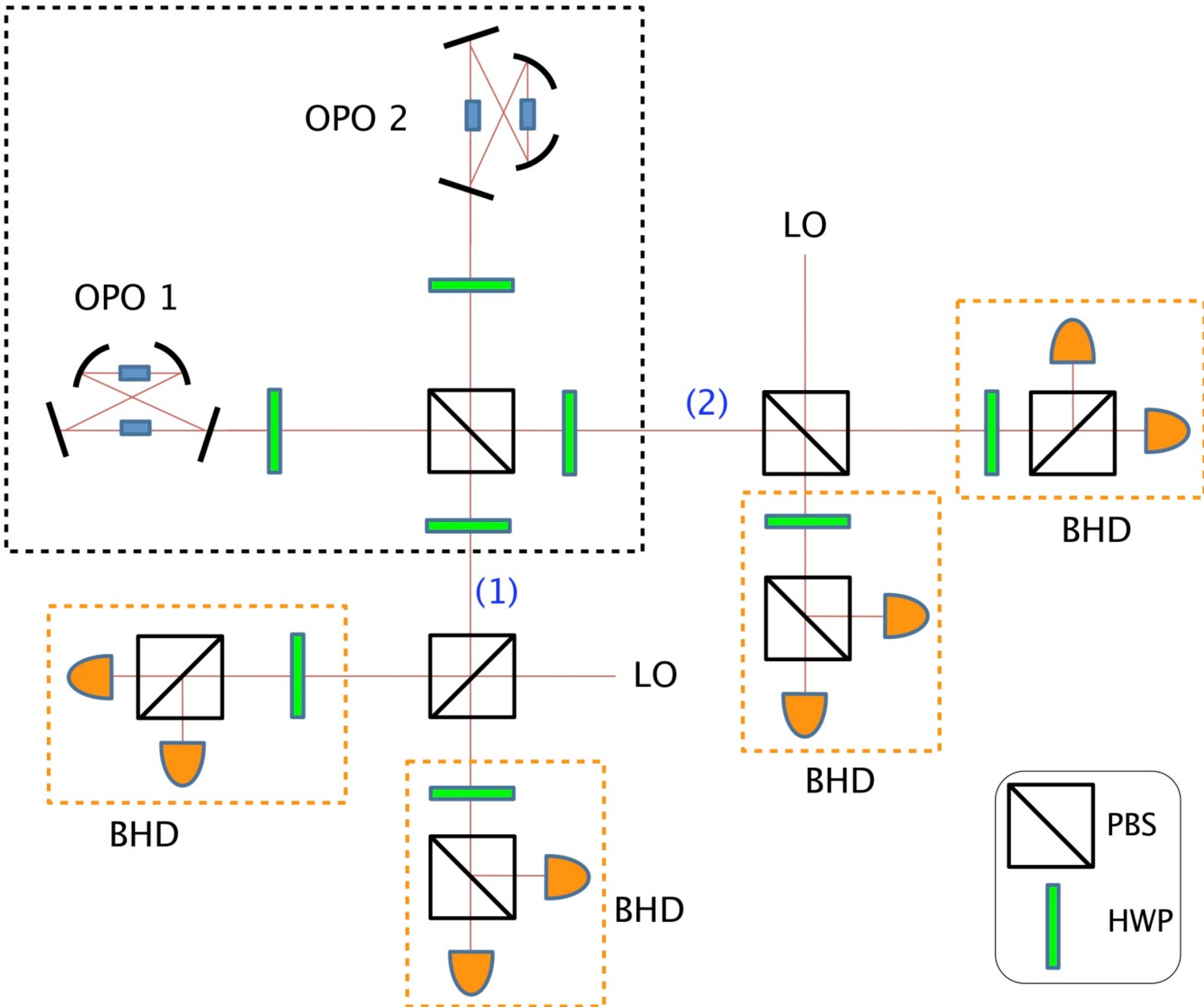


# Measurements on cluster state: graph shaping

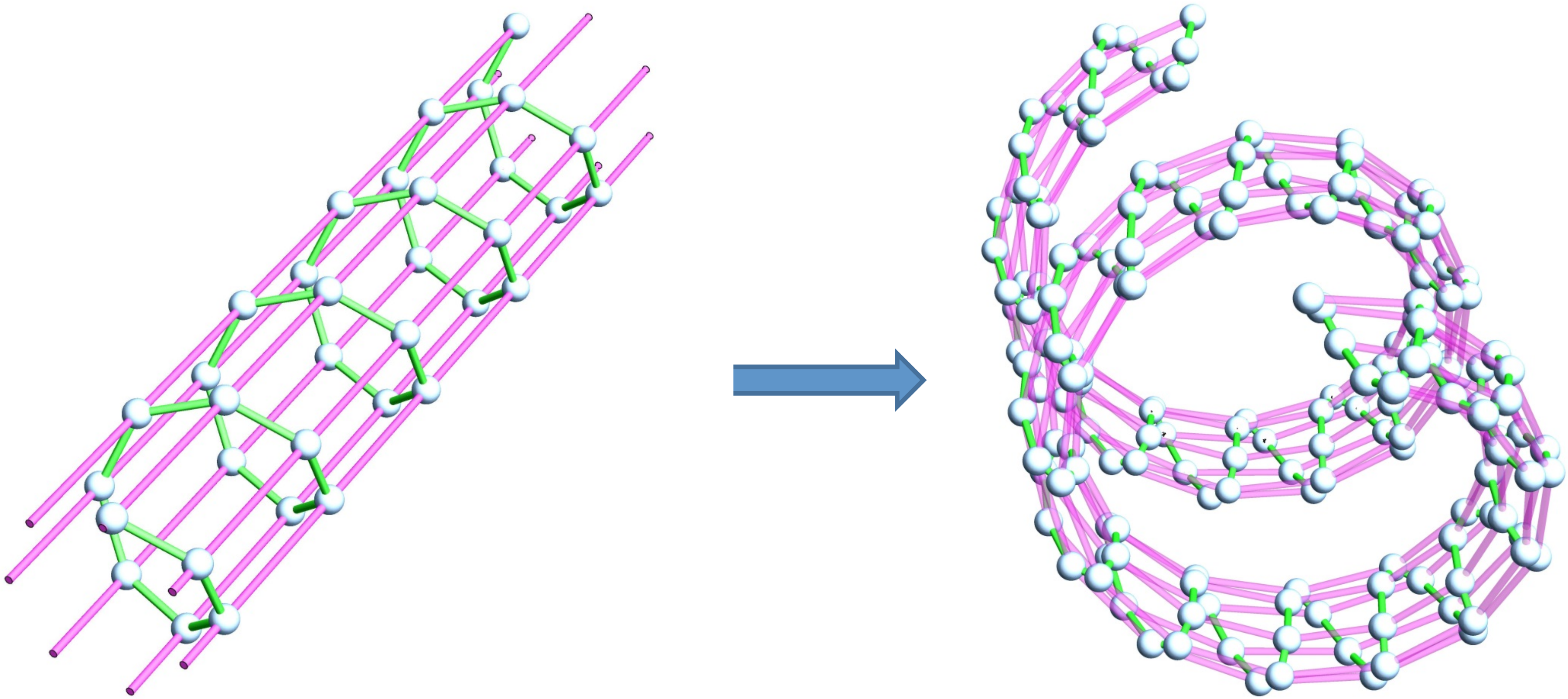


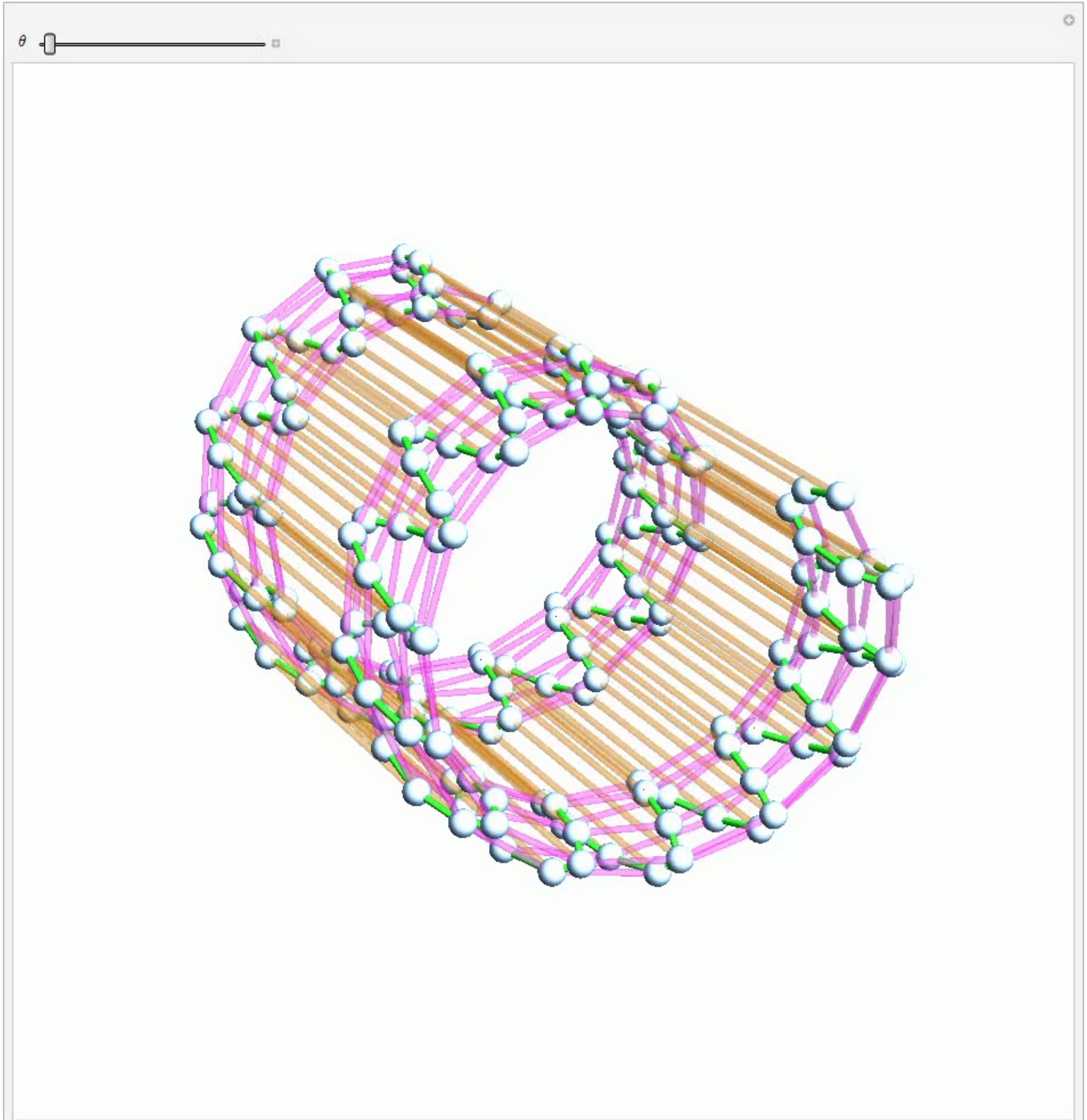

q
q
q
q
q

# Square lattice cluster state: experimental setup



# Cubic lattice cluster state

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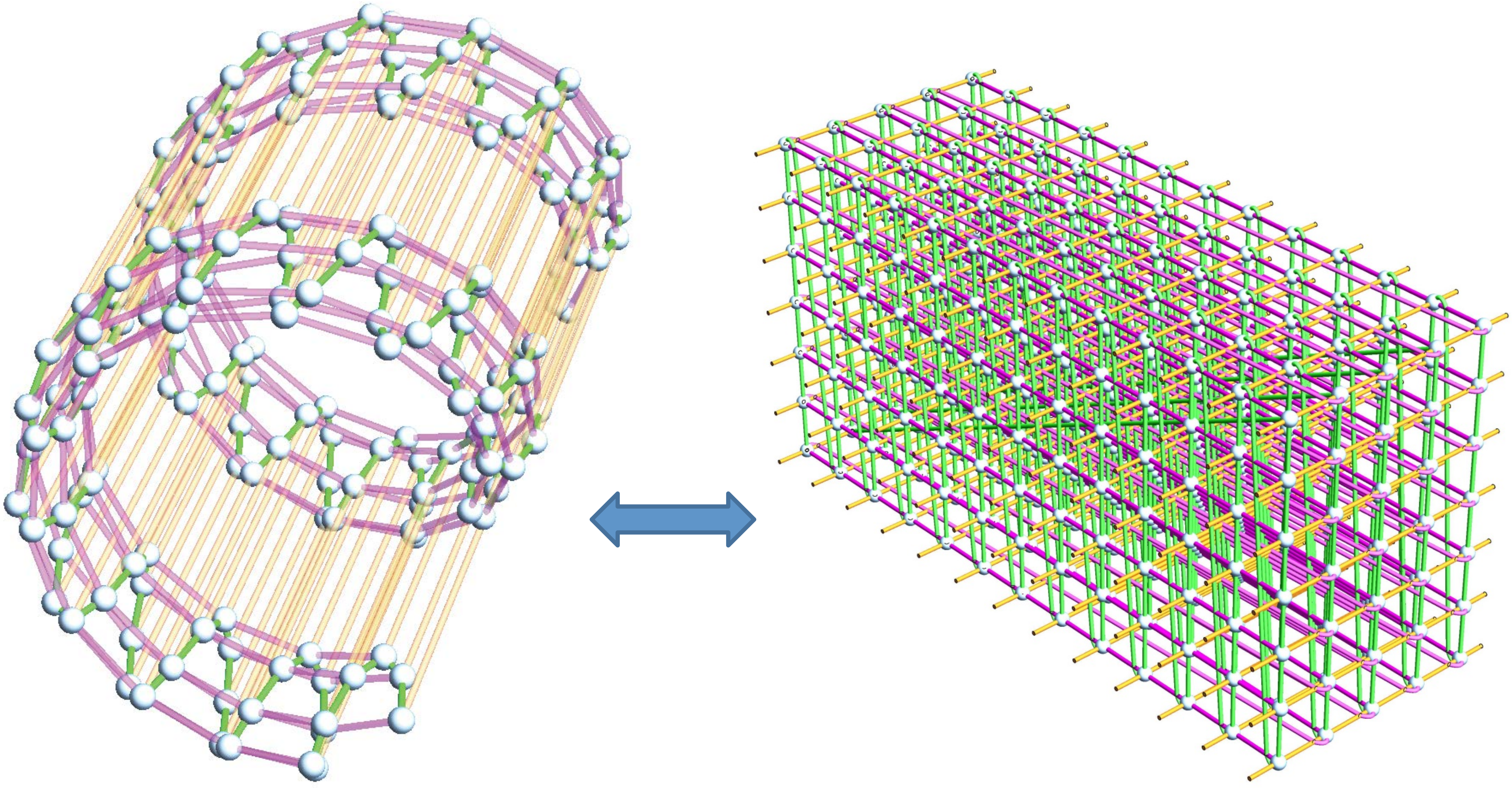
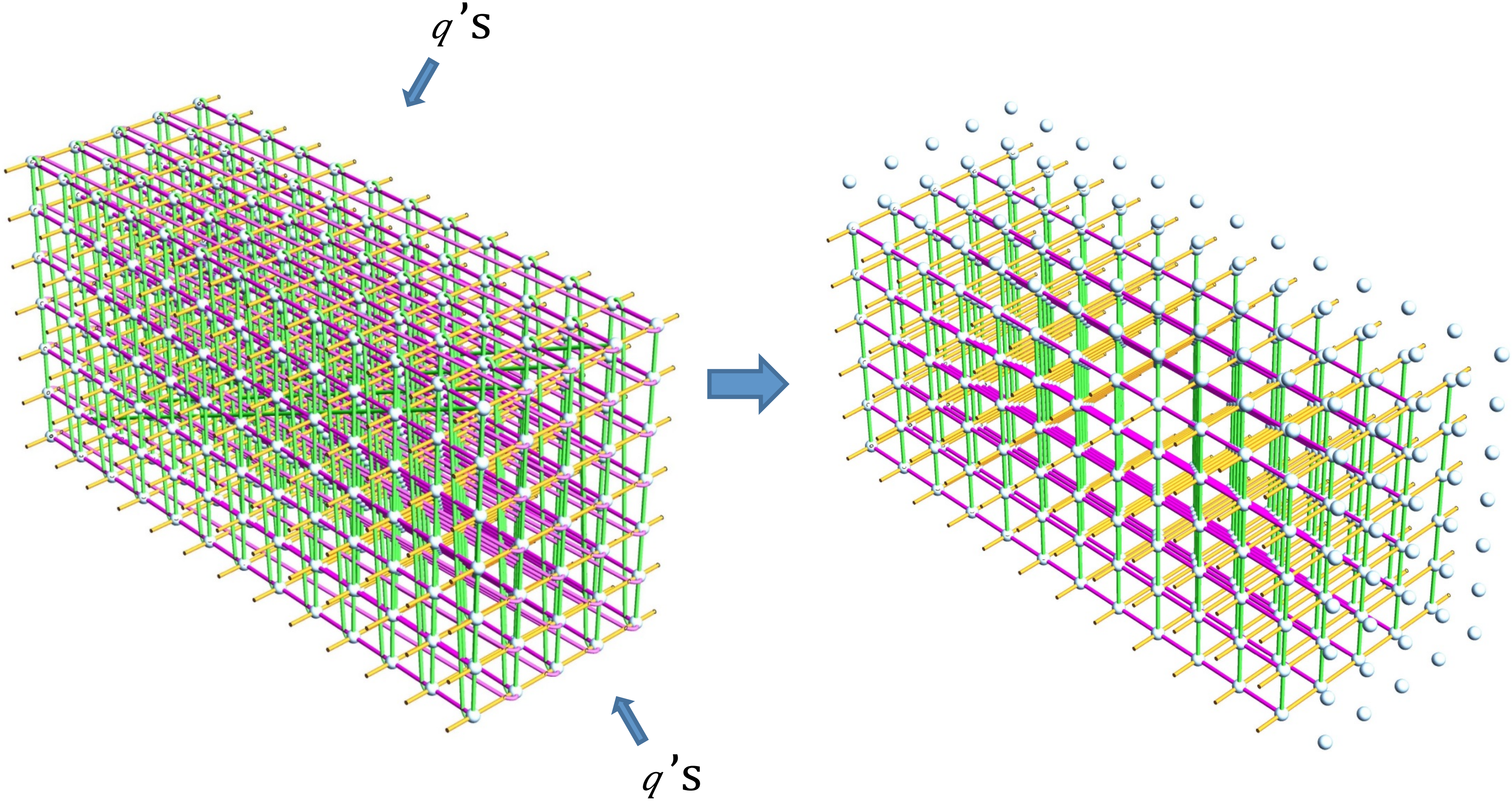
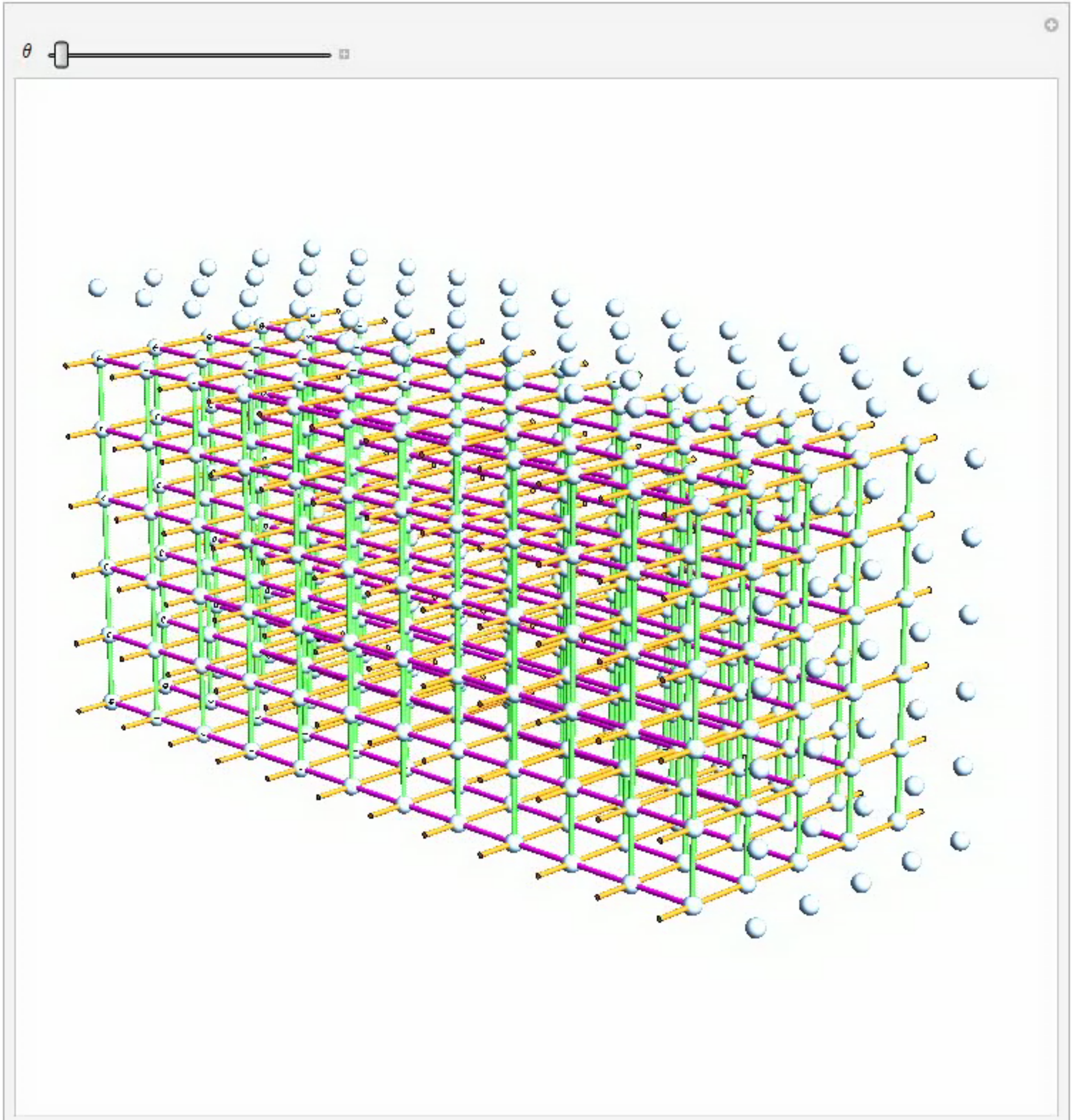
# Cubic lattice cluster state





$q$'s
$q$'s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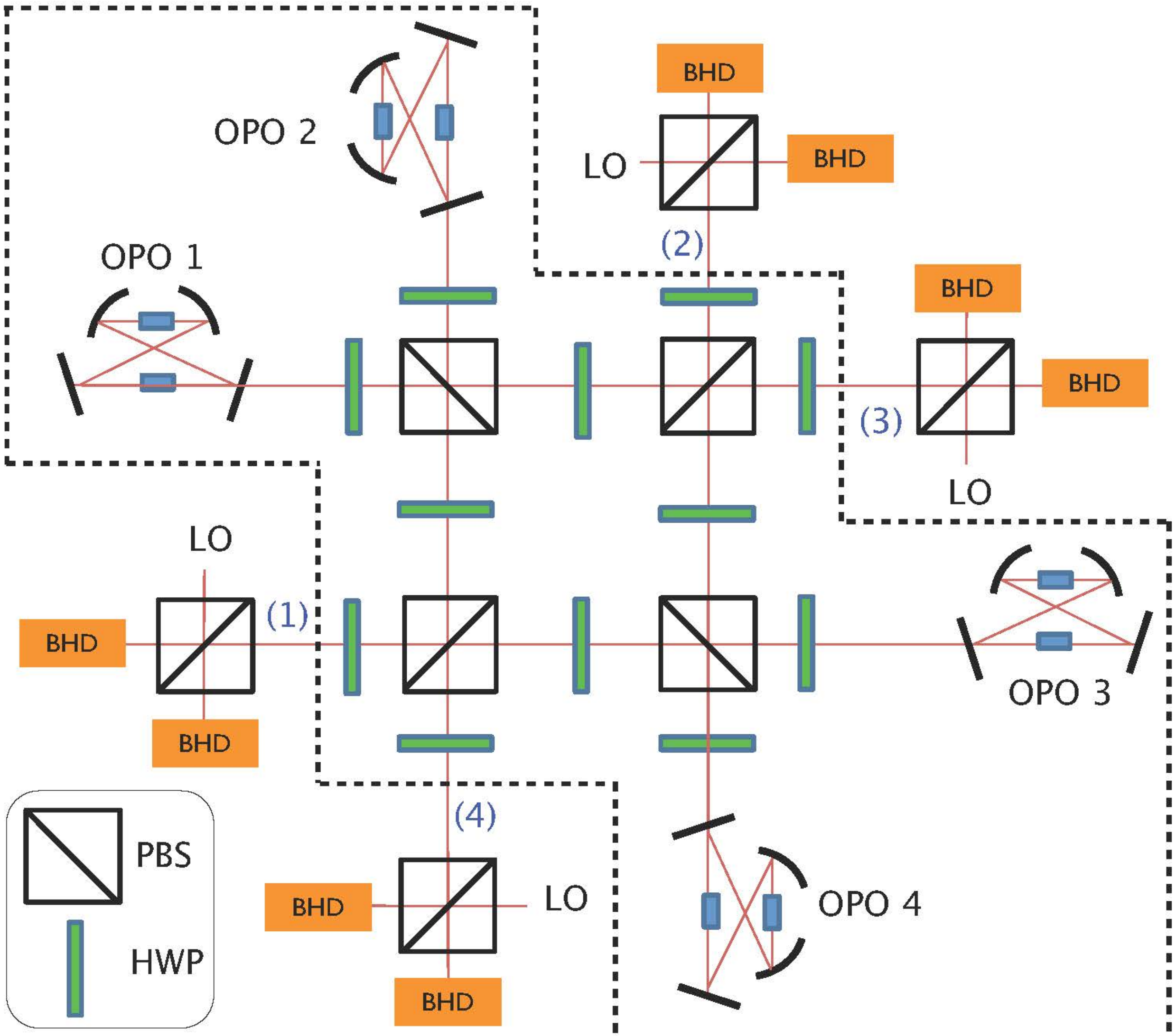
# Hypercubic lattice cluster state

- Continue the same procedure, in general:

  $D$-dimensional hypercubic lattice cluster state can be created by:
  - $D$ two-frequency pumped OPOs
  - one $D \times D$ interferometer

- *Multiple copies* of hypercubic lattice cluster state can be created by the same setup, just will more pump spacing!

# 4-D Hypercubic lattice cluster state: experimental setup



# Potential applications

- Linear & square lattice: universal one-way quantum computing

  Menicucci *et al*., PRL (2006)

- Cubic lattice: topological error correction

  Raussendorf *et al*., Ann.Phys. (N.Y.) 321, 2242–2270 (2006)

  Dennis *et al*., J.Math.Phys. (2001)

- Hypercubic lattice?

# Summary

- Circuit-based *vs* measurement-based QC
- Discrete variables *vs* Continuous variables
- Generating scalable optical hypercubic lattice cluster states
- Experimental creation and verification
- Potential use
- For more information:

P.Wang, M.Chen, N.C.Menicucci,and O.Pfister,"Weaving quantum optical frequency combs into hypercubic cluster states", arXiv:1309.4105[quant-ph],(2013)

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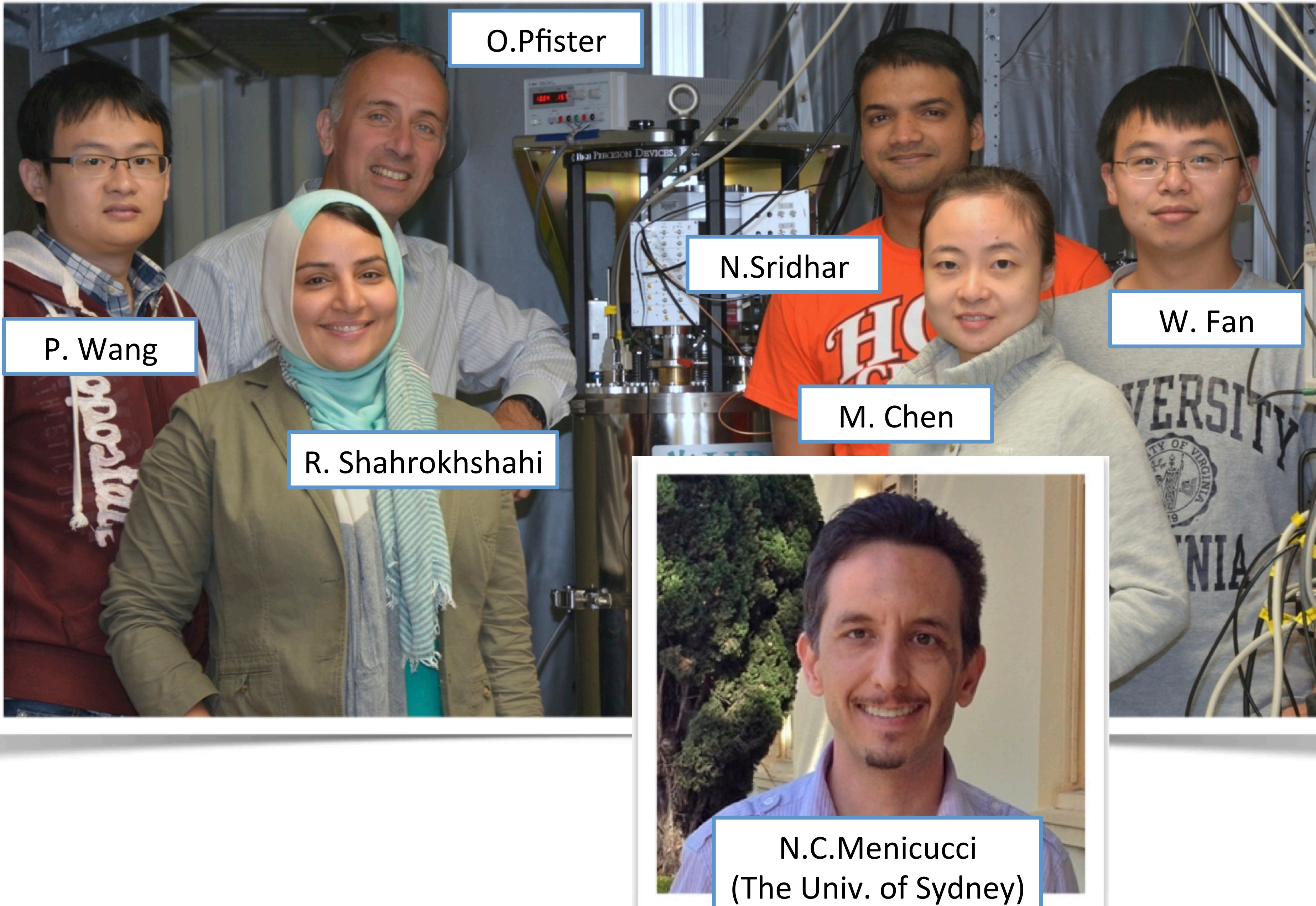

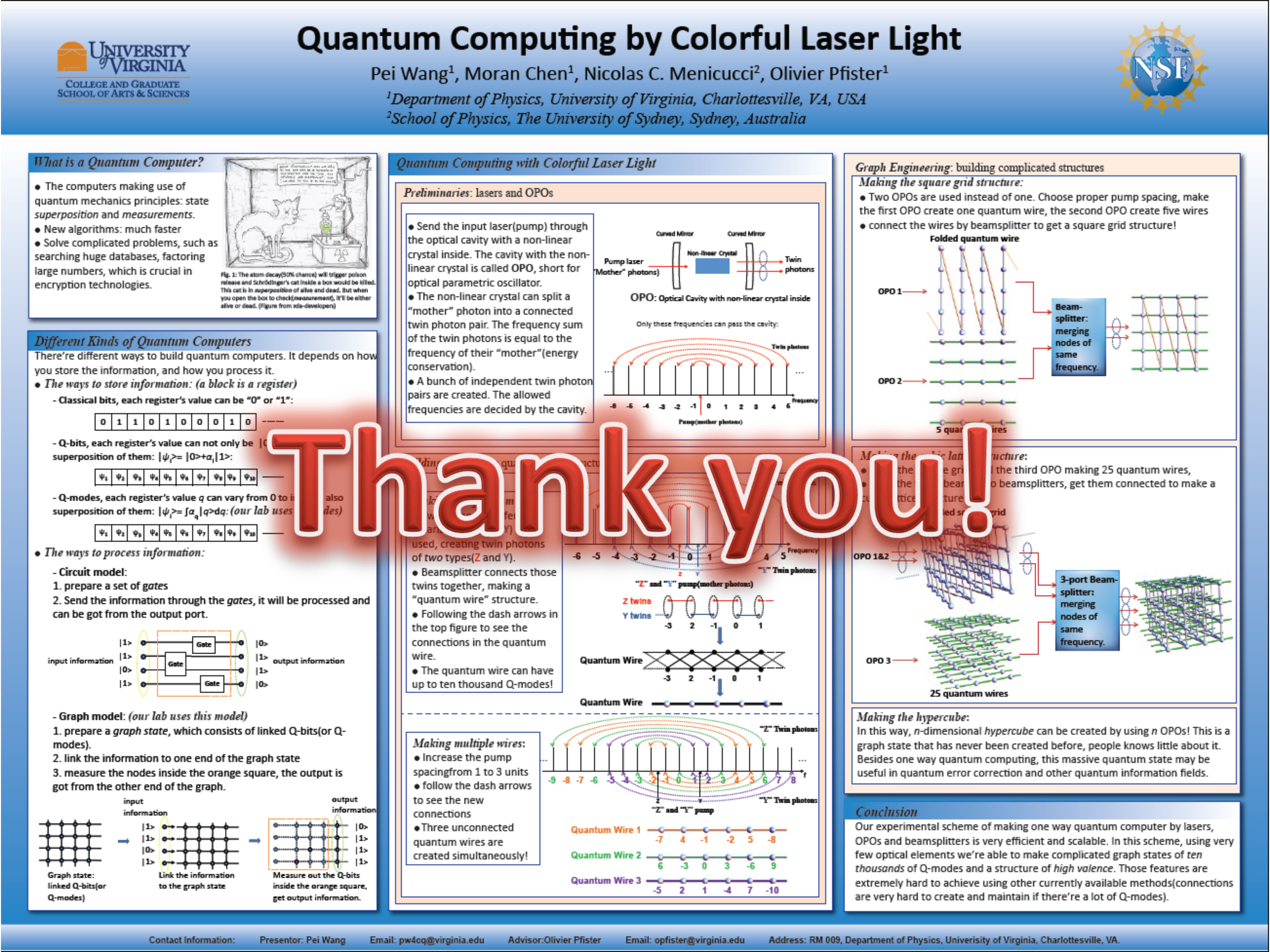
Quantum Computing by Colorful Laser Light
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University of Virginia College and Graduate School of Arts & Sciences
NSF
Thank you!
What is a Quantum Computer?
• The computers making use of quantum mechanics principles: state superposition and measurements.
• New algorithms: much faster
• Solve complicated problems, such as searching huge databases, factoring large numbers, which is crucial in encryption technologies.
Fig. 1: The atom decay(50% chance) will trigger poison release and Schrödinger's cat inside a box would be killed. This cat is in superposition of alive and dead. But when you open the box to check(measurement), it'll be either alive or dead. (Figure from xkcd-developers)
Different Kinds of Quantum Computers
There're different ways to build quantum computers. It depends on how you store the information, and how you process it.
• The ways to store information: (a block is a register)
- Classical bits, each register's value can be "0" or "1":
0 1 1 0 1 0 0 0 1 0
- Q-bits, each register's value can not only be |0> or |1>, but also superposition of them: |ψ>= |0>+α|1>:
- Q-modes, each register's value q can vary from 0 to infinity, also superposition of them: |ψ>= ∫α|q>dq: (our lab uses Q-modes)
• The ways to process information:
- Circuit model:
1. prepare a set of gates
2. Send the information through the gates, it will be processed and can be got from the output port.
input information
output information
Gate
- Graph model: (our lab uses this model)
1. prepare a graph state, which consists of linked Q-bits(or Q-modes).
2. link the information to one end of the graph state
3. measure the nodes inside the orange square, the output is got from the other end of the graph.
Graph state: linked Q-bits(or Q-modes)
Link the information to the graph state
Measure out the Q-bits inside the orange square, get output information.
Quantum Computing with Colorful Laser Light
Preliminaries: lasers and OPOs
• Send the input laser(pump) through the optical cavity with a non-linear crystal inside. The cavity with the non-linear crystal is called OPO, short for optical parametric oscillator.
• The non-linear crystal can split a "mother" photon into a connected twin photon pair. The frequency sum of the twin photons is equal to the frequency of their "mother"(energy conservation).
• A bunch of independent twin photon pairs are created. The allowed frequencies are decided by the cavity.
Curved Mirror
Non-linear Crystal
Pump laser ("Mother" photons)
Twin photons
OPO: Optical Cavity with non-linear crystal inside
Only these frequencies can pass the cavity:
Pump(mother photons)
• Beamsplitter connects those twins together, making a "quantum wire" structure.
• Following the dash arrows in the top figure to see the connections in the quantum wire.
• The quantum wire can have up to ten thousand Q-modes!
"Z" and "Y" pump(mother photons)
Z twins
Y twins
Quantum Wire
Making multiple wires:
• Increase the pump spacingfrom 1 to 3 units
• follow the dash arrows to see the new connections
• Three unconnected quantum wires are created simultaneously!
"Z" Twin photons
"Y" Twin photons
Quantum Wire 1
Quantum Wire 2
Quantum Wire 3
Graph Engineering: building complicated structures
Making the square grid structure:
• Two OPOs are used instead of one. Choose proper pump spacing, make the first OPO create one quantum wire, the second OPO create five wires
• connect the wires by beamsplitter to get a square grid structure!
Folded quantum wire
OPO 1
OPO 2
Beam-splitter: merging nodes of same frequency.
• Using the third OPO making 25 quantum wires,
• By two beamsplitters, get them connected to make a cubic lattice structure.
OPO 1&2
OPO 3
3-port Beam-splitter: merging nodes of same frequency.
25 quantum wires
Making the hypercube:
In this way, n-dimensional hypercube can be created by using n OPOs! This is a graph state that has never been created before, people knows little about it. Besides one way quantum computing, this massive quantum state may be useful in quantum error correction and other quantum information fields.
Conclusion
Our experimental scheme of making one way quantum computer by lasers, OPOs and beamsplitters is very efficient and scalable. In this scheme, using very few optical elements we're able to make complicated graph states of ten thousands of Q-modes and a structure of high valence. Those features are extremely hard to achieve using other currently available methods(connections are very hard to create and maintain if there're a lot of Q-modes).
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