Confronting electron and neutrino scattering
Can the axial mass controversy be resolved?

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Outline

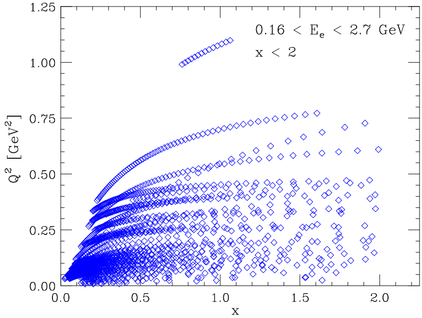
- ⋆ Preamble & motivation
- ⋆ Lessons from electron scattering
  - ▹ the $e + A \to e' + X$ cross section
  - ▹ impulse approximation and beyond
- ⋆ Neutrino-nucleus scattering at beam energy $\lesssim 1$ GeV: what do accelerator based neutrino oscillation experiments actually measure?
- ⋆ Interpretation and theoretical description of the detected signal in the charged current quasi elastic (CCQE) channel
- ⋆ Summary & Outlook

Preamble & motivation

★ Vast supply of precise $e - A$ data available

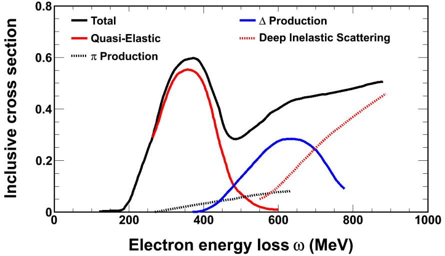
$$Q^2 = 4E_e E_{e'} \sin^2\frac{\theta_e}{2} \quad , \quad x = \frac{Q^2}{2M\omega}$$

★ Carbon target

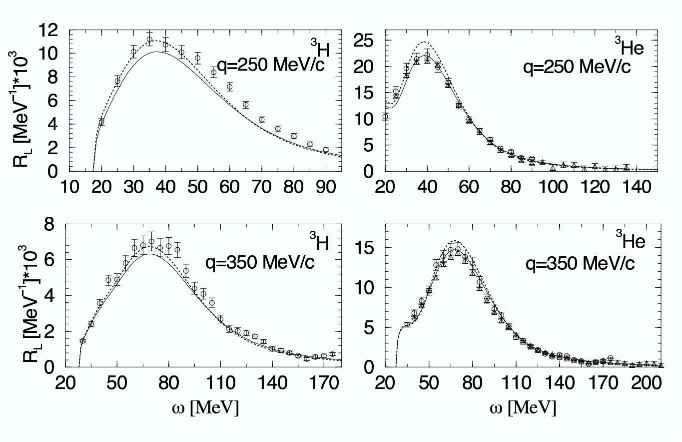


★ Different rection mechanisms contributimg to the mesured cross sections can be readily identified

$$e + A \longrightarrow e' + X$$



★ *Ab initio* calculations of the electron scattering cross section (mostly in the quasielastic channel) – carried out for *lightest* nuclei in the *low-energy* region – provide a remarkably good description of the data



- ⋆ Models based on the same dynamical input and the *impulse approximation* (IA) scheme have been remarkably successful in describing electron scattering data at larger beam energies, up to few GeV, for a variety of targets
- ⋆ The extension of these models to the case of neutrino scattering is needed to reduce the systematic uncertainty of LBL neutrino oscillation experiments
- ⋆ Oscillation probablity (two flavors, for simplicity)

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\theta \ \sin^2 \left(\frac{\Delta m^2 L}{4 E_\nu} \right)$$

Sensitivity reaches its maximum at

$$E_\nu \sim 0.5 \ \ \mathrm{GeV} \times \left[\frac{L}{250 \ \mathrm{km}} \right]$$

Theory of electron-nucleus scattering

- Double differential electron-nucleus cross section

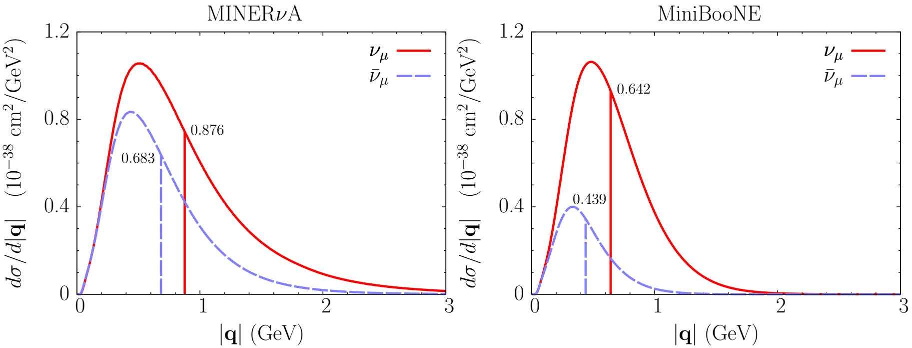
$$\frac{d\sigma_{eA}}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e} L_{\mu\nu} W_A^{\mu\nu}$$

  - $L_{\mu\nu}$ is fully specified by the *measured* electron kinematical variables
  - the determination of the target response tensor

$$W_A^{\mu\nu} = \sum_n \langle 0|J_A^\mu|n\rangle\langle n|J_A^\nu|0\rangle \delta^{(4)}(P_0 + k_e - P_n - k_{e'})$$

  requires the description of the target internal dynamics and electromagnetic current → approximations needed to go beyond the non relativistic regime

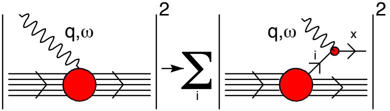
Why worry about relativity



- ★ $|\mathbf{q}|$-dependence of the CCQE cross section averaged with the Minerνa (left) and MiniBooNE (right) fluxes
- ★ Unlike the initial state, the nuclear current and the final hadronic state *can not* be described using non relativistic many-body theory

The impulse approximation

- The IA amounts to the replacement



$$J_A^\mu = \sum_i j_i^\mu \quad , \quad |X\rangle \to |\mathbf{p}\rangle \otimes |X_{(A-1)}\rangle$$

- Nuclear dynamics and electromagnetic interactions are decoupled

$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E)$$

  - The electron-nucleon cross section $d\sigma_N$ can be written in terms of stucture functions extracted from electron-proton and electron-deuteron scattering data
  - The *hole* spectral function $P_h(\mathbf{k}, E)$, momentum *and* energy distribution of the knocked out nucleon, can be obtained from *ab initio* many-body calculations

Hole state spectral function

- Definition

$$P_h(\mathbf{k}, E) = \sum_n |\langle n|a_\mathbf{k}|0\rangle|^2 \, \delta(E_0 + E - E_n)$$

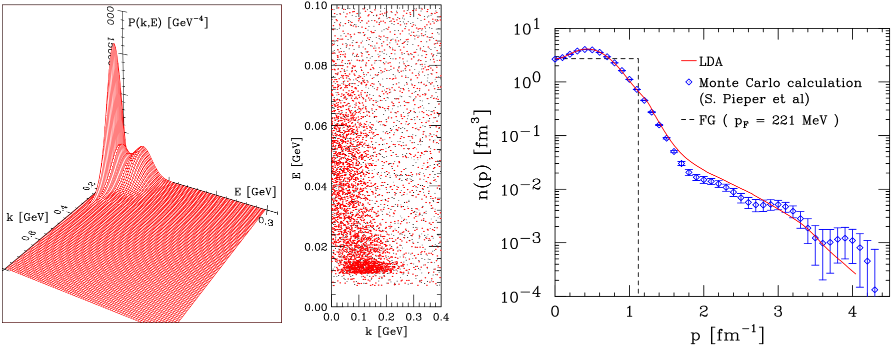
- Exact calculations have been carried out for $A = 2, \ 3$. Accurate results, obtained using correlated wave functions, are also available for nuclear matter.
- For medium-haevy nuclei, approximated spectral functions have been constructed combining nuclear matter results and experimental information from $(e, e'p)$ experiments in the local density approximation (LDA)

$$P_h(\mathbf{k}, E) = P_{\rm exp}(\mathbf{k}, E) + P_{\rm corr}(\mathbf{k}, E)$$

$$P_{\rm exp}(\mathbf{k}, E) = \sum_n Z_n |\phi_n(\mathbf{k})|^2 \, F_n(E - E_n)$$

$$P_{\rm corr}(\mathbf{k}, E) = \int d^3r \, \rho_A(r) \, P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

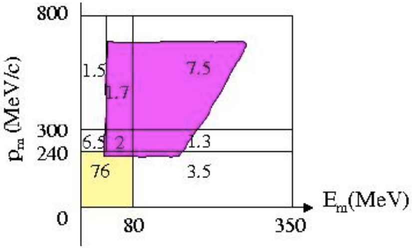
Spectral function and momentum distribution of Oxygen

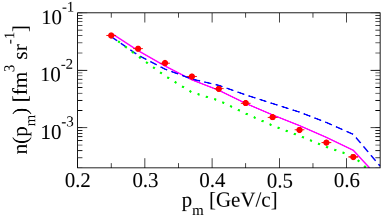


- shell model states account for $\sim$ 80% of the strenght
- the remaining $\sim$ 20%, arising from NN correlations, is located at high momentum *and* large removal energy ($\mathbf{k} \gg k_F, E \gg \epsilon$)

Measured correlation strength

- the correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target
- strong energy-momentum correlation: $E \sim E_{thr} + \frac{A-2}{A-1}\frac{\mathbf{k}^2}{2m}$

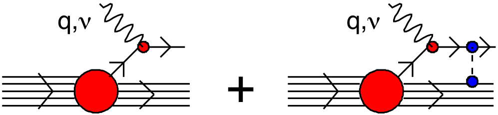




- Measured correlation strength 0.61 ± 0.06, to be compared with the theoretical predictions 0.64 (CBF) and 0.56 (G-Matrix)

Beyond IA: final state interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the importance of FSI



$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ with the particle spectral function of the non interacting system

FSI (continued)

- effects of FSI on the inclusive cross section
  - (A) shift in energy transfer → mean field of the spectators
  - (B) redistributions of the strenght → coupling of $1p-1h$ final state to $np-nh$ final states
- high energy approximation
  - (A) the struck nucleon moves along a straight trajectory with constant velocity
  - (B) the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers.

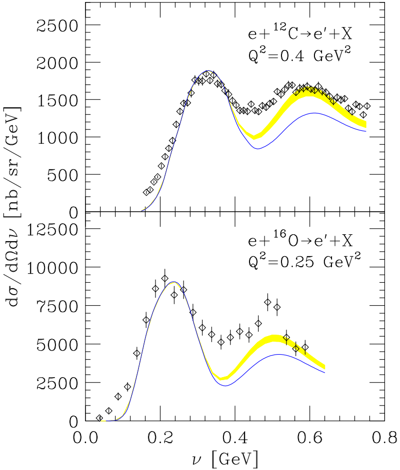
$$\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}) \to \sqrt{T}\delta(\widetilde{\omega} - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$

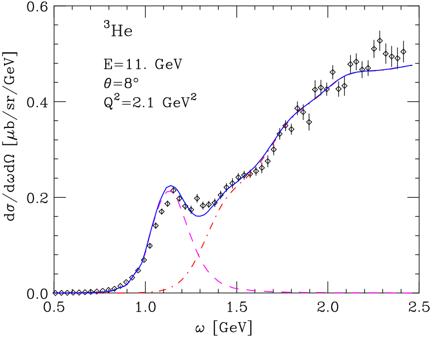
$$+(1-\sqrt{T})f(\widetilde{\omega} - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}))$$

- the nuclear transparency T and the folding function f can be computed within nuclear many-body theory using the *measured* nucleon-nucleon scattering amplitude

Theory *vs* data

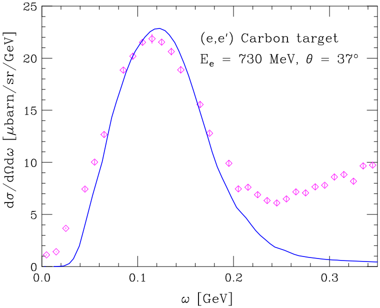
- *Recall*: theoretical calculations involve no adjustable parameters





The quasi elastic (QE) sector

- Elementary interaction vertex described in terms of the vector form factors, $F_1^{(p,n)}$ and $F_2^{(p,n)}$, precisely measured over a broad range of Q^2



- Position and width of the peak are determined by $P_h(\mathbf{k}, E)$
- The tail extending to the region of high energy loss is due to nucleon-nucleon correlations in the initial state, leading to the occurrence of two particle-two hole (2p2h) final states

Extension to CCQE neutrino nucleus scattering

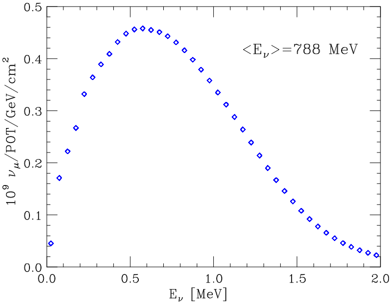
- Consider the data sample of charged current QE data collected by the MiniBooNE Collaboration using a CH_2 target
- The measured double differential cross section is averaged over the energy of the incoming neutrino, distributed according to the flux Φ

$$\frac{d\sigma_A}{dT_\mu d\cos\theta_\mu} = \frac{1}{N_\Phi}\int dE_\nu \Phi(E_\nu)\frac{d\sigma_A}{dE_\nu dT_\mu d\cos\theta_\mu}$$

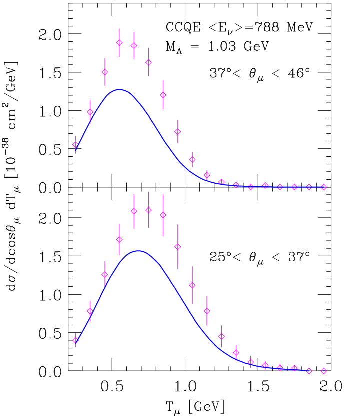
- In addition to F_1 and F_2 , the QE electron-nucleon cross section is determined by the axial form factor F_A, assumed to be of dipole form and parametrized in terms of the axial mass M_A
- According to the *paradigm* succesfully employed to describe electron scattering data, in order to minimize the bias associated with nuclear effetcs, M_A must be determined from measurement carried out using a deuterium target. The resulting value is $M_A = 1.03$ GeV

Analysis of CCQE data

- MiniBooNE flux



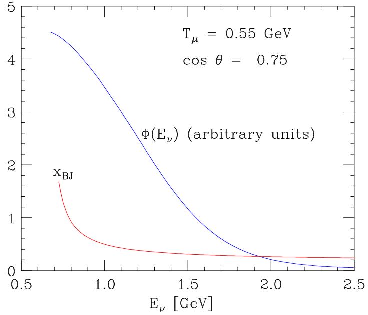
- MiniBooNe CCQE data



- Theoretical calculations carried out setting $M_A = 1.03$ GeV . A fit to the data within the Relativistic Fermi Gas Model yields $M_A = 1.25$ GeV.

Contribution of different reaction mechanisms

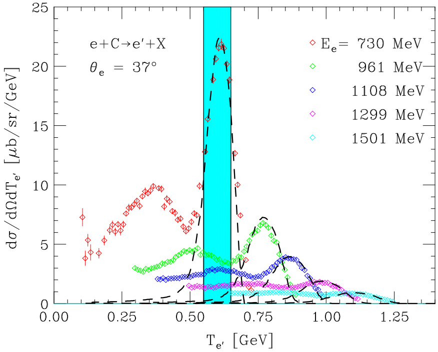
- In neutrino interactions the lepton kinematics is *not* determined. The flux-averaged cross sections at fixed T_μ and $\cos\theta_\mu$ picks up contributions at different beam energies, corresponding to a variety of kinematical regimes in which different reaction mechanisms dominate



- $x = 1 \to E_\nu = 0.788$ GeV , $x = 0.5 \to E_\nu = 0.975$ GeV
- $\Phi(0.975)/\Phi(0.788) = 0.83$

“Flux averaged” electron-nucleus x-section

- The electron scattering x-section off Carbon at $\theta_e = 37°$ has been measured for a number of beam energies



- MIT-Bates data compared to theoretical calculations including QE scattering only

Where does the “excess” strength come from?

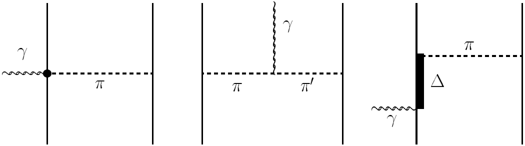
- It has been suggested that 2p2h (CCQE like) final states provide a large contribution to the measured neutrino cross section
- Two particle-two hole final states may be produced through different mechanisms
  - Initial state correlations : lead to the tail extending to large energy loss, clearly visible in the calculated QE cross section. The corresponding strength is consistent with the measurements of the coincidence $(e, e'p)$ x-section carried out by the JLAB E97-006 Collaboration.
  - Final state interactions : lead to a redistribution of the inclusive strength, mainly affecting the region of $x > 1$, i.e. low energy loss, where the cross section is small
  - Coupling to the two-body current : leads to the appearance of strength at $x < 1$, mainly in the *dip* region between the QE and Δ-excitation peaks
- the description of the measured neutrino cross sections requires that all the above mechanism be taken into account in a consistent fashion

The nuclear current operator

- ★ The nuclear current includes one- and two-nucleon contributions

$$J_A^\mu = \sum_{i=1}^{A} j_i^\mu + \sum_{j>i=1}^{A} j_{ij}^\mu$$

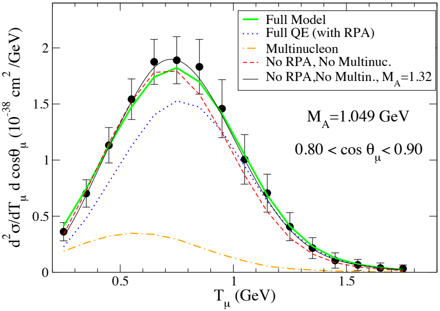
  - ▹ j_i^μ defined in terms of nucleon structure functions
  - ▹ j_{ij}^μ takes into acount interactions in which the momentum transfer is shared between two nucleons



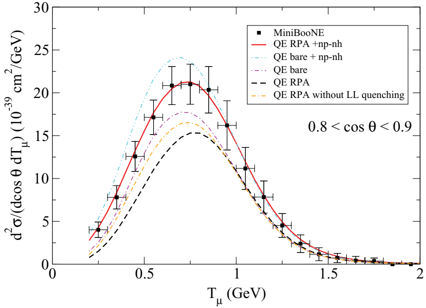
- ★ Recall: unlike the ground state, the nuclear current operator depends on momentum trasfer. Hence, the relativistic description may become inadeqaute

MEC within the independent particle model (IPM)

- J. Nieves *et al* , Phys. Lett. B **707**, 72 (2012)



- M. Martini *et al*, Phys. Rev.C **80**, 065501 (2009);



- After the inclusion of fully relativistic MEC, calculations based on the IPM provide a quantitative description of the MiniBooNE data
- However, within these approaches NN correlations are neglected altogether, or included using oversimplified *ad hoc* procedures

MEC within realistic nuclear models

- the interplay of MEC and correlation effects can be studied within nonrelativistic models, that can be solved exactly using stochastic methods for nuclei as heavy as as ^{12}C
- inclusion of the two-body current leads to an enhancement of the integrated electromagnetic response of light nuclei in the transverse channel

$$S_T(\mathbf{q}) = \int d\omega S_T(\mathbf{q},\omega) \ ,$$

where

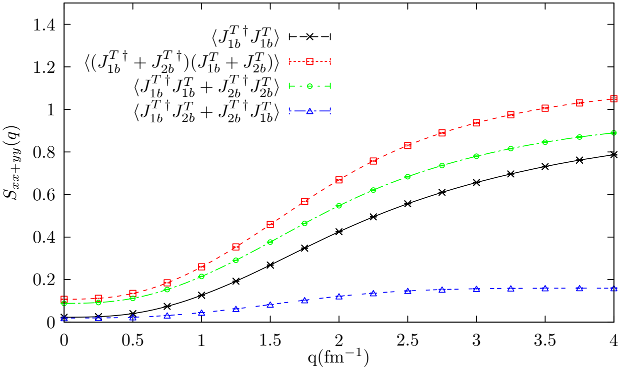
$$S_T(\mathbf{q},\omega) = S^{xx}(\mathbf{q},\omega) + S^{yy}(\mathbf{q},\omega) \ ,$$

and

$$S^{\alpha\beta} = \sum_N \langle 0|J_A^\alpha|N\rangle\langle N|J_A^\beta|0\rangle\delta(E_0+\omega-E_N) \ .$$

Interference between MEC and correlation amplitudes

- Sum rule of the electromagnetic response of ${}^{12}C$ in the transverse channel



- Interference terms are large. MEC and correlations must be treated consistently

Generalization of IA factorization

- ⋆ Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to two-nucleon emission amplitudes
  - ▹ Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|X\rangle \to |\mathbf{p}, \mathbf{p}'\rangle \otimes |X_{(A-2)}\rangle$$

$$\langle X|j^{\mu}_{ij}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k}, \mathbf{k}') \langle \mathbf{p}\mathbf{p}'|j^{\mu}_{ij}|\mathbf{k}\mathbf{k}'\rangle$$

The amplitude

$$M_n(\mathbf{k}, \mathbf{k}') = \langle n_{(A-2)}, \mathbf{k}, \mathbf{k}'|0\rangle$$

is independent of q and can be obtained from non relativistic many-body theory

Two-nucleon spectral function

- ⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter

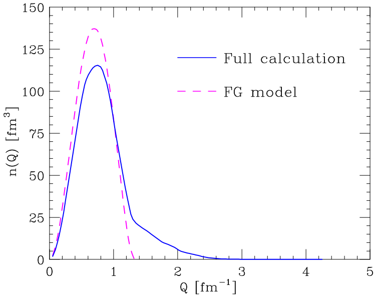
$$P(\mathbf{k}_1, \mathbf{k}_2, E) = \sum_n |M_n(k_1, k_2)|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}_1, \mathbf{k}_2) = \int dE \, P(\mathbf{k}_1, \mathbf{k}_2, E)$$

- ⋆ Relative momentum distribution

$$n(\mathbf{Q}) = 4\pi|\mathbf{Q}|^2 \int d^3q \, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 \quad , \quad \mathbf{Q} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$



Summary & Outlook

- ⋆ Ab initio calculations based on nuclear many-body theory and the available experimantal information on electron-nucleon interactions provide a remarkably accurate description of the nuclear cross sections in the impulse approximation regime
- ⋆ The generalization to neutrino-nucleus scattering, needed to reduce the systematic uncertainty of LBL neutrino oscillation experiments, involves additional difficulties, arising from the flux average, and requires a consistent description of all relevant reaction mechanisms
- ⋆ Calculations carried out using realistic models of nuclear effects suggest that the excess cross section observed in the quasi elastic sector can be explained without advocating medium modifications of the axial form factor.
- ⋆ The implementation of realistic nuclear models in event generation codes employed for data analysis of neutrino oscillation searches is under way

Background slides

ab initio many-body approach

⋆ The nucleus is seen as a collection of pointlike protons and neutrons interacting through the hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} U_{ijk}$$

- the potentials are determined by a fit to the properties of the *exactly solvable* two- and three-nucleon systems
- once the eigenstates of H are known from the solution of the Schrödinger equation

$$H\,|n\rangle = E_n\,|n\rangle$$

calculations of nuclear observables do not involve any additional parameters

Correlated basis function formalism

- ⋆ The eigenstates of the nuclear hamiltonian are approximated by the set of correlated states, obtained from independent particle model states [e.g. Fermi Gas (FG) states for nuclear matter]

$$|n\rangle = \frac{F|n_{MF}\rangle}{\langle n_{MF}|F^\dagger F|n_{MF}\rangle^{1/2}} = \frac{1}{\sqrt{\mathcal{N}_n}} F\,|n_{MF}\rangle \quad , \quad F = \mathcal{S} \prod_{j>i} f_{ij}$$

- ⋆ the structure of the two-nucleon correlation operator reflects the complexity of nuclear dynamics

$$f_{ij} = \sum_{S,T=0,1} [f_{TS}(r_{ij}) + \delta_{S1} f_{tT}(r_{ij}) S_{ij}] P_{ST}$$

$$P_{ST} \ \text{spin} - \text{isospin projector operator} \ , \ S_{ij} = \sigma_i^\alpha \sigma_j^\beta \left(\frac{r_{ij}^\alpha r_{ij}^\beta}{r_{ij}^2} - \delta^{\alpha\beta} \right)$$

- ⋆ shapes of $f_{TS}(r_{ij})$ and $f_{tT}(r_{ij})$ determined form minimization of the ground-state energy

Nucleon-nucleon potential and correlation functions