Studying Topological Insulators

via

time reversal symmetry breaking and proximity effect

Roni Ilan UC Berkeley

Joel Moore,
Jens Bardarson,
Jerome Cayssol,
Heung-Sun Sim

Topological phases

Insulating phases with metallic surface states.

Bulk is insulating but non-trivial!

Properties/existence of surface states protected by bulk properties.

Topological invariants of the bulk are immune to continuous deformations.

Probes of topological properties

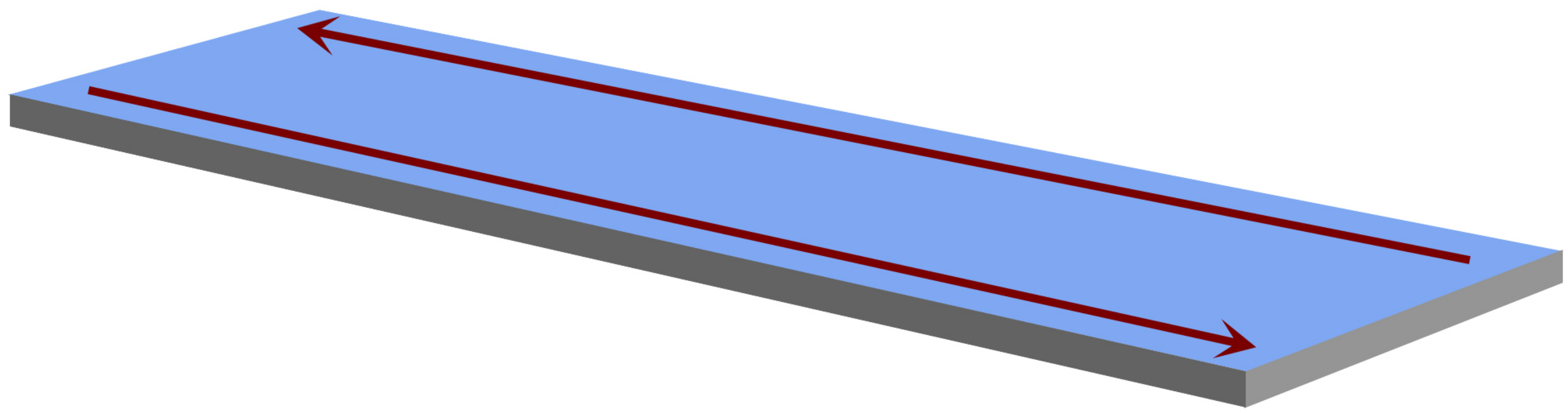
Many features are attributed to surface states as mirrors of topological properties of the bulk

How can we observe this in experiment?

Topological phases and symmetry

Some topological phases require no symmetry:

Quantum Hall effect



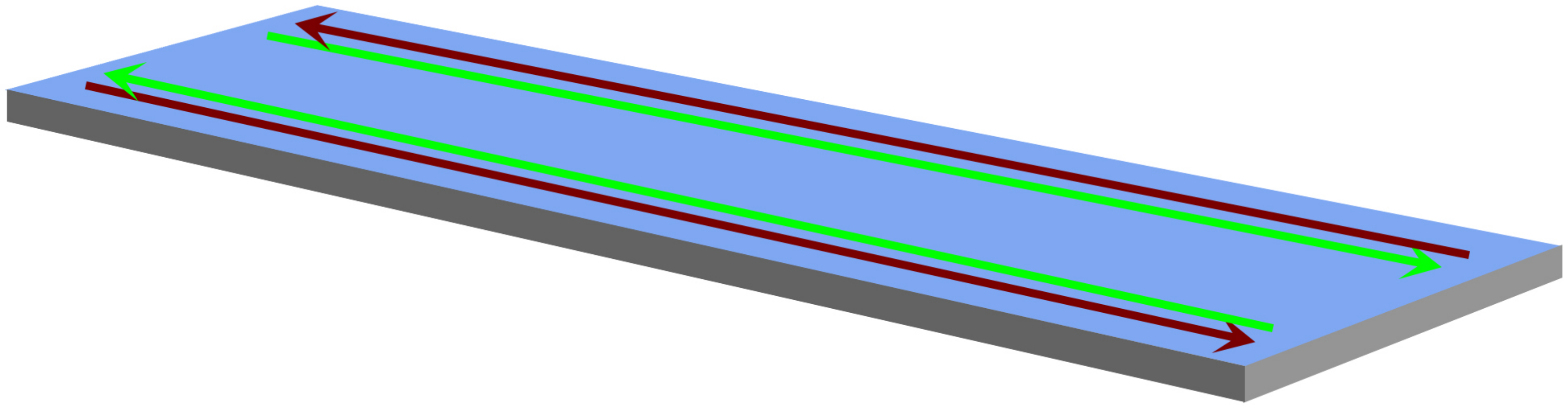
Bulk topological invariant = σ_{xy} = Chern number

Chiral edge modes protected due to spatial separation

Topological phases and symmetry

TI surface states are protected
by time reversal symmetry

Quantum Spin Hall effect (2DTI)



An opportunity to study surface states by
breaking that symmetry

Surface states of 2 and 3D TI's

2D 3D

Spin-momentum locking

Dirac fermions

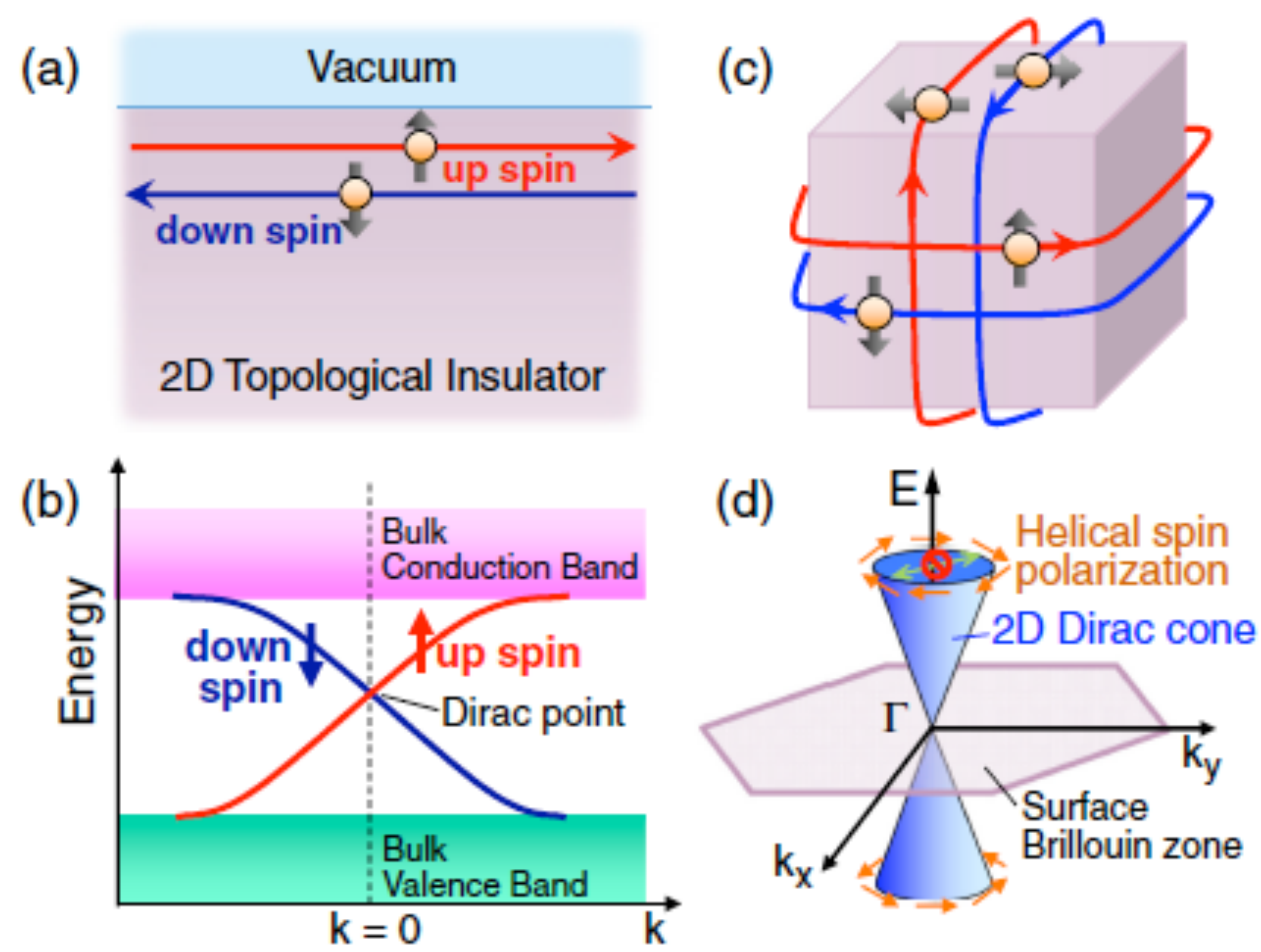


Figure from Y. Ando J. Phys. Soc. Jpn. 82, 102001 (2013)

Time reversal symmetry

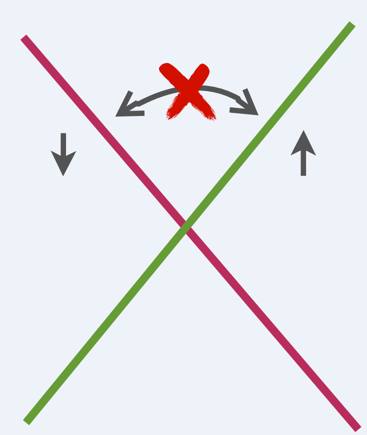
Half integral spin $\qquad T^2 = -1$

$$[H, T] = 0 \quad \Rightarrow \quad |\psi\rangle \qquad |T\psi\rangle$$

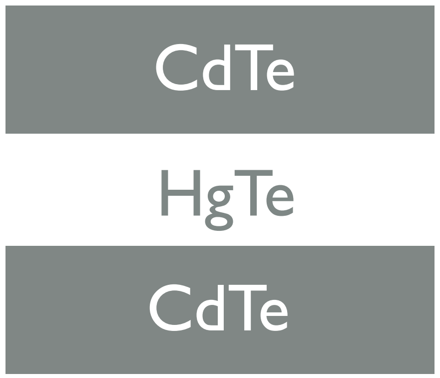
$$\langle T\psi|\psi\rangle = \langle T^2\psi|T\psi\rangle^* = -\langle\psi|T\psi\rangle^* = -\langle T\psi|\psi\rangle$$

Absence of backscattering

$$\langle\psi|V|T\psi\rangle = 0$$

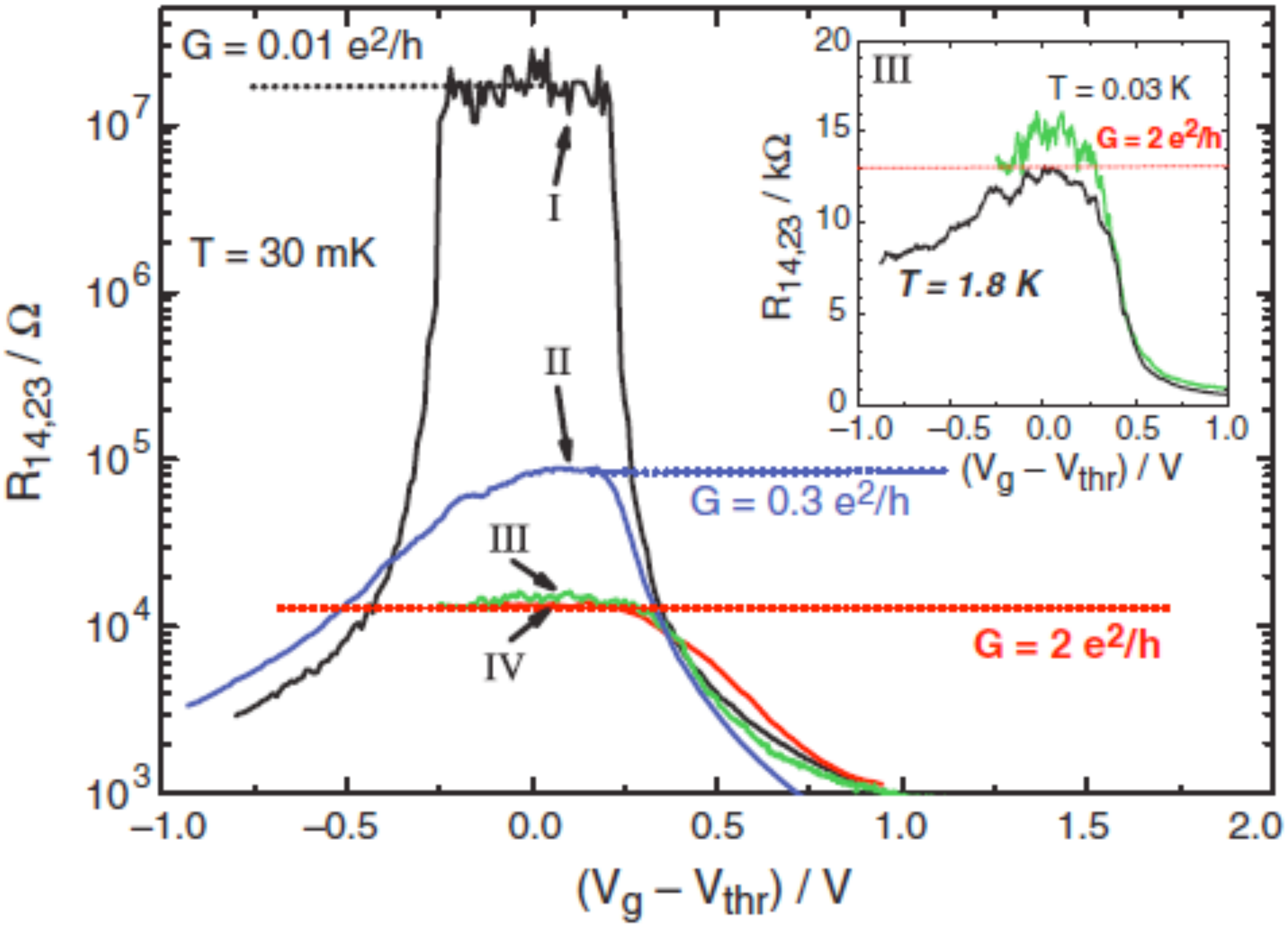


Experimental detection: 2DTI



I. Trivial
II-IV. Non trivial

L=20 microns
(L,W) =(1,1) micron
(L,W) = (1,0.5) micron



[Konig et al 07]

Experimental detection: 3DTI

Bi2X3

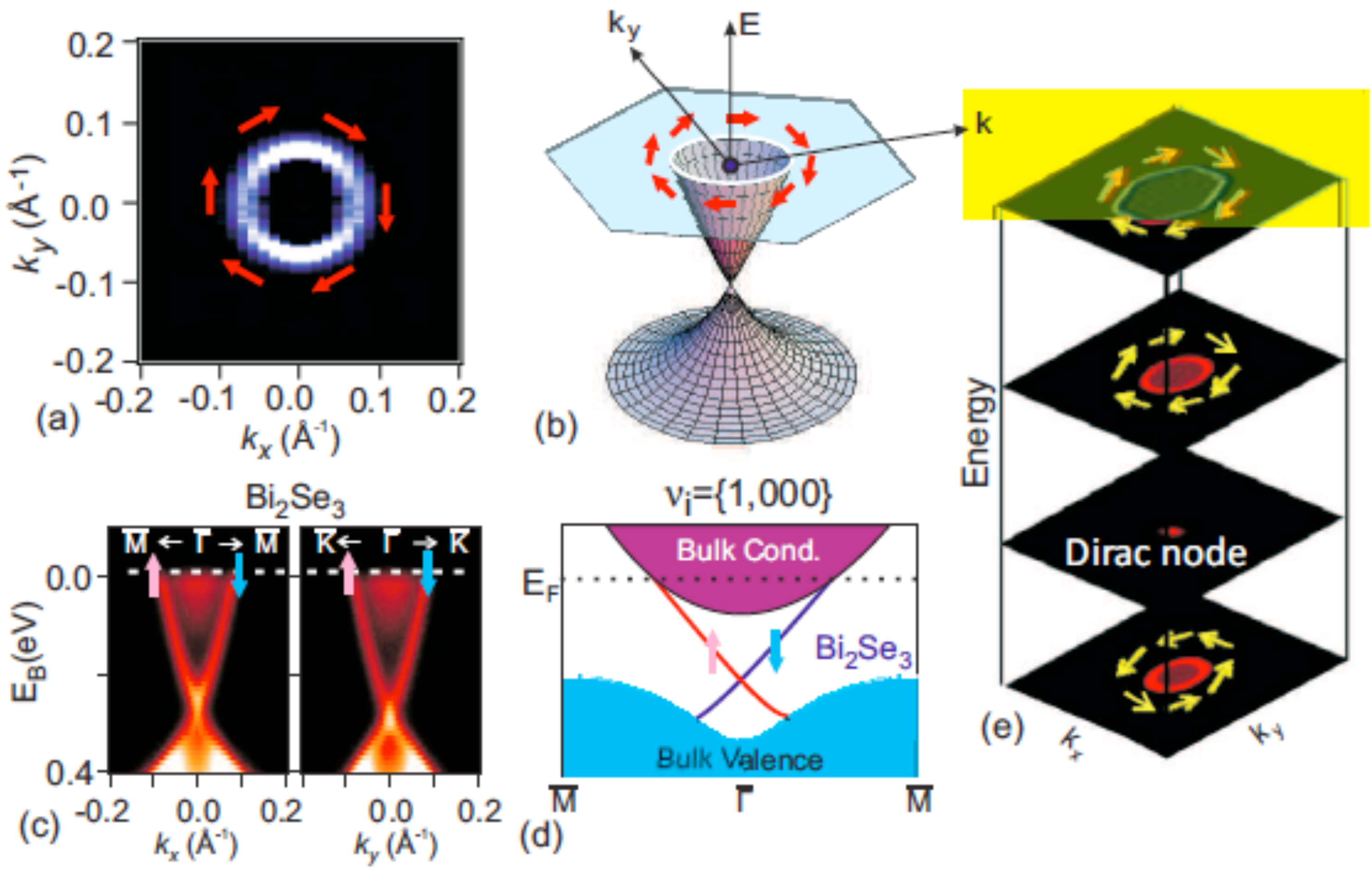


Figure from Moore and Hasan, Annu. Rev. Condens. Matter Phys. 2011.
Experiments by Xia et al. 2008, 2009 and Hsieh et al. 2009

Interesting questions

2D TI:

Surface states are 1D.

$$G = 2e^2/h$$ ✓

Helical states. ✓

Evidence of luttinger liquid physics?

Impurity physics?

Exact solutions for non-equilibrium transport?

Interesting questions

3D TI:

Surface states are 2D.

Dirac Fermions.✓

Transport?

Topological effects?

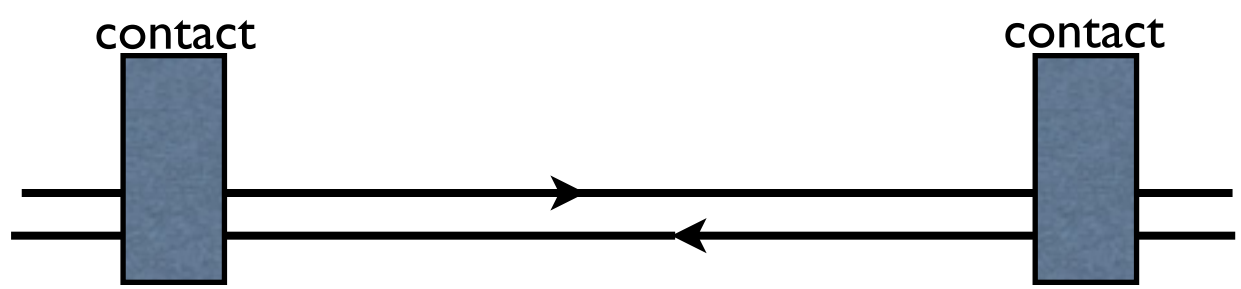
Majorana fermion physics?

Luttinger liquid physics on the edge of a 2DTI

Does $\frac{2e^2}{h}$ mean edge is non-interacting?

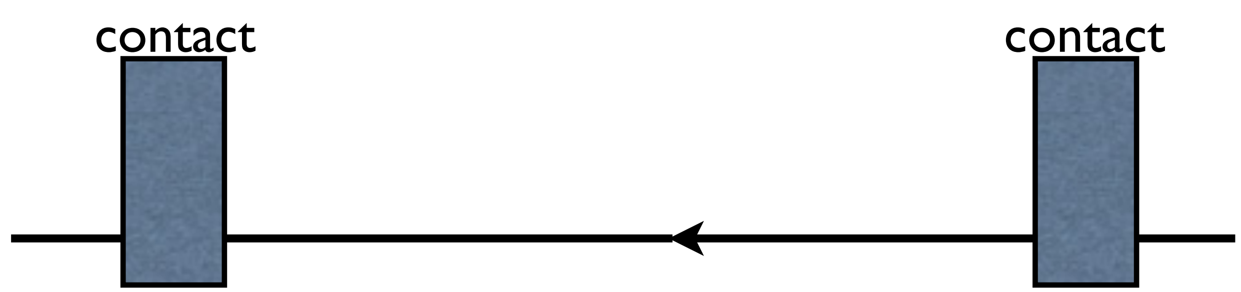
No!

QSHE



$$G = \cancel{g}\frac{e^2}{h}$$

QHE



$$G = \nu\frac{e^2}{h}$$

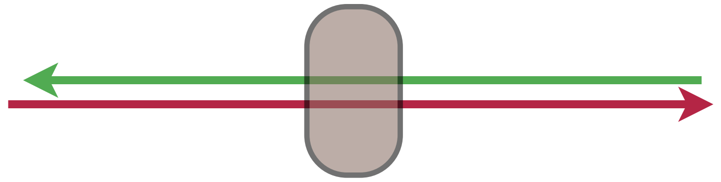
[Maslov and Stone 95, Safi and Schulz 95,97; Furusaki and Nagaosa 96; Oreg and Finkel'stein 96; Alekseev et al. 96; Egger and Grabert 98..... Wen 90, Milliken et al 96]

Luttinger liquid physics on the edge of a 2DTI

To observe interaction effects we must have backscattering.

Need to break time reversal symmetry:

Local Magnetic impurity



Local breaking of TR

Global magnetic field + impurity

[Maciejko et al 09; Tanaka et al 11; Beri and Cooper 12; Soori et al. 12; Timm. 12, Lezmy Oreg and Berkooz, 12]

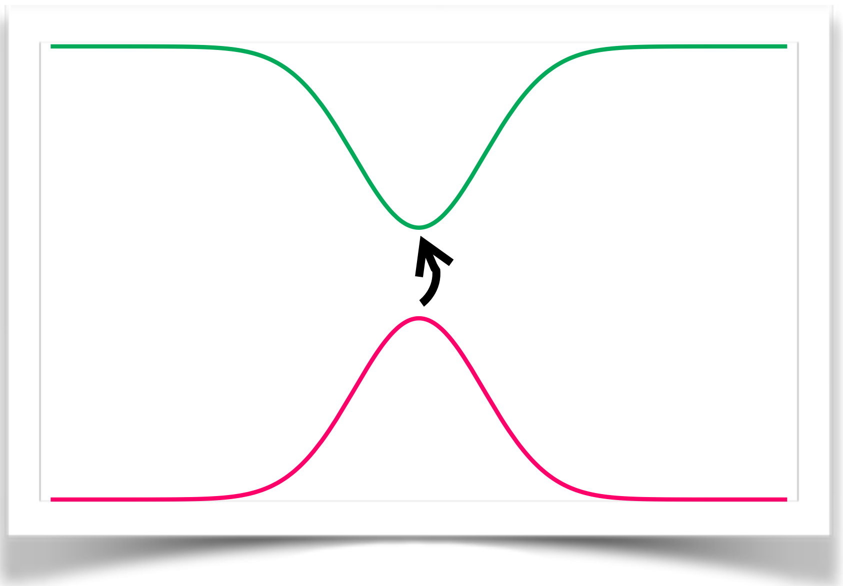
Goal

To find a controlled way to induce local backscattering

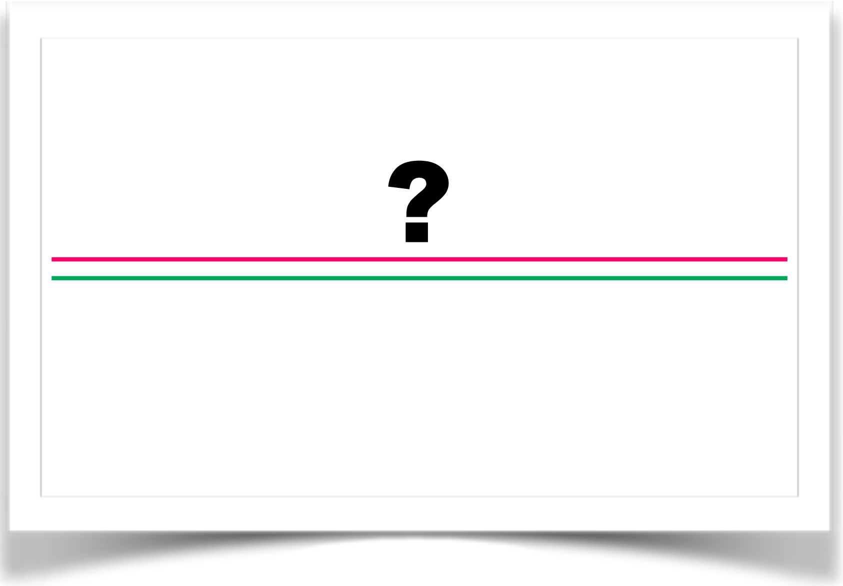
Use the special properties of 2DTI
to obtain exact solution for noneq. conductance

designing a tunable impurity

Point contact in the QHE



Point contact in the QSHE

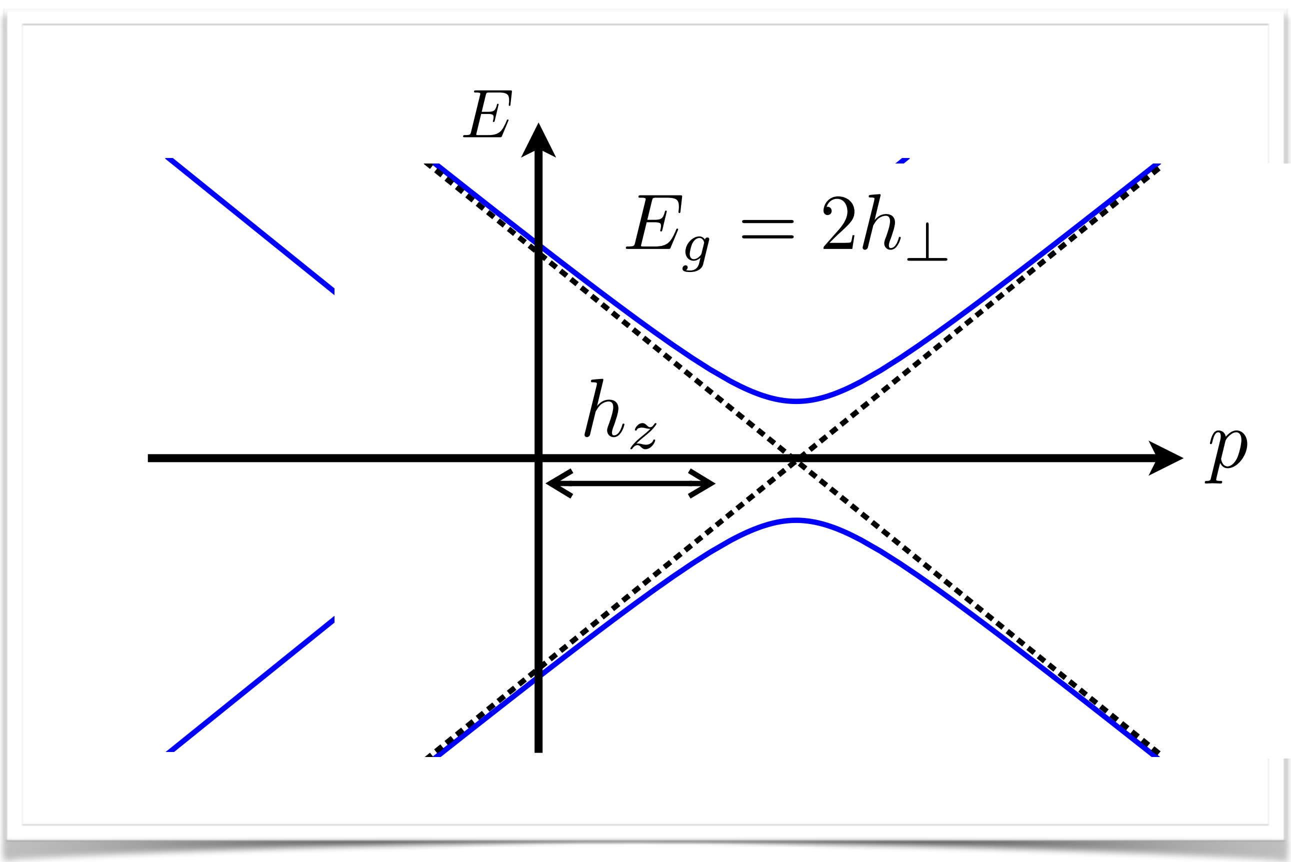


magnetic field

Breaks time reversal symmetry

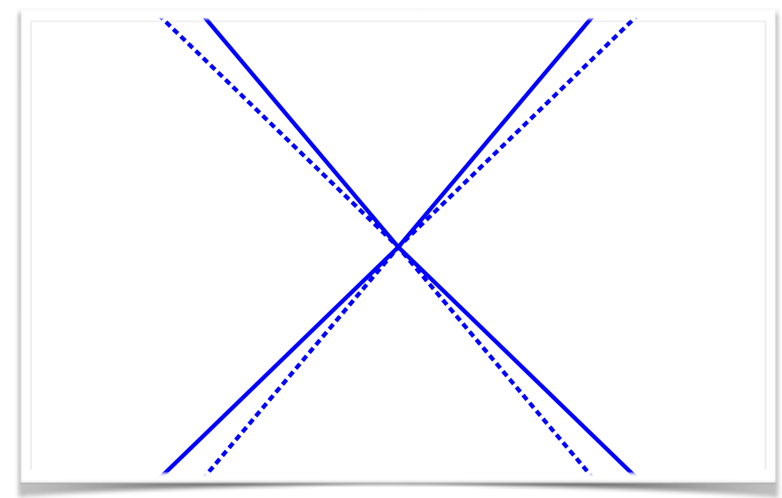
$$H = -i\hbar v_F \sigma_z \partial_x + \mu_B \frac{g_e}{2} \vec{B} \cdot \vec{\sigma}$$

$$E^2 = (vp - h_z)^2 + h_\perp^2$$

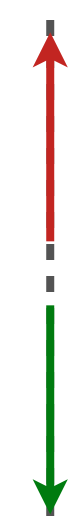


Rashba spin orbit coupling

$$H = -i\hbar v_F \sigma_z \partial_x - \frac{i\hbar}{2}\{\alpha, \partial_x\}\sigma_y$$



$$v \to v_\alpha = \sqrt{\alpha^2 + v^2}$$

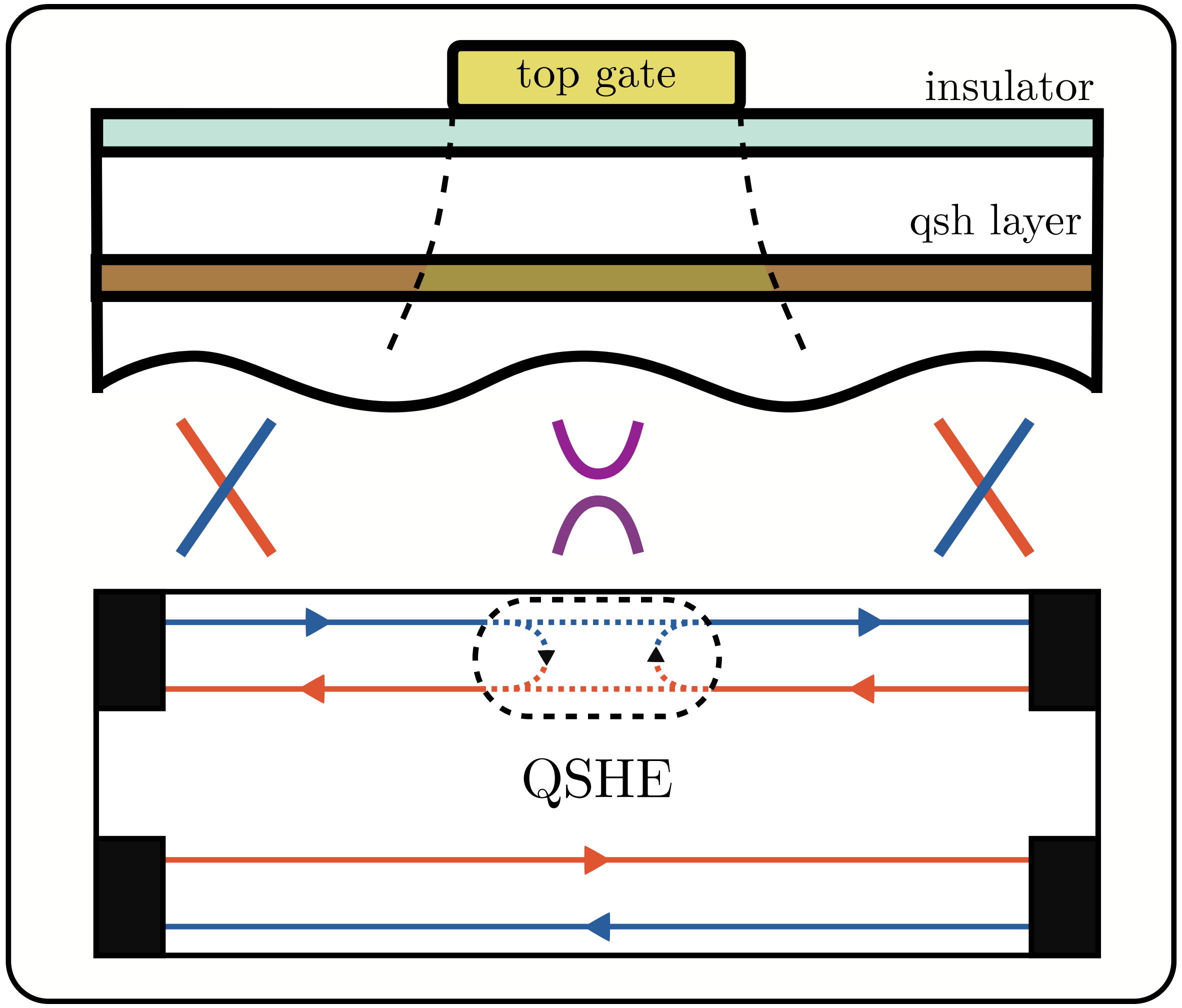


$$\theta = \tan^{-1}(\alpha / v_F)$$

tunable impurity

$$H = -i\hbar v_F \sigma_z \partial_x + h\sigma_z - \frac{i\hbar}{2}\{\alpha, \partial_x\}\sigma_y$$

$\alpha = 0 \qquad \alpha \neq 0 \qquad \alpha = 0$

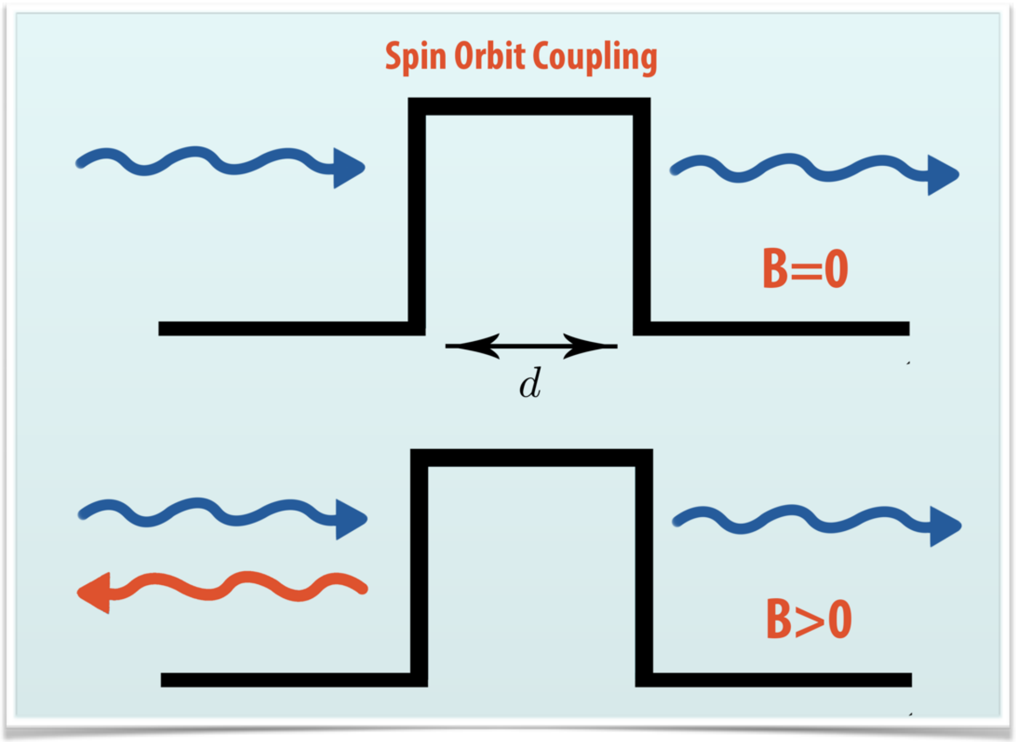


$$E_g = 2\alpha h / v_F$$

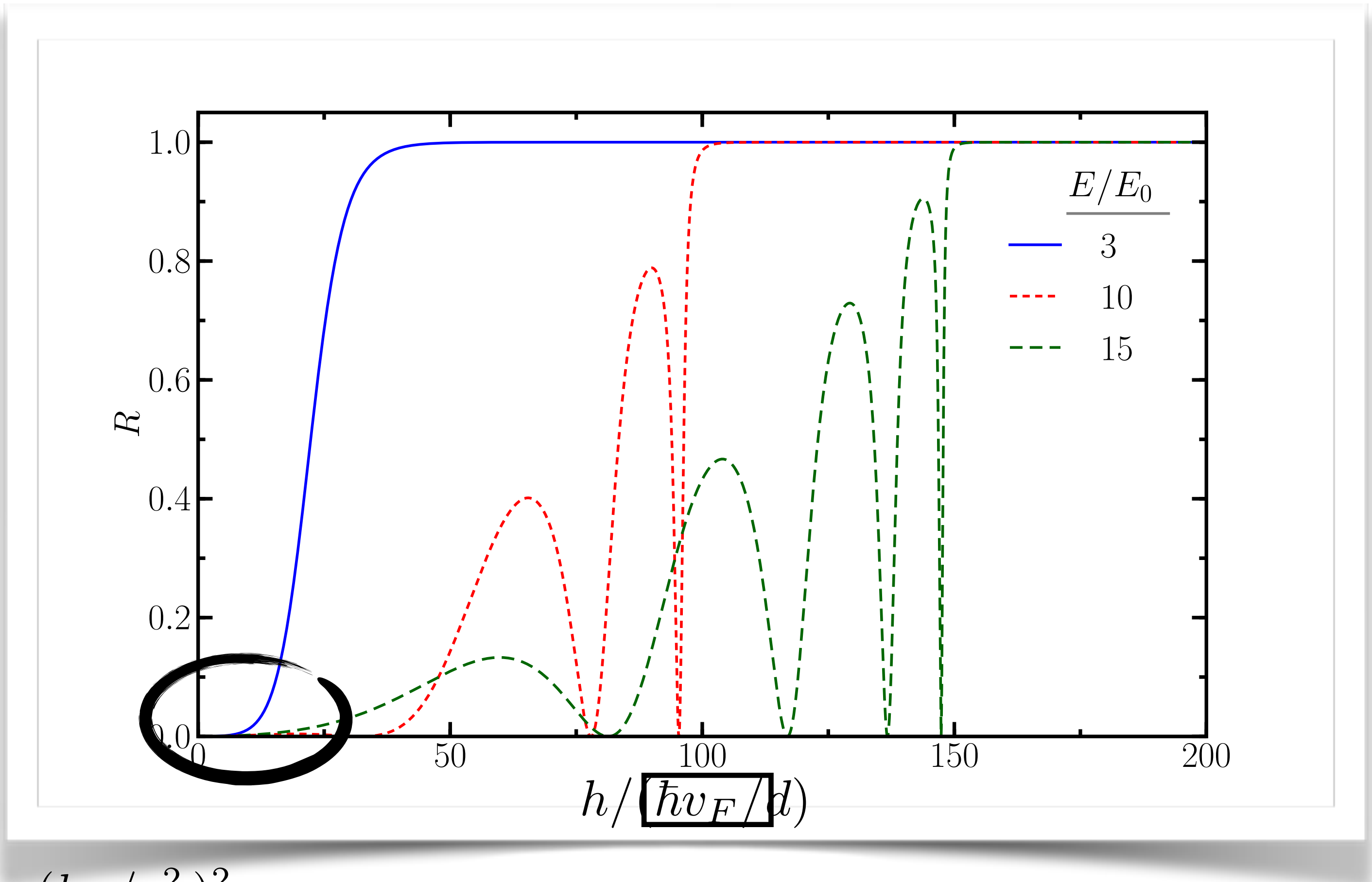
Rashba SO controllable by gate

[Nitta et al 97;Hinz et al 06]

Non interacting electrons



$$\Psi(x) = \begin{cases} \psi_R e^{ip_R x} + r\psi_L e^{ip_L x} & x < 0 \\ a_+\psi_+ e^{ip_+ x} + a_-\psi_- e^{ip_- x} & 0 < x < d \\ t\psi_R e^{ip_R x} & x > d \end{cases}$$



$R \sim (h\alpha/v_F^2)^2$

Interacting edge

$$H_0 = iv(\psi_\uparrow^\dagger \partial_x \psi_\uparrow - \psi_\downarrow^\dagger \partial_x \psi_\downarrow)$$

$$H_{int} = u_2 \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow + u_4 \left[(\psi_\uparrow^\dagger \psi_\uparrow)^2 + (\psi_\downarrow^\dagger \psi_\downarrow)^2 \right]$$

$$H_{BS} = \lambda \psi_L^\dagger(0) \psi_R + h.c$$

$$\uparrow = R \quad \downarrow = L$$

Helicity reduces the number of degrees of freedom by locking the spin to the momentum

Interacting edge

Maps to a <u>*spinless*</u> Luttinger Liquid

$$H = \frac{v}{4\pi g}((\partial_x \phi_R)^2 + (\partial_x \phi_L))^2$$

$$H_{BS} = \lambda \cos(\phi_R - \phi_L)$$

$$H_{BS} = \frac{1}{2}\sum_n \lambda_n e^{in\phi_R(0)} e^{-in\phi_L(0)} + h.c.$$

$$\frac{d\lambda_n}{d\ell} = (1 - n^2 g)\lambda_n$$

$1/4 < g < 1$ **One relevant term**

$$G(T) \sim \frac{e^2}{h}\left[1 - \left(\frac{T_B}{T}\right)^{2(1-g)}\right] \qquad T_B/T \to 0$$

$$G(T) \sim \frac{e^2}{h}\left(\frac{T}{T_B}\right)^{2(1/g-1)} \qquad T_B/T \to \infty$$

$T_B \sim \lambda^{1/(1-g)}$

[Kane and Fisher 92, Kane and Teo 09, and many others]

Spinless luttinger liquid with impurity

ϕ_R
ϕ_L

$$\phi^o(x,t) = \tfrac{1}{\sqrt{2}}(\phi_L(x,t) - \phi_R(-x,t))$$
$$\phi^e(x,t) = \tfrac{1}{\sqrt{2}}(\phi_L(x,t) + \phi_R(-x,t))$$

$$\phi_R(x,t) \equiv \phi^o(x+t)$$
$$\phi_L(x,t) \equiv \phi^o(-x+t)$$

$$H_{SG} = \tfrac{1}{8\pi g} \int_0^\infty [\Pi^2 + (\partial_x \phi^o)^2] + \lambda \cos \phi^o(0)$$

Integrable model, can be solved exactly

[Fendley, Ludwig, Saleur 95; Fendley, Lesage, Saleur 96]

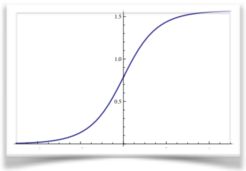
Integrability

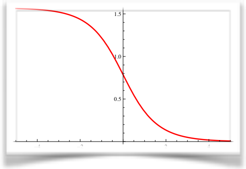
[Ghoshal and Zamolodchikov 94; Fandley Ludwig and Saleur 95; Fendley Lesage and Saleur 96]

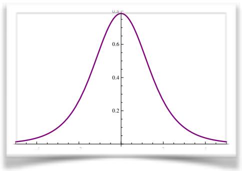
Infinite set of conserved quantities **with impurity**

$$H_{SG} = \frac{1}{8\pi g}\int_0^\infty [\Pi^2 + (\partial_x \phi^o)^2] + \lambda \cos\phi^o(0)$$

find "quasiparticle" basis







Additive energies

$H = v\sum_i p_i$

One body S matrices

Particles scatter one-by-one

Two body S matrices in the bulk

Particles scatter elastically off each other

Does not mean scattering is trivial!

Permute quantum numbers, phase shifts

$$f_j = \frac{1}{1+e^{-\mu_j/T+\epsilon_j}}$$

$\epsilon_j(\theta)$

TBA equations for

EXACT SOLUTION AT $g = 1 - 1/m$

$$I(V, T_B, T) = \frac{T(m-1)}{2} \int \frac{d\theta}{\cosh^2[\theta - \ln(T_B/T)]} \ln\left(\frac{1 + e^{(m-1)V/2T - \epsilon_+(\theta)}}{1 + e^{-(m-1)V/2T - \epsilon_+(\theta)}} \frac{1 + e^{-(m-1)V/2T - \epsilon_+(\infty)}}{1 + e^{(m-1)V/2T - \epsilon_+(\infty)}} \right)$$

$$G(V, T_B, T) = \frac{dI_{BS}}{dV} = G(V/2T, T_B/T)$$

$$G(V/2T, 0) = g$$

Include contact correction

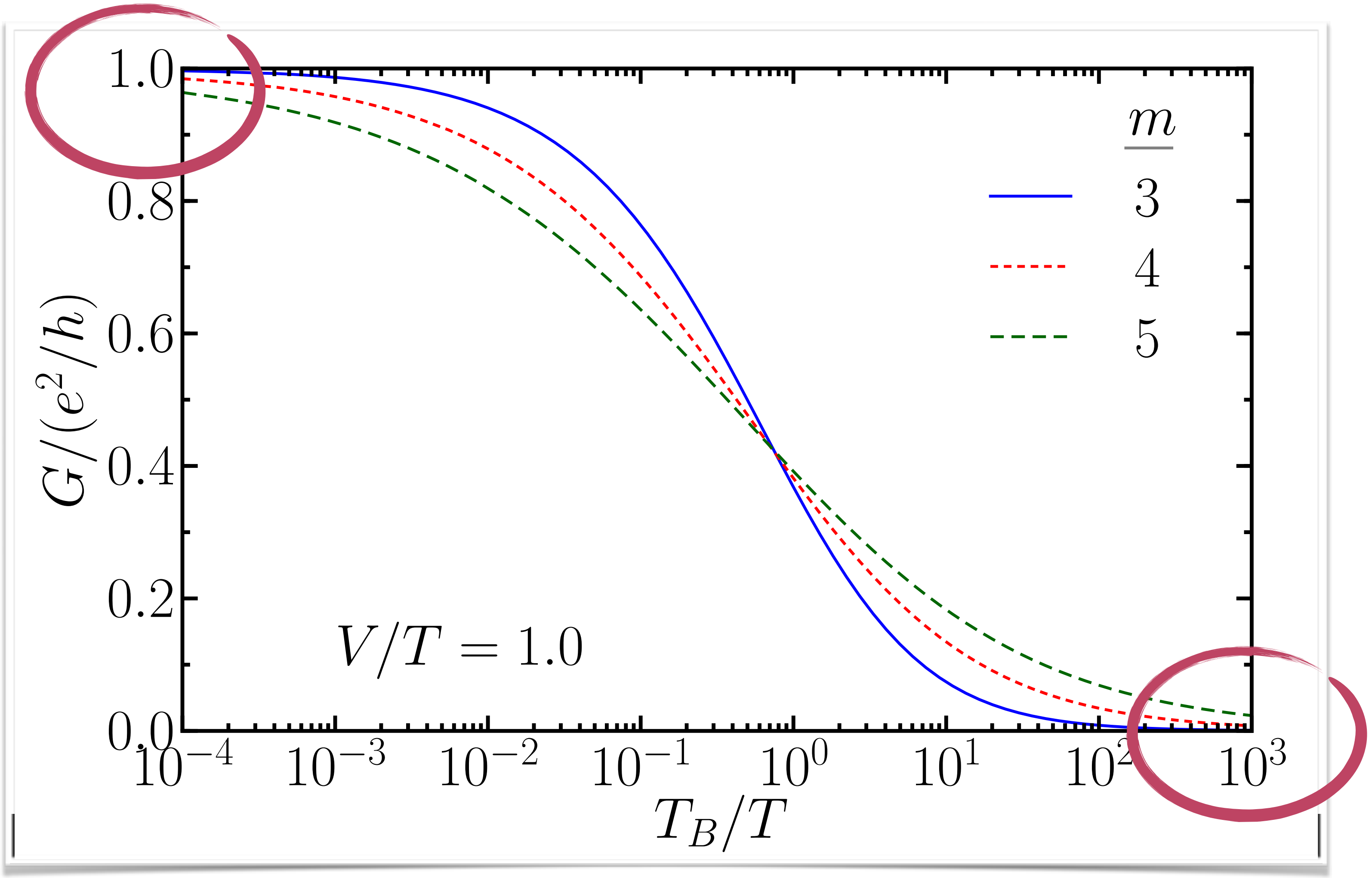
$$V = -\left(1 - \frac{1}{g}\right) I(W) + W$$

[Fendley Lesage and Saleur 96, Koutouza, Siano and Saleur 01]

Conductance vs. temperature

$$g = 1 - \frac{1}{m}$$

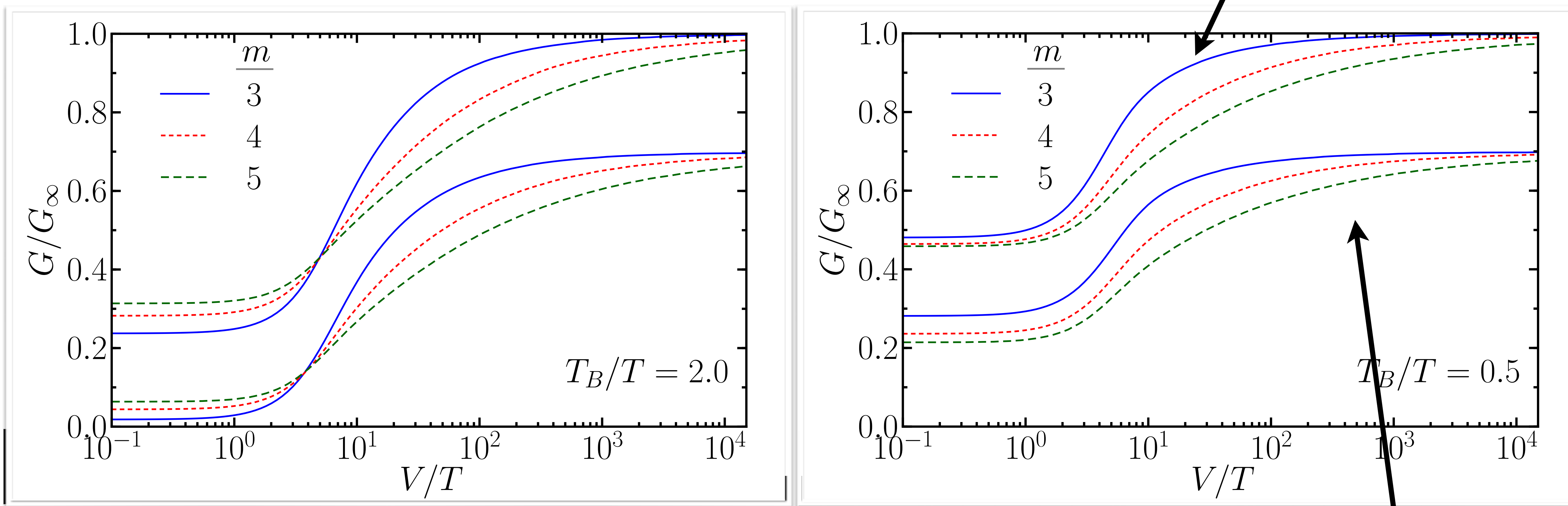
$G(T) \sim \frac{e^2}{h}\left[1 - \left(\frac{T_B}{T}\right)^{2(1-g)}\right]$



$G(T) \sim \frac{e^2}{h}\left(\frac{T}{T_B}\right)^{2(1/g-1)}$

VOLTAGE DEPENDENCE

With FL contact



W/O FL contact
(shifted down)

Conclusion

A setup for transport experiment on the QSHE edge:
controlled TR symmetry breaking
+ controlled SO coupling
= "point contact"

Interacting edge theory is integrable model:
Exact solutions for conductance with FL leads

From experiment:
estimate LL parameter
Confirm integrability
Confirm effects of leads

Perfect transmission and Majorana fermions in 3DTI

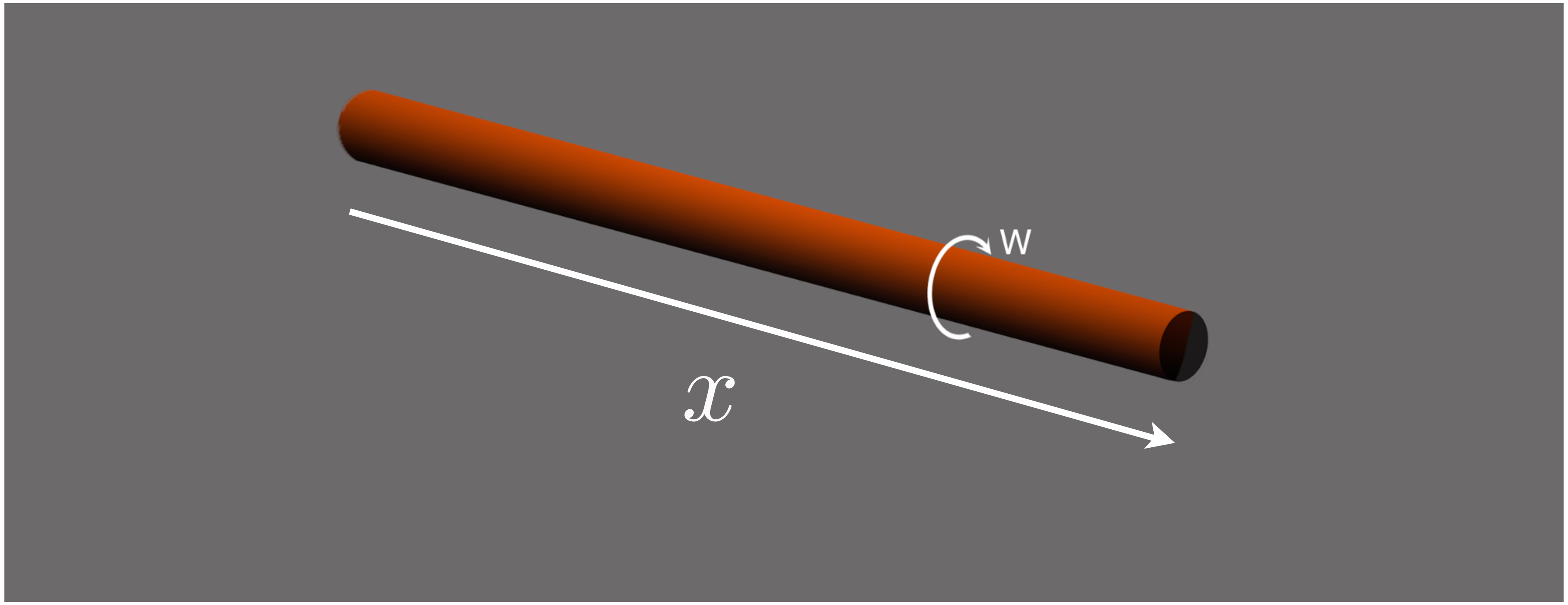
Detect a perfectly transmitted mode and signature of Majorana fermions

Problem with current setups

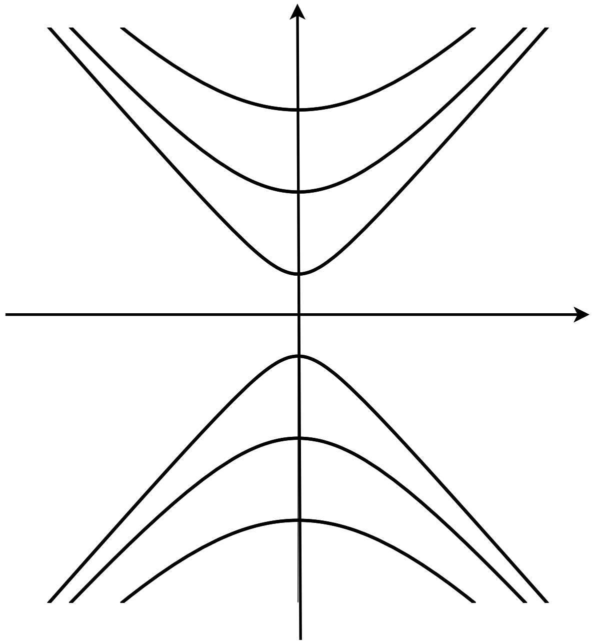
Large number of modes and finite chemical potential

Large contribution to transport from bulk states

Dirac fermion on a curved surface- TI nanowires

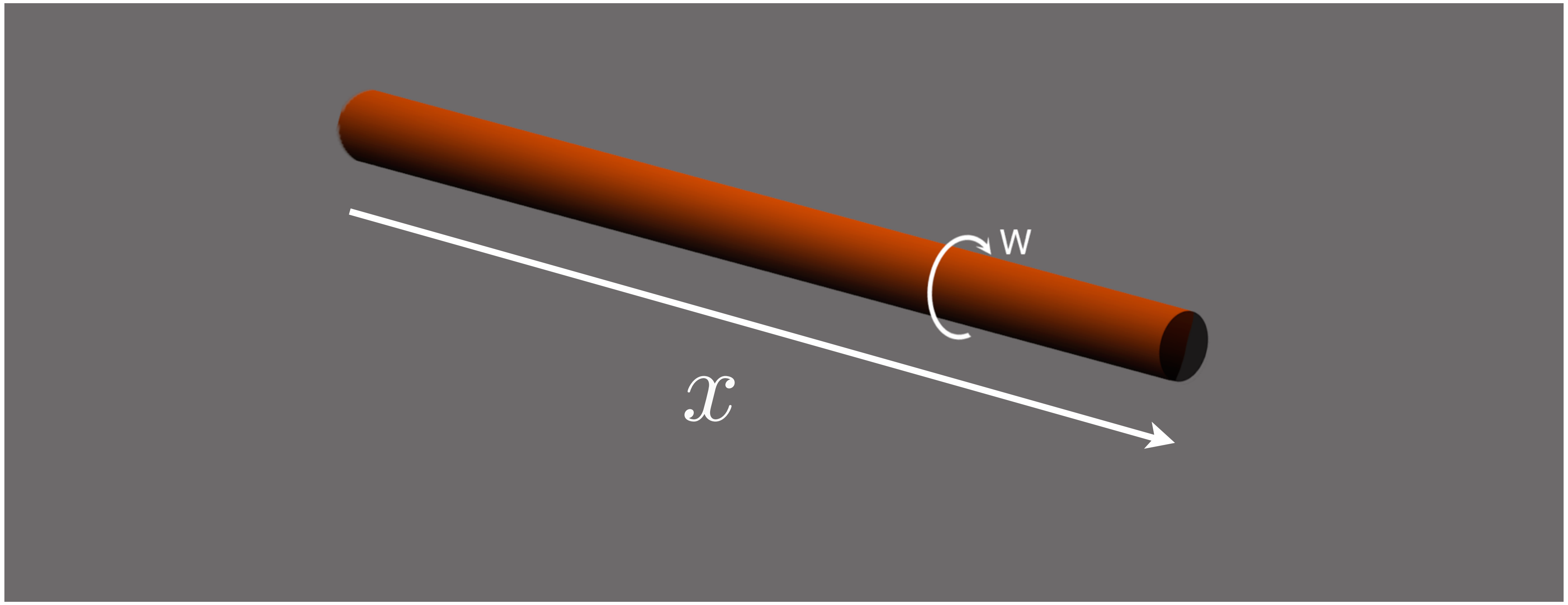


$$\psi(x, y+W) = e^{i\pi}\psi(x, y)$$



[Ando and Suzuura 2002, Ran, Vishwanath, and Lee 2008, Rosenberg, Guo, and Franz 2010, Ostrovsky, Gornyi, and Mirlin 2010, Bardarson and Moore 2013]

Dirac fermion on a curved surface- TI nanowires



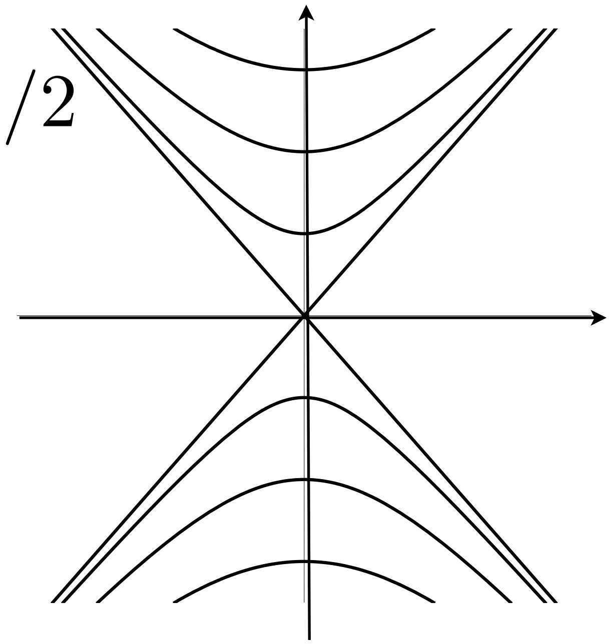
$$\psi(x, y+W) = e^{i\pi + i2\pi\Phi/\Phi_0}\psi(x,y)$$

odd number of modes

$$T^2 = -1$$
$$S^T = -S$$

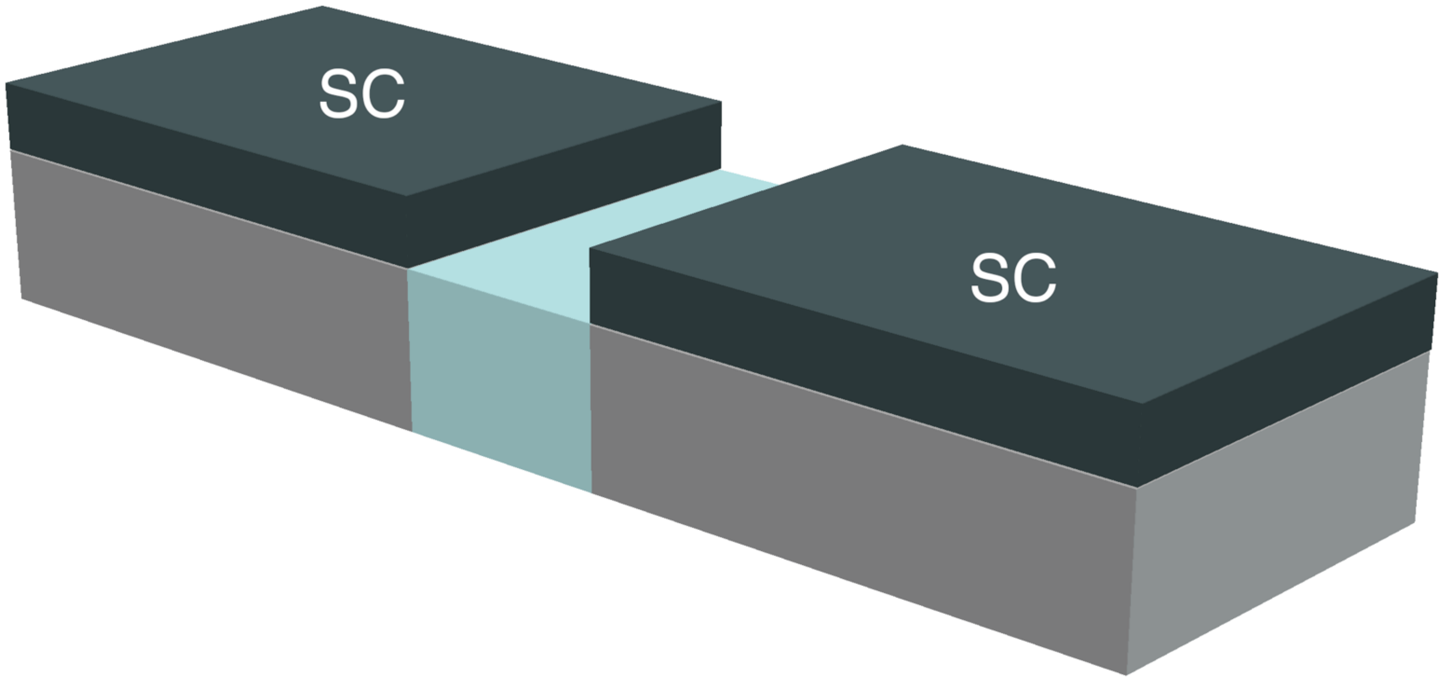
Perfectly transmitted mode

$\Phi = \Phi_0/2$



[Ando and Suzuura 2002, Ran, Vishwanath, and Lee 2008, Rosenberg, Guo, and Franz 2010, Ostrovsky, Gornyi, and Mirlin 2010, Bardarson and Moore 2013]

Proximity effect - Josephson Junctions



"Whereas bulk transport tends to obscure
the observation of surface state transport in the normal state, the
supercurrent is found to be carried mainly by the surface states"

[Veldhorst et al. Nature Mater. 2012]

Majorana states $\Delta\phi = \pi$

[Fu and Kane, PRL 2008]

[Alicea Rep Prog Phys (2012), Beenakker, Annu. Rev. Cond. Mat. Phys. 2013, Grosfeld and Stern 2011, Potter and Fu 2013, Weider et al 2013]

3DTI and Majorana fermions

Short JJ $L \ll \xi$

[Beenakker 2006,
Titov and Beenakker 2006]

$$E_n(\phi) = \Delta\sqrt{1 - \tau_n \sin^2(\phi/2)}$$

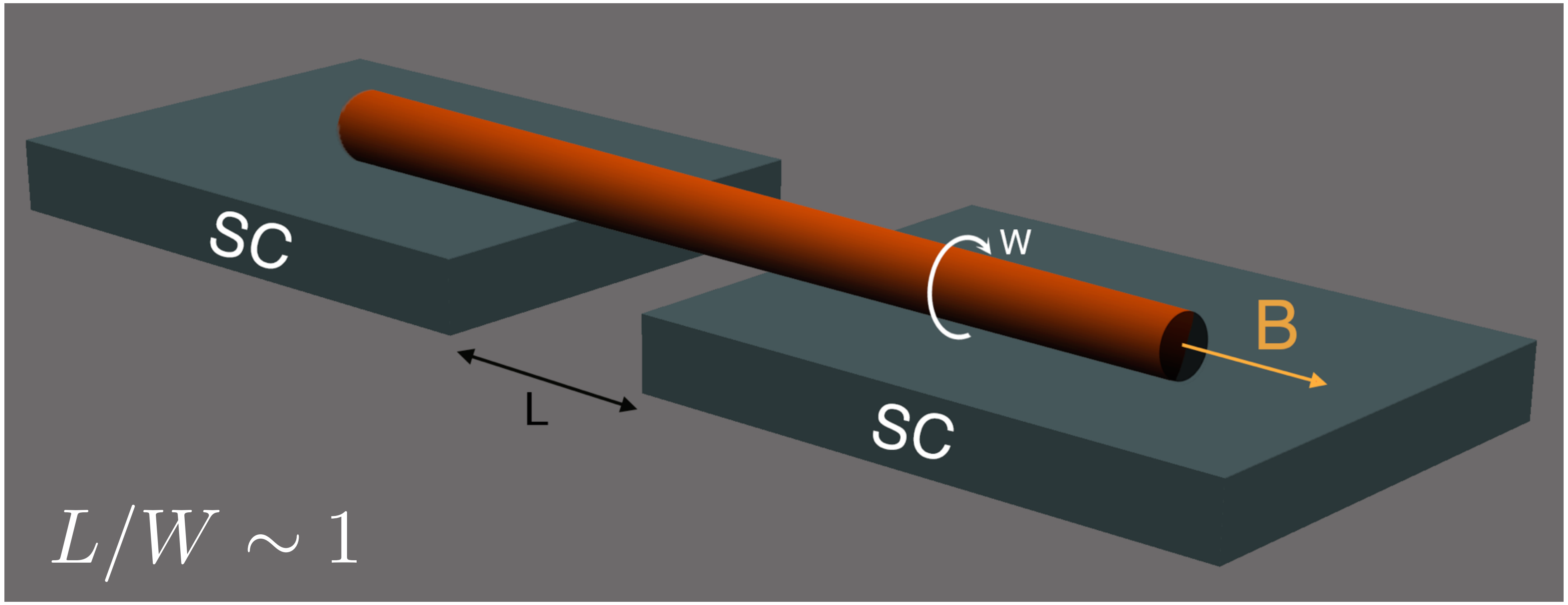
$$\boxed{\begin{array}{c}\phi = \pi \\ + \\ \tau = 1\end{array}}$$

$$\overline{}$$

$$E = 0$$

Zero energy Majorana state requires a
perfectly transmitted mode

Josephson junction on a 3DTI nanowire



Hosts a PTM $\Phi = \phi_0/2$

Hosts Majorana fermions $\Delta\phi = \pi$

[Ando and Suzuura 2002, Ran, Vishwanath, and Lee 2008, Rosenberg, Guo, and Franz 2010, Ostrovsky, Gornyi, and Mirlin 2010, Bardarson and Moore 2013, Fu and Kane PRL 2008, Cook and Franz 2011, Cook at al 2012]

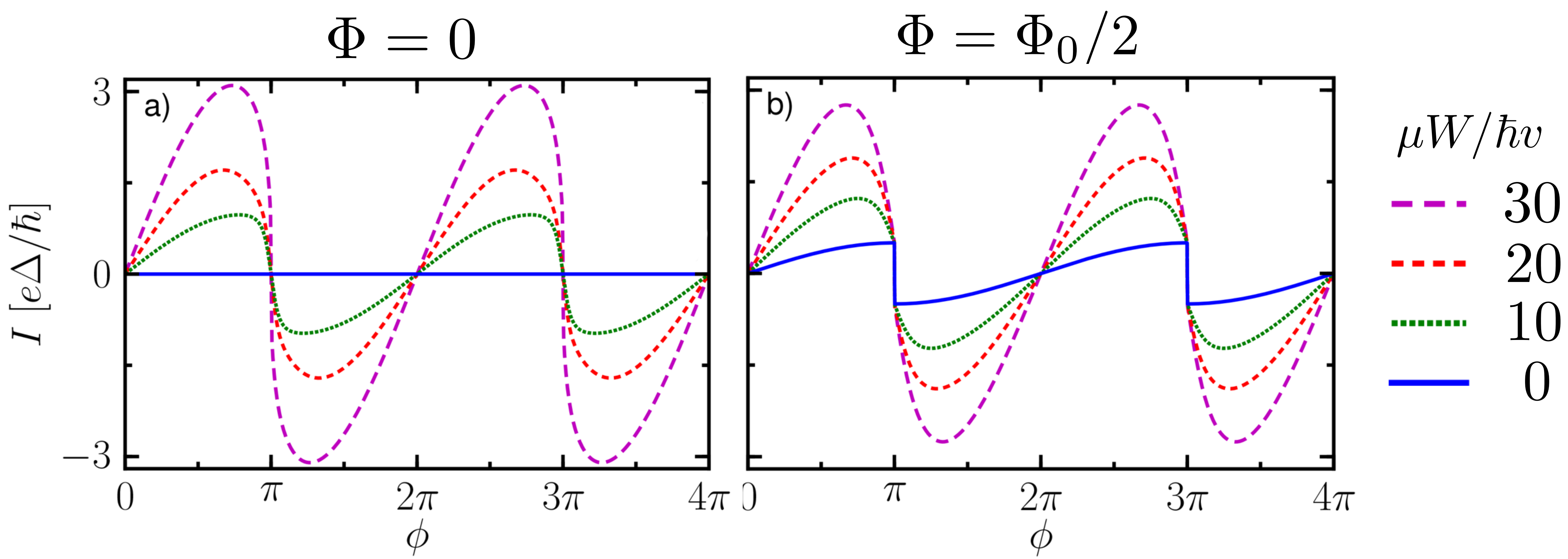
Current Phase relation - clean wire

[Titov and Beenakker 2006]

$$E_n(\phi) = \Delta\sqrt{1 - \tau_n \sin^2(\phi/2)}$$

$$\tau_n = \frac{k_n^2}{k_n^2 \cos^2(k_n L) + (\mu/\hbar v)^2 \sin^2(k_n L)} \qquad k_n = \sqrt{(\mu/\hbar v)^2 - q_n^2}$$

$$I \propto \sum_n \partial E_n / \partial\phi$$



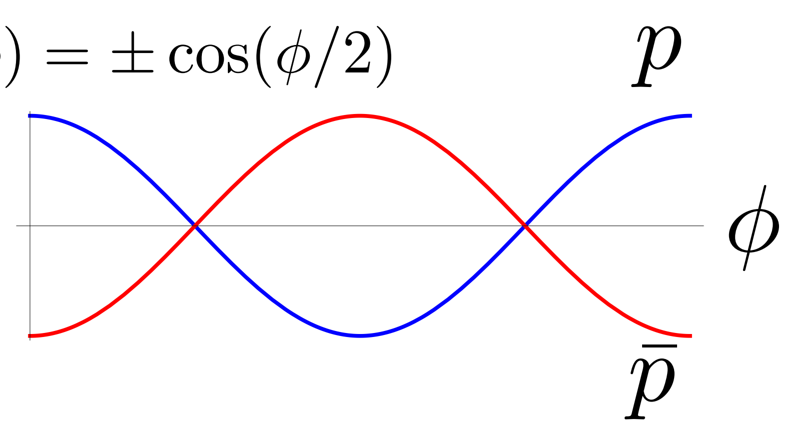
[RI, Jens Bardarson, H.S. Sim, Joel Moore, 2013]

Current Phase relation - clean wire

Parity anomaly

$$\gamma^\dagger = \gamma$$

$E(\phi) = \pm\cos(\phi/2)$

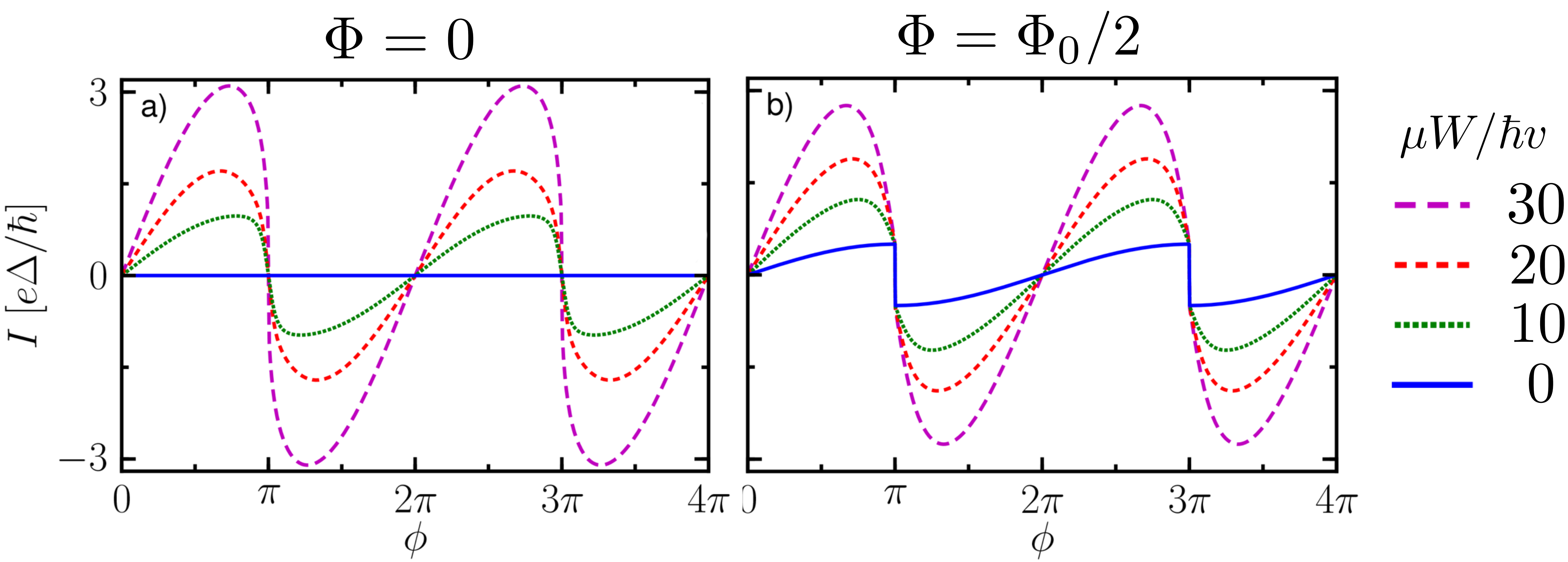


γ_2 ●

γ_1 ●

$$c^\dagger = \gamma_1 - i\gamma_2$$

$$c = \gamma_1 + i\gamma_2$$



[RI, Jens Bardarson, H.S. Sim, Joel Moore,2013]

Current Phase relation - clean wire

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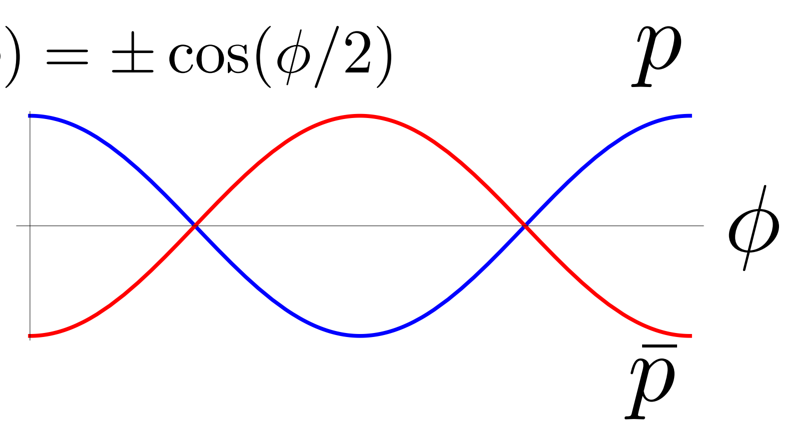


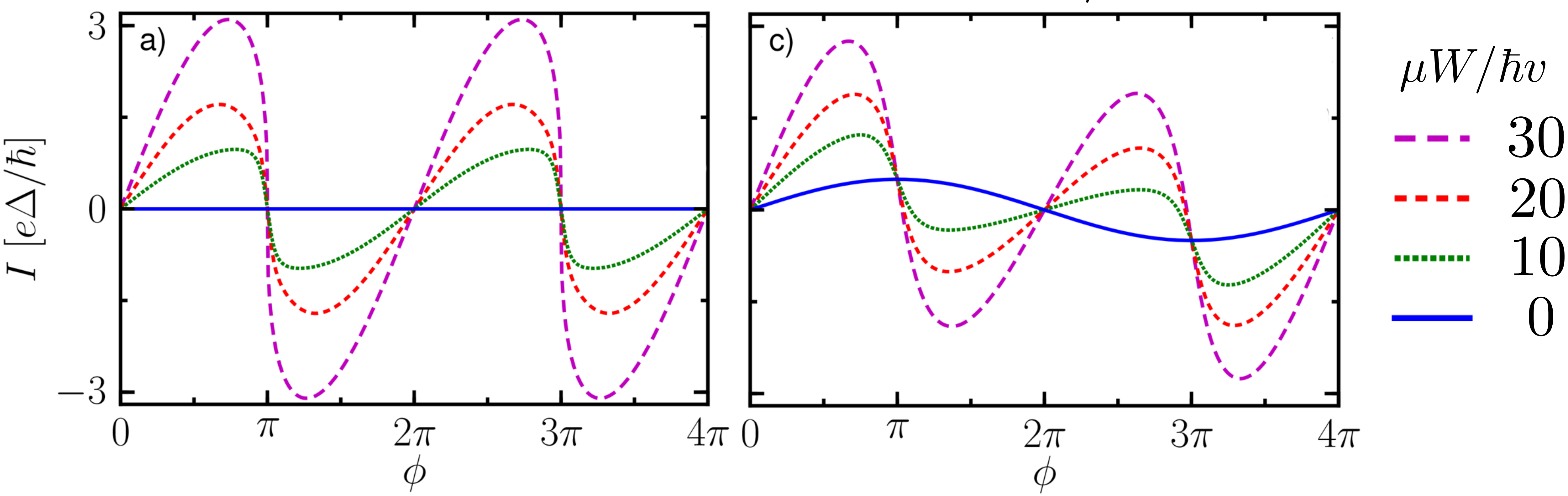
$$\gamma^\dagger = \gamma$$

γ_2

γ_1

$$c^\dagger = \gamma_1 - i\gamma_2$$
$$c = \gamma_1 + i\gamma_2$$

$\Phi = 0$ $\Phi = \Phi_0/2$



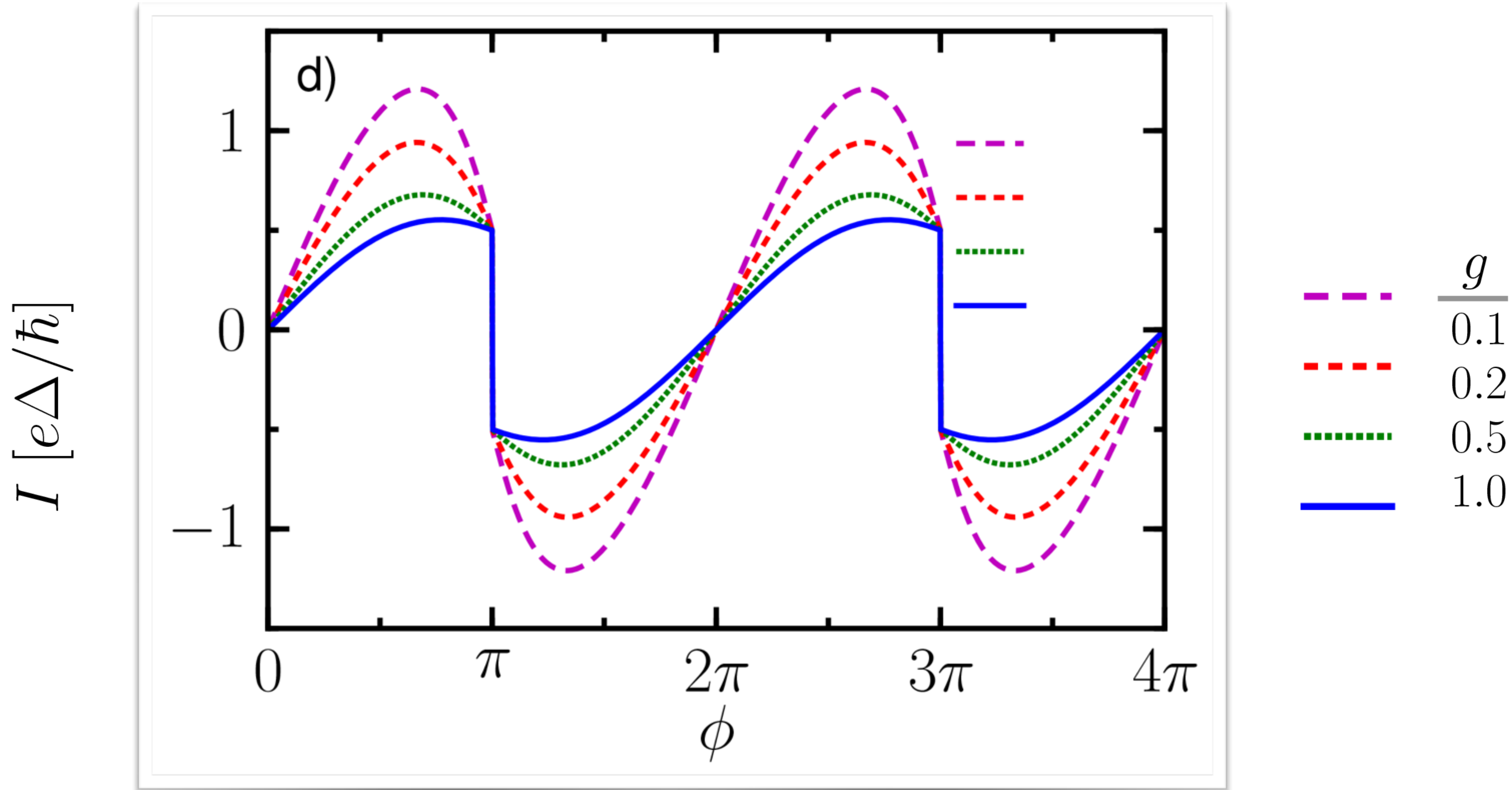
[RI, Jens Bardarson, H.S. Sim, Joel Moore,2013]

Current Phase relation - disorder

$$\langle V(r)V(r')\rangle = g\frac{\hbar v}{2\pi\xi_D^2}e^{|r-r'|/2\xi_D^2}$$

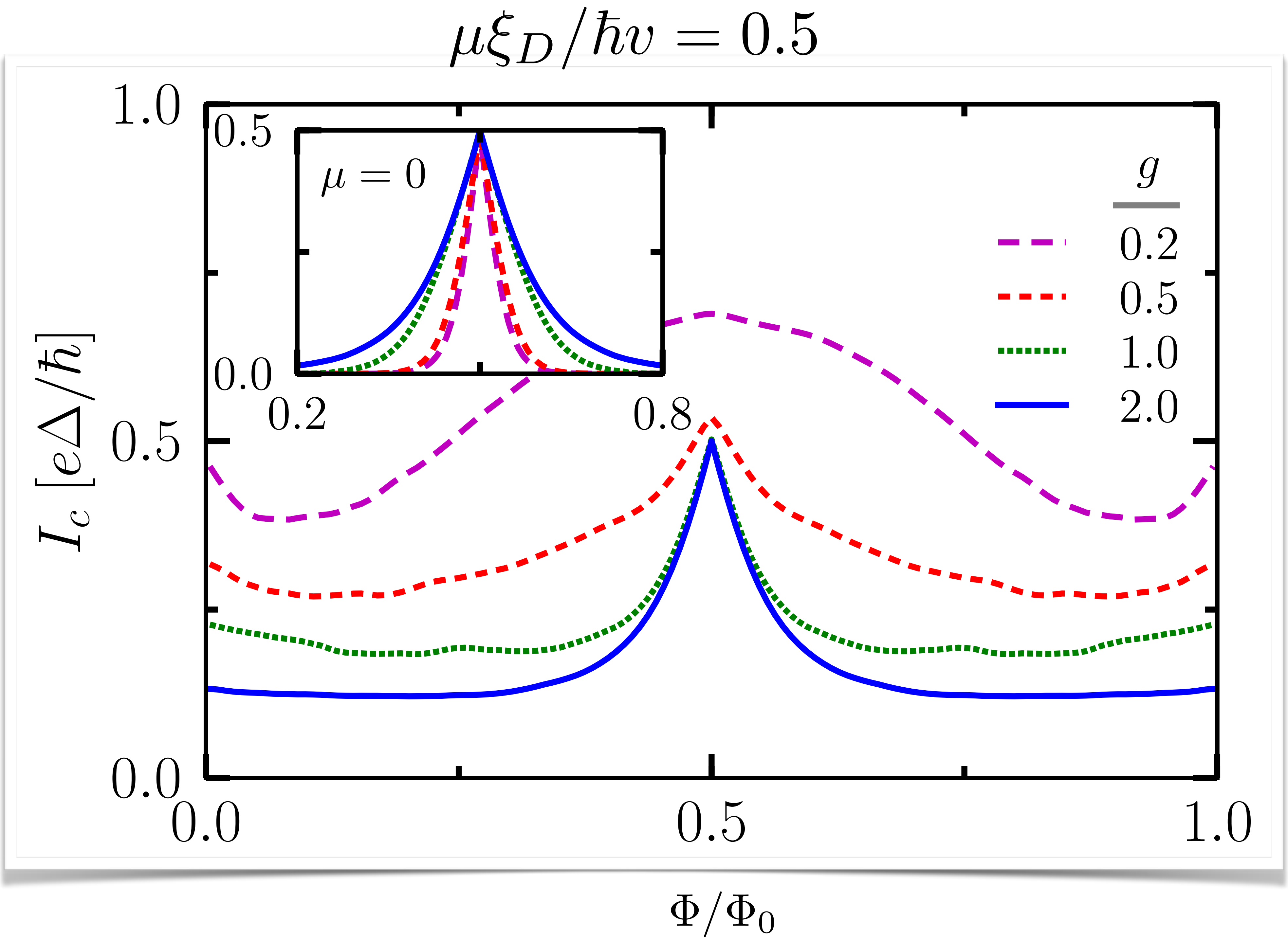
[Bardarson et al. PRL 2007]

$\Phi = \Phi_0/2$ $\mu W/\hbar v = 20$



[RI, Jens Bardarson, H.S. Sim, Joel Moore,2013]

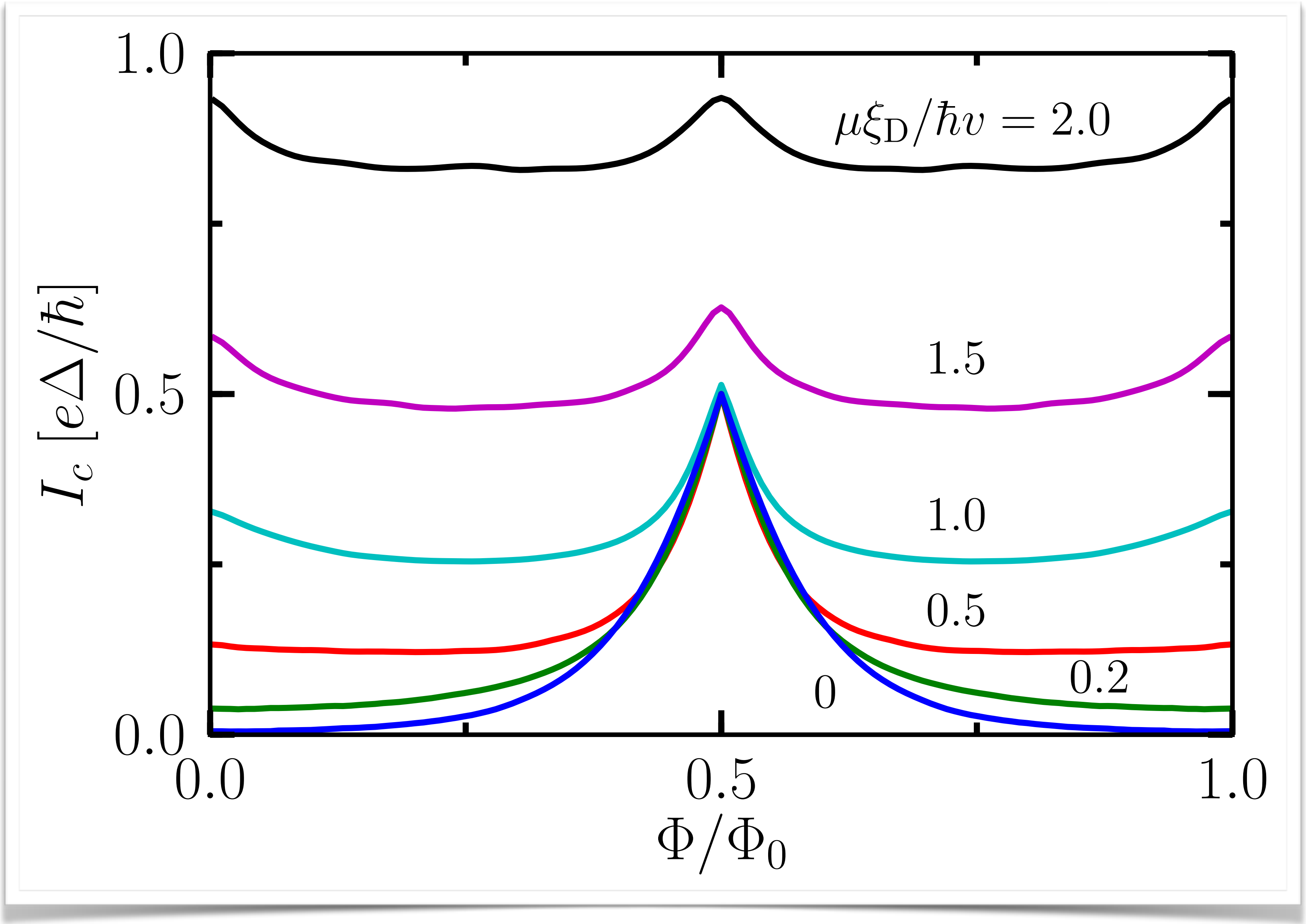
Critical current - disorder



[RI, Jens Bardarson, H.S. Sim, Joel Moore, 2013]

Critical current - disorder

$g = 2$



[RI, Jens Bardarson, H.S. Sim, Joel Moore,2013]

Conclusion

Josephson junctions on 3DTI nanowires are
a great place to look for signatures of surface
physics:

PTM , Majorana fermions

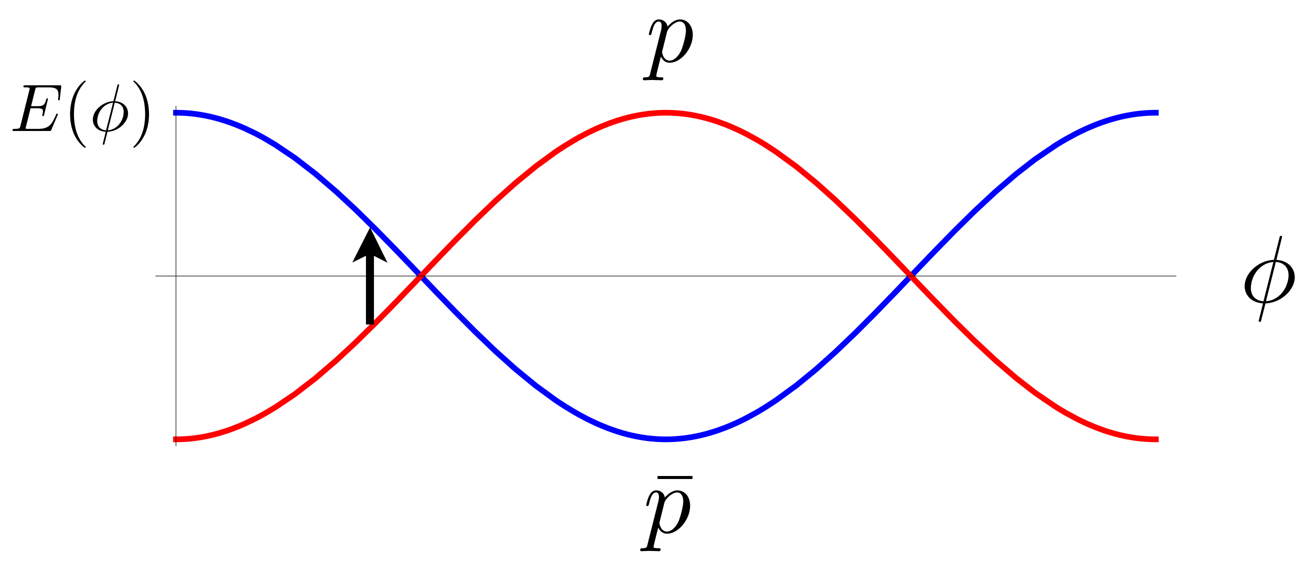
At finite chemical potential, enhanced by disorder
(and at finite temperature)

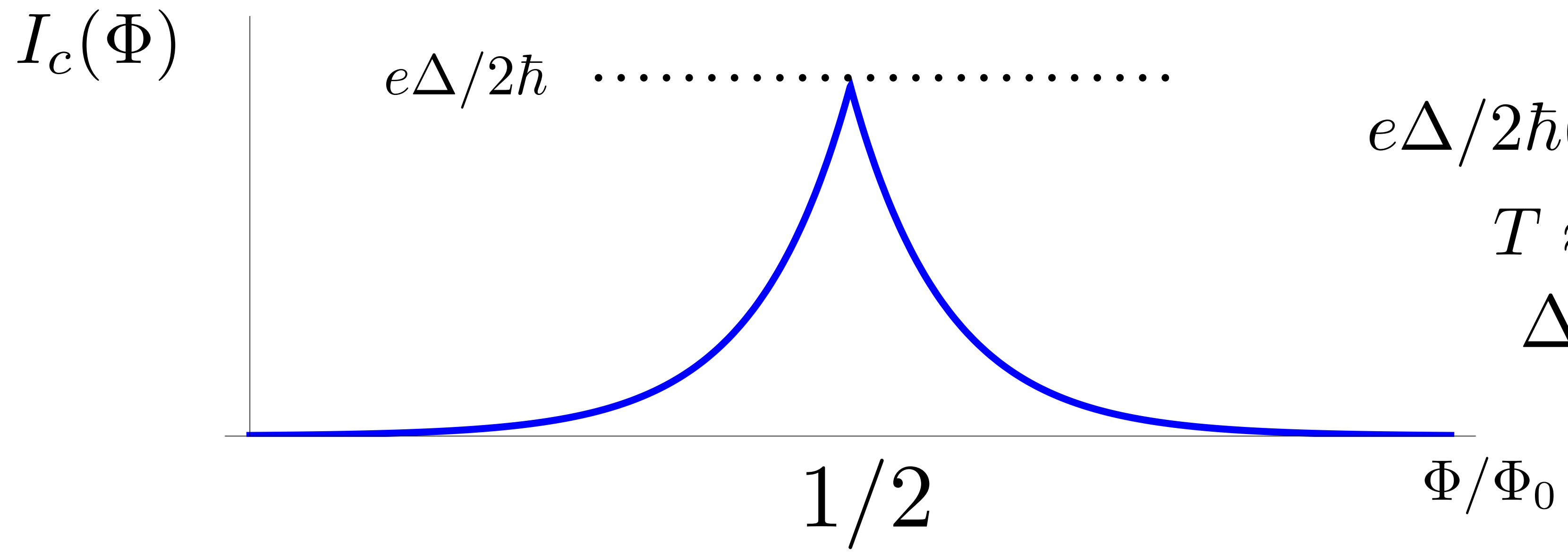
Not constrained by parity conservation

Thanks

- Paul Fendley (UVA)
- David Goldhaber Gordon Group (Stanford)
- Kam Moler Group (Stanford)
- Fernando De Juan (Berkeley)

Effects of temperature



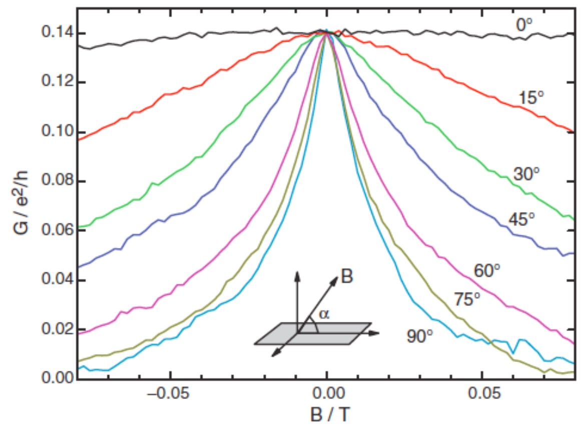


$$e\Delta/2\hbar(1-(T/\Delta)^2)$$

$$T \approx 30mK$$

$$\Delta \approx 1K$$

Some numbers



$$E = \pm\sqrt{(v_\alpha p + v_F h/v_\alpha)^2 + \alpha_0^2 h^2/v_\alpha^2}$$

$$B = 10mT$$

$$v_F = 5.5 \times 10^5 m/s$$

$$\alpha_0 \approx 5 \times 10^4 m/s$$

$$E_g \approx 100 - 300 mK$$

Temperature in experiment ~30mK (taken from Konig et al)

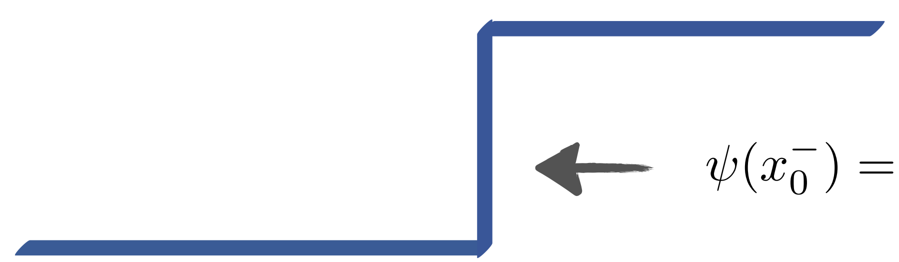
Non interacting electrons

$$H = -i\hbar v_F \sigma_z \partial_x - \frac{i\hbar}{2}\{\alpha, \partial_x\}\sigma_y$$

$$\Psi(x_1) = T_{x_1,x_0}\Psi(x_0) \qquad T_{x_1,x_0} = P_x e^{i\int_{x_0}^{x_1} dx \frac{(v_F\sigma_z + \alpha\sigma_y)}{v_F^2+\alpha^2}\left[E + \frac{i}{2}(\partial_x \alpha)\sigma_y\right]}$$

$$\psi_R(x) = (v_F^2 + \alpha^2)^{-1/4} e^{i\phi(x)} \begin{pmatrix} \cos\theta \\ i\sin\theta \end{pmatrix} \qquad \phi(x) = \int_0^x \frac{E}{\sqrt{v_F^2 + \alpha(x)^2}}.$$

$$\psi_L(x) = (v_F^2 + \alpha^2)^{-1/4} e^{-i\phi(x)} \begin{pmatrix} i\sin\theta \\ \cos\theta \end{pmatrix} \qquad 2\theta(x) = \tan^{-1}(\alpha/v_F)$$

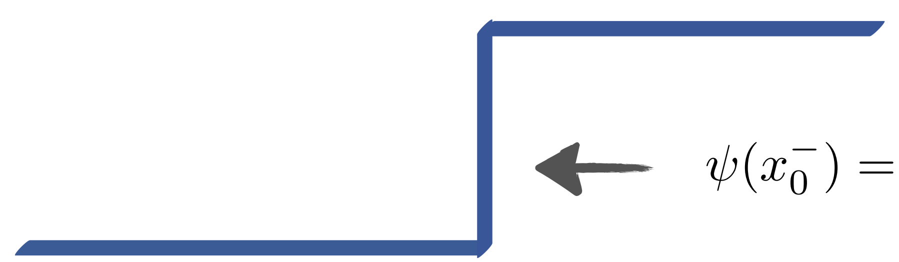


$$\psi(x_0^-) = \left(\frac{v_F}{v_\alpha}\right)^{1/2} e^{i\theta\sigma_x}\psi(x_0^+)$$

Non interacting electrons

$$H = -i\hbar v_F \sigma_z \partial_x + h\sigma_z - \frac{i\hbar}{2}\{\alpha, \partial_x\}\sigma_y$$

$$\Psi(x_1) = T_{x_1,x_0}\Psi(x_0) \qquad T_{x_1,x_0} = P_x e^{i\int_{x_0}^{x_1} dx \frac{(v_F\sigma_z + \alpha\sigma_y)}{v_F^2+\alpha^2}\left[E + h\sigma_z + \frac{i}{2}(\partial_x \alpha)\sigma_y\right]}$$



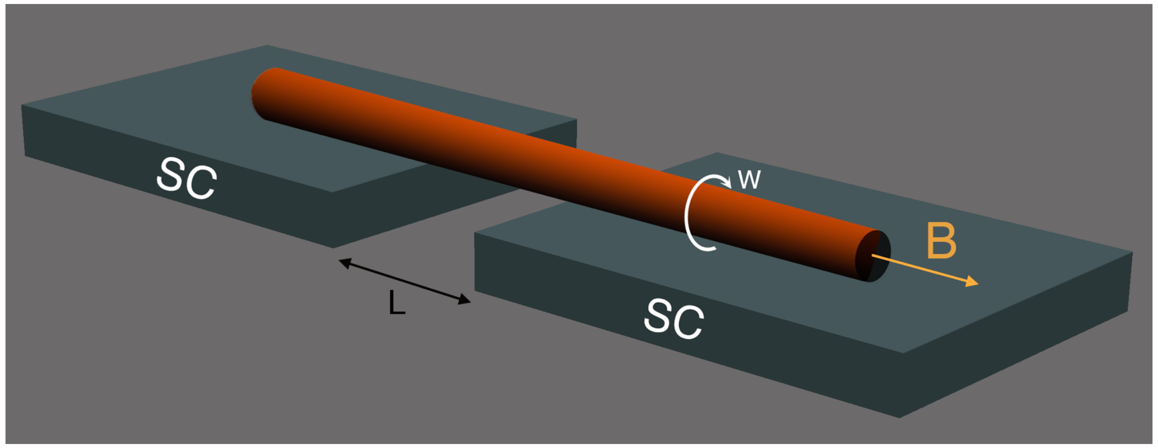
$$\psi(x_0^-) = \left(\frac{v_F}{v_\alpha}\right)^{1/2} e^{i\theta\sigma_x}\psi(x_0^+)$$

Experimental feasibility

Short junction limit	$L \ll \xi$
Mode suppression	$L/W \sim 1$
Flux through wire	$\Phi > \Phi_0/2$
Distance from Dirac point	$\mu\xi_D/\hbar v$
Disorder strength	g

$W \sim 400nm$

$B \approx 0.2T$



$g \approx 1 \quad \xi_D \sim 10nm \quad n \sim 8 \times 10^{10}\ cm^{-2}$

[Sacepe et al. Nature Commun 2011, Kong et al. Nano Letters 2010, Beidenkophf et al. 2011, ,Williams et al. PRL 2012, Tian et al. Sci. Rep. 2012, Veldhorst et al Nature Mater. 2012, Kim et al Nature Phys. 2012, Cho et al. Nature Commun 2013...]

What's the big deal about Majorana fermions?

$$\gamma^\dagger = \gamma$$

γ_2

γ_1

$$c^\dagger = \gamma_1 - i\gamma_2$$
$$c = \gamma_1 + i\gamma_2$$

Ground state degeneracy

Topological protection

Potential for the perfect qubit

Non Abelian statistics