Observability of Partonic Orbital Angular Momentum

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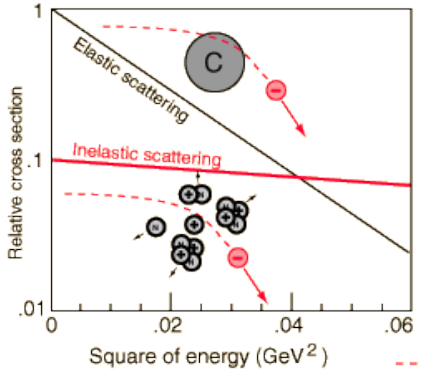
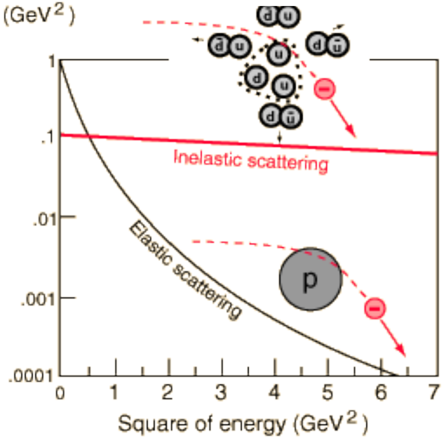
From Nucleons to Partons

- Scattering experiments probe internal structure of matter.
- By increasing energy we increase resolution.
- You start with the whole nucleus that seems point like.
- Then you observe the protons and neutrons
- Increase the energy further and seems like we are scattering off point like

  particles again - quarks!





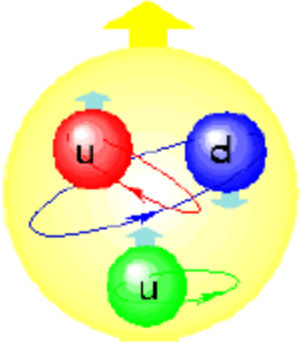
Spin Puzzle

The EMC experiment in late 1980s showed that the quark spin contribution to proton spin is only 30%

Look for other sources of spin

-Orbital Angular Momentum of Quarks and Gluons (due to internal motion)

-Gluon Spin



Angular Momentum

Lagrangian Density $\mathcal{L}$

↓

Energy Momentum Tensor T $T^{\mu}{}_{\nu} = \sum_i \frac{\partial \mathcal{L}}{\partial(\partial\phi_i/\partial x^{\mu})} \frac{\partial \phi_i}{\partial x^{\nu}} - \delta^{\mu}_{\nu} \mathcal{L}.$

↓

Angular Momentum Tensor M $M^{\mu\nu\lambda} \equiv x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\lambda}$

↓

Angular Momentum J $J^k \equiv \frac{1}{2} \epsilon^{kij} \int \mathrm{d}^3 x M^{0ij}$

Free Spin ½ particles

Dirac Lagrangian $\bar{\psi}(x)\ (i\gamma^\mu\partial_\mu - m)\ \psi(x)$

Stress Tensor $T_{\mu\nu} = \frac{1}{4} i\overline{\psi}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu)\psi + h.c.$

Angular Momentum Tensor

$$M^{\mu\nu\lambda} = \frac{1}{2} i\overline{\psi}\gamma^\mu(x^\nu\partial^\lambda - x^\lambda\partial^\nu)\psi + \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\overline{\psi}\gamma_\sigma\gamma_5\psi$$

Angular Momentum

$$J^k = \int d^3x i\psi^\dagger(x\times\nabla)^k\psi + \frac{1}{2}\int d^3x \psi^\dagger\sigma^k\psi$$

This is all done for free quarks

Ji's Picture

QCD Lagrangian $$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_f \bar{\psi}_f \left(i\not\partial + g\not A - m_f \right) \psi_f$$

The quarks are bound : use covariant derivative

takes gluon action into account

$$T_{(1/2)\mu\nu} = \frac{1}{4} i \bar{\psi} (\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi + \text{h.c.} \longrightarrow T_q^{\mu\nu} = \frac{1}{2} [\overline{\psi} \gamma^{(\mu} i \overrightarrow{D^{\nu)}} \psi + \overline{\psi} \gamma^{(\mu} i \overleftarrow{D^{\nu)}} \psi]$$

$$\langle P' | T^{\mu\nu}_{q,g} | P \rangle = \overline{U}(P') [A_{q,g}(\Delta^2) \gamma^{(\mu} \overline{P}^{\nu)} + B_{q,g}(\Delta^2) \overline{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M + C_{q,g}(\Delta^2) (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2)/M + \overline{C}_{q,g}(\Delta^2) g^{\mu\nu} M] U(P) ,$$

$$J^i_{q,g} = \frac{1}{2} \epsilon^{ijk} \int d^3x (T^{0k}_{q,g} x^j - T^{0j}_{q,g} x^k) \qquad J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)]$$

X Ji, PRD 78, 1996

$$A_q(t) + B_q(t) = \int_{-1}^{1} \mathrm{d}x \, x [H_q(x, \xi, t) + E_q(x, \xi, t)] \longleftarrow$$

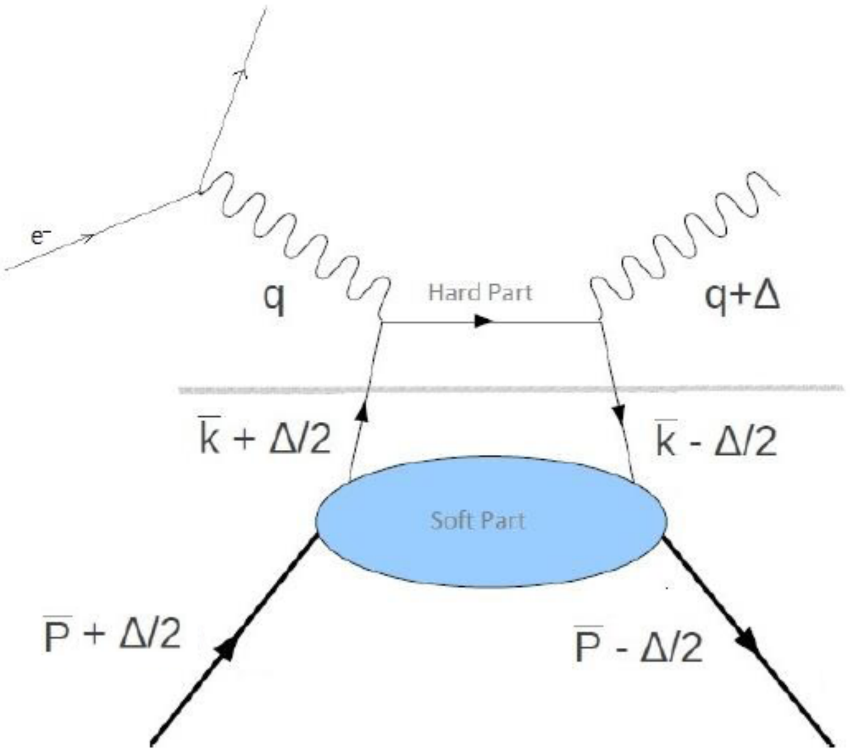
GPDs !!!

Deeply Virtual Compton Scattering

$$ep \longrightarrow ep\gamma$$

- Hard Part : Quark Photon Scattering
- Soft Part : Probe Proton's internal structure

X Ji, PRD 78, 1996



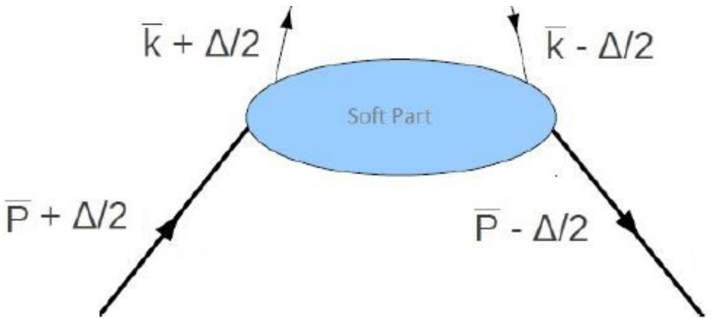
Generalised Parton Distributions

Describing the soft part in DVCS

$$F^{\Gamma}_{\Lambda',\Lambda}(k^+, p, \Delta) = \int \frac{dz^-}{2\pi} e^{ik.z} \langle \Lambda', P' | \bar{\Psi}(z) \Gamma \Psi(0) | P, \Lambda \rangle |_{z^+, z_\perp = 0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \Lambda') [\gamma^+ H(x, \zeta, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2m} E(x, \zeta, t)] u(p, \Lambda)$$

$$x = k^+/P^+ \quad \zeta = \Delta^+/P^+ \quad t = \Delta^2$$

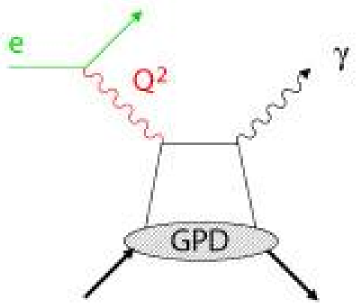


GPDs H and E connect to angular momentum of quarks by Ji's sum rule

$$\langle J_z \rangle = \int_{-1}^{1} dx x [H(x, \zeta, 0) + E(x, \zeta, 0)]$$

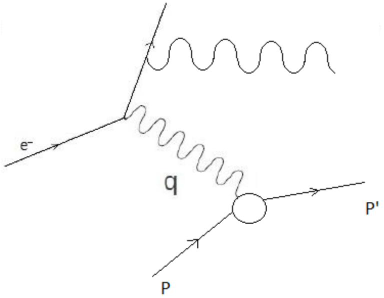
Experimentally Observing GPDs

$$ep \longrightarrow ep\gamma$$



DVCS

+



Bethe Heitler

$$\sigma \propto |T_{DVCS} + T_{BH}|^2$$

Guidal and Moutarde, Eur Phys J A 42, (2009) (Hermes Data)
Gary Goldstein, Osvaldo Hernandez, Simonetta Liuti , Phys Rev D 84 (2007)

Experimentally Observing GPDs

$$A_C = \frac{\frac{d^4\sigma^+}{d\Phi} - \frac{d^4\sigma^-}{d\Phi}}{\frac{d^4\sigma^+}{d\Phi} + \frac{d^4\sigma^-}{d\Phi}} = \frac{I}{|T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2},$$

$$A_{UT}^{\mathrm{DVCS}} = \frac{1}{S_\perp} \frac{\left(\frac{d^4\sigma^+_{\Leftarrow}}{d\Phi} - \frac{d^4\sigma^+_{\Rightarrow}}{d\Phi}\right) + \left(\frac{d^4\sigma^-_{\Leftarrow}}{d\Phi} - \frac{d^4\sigma^-_{\Rightarrow}}{d\Phi}\right)}{\frac{d^4\sigma^+}{d\Phi} + \frac{d^4\sigma^-}{d\Phi}} = \frac{|T_{TP}^{\mathrm{DVCS}}|^2}{|T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2},$$

beam polarization

target polarization

$$A_{UT}^{I} = \frac{1}{S_\perp} \frac{\left(\frac{d^4\sigma^+_{\Leftarrow}}{d\Phi} - \frac{d^4\sigma^+_{\Rightarrow}}{d\Phi}\right) - \left(\frac{d^4\sigma^-_{\Leftarrow}}{d\Phi} - \frac{d^4\sigma^-_{\Rightarrow}}{d\Phi}\right)}{\frac{d^4\sigma^+}{d\Phi} + \frac{d^4\sigma^-}{d\Phi}} = \frac{I_{TP}}{|T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2},$$

$$A_{LU} = \frac{\frac{d^4\sigma^{\uparrow}}{d\Phi} - \frac{d^4\sigma^{\downarrow}}{d\Phi}}{\frac{d^4\sigma^{\uparrow}}{d\Phi} + \frac{d^4\sigma^{\downarrow}}{d\Phi}} = \frac{I^{\uparrow} - I^{\downarrow}}{|T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2 + I}$$

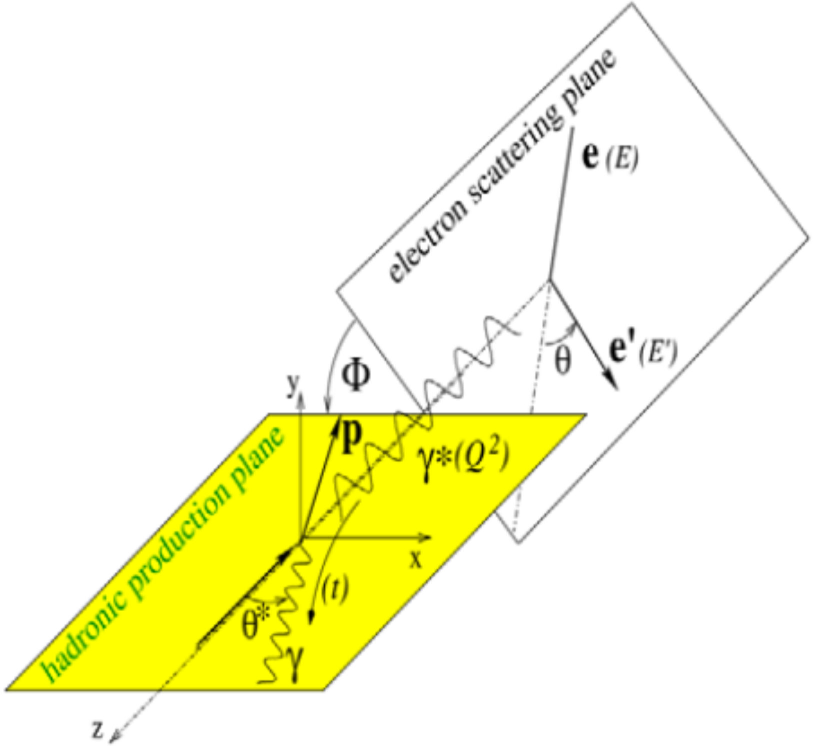
Experimentally Observing GPDs

$$A_{LU} = \frac{A(\phi)\sin\phi}{B(\phi) + C(\phi)\cos\phi}$$

$$A_C = \frac{D(\phi) + C(\phi)\cos\phi}{B(\phi) - D(\phi)}$$

$$A_{UT}^{\mathrm{DVCS}} = \frac{E(\phi)\sin(\phi - \phi_S)}{B(\phi) - D(\phi)}$$

to leading order



Experimentally Observing GPDs

$$A = K_I^1(\phi)\Im m C_I,$$
$$B = C_{\mathrm{BH}}^0(\phi) + K_{\mathrm{DVCS}}^0 C_{\mathrm{DVCS}} + K_I^0(\phi)\Re e C_I,$$
$$C = K_I^2(\phi)\Re e C_I,$$
$$D = K_I^0(\phi)\Re e C_I,$$
$$E = K_{\mathrm{DVCS}}^{UT} C_{\mathrm{DVCS}}^{UT},$$
$$F = \frac{-t}{Q^2} K_I^{UT}(\phi) C_{I,0}^{UT},$$
$$G = K_I^{UT}(\phi) C_{I,0}^{UT},$$
$$H = K_I^{UT}(\phi) C_{I,1}^{UT}.$$

$$A_{LU} = \frac{A(\phi)\sin\phi}{B(\phi) + C(\phi)\cos\phi}$$

$$A_C = \frac{D(\phi) + C(\phi)\cos\phi}{B(\phi) - D(\phi)}$$

$$A_{UT}^{\mathrm{DVCS}} = \frac{E(\phi)\sin(\phi - \phi_S)}{B(\phi) - D(\phi)}$$

Factorize coefficients into
kinematic and dynamic part

Experimentally Observing GPDs

$$C_I = F_1\mathcal{H} + \frac{x_{Bj}}{2-x_{Bj}}(F_1+F_2)\tilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}, \quad (86)$$

$$C_{\mathrm{DVCS}} = \Re e^2\mathcal{H} + \Im m^2\mathcal{H} + \Re e^2\tilde{\mathcal{H}} + \Im m^2\tilde{\mathcal{H}}, \quad (87)$$

$$C_{\mathrm{DVCS}}^{UT} = \Re e\mathcal{E}\Im m\mathcal{H} - \Re e\mathcal{H}\Im m\mathcal{E}, \quad (88)$$

$$C_{I,0}^{UT} = \frac{t}{4M^2}(2-x_{Bj})F_1\Im m\mathcal{E} - \frac{t}{M^2}\frac{1-x_{Bj}}{2-x_{Bj}}F_2\Im m\mathcal{H}, \quad (89)$$

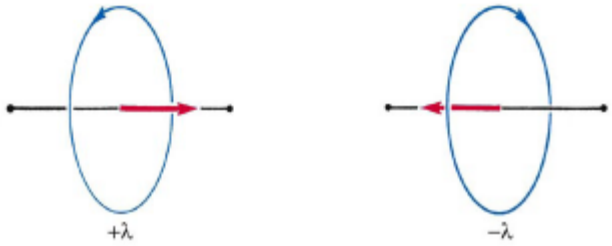
$$C_{I,1}^{UT} = (1-x_{Bj})\frac{t}{M^2}F_2\Im m\tilde{\mathcal{H}}. \quad (90)$$

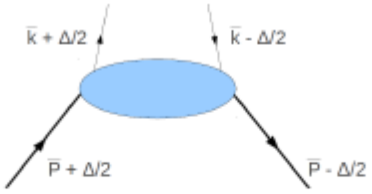
Fourier Transforms of GPDs !!

Gary Goldstein, Osvaldo Hernandez, Simonetta Liuti , Phys Rev D 84 (2007)

Generalised Transverse Momentum Distributions

Include Transverse Momentum of Quarks. Hence, expected to descibe quarks longitudianl orbital angular momentum





$$G^{\Gamma}_{\Lambda',\Lambda}(k^+, k_\perp, p, \Delta) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik.z} \langle \Lambda', P' | \bar{\Psi}(z) \Gamma \Psi(0) | P, \Lambda \rangle |_{z^+=0}$$

$$G^{\gamma^+}_{\Lambda',\Lambda} = \frac{1}{2M} \bar{u}(p, \Lambda') [F_{11} + \frac{i\sigma^{i+} k^i_T}{\bar{p}_+} F_{12} + \frac{i\sigma^{i+} \Delta^i_T}{\bar{p}_+} F_{13} + \frac{i\sigma^{ij} k^i_T \Delta^j_T}{M^2} F_{14}] u(p, \Lambda)$$

Meissner Metz Schlegel, JHEP08 (2009)

The GTMDs are scalars that depend on k perp

GTMDs

$$W^{\Gamma}_{\Lambda',\Lambda}(k,p,\Delta) = \int \frac{d^4z}{(2\pi)^4} e^{ik.z} \langle \Lambda', P' | \underbrace{\bar{\Psi}(-\frac{z}{2})\Gamma\Psi(\frac{z}{2})}_{\text{Quark Current}} | P, \Lambda \rangle$$

unintegrated soft part

integrate over k $^-$

$$G^{\Gamma}_{\Lambda',\Lambda}(k^+, k_\perp, p, \Delta) = \int \frac{dz^- d^2z_\perp}{(2\pi)^3} e^{ik.z} \langle \Lambda', P' | \bar{\Psi}(z)\Gamma\Psi(0) | P, \Lambda \rangle |_{z^+=0}$$

GTMDs

integrate over k $^\perp$

$$F^{\Gamma}_{\Lambda',\Lambda}(k^+, p, \Delta) = \int \frac{dz^-}{2\pi} e^{ik.z} \langle \Lambda', P' | \bar{\Psi}(z)\Gamma\Psi(0) | P, \Lambda \rangle |_{z^+, z_\perp=0}$$

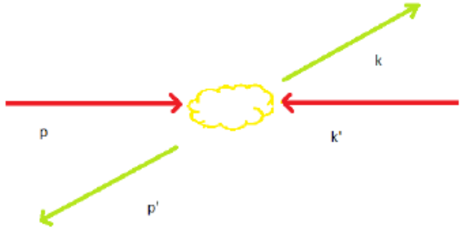
GPDs

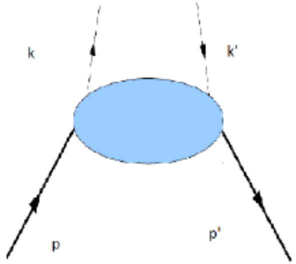
GTMDs and OAM

- It is suggested that F14 connects to OAM
- We argue that at leading twist the soft part is like two body scattering which must be planar

$$\frac{i\sigma^{ij} k_T^i \Delta_T^j}{M^2} F_{14}$$

The coefficient of F14 is a cross product that must go to zero if the interaction is planar



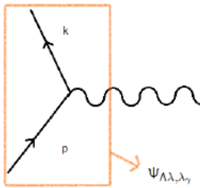


Lorce and Pasquini, PRD 84 (2011)

Model Calculations of F14

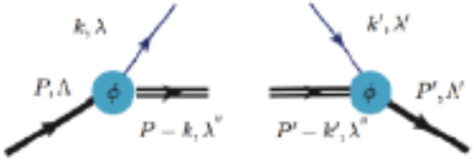
Quark Target : Treat the proton like a free quark

Brodsky, Diehl and Hwang arXiv:
hep-ph/0009254



Quark Diquark (spectator model) : The proton splits into a quark and a diquark structure

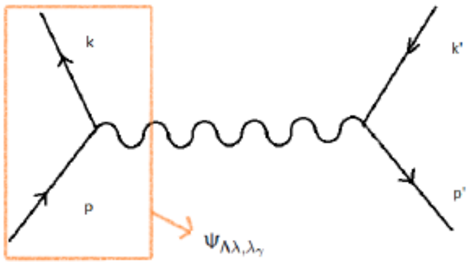
Gary R. Goldstein, J. Osvaldo Gonzalez Hernandez,
Simonetta Liuti arXiv:1012.3776



MIT-Bag : Three free quarks confined to a given volume, one participates in the scattering

Chang HM, Manohar AV, Waalewijn WJ,
Phys. Rev. D 87, 034009 (2013)

Quark Target Model



Treat the proton like a free quark
The soft part looks exactly like
two body scattering

$$\Psi^*_{\Lambda'\lambda'\lambda_\gamma}\Psi_{\Lambda\lambda\lambda_\gamma}$$

scattering amplitude of whole process

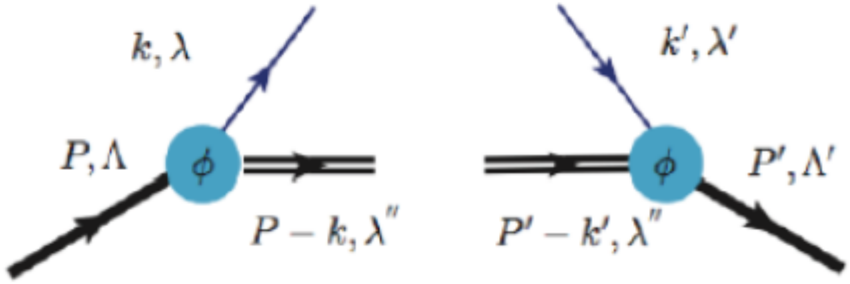
$$F_{14} = 4m^3\left(\frac{1}{x^2} - 1\right)\frac{\phi'^*\phi}{P+}$$

$$\phi'\phi = \frac{x^2(1-x)}{(m^2(x-1)^2 - k_\perp^2)(m^2(x-1)^2 - k_\perp'^2)}$$

Diquark model

The proton splits into quark and a diquark

The diquark can have spin 0 or 1

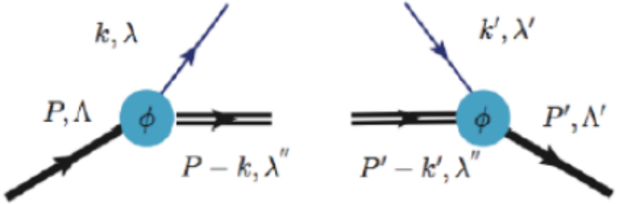


Scalar Diquark

The proton splits into a quark and a scalar diquark (spin zero) at the vertex. □ is the scalar coupling at the proton quark diquark vertex is given by

$$\Gamma = g_s \frac{k^2 - m^2}{(k^2 - M_\Lambda^2)^2}$$

$$A_{\Lambda',\lambda';\Lambda,\lambda} = \phi^*_{\lambda',\Lambda'}(k', P')\phi_{\lambda,\Lambda}(k, P)$$



$$\phi_{\lambda,\Lambda}(k, P) = \Gamma(k)\bar{u}(k, \lambda)U(P, \Lambda)$$

$$\phi^*_{\lambda',\Lambda'}(k', P') = \Gamma(k)\bar{u}(k, \lambda)U(P, \Lambda)$$

$$F_{14} = \frac{2m^3 Im(\phi^*_{+,-}(k',p')\phi_{+,-}(k,p))}{\bar{P}^+(k^1\Delta^2 - k^2\Delta^1)}$$

Courtoy, Gonzalez, Goldstein, Liuti and Rajan , Phys Lett B (2014)

Connect an Observable to OAM

$$\langle J_z \rangle = \int_{-1}^{1} dx x [H(x,\zeta,0) + E(x,\zeta,0)]$$

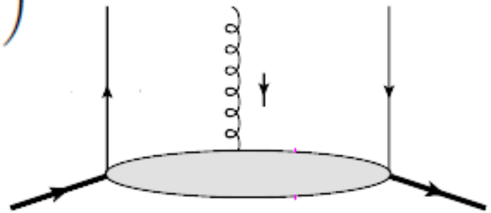
$$L_q = \frac{1}{2}\int_{-1}^{1} dx \quad x \quad (H_q(x,0,0) + E_q(x,0,0)) - \frac{1}{2}\int_{-1}^{1} dx\, \tilde{H}(x,0,0)$$

D.V. Kiptily and M.V. Polyakov, Eur. Phys. J. C. 37, 105 (2004)

$$\int dx\, x\, G_2^q(x,0,0) = \frac{1}{2}\left[-\int dx x (H^q(x,0,0) + E^q(x,0,0)) + \int dx \tilde{H}^q(x,0,0)\right]$$

Information on G2 is contained in $\quad \widetilde{\mathcal{H}}^{eff} = -2\xi\left(\frac{1}{1+\xi}\widetilde{\mathcal{H}} + \widetilde{\mathcal{H}}_3^+ - \widetilde{\mathcal{H}}_3^-\right)$

$$\widetilde{\mathcal{H}} = C^+ \otimes \tilde{H}, \quad \widetilde{\mathcal{H}}_3^+ = C^+ \otimes \tilde{E}'_{2T}, \quad \widetilde{\mathcal{H}}_3^- = C^- \otimes \tilde{E}_{2T}$$



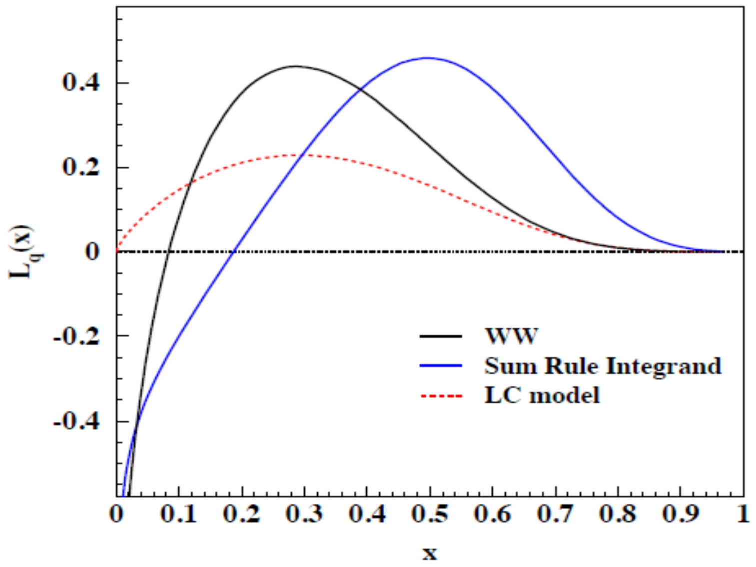
But these are all twist 3 GPDs !!

Comparing Twist Three GPDs

Polyakov et al. [13]	$2G_1$	G_2	G_3	G_4
Meissner et al. [3]	$2\widetilde{H}_{2T}$	$\widetilde{E}_{2T}$	E_{2T}	H_{2T}
Belitsky et al. [16]	E^3_+	$\widetilde{H}^3_-$	$H^3_+ + E^3_+$	$\frac{1}{\xi}\widetilde{E}^3_-$

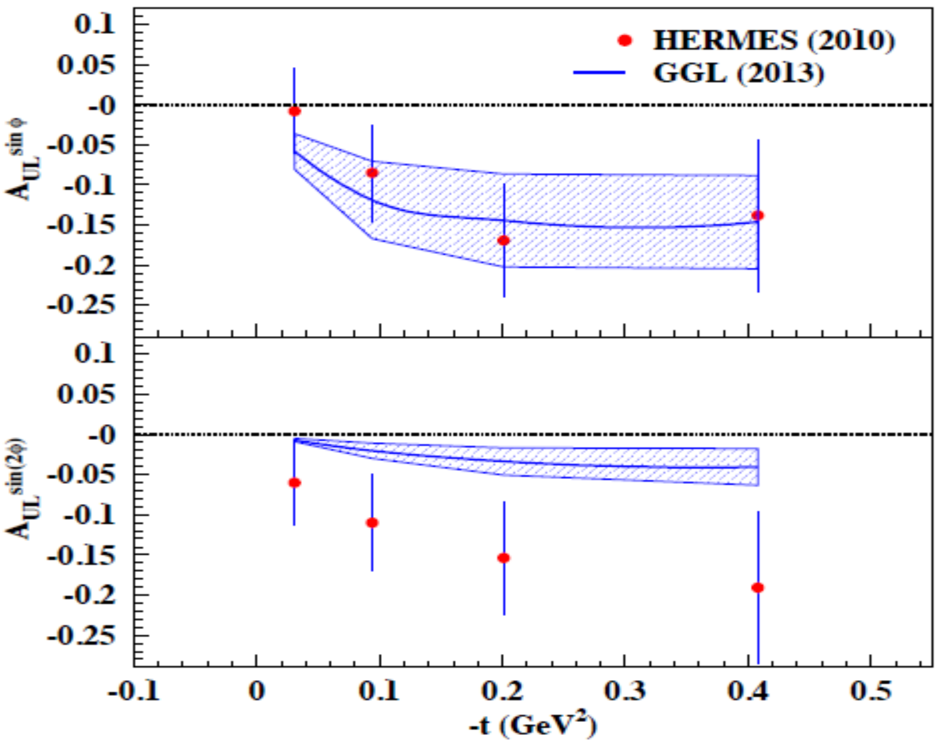
These GPDs have the same helicity structure

Comparison of various integrands



$$L_q^{WW}(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0)$$

Connection to twist 3



$$A_{UL} = \frac{a\sin\phi + b\sin 2\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi}$$

Conclusion

We have shown that Partonic Orbital Angular Momentum is a twist three object.

We have suggested an observable that connects to OAM to an experimental observable

Future work would involve further twist 3 calculations and verification of the sum rule in the non Wandzura Wilzek regime

Experimentally Observing GPDs

$$T^{\mu\nu} = -i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{\gamma^\mu i(\not{k} + \not{q})\gamma^\nu}{(k+q)^2 + i\epsilon} + \frac{\gamma^\nu i(\not{k} - \not{\Delta} - \not{q})\gamma^\mu}{(k-\Delta-q)^2 + i\epsilon}\right)\mathcal{M}(k, P, \Delta)\right]$$

$$\mathcal{M}_{ij}^{\Lambda\Lambda'}(k, P, \Delta) = \int d^4y e^{iky} \langle P', \Lambda' | \bar{\psi}_j(0) \psi_i(y) | P, \Lambda \rangle$$

$$T^{\mu\nu} = \frac{1}{2} g_T^{\mu\nu} \mathcal{F}_S^{\Lambda\Lambda'} + \frac{i}{2} \epsilon_T^{\mu\nu} \mathcal{F}_A^{\Lambda\Lambda'}$$

$$\mathcal{F}_{\Lambda,\Lambda'}^S(\zeta, t) = \int_{-1+\zeta}^{1} dX \left(\frac{1}{X - \zeta + i\epsilon} + \frac{1}{X - i\epsilon}\right) F_{\Lambda,\Lambda'}^S(X, \zeta, t),$$

$$F_{\Lambda,\Lambda'}^S(X, \zeta, t) = \frac{1}{2\bar{P}^+}\left[\bar{U}(P', \Lambda')\left(\gamma^+ H(X, \zeta, t) + \frac{i\sigma^{+\mu}(-\Delta_\mu)}{2M} E(X, \zeta, t)\right) U(P, \Lambda)\right]$$