Analysis of α_s from the realization of quark-hadron duality

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Seminar at UVa
March 21st, 2013

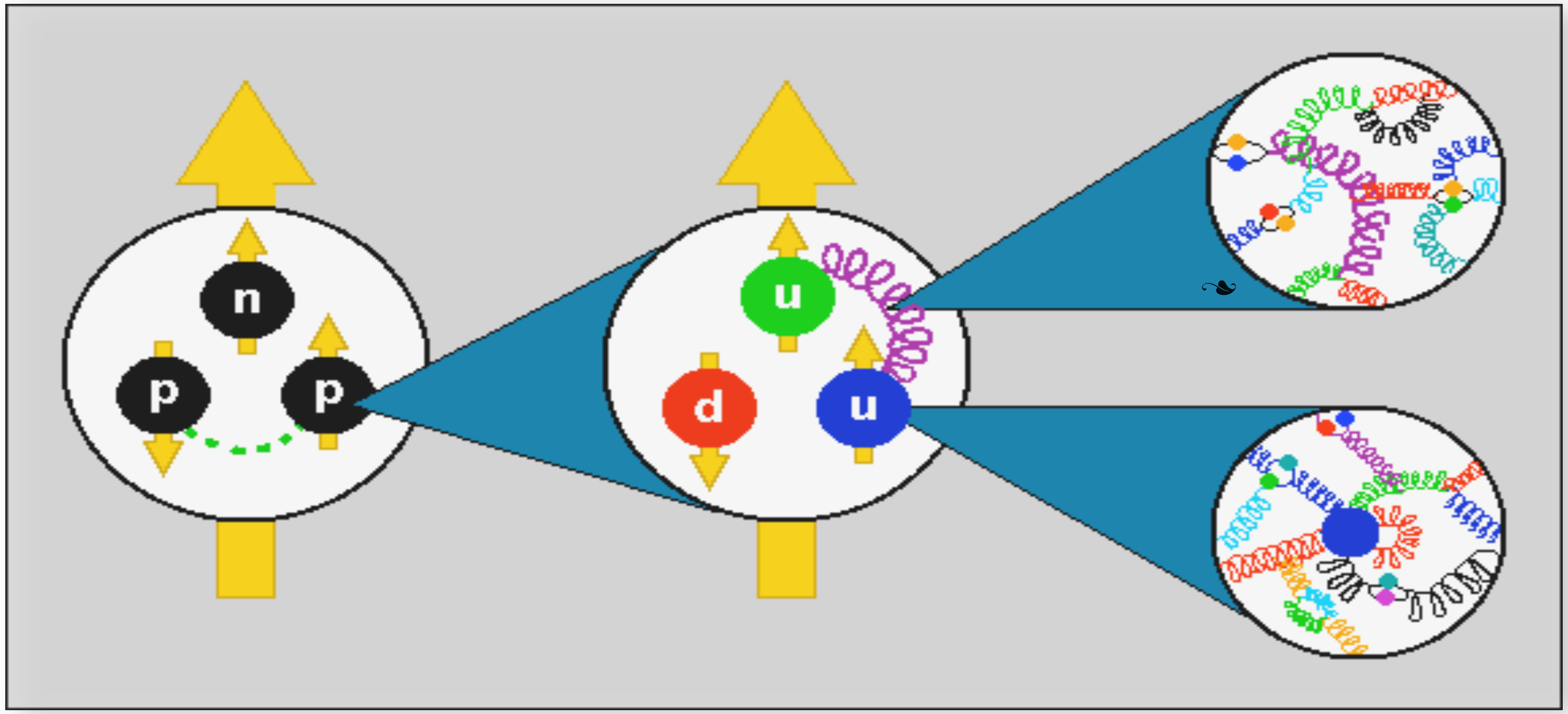
Outline

- **Hadron Phenomenology**
  - PDFs at large-x
- **Parton-Hadron Duality**
  - How to explain it?
  - Rôle of perturbative QCD
  - New data analysis: JLab
  - Intersection of pQCD and nonperturbative QCD
- **Strong coupling constant at low energy**

Hadron Phenomenology

Hadron Structure

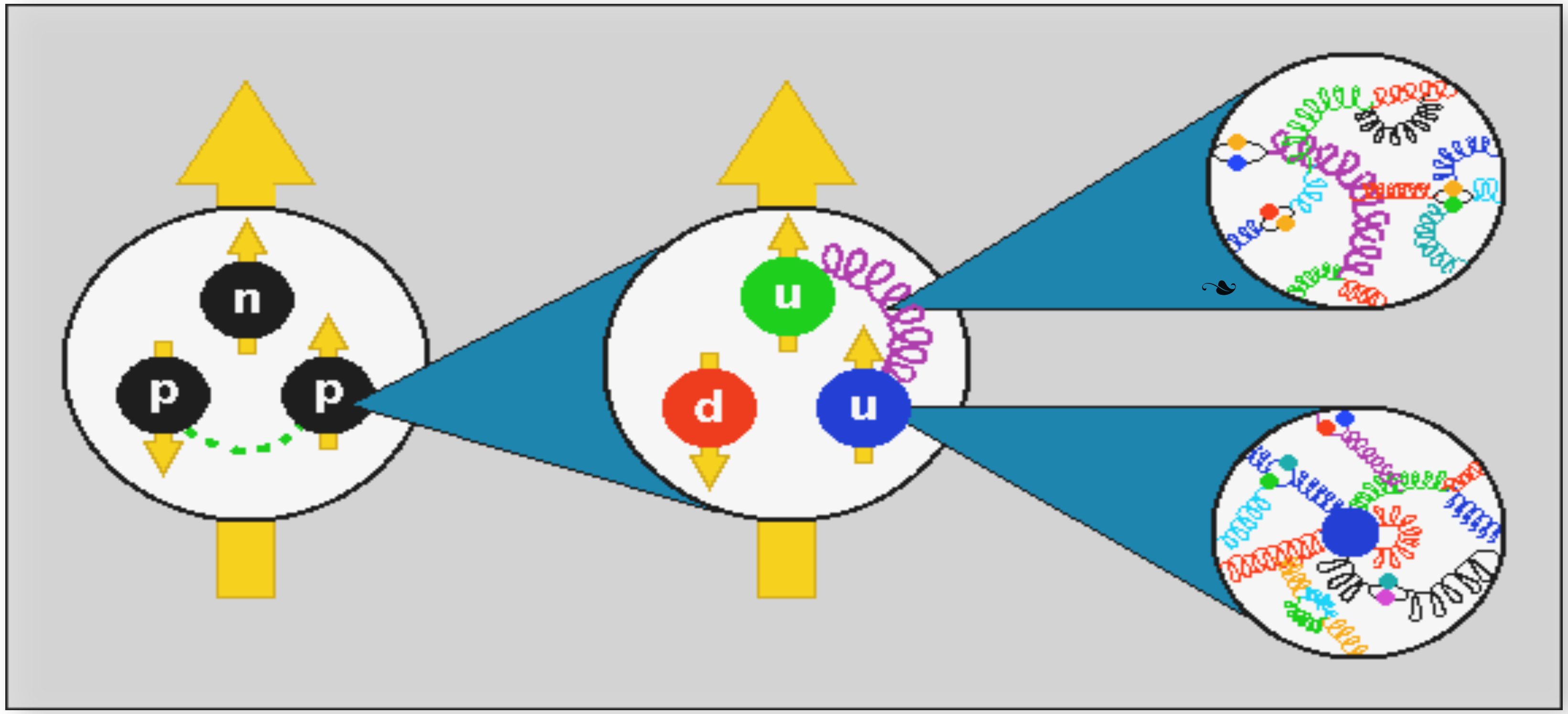
Hadron ⇔ Constituent quarks ⇔ Current quarks



Energy scale

Hadron Structure

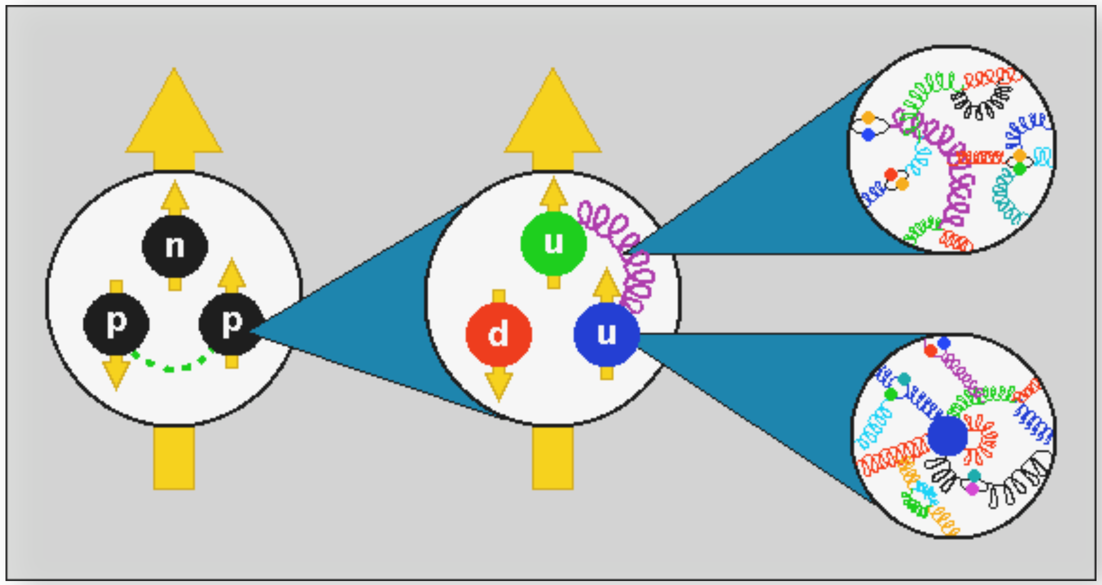
Hadron ⇔ Constituent quarks ⇔ Current quarks



Nonperturbative vs. Perturbative QCD

Energy scale

Hadron Structure



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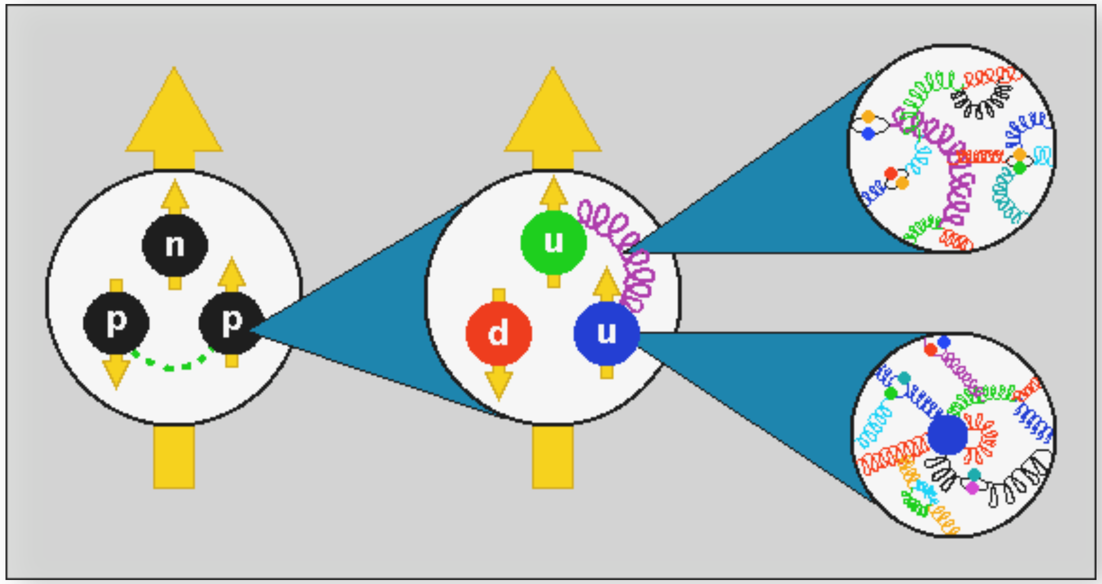

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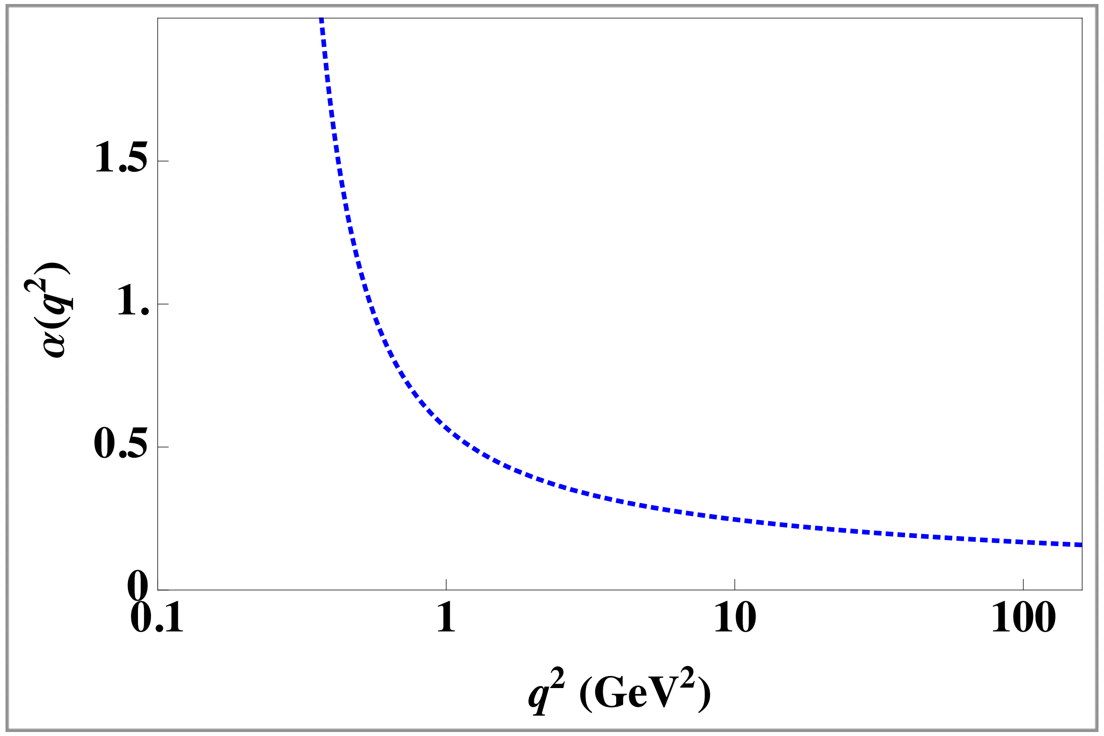


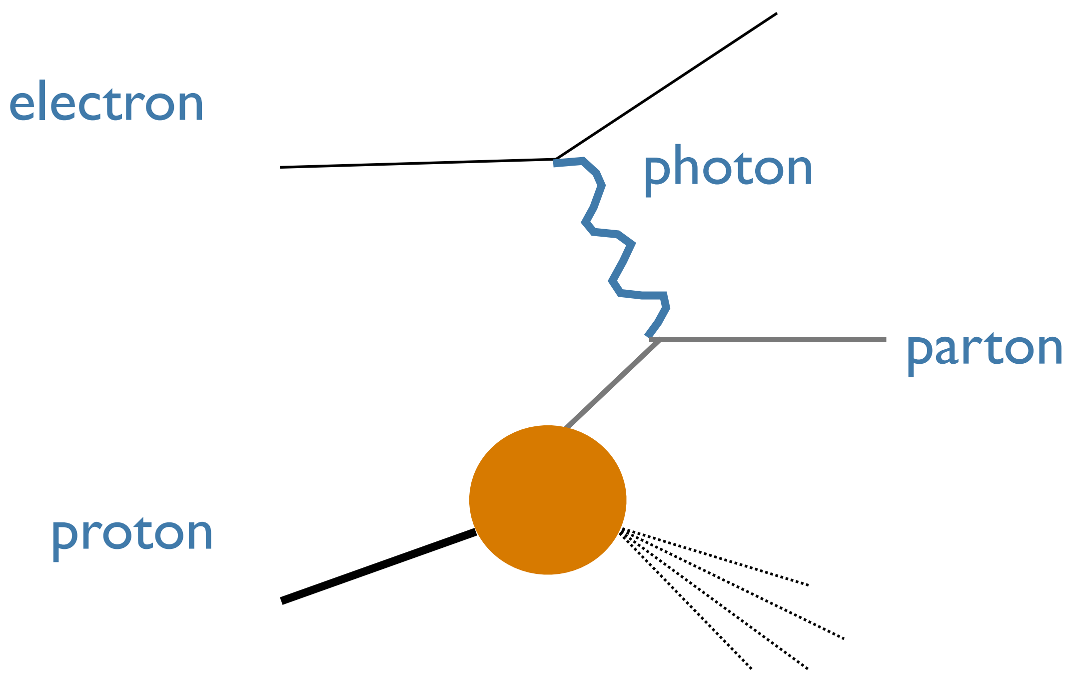
Nonperturbative vs. Perturbative QCD

Energy scale

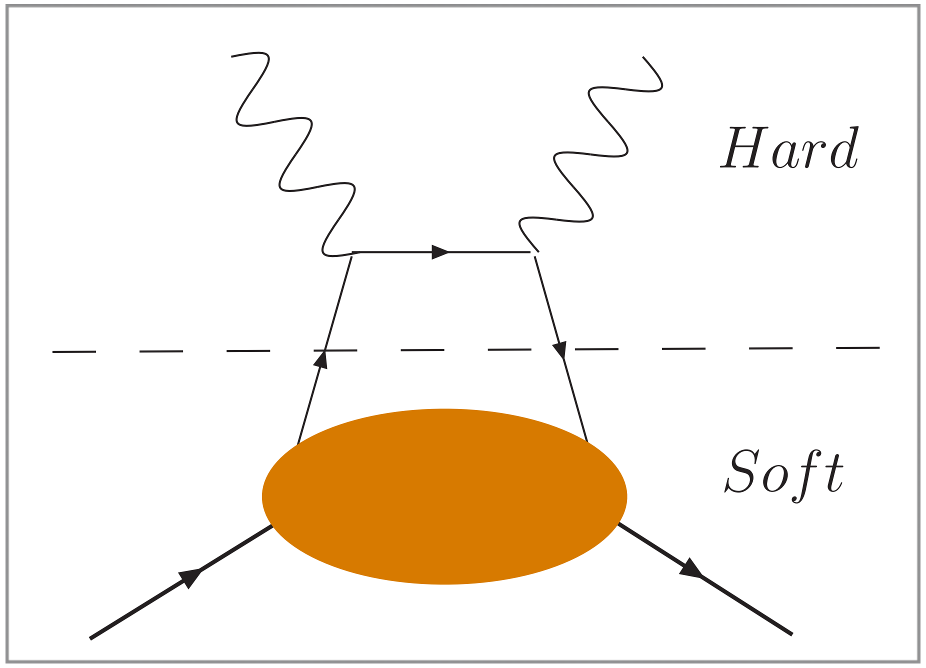
Hard Probes and Factorization

Small size configuration ⇒ Hard Probes ⇒ Hard processes

Deep Inelastic Scattering



Hadronic tensor ⇒



Parton Model
High energy photon Q^2
Fast-moving proton
Bjorken scaling

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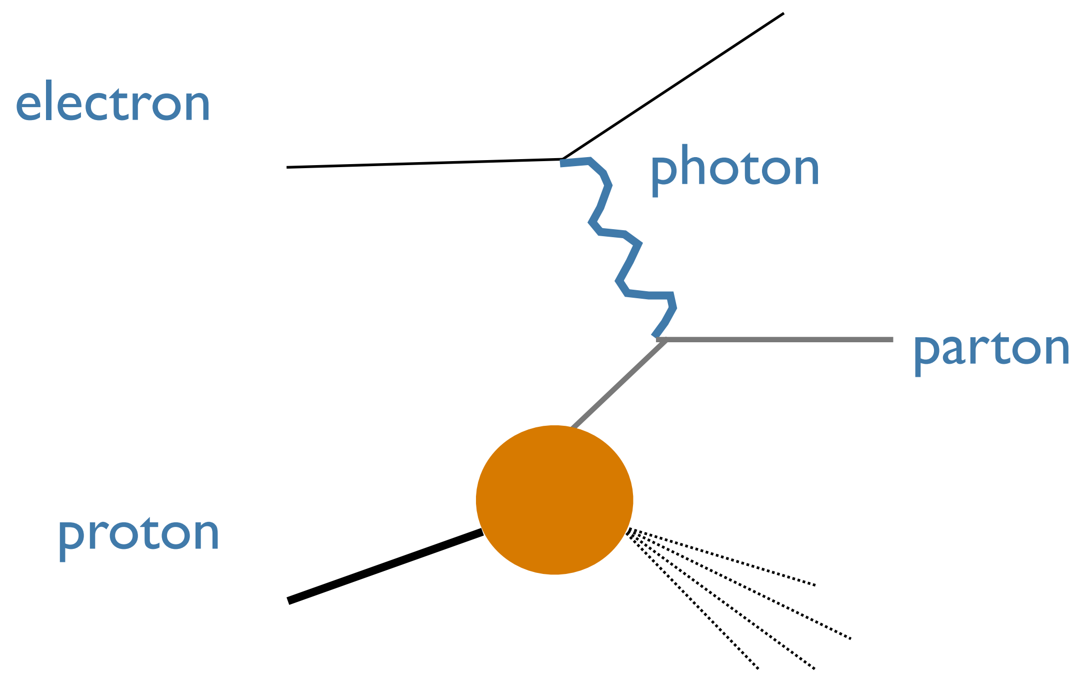
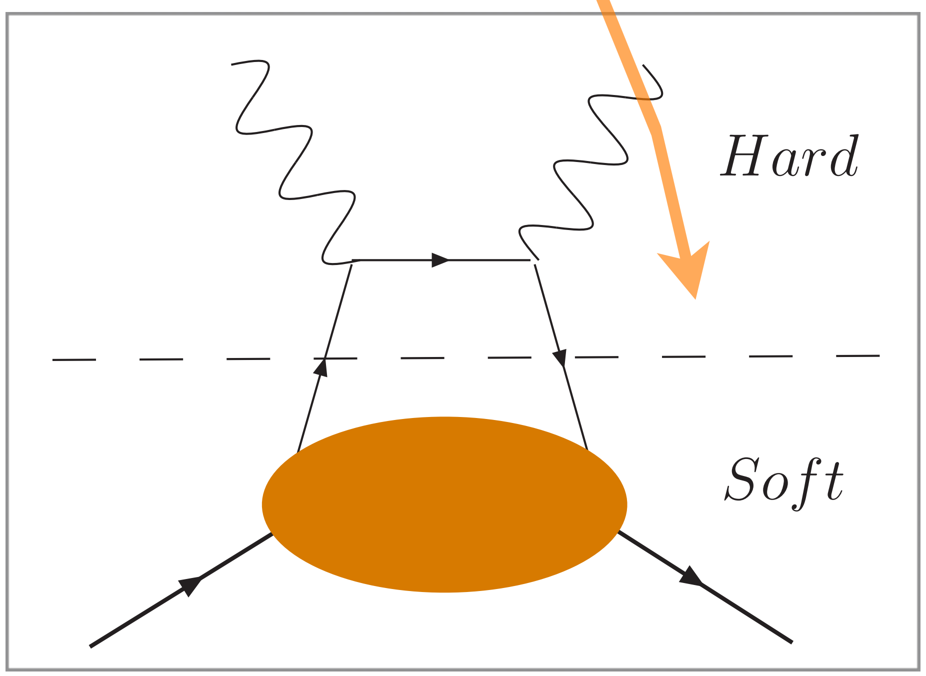


Factorization
& factorization scale

Hadronic tensor ⇒



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Structure Functions and DIS

Parton Model
Bjorken scaling

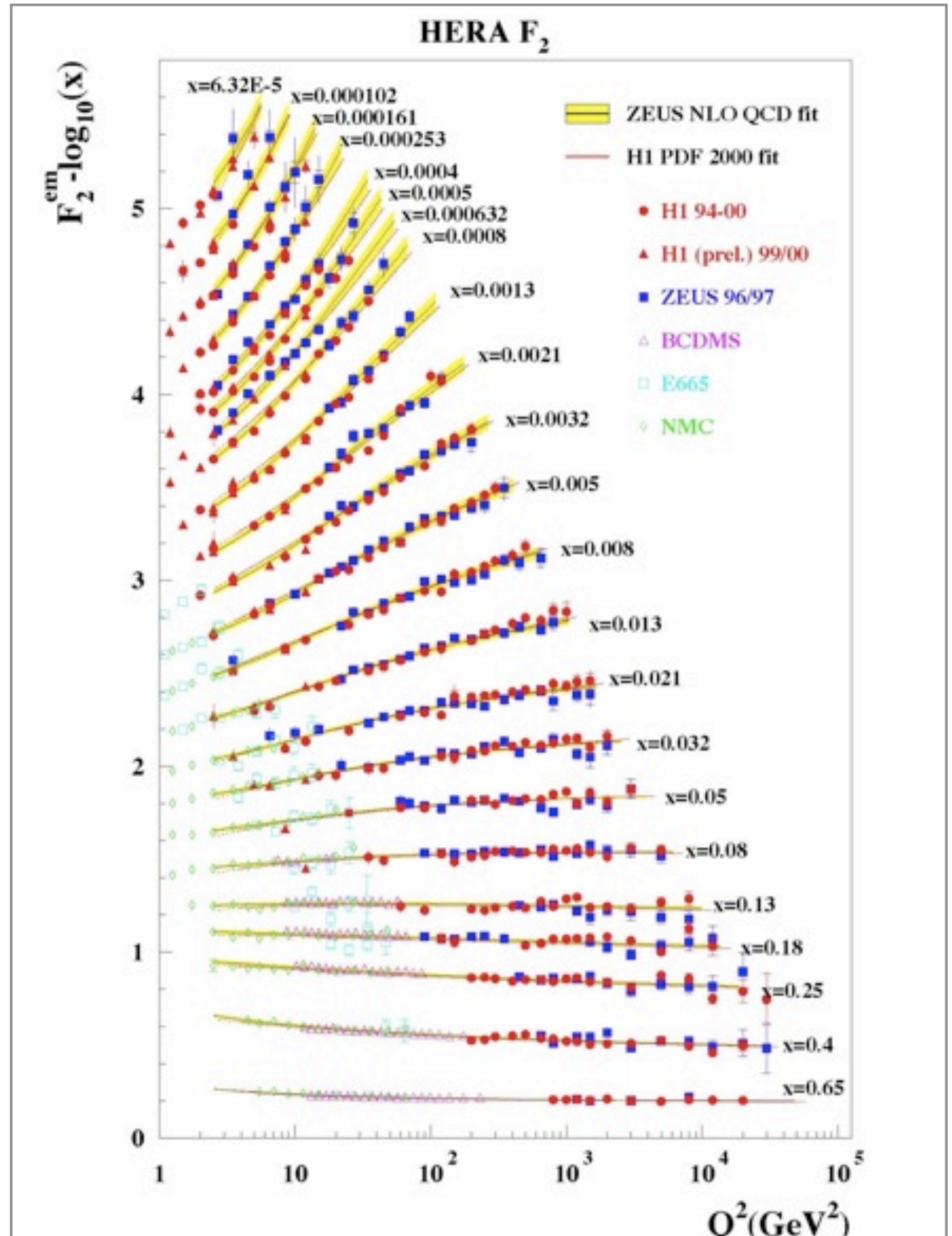
$$F_2(x,Q^2) = \sum_{q\bar{q}} \int_0^1 d\xi\, f_1(\xi,Q^2)\, x e_q^2\, \delta(x-\xi)$$

$$F_2(x) \equiv F_2(x,Q^2)$$

Scaling violations
lead to
Q^2-dependence of the Structure Functions

→ DGLAP equations
[Dokshitzer–Gribov–Lipatov Altarelli-Parisi]

→ Jargon: “Q^2 or QCD evolution”



F_2 in perturbative QCD

$$F_2(x, Q^2) = x \sum_{q,\bar{q}} e_q^2 \left[q_0(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left\{ P\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{\kappa^2} + C\left(\frac{x}{\xi}\right) \right\} + \ldots \right]$$

$$q(x, \mu^2) = q_0(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left\{ P\left(\frac{x}{\xi}\right) \ln \frac{\mu^2}{\kappa^2} + C\left(\frac{x}{\xi}\right) \right\}$$

$q_0 \rightarrow$ input PDFs

$P \rightarrow$ splitting functions

$C \rightarrow$ coefficient functions

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DIS scheme $\rightarrow$

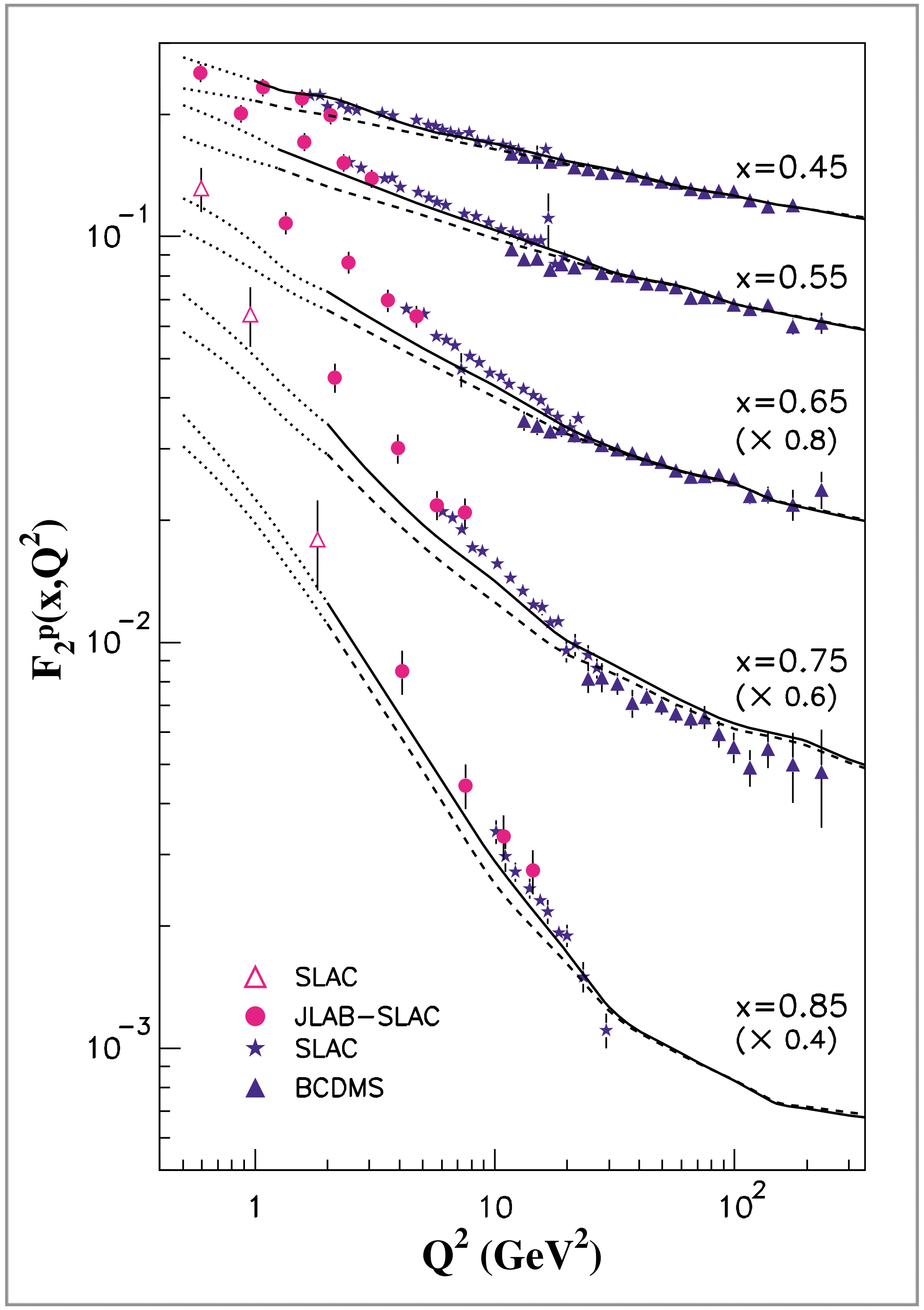
$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi}\, q(\xi,\mu^2) \times \left\{ \delta(1-\frac{x}{\xi}) + \frac{\alpha_S}{2\pi} P\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu^2} + \ldots \right\}$$

$\overline{\text{MS}}$ scheme $\rightarrow$

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi}\, q(\xi,Q^2) \left\{ \delta(1-\frac{x}{\xi}) + \frac{\alpha_S}{2\pi} C_{\overline{\text{MS}}}\left(\frac{x}{\xi}\right) + \ldots \right\}$$

Large-x region

- When x→1, →elastic scattering
- Exclusive scattering
- Intertwine with resonance region
- How to obtain clean PDFs?



Liuti et al., PRL89

Large-x region

- When x→1, →elastic scattering
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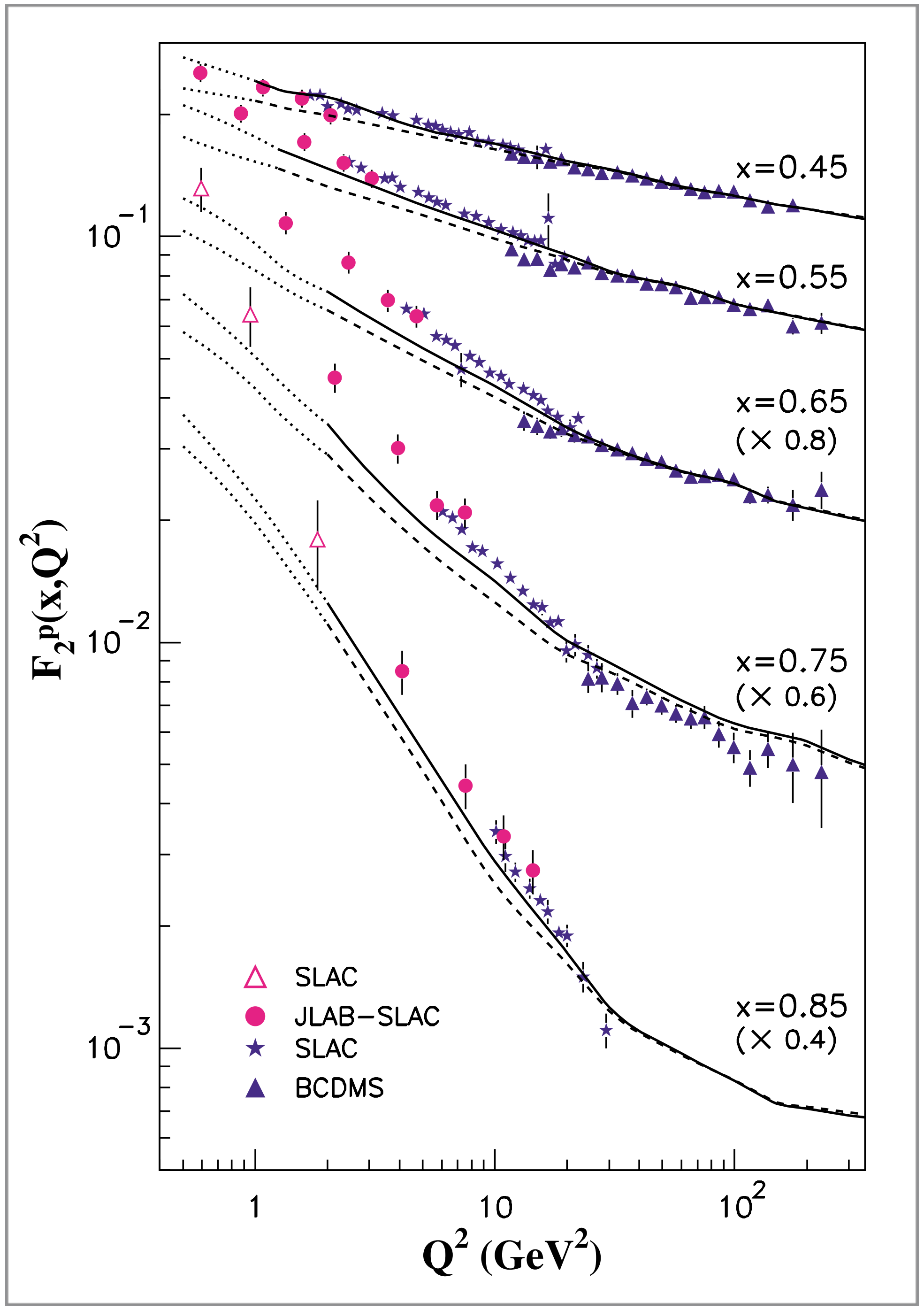
- How to obtain clean PDFs?

- Order in pQCD?
- Higher order in PDFs?
- Corrections due to target mass
- Tuning of pQCD?

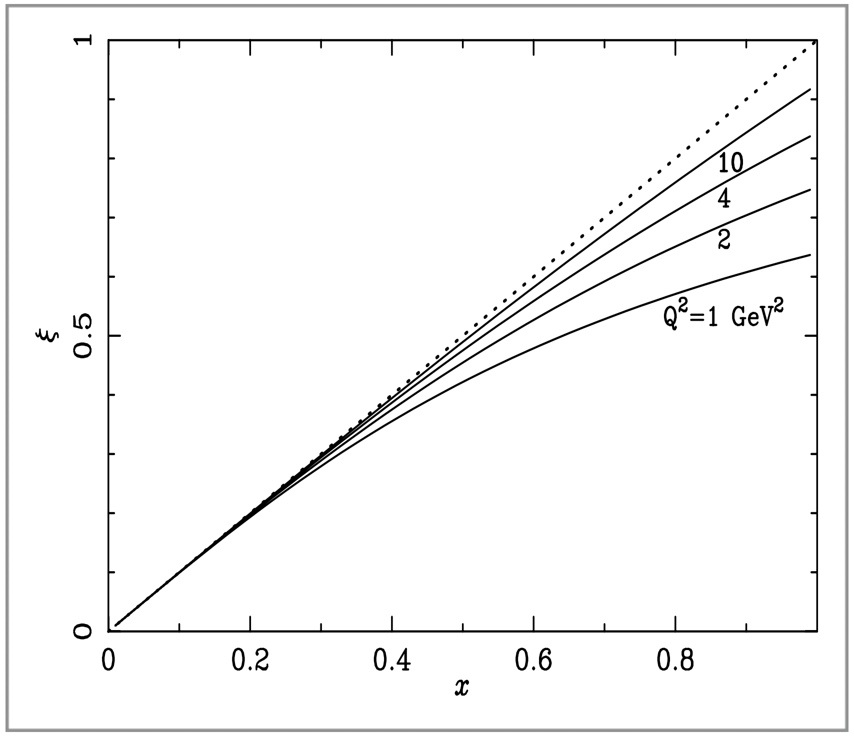


Liuti et al., PRL89

Target Mass Corrections

- Effects associated with the nonzero mass of the target
- infinite vs. finite target mass ⇒ Bjorken vs. Nachtmann variable

$$x = \frac{Q^2}{2P.q} \Leftrightarrow \xi = \frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}}$$



$$\boxed{F_2^{NS(TMC)}(x,Q^2) \;=\; \frac{x^2}{\xi^2\gamma^3}F_2^{\infty}(\xi,Q^2) + 6\frac{x^3M^2}{Q^2\gamma^4}\int_{\xi}^{1}\frac{d\xi'}{\xi'^2}F_2^{\infty}(\xi',Q^2)}$$

Georgi & Politzer (1976)

$$\boxed{F(x,Q^2,M^2) \propto \int_{\xi}^{\xi/x}\frac{dx}{x}H(\xi/x,Q^2)q(x,Q^2)}$$

, ..., Accardi & Qiu (2008)

J. Phys. G: Nucl. Part. Phys. 35

Parton-Hadron Duality

Bloom-Gilman duality

- **Inclusive electroproduction**

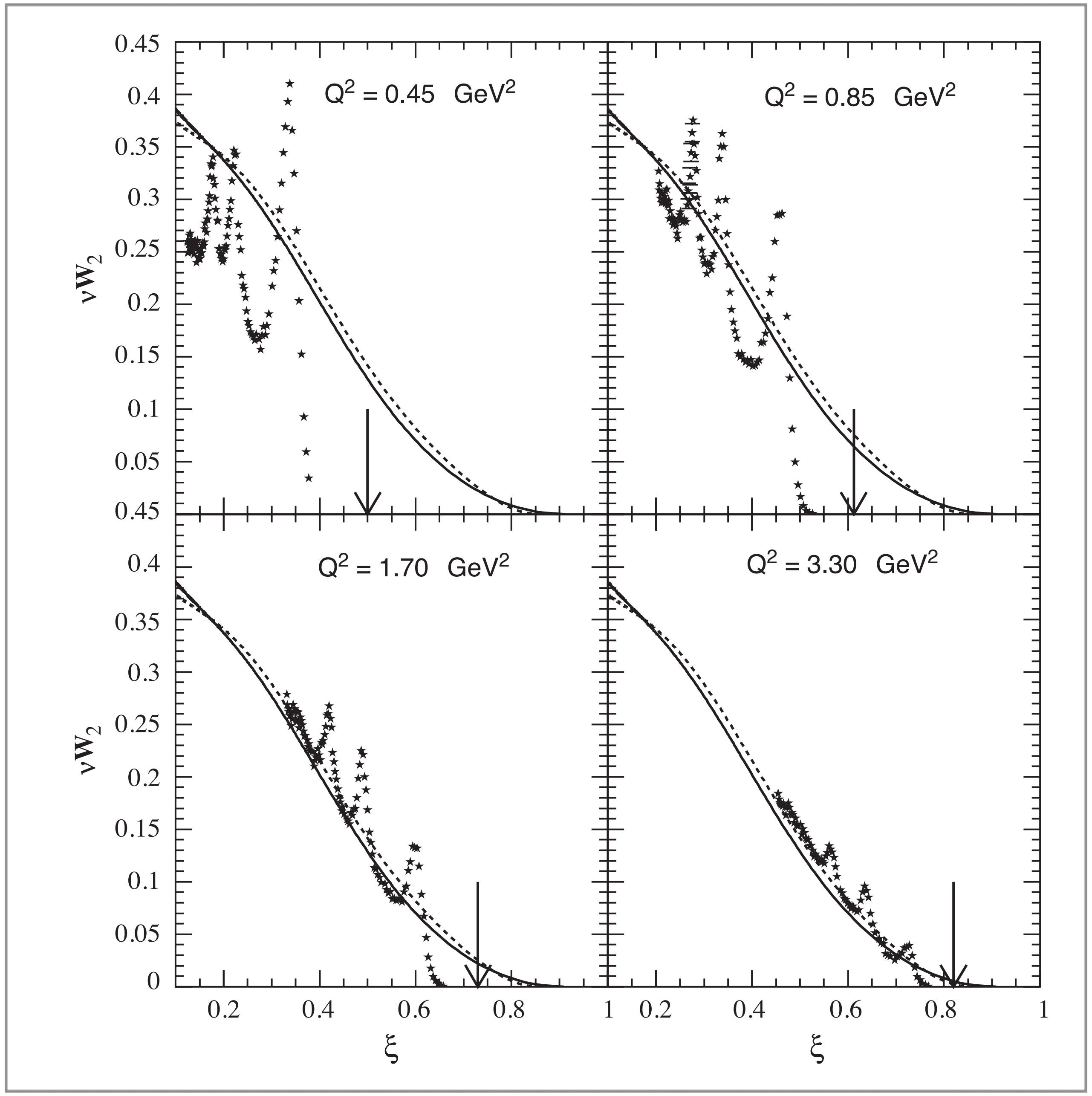
  can be studied in both the resonance and the scaling region

- **Connection in the data between structure function**
  - **in resonance region**
  - **in the scaling region**

$W^2=Q^2(1/x-1)+M^2$

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

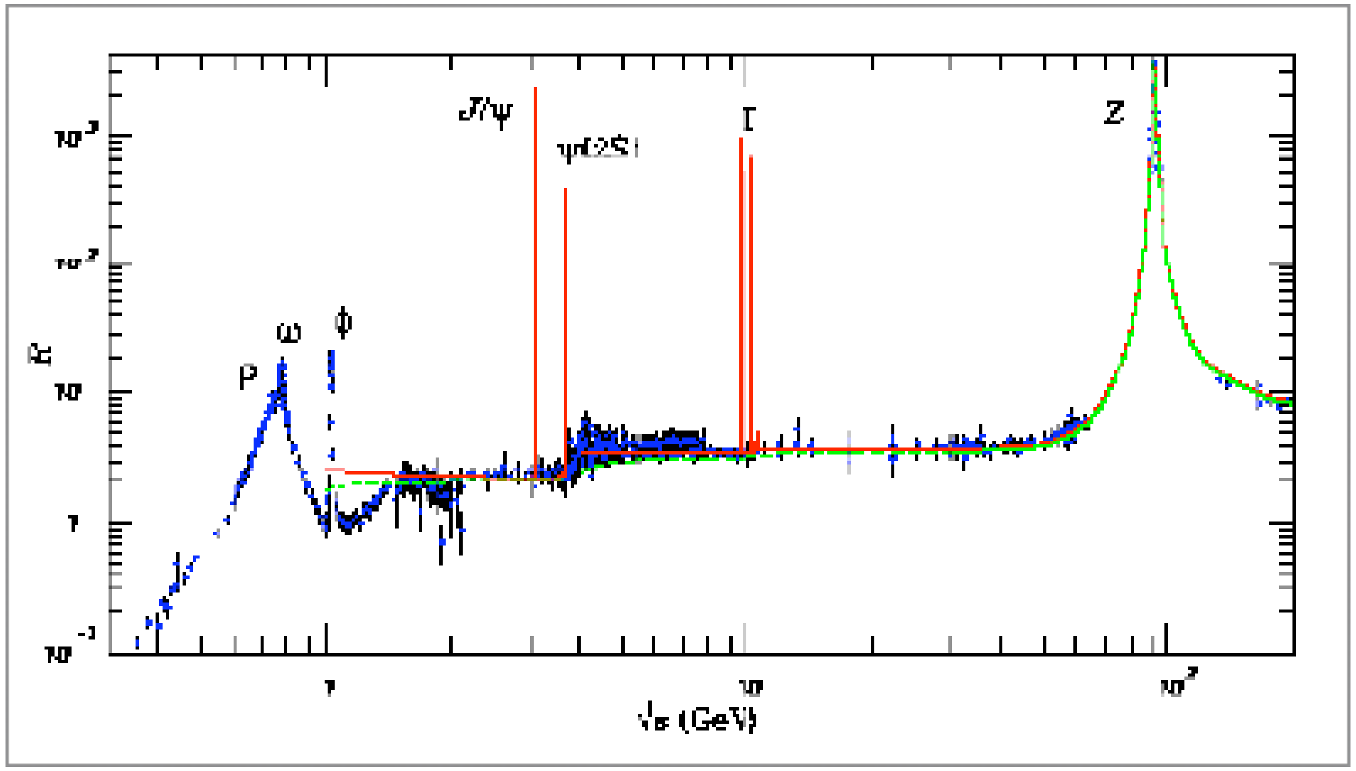
PreQCD



Parton-Hadron Duality

[Poggio, Quinn & Weinberg, Phys Rev D13]

e$^+$ - e$^-$ $\longrightarrow$ hadrons $\equiv \sum_q (e^+e^- \rightarrow q\bar{q}) \Rightarrow$ $\sigma_{hadrons} \equiv \sum_q \hat{\sigma}_q$



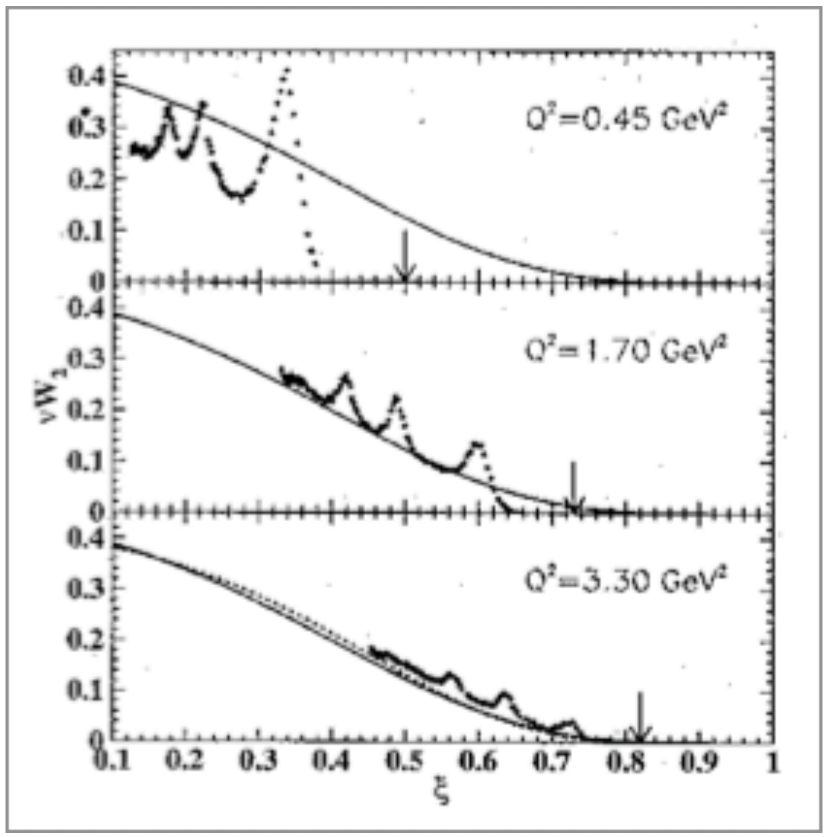
averaged hadronic cross section ⇔ averaged quark cross section

⇒ **Smearing techniques**

Complementarity between Parton and Hadron descriptions of observable

Bloom-Gilman: what do we understand?

Jefferson Lab



[Niculescu et al., PRL85]

Structure functions

Resonance region ⇔ Scaling region

x_{Bj}>0.5, Q^2 multi-GeV region ⇒ 1.2<W^2≤4GeV^2

[Bloom & Gilman, Phys.Rev.Lett.25]

- The resonance region data oscillate around the scaling curve.
- The resonance data are on average equivalent to the scaling curve
- The resonance region data “slide” along the deep inelastic curve with increasing Q2.

Review [Melnitchouk et al., Physics Reports 406]

Duality and QCD

$$I^{res}(Q^2) = \int_{x_m}^{x_M} F_2^{\mathrm{Res}}(x, Q^2)\, dx$$

$$I^{DIS}(Q^2) = \int_{x_m}^{x_M} F_2^{\mathrm{DIS}}(x, Q^2)\, dx$$

experiment

perturbative QCD

Global duality: $x_M \div x_m \Leftrightarrow W^2_m \div W^2_M \Rightarrow 1.2 \div 4\ \mathrm{GeV}^2$

- **Nonperturbative models analysis**
- **Perturbative analysis**

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[Bianchi, Fantoni & Liuti, PRD69]

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Start with NLO PDF and then ...

- Target Mass Corrections (TMC)
- Higher-order in pQCD
- Higher-Twists
- Large-x Resummation (LxR)

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Start with NLO PDF and then ...

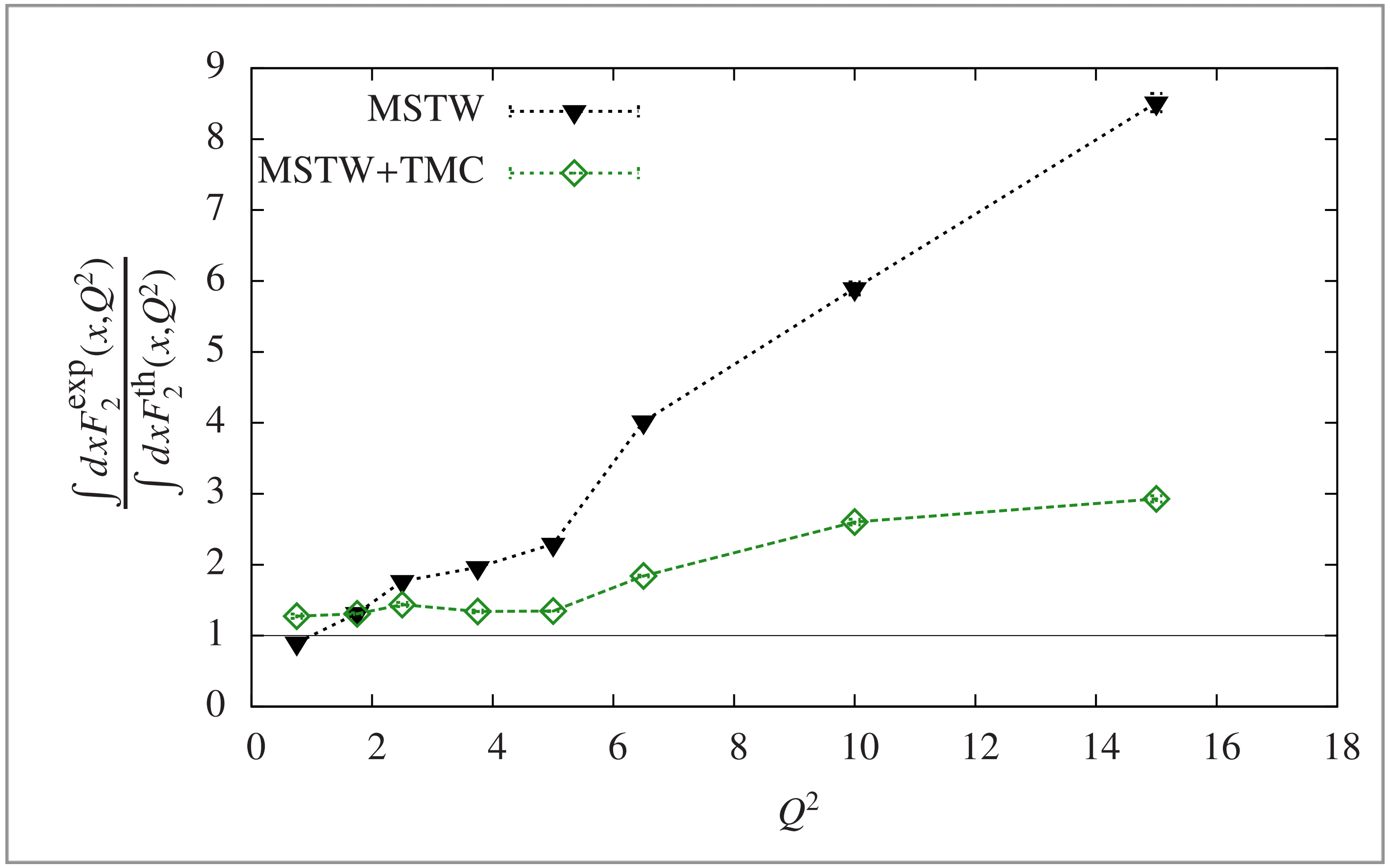
- Target Mass Corrections (TMC) ⟶ **Ok**
- Higher-order in pQCD ⟶ **?**
- Higher-Twists
- Large-x Resummation (LxR) ⟶ **pQCD**

Data analysis: F_2 at JLab

$$R^{\mathrm{exp/th}}(Q^2) = \frac{\displaystyle\int_{x_{\min}(W^2=4\mathrm{GeV}^2)}^{x_{\max}(W^2=1.2\mathrm{GeV}^2)} dx\, F_2^{\mathrm{exp}}(x,Q^2)}{\displaystyle\int_{x_{\min}(W^2=4\mathrm{GeV}^2)}^{x_{\max}(W^2=1.2\mathrm{GeV}^2)} dx\, F_2^{\mathrm{th}}(x,Q^2)}$$

Hall C E94-110 reanalyzed by Monaghan [1209.4542]

=1 if duality fulfilled



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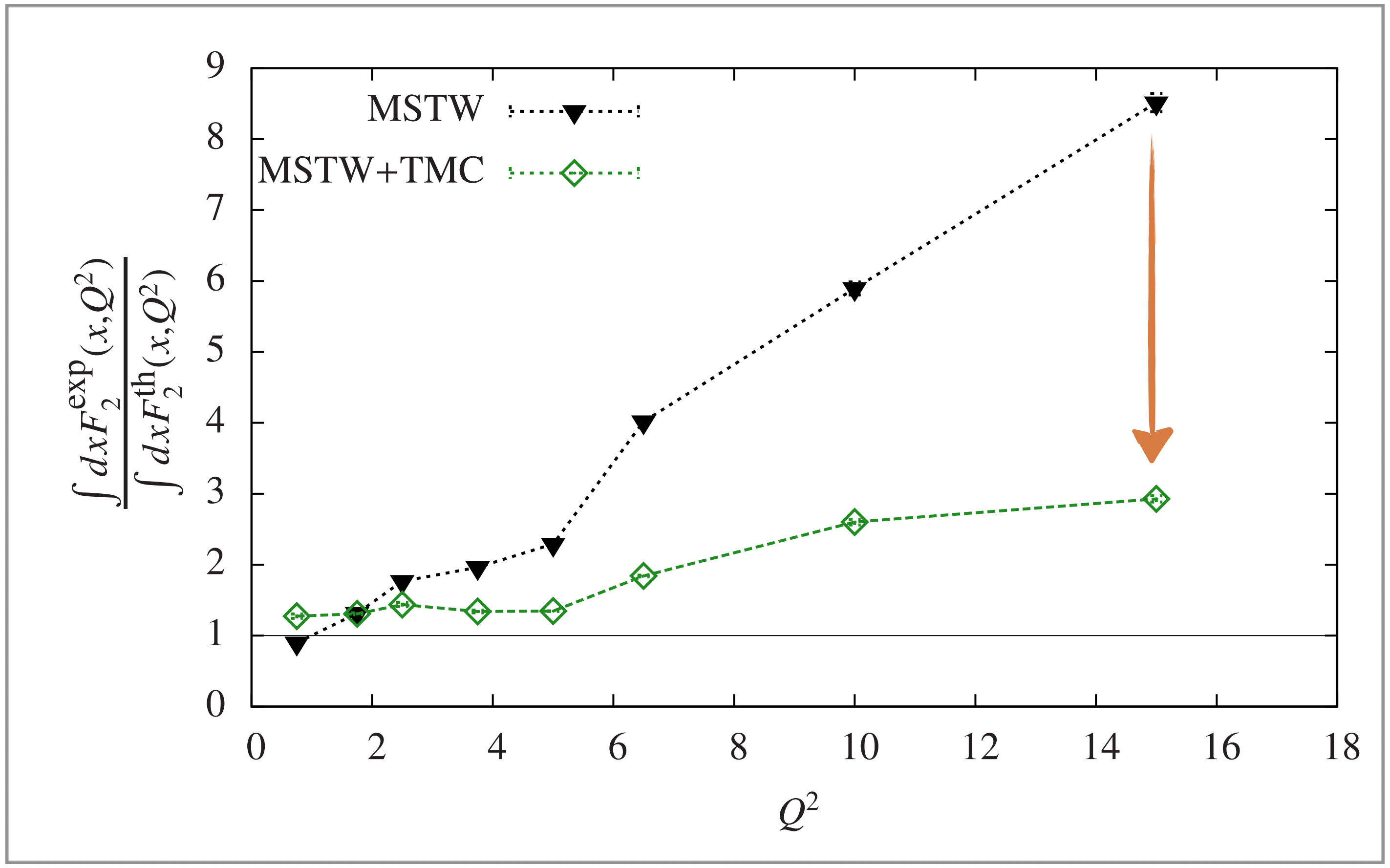

TMC effect

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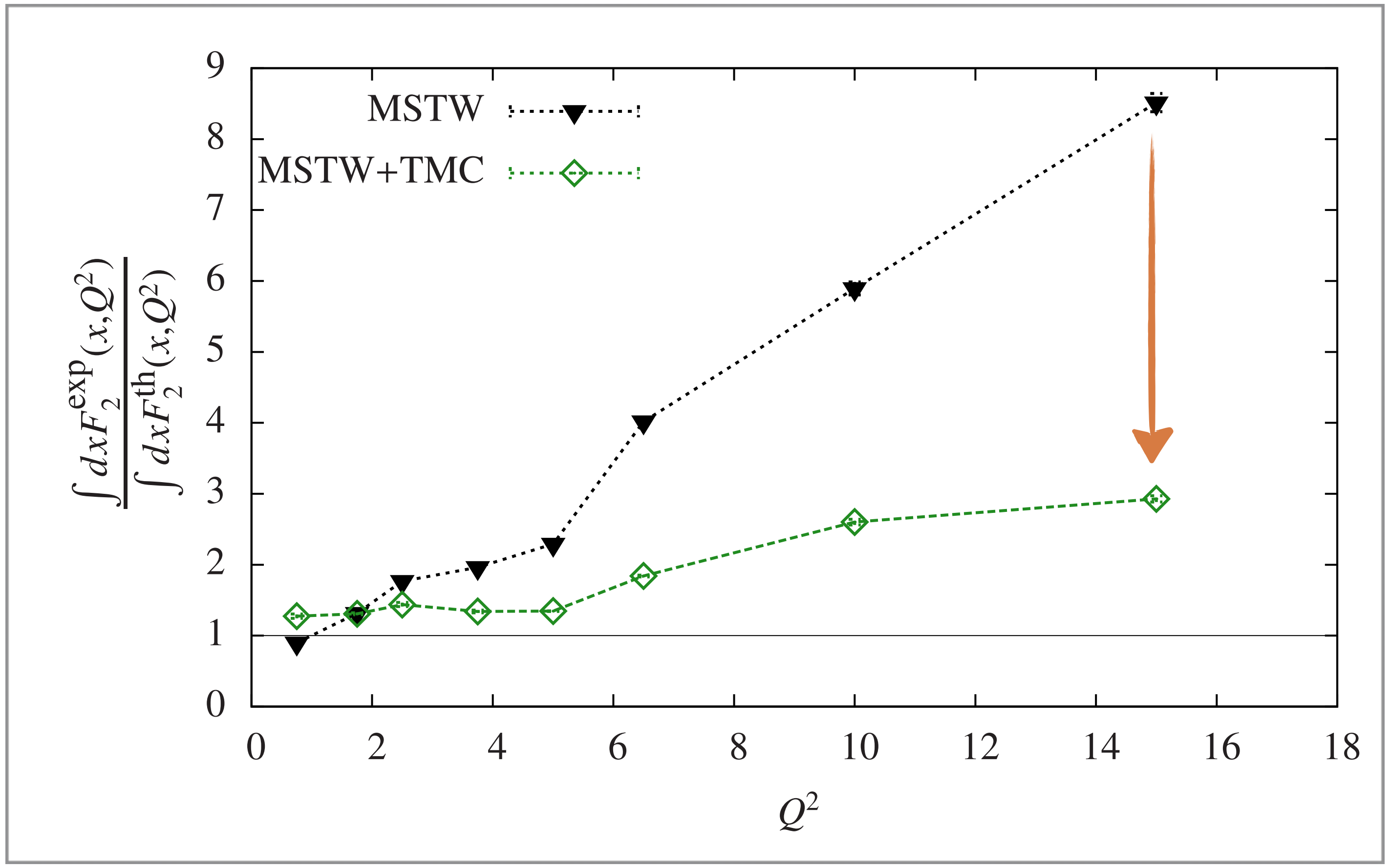

TMC effect

Still missing something...

F_2 in perturbative QCD

$\overline{\text{MS}}$ scheme →

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In practice:

1. DGLAP
2. convolution with coefficient functions

1. $q_0 \rightarrow$ leading-twist PDFs here MSTW08NLO
2. $q_0 \rightarrow$ evolved to $q(x, Q^2)$ via DGLAP with $P \rightarrow$ splitting functions, to NLO
3. $C \rightarrow$ coefficient functions, to NLO

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Is it still true at large-x ?

Large-x Resummation

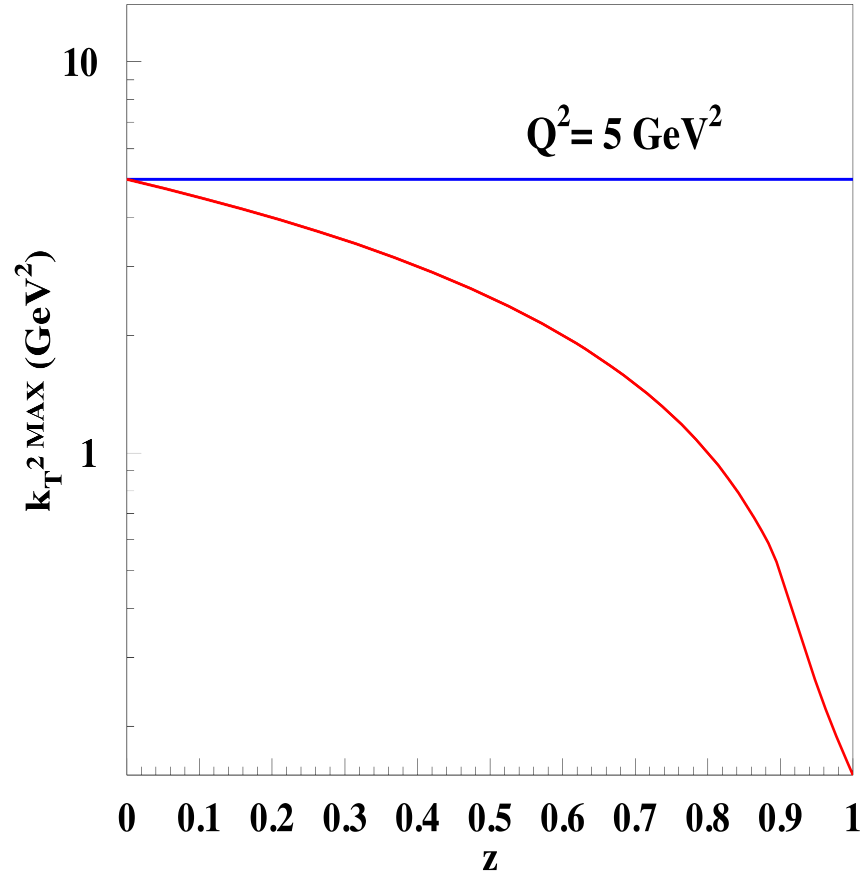
Amati et al., Nucl.Phys. B173 (1980) 429

- Large invariants: $\Lambda^2 \ll W^2 \sim Q^2$
- Argument for α_s is ω^2, mass square of final state of γ^* parton collision

$$\omega^2 = \frac{Q^2}{z}(1-z)$$

Without LxR, upper limit $=Q^2$

$$q(x,Q^2) = \int_x^1 \frac{dz}{z} \int_{\mu^2}^{Q^2 \frac{1-z}{4z}} dk_T^2\, \alpha_S(k_T^2)\, P_{qq}(z)\, q\left(\frac{x}{z}, k_T^2\right)$$



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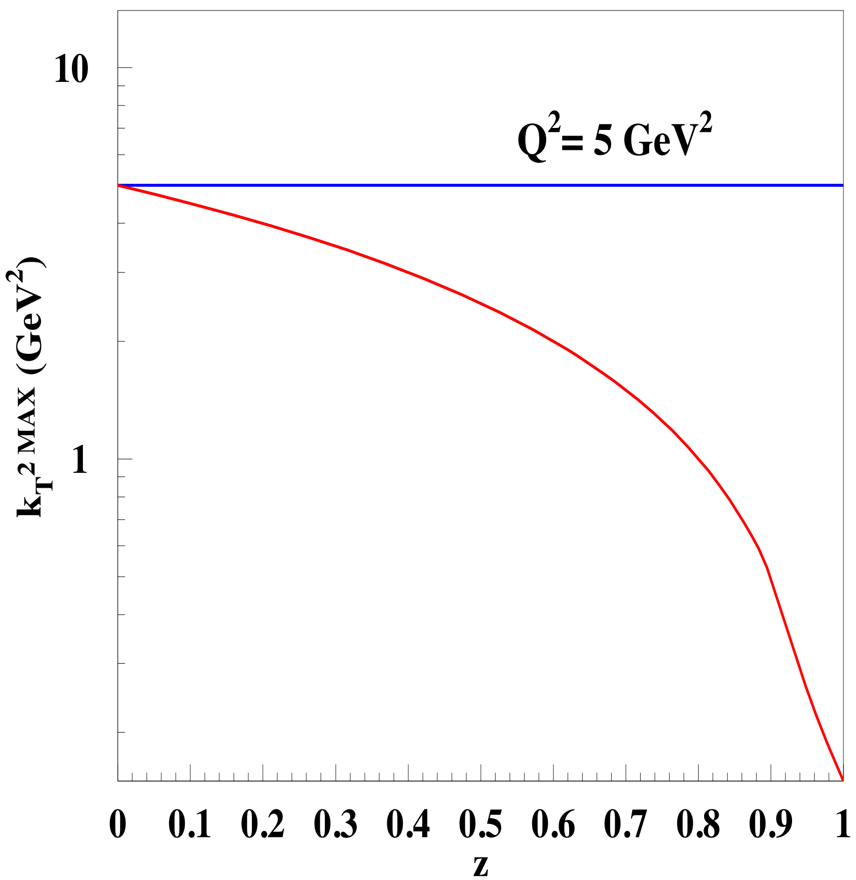
The structure functions become

$$F_2^{NS}(x,Q^2) = \sum_q \int_x^1 dz\, \frac{\alpha_s\left(\frac{Q^2(1-z)}{4z}\right)}{2\pi}\, C_{NS}(z)\, q_{NS}\left(\frac{x}{z}, Q^2\right)$$



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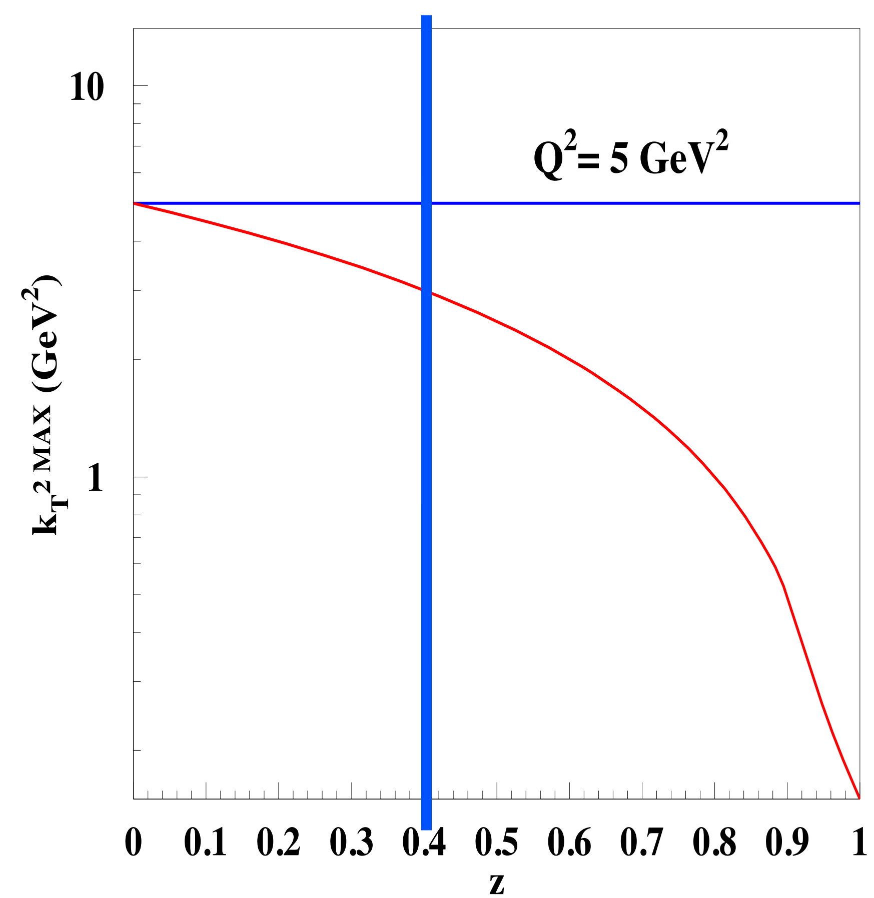
x-values



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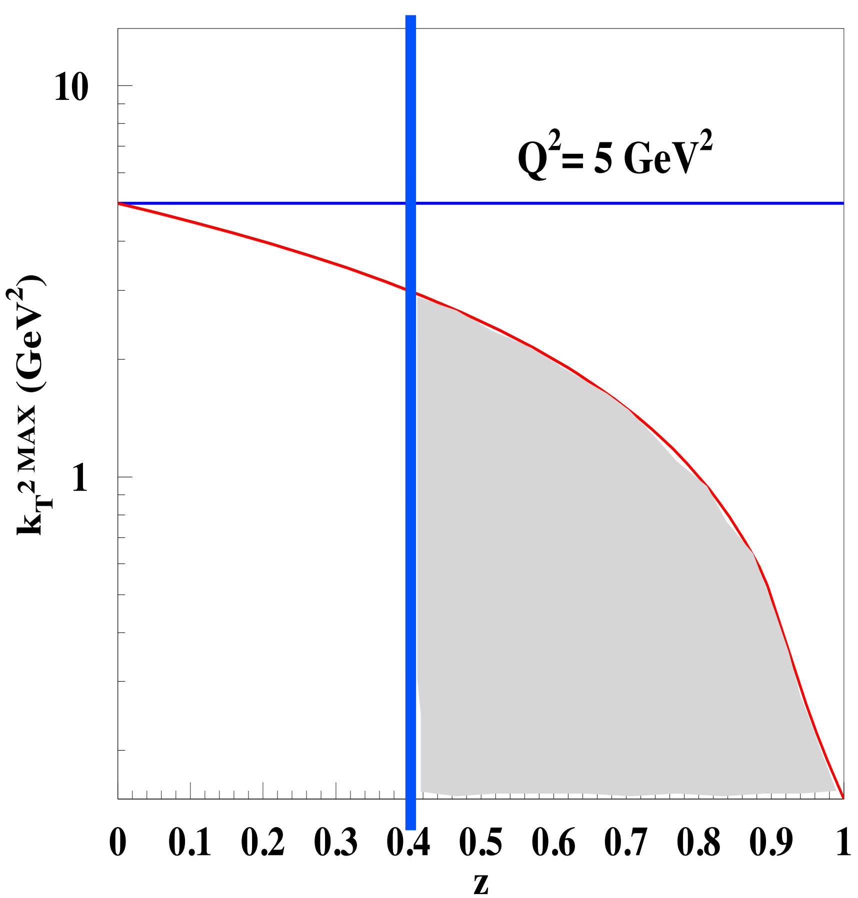

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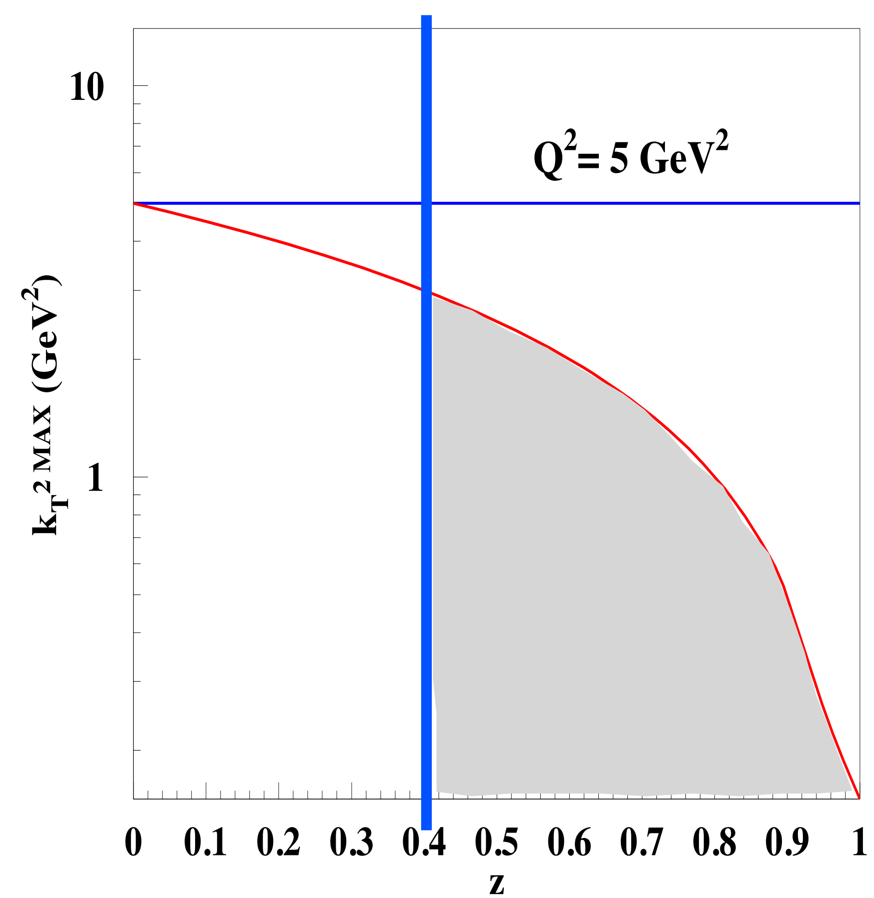
Without LxR, upper limit $=Q^2$

$$q(x,Q^2) = \int_x^1 \frac{dz}{z} \int_{\mu^2}^{Q^2\frac{1-z}{4z}} dk_T^2\, \alpha_S(k_T^2)\, P_{qq}(z)\, q\left(\frac{x}{z}, k_T^2\right)$$

The structure functions become

$$F_2^{NS}(x,Q^2) = \sum_q \int_x^1 dz\, \frac{\alpha_s\left(\frac{Q^2(1-z)}{4z}\right)}{2\pi}\, C_{NS}(z)\, q_{NS}\left(\frac{x}{z}, Q^2\right)$$

x-values



restricted phase space for real gluon emission

Many ways to implement LxR

Our strategy

- We don't touch the DGLAP part
- Resummation at the coefficient function level :

$$\boxed{F_2^{NS}(x, Q^2) = xq(x, Q^2) + \frac{\alpha_s}{4\pi} \sum_q \int_x^1 dz\, B^q_{\rm NS}(z)\, \frac{x}{z}\, q\left(\frac{x}{z}, Q^2\right)}$$

- Divergent term at x→1,

$$\boxed{B^q_{\rm NS}(z) = \left[\hat{P}^{(0)}_{qq}(z) \left\{\ln\left(\frac{1-z}{z}\right) - \frac{3}{2}\right\} + \text{E.P.}\right]_+}$$

[Courtoy & Liuti, 1302.4439]

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$$\alpha_s(Q^2) \to \alpha_s\left(Q^2\frac{(1-z)}{z}\right)$$

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[Courtoy & Liuti, 1302.4439]

- Resummed as :

$$\ln(1-z) \;=\; \frac{1}{\alpha_{s,\rm LO}(Q^2)}\int^{Q^2} d\ln Q^2\;\left[\alpha_{s,\rm LO}(Q^2(1-z)) - \alpha_{s,\rm LO}(Q^2)\right] \equiv \ln_{\rm LxR}$$

Rôle of the coupling constant

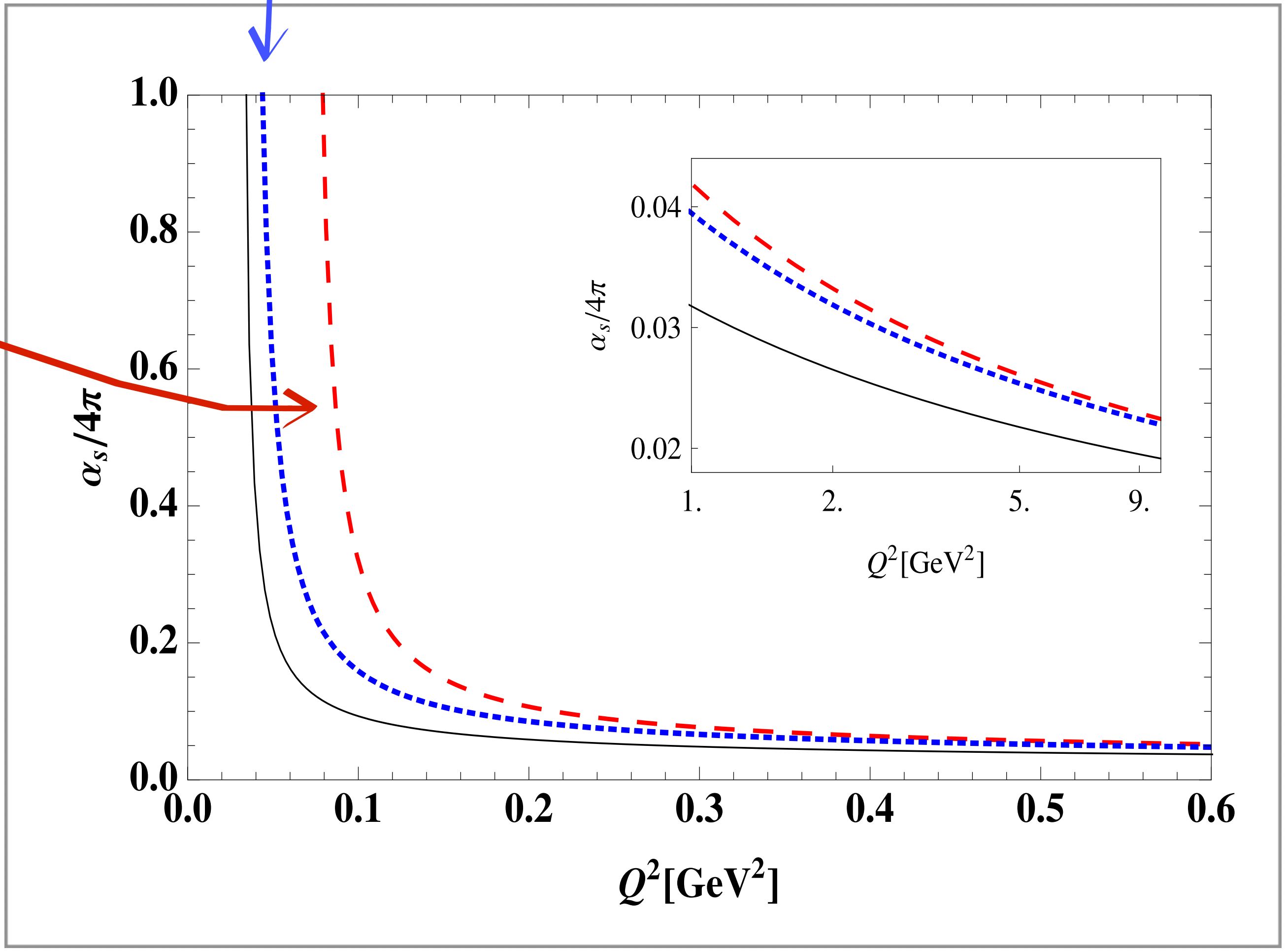
Example

- LO exact solution, Λ=174MeV → reaches Landau pole at Q=174MeV

- expansion in α_s $$\alpha_s(\tilde{W}^2) = \alpha_s(Q^2) - \frac{\beta_0}{4\pi} \ln\left(\frac{1-z}{z}\right) \alpha_s^2(Q^2)$$

- full dependence in z,

  Λ=174MeV → reaches Landau pole at Q >174MeV



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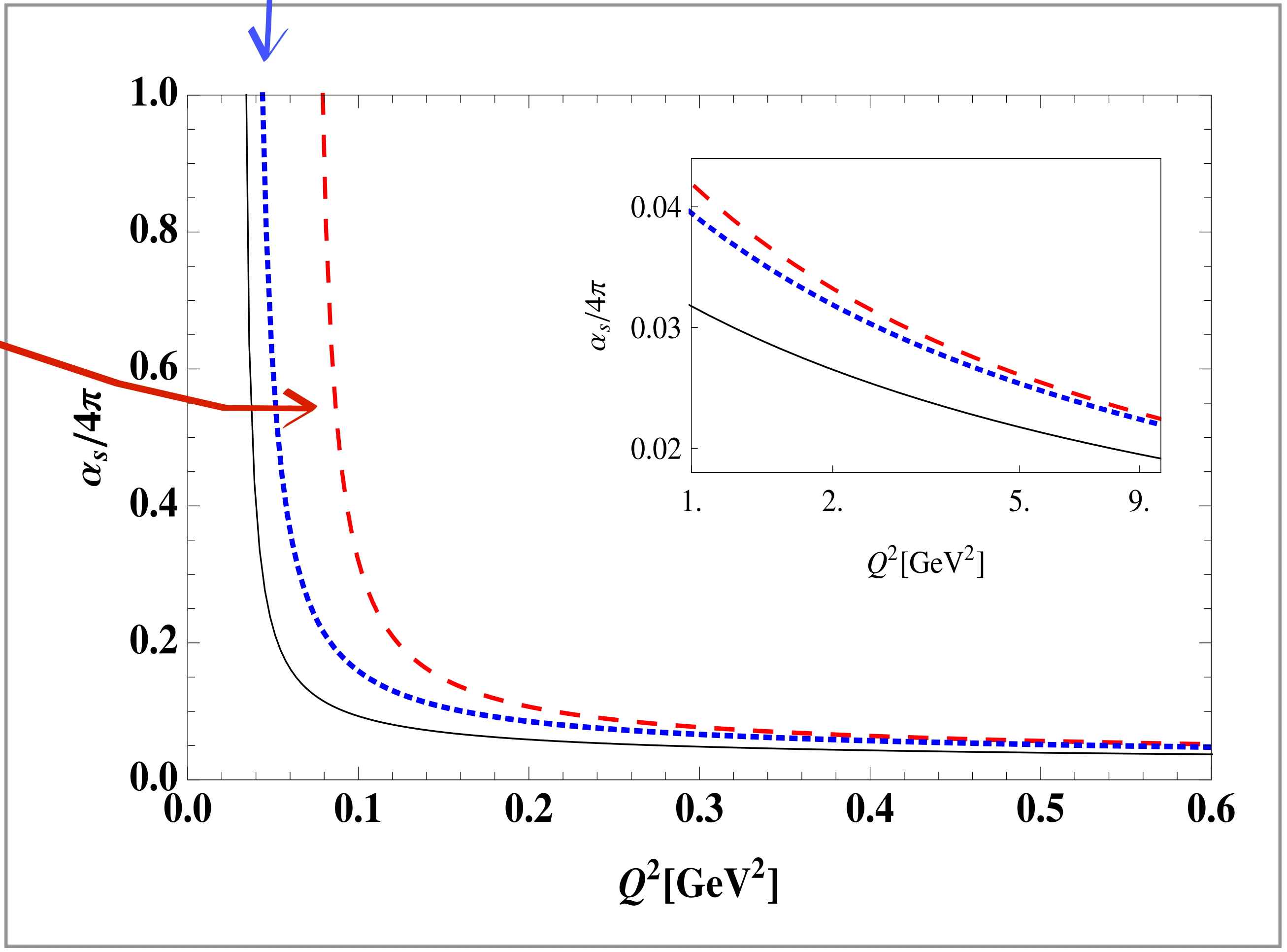
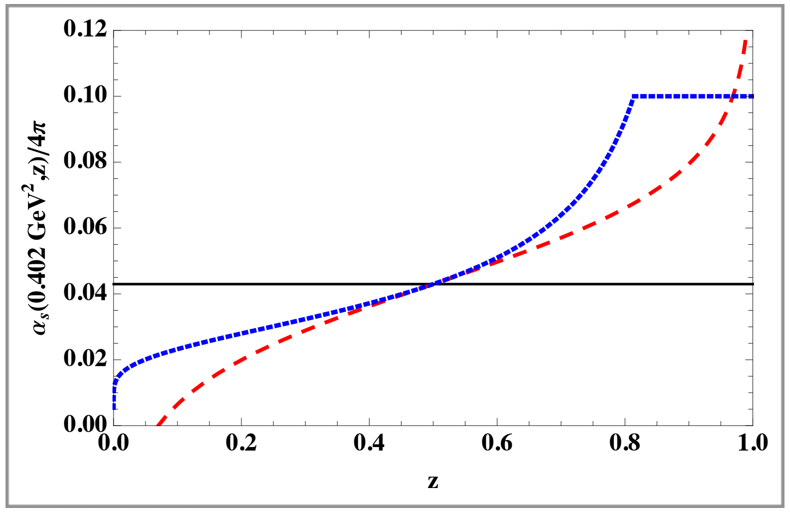
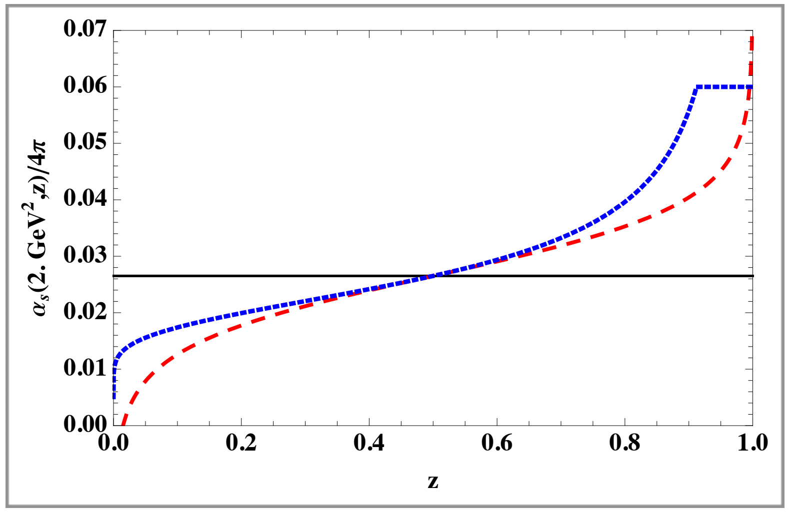


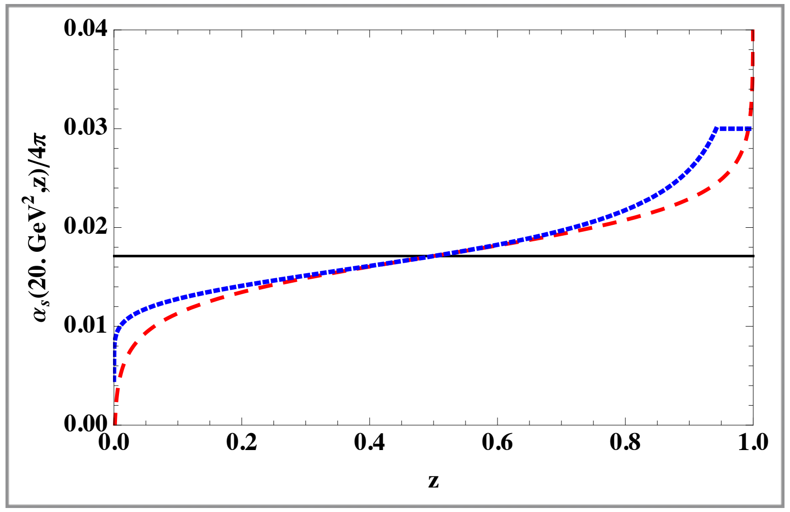
α_s might blow up

Large-x Resummation: α_s as (hidden) free parameter

[Courtoy & Liuti, 1208.5636]







- $\alpha_s(Q^2)$;
- an expansion of $\alpha_s(\tilde{W}^2)$ in $\ln((1-z)/z)$, to NLO,

$$\alpha_s(\tilde{W}^2) = \alpha_s(Q^2) - \frac{\beta_0}{4\pi} \ln\left(\frac{1-z}{z}\right) \alpha_s^2(Q^2)$$

- the complete z dependence of $\alpha_s(\tilde{W}^2)$

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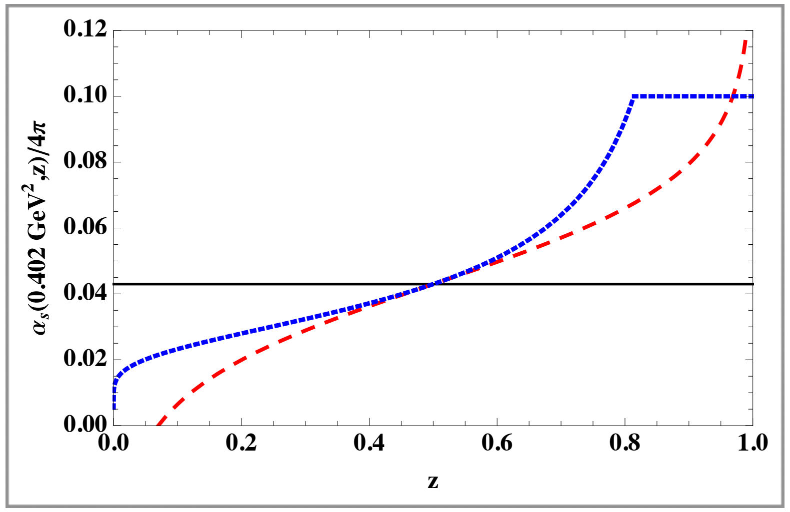

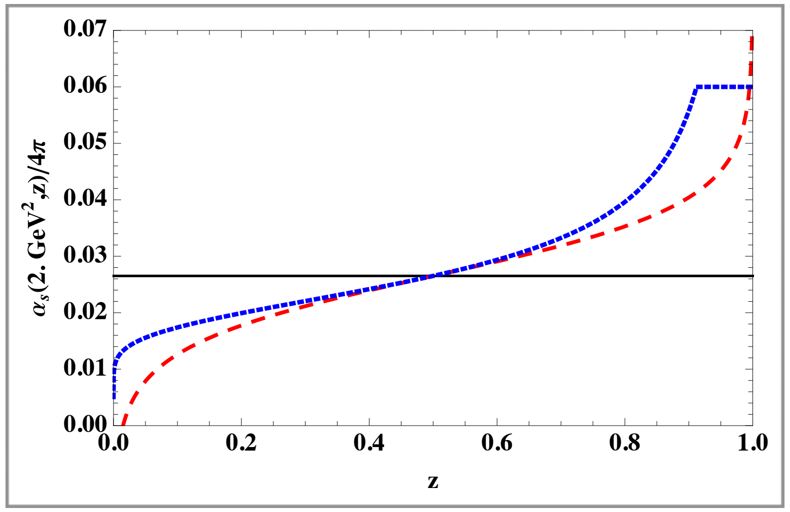

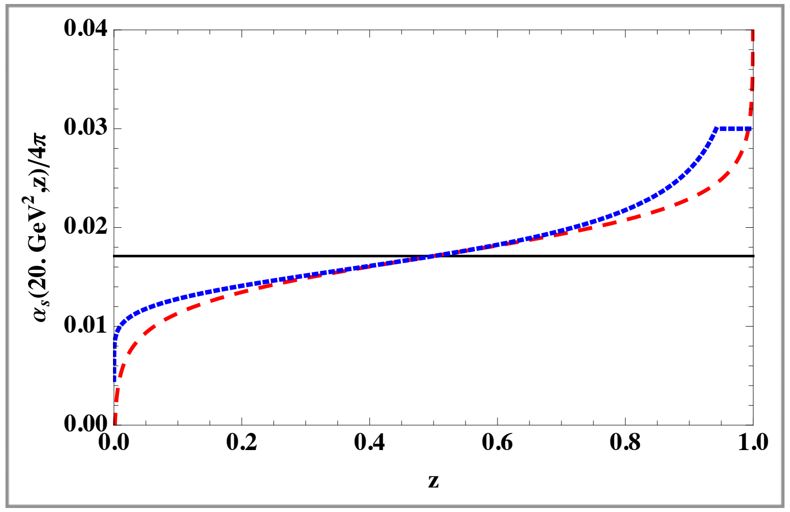

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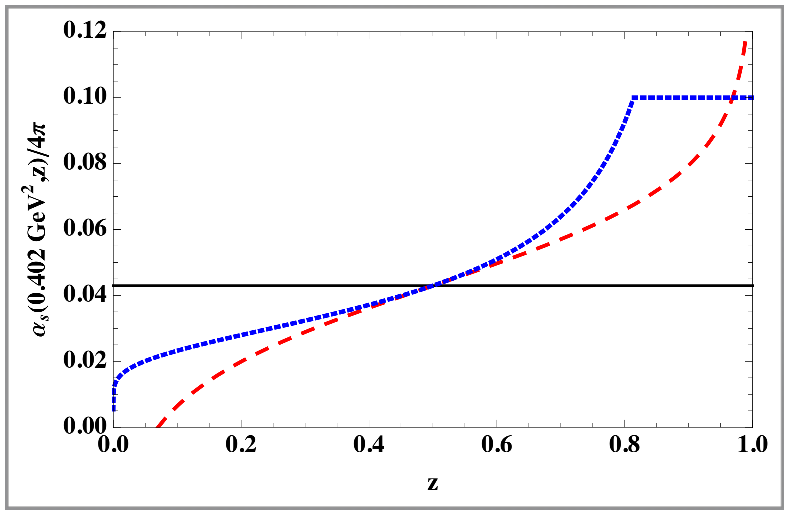

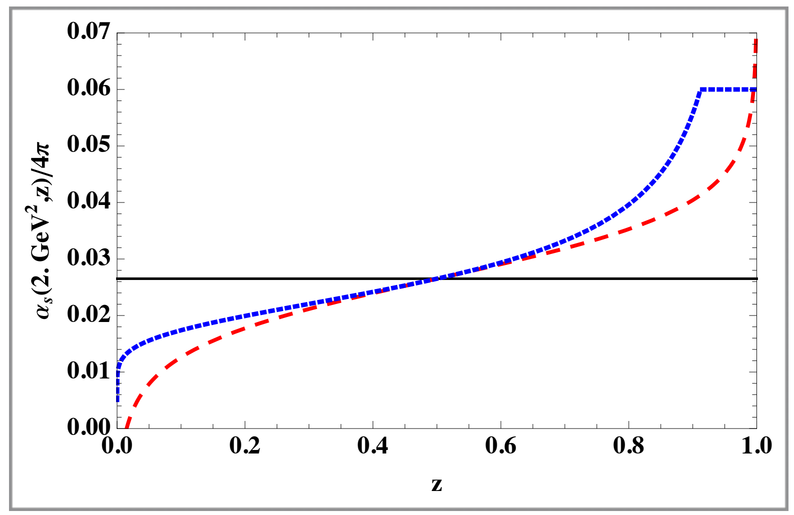

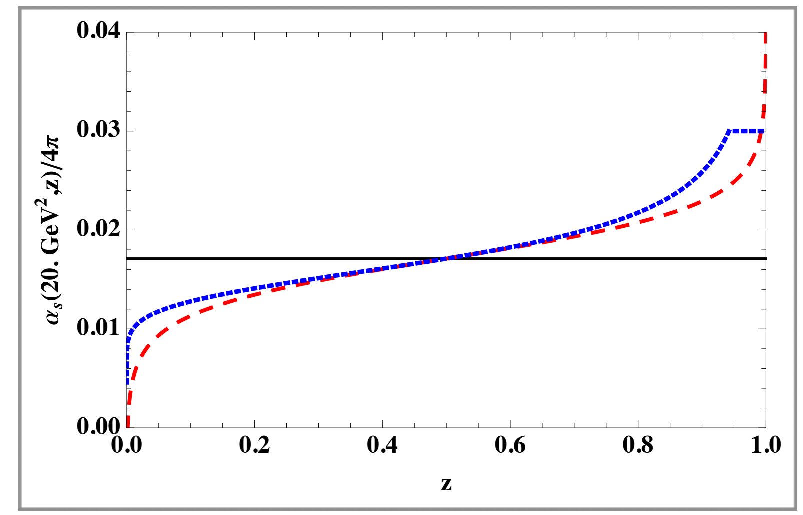

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What does a cut in α_s means?

Running Coupling Constant

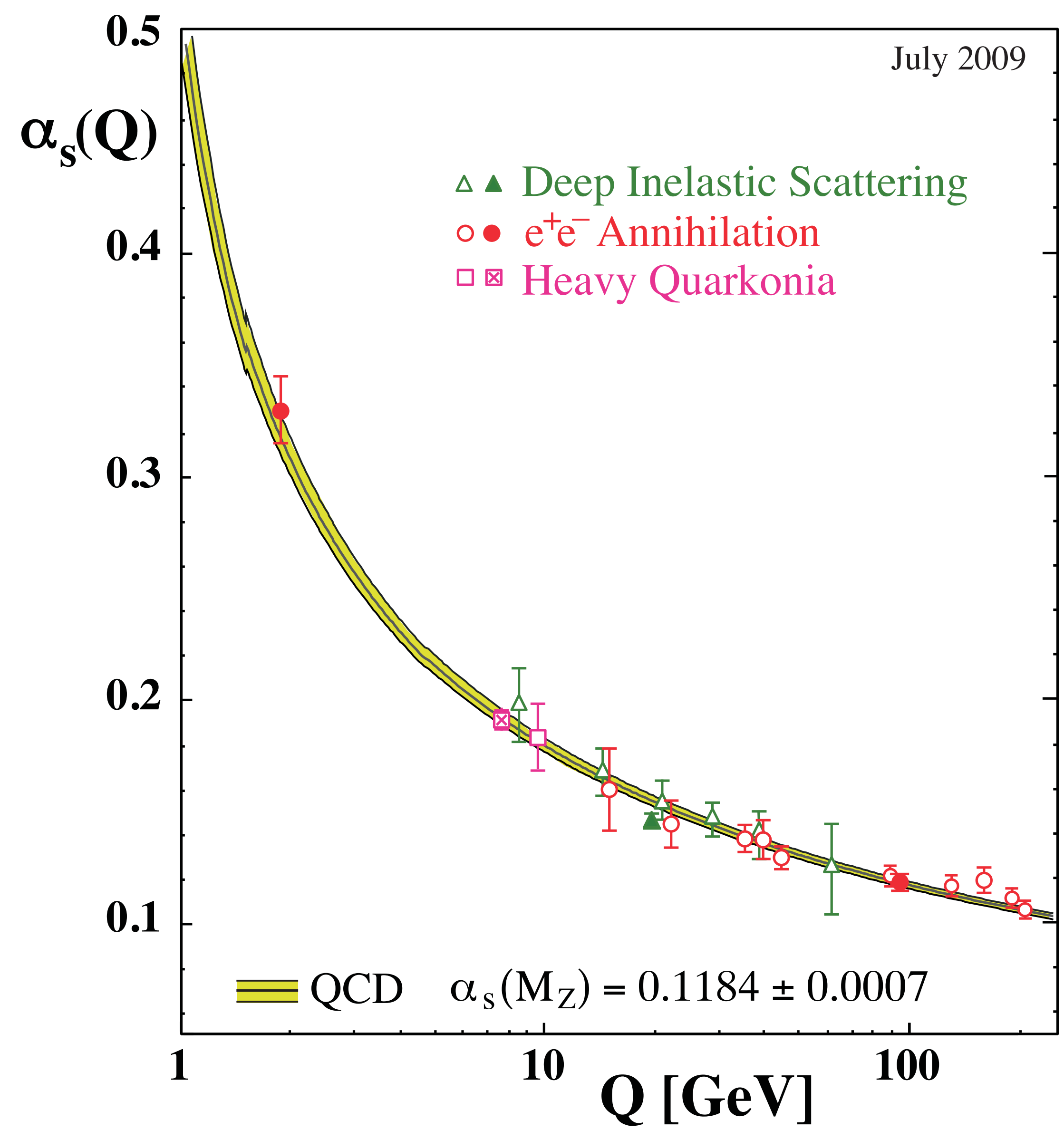
QCD Coupling Constant in pQCD

- QCD with massless quarks
  - ➡ no scale parameters
- RGE introduces a momentum scale Λ
  - ➡ interaction strength =1
- Renormalization scheme dependence of Λ
- World data average (2009)

$$\alpha_s(M_{Z^0}) = 0.1184 \pm 0.0007$$

that corresponds to (to NNLO)

$$\Lambda^{(5)}_{\overline{MS}} = (213 \pm 9\ )\,\mathrm{MeV}$$



Bethke, Eur.Phys.J. C64

QCD Running Coupling Constant

$$\frac{d\,a(Q^2)}{d(\ln\ Q^2)} = \beta_{N^m LO}(\alpha) = \sum_{k=0}^{m} a^{k+2}\beta_k$$

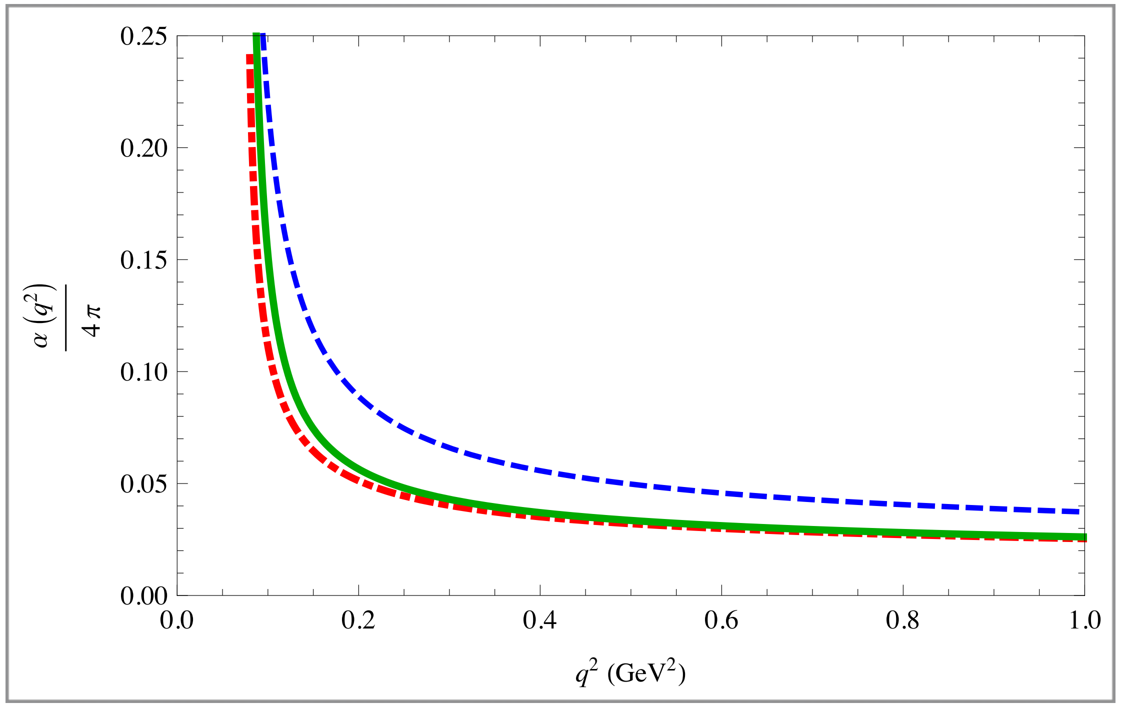
$\overline{\text{MS}}$ scheme

$a=\alpha_s/4\pi$

LO exact perturbative solution Λ=250 MeV

NLO exact perturbative solution Λ=250 MeV

NNLO exact perturbative solution Λ=250 MeV



QCD predicts the shape of the running coupling constant, not its value

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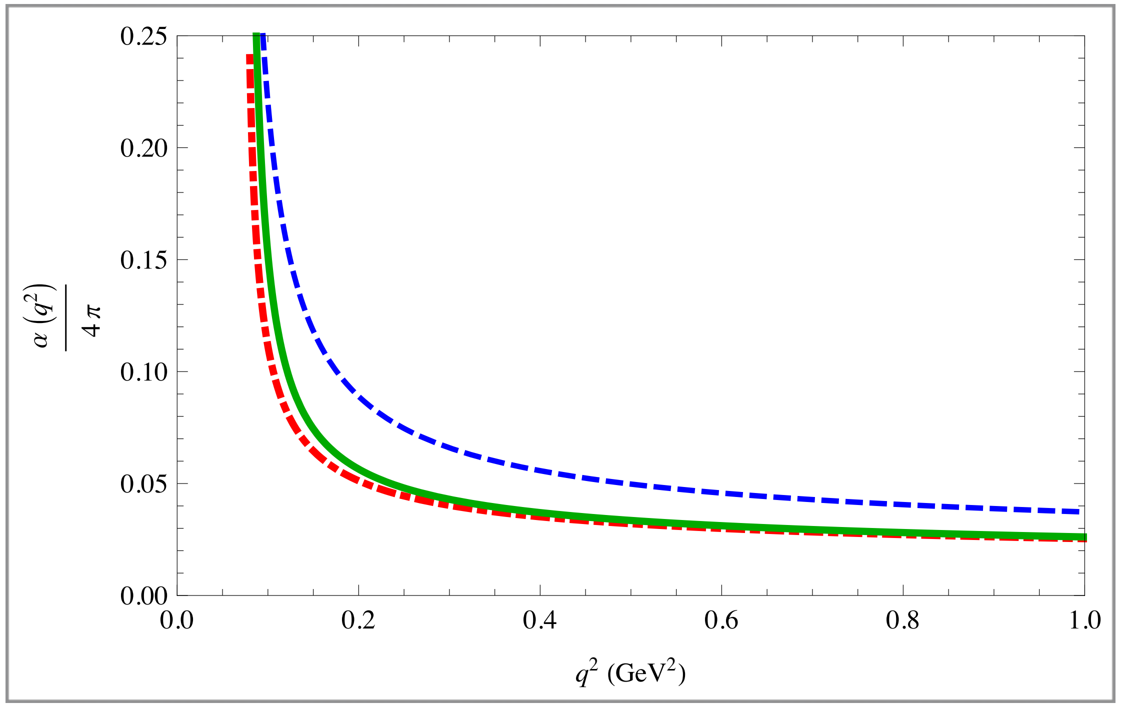


QCD predicts the shape of the running coupling constant, not its value

Intermediate energy?
Perturbative to non-perturbative transition?

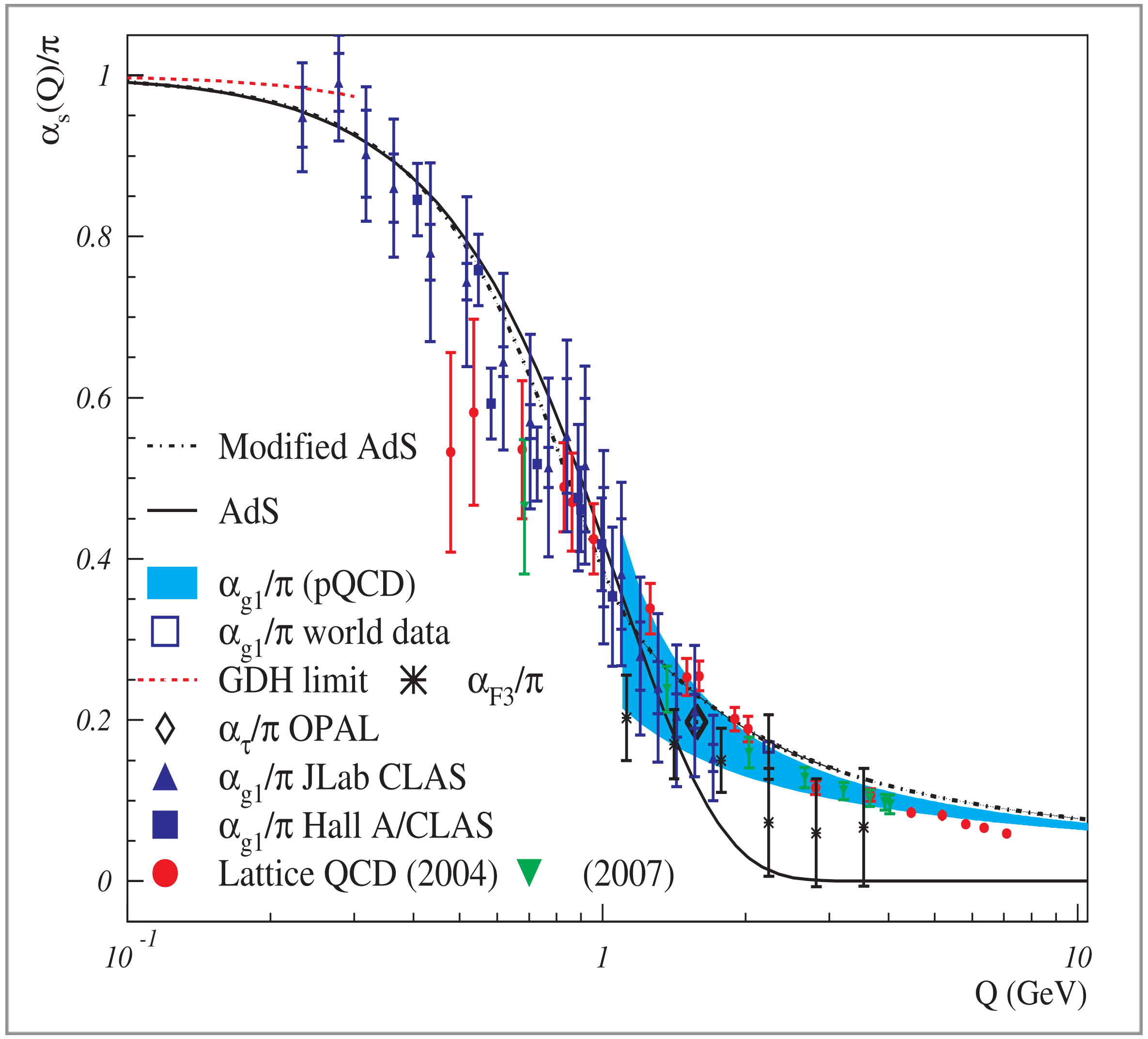
Effective Charges

The non-perturbative approach:

- Importance of finite couplings
- Taming the Landau pole

The non-perturbative extraction:

- Effective couplings from phenomenology
- Dimensional transmutation (RG-improved)
  - ➡ from RS dependence to Observable dependence (à la Grunberg)



[Brodsky et al., Phys.Rev.D81]
[Deur et al., Phys.Lett.B60]

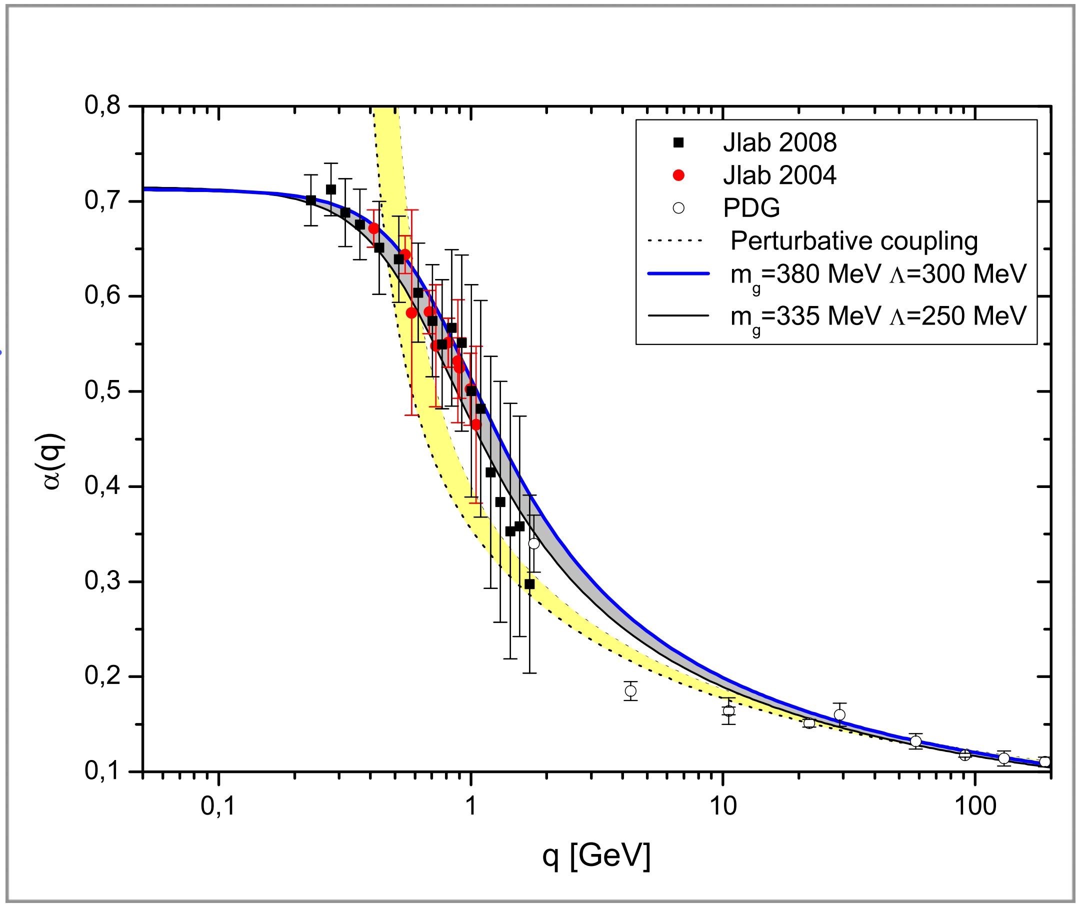
Non-perturbative analysis

Qualitative analysis

➡ **Implications of IR finite α_s in hadronic physics**

Examples of non-perturbative approaches:

- ➡ Cornwall: *gluon propagator*
- ➡ Shirkov: *analytic perturbative theory*
- ➡ Fischer & Alkofer: *ghost-gluon vertex*



Importance and applications of finite couplings:

e.g.

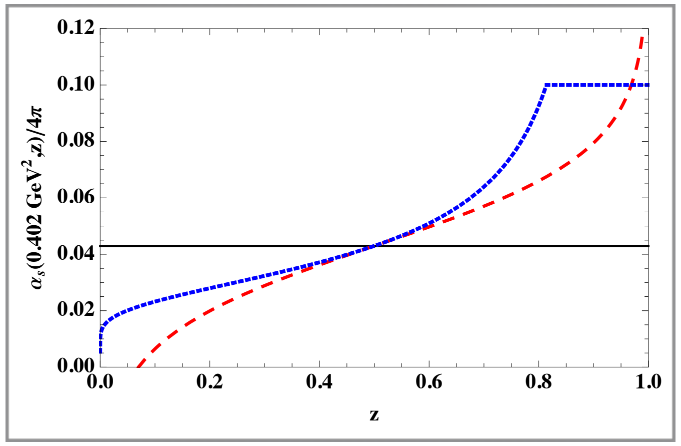
Dokshitzer et al., Nucl.Phys.B469 (1996) 93

Plot by Arlene C. Aguilar

Back to duality

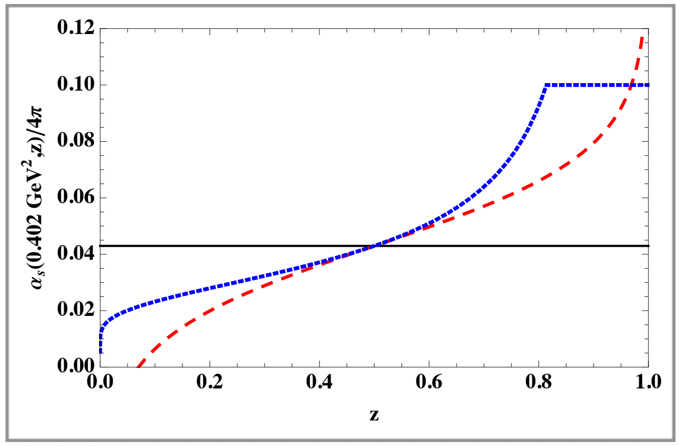
Parametrize the realization of duality

- Freeze α_s by imposing a z_{max}: $\widetilde{W}^2(z_{\max}) = Q^2(1 - z_{\max})/z_{\max}$
- Changes the behavior of the coefficient function $x \to 1$



Back to duality

Parametrize the realization of duality



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- **Changes the behavior of the coefficient function $x \rightarrow 1$**
- **Realization of duality depends on z_{max} :**

$$\boxed{R^{\mathrm{exp/th}}(z_{\max}, Q^2) = \frac{\int_{x_{\min}}^{x_{\max}} dx\, F_2^{\mathrm{exp}}(x, Q^2)}{\int_{x_{\min}}^{x_{\max}} dx\, F_2^{NS,\mathrm{Resum}}(x, z_{\max}, Q^2)} = \frac{I^{\mathrm{exp}}}{I^{\mathrm{Resum}}} = 1}$$

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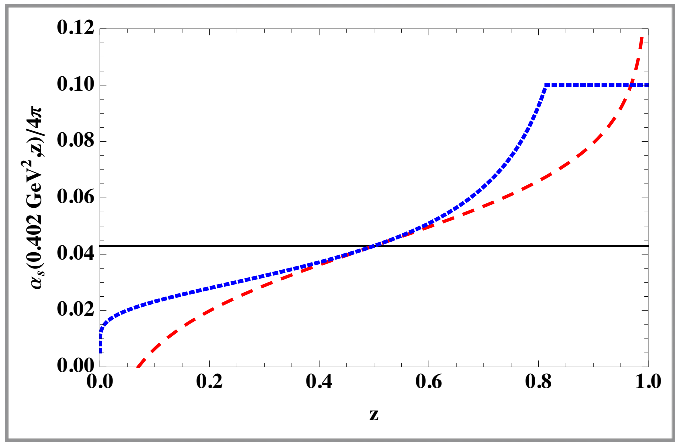

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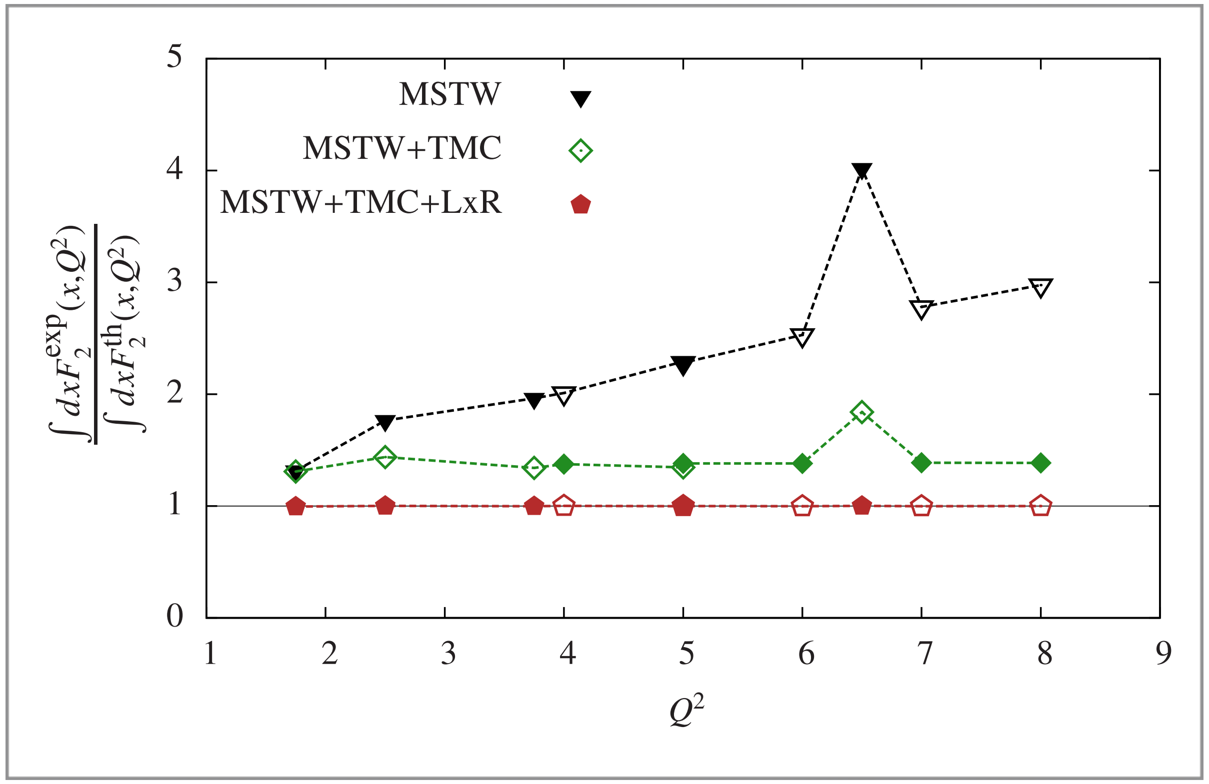
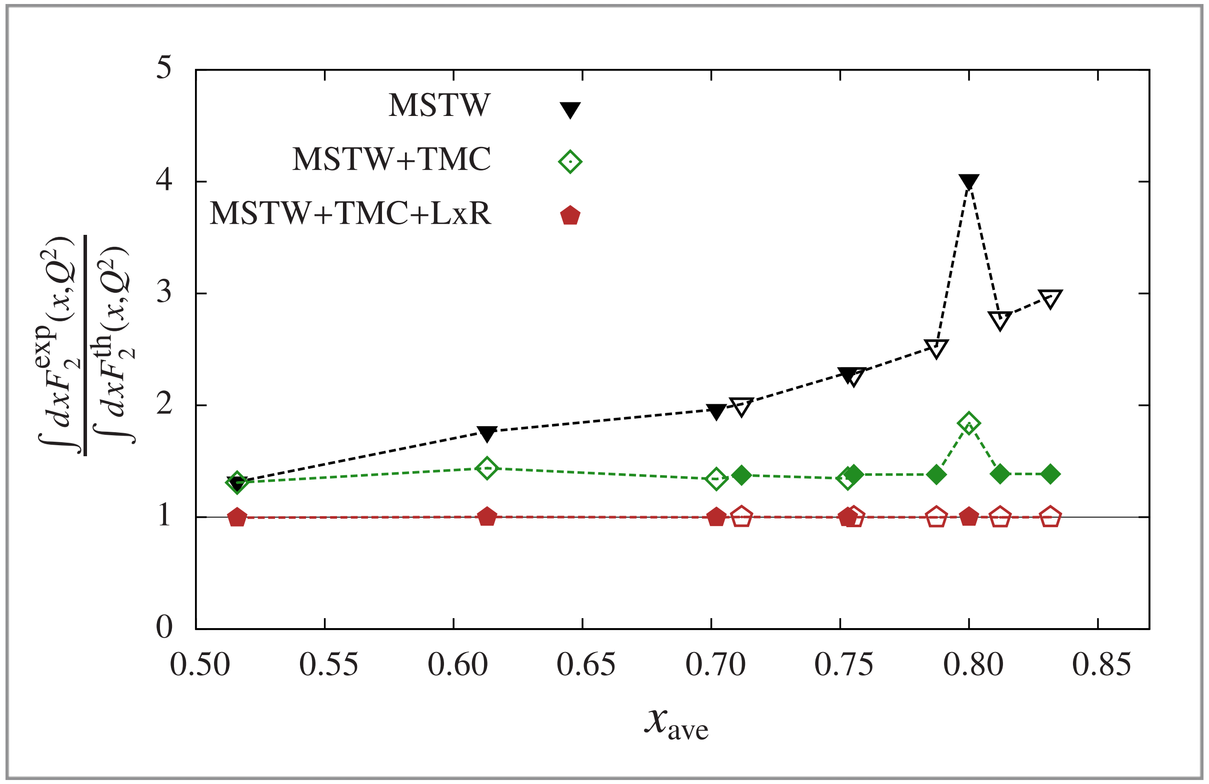
- Adjust z_{max} according to the data

Results





	Q^2 [GeV^2]	$I^{\rm exp}(Q^2)$	$I^{(0),\rm DIS}(Q^2)$	$I^{(0),\rm DIS+TMC}(Q^2)$	$I^{\rm Resum}(z_{\rm max}, Q^2)$	$z_{\rm max}$
JLab data	1.75	6.994×10^{-2}	5.316×10^{-2}	5.345×10^{-2}	7.025×10^{-2}	0.63
	2.5	4.881×10^{-2}	2.765×10^{-2}	3.393×10^{-2}	4.872×10^{-2}	0.745
	3.75	2.356×10^{-2}	1.201×10^{-2}	1.756×10^{-2}	2.359×10^{-2}	0.76
	5.	1.267×10^{-2}	0.553×10^{-2}	0.942×10^{-2}	1.270×10^{-2}	0.79
	6.5	0.685×10^{-2}	0.170×10^{-2}	0.372×10^{-2}	0.683×10^{-2}	0.9
SLAC data Phys.Lett. B282	4.	2.045×10^{-2}	1.017×10^{-2}	1.487×10^{-2}	2.041×10^{-2}	0.79
	5.	1.255×10^{-2}	0.550×10^{-2}	0.909×10^{-2}	1.255×10^{-2}	0.811
	6.	0.802×10^{-2}	0.317×10^{-2}	0.581×10^{-2}	0.803×10^{-2}	0.825
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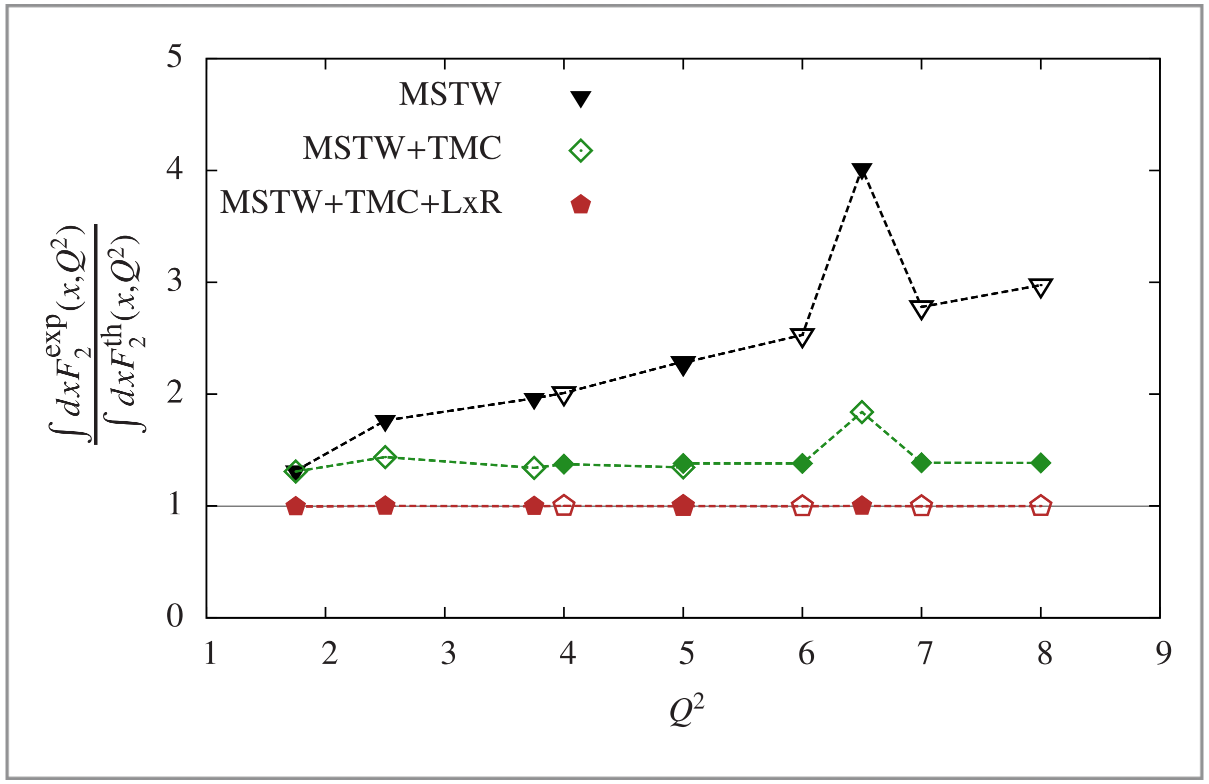

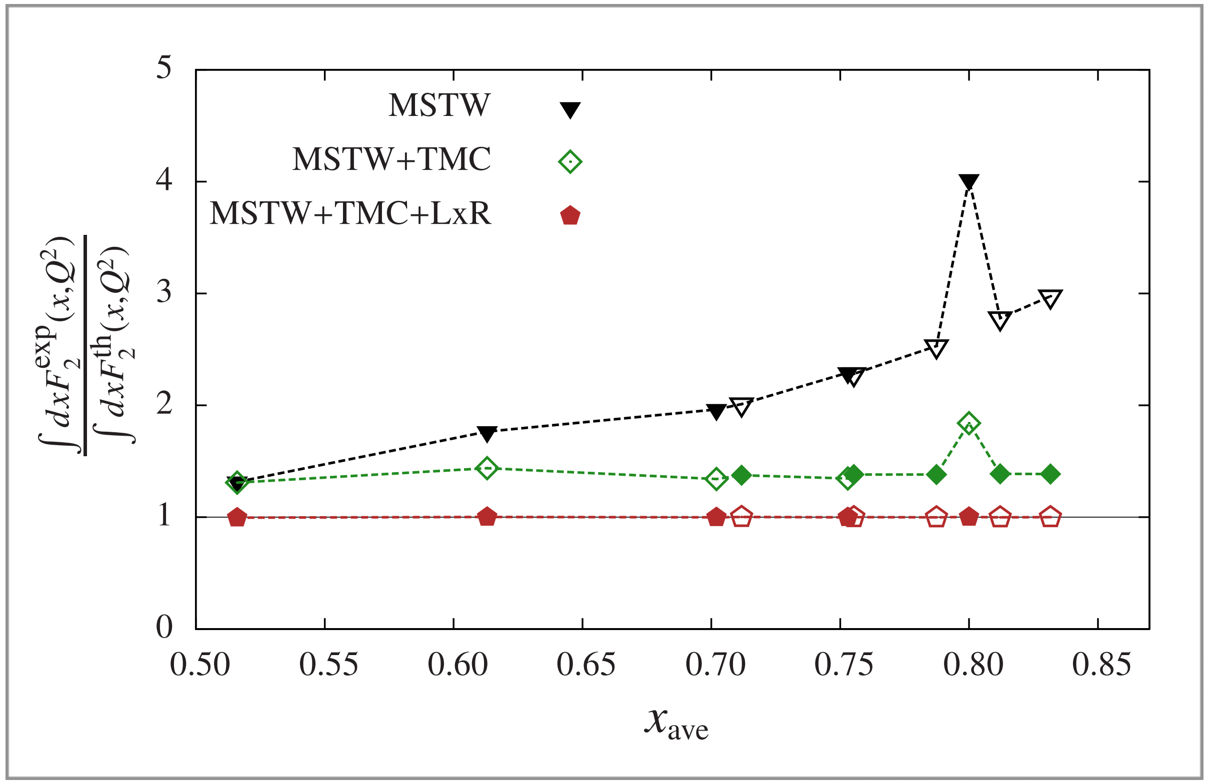

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Phys.Lett. B282

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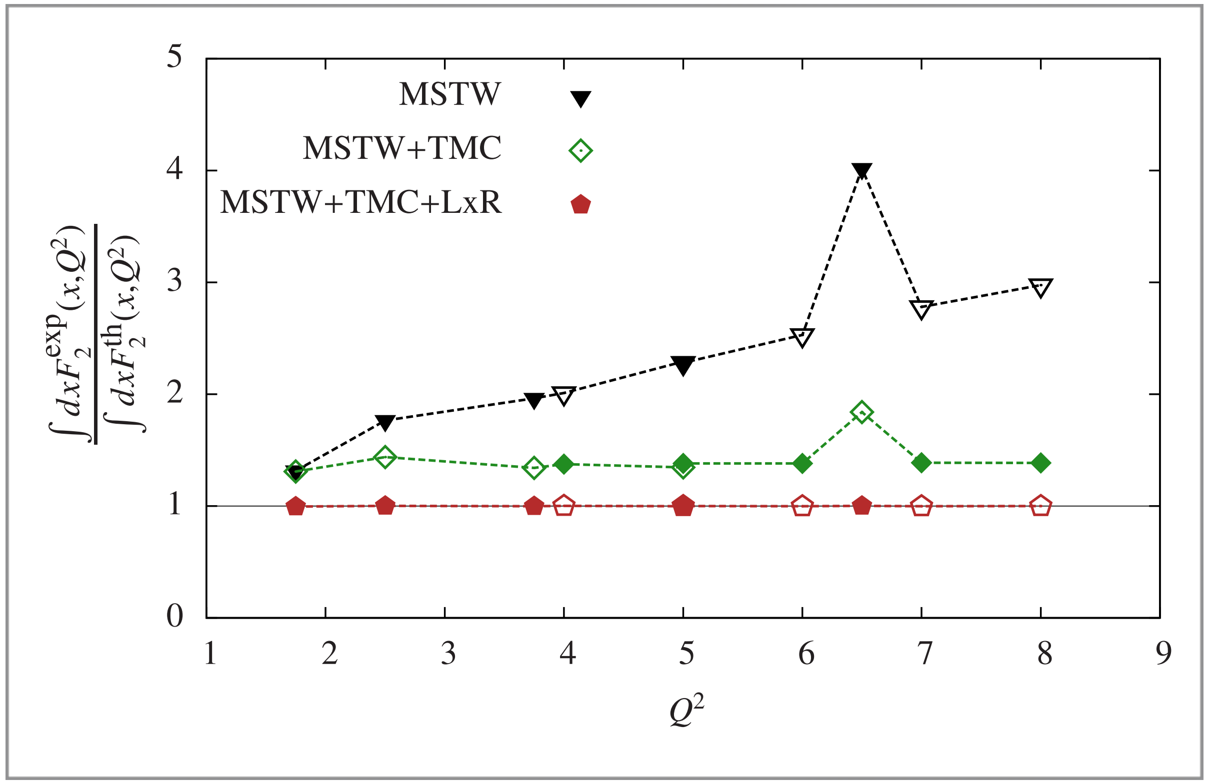

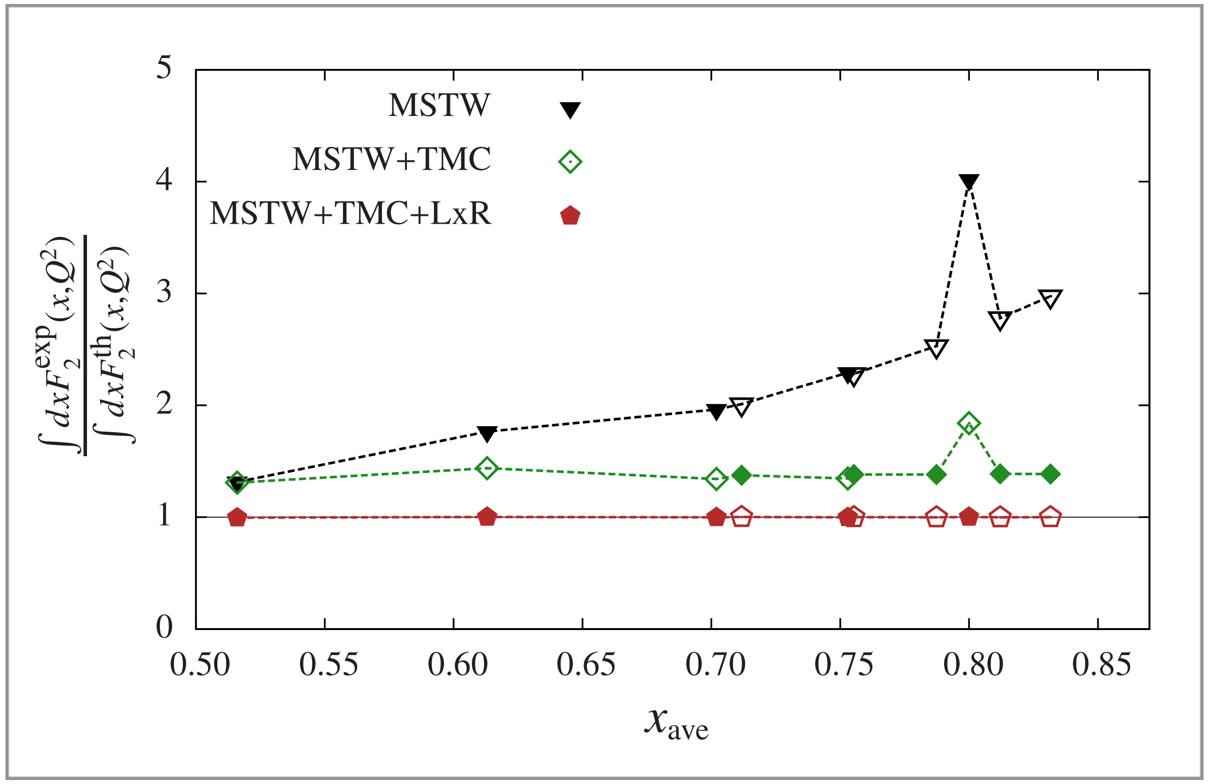

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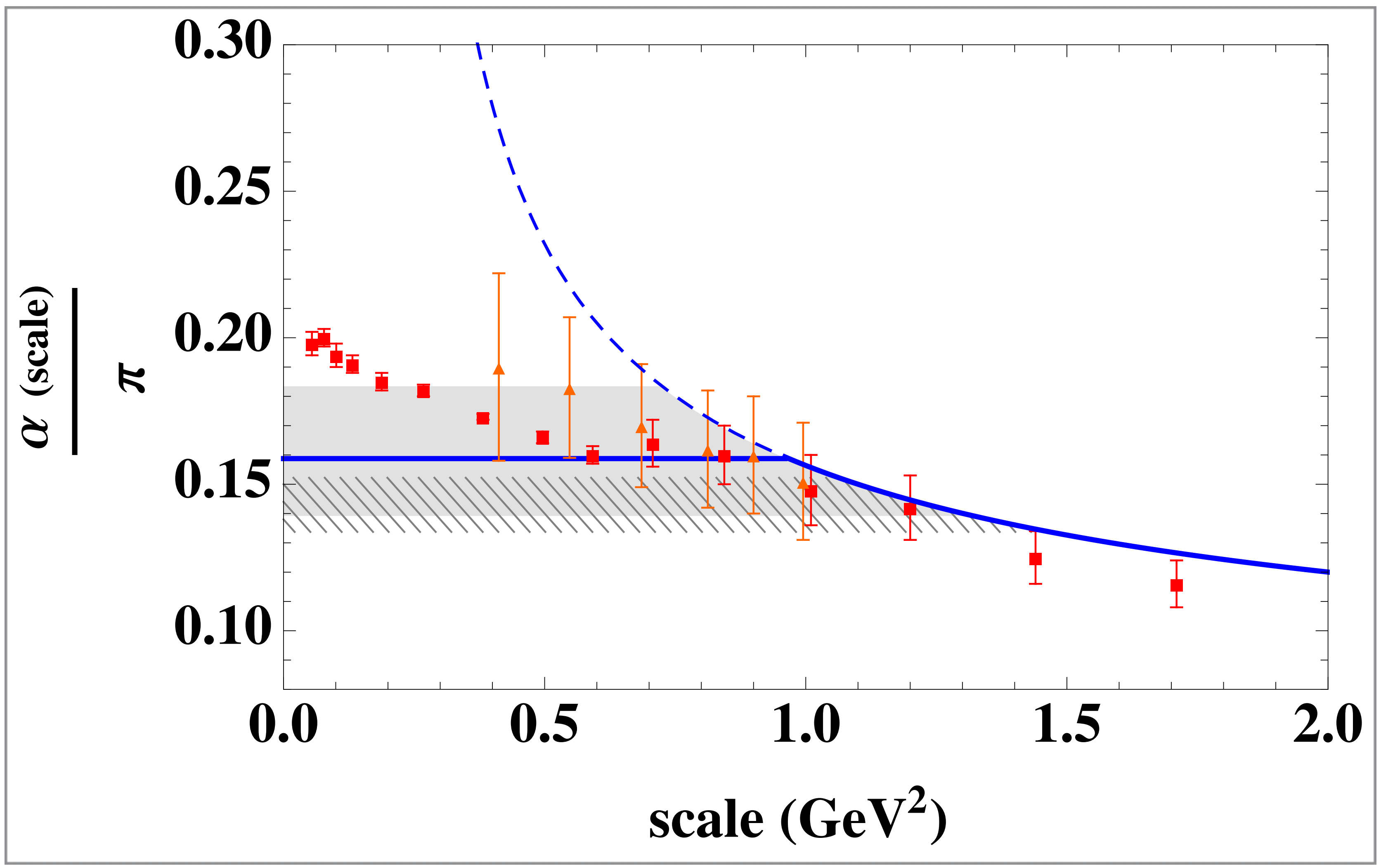
?

$$\boxed{\alpha_s\left(Q^2\frac{(1-z)}{z}\right)}$$

Effective behavior of α_s



Our extraction α_s

Exact NLO α_s

Error band z_{max} JLab

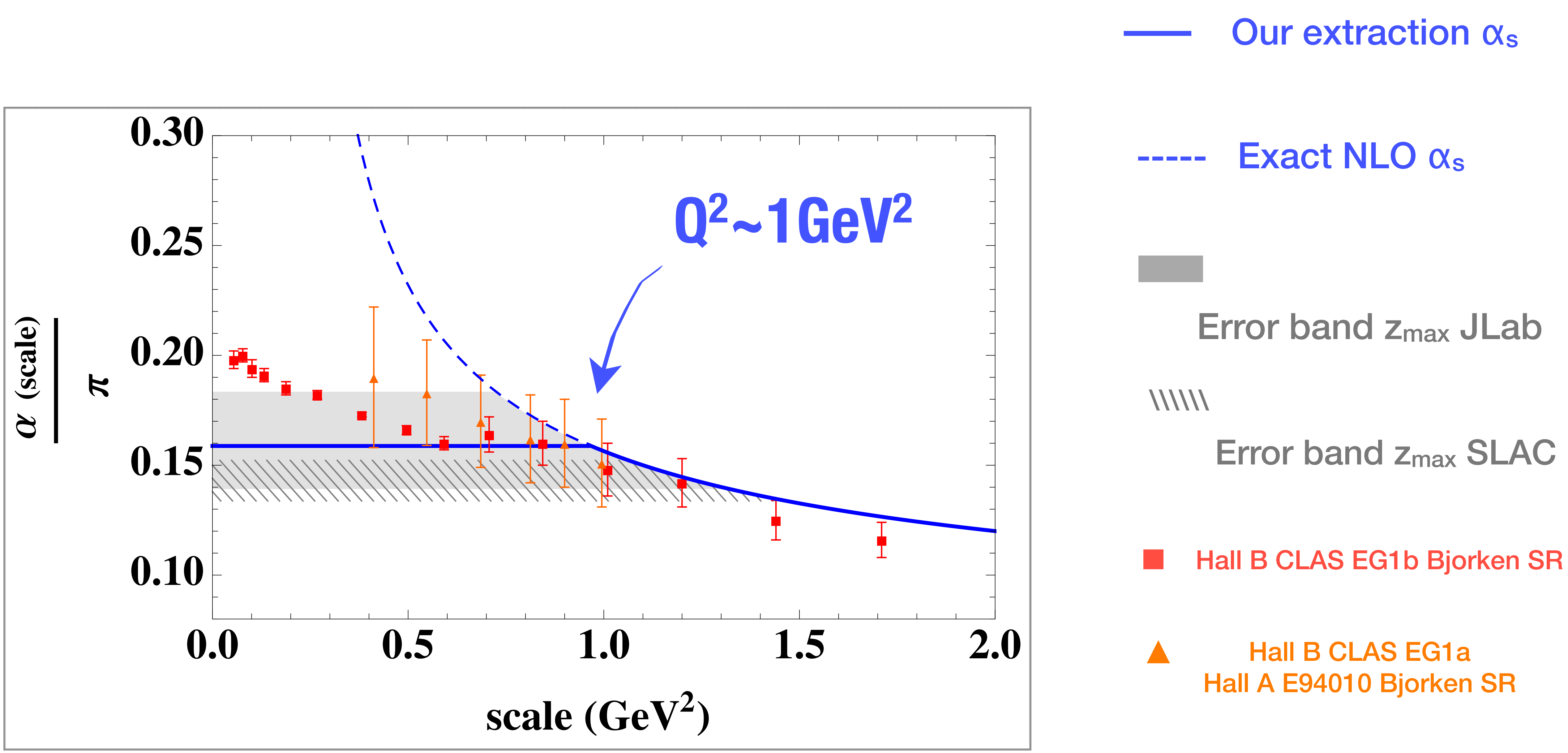
Error band z_{max} SLAC

Hall B CLAS EG1b Bjorken SR

Hall B CLAS EG1a
Hall A E94010 Bjorken SR

[Courtoy & Liuti, 1302.4439]

Effective behavior of α_s



[Courtoy & Liuti, 1302.4439]

Possible twist-3 effects

- Higher twist effects are expect to dominate at $x \rightarrow 1$
- de Rújula et al: Duality means suppression of higher-twist
- Intricate rôle of higher-twist at the frontier with NP QCD

  → compatibility with confinement?

- Here: all the nonperturbative effects into α_s

  → smooth transition from perturbative to nonperturbative physics

Conclusions and more

- Analyzis of the Bloom-Gilman duality in perturbative QCD
- Parametrized by the freezing of the running coupling constant
- $\alpha_s\ (Q^2 < 1 GeV^2)/\pi = 0.1588$

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Go deeper into the Qualitative analysis

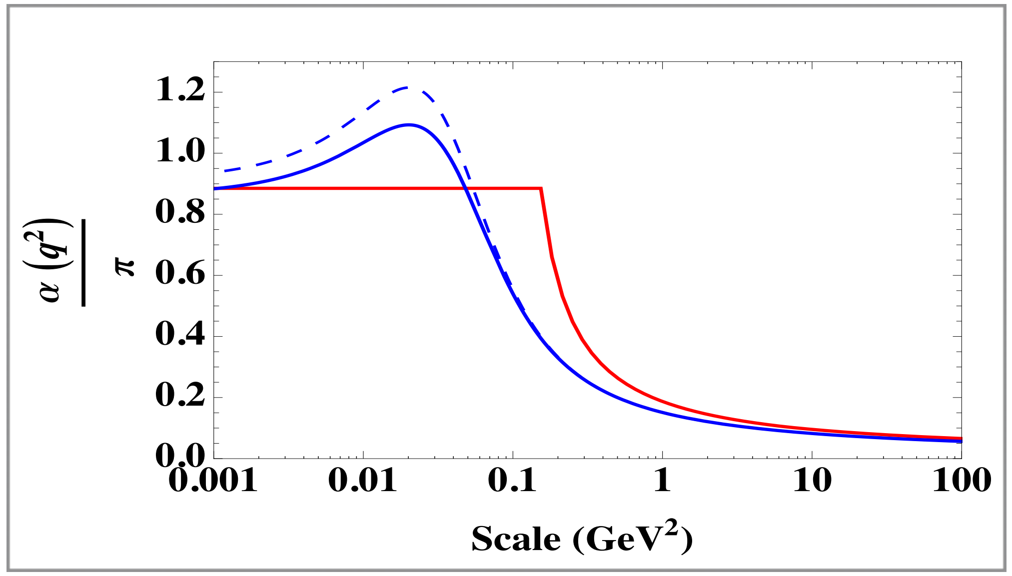
- from pQCD: systematic study of all input PDF sets
  - Self Organizing Map analysis of PDF with/without Large-x physics
  - UVa: S. Liuti and E. Askanazi and D. Day
- from NP QCD: systematic study of different approaches to effective charge

Nonperturbative QCD coupling from Phenomenology

Joint analysis: Chen, Courtoy, Deur, Liuti & Vento

Nonperturbative Coupling Constant & LxR

How we go further : Nonperturbative Coupling Constant from DSE



Cornwall α_s^{NP}

3-4 free parameters

(up to physical constrains)

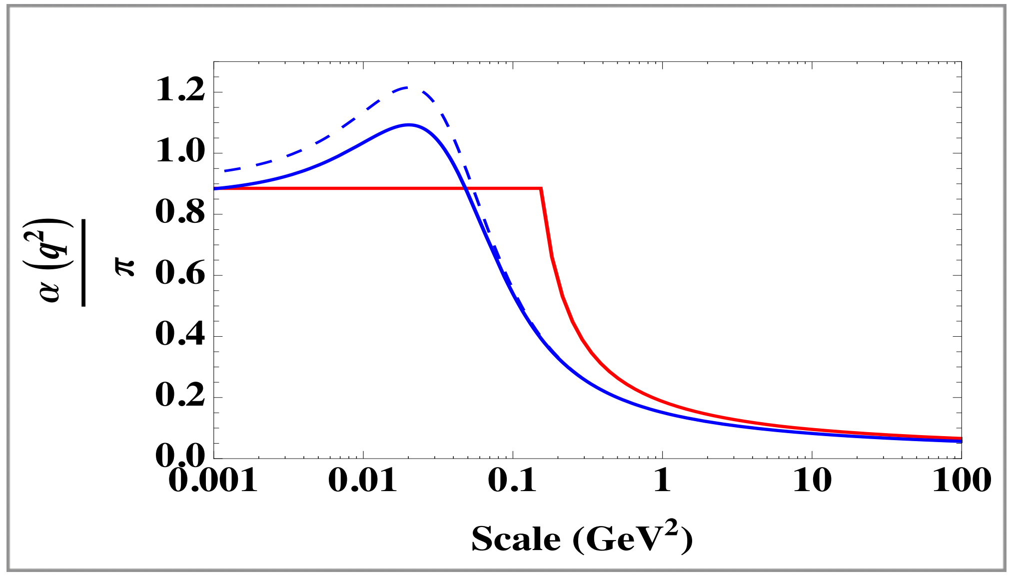
- Nonperturbative effects gathered in effective coupling α_s^{NP}
- Use of NP running coupling that scales to LO pQCD result
- Include in LxR

[Courtoy, Liuti & Vento, in progress]

- Apply with Shirkov and Fischer effective coupling as well

Nonperturbative Coupling Constant & LxR

How we go further : Nonperturbative Coupling Constant from DSE



Cornwall α_s^{NP}

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(up to physical constrains)

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[Courtoy, Liuti & Vento, in progress]

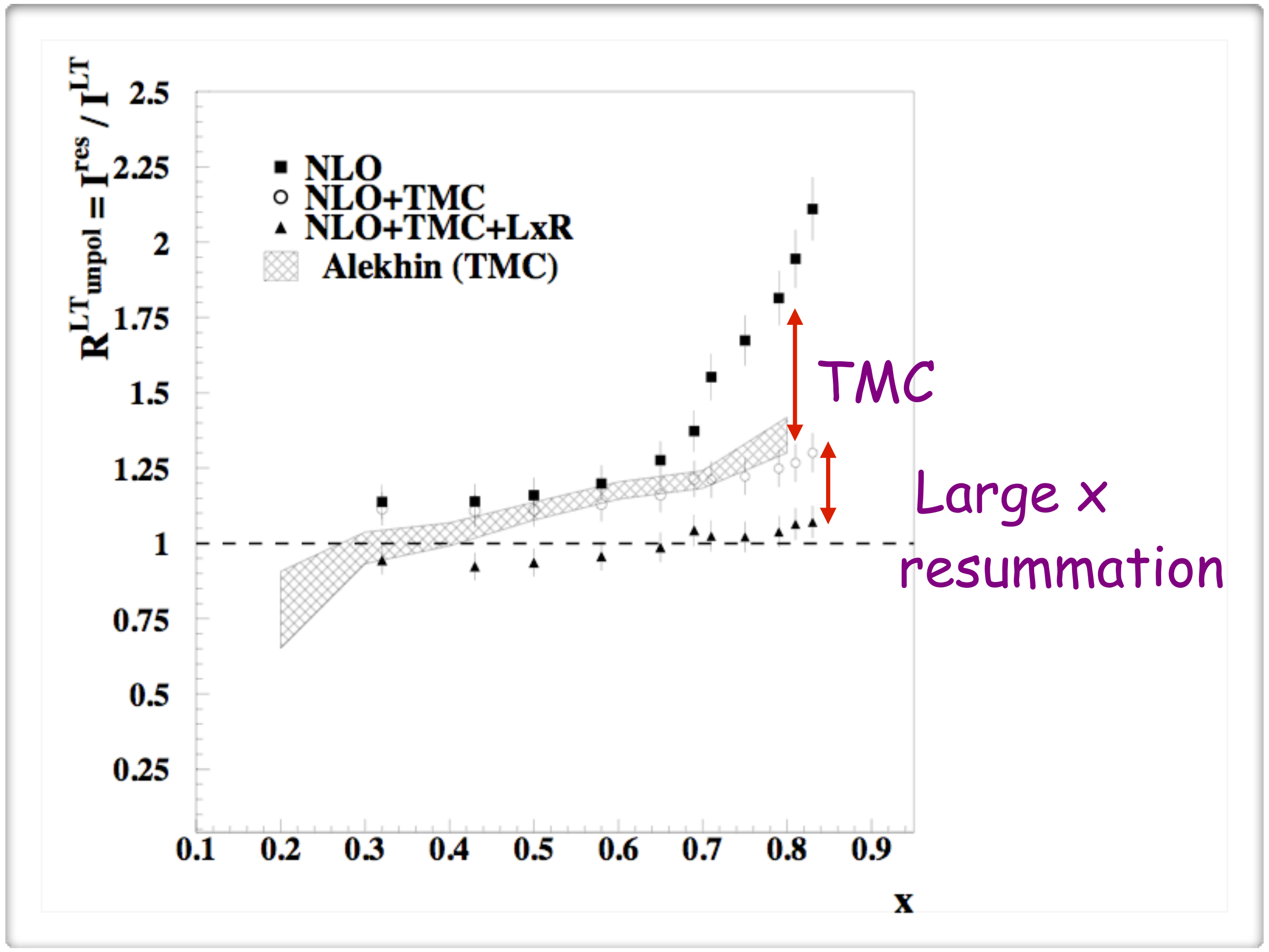
- Apply with Shirkov and Fischer effective coupling as well

How to relate the coupling constant?

- **Commensurate Scale Relations?** [Brodsky & Lu, Phys. Rev. D251]
- **RG-improved perturbation theory?** [Grunberg, Phys. Rev. D29]

Backup

Size of Nonperturbative Contributions

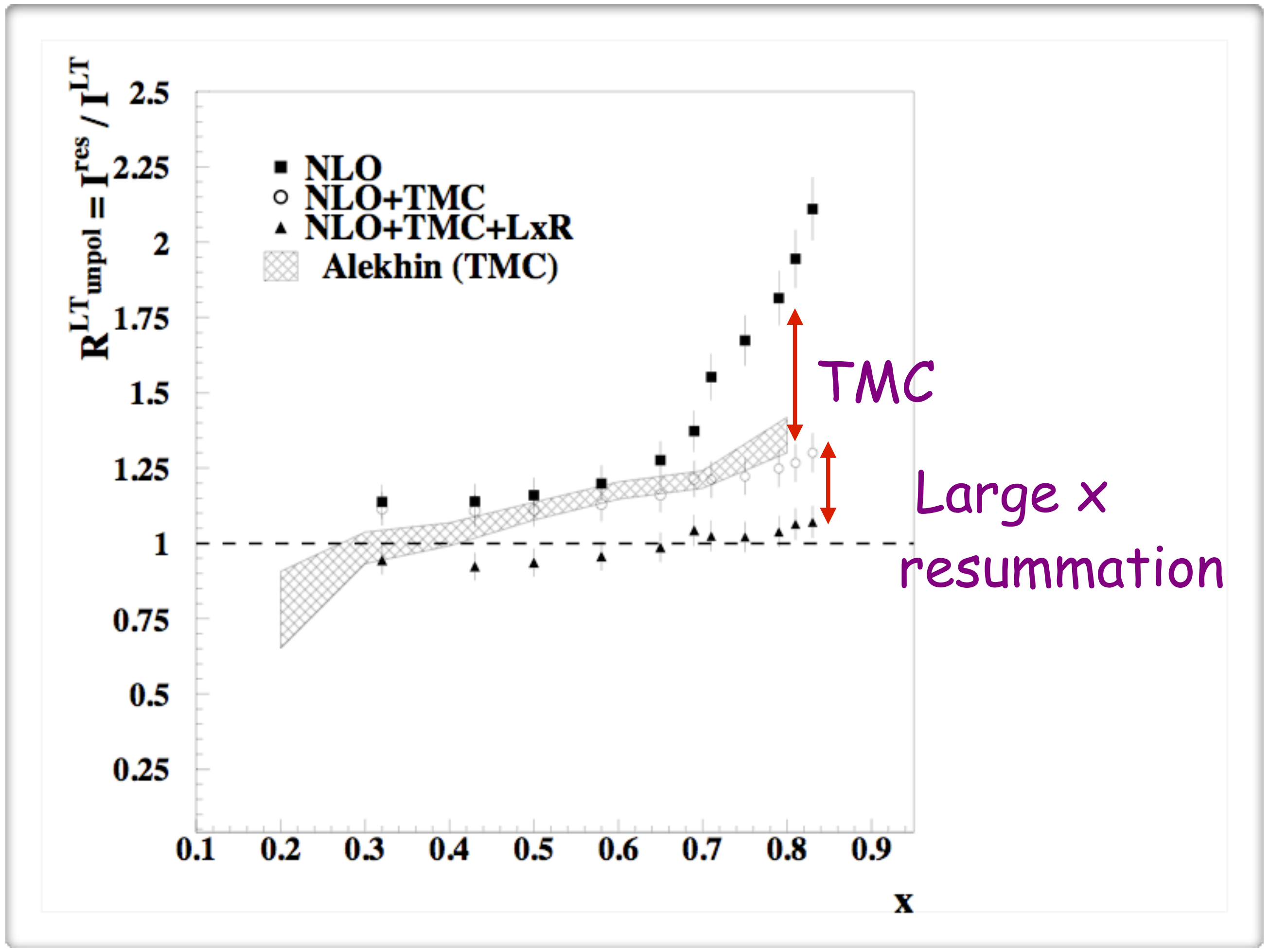


[Niculescu et al., PRD60]

[Bianchi, Fantoni & Liuti, PRD69]

$$R \equiv \frac{I^{\mathrm{res}}(Q^2)}{I^{\mathrm{DIS}}(Q^2)} \quad \Leftrightarrow \quad \text{Duality fulfilled if R=1}$$

Size of Nonperturbative Contributions



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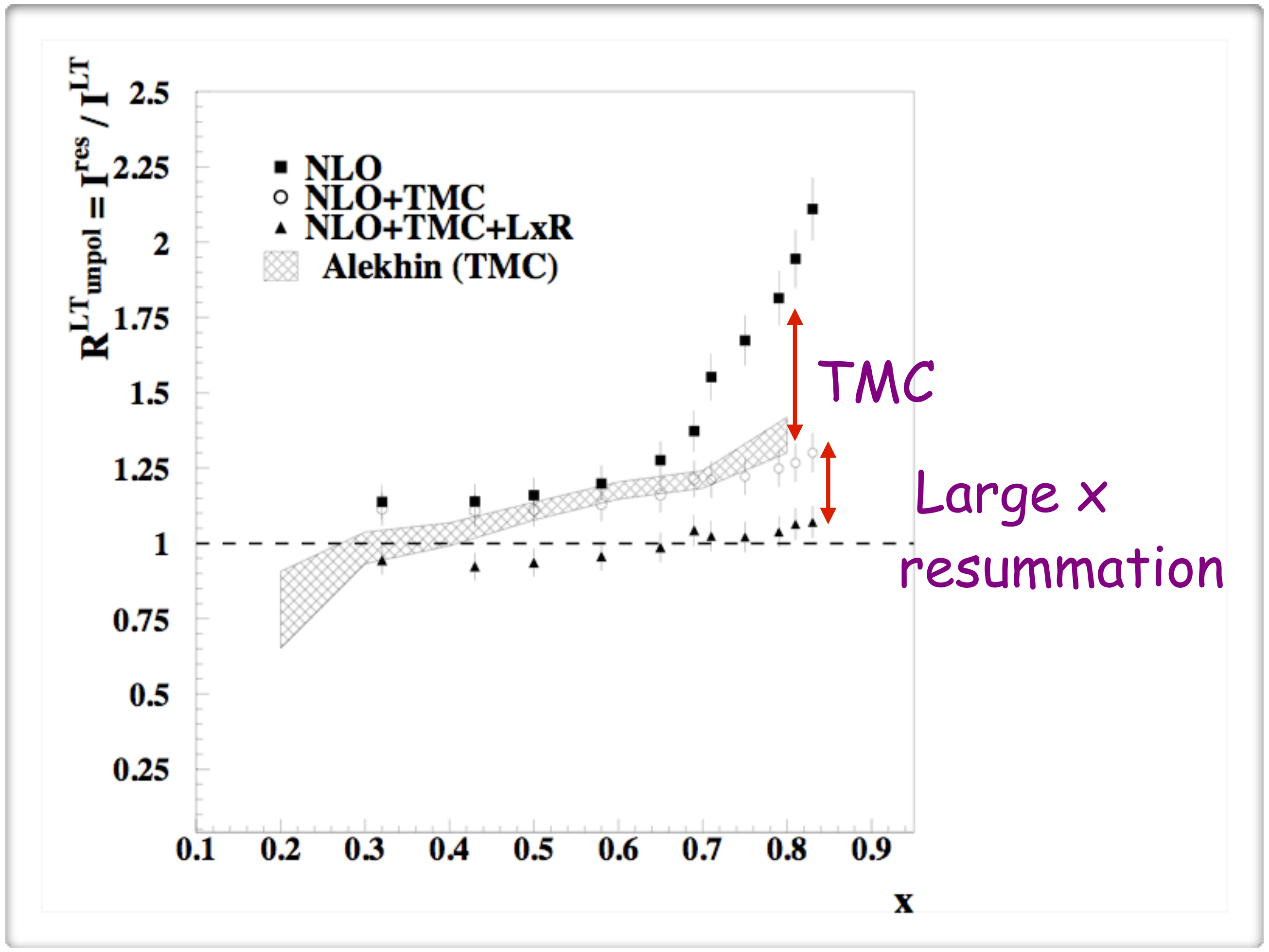

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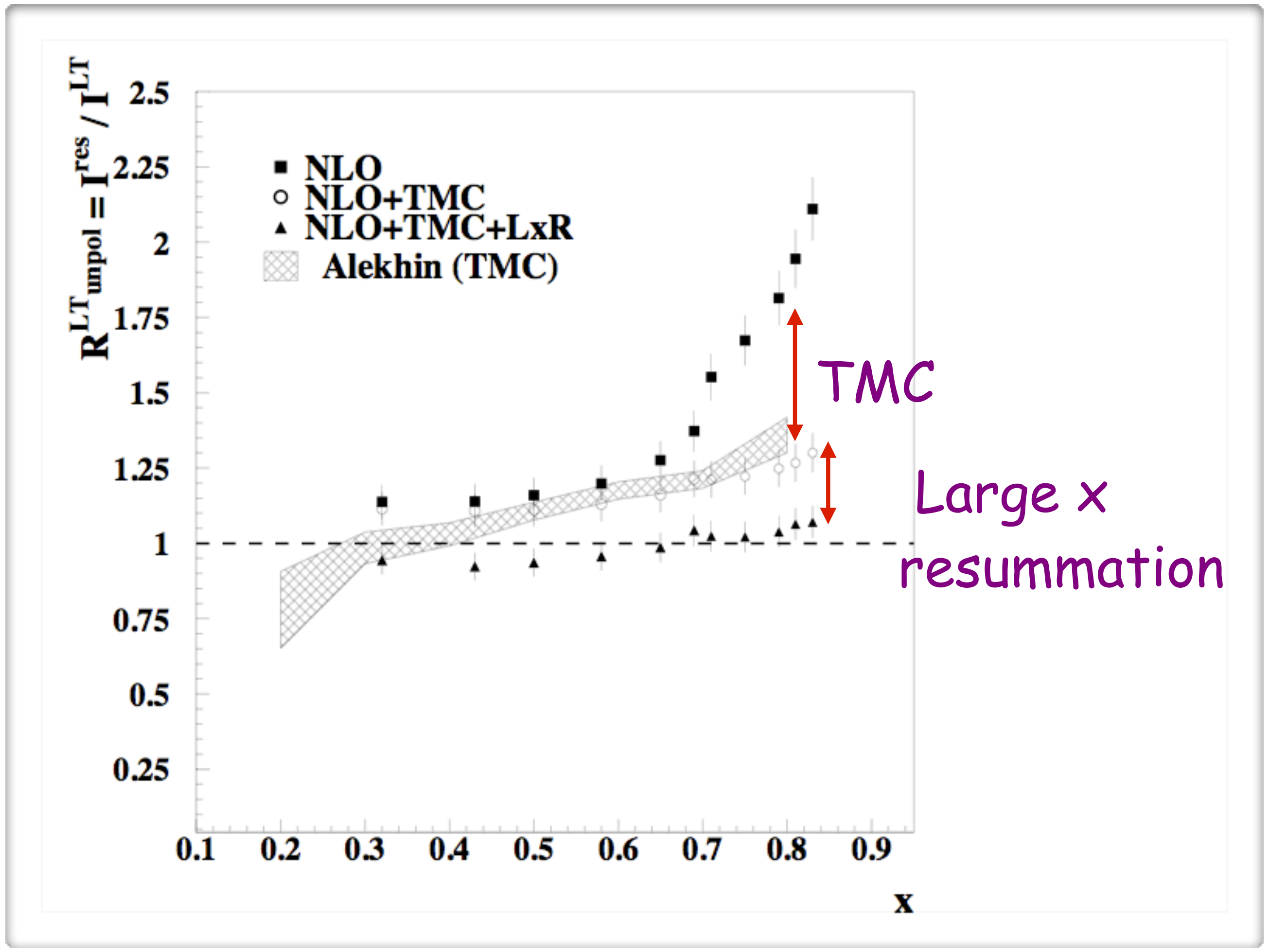
Size of Nonperturbative Contributions



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Jefferson Lab

New JLab data has been analyzed **(P. Monaghan)**

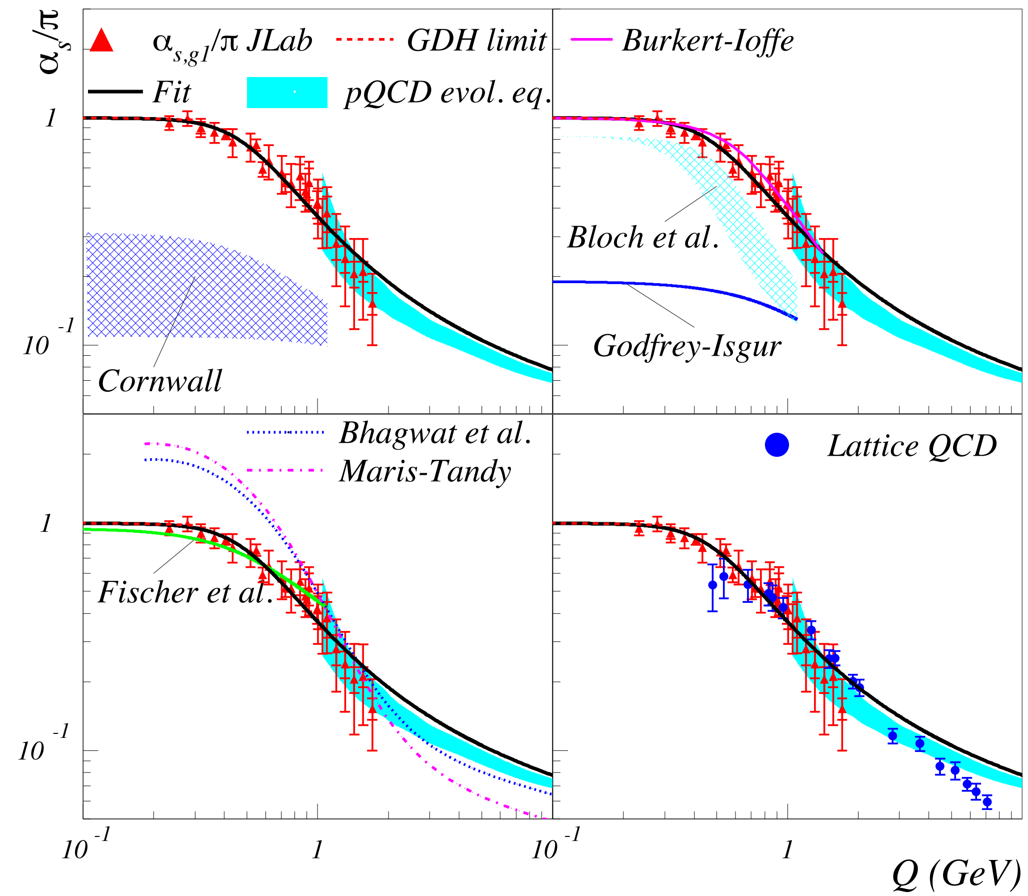


FIGURE 5. The effective coupling α_{s,g_1} extracted from JLab data, its fit, and its extraction using the Burkert and Ioffe [24] model to obtain Γ_1^{p-n}. The α_s calculations are: Top left: Schwinger-Dyson equations (Cornwall [35]); Top right: Schwinger-Dyson equations (Bloch) [36] and α_s used in a quark constituent model [37]; Bottom left: Schwinger-Dyson equations (Maris-Tandy [38]), Fischer, Alkofer, Reinhardt and Von Smekal [39] and Bhagwat et al. [40]; Bottom right: Lattice QCD [41].

LxR

The functional form $\ln_{\text{LxR}}$ is therefore slightly changed. Two distinct regions can be studied: the "running" behavior in $x < z < z_{\max}$ and the "steady" behavior $z_{\max} < z < 1$,

$$F_2^{NS,\text{Resum}}(x, z_{\max}, Q^2) \;=\; xq(x,Q^2) + \frac{\alpha_s}{4\pi}\sum_q \left\{ \int_x^1 dz\, \left[B_{\text{NS}}^q(z) - \hat{P}_{qq}^{(0)}(z)\, \ln(1-z) \right] \right.$$
$$\left. + \int_x^{z_{\max}} dz\, \hat{P}_{qq}^{(0)}(z)\, \ln_{\text{LxR}} + \ln_{\text{LxR, max}} \int_{z_{\max}}^1 dz\, \hat{P}_{qq}^{(0)}(z) \right\} \frac{x}{z}\, q\left(\frac{x}{z}, Q^2\right).$$

Extraction of α_s at low energy

- Polarized scattering from both proton and neutron

Deur et al. Phys.Lett. B650 (2007) 244-248

Natale, PoS QCD-TNT09 (2009) 031

Bjorken Sum Rule from JLab & GDH Sum Rule at Q^2=0 GeV^2

- Deep Inelastic Scattering (DIS) at large Bjorken-x & parton-hadron duality

Liuti, [arXiv:1101.5303 [hep-ph]].

- Semi-Inclusive DIS & Extraction of T-odd TMDs from SSAs

A.C., Vento & Scopetta, Eur. Phys. J. A47, 49 (2011)

Joint analysis: Chen, Courtoy, Deur, Liuti & Vento

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Nonperturbative Gluon Propagator

Solving the Schwinger-Dyson eqs ...

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)
A. C. Aguilar and J. Papavassiliou, JHEP0612, 012 (2006)

$$\boxed{\Delta^{-1}(Q^2) = Q^2 + m^2(Q^2)}$$

$$\boxed{m^2(Q^2) = m_0^2 \left[\ln\left(\frac{Q^2+\rho m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho m_0^2}{\Lambda^2}\right)\right]^{-1-\gamma}}$$

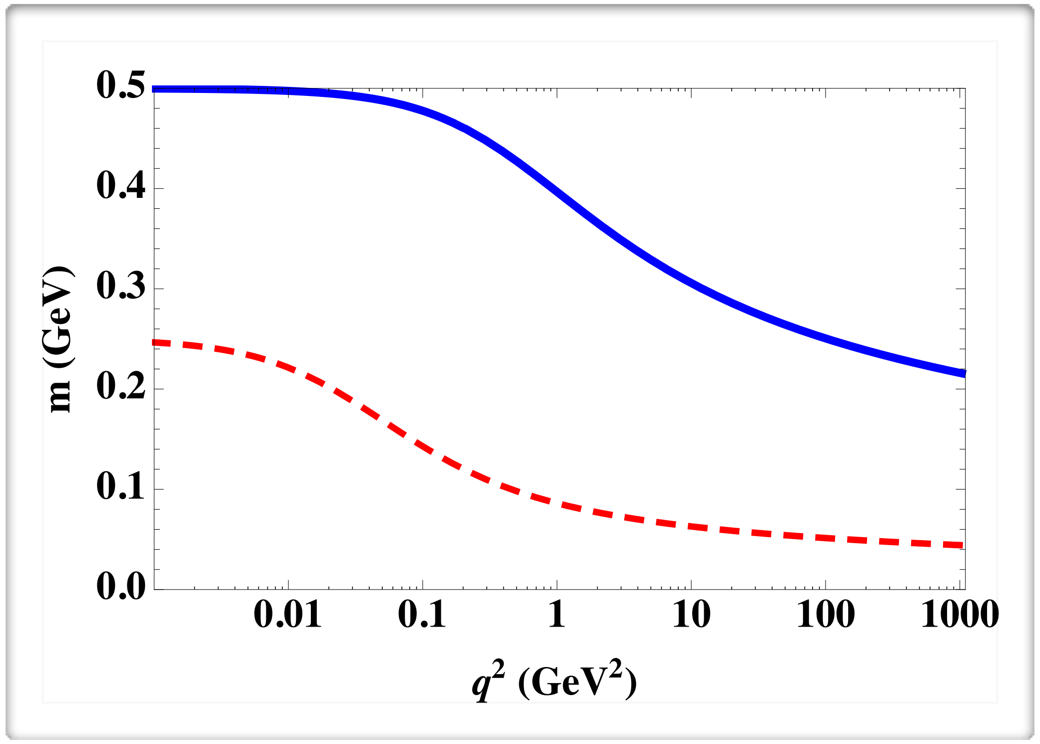
Gluon Mass as IR Regulator

- **effective gluon mass**

phenomenological estimates

$$\boxed{m_0 \sim \Lambda - 2\Lambda}$$

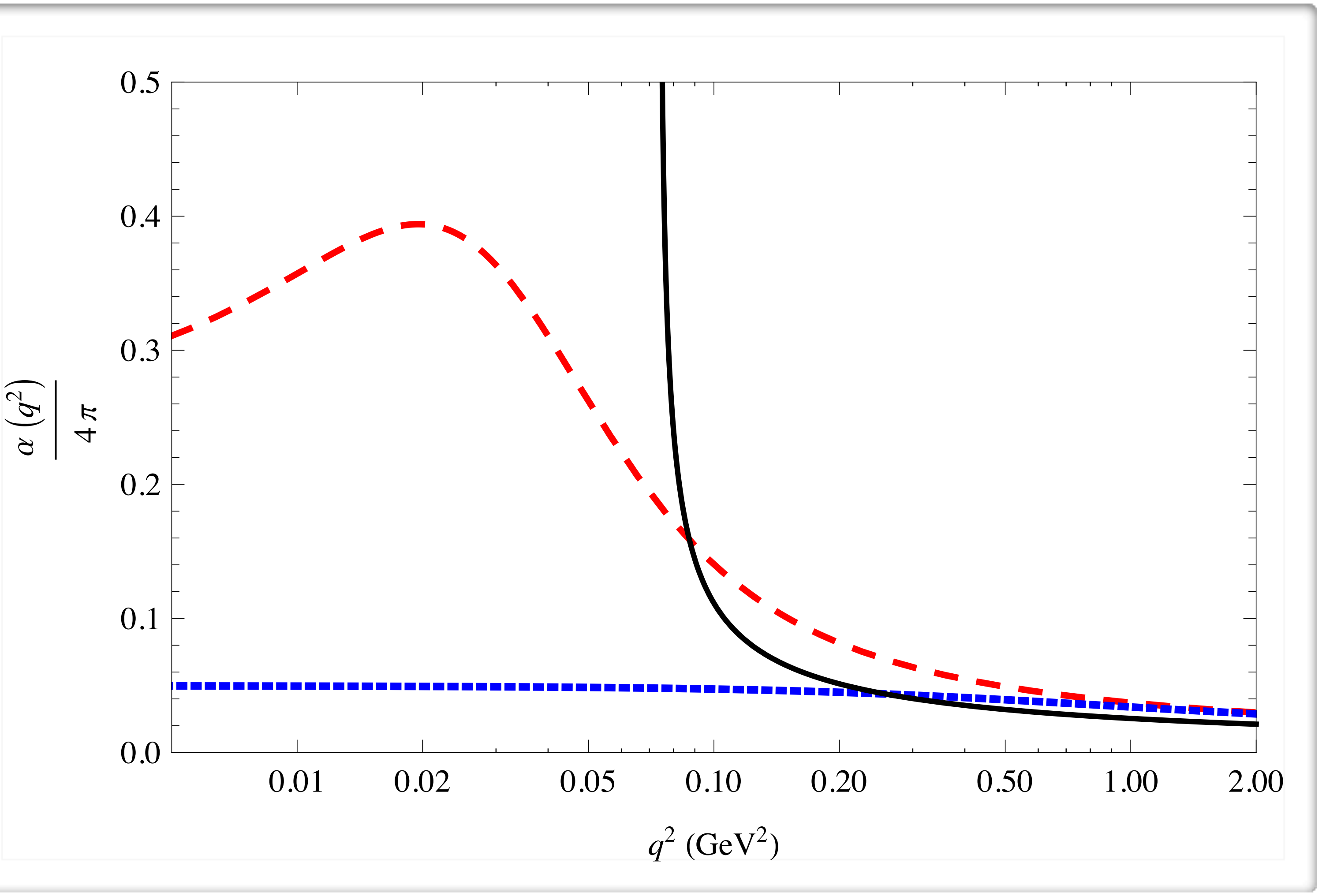
- **Solution free of Landau pole**
- **Freezes in the IR**



Low mass scenario
High mass scenario

NP Momentum-dependence of the Coupling Constant

$$\frac{\alpha_{\rm NP}(Q^2)}{4\pi} = \left[\beta_0 \ln\left(\frac{Q^2 + \rho m^2(Q^2)}{\Lambda^2}\right)\right]^{-1}$$



NLO exact perturbative evolution
Λ=250 MeV ; $\overline{MS}$ scheme

Low mass scenario NP coupling constant
m_0=250 MeV ; Λ=250 MeV ; ρ=1.5

High mass scenario NP coupling constant
m_0=500 MeV ; Λ=250 MeV ; ρ=2.