Continuity of the Deconfinement Transition in (Super) Yang Mills Theory

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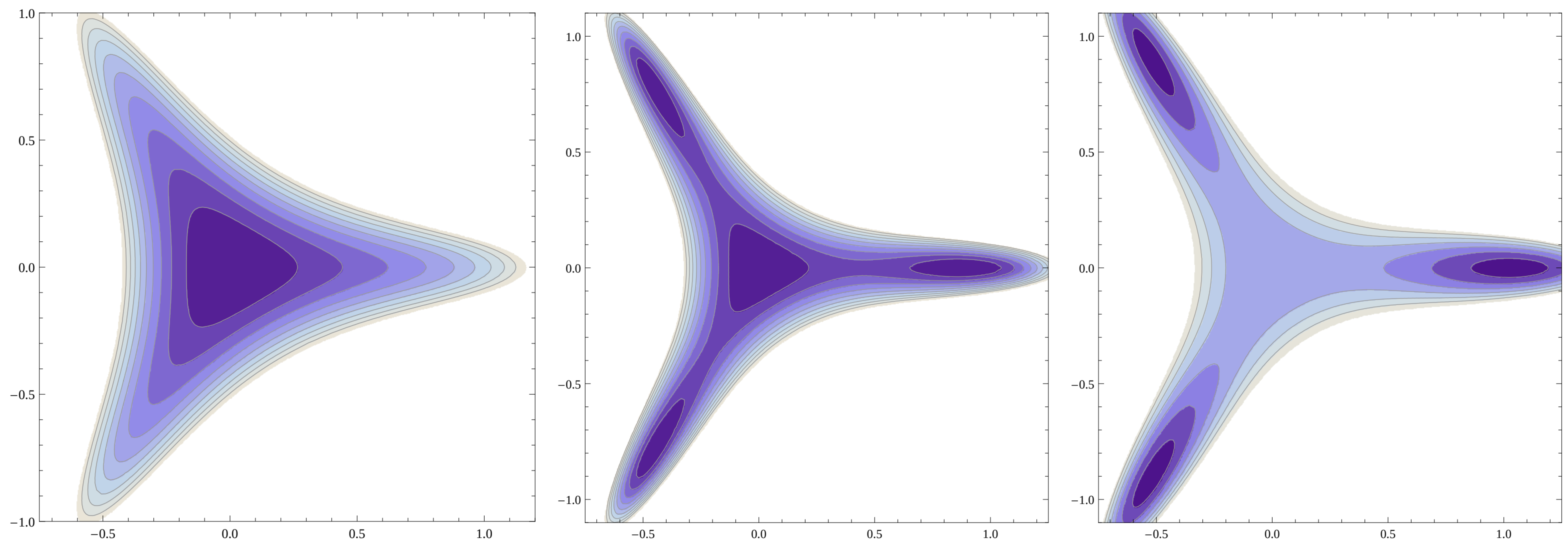
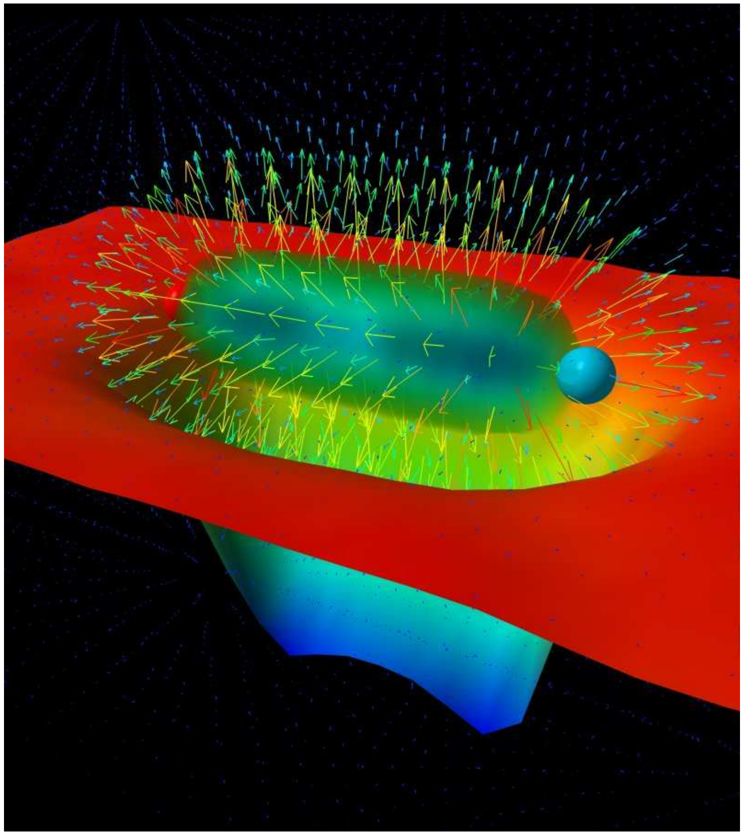
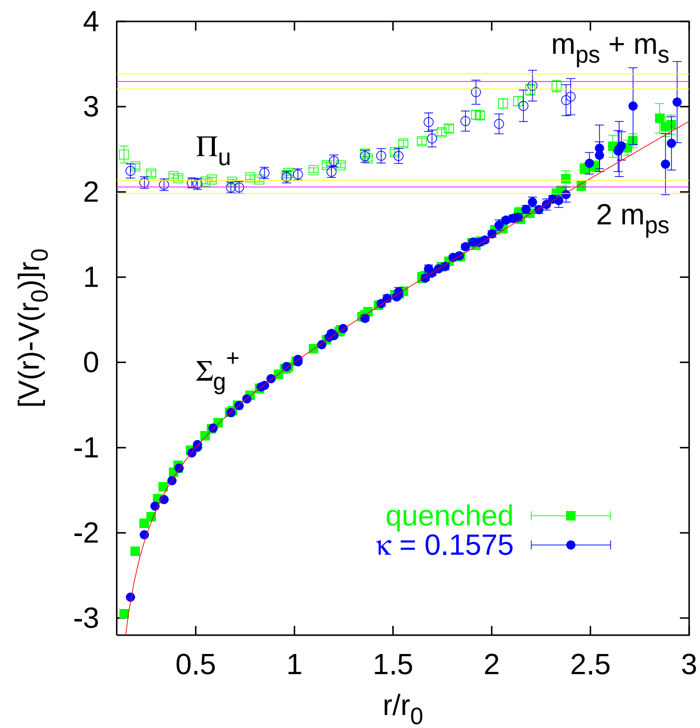


with Mithat Ünsal and Erich Poppitz

Confinement and the QCD string



Leinweber (2001)



Bali (2001)

Confinement well established numerically (and empirically)

Confinement and the QCD string

Challenge: Understand confinement analytically

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Not just a problem in pure mathematics: Understand dynamics, suggest new observables, . . .

Some successes (QCD-like gauge theories)

- Polyakov model (compact QED in 2+1)
- $\mathcal{N} = 2$ SUSY YM softly broken to $\mathcal{N} = 1$

Confinement: Goals

- Mass gap in the pure gauge theory: $m_{0^{++}}$

$$\left\langle \mathrm{Tr}[F^2(x)]\mathrm{Tr}[F^2(0)] \right\rangle \sim f^2 \exp(-m_{0^{++}} x)$$

- String tension, effective theory of the QCD string.

$$\langle W(C) \rangle = \left\langle \mathrm{Tr} \exp \left[i \int_C A^\mu dx_\mu \right] \right\rangle \sim \exp(-\sigma A(C))$$

- Polykaov line: Effective potential, correlation functions.

$$\langle \Omega(\vec{x}) \rangle = \left\langle \mathrm{Tr} \exp \left[i \int_0^\beta A_4 dx_4 \right] \right\rangle \sim 0$$

- Critical temperature, center symmetry breaking

$$\Omega \rightarrow z\Omega \qquad z \in Z_N$$

- Theta dependence, $d^2E/d\theta^2 \neq 0$.

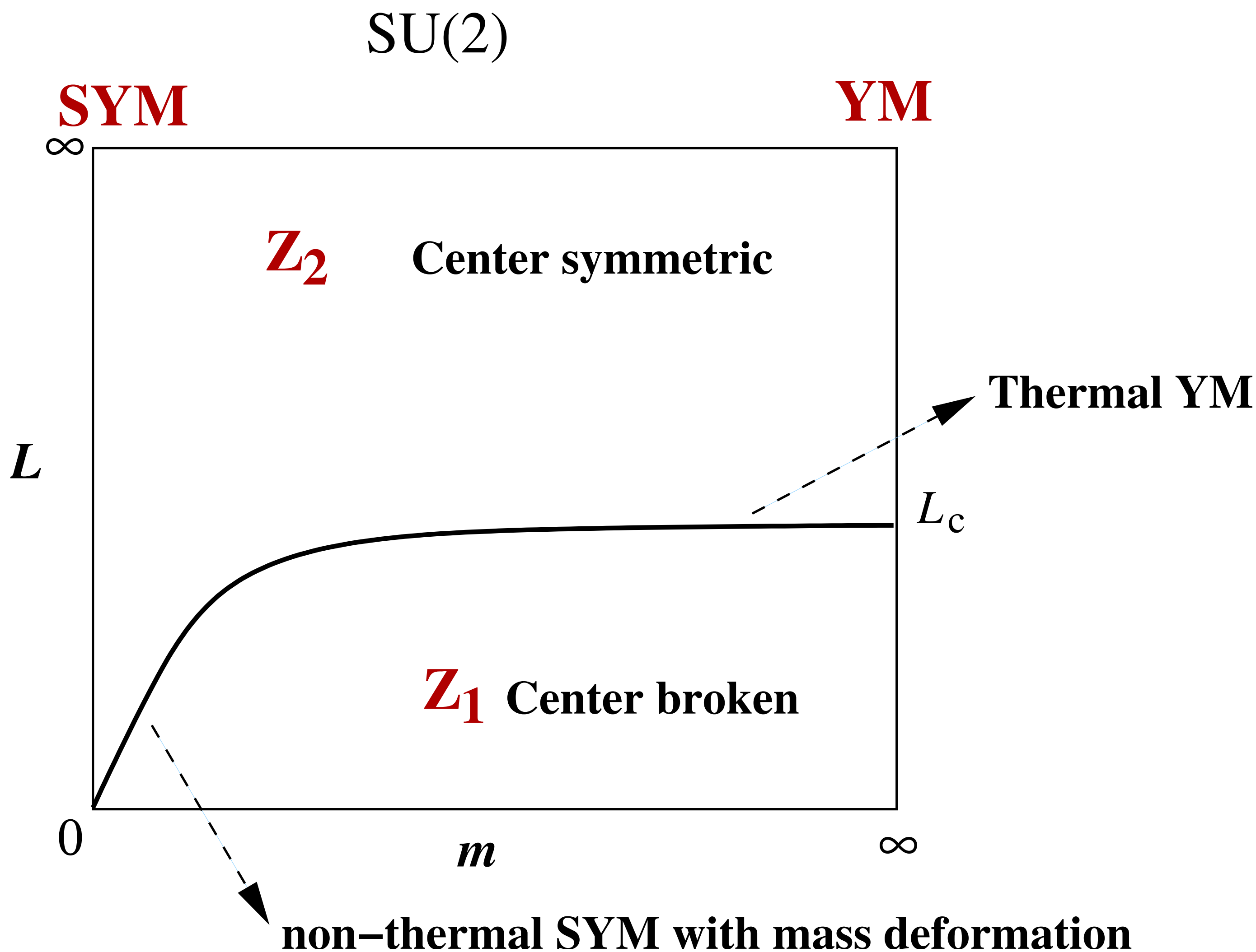
In this work we will pursue a more modest goal.

We will study confinement and the deconfinement phase transition in a non-abelian gauge theory which is weakly coupled (by using a suitable compactification).

We will argue that this theory is continuously connected (by decoupling an extra matter field) to pure gauge theory.

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L-m plane



Ingredients

- $R^3 \times S_1$ circle-compactified gauge theory.
- Small S_1: Effective 3d theory involving holonomy and (dual) photon.
- Double expansion: Perturbative and non-perturbative effects (monopoles, topological molecules).
- Topological molecules: supersymmetry versus BZJ.
- Competition: Center stabilizing molecules, center breaking perturbative (and monopole) effects.

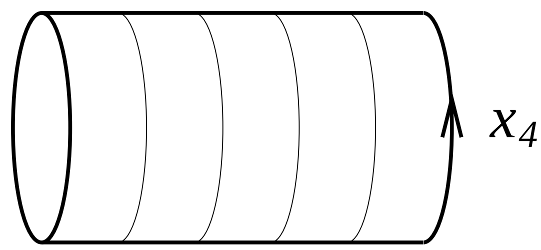
Gauge theory on $R^3 \times S_1$

SU(2) gauge theory, $n_f = 1$ adjoint Weyl fermion

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{i}{g^2}\lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2}\lambda^a\lambda^a$$

$$A^a_\mu(0) = A^a_\mu(L)$$

$$\lambda^a(0) = \lambda^a(L)$$



Vacua labeled by Polyakov line

$$\Omega = \exp\left[i \int A_4 dx_4 \right]$$

Center symmetry $\quad \Omega \to z\Omega \qquad z \in Z_2$

Small S_1: Effective Theory

Consider small S_1 and $\Omega \neq 1$: A_4 is a Higgs field, theory abelianizes.
Bosonic sector of effective 3d theory

$$\mathcal{L} = \frac{g^2}{32\pi^2 L}\left[(\partial_i b)^2 + (\partial_i \sigma)^2\right] + V(\sigma, b)$$

$$\Omega = \begin{pmatrix} e^{i\Delta\theta/2} & 0 \\ 0 & e^{-i\Delta\theta/2} \end{pmatrix} \quad b = \frac{4\pi}{g^2}\Delta\theta \qquad \epsilon_{ijk}\partial_k \sigma = \frac{4\pi L}{g^2} F_{ij}$$

holonomy b dual photon σ

Note: $m = 0$ effective theory can be super-symmetrized

$$\Phi = b + i\sigma + \sqrt{2}\theta^\alpha \lambda^\alpha$$

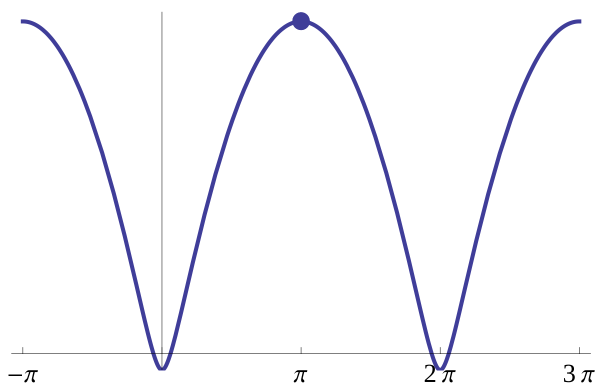
Perturbation Theory

Perturbative potential for holonomy (Gross, Pisarski, Yaffe, 1981)

$$V(\Omega) = -\frac{m^2}{2\pi^2 L^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \, |\mathrm{tr}\, \Omega^n|^2 = -\frac{m^2}{L^2} B_2\left(\frac{\Delta\theta}{2\pi}\right)$$

$m = 0$: Bosonic and fermionic terms cancel.

$m \neq 0$: Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ unstable.



Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x \, F\tilde{F}$$

2. Holonomy (eigenvalues q^α of Polyakov line at spatial infinity)

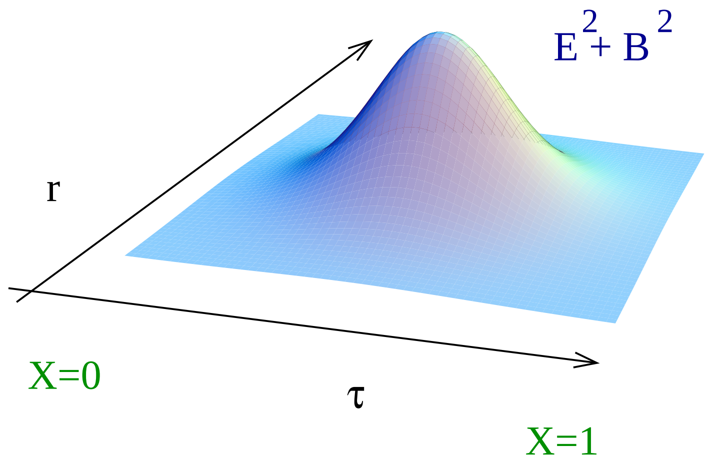
$$\langle \Omega(\vec{x}) \rangle = \left\langle \mathrm{Tr} \exp \left[i \int_0^\beta A_4 dx_4 \right] \right\rangle$$

3. Magnetic charges

$$Q_M^\alpha = \frac{1}{4\pi} \int d^2S \, \mathrm{Tr} \left[P^\alpha B \right]$$

Periodic instantons (calorons)

Instanton solution in R^4 can be extended to solution on $R^3 \times S^1$



$$Q_{top} = \pm 1$$

$$P_\infty = 1 \quad Q_M^\alpha = 0$$

$SU(2)$ solution has $1+3+1+3=8$ bosonic zero modes

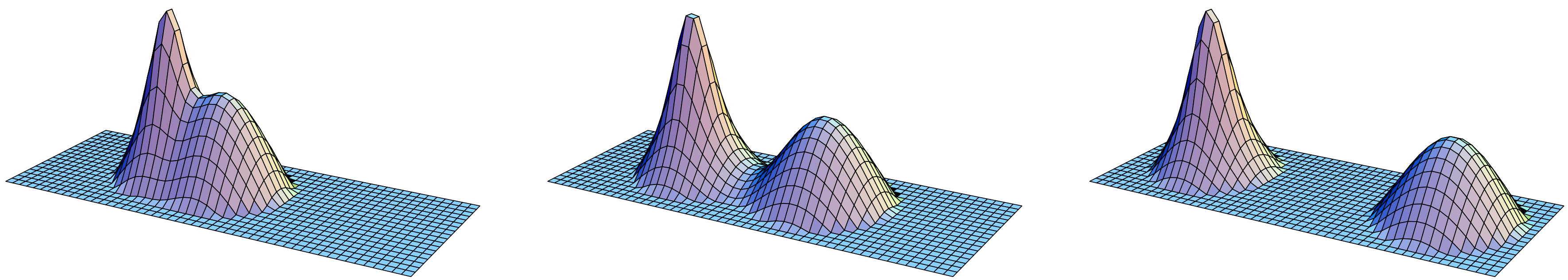
$$\int \frac{d\rho}{\rho^5} \int d^3x \, dx_4 \int dU \, e^{-2S_0} \qquad 2S_0 = \frac{8\pi^2}{g^2}$$

$4n_{adj}$ fermionic zero modes

$$\int d^2\zeta d^2\xi$$

Calorons at finite holonomy: monopole constituents

KvBLL (1998) construct calorons with non-trivial holonomy



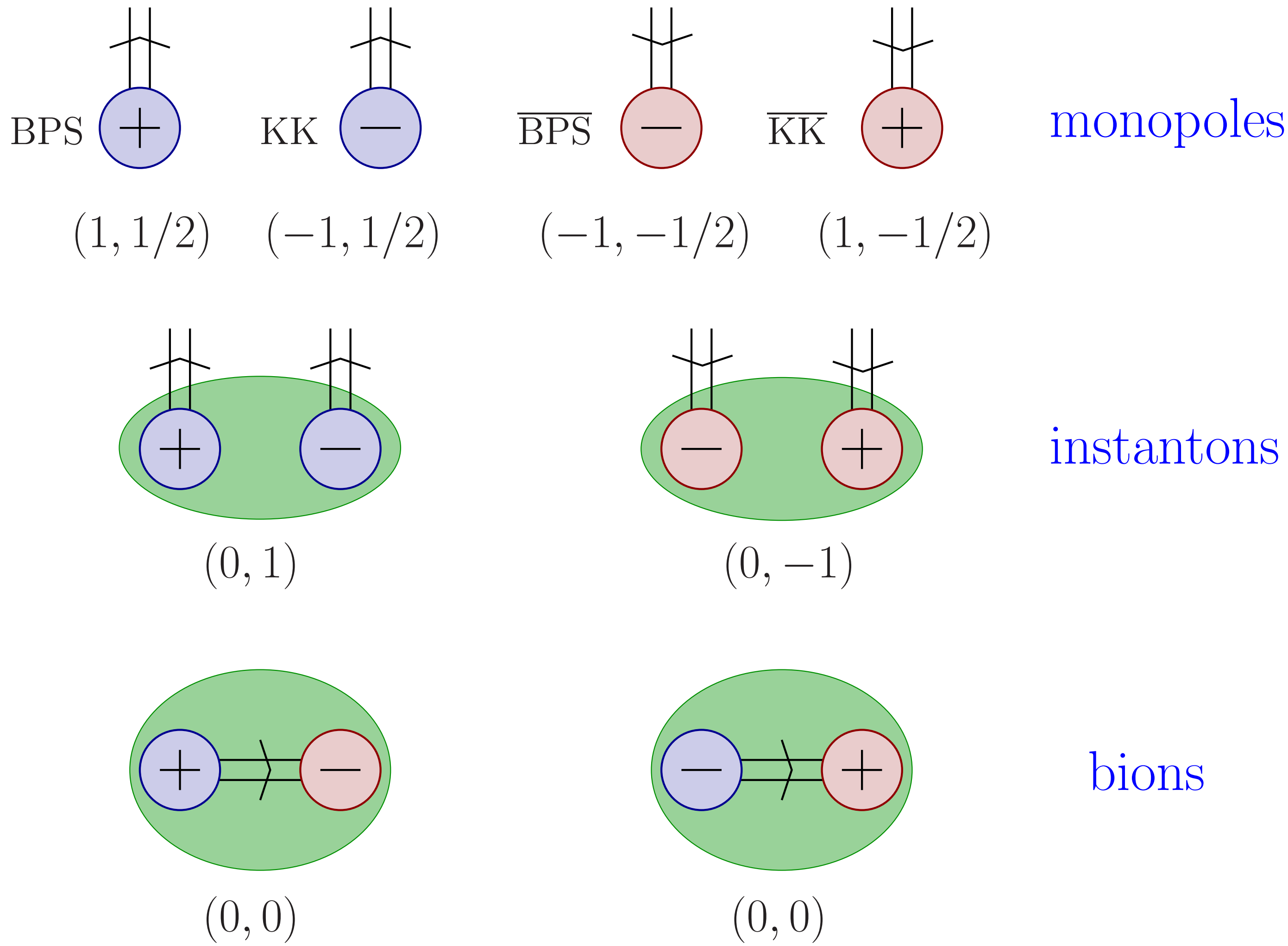
BPS and KK monopole constituents. Fractional topological charge, $1/2$ at center symmetric point.

$2 \times (3+1) = 8$ bosonic zero modes, 2×2 fermionic ZM.

$$\int d\phi_1 \int d^3x_1 \int d^2\zeta \, e^{-S_1} \int d\phi_2 \int d^3x_2 \int d^2\xi \, e^{-S_2}$$

Topological objects

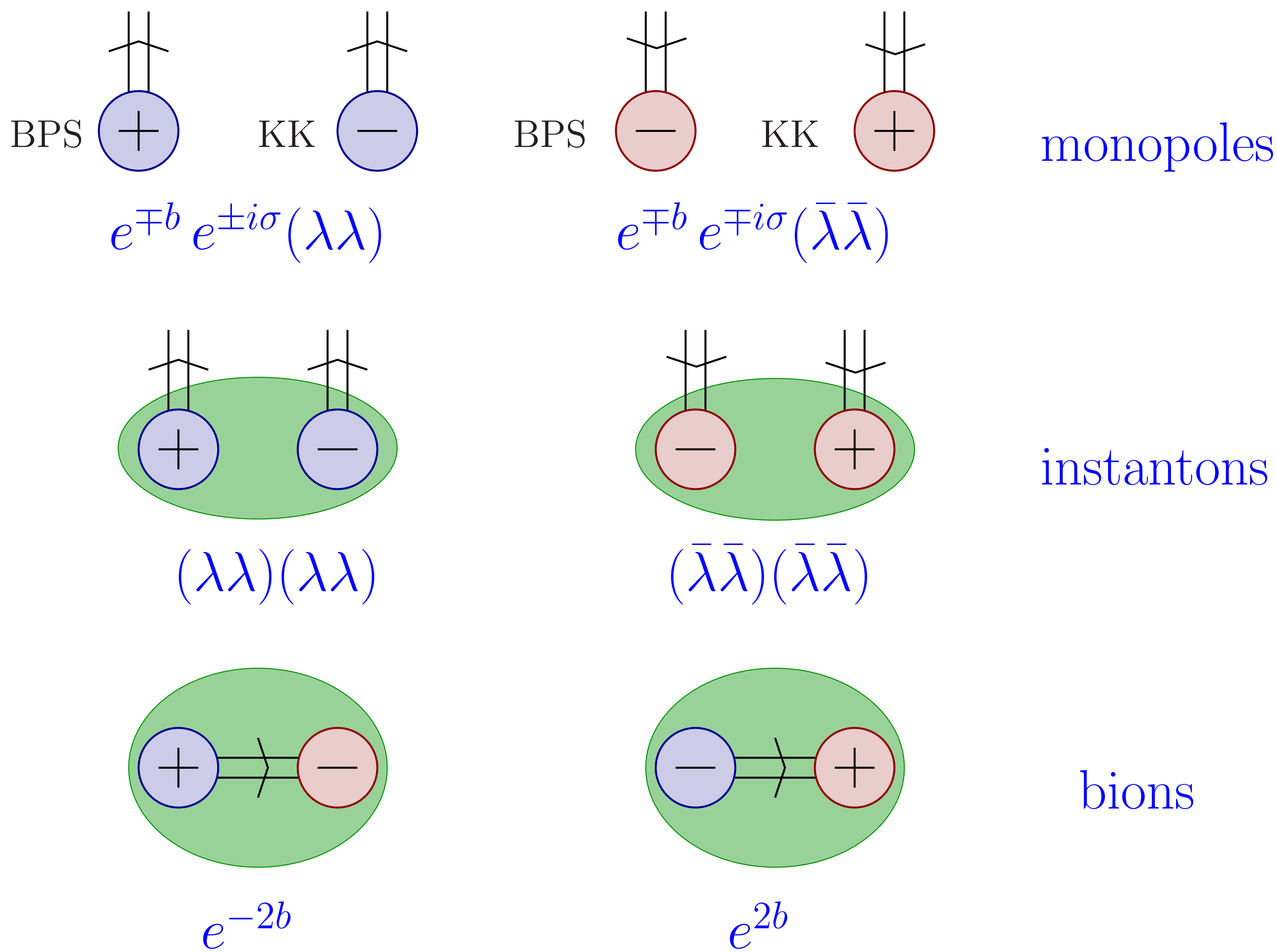
$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

Topological objects: Coupling to low energy fields

$$(Q_M, Q_{top}) = \left(\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F}\right)$$



Kraan, van Baal (1998); Lee, Lu (1998); Unsal (2008)

Non-perturbative effects at $m = 0$ from supersymmetry

Monopoles contribute to superpotential: $(\lambda\lambda)e^{-b+i\sigma} \sim \int d^2\theta e^{-\Phi}$

$$\mathcal{W} = \frac{M_{PV}^3 L}{g^2}\left(e^{-b} + e^{-2S_0}e^{b}\right)$$

Scalar potential

$$V(b,\sigma) \sim \left|\frac{\partial \mathcal{W}}{\partial \Phi}\right|^2 \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6}\left[\cosh\left(\frac{8\pi}{g^2}\left(\Delta\theta - \pi\right)\right) - \cos(2\sigma)\right]$$

Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\rightarrow$ confinement)

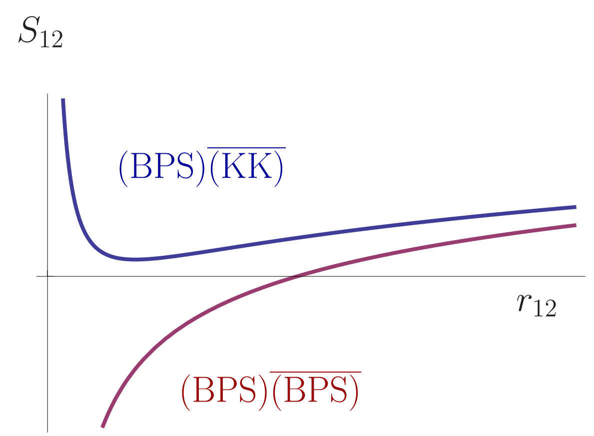
Davies, Hollowood, Khoze, Mattis (1999)

Non-perturbative effects at $m = 0$ from BZJ

Consider magnetically neutral topological molecules. Integrate over near zero-mode:

$$V_{BPS,\overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r\, e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r}(q_m^1 q_m^2 - q_b^1 q_b^2) + 4\log(r)$$



Saddle point integral after analytic continuation $g^2 \to -g^2$ (BZJ)

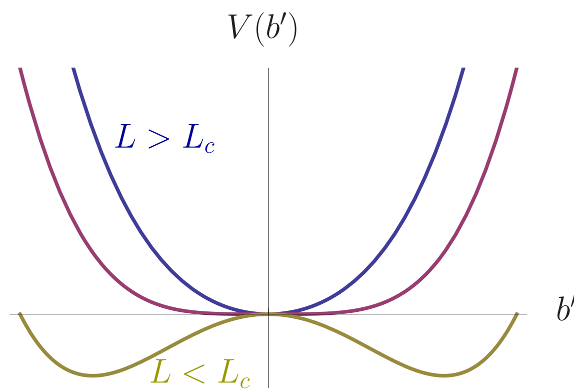
$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \cosh\left(\frac{8\pi}{g^2}(\Delta\theta - \pi)\right)$$

Same for magnetically charged molecules: $V \sim \cos(2\sigma)$.

Effective potential for $m \neq 0$

Effective potential: molecules, monopoles, perturbation theory

$$
\begin{aligned}
\tilde{V} &= \cosh 2b' - \cos 2\sigma \\
&+ \quad \frac{\tilde{m}}{2\tilde{L}^2} \cos\sigma \left(\cosh b' - \frac{b' \sinh b'}{3 \log \tilde{L}^{-1}} \right) \\
&- \quad \frac{1}{1728} \left(\frac{\tilde{m}}{\tilde{L}^2} \right)^2 \frac{1}{\log^3 \tilde{L}^{-1}} \, (b')^2 \; .
\end{aligned}
$$



$\tilde{L} = L\Lambda$, $\tilde{m} = m/\Lambda$, $b' = \frac{4\pi}{g^2}(\Delta\theta - \pi)$

Critical S_1 size $\quad \tilde{L}_c^2 = \frac{\tilde{m}}{8}\left[1 + \mathcal{O}\left(\frac{1}{\log \tilde{L}}, \frac{\tilde{m}}{\tilde{L}^2}\right)\right]$,

Corresponds to $T_c = \sqrt{\frac{8}{\tilde{m}}}\Lambda_{QCD}$

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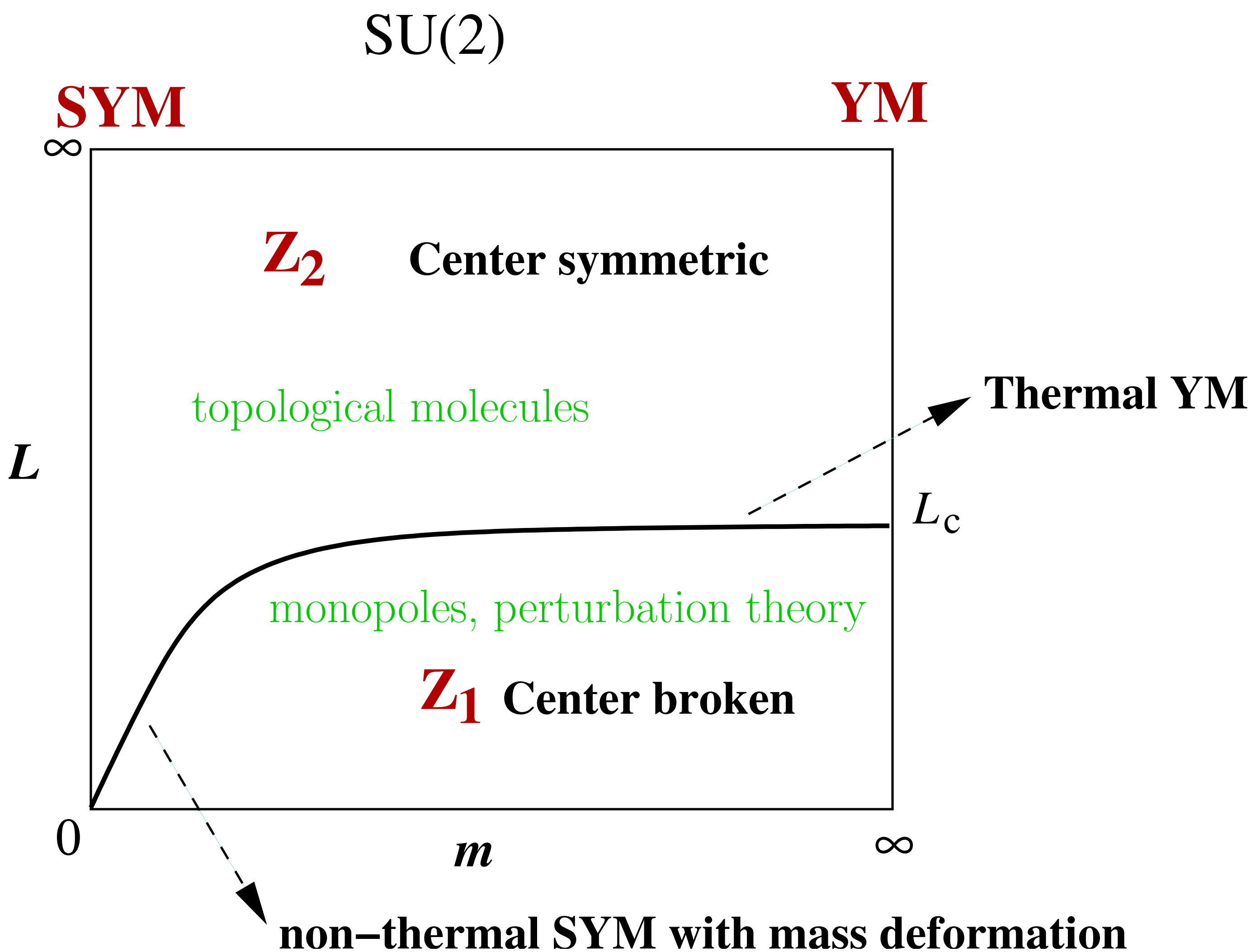
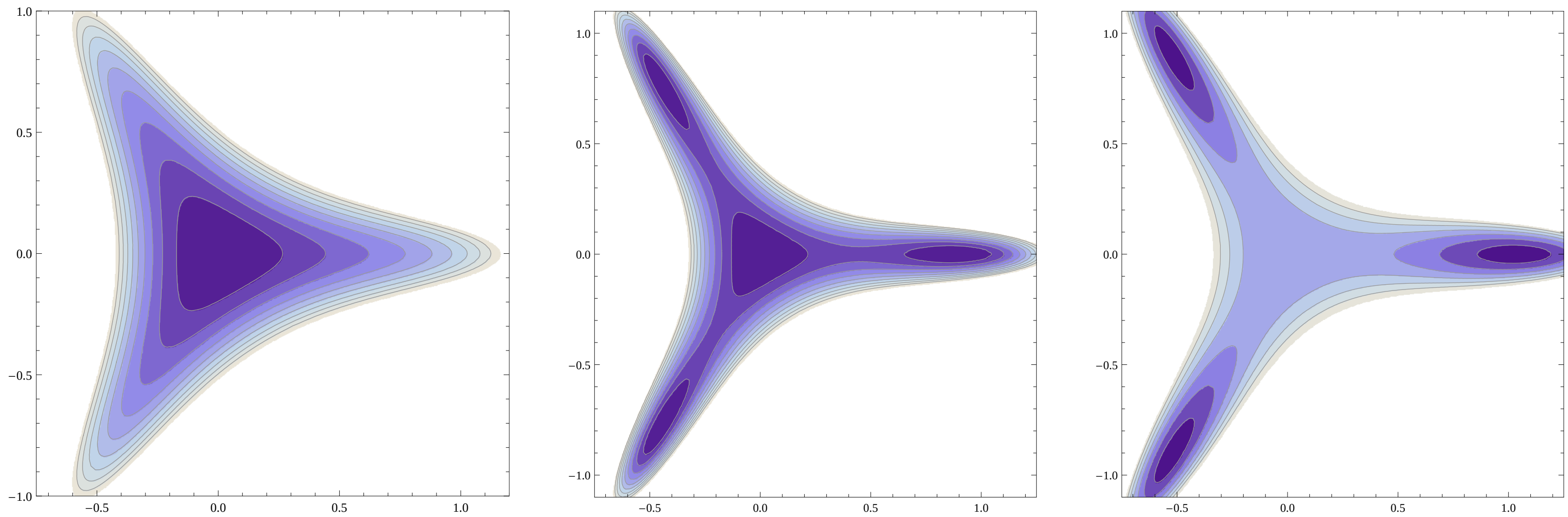
Phase diagram in L-m plane



Outlook: higher rank gauge groups, θ dependence, pure gauge

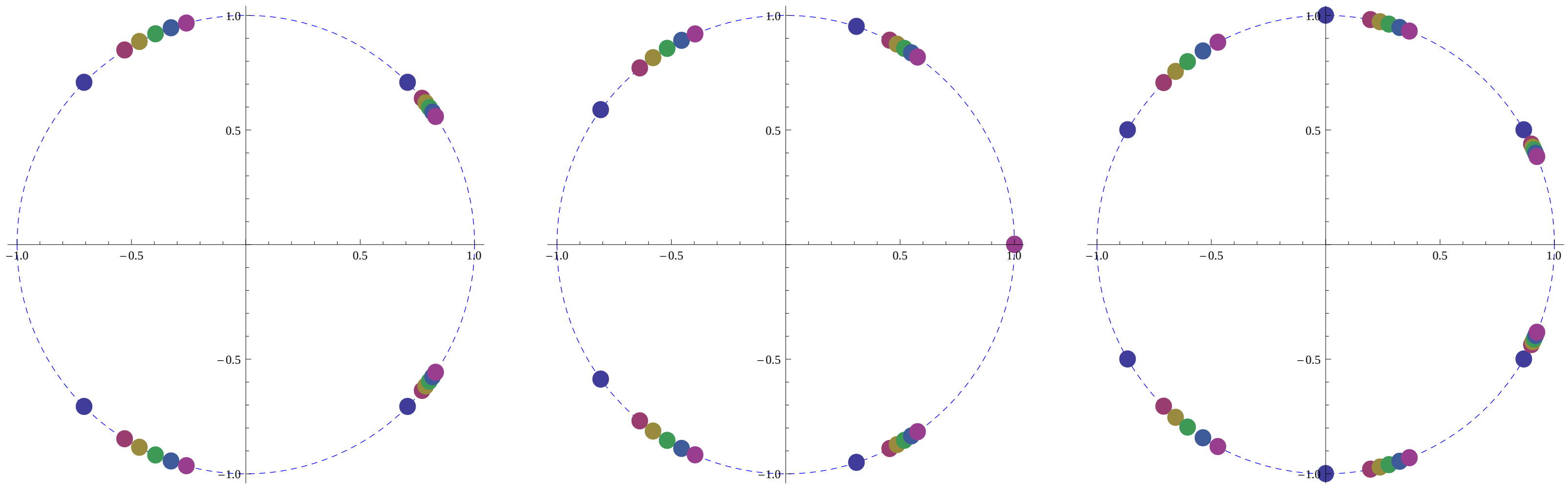
$SU(N \geq 3)$: First order transition $Z_N \to \emptyset$



Smooth $N_c \to \infty$ limit (because $Q_{top} \sim 1/N_c$)

$$\mu_{PV}^3 e^{-\frac{8\pi^2}{g^2 N_c}} \sim \Lambda^3$$

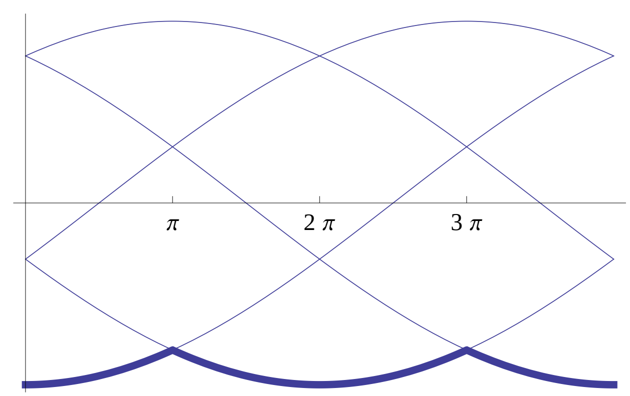
Large N_c: Eigenvalues of P for $N_c = 4, 5, 6$



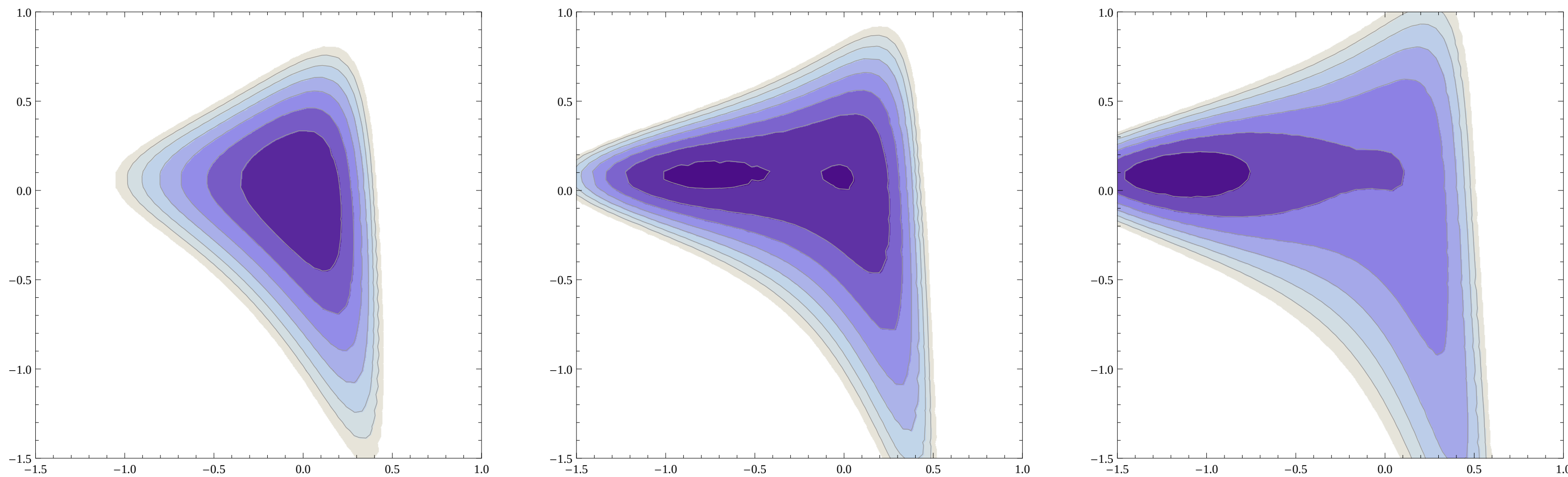
$\theta \neq 0$: Get $V_k \sim \cos\left(\frac{2\pi k + \theta}{N_c}\right)$, $k = 1, \ldots, N - 1$.

2π periodicity + $1/N_c$ scaling

$\rightarrow$ mulitiple branches



G_2: First order transition without change of symmetry.



Pure gauge theory: Find center stabilizing molecules from BZJ.

But: Semi-classical approximation not reliable.