Hawking Radiation in AdS_5

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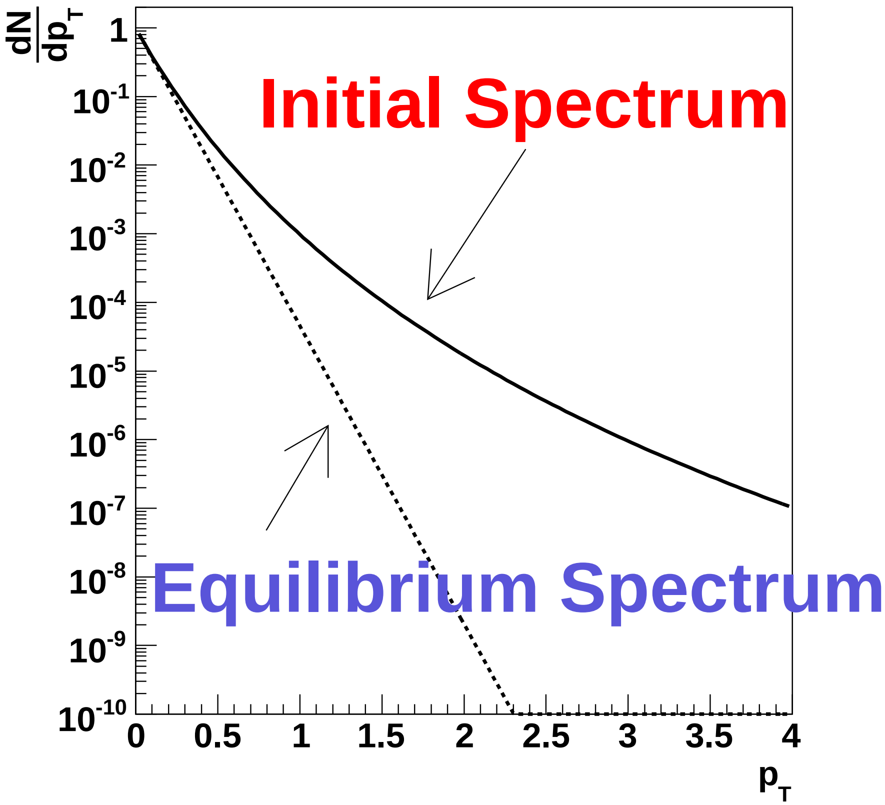
- Dam T. Son, DT; arXiv:0901.2338
- Simon Caron-Huot, DT, Paul Chesler; arXiv:1102.1073 (yesterday,yeah!)

Two important RHIC Data

(Or part of why I came to talk about AdS_5)

Energy Loss of Fast Partons – Cartoon



- Power law initial spectrum:

$$\frac{dN}{dp_T} \propto \left(\frac{1}{p_T}\right)^{10}$$

- Exponential equilib. spectrum:

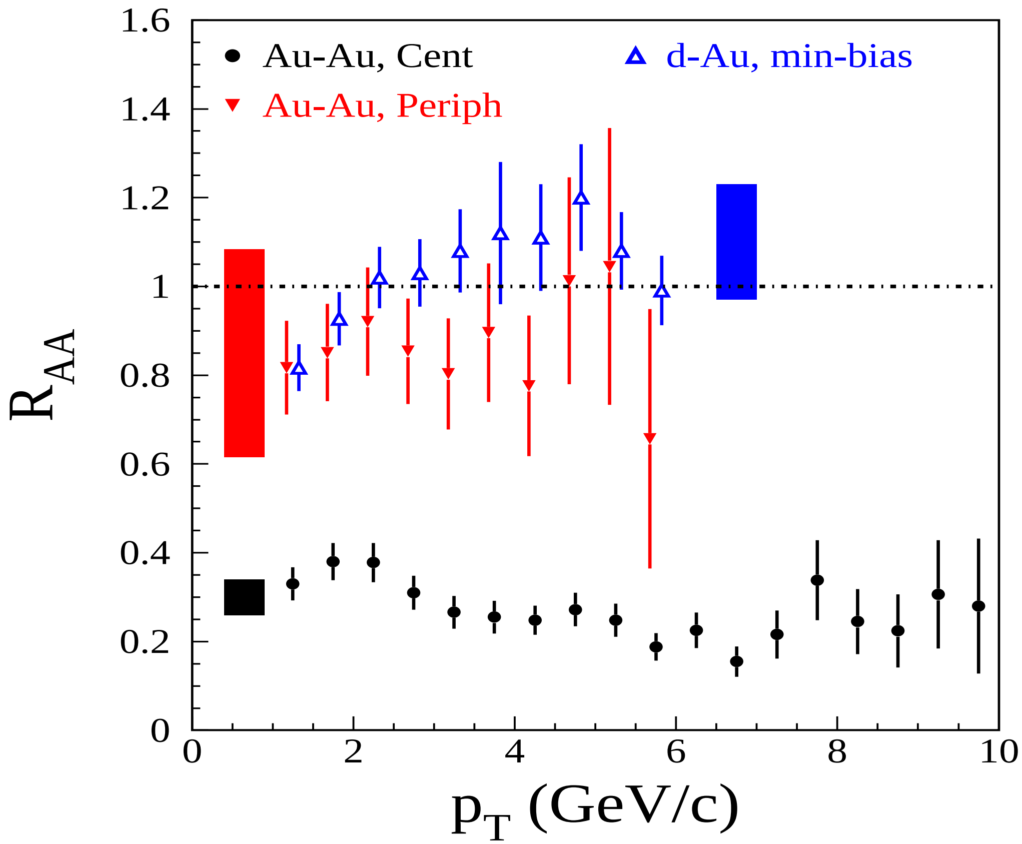
$$\frac{dN}{dp_T} \propto e^{-\frac{p_T}{T}}$$

The initial spectrum will lose energy and approach the equilibrium spectrum

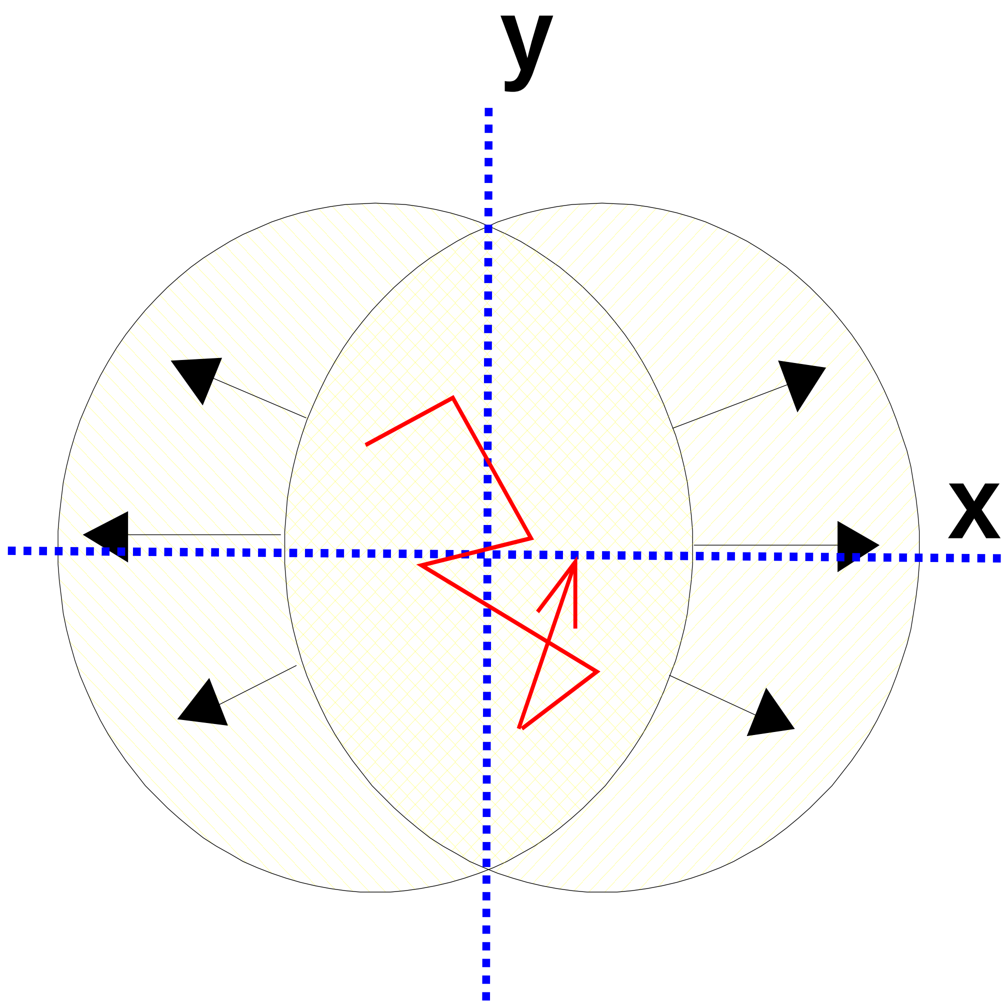
Tells something about density and interaction rates

Data on π^0 p_T spectrum

$$R_{AA} \equiv \frac{\left(\frac{dN}{p_T dp_T}\right)_{\text{In AuAu}}}{N_{\text{coll}} \left(\frac{dN}{p_T dp_T}\right)_{\text{In pp}}}$$



Heavy Quarks



The heavy quarks will either relax to the thermal spectrum and show the same v_2 as all thermal particles or not depending on the Drag/Diffusion coefficients and p_T.

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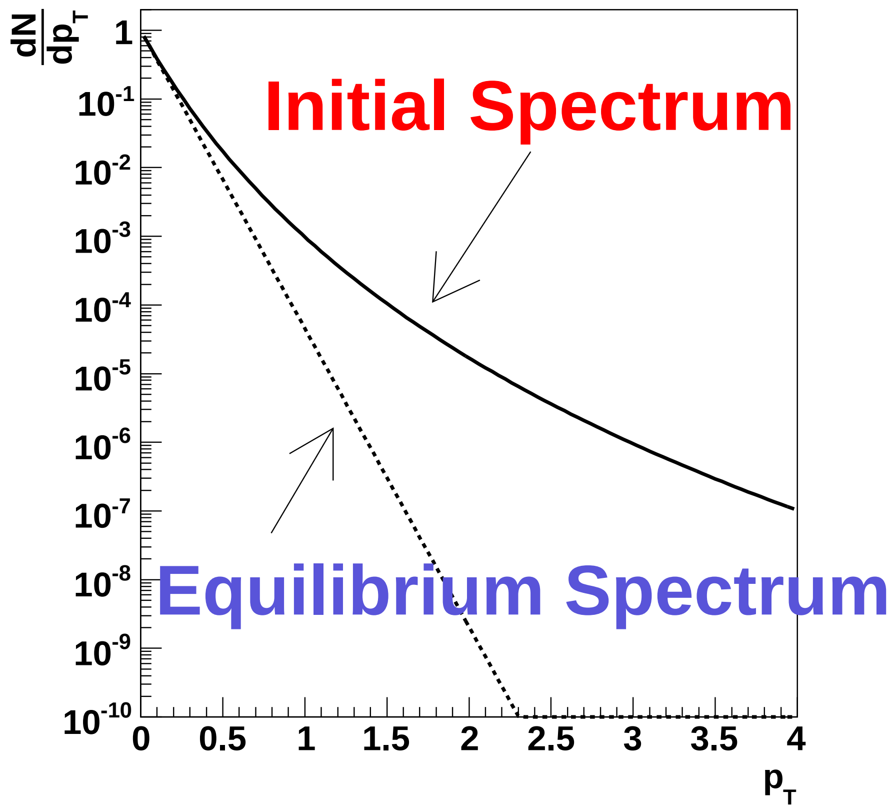

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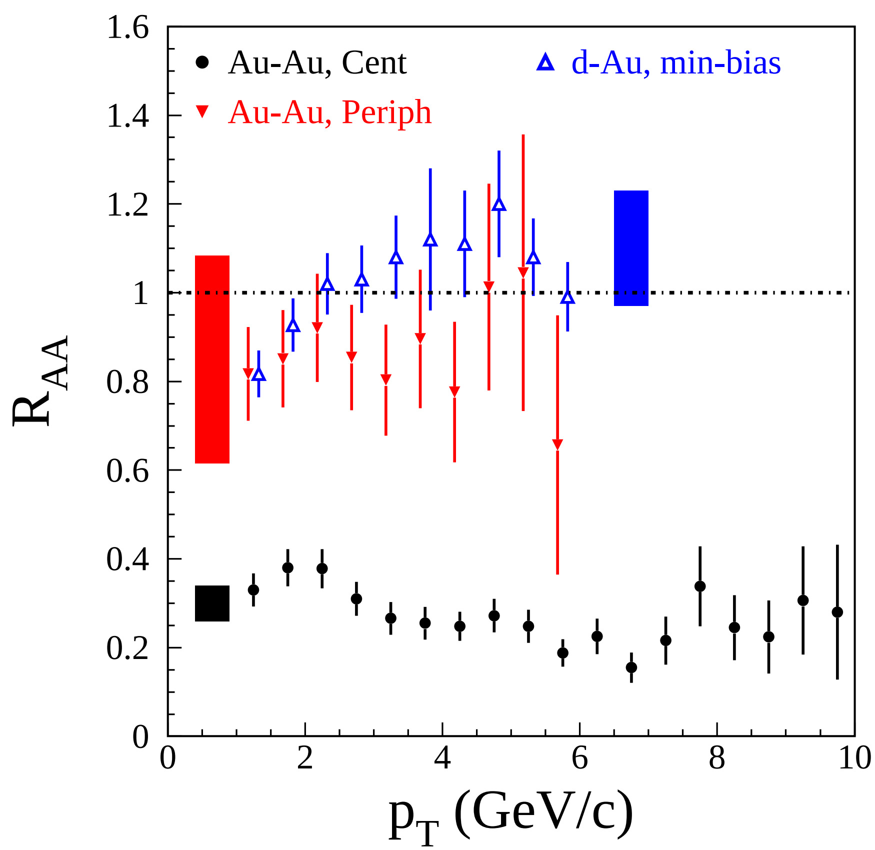

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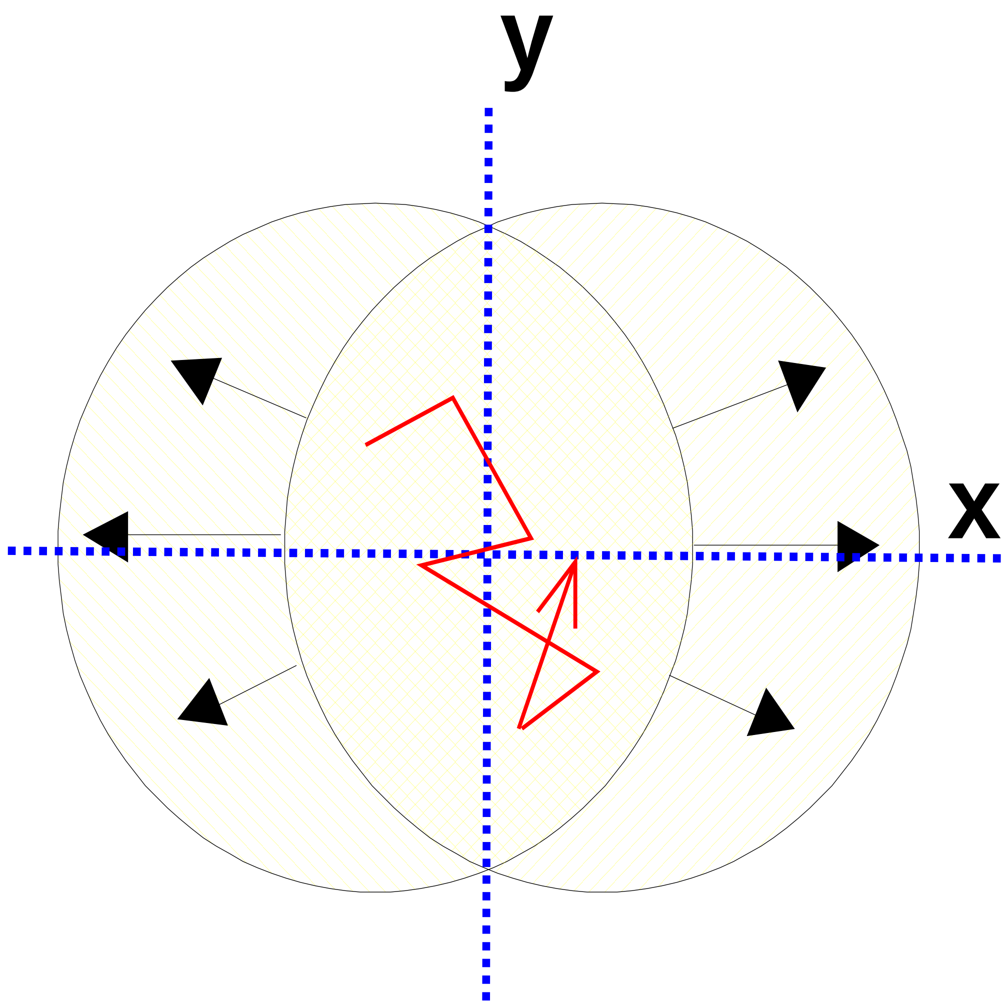

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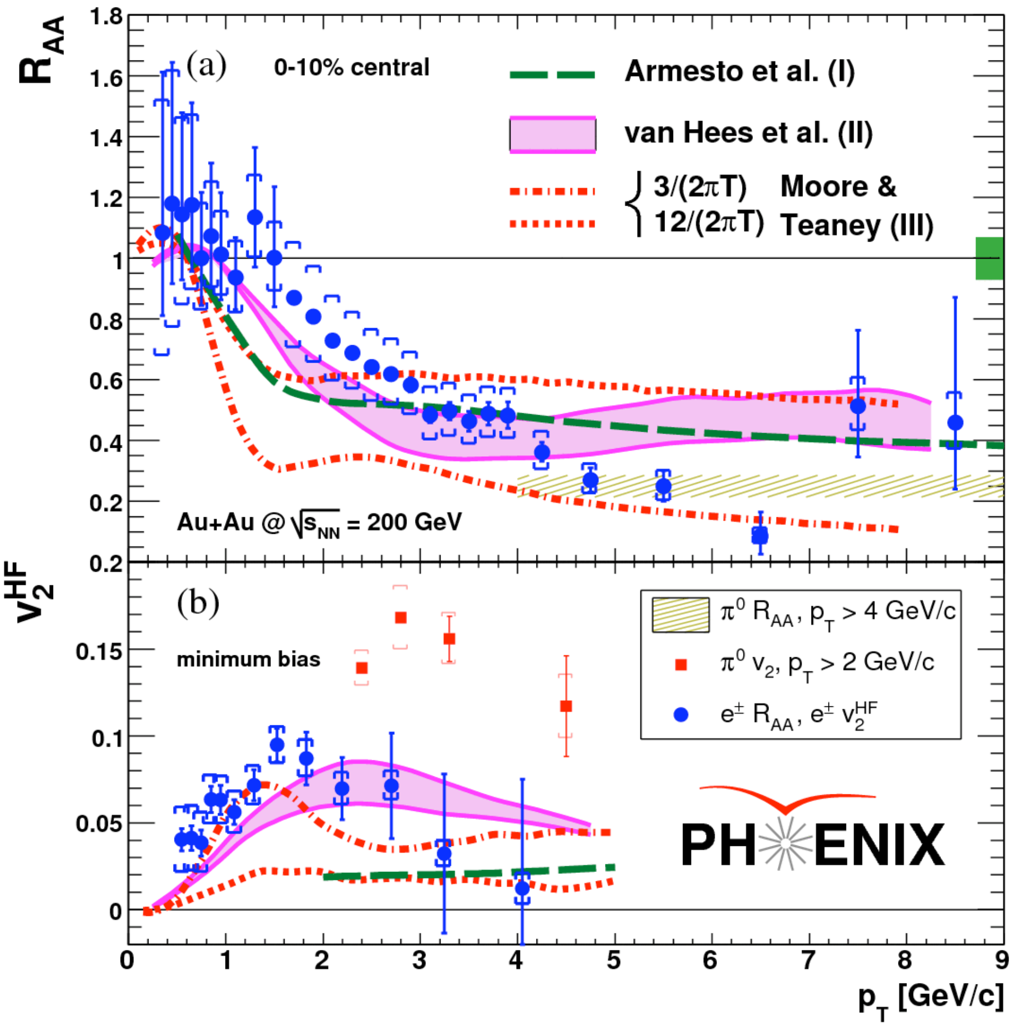


The diffusion coefficient is of order

$$D \sim \frac{1}{T}$$

Data Recapitulation

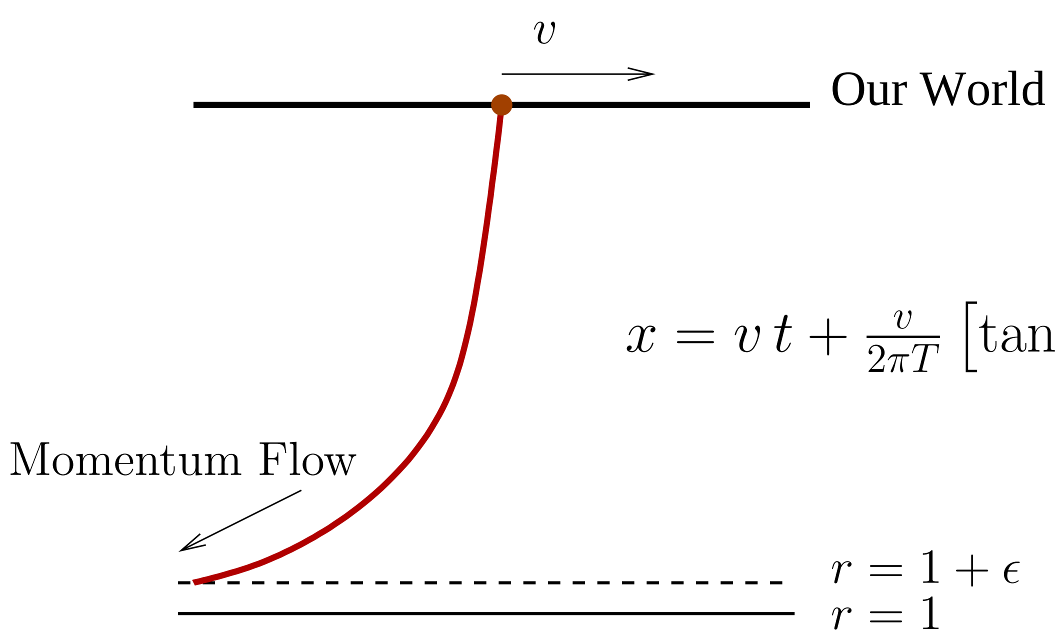
1. Energy loss significant

2. Heavy Quarks – Suppressed and flowing

Heavy Quarks in AdS/CFT

$$\frac{dP}{dt} = -\underbrace{\frac{\pi}{2}\sqrt{\lambda}T^2}_{\equiv\eta}\, v$$

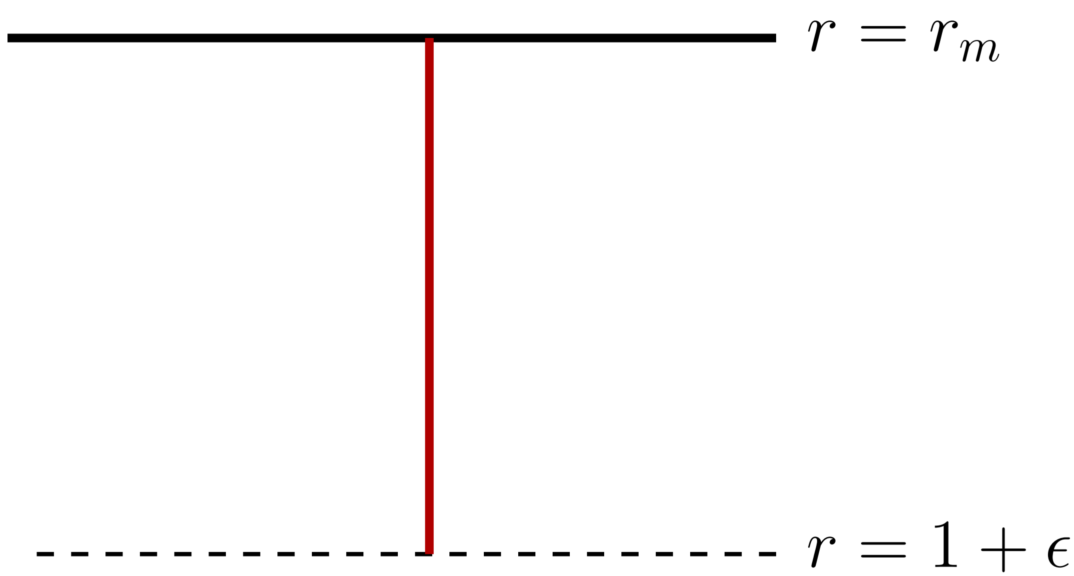
(a) HKKKY (b) DT, J. Casalderrey (c) S. Gubser



$$x = v\,t + \frac{v}{2\pi T}\left[\tan^{-1}(z) - \tanh^{-1}(z)\right]$$

Large drag of quarks in AdS/CFT

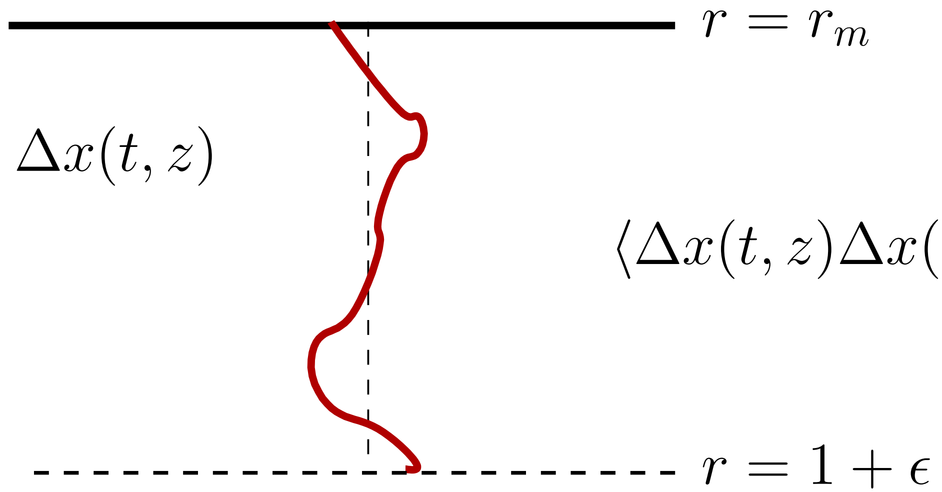
A heavy quark in AdS/CFT



Not the dual of an equilibrated quark!

A heavy quark in AdS/CFT with noise

$$M\frac{d^2\mathbf{x}}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x} + \underbrace{\xi}_{\text{Noise}} \qquad \langle \xi(t)\xi(t')\rangle = 2T\eta\,\delta(t-t')$$



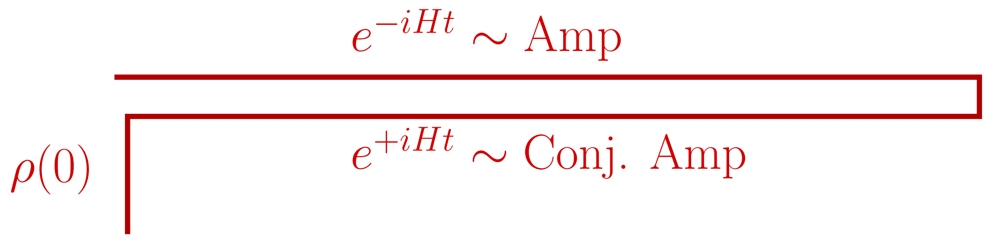
$$\langle \Delta x(t,z)\Delta x(t',z')\rangle \; = ?$$

Where is the noise? Hawking Radiation?

Brownian Motion

Brownian Motion

$$\rho(t) = e^{iHt}\rho(0)e^{-iHt}$$



- Consider a heavy particle coupled to bath a force on the contour

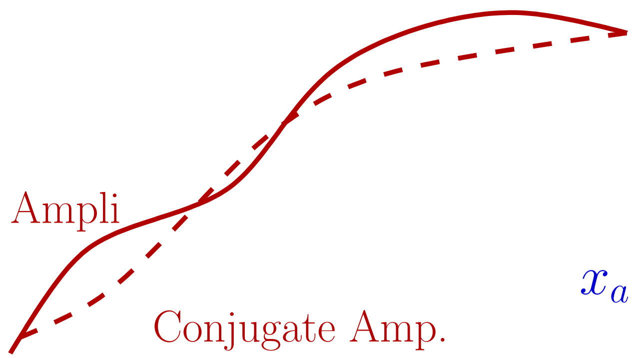
$$Z_Q = \left\langle \int Dx_1 Dx_2 \, e^{i\int \frac{1}{2}Mv_1^2 - i\int \frac{1}{2}Mv_2^2} \; e^{i\int dt_1 F_1 x_1} \; e^{-i\int dt_2 F_2 x_2} \right\rangle_{\mathrm{Bath}}$$

- Different correlators measure different time orderings

$$G_{11}(t,t') = \langle F_1 F_1 \rangle \quad = \quad \left\langle T[\hat{F}(t)\hat{F}(t')] \right\rangle$$

$$G_{21}(t,t') = \langle F_1 F_2 \rangle \quad = \quad \left\langle \hat{F}(t)\hat{F}(t') \right\rangle$$

Quasi-Classical Motion – "ra" basis



$$x_r \equiv \frac{x_1+x_2}{2}$$

$$x_a \equiv x_1 - x_2$$

x_a is small in classical limit

- Motion classical

$$Z_Q = \left\langle \int Dx_r Dx_a \, e^{i\int M\dot{x}_r\dot{x}_a} \; e^{i\int dt_1 F_r x_a} \; e^{-i\int dt_2 F_a x_r} \right\rangle_{\rm Bath}$$

- Only two point functions are:

$$\begin{aligned} \langle F_r F_a \rangle = G_R\;(t,t') &= \theta(t)\left\langle [F(t),F(t')] \right\rangle \\ \langle F_r F_r \rangle = G_{\rm sym}(t,t') &= \left\langle \left\{ F(t),F(t') \right\} \right\rangle \end{aligned}$$

$$G_{\rm sym}(\omega) = -(1+2n)\,{\rm Im}G_R(\omega)$$

Brownian Motion

1. The Partition Function in a heavy quark approximation

$$Z_Q = \int Dx_r Dx_a \, e^{-i\int x_a\left[-M\omega^2\right]x_r} \underbrace{e^{-i\int d\omega\, x_a G_R(\omega) x_r} e^{-\frac{1}{2}\int d\omega\, x_a\, G_{\rm sym}(\omega)\, x_a}}_{e^{iS_{\rm eff}}}$$

2. Replace Gaussian with fourier transform

$$e^{-\frac{1}{2}\int d\omega\; x_a G_{\rm sym}(\omega) x_a} = \int D\xi e^{i\int \xi x_a} e^{-\frac{1}{2}\int \xi G^{-1}_{\rm sym}(\omega)\xi}$$

3. Finally do the integrals over x_a

$$Z_Q = \int Dx_r D\xi\; \delta\left(-M\omega^2 x_r + \underbrace{G_R(\omega)}_{\rm Drag} x_r(\omega) + \underbrace{\xi(\omega)}_{\rm Noise}\right) e^{-\frac{1}{2}\xi G^{-1}_{\rm sym}(\omega)\xi}$$

Result: Generalized Langevin with memory

$$M_Q \frac{d^2 \bar{X}}{dt^2} + \int^t \underbrace{G_R(t-t')}_{\text{Drag}} \bar{X}(t') = \underbrace{\xi(t)}_{\text{noise}}$$

1. Drag = retarded force-force correlator

$$G_R(t) = \theta(t) \, \langle [F(t), F(0)] \rangle$$

2. Noise= symmetrized force-force correlator

$$\langle \xi(t)\xi(0) \rangle = \langle \{ F(t), F(0) \} \rangle$$

3. Fluctuation - Dissipation

$$\langle \xi(t)\xi(0) \rangle = -(1+2n)\mathrm{Im}G_R(\omega)$$

Fluctuation Dissipation - Oscillator

1. Commutator (or spectral density) is <u>*independent*</u> of wave function

$$\underbrace{[\hat{x}(\omega), \hat{x}(0)]}_{\text{spectral density} \equiv \rho(\omega)} \qquad \propto \delta(\omega - \omega_o) - \delta(\omega + \omega_o)$$

2. Anti-commutator measures how modes are occupied

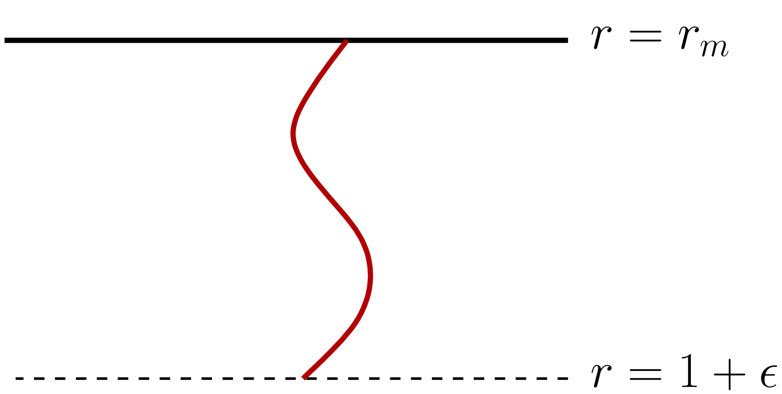
$$\frac{1}{2}\langle\{\hat{x}(\omega)\hat{x}(0)\}\rangle = \left(\frac{1}{2} + n(\omega)\right)\rho(\omega)$$

for equilibrium $n(\omega) = 1/(e^{\omega/T} - 1)$

Equilbrium is when the FDT is satsified

AdS/CFT

Small Fluctuations of the Straight String



- Action

$$S = -\int \mathrm{d}t\mathrm{d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x)^2 - \frac{\overbrace{m}^{\text{mass}}}{2f} (\partial_t x)^2 \right]$$

- Tension depends on radius

$$\text{Tension} \equiv T_o(r) \propto \sqrt{\lambda}\, f(r) r^4 \to \sqrt{\lambda}\, 4(r-1)$$

- Find two solutions: infalling $(-)$ and one outgoing $(+)$

$$x(\omega, r) \sim (r-1)^{\mp i\omega/4\pi T}$$

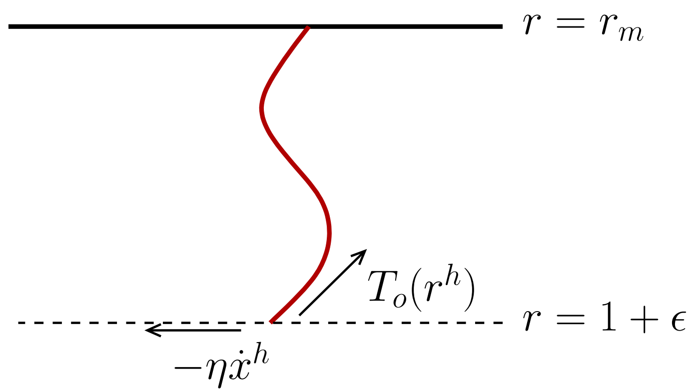
Infalling solution has a physical interpretation:

- Near the horizon we have

$$x(\omega, r) \approx (r-1)^{-i\omega/4\pi T} \qquad \to \qquad \overbrace{(r-1)}^{\propto T_o(r)}\, \partial_r x(\omega, r) = \frac{-i\omega}{4\pi T} x(\omega, r)$$

- Multiply by constant $2\pi\sqrt{\lambda}T^3$

$$\underbrace{T_o(r_h)\,\partial_r x(r_h, t)}_{\text{Tension}} = \underbrace{\eta \dot{x}(r_h, t)}_{\text{Drag}}$$



Horizon motion is overdamped – no acceleration on stretched horizon

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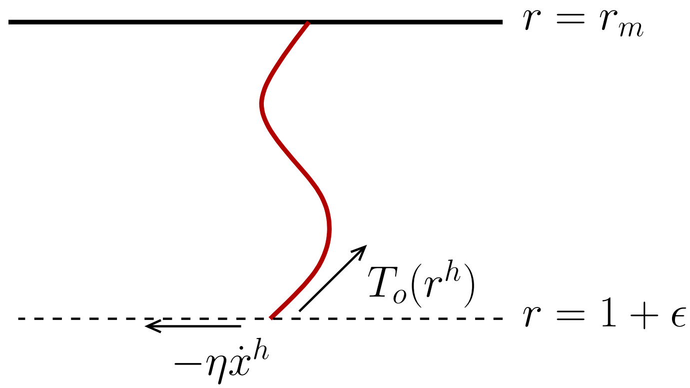
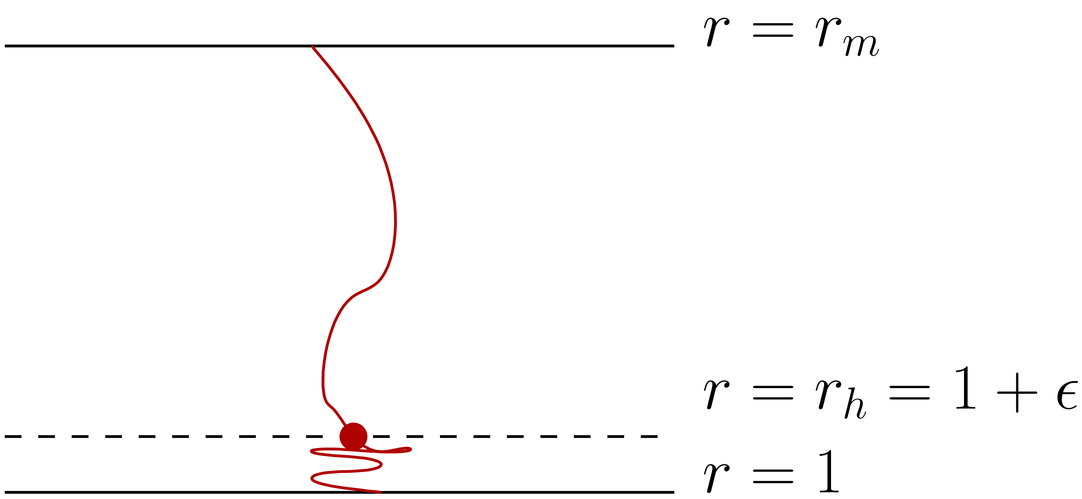
- Multiply by constant $2\pi\sqrt{\lambda}T^3$

$$\underbrace{T_o(r_h)\,\partial_r x(r_h, t)}_{\text{Tension}} = \underbrace{\eta \dot{x}(r_h, t)}_{\text{Drag}} + \underbrace{???}_{\text{noise}}$$



Horizon motion is overdamped – no acceleration on stretched horizon

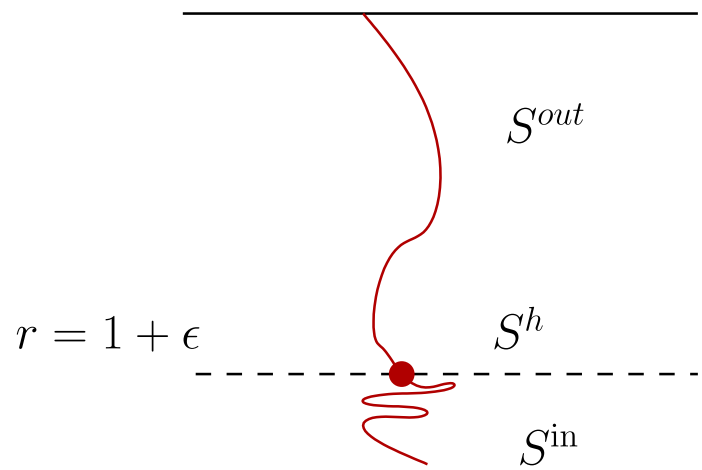
Strategy: Integrate out the fluctuations below the stretched horizon



Find an effective action and an EOM for the horizon endpoint x^h

$$S_{\mathrm{eff}}^h$$

Classical Membrane Paradigm



- Re write the action

$$
\begin{aligned}
S =& S^{\text{out}} + S^{\text{in}} \\
=& (S^{\text{out}} + S^h) + (S^{\text{in}} - S^h)
\end{aligned}
$$

- Vary the pieces

$$\delta S = (\delta S^{\text{out}} + \delta S^h) + (\delta S^{\text{in}} - \delta S^h)$$

Choose S^h so that the out and in actions are separately minimized

Actually doing the membrane paradigm

$$S^{\rm in} - S^h = -\int \mathrm{d}t\mathrm{d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x_r)(\partial_r x_a) - \frac{\overbrace{m}^{\text{mass}}}{2f} \dot{x}_r \dot{x}_a \right] - S^h$$

- Varying with respect to x_a gives

$$\left[\partial_r \left(T_o(r) \partial_r x_r(\omega, r) \right) + \frac{m\omega^2}{f} x_r(\omega, r) \right] = 0$$

$$T_o(r_h) \partial_r x_r(\omega, r) = \frac{\delta S^h}{\delta x_a} \qquad \Leftarrow \text{Horizon boundary condition}$$

- Recall: the classical solution obeys infalling boundary condition: $x = C(r-1)^{-i\omega/4\pi T}$

$$T_o(r) \partial_r x_r = -i\omega\eta \, x_r^h \qquad \Leftarrow \text{Infalling bc}$$

So at a classical level the horizon action

$$S^h_{\rm eff} = \int_\omega x_a^h \, [\underbrace{-i\omega\eta}_{G_R^h(\omega)}] \, x_r^h$$

Actually doing the membrane paradigm

$$S^{\rm in} - S^h = -\int {\rm d}t{\rm d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x_r)(\partial_r x_a) - \frac{\overbrace{m}^{\text{mass}}}{2f} \dot{x}_r \dot{x}_a \right] - S^h$$

- Varying with respect to x_a gives

$$\left[\partial_r \big(T_o(r) \partial_r x_r(\omega, r) \big) + \frac{m\omega^2}{f} x_r(\omega, r) \right] = 0$$

$$T_o(r_h) \partial_r x_r(\omega, r) = \frac{\delta S^h}{\delta x_a} \qquad \Leftarrow \text{Horizon boundary condition}$$

- The classical solution obeys infalling boundary condtion: $x = C(r-1)^{-i\omega/4\pi T}$

$$T_o(r) \partial_r x_r = -i\omega\eta x_r^h \qquad \Leftarrow \text{Infalling bc}$$

So at a quantum level guess the horizon action:

$$iS^h_{\rm eff} = -i \int_\omega x^h_a [\underbrace{-i\omega\eta}_{G^h_R(\omega)}] x^h_r - \frac{1}{2} \int_\omega x^h_a [\underbrace{(1+2n)\omega\eta}_{G^h_{\rm sym}(\omega)}] x^h_a$$

Brownian Motion of the horizon

$$iS^h_{\rm eff} \quad = \quad -i\int_\omega x^h_a\,[\underbrace{-i\omega\eta}_{G_R(\omega)}]\,x^h_r - \frac{1}{2}\int_\omega x^h_a\,[\underbrace{(1+2n)\omega\eta}_{G_{\rm sym}(\omega)}]\,x^h_a$$

1. Dynamics of String in AdS_5

$$Z = \int \left[\mathbb{D}x_1\,\mathrm{D}x_1^h\right]\,\left[\mathbb{D}x_2\,\mathrm{D}x_2^h\right]\,e^{iS_1-iS_2}\,e^{iS^h_{\rm eff}}$$

2. Particle Dynamics

$$Z_Q \quad = \quad \int Dx_r Dx_a\,e^{-i\int x_a\left[-M\omega^2\right]x_r}\,\underbrace{e^{-i\int d\omega x_a G_R(\omega)x_r}\,e^{-\frac{1}{2}\int d\omega x_a\,G_{\rm sym}(\omega)\,x_a}}_{e^{iS_{\rm eff}}}$$

3. Go through the same steps as with the particle

$$\left[\partial_r\left(T_o(r)\partial_r x_r(\omega,r)\right) + \frac{m\omega^2}{f}x_r(\omega,r)\right] = 0 \qquad \Leftarrow \text{Bulk equation of motion}$$

$$T_o(r_h)\partial_r x_r(\omega,r_h) + \xi^h(\omega) = -i\omega\eta x^h_r(\omega) \qquad \Leftarrow \text{Horizon boundary condition}$$

$$T_o(r_m)\partial_r x(\omega,r_m) = 0 \qquad \Leftarrow \text{Neumann boundary condition}$$

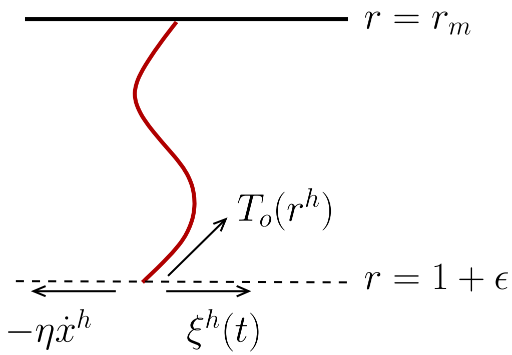
Summary

Hawking radiation and the random force

- Still overdamped motion with a random force

$$\overbrace{T_o(r_h)\partial_r x(t, r_h)}^{\text{Tension}} + \overbrace{\xi^h(t)}^{\text{Random force}} = \overbrace{\eta\, \dot{x}^h(t)}^{\text{Drag}}$$

- Picture



- Random force satisfies a horizon fluctuation dissipation theorem

$$\left\langle \xi^h(t)\xi^h(t') \right\rangle = 2T\eta\delta(t - t')$$

- More Generally when times of order $1/\pi T$

$$\left\langle \xi^h(\omega)\xi^h(-\omega) \right\rangle = (1 + 2n)\omega\eta$$

Gives rise to random motion on the string

Preliminaries

- Action

$$S = \frac{\sqrt{\lambda}}{2\pi} \int \mathrm{d}t\mathrm{d}r \, g_{xx} \left[-\frac{1}{2}\sqrt{h} h^{\mu\nu} \partial_\mu x \partial_\nu x \right] ,$$

- Coordinates and Metric: $t, r \to v, r$

$$h_{\mu\nu}\mathrm{d}\sigma^\mu \mathrm{d}\sigma^\nu = (\pi T)^2 \left[-A(r)\mathrm{d}v^2 + \frac{2}{\pi T}\mathrm{d}r\mathrm{d}v \right] .$$

- Drag

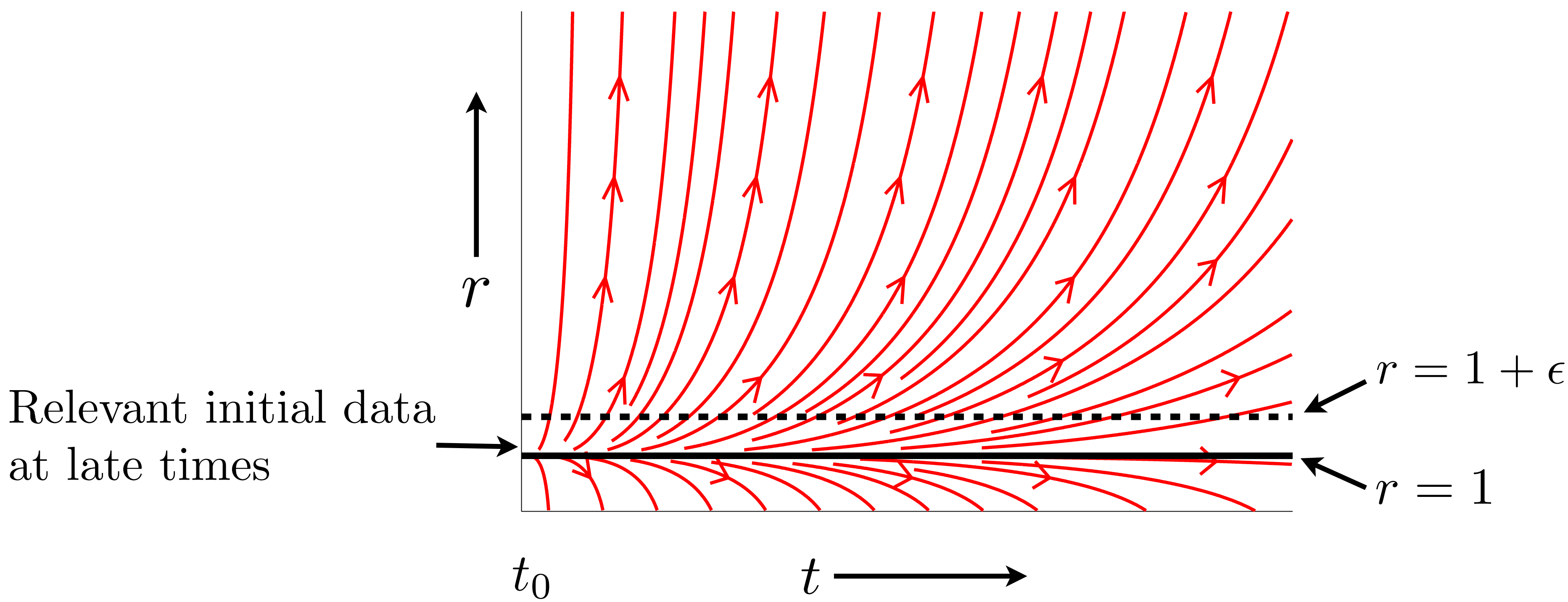
$$\eta \equiv \frac{\sqrt{\lambda}}{2\pi} g_{xx}(r_h) = \frac{\sqrt{\lambda}}{2\pi}(\pi T)^2 .$$

- Tension

$$T_o(r) = \frac{\sqrt{\lambda}}{2\pi} g_{xx}(r)\sqrt{h} h^{rr}(r)$$

Formal solution and initial data

$$G_{\rm sym}(1|2) = \underbrace{\left[\frac{\sqrt{\lambda}}{2\pi}\int \mathrm{d}r'_1\; g_{xx}\sqrt{h}h^{tt}(r'_1)\,G_R(1|1')\overleftrightarrow{\partial_{t'_1}}\right] \times \left[\frac{\sqrt{\lambda}}{2\pi}\int \mathrm{d}r'_2\; g_{xx}\sqrt{h}h^{tt}(r'_2)\,G_R(2|2')\overleftrightarrow{\partial_{t'_2}}\right]}_{\text{formal sol}} G_{\rm sym}(1'|2')\,,$$



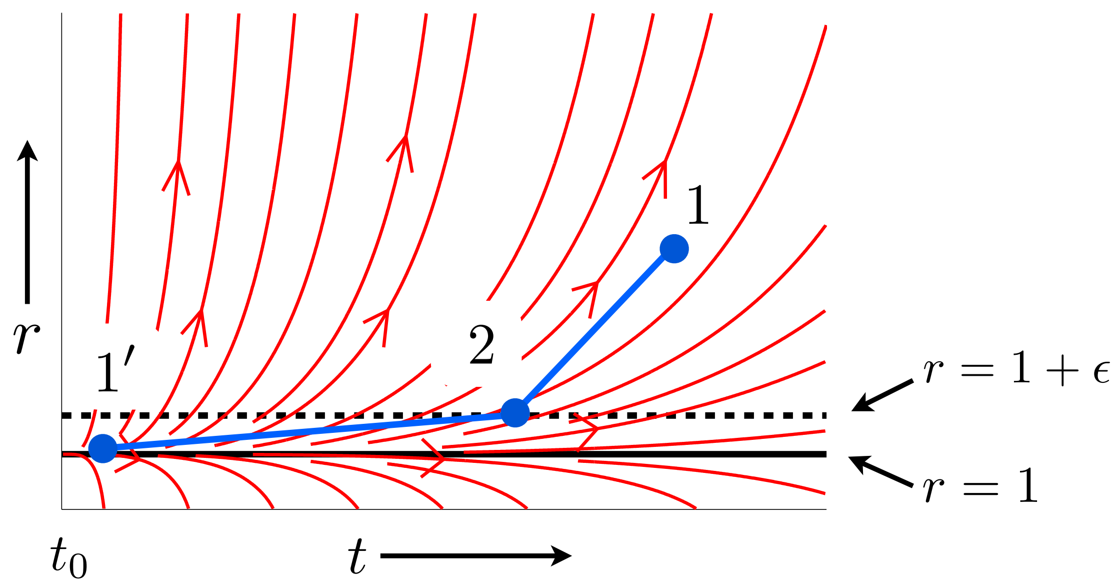
Two step evolution:

(a) From horizon to stretched horizon

(b) The stretched horizon to boundary

- Take (choice) reflective boundary conditions $g_R(t_1 r_1 = 1+\epsilon | t_2 r_2) = 0$

$$G_R(1|1') = \underbrace{\int \mathrm{d}t_2\, G_R(1|2)\ T_o(r_2) \frac{\partial}{\partial r_2}\, g_R(t_2 r_2|1')\Big|_{r_2=1+\epsilon}}_{\text{composition rule}}$$

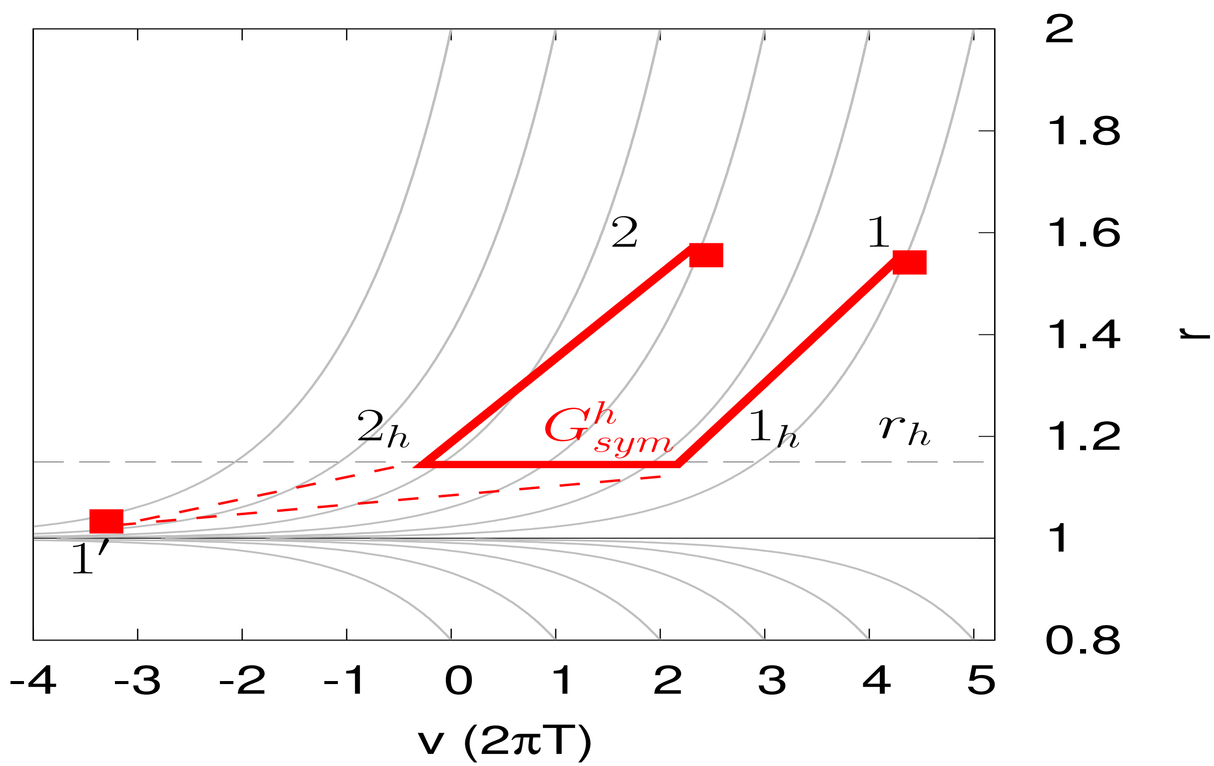


Result

$$G_{\text{sym}}(1|2) = \overbrace{\int \mathrm{d}t_{1h}\mathrm{d}t_{2h}\, G_R(1|1_h)\; G_R(2|2_h)\; G^h_{\text{sym}}(1_h|2_h)}^{\text{looks like a feynman graph}}\,,$$

where

$$G^h_{\text{sym}}(t_1|t_2) = T_o(r_1)T_o(r_2)\;\partial_{r_1}\partial_{r_2} g_{\text{sym}}(1|2)\Big|_{r_1,r_2=1+\epsilon}\,.$$



Need to show that $G^h_{\text{sym}}(\omega) = (1+2n)\omega\eta$

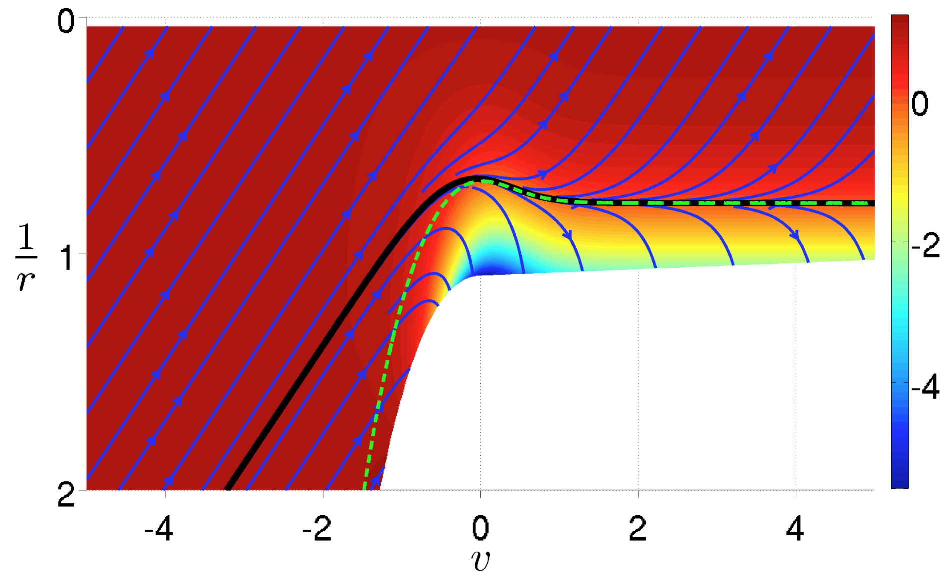
Solution to Geodesic Equations

$$
\begin{aligned}
g_{rr}(v_1 r_1 | v_2 r_2) = &+ f(\underbrace{e^{-2\pi T v_1}(r_1 - 1)}_{\text{out}}, \underbrace{e^{-2\pi T v_2}(r_2 - 1)}_{\text{out}}) \\
&- f(\underbrace{e^{-2\pi T v_1}(r_1 - 1)}_{\text{out}}, \underbrace{e^{-2\pi T v_2}(r_h - 1)}_{\text{in}}) \\
&- f(\underbrace{e^{-2\pi T v_1}(r_h - 1)}_{\text{in}}, \underbrace{e^{-2\pi T v_2}(r_2 - 1)}_{\text{out}}) \\
&+ f(\underbrace{e^{-2\pi T v_1}(r_h - 1)}_{\text{in}}, \underbrace{e^{-2\pi T v_2}(r_h - 1)}_{\text{in}}),
\end{aligned}
$$

Non-equilibrium

- In 4D world a strong gravitation pulse rips up vacuum creating plasma

$$ds^2 = A dv^2 + 2 dr dv + \Sigma^2 \left(e^{B} d\mathbf{x}_\perp^2 + e^{-2B} dx_\parallel^2 \right) \ ,$$



- Surface Properties

$$\underbrace{2\pi T_{\text{eff}}(v)}_{\text{Lyapunov exponent}} \equiv \frac{1}{2} \left. \frac{\partial A(r,v)}{\partial r} \right|_{r=r_h(v)} \qquad \underbrace{\eta(v) \equiv \frac{\sqrt{\lambda}}{2\pi} g_{xx}(r_h(v), v)}_{\text{non-equil drag}} \ .$$

Results

1. Anti-commutator

$$G_{\rm sym}(1|2) = \int {\rm d}v_{1h}{\rm d}v_{2h}\, G_R(1|1_h)\; G_R(2|2_h)\; G^h_{\rm sym}(1_h|2_h)\,.$$

where

$$G^h_{rr}(v_1|v_2) = -\frac{\sqrt{\eta(v_1)\eta(v_2)}}{\pi}\partial_{v_1}\partial_{v_2}\log\left|1-e^{-\int_{v_1}^{v_2} 2\pi T_{\rm eff}(v')dv'}\right|\,.$$

2. Commutator – initial conditions from canonical commutation relations

$$\rho(1|2) = \int {\rm d}v_{1h}{\rm d}v_{2h}\, G_R(1|1_h)\, G_R(2|2_h) \qquad \underbrace{\rho^h(v_{1h}|v_{2h})}_{\text{from commutation rel.}} \qquad ,$$

where

$$\rho^h(v_{1h}|v_{2h}) = 2\sqrt{\eta(v_{1h})\eta(v_{2h})}\delta'(v_1-v_2)\,,$$

Equilibration

- Take Wigner transforms of horizon correlator. Dissipation is local

$$\rho^h(\bar{v},\omega) = \int_{-\infty}^{\infty} \mathrm{d}(v_1 - v_2) e^{+i\omega(v_1 - v_2)} \rho^h(v_1, v_2)\,,$$
$$= 2\eta(\bar{v})\,\omega\,.$$

- For a non equilibrium timescale τ we have

$$G^h_{\mathrm{sym}}(\bar{v},\omega) \simeq \left(\frac{1}{2} + n(\omega)\right)\rho^h(\bar{v},\omega) \,+\, O\left(\frac{1}{\tau^2\omega^2}\right)\,,$$

High frequencies are born into equilibrium on the event horizon

Conclusions

1. Quantum Mechanics of AdS_5 leads to thermal noise
   - Prototypical Example - Brownian Motion
2. Other fields also fluctuate: the dilation ϕ, the graviton $h^{\mu\nu}$, etc, fluctuate
   - Applications?
3. Gave a different derivation of the Hawking flux that extends to non-equilibrium
   1. Relation to Tunneling?
   2. Entanglement entropy?
   3. Fluctuation Dissipation: gravity pulls down, and fields fluctuate