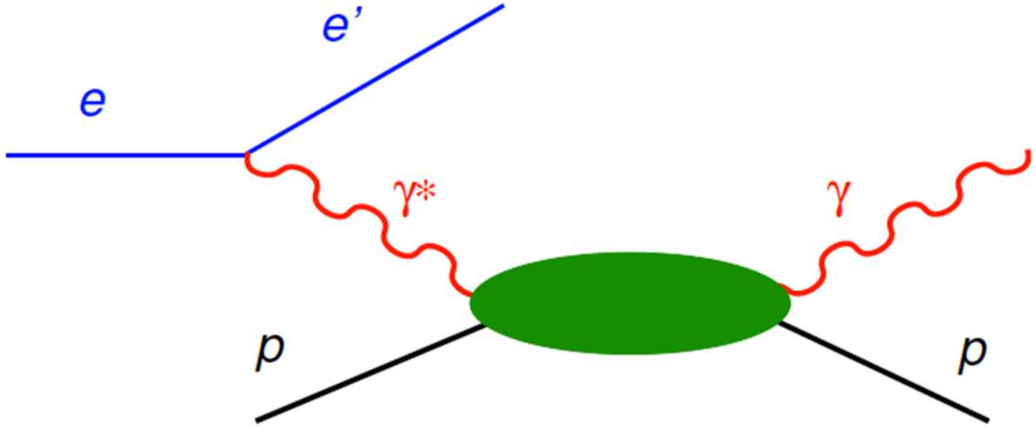
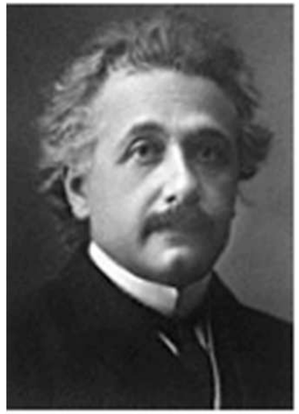
Kinematic Issue of GPDs in DVCS



B.L.G. Bakker and C.-R.Ji, arXiv:1002.0443[hep-ph];
PRD83,091502(R) (2011).

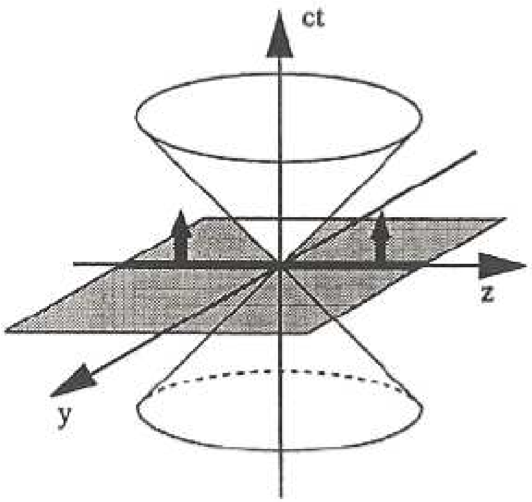
Univ. of Virginia, December 7, 2011

Relativistic Particle Physics

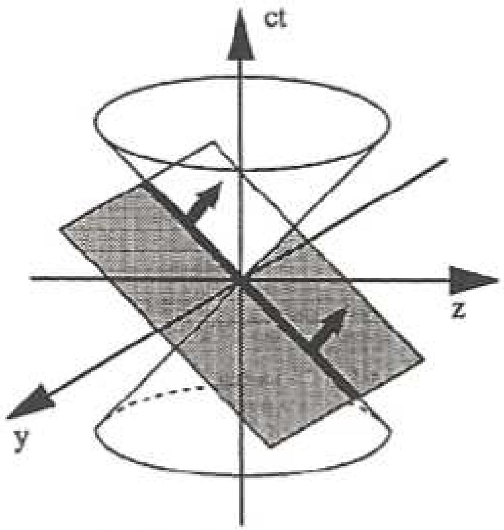


- Einstein's Special Relativity in 1905
  - -World Year of Physics (2005)
  - -Important Role (Hadron Physics)
  - -Dirac's Proposition (1949)

World Year of PHYSICS 2005
www.physics2005.org



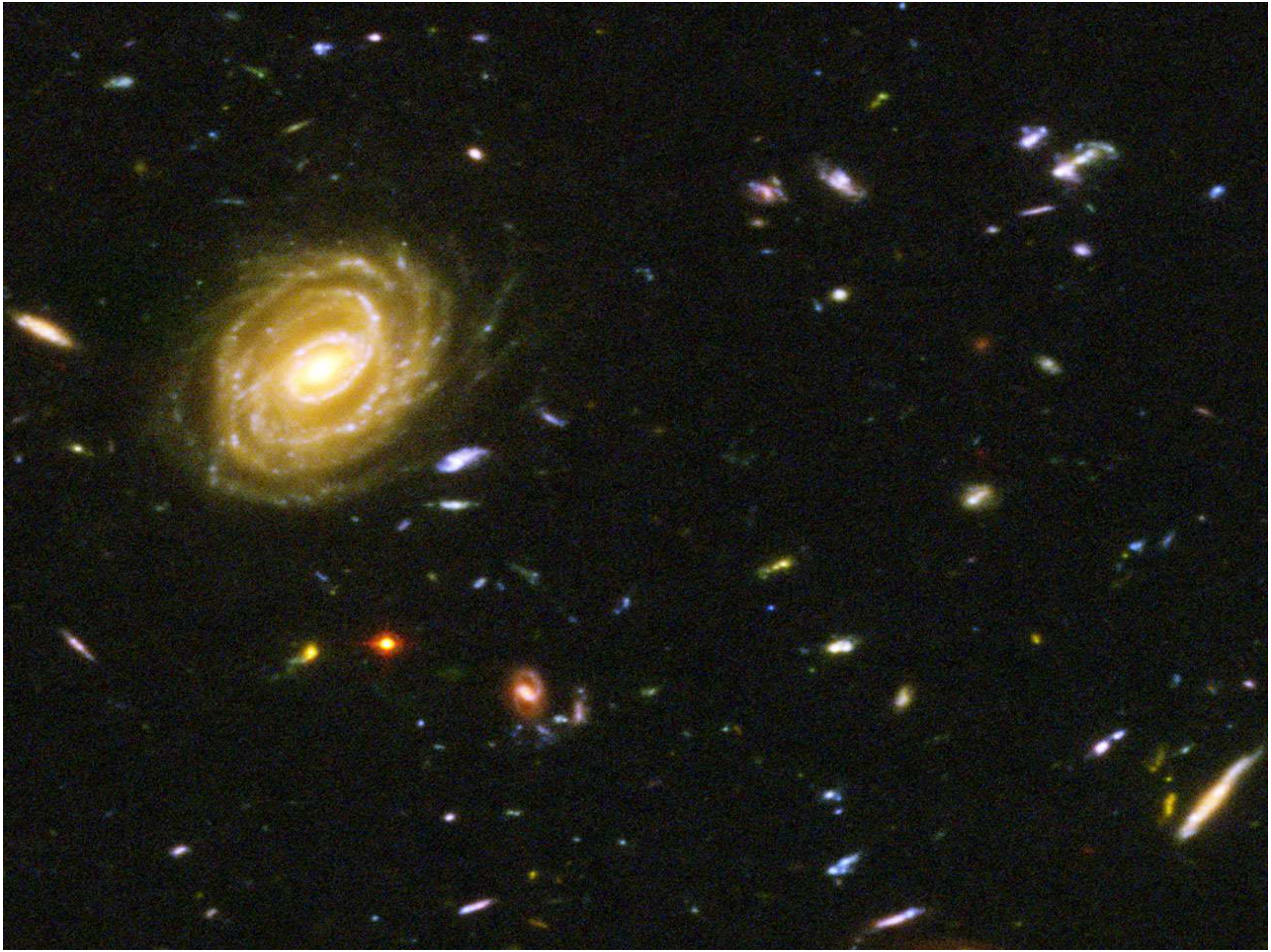
The instant form



The front form

Outline

- Why LFD?
  - Distinguished Features in LFD
  - Application to Hadron Phenomenology
  - Treacherous Points: Zero-Modes
- Kinematic Issue in DVCS and GPDs
  - Original Formulation of DVCS with GPDs
  - Tree Level Calculation
  - JLab Kinematics
- Conclusions

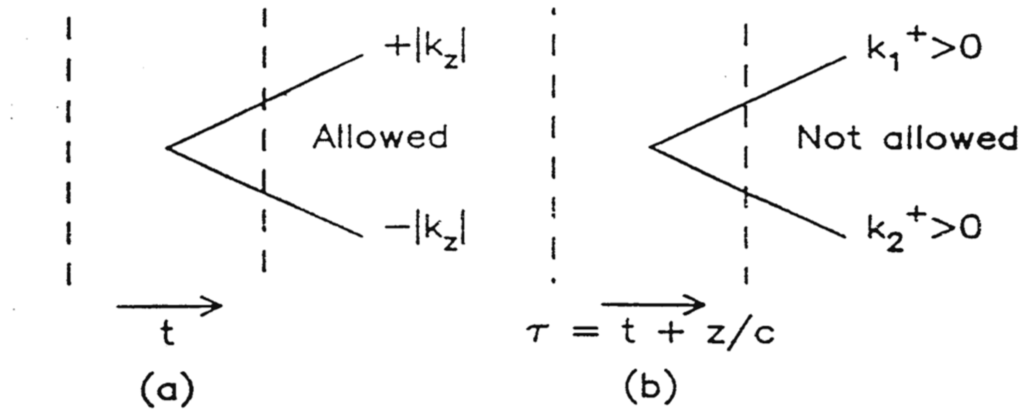


Distinguished Features in LFD

$$
\begin{array}{ccc}
\textit{Equal } t & & \textit{Equal } \tau \\
p^0 & \leftrightarrow & p^- = p^0 - p^3 \\
(p^1, p^2) & \leftrightarrow & \vec{p}_\perp \\
p^3 & \leftrightarrow & p^+ = p^0 + p^3
\end{array}
$$

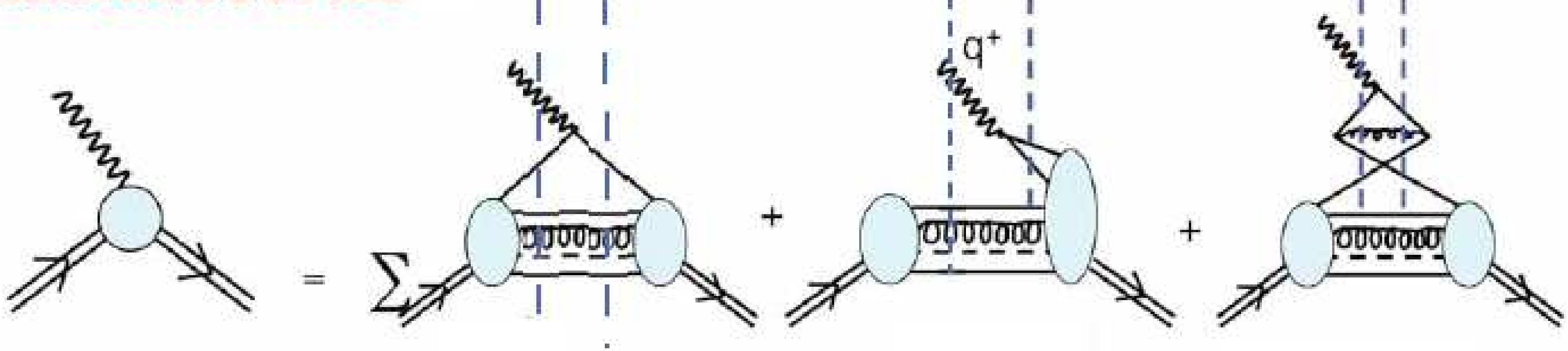
Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad\qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Calculation of Form Factors in Equal-Time Theory

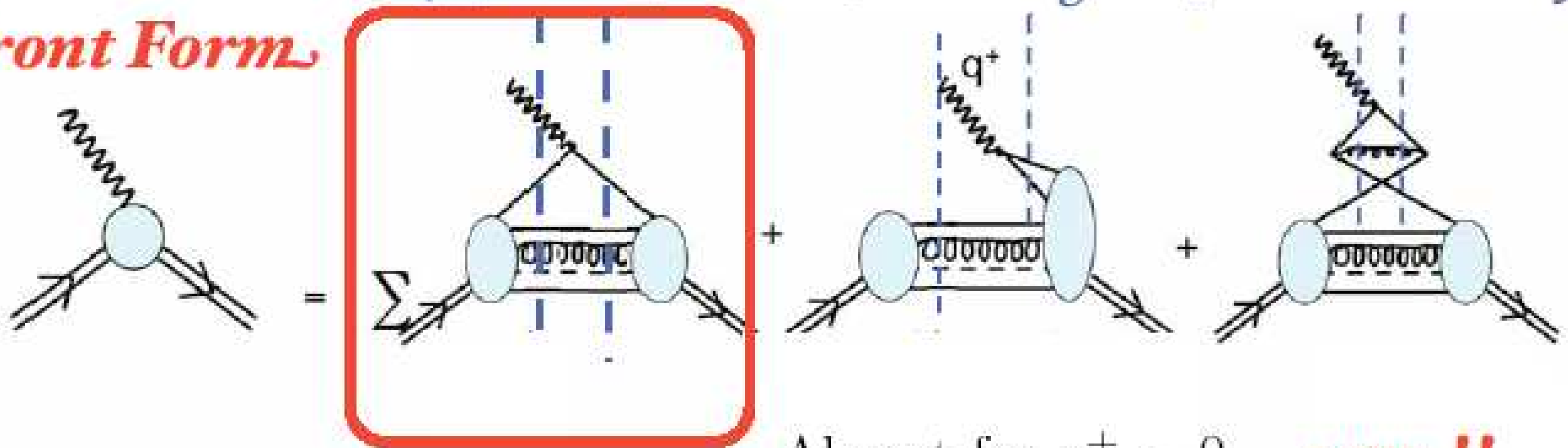
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form

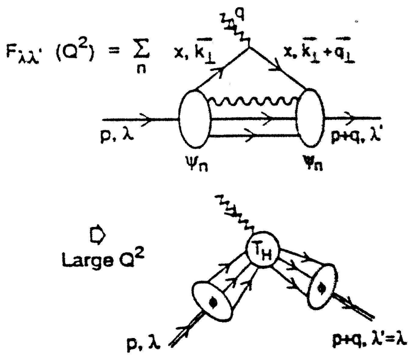


Absent for $q^+ = 0$ **zero !!**

Applications to Hadron Phenomenology

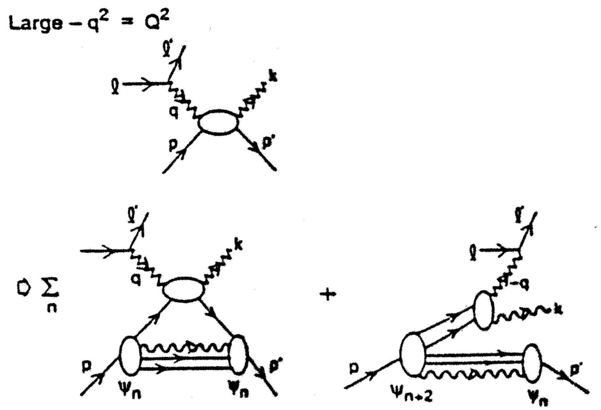
Form Factors l p →l' p'

<p' λ'|J⁺(0)|p λ>

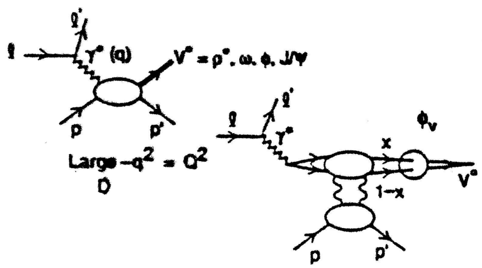


Virtual Compton γ* p→γ' p'

<p'λ'|J^μ(z)J^ν(0)|pλ>

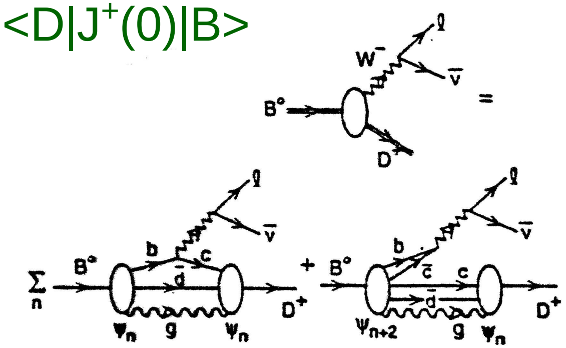


Vector Meson Leptoproduction γ*p→V*p'



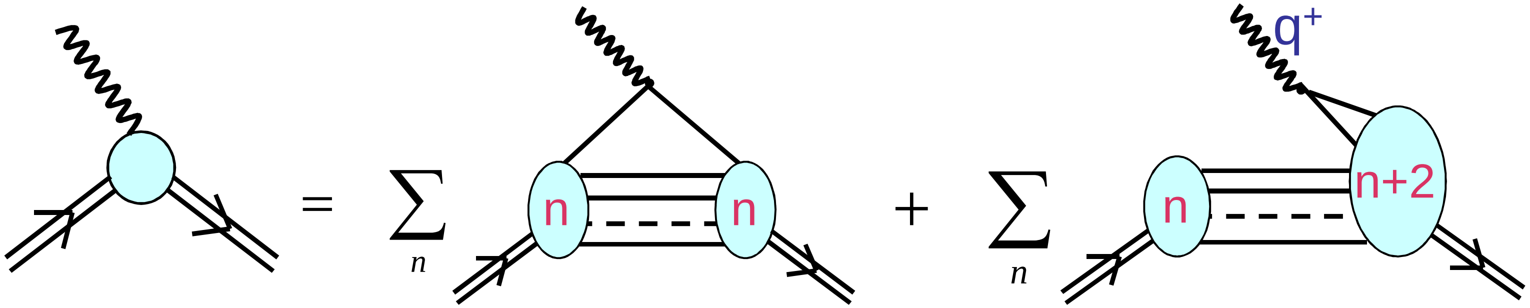
Weak Decay

<D|J⁺(0)|B>



Zero-Mode Issue in LFD

- Valence and Nonvalence Contributions

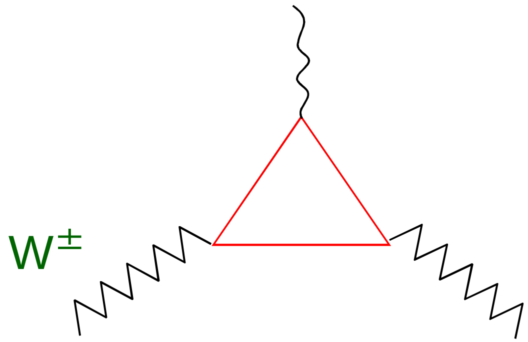


- Even if $q^+ \to 0$, the off-diagonal elements do not go away in some cases.

$$\lim_{q^+ \to 0} \int_{p^+}^{p^+ + q^+} dk^+ (....) \neq 0$$

For example, G^+_{00} has the zero-mode contribution in the Standard Model $W^{\pm}$ form factors.

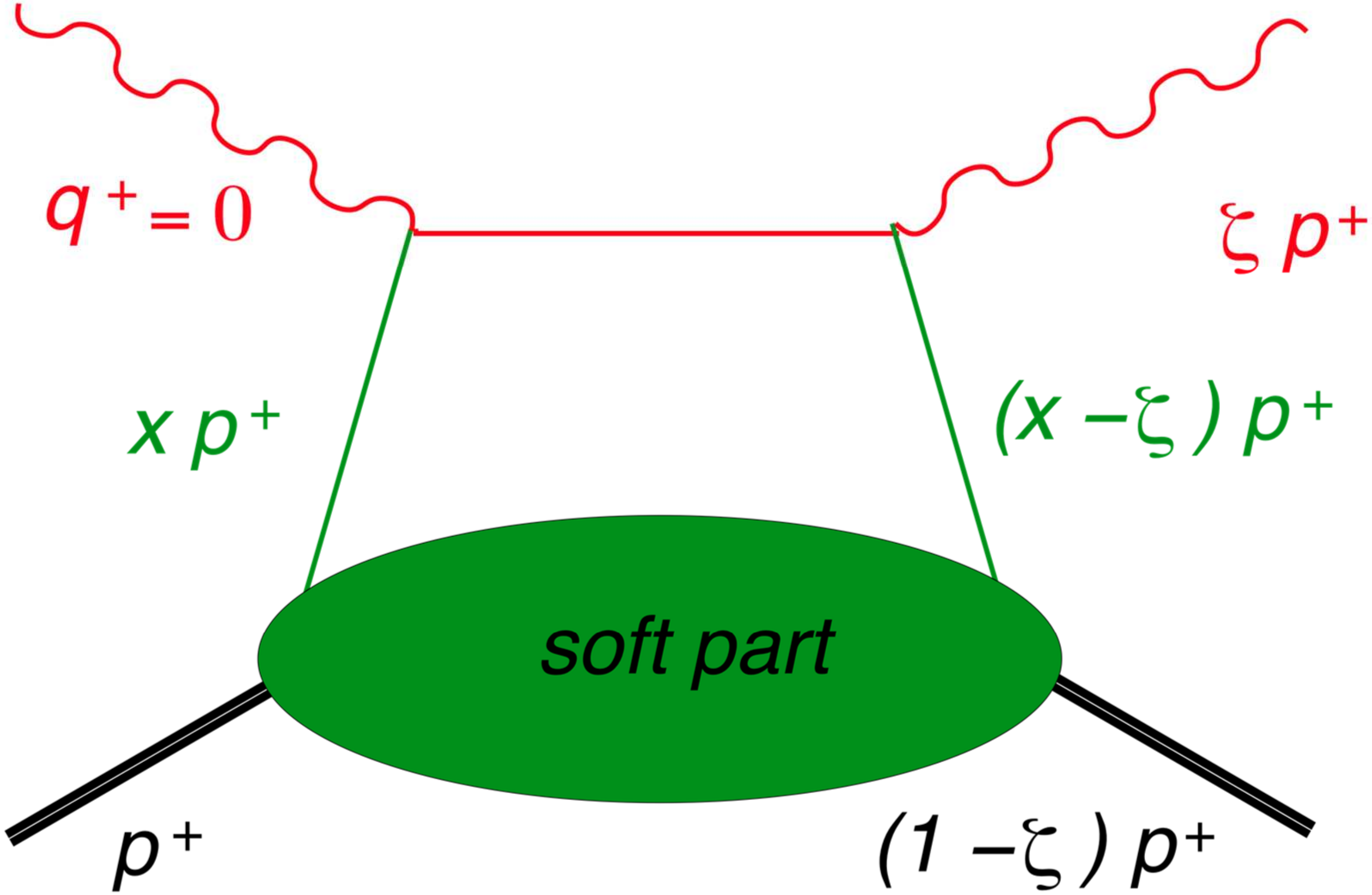
$$\left(G^+_{00}\right)_{Z.M.} = \frac{g^2 Q_f p^+}{2\pi^3 M_W^2} \int_0^1 dx \int d^2 k_\perp \frac{k_\perp^2 + m_1^2 - x(1-x)Q^2}{k_\perp^2 + m_1^2 + x(1-x)Q^2} \neq 0$$



B.Bakker and C.Ji, PRD71,053005(2005)

GPDs rely on the handbag dominance in DVCS; i.e.

Q^2 >> any soft mass scale



$q^2 = q^+q^- - q_\perp^2 = -q_\perp^2 = -Q^2 < 0$, e.g.

S.J.Brodsky,M.Diehl,D.S.Hwang, NPB596,99(01)

Nucleon GPDs in DVCS Amplitude

X.Ji,PRL78,610(1997): Eqs.(14) and (15)

$$p^\mu = \Lambda(\overset{ct}{1}, \overset{x}{0}, \overset{y}{0}, \overset{z}{1}) \quad ,$$

$$n^\mu = (\overset{ct}{1}, \overset{x}{0}, \overset{y}{0}, \overset{z}{-1})/(2\Lambda) \quad ,$$

$$\overline{P}^\mu = \frac{1}{2}(P + P')^\mu = p^\mu + \frac{M^2 - \Delta^2/4}{2} n^\mu,$$

$$q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi} n^\mu \quad , \quad \xi = \frac{Q^2}{2\overline{P}\cdot q} \quad ,$$

$$\Delta^\mu = -\xi\left[p^\mu - \frac{M^2 - \Delta^2/4}{2} n^\mu\right] + \Delta_\perp^\mu \ .$$

$$T^{\mu\nu}(p,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} + \frac{1}{x + \frac{\xi}{2} - i\varepsilon}\right)$$

$$\times\left[H(x,\Delta^2,\xi)\overline{U}(P')\not{n}U(P) + E(x,\Delta^2,\xi)\overline{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$

$$-\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta \int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} - \frac{1}{x + \frac{\xi}{2} - i\varepsilon}\right)$$

$$\times\left[\tilde{H}(x,\Delta^2,\xi)\overline{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\xi)\frac{\Delta\cdot n}{2M}\overline{U}(P')\gamma_5 U(P)\right]$$

Just above Eq.(14),

``To calculate the scattering amplitude, it is convenient to define <u>a special system of coordinates</u>.”

Note here that $q'^2 = -\mathbf{\Delta}_\perp^2$ = 0 , i.e. t = 0 .

Nucleon GPDs in DVCS Amplitude

A.V.Radyushkin, PRD56, 5524 (1997): Eq.(7.1)

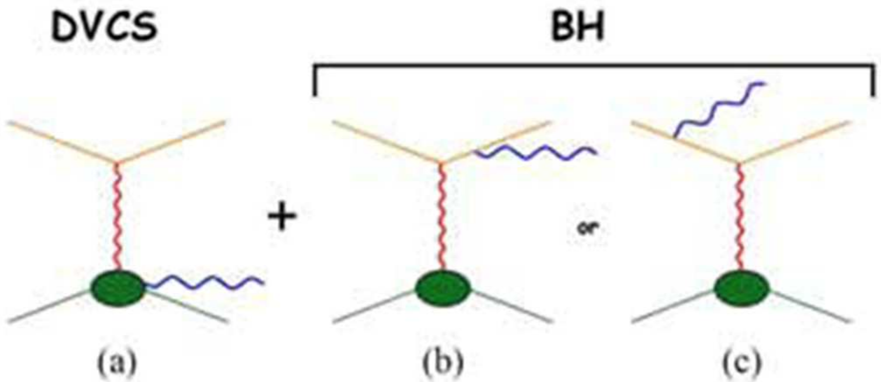
$$q = q' - \zeta p \quad ,$$
$$\zeta = \frac{Q^2}{2p \cdot q'} \quad ,$$
$$r = p - p'$$

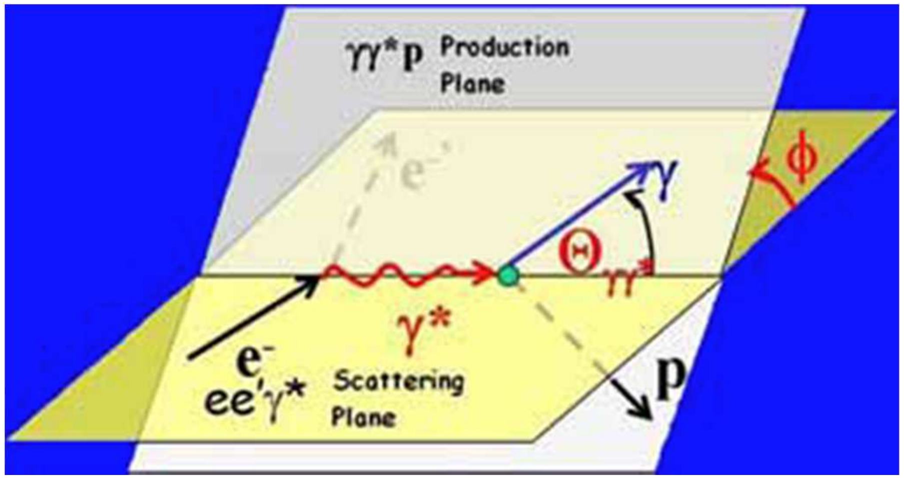
$$\begin{aligned} T^{\mu\nu}(p,q,q') = \frac{1}{2(p \cdot q')} \sum_a e_a^2 \Big[&\left(-g^{\mu\nu} + \frac{1}{p \cdot q'}(p^\mu q'^\nu + p^\nu q'^\mu) \right) \\ &\times \left\{ \bar{u}(p') \not{q}' u(p) T_F^a(\zeta) + \frac{1}{2M} \bar{u}(p')(\not{q}' \not{r} - \not{r} \not{q}') u(p) T_K^a(\zeta) \right\} \\ &+ i\varepsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q'_\beta}{p \cdot q'} \left\{ \bar{u}(p') \not{q}' \gamma_5 u(p) T_G^a(\zeta) + \frac{q' \cdot r}{2M} \bar{u}(p') \gamma_5 u(p) T_P^a(\zeta) \right\} \Big] \end{aligned}$$

At the beginning of Section 2E (Nonforward distributions),
``Writing the momentum of the virtual photon as q=q'-ζp is equivalent to using the Sudakov decomposition in the light-cone `plus'(p) and `minus'(q') components in a situation <u>when there is no transverse momentum</u> ."

Note here that $t = \Delta^2 = (\zeta P)^2 = \zeta^2 M^2 > 0$,i.e. only consistent at t=0, neglecting nucleon mass.

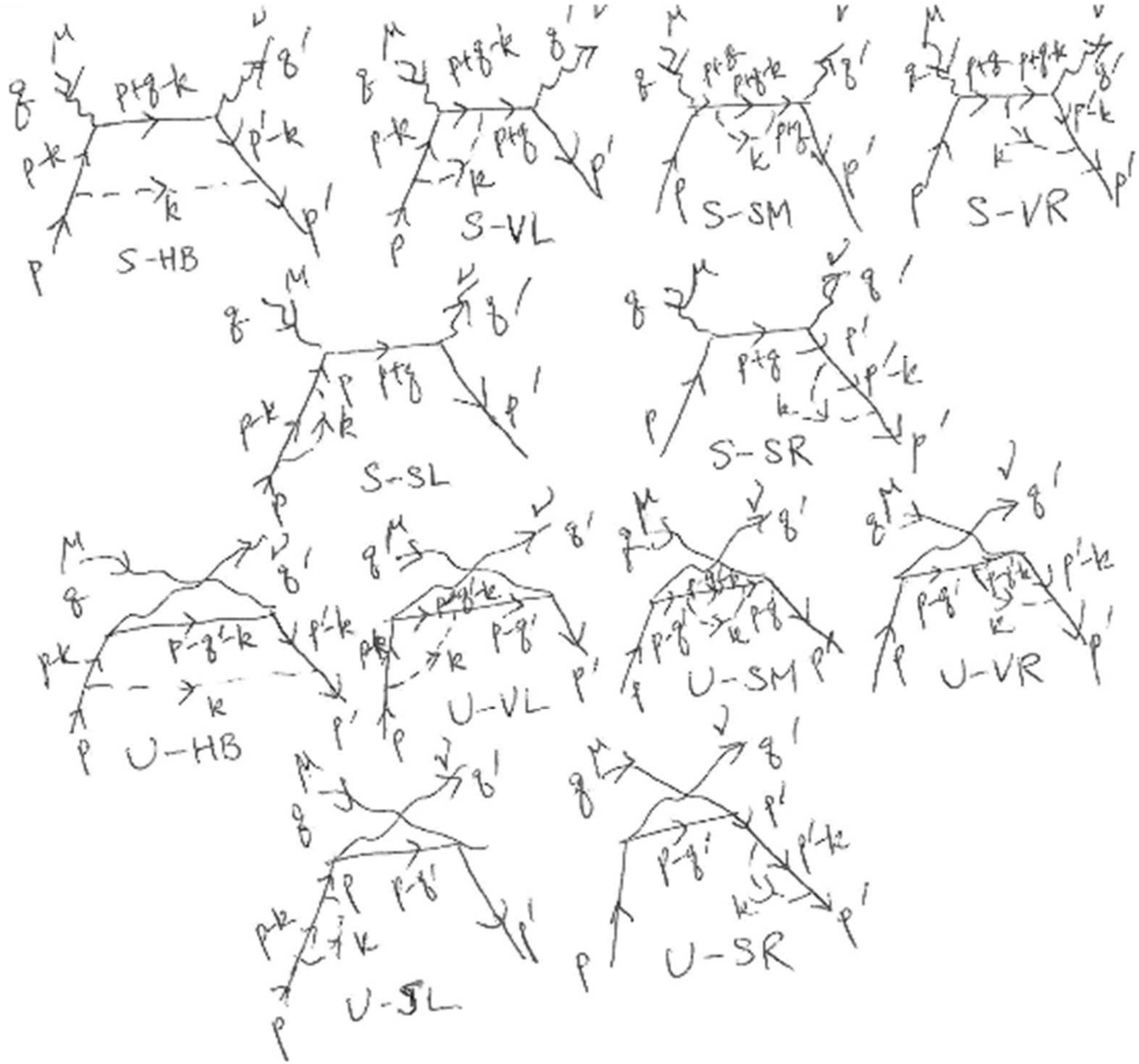
JLab Kinematics $t < -|t_{min}| \;\; \overline{0}$



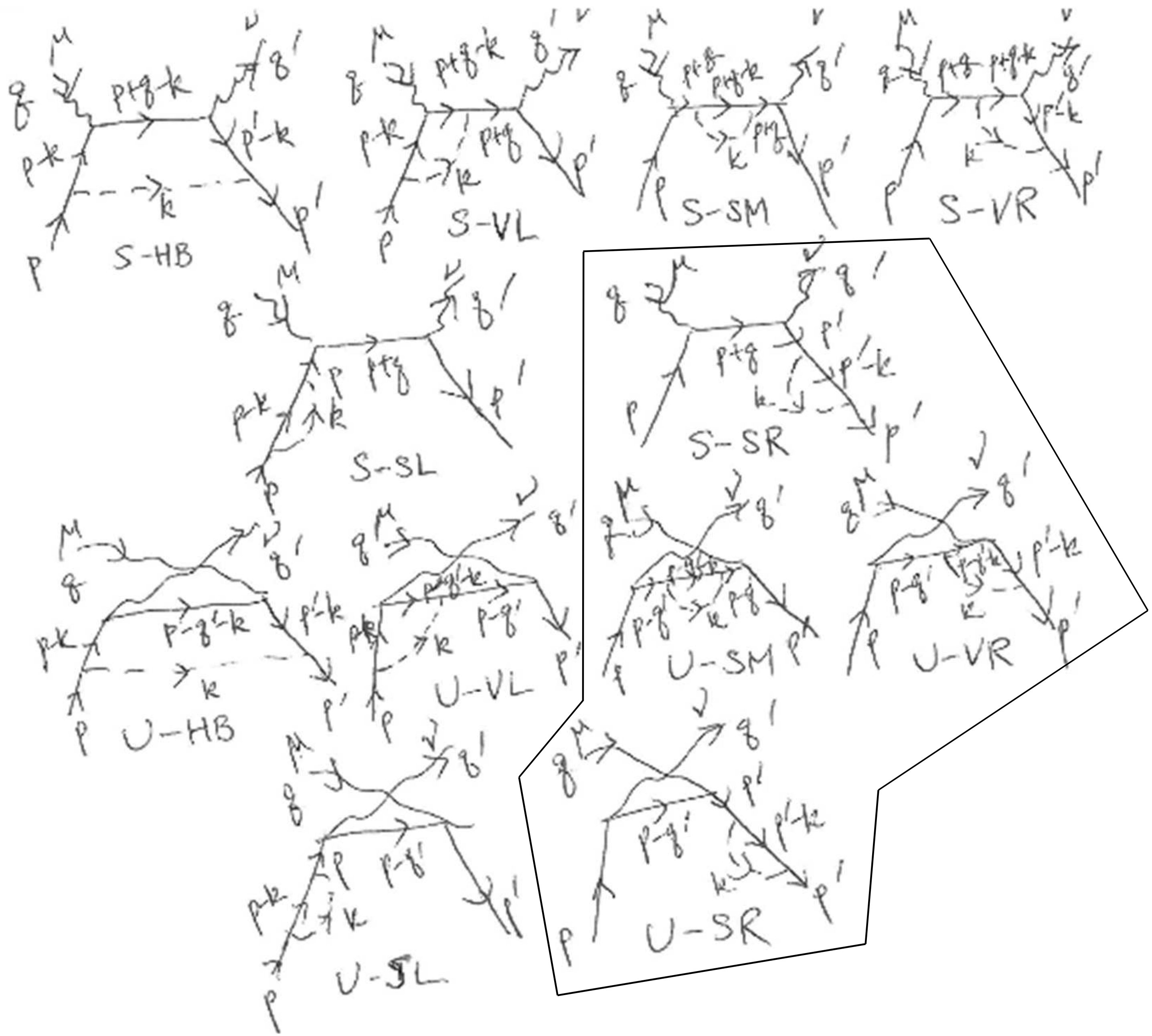


Original formulation of DVCS in terms of GPDs due to X.Ji and A.Radyushkin is applied only at $t=0$.

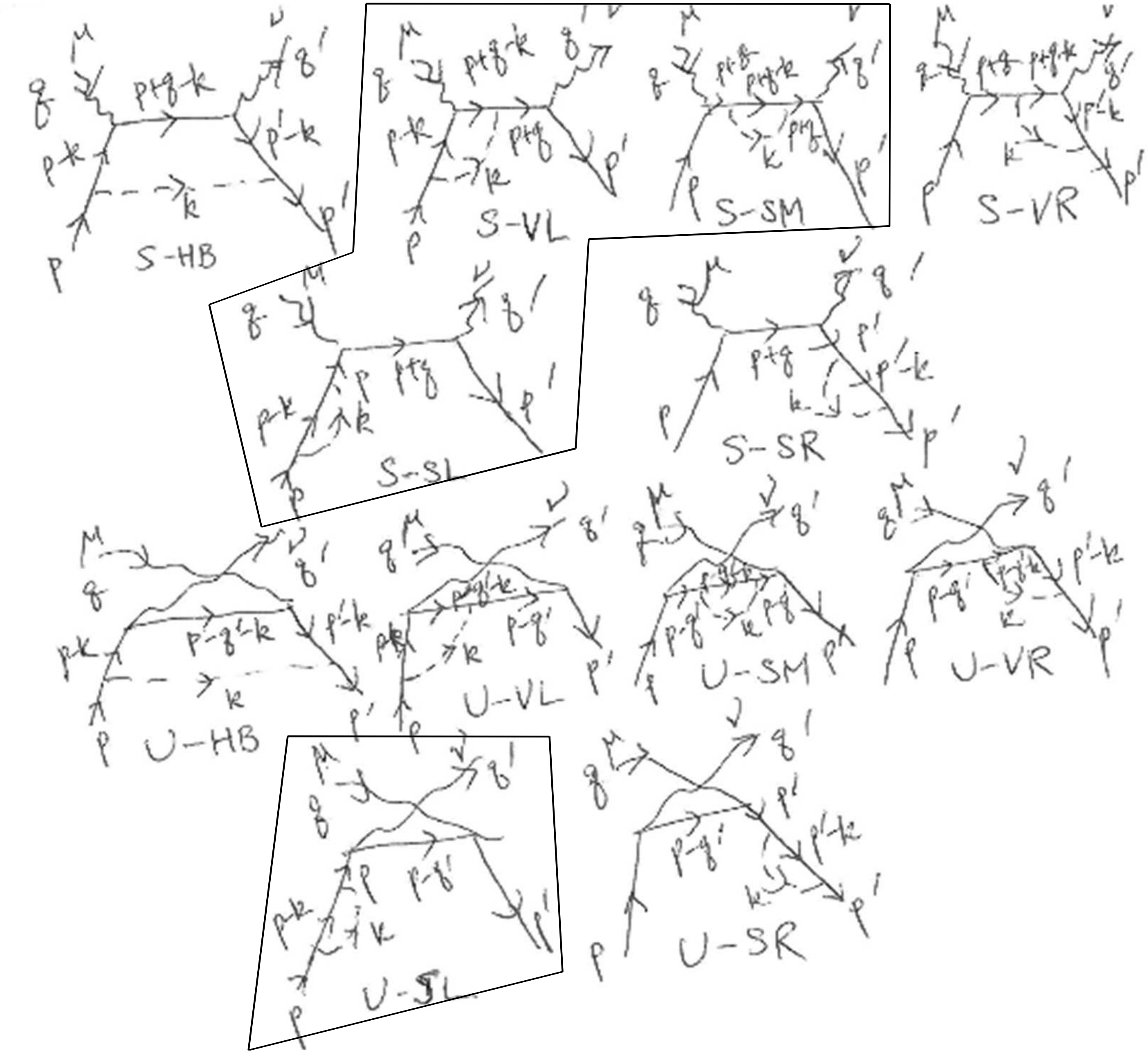
Gauge Invariance Check in One-Loop Level



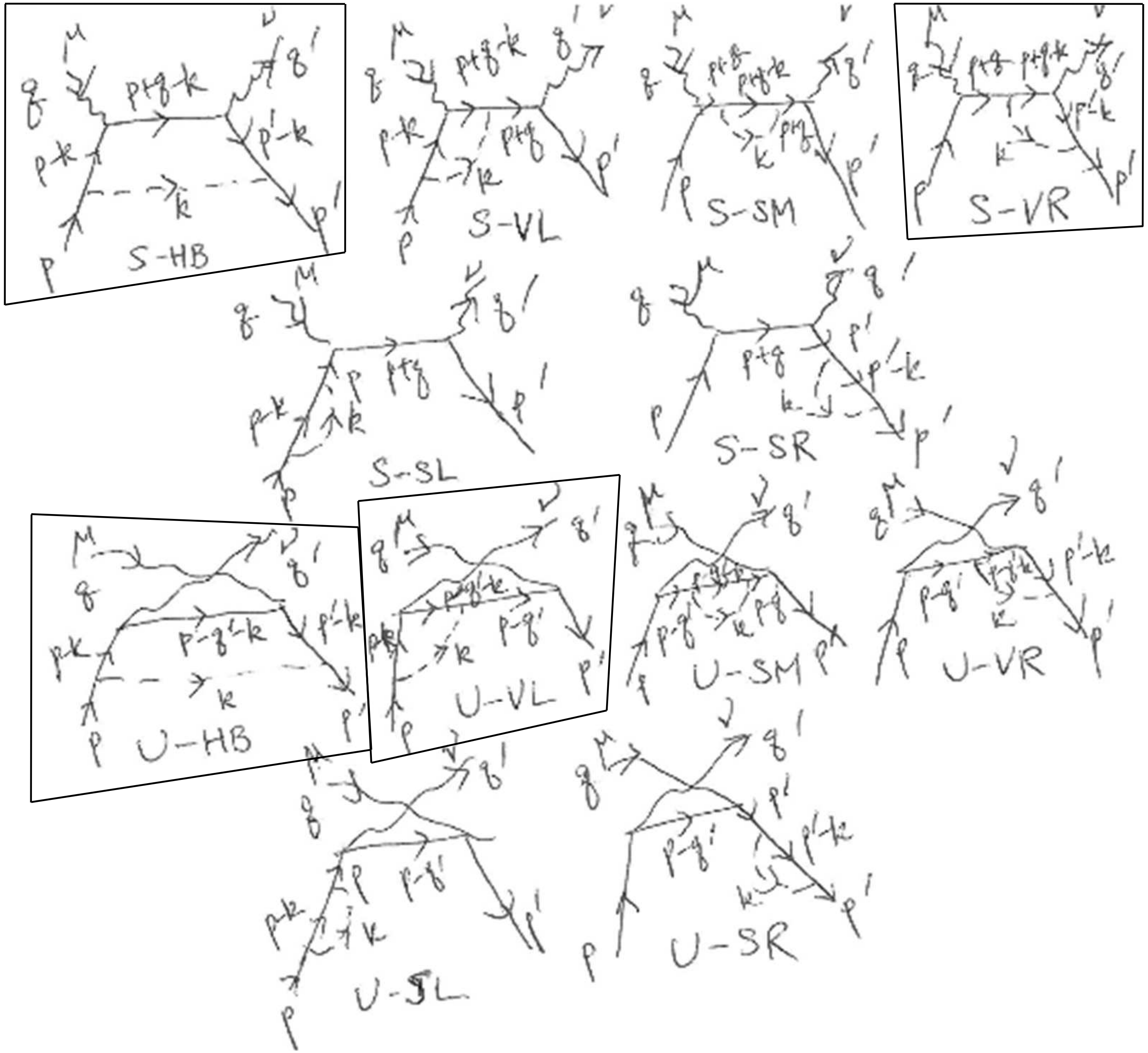
Gauge Invariance Check in One-Loop Level



Gauge Invariance Check in One-Loop Level



Gauge Invariance Check in One-Loop Level



“Bare Bone” VCS Amplitude at Tree Level

$$S=(k+q)^2$$

$$U=(k-q')^2$$

$$=k'$$

Hadron Helicity Amplitude:

$$\mathrm{H}(h_q,h_{q'},s_k,s_{k'})=\varepsilon^{*}_{\mu}(q',h_{q'})\,\varepsilon_{\nu}(q,h_q)\left(T_S^{\mu\nu}+T_U^{\mu\nu}\right)$$

Neglecting masses,

$$T_S^{\mu\nu}=\frac{k_\alpha+q_\alpha}{S}\,\overline{u}(k',s_{k'})\gamma^\mu\gamma^\alpha\gamma^\nu u(k,s_k)$$

$$T_U^{\mu\nu}=\frac{k_\alpha-q'_\alpha}{U}\,\overline{u}(k',s_{k'})\gamma^\nu\gamma^\alpha\gamma^\mu u(k,s_k)$$

Identity: $\gamma^\mu\gamma^\alpha\gamma^\nu=g^{\mu\alpha}\gamma^\nu+g^{\alpha\nu}\gamma^\mu-g^{\mu\nu}\gamma^\alpha+i\varepsilon^{\mu\alpha\nu\beta}\gamma_\beta\gamma_5$

JLab Kinematics t < 0

- We want to see the effect of taking t<0.
- Therefore we mimic the kinematics at JLab.
- In JLab kinematics, the final hadron and final photon move off the z-axis.

$$k'^{\mu} = \left((x - \zeta_{\mathrm{eff}})P^+, \mathbf{\Delta}_\perp, \frac{\mathbf{\Delta}_\perp^2}{2(x - \zeta_{\mathrm{eff}})P^+)} \right)$$

$$q'^{\mu} = \left(\alpha \frac{\mathbf{\Delta}_\perp^2}{Q^2} P^+, -\mathbf{\Delta}_\perp, \frac{Q^2}{2\alpha P^+} \right)$$

The quantity ζ_{eff} is given by

$$\zeta_{\mathrm{eff}} = \zeta + \alpha \frac{\mathbf{\Delta}_\perp^2}{Q^2} \to \zeta \text{ for } Q \to \infty$$

$$\alpha = \frac{x - \zeta}{2} \left(1 - \sqrt{1 - \frac{4\zeta}{x - \zeta} \frac{\mathbf{\Delta}_\perp^2}{Q^2}} \right) \to 0 \text{ for } Q \to \infty$$

Using Sudakov vectors

$$n(+)^{\mu} = (1,0,0,0), \; n(-)^{\mu} = (0,0,0,1)$$

we find

$$\begin{aligned} T_s{}^{\mu\nu} &= \frac{1}{s}\left[(\{(k^+ + q^+)n^{\mu}(+) + q^- n^{\mu}(-) + q_{\perp}{}^{\mu}\}n^{\nu}(+)\right. \\ &\quad + \{(k^+ + q^+)n^{\nu}(+) + q^- n^{\nu}(-) + q_{\perp}{}^{\nu}\}n^{\mu}(+) - g^{\mu\nu}q^-) \\ &\quad \times \bar{u}(k';s')\not{n}(-)u(k;s) \\ &\quad - i\epsilon^{\mu\nu\alpha\beta}\{(k^+ + q^+)n_{\alpha}(+) + q^- n_{\alpha}(-) + q_{\perp\alpha}\}n_{\beta}(+) \\ &\quad \left.\times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\right]. \end{aligned}$$

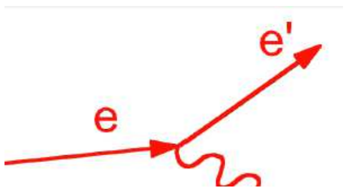
Keeping no transverse momentum in DVCS, we agree on

$$\begin{aligned} T_s{}^{\mu\nu} &= \frac{q^-}{s}\left[\{n^{\mu}(-)n^{\nu}(+) + n^{\nu}(-)n^{\mu}(+) - g^{\mu\nu}\}\right. \\ &\quad \times \bar{u}(k';s')\not{n}(-)u(k;s) \\ &\quad \left. - i\epsilon^{\mu\nu\alpha\beta}n_{\alpha}(-)n_{\beta}(+) \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\right] \end{aligned}$$

equivalent to the expression given by X. Ji and A.V. Radyushkin.

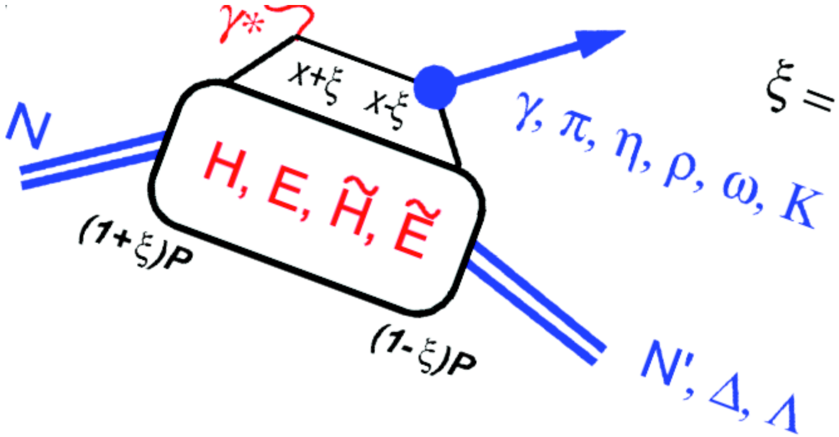
Investigation of Complete Ampltude

Attach the lepton current and check the spin filter for the DVCS amplitude.



$$\epsilon_{\mathrm{LF}}(q;\pm 1) = \frac{1}{\sqrt{2}}\left(0, \mp 1, -i, \mp \frac{q_x \pm i q_y}{q^+}\right)$$

$$\epsilon_{\mathrm{LF}}(q;0) = \frac{1}{\sqrt{q^2}}\left(q^+, q_x, q_y, \frac{q_\perp^2 - q^2}{2q^+}\right)$$



$$\xi = \frac{x_B}{2 - x_B}$$

Singularities develop in the polarization vector as $q^+ \rightarrow 0$.

The amplitudes being obtained by contraction with the polarization vectors may be sensitive to the neglected parts.

Calculation for massless spinors

Complete amplitude

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\}h) \frac{1}{q^2} \mathcal{H}(\{s', s\}\{h', h\}),$$

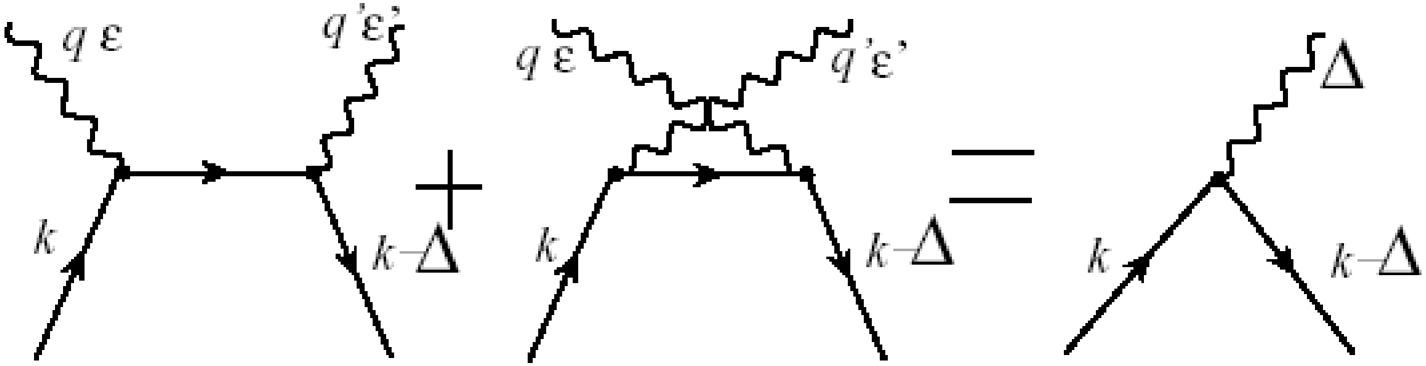
Leptonic and hadronic parts

$$\begin{aligned} \mathcal{L}(\{\lambda', \lambda\}h) &= \bar{u}_{\mathsf{LF}}(\ell'; \lambda') \not{\epsilon}^*(q; h) u_{\mathsf{LF}}(\ell; \lambda), \\ \mathcal{H}(\{s', s\}\{h', h\}) &= \bar{u}_{\mathsf{LF}}(k'; s')(\mathcal{O}_s + \mathcal{O}_u) u_{\mathsf{LF}}(k; s), \end{aligned}$$

Operators

$$\begin{aligned} \mathcal{O}_s &= \frac{\not{\epsilon}^*_{\mathsf{LF}}(q'; h')(\not{k} + \not{q})\not{\epsilon}_{\mathsf{LF}}(q; h)}{(k+q)^2}, \\ \mathcal{O}_u &= \frac{\not{\epsilon}_{\mathsf{LF}}(q; h)(\not{k} - \not{q}')\not{\epsilon}^*_{\mathsf{LF}}(q'; h')}{(k-q')^2} \end{aligned}$$

Full Amp vs. Reduced Amp

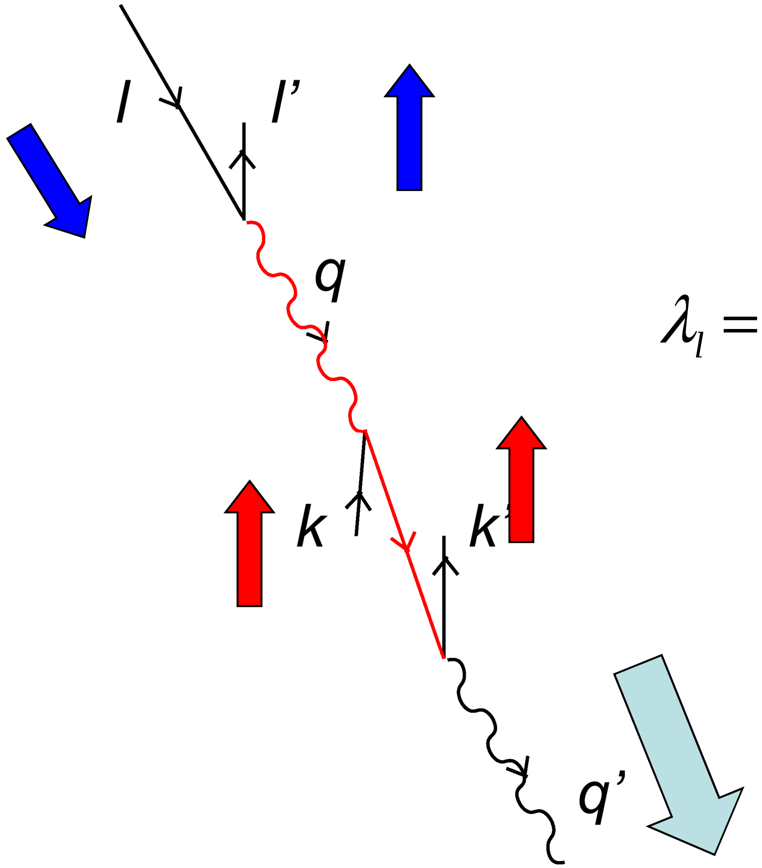


S-channel: $$\frac{\not\varepsilon^*(\lambda')(\not k+\not q+m)\not\varepsilon(\lambda)}{(k+q)^2-m^2} \longrightarrow \frac{\not\varepsilon^*(\lambda')q^-\gamma^+\not\varepsilon(\lambda)}{(x-\zeta)P^+q^-}$$

U-channel: $$\frac{\not\varepsilon(\lambda)(\not k'-\not q+m)\not\varepsilon^*(\lambda')}{(k'-q)^2-m^2} \longrightarrow -\frac{\not\varepsilon(\lambda)q^-\gamma^+\not\varepsilon^*(\lambda')}{x\,P^+q^-}$$

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.

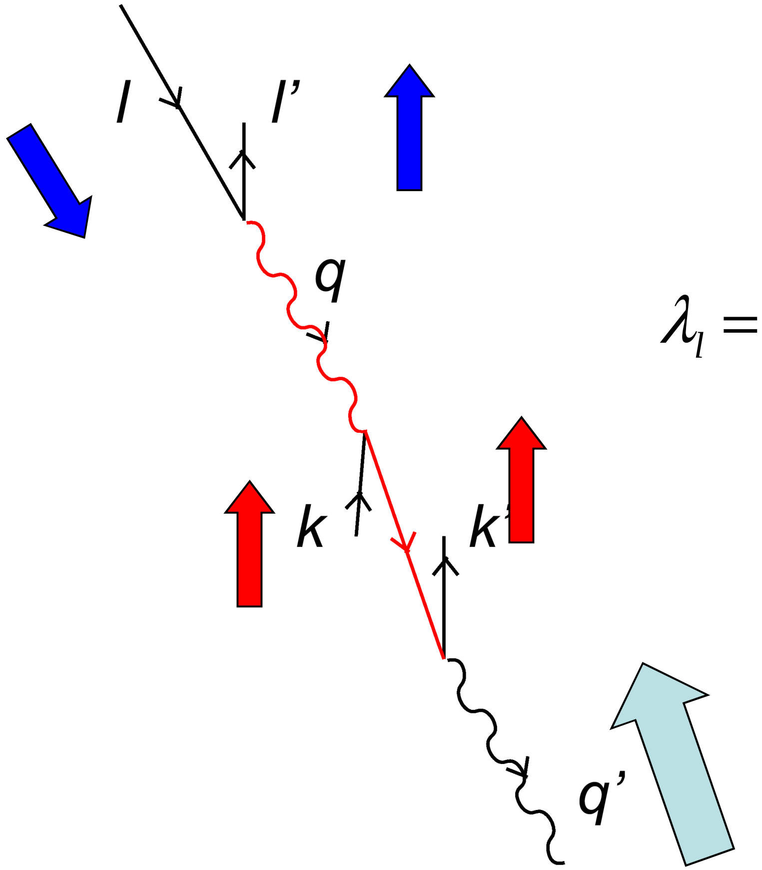


$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\; s_k = s_{k'} = +\frac{1}{2}\,,\; h_{q'} = +1\,;$$

Allowed !

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\; s_k = s_{k'} = +\frac{1}{2}\,,\; h_{q'} = -1\,;$$

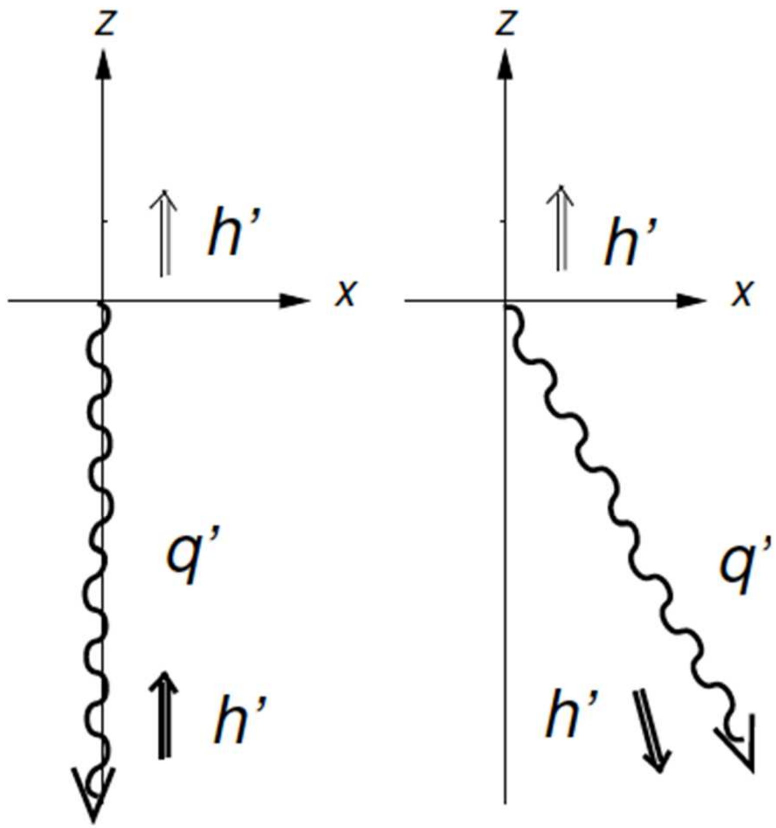
Prohibited !

Comparison

Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda',\lambda\},h)\frac{1}{q^2}\mathcal{H}(\{h',h\}\{s',s\})$ in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless, $\lambda'=\lambda$ and $s'=s$.

λ	h'	s	this work	AVR	XJ
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{4}{Q^3}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_\perp^2}{Q^2}\right)$

Swap



Definition of Light-Front Helicity

C.Carlson and C.Ji, Phys.Rev.D67,116002(03)

Photon Kinematics

K2 Kinematics (effectively, '1+1' dim.)

$$
\begin{aligned}
q^\mu &= \left(-\zeta p^+, 0, 0, \frac{Q^2}{2\zeta p^+}\right), \\
q'^\mu &= \left(0, 0, 0, \frac{Q^2}{2\zeta p^+}\right).
\end{aligned}
$$

K3 Nonvanishing q^+ and q'^+ Kinematics

$$
\begin{aligned}
q^\mu &= \left(-\frac{\zeta}{2} p^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta p^+}\right), \\
q'^\mu &= \left(\frac{\zeta}{2} p^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta p^+}\right).
\end{aligned}
$$

The Mandelstam variables in these two kinematics are the same, in particular $t = 0$.

Swap

Owing to the definition of the LF helicity, it is opposite to the IF helicity if the momentum of the photon in the final state is pointing strictly in the $-z$-direction.

This explains the differences among the results in kinematics **K2**, **K3**, and **K4** at $t = 0$.

$$
\begin{array}{|ccccc|}
\hline
\textbf{K2} & & \textbf{K3} & & \textbf{K4}\ (t=0) \\
\hline
\mathrm{BC2}(\pm) & = & \mathrm{BC3}(\mp) & = & \mathrm{BC4}(\mp) \\
\| & & \| & & \| \\
\mathrm{AVR2}(\pm) & = & \mathrm{AVR3}(\mp) & = & \mathrm{AVR4}(\mp) \\
\| & & \| & & \| \\
\mathrm{XJ2}(\pm) & = & \mathrm{XJ3}(\pm) & = & \mathrm{XJ4}(\pm) \\
\hline
\end{array}
$$

Conclusions

- For the good hadron phenomenology, treacherous points such as zero-modes and singularities should be taken into account correctly.
- As a consequence, we find that the XJ and AVR amplitudes for DVCS in terms of GPDs for $t < 0$ are not satisfactory.
- More careful investigation on the GPD formulation and the corresponding sum rules is necessary.