The Large Nc Limits of QCD

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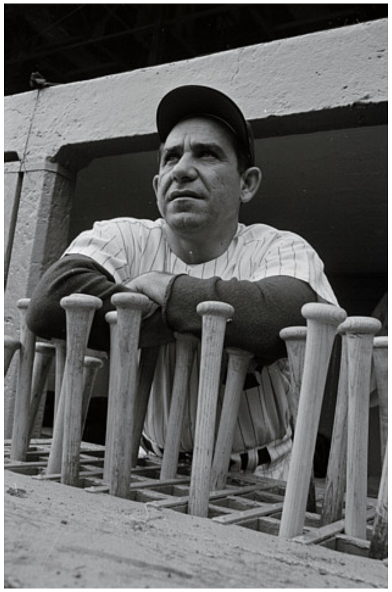
Outline

- QCD and its large Nc limits: different treatments of fermions yield distinct large Nc limits.
  - Quarks: fundamental (F)
  - 2-index anti-symmetric (AS)
  - Hybred or Corrigan-Ramond (CR)
- Generic properties
- Baryons & baryon models
- Nuclear interactions & dense matter



Quarks in Fundamental

Quarks in 2-index anti-symmetric

“When you come to a fork in the road, take it.”
---Yogi Berra,
American baseball player, coach and part-time philosopher

“Two roads diverged in a wood, and I—
I took the one less traveled by
And that has made all the difference.”
---Robert Frost,
American poet

Large Nc QCD

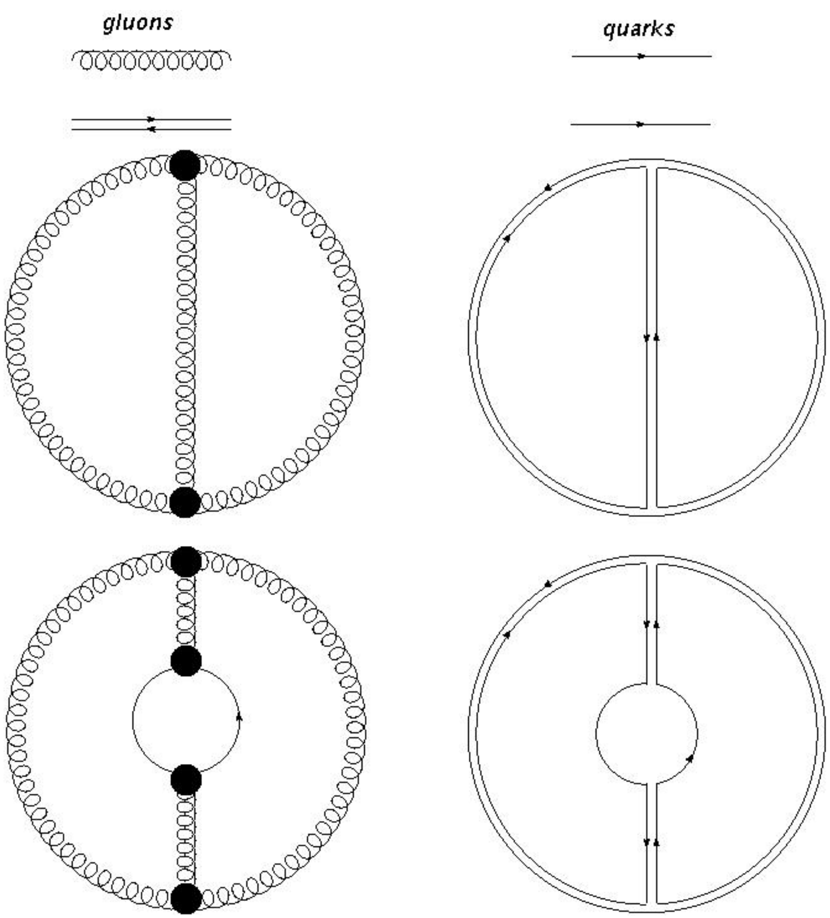
QCD and its large Nc limits:

- The large Nc limit of QCD is not unique
  - For gluons there is a unique prescription SU(3)➔SU(Nc)
  - However for quarks, we can choose different representations of the gauge group
  - Asymptotic freedom restricts the possibilities to the fundamental (F), adjoint (Adj), two index symmtetric (S), two index anti-symmtetric (S),
    - Adj transforms like gluons (traceless fundamental color-anticolor); dimension Nc^2-1; 8 for Nc=3 (unlike our world).
    - S transforms like two colors (eg fundamental quarks) with indices symmetrized; dimension Nc^2-Nc; 6 for Nc=3 (unlike our world).
    - AS transforms like two colors (eg fundamental quarks) with indices antisymmetrized; dimension ½Nc(Nc-1); 3 for Nc=3 (just like our world).

- Note that Nc=3 quarks in the AS representation are indistinguishable from the (anti-) fundamental.
- However quarks in the AS and F extrapolate to large Nc in different ways.
  - The large Nc limits are physically different
  - The 1/Nc expansions are different.
  - A priori it is not obvious which expansion is better
  - It may well depend on the observable in question
- The idea of using QCD (AS) at large Nc is old
  - Corrigan &Ramond (1979)
  - Idea was revived in early part of this decade by Armoni, Shifman and Veneziano who discovered a remarkable duality that emerges at large Nc.

Principal difference between QCD(AS) and QCD(F) at large Nc is in the role of quarks loops

Easy to see this using `t Hooft color flow diagrams



$$\left.\begin{array}{r} g^2 \sim 1/N_c \\ 3\ \text{color loops}\ N_c^3 \end{array}\right\} N_c^2$$

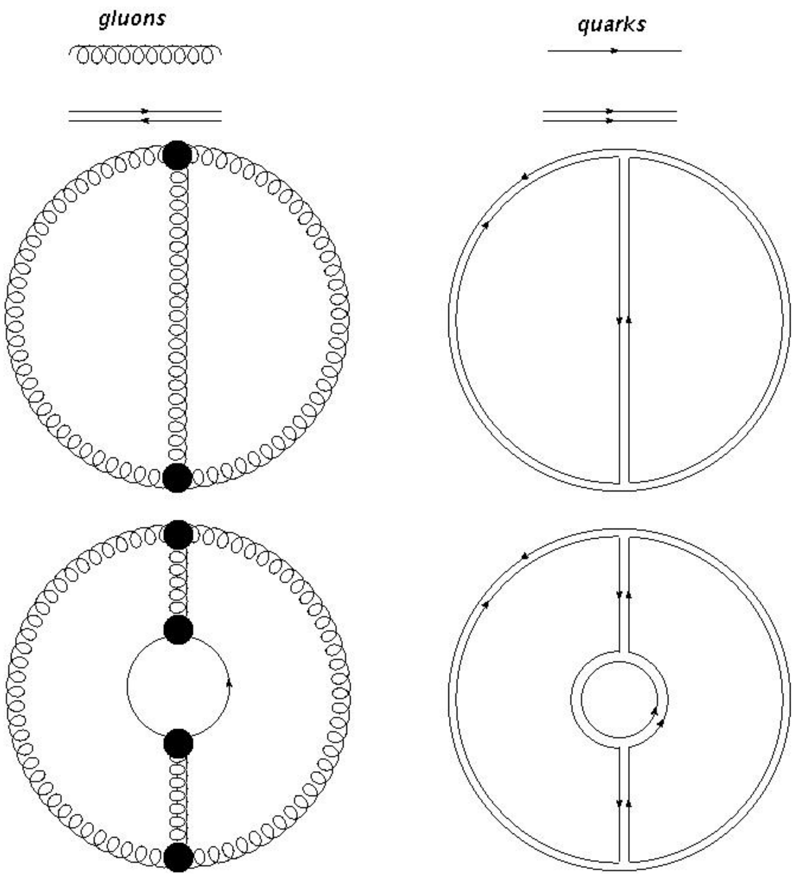
$$\left.\begin{array}{r} g^4 \sim 1/N_c^2 \\ 3\ \text{color loops}\ N_c^3 \end{array}\right\} N_c$$

QCD(F)

Insertion of a planar quark loops yields a 1/Nc suppression.

Leading order graphs are made of planar gluons

QCD(AS)



$$\left.\begin{array}{r} g^2 \sim 1/N_c \\ \text{3 color loops } N_c^3 \end{array}\right\} N_c^2$$

Insertion of a planar quark loops does not lead to a 1/Nc suppression.

$$\left.\begin{array}{r} g^4 \sim 1/N_c^2 \\ \text{4 color loops } N_c^4 \end{array}\right\} N_c^2$$

Leading order graphs are made of planar gluons and quarks

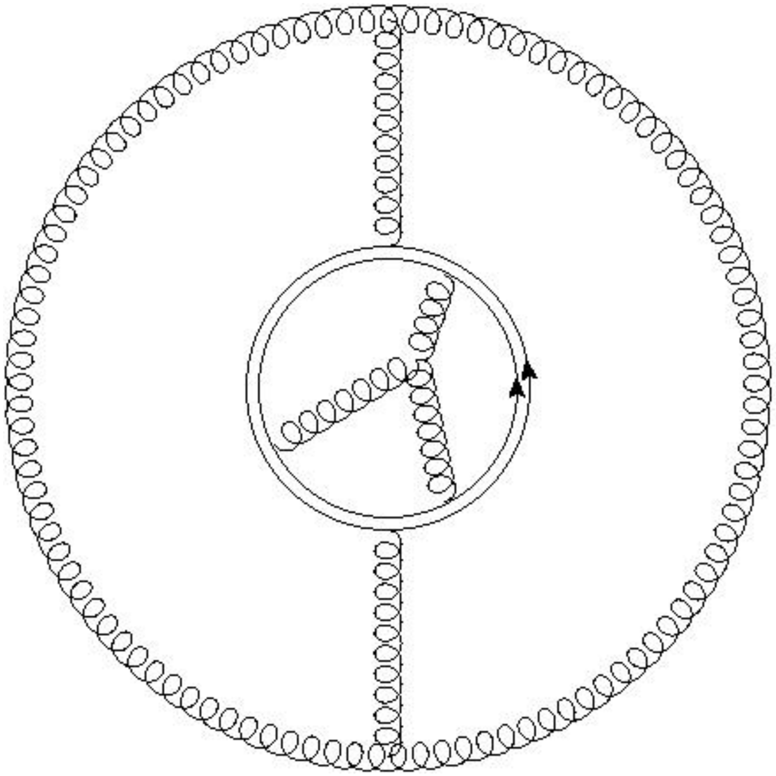
Principal phenomenological difference between the two is the inclusion of quark loop effects at leading order in QCD (AS)

A remarkable fact about QCD(AS):
At large Nc, QCD(AS) with Dirac fermions becomes equivalent to QCD(Adj) with Majorona fermions for a certain class of observables. These "neutral sector" observables include $\langle \bar{q} q \rangle$.

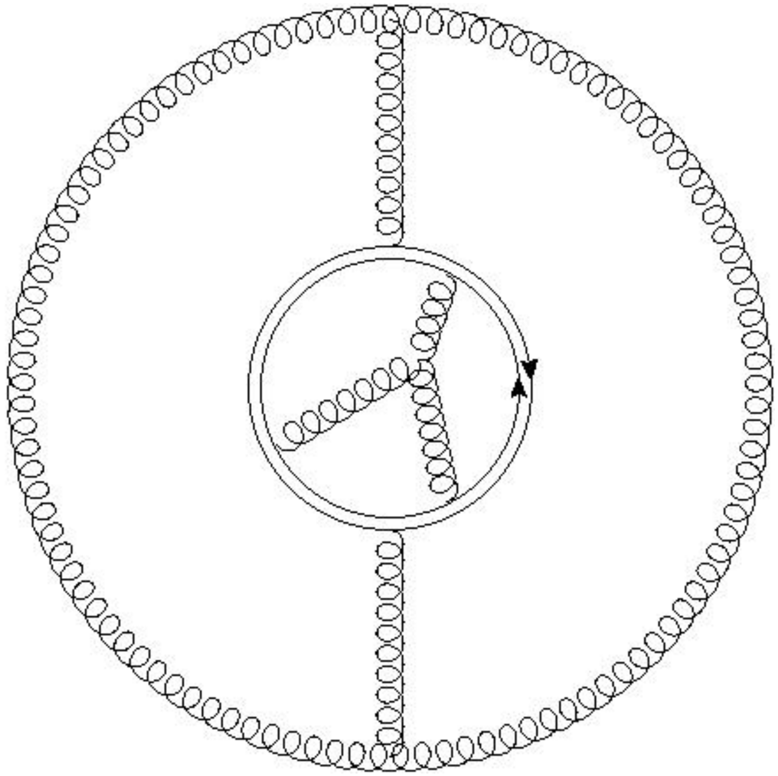
The full nonperturbative demonstration of this by Armoni, Shifman and Venziano (ASV)is quite beautiful and highly nontrivial. There is a simple hand waving argument which gets to the guts of it

Due to large Nc planarity, any fermion loops divide any gluons in a diagram into those inside and those outside.

With two index representations the "inside" gluons couple to the inner color line of the quark and "outside" gluons to the outer ones



QCD(AS)



QCD(Adj)

Since the inside gluons don't know about what happens outside, one can flip the direction of color flow on the outside without changing the dynamics.

This equivalence is pretty but can you make any money on it?

If all you can do is relate one intractable theory to another, it would be of limited utility.

However: QCD(Adj) with a single massless quark is $\mathcal{N}$=1 SUSY Yang-Mills. Thus, at large Nc a non-Supersymmetric theory (QCD(AS) with one flavor) is equivalent to a supersymmetric theory. Thus one can use all the power of SUSY to compute observables in $\mathcal{N}$=1 SYM and at large Nc one has predicted observables in QCD(AS) !

Can you make any *phenomenological* money on it?

Real QCD has more than one flavor!!!

ASV scheme: Suppose you put the quarks one flavor in the AS representation and the other flavor(s) in the F. For example put up quarks in AS and down quarks in F. The ones in the F are dynamically suppressed at large Nc and the theory again becomes equivalent to $\mathcal{N}$=1 SYM. In fact this is precisely the Corrigan-Ramond scheme introduced long ago to ensure baryons with 3 quarks at any Nc.

But…

In my view, the scheme is likely not be viable phenomenologically. The 1/Nc expansion is based on the assumption that the large Nc world is similar to the Nc=3 one. In this case they are radically different .

Isospin (or more generally flavor symmetry) is badly broken at large Nc since the flavors are treated different.

At any Nc≠3, this isospin violation is large!!!

For example while you can form $\bar{u}u$ mesons and $\bar{d}d$ mesons for arbitrary Nc, $\bar{u}d$ and $\bar{d}u$ only exist for Nc=3; for all other Nc, they are not color singlets. Don't get an isotriplet of pions except at Nc=3.

Large isospin violations occur as soon as one departs Nc=3; one does not have the isospin violation smoothly turning off as Nc approaches 3.

Accordingly in the remainder of this talk I will focus entirely on the cases where all flavors are either AS or F.

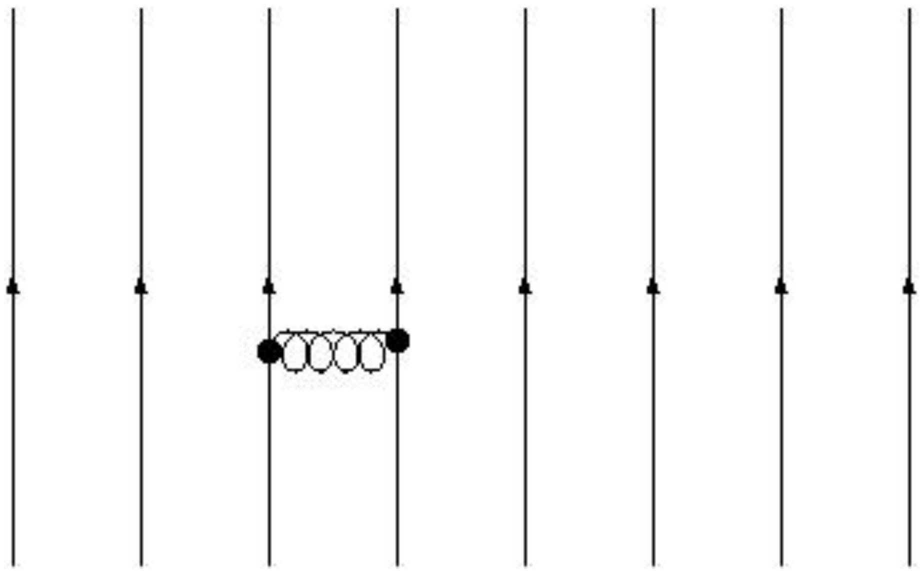
Generic Virtues and Vices of QCD(AS) and QCD(F) at large Nc

		
QCD(F)	Explains the success of the OZI rule in a natural way	Fails to explain effects involving the anomaly (eg. η')
QCD(AS)	Naturally includes effects involving the anomaly	Fails to explains the success of the OZI rule

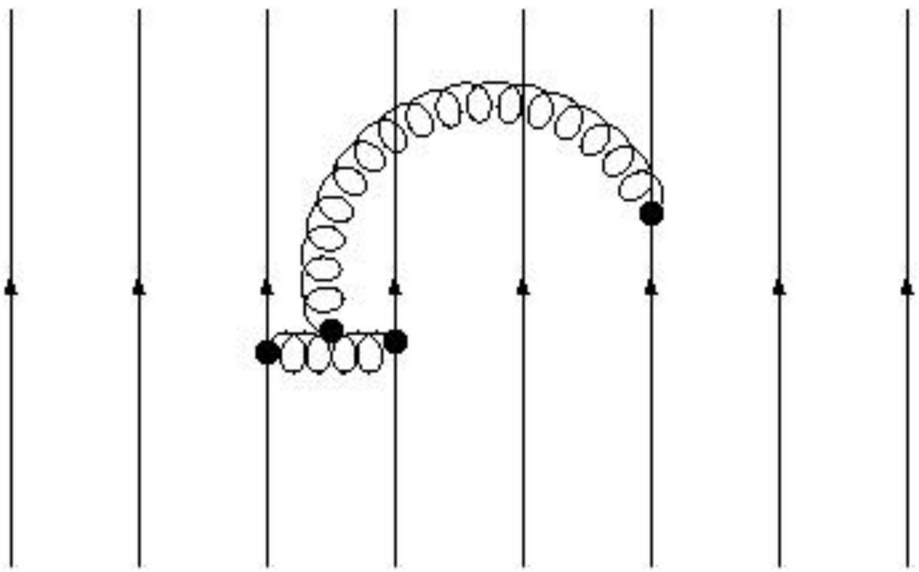
Implication for Baryons and Baryon Models

- Baryons are heavy
  - QCD(F) $M_N \sim Nc$ (Consistency shown by Witten 1979)
  - QCD(AS) $M_N \sim Nc^2$ (Consistency shown by Cherman&TDC 2006, Bolognesi 2006; TDC, Lebed, Schafer 2010)

QCD(F): There are Nc quarks each of which contributes to the energy as it propagates. The interactions between between quarks also contribute of order Nc.



$$\left.\begin{array}{c} g^2 \sim 1/N_c \\ \text{Combinatoric Factor } N_c^2 \\ \text{(each vertex has } N_c \text{ possibilities} \end{array}\right\} N_c$$



$$\left.\begin{array}{c} g^4 \sim 1/N_c^2 \\ \text{Combinatoric Factor } N_c^3 \\ \text{(each vertex has } N_c \text{ possibilities)}\end{array}\right\} N_c$$

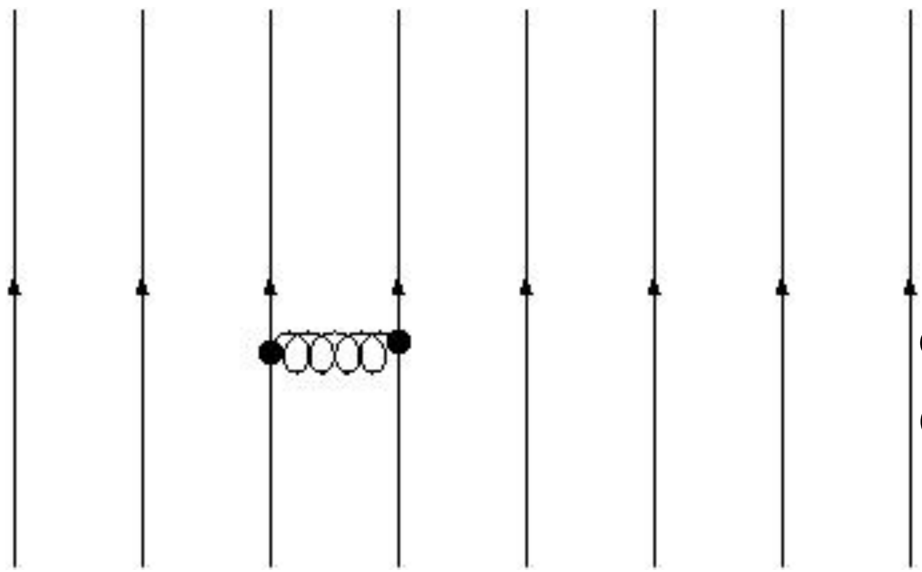
Relatively easy to see that all classes of connected diagram contribute at order Nc or less to the mass in QCD(F).

What about QCD(AS)?

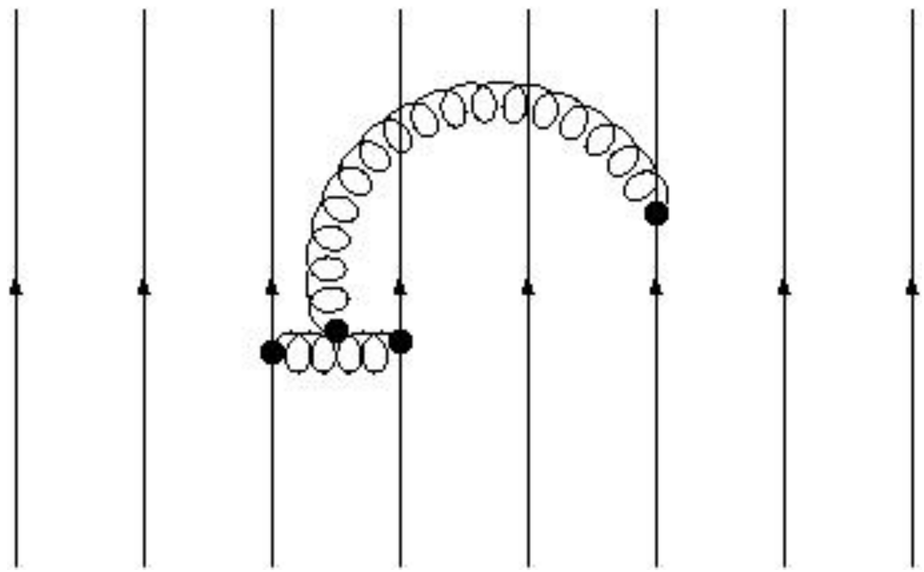
Bolognesi showed that a color singlet baryon had each kind quark color once and only once: Nc(Nc-1)/2 quarks. Thus one expects baryon mass to scale as Nc^2

- There is a problem: apply Witten's reasoning and there is an inconsistency---the interactions don't appear to scale as Nc^2

Look at the one-gluon contribution



$$\left.\begin{array}{c} g^2 \sim 1/N_c \\ \text{Combinatoric Factor } \sim N_c^4 \\ \text{(each vertex has } \sim N_c^2 \text{ possibilities)} \end{array}\right\} N_c^3 \neq N_c^2$$



$$g^4 \sim 1/N_c^2$$
Combinatoric Factor $\sim N_c^6$
(each vertex has $\sim N_c^2$ possibilities)

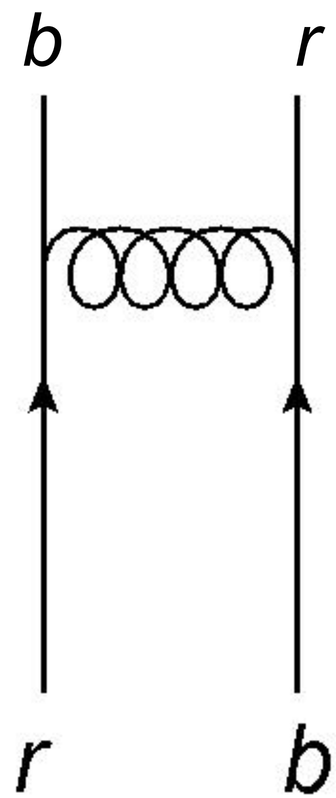
$$\left.\begin{array}{l} g^4 \sim 1/N_c^2 \\ \text{Combinatoric Factor } \sim N_c^6 \\ \text{(each vertex has } \sim N_c^2 \text{ possibilities)} \end{array}\right\} N_c^4 \neq N_c^2$$

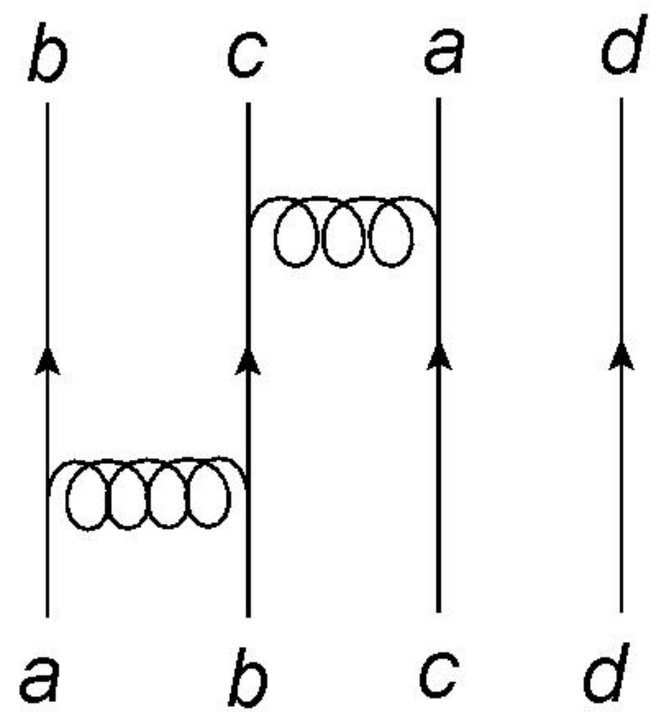
Even worse!!

What's going on?

The combinatorics are wrong. There is a subtlety which does not arise in the case of QCD(F)

Examples (in QCD_F)





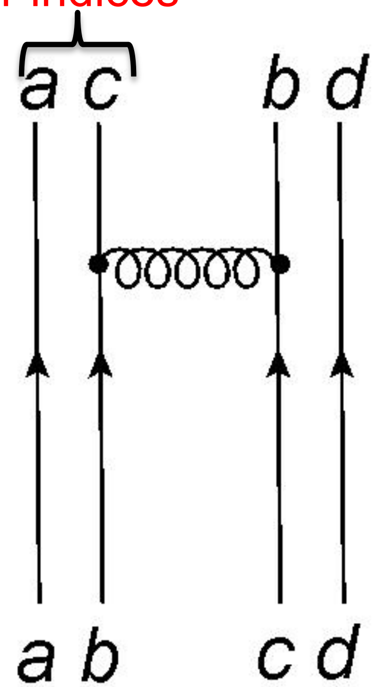
Gluon exchange simply flips colors of quarks

Final quark colors are same as initial ones; all such exchanges are allowed for color singlets.

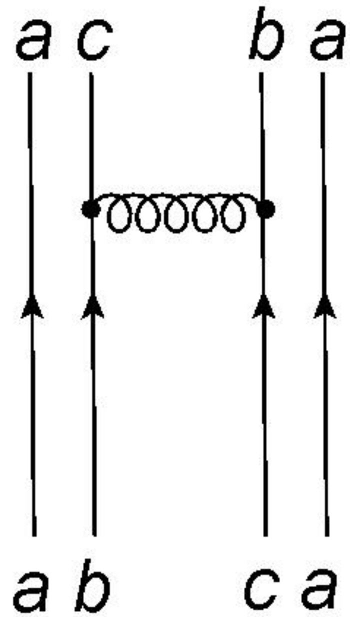
The Case of QCD(AS)

Not all exchanges contribute in a color singlet, (which requires each color combination once and only once).

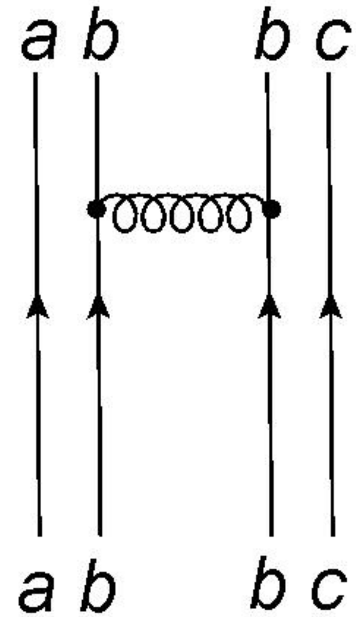
Each quark has 2 color indices



Naively, $O(Nc^3)$ but
No contribution



Contributes
$O(Nc^2)$



Contributes
$O(Nc^2)$

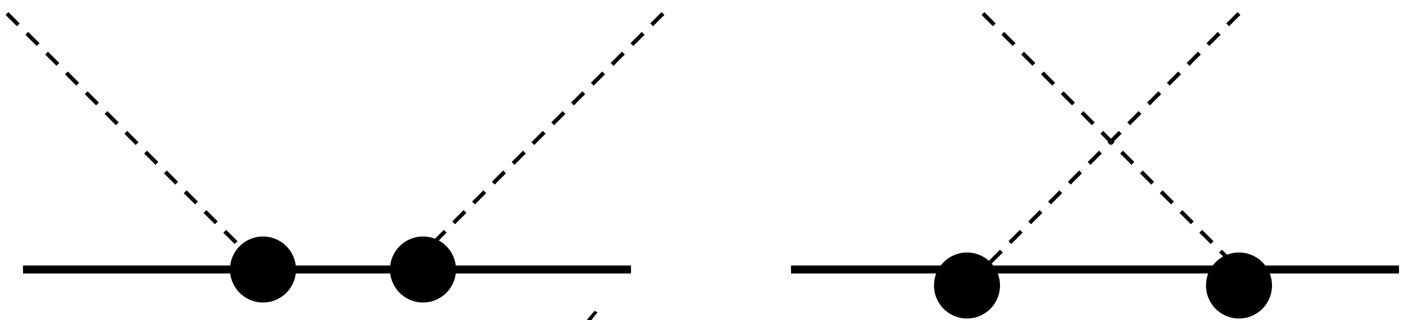
- This fact suppresses many of the combinatoric factors.
- A Cherman & TDC(2006) showed that for a wide a class of diagrams the total contributions are ~Nc^2 as needed.
  - However general proof was lacking due to the complexity of the general case
- Recently, some new diagrammatic tools were developed which allowed for a full proof. Even with these tools the demonstration is rather intricate TDC, RF Lebed and D.L. Shafer(2010).
  - The scaling of the baryon mass as Nc^2 for QCD(AS) is now on as solid ground as Witten's demonstration that it scales as Nc in QCD(F)

- Generic meson-baryon coupling is strong
  - QCD(F) $g_{Nm} \sim Nc^{1/2}$ (Witten 1979)
  - QCD(AS) $g_{Nm} \sim Nc$ (Cherman&TDC 2006)

- If pion coupling to the nucleon g_A/f_π has a generic strength ($g_A/f_\pi \sim Nc^{1/2}$ for QCD(F); $g_A/f_\pi \sim Nc$ for QCD(AS)) then an $S(2N_f)$ spin-flavor symmetry emerges at large Nc. This is a consequence of demanding "large Nc consistency" in which the π-N scattering amplitude is Nc^0 while the Born and cross-born contributions are Nc^1 (F) or Nc^2 (AS) (Gervais& Sakita 1984; Dashen&Manohar 1993)

- Spin-Flavor (Gervais&Sakita84, Dashen&Manohar92)

Consider pion-nucleon scattering



$$A = i g_A^2 p_i p_j \left(\frac{\sigma_i \tau_a \sigma_j \tau_b}{f_\pi^2(-\omega)} + \frac{\sigma_i \tau_a \sigma_j \tau_b}{f_\pi^2 \omega} \right)$$

$\sim N_c^2$ QCD(F)

$\sim N_c^4$ QCD(AS)

$\sim N_c^1$ QCD(F)

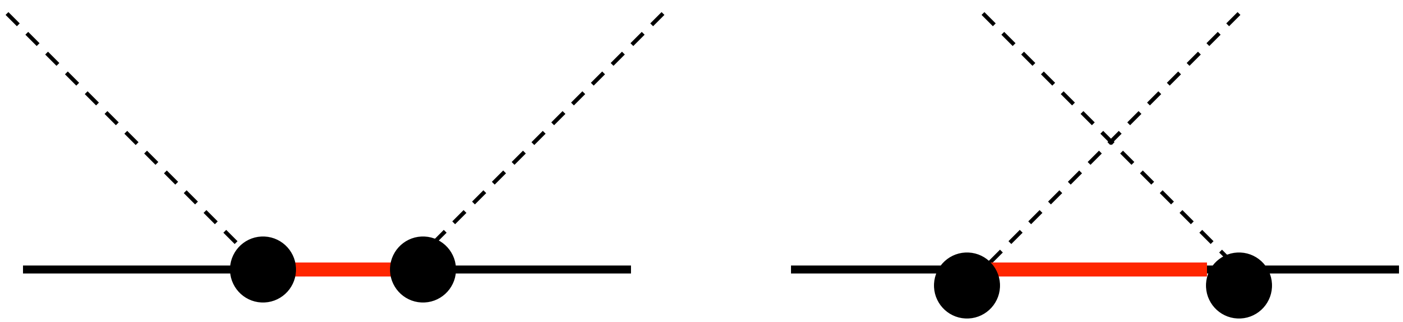
$\sim N_c^2$ QCD(AS)

$$\mathrm{A} \sim N_c [\sigma_i \tau_a, \sigma_j \tau_b] \quad \text{QCD(F)}$$

$$\mathrm{A} \sim N_c^2 [\sigma_i \tau_a, \sigma_j \tau_b] \quad \text{QCD(AS)}$$

This violates unitarity (and Witten scaling rules)

To get sensible results this needs to be canceled



Cancellations require

• Other baryons in intermediated state which are degenerate with nucleon at large N_c. (eg. Δ)

•Conspiracy between vertices

Group Theory

- Assume family of degenerate baryons at large N_c.
- Assume coupling constants X_{ia} between these baryons. Consistency requires

$$[X_{ia}, X_{jb}] = 0$$

- Full group structure follows from spin and flavor transformation properties; contracted $SU(2\,N_f)$
- Scale of the corrections fixed:

$[X_{ia}, X_{jb}] \sim N_c^{-1}$ QCD(F) $\qquad$ $[X_{ia}, X_{jb}] \sim N_c^{-2}$ QCD(AS)

Contracted SU(2N_f) Symmetry

$$[J_i, J_j] = i\varepsilon_{ijk} J_k$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$[T_i, X_{jb}] = i\varepsilon_{ijk} X_{kb}$$

$$[T_a, X_{jb}] = i f_{abc} X_{jc}$$

$$[X_{ia}, X_{jb}] = 0$$

Degenerate baryons fall in irreps of this group at large Nc

Such a symmetry implies that there is an infinite tower of baryon states with I=J which are degenerate at large Nc and with relative matrix elements fixed by CG coefficients of the group.

For Nc=3 the N& Δ are identified as members of the band. (Other states are large Nc artifacts)

Corrections to this:

$$\text{QCD(F)}: \quad M_\Delta - M_N \sim \frac{1}{N_c} \qquad \text{Fractional correction to ratio of ME's} \sim \frac{1}{N_c}$$

$$\text{Fractional correction to ratio of "Golden" ME's} \sim \frac{1}{N_c^2}$$

$$\text{QCD(AS)}: \quad M_\Delta - M_N \sim \frac{1}{N_c^2} \qquad \text{Fractional correction to ratio of ME's} \sim \frac{1}{N_c^2}$$

$$\text{Fractional correction to ratio of "Golden" ME's} \sim \frac{1}{N_c^4}$$

Phenomenologically the predictions of the contracred SU($2N_f$) symmetry and the scale of its breaking do very well

Eg. Axial couplings Dashen & Manohar 1993

Baryon mass relations and SU(3) flavor breaking Jenkins &Lebed 1995

Cherman,Cohen &Lebed 2009

Isoscalar mass combinations

$$N_0 = \frac{1}{2}(p+n)\,, \quad \text{and } \Lambda$$

$$\Sigma_0 = \frac{1}{3}(\Sigma^+ + \Sigma^0 + \Sigma^-)\,,$$

$$\Xi_0 = \frac{1}{2}(\Xi^0 + \Xi^-)\,,$$

$$\Delta_0 = \frac{1}{4}(\Delta^{++} + \Delta^+ + \Delta^0 + \Delta^-)\,,$$

$$\Sigma_0^* = \frac{1}{3}(\Sigma^{*+} + \Sigma^{*0} + \Sigma^{*-})\,,$$

$$\Xi_0^* = \frac{1}{2}(\Xi^{*0} + \Xi^{*-})\,. \text{ and } \Omega$$

Scale of SU(3) flavor breaking

- One of many possible measures:

$$\epsilon \equiv \frac{1}{3}\sum_{i=1}^{3}\frac{B_i - N_0}{(B_i + N_0)/2} \approx 0.25$$

with $B_i = \Sigma_0, \Lambda, \Xi_0$

- Any other reasonable definition should give $\varepsilon \approx 0.25–0.30$

The I = 0 Mass Combinations Special to $1/N_c$

	Mass Combination	Large N_c^{F} suppression	Large N_c^{AS} suppression
M_1	$5(2N_0+3\Sigma_0+\Lambda+2\Xi_0)-4(4\Delta_0+3\Sigma_0^*+2\Xi_0^*+\Omega)$	$1/N_c$	$1/N_c^2$
M_2	$5(6N_0-3\Sigma_0+\Lambda-4\Xi_0)-2(2\Delta_0-\Xi_0^*-\Omega)$	ϵ	ϵ
M_3	$N_0-3\Sigma_0+\Lambda+\Xi_0$	ϵ/N_c	ϵ/N_c^2
M_4	$(-2N_0-9\Sigma_0+3\Lambda+8\Xi_0)+2(2\Delta_0-\Xi_0^*-\Omega)$	ϵ/N_c^2	ϵ/N_c^4
M_5	$35(2N_0-\Sigma_0-3\Lambda+2\Xi_0)-4(4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega)$	ϵ^2/N_c	ϵ^2/N_c^2
M_6	$7(2N_0-\Sigma_0-3\Lambda+2\Xi_0)-2(4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega)$	ϵ^2/N_c^2	ϵ^2/N_c^4
M_7	$\Delta_0-3\Sigma_0^*+3\Xi_0^*-\Omega$	ϵ^3/N_c^2	ϵ^3/N_c^4

ε Is SU(3) flavors breaking scale

Can we see evidence of large Nc behavior behavior in these relations beyond mere SU(3) flavor and its breaking?

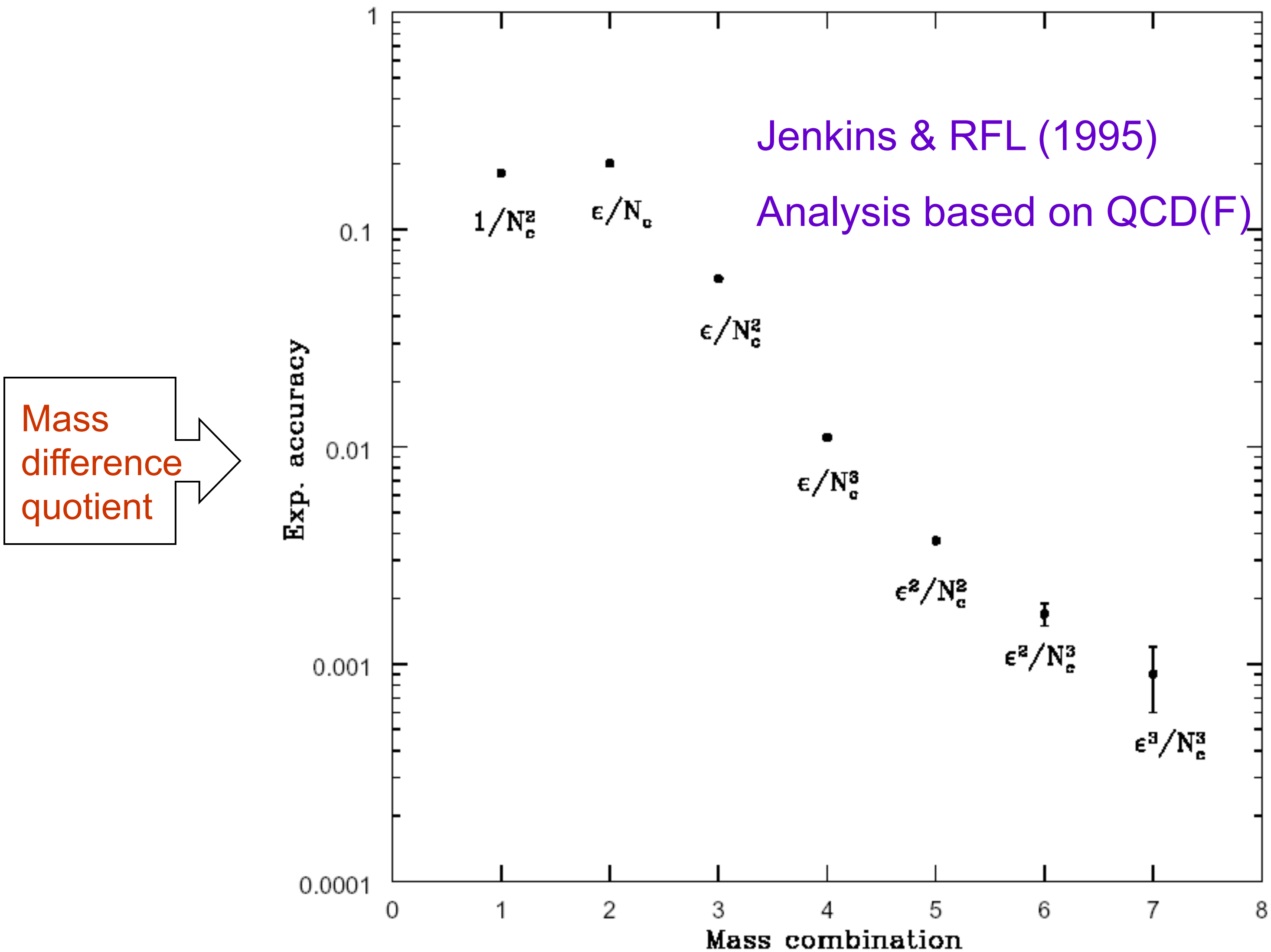
Cherman, Cohen & RFL, Phys. Rev. D **80**, 036002 [2009]:
Compare these results for N_C^F and N_C^{AS}

To test the quality of the large Nc predcitions of mass relations quantitatively we need quantitative measure of their accuracy.

- Take each M_i and form M_i', the same combination with all "–" signs turned to "+" (Note that M_i' is $O(N_C)$ $[N_C^F]$, $O(N_C^2)$ $[N_C^{AS}]$)
- Define the scale-independent ratios $R_i \equiv M_i/(\frac{1}{2} M_i')$

  e.g., $M_3 = N_0 - 3\Sigma_0 + \Lambda + \Xi_0$

  $\rightarrow R_3 = (N_0 - 3\Sigma_0 + \Lambda + \Xi_0)/[\frac{1}{2}(N_0 + 3\Sigma_0 + \Lambda + \Xi_0)]$



Large Nc has real predictive power: the relations are MUCH better than pure SU(3)!!

Note that depending on the choice taken for ε this has "natural" coefficients as an expansion in $1/N_c$ QCD(F) or$1/N_c^2$ QCD (AS) (A. Cherman, TDC & R. F. Lebed 2009)

An analogous study for magnetic moments recently completed by Rich Lebed indicates that QCD(F) works much more naturally than QCD (AS)(R. F. Lebed 2010)

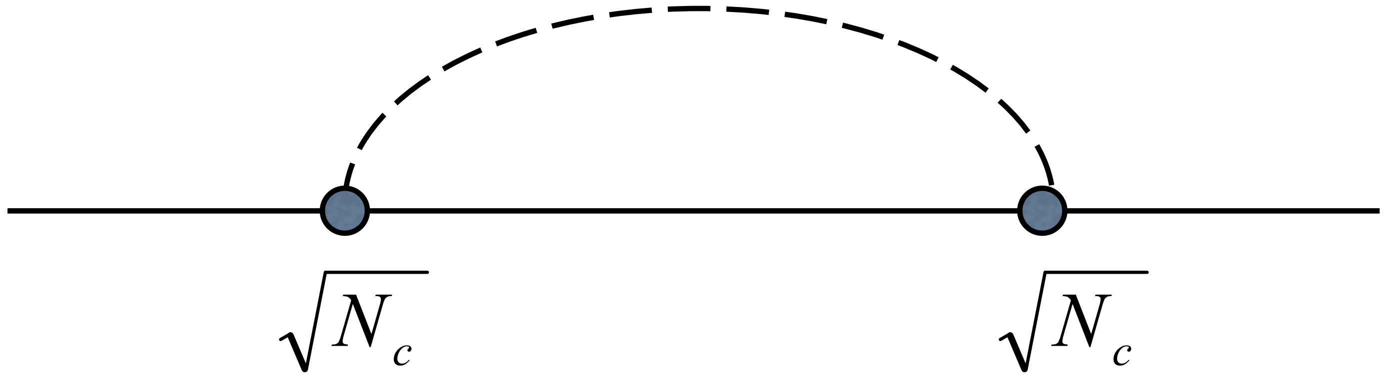
As noted above which expansion works better may depend snsitively on which observable is being studied

The role of meson loops in baryon properties

- In both the case of QCD(F) and QCD(AS) baryons include effects which at the hadronic level appear to be due to meson loops

- This fact is often not fully appreciated but is clearly true for both QCD(AS) and QCD(F).

Consider QCD(F)



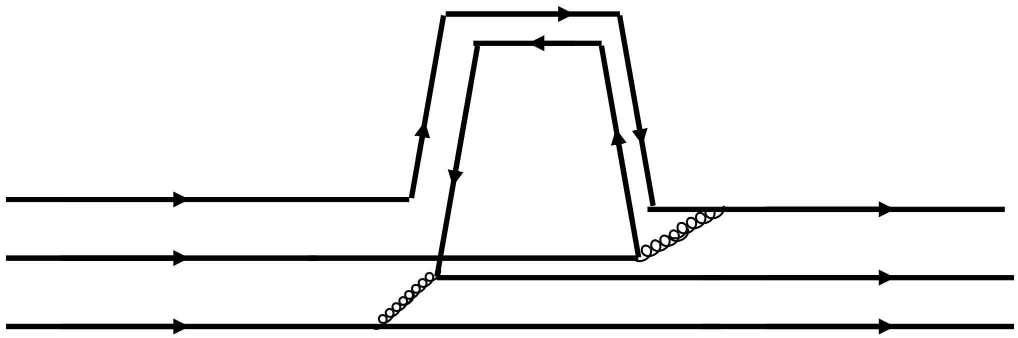
Meson loop contribution to the nucleon self-energy is order N_c. This is leading order since $M_N \sim N_c$.

(Analogous behavior in QCD(AS) with $N_c^{1/2} \rightarrow N_c$.)

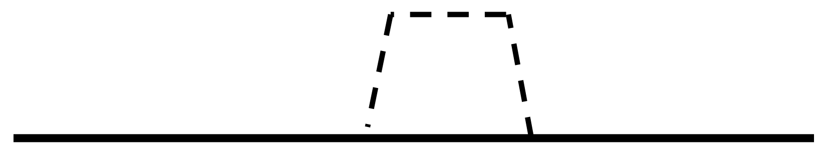
How can this be? Quark loops are suppressed at large Nc for QCD(F) and surely meson loops involve quark loops.

Actually this is not true.

While meson loops in meson do involve quark loops for baryons they need not (TDC & D.B. Leinweber 1992):consider “z-graphs” in “old fashioned” perturbation theory for quarks in a nucleon



At hadronic level this looks like



Very strong evidence for this: Skyrme and other large Nc chiral soliton models exactly reproduce the non-analytic dependence on m_π which emerge from pion loops in chiral perturbation theory(TDC& W. Broniowski 1992)

QCD(AS) also has contribution at leading order from internal quark loops. This yields some qualitative differences:

Eg. strange quark form factors in the nucleon

$$G_E^s(Q^2) \sim N_c^0 \qquad \text{QCD(F)}$$

$$G_E^s(Q^2) \sim N_c^1 \qquad \text{QCD(AS)}$$

(Cherman&TDC 2007)

All sensible models which are supposed to encode large Nc physics should reproduce these generic features in a self-consistent way

Often, models build in Nc scaling implicitly through parameters. For example in the Skyrme model f_π is a parameter and encodes the correct QCD(F) scaling if one takes $f_\pi \sim Nc^{1/2}$.

Most of the models on the market (eg. Skyrme, NJL, Holographic etc) are self-consistent in that if you impose the correct Nc scaling for the input parameters, you will get the correct scaling for the predictions; eg. $M_N \sim Nc$ for QCD(F)

The same models will correctly reproduce QCD(AS) scaling for the predictions if one imposes QCD(AS) scaling for the input paramters; simple subsitution $N_c^{1/2} \rightarrow N_c$

Models for QCD(AS) can differ in form QCD(F) since at leading order they are allowed terms associated with internal quark loops (eg.~ terms with more than one flavor trace in Skyrme type models.)

Sensible models should also correctly encode the leading order contributions from meson loops in baryons discussed above.

For generic mesons this is hard to pick out. However for observables dominated by long distance behavior this is controlled by pion loop physics and is fixed by chiral symmetry, the contracted SU(2Nf) symmetry and the value of g_A/f_π ; the leading behavior is model independent and calculable in large Nc chiral perturbation theory.

For example the long range part of the isoscalar and isovector electromagnetic form factors are dominated by 3 pion and 2 pion contribtions respectively

For models in the chiral limit of m_π=0, there is a remarkable combination of form factors in which all model dependent parameters cancel Cherman, TDC, Nielsen (2009)

$$\lim_{r\to\infty} \frac{\tilde{G}^E_{I=0}\tilde{G}^E_{I=1}}{\tilde{G}^M_{I=0}\tilde{G}^M_{I=1}} = \frac{18}{r^2}$$

$\tilde{G}(r)$ is the Fourier transform of the standard momentum space fom factors

This ratio is valid for both QCD(F) & QCD(AS) and is a good probe of whether a model correctly incorporates the leading order large Nc physics associated with meson loops in the baryon. **All** chiral soliton models (Skyrme, NJL) when treated at leading order in 1/N (mean-field or classical hedgehogs semi-classicaly quantized) satisfy this.

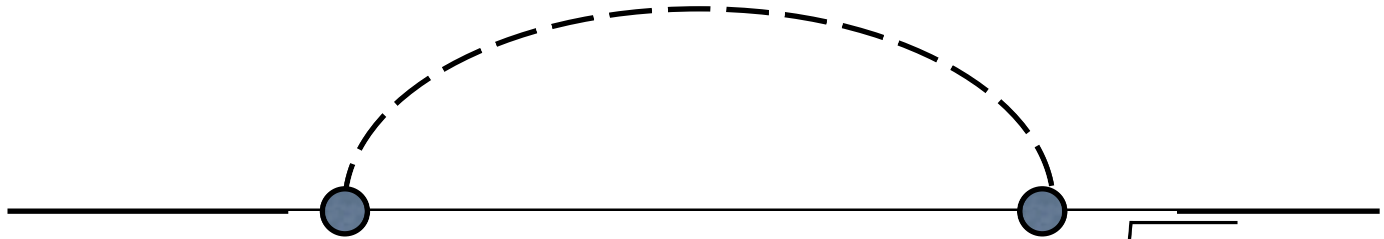
Bottom up holographic models of baryons as 5-d Skyrmions (Pomarol-Wulzer, 2008) also satisfy this relation. The have correctly built in the meson loop physics present at leading order in 1/Nc

However the top-down Sakai&Sugmato model derived from a stringy construction is problematic. It has in additon to Nc and a scale parameter, a strength parameter λ, which must taken as large to derive a gravity theory from the stringy construction.

Taking large λ in a baryon model, yields small size objects treatable as 5-d instantons (Hata et al 2007; Hashimoto, Sakai, Sugimoto 2008; Hong et al 2008)

Hadronic couplings in the SS model

$$f_\pi = \sqrt{\frac{\lambda N_c}{54\pi^4}} M_{KK} \qquad g_A = N_c \sqrt{\frac{24}{45\pi^2}} \qquad \frac{g_A}{f_\pi} \sim \sqrt{\frac{N_c}{\lambda}}$$



$$\frac{g_A}{f_\pi} \sim \sqrt{\frac{N_c}{\lambda}} \qquad \frac{g_A}{f_\pi} \sim \sqrt{\frac{N_c}{\lambda}}$$

If large *Nc* limit is implicitly taken first in the construction of the model then pion cloud effect contributes at leading order (*Nc*) albeit with a coefficient which is numerically small ($\sim 1/\lambda$)

However if the large λ limit is implicitly taken first in the construction of the model then pion cloud effect vanishes at the outset. This would be very troubling since unlike the large *Nc* limit, the large λ limit is an artifact of the model which has no analog in QCD. Thus an artificial limit would eliminate leading order QCD effects in the 1/Nc expansion.

Which is it? Use model independent form factor relations to tell.

Expressions for form factors for solitons in the Sakai-Sugimoto model are known. The ratio can be evaluated:

$$\lim_{r\to\infty} \frac{\tilde{G}^E_{I=0}\tilde{G}^E_{I=1}}{\tilde{G}^M_{I=0}\tilde{G}^M_{I=1}} = \frac{\lambda\sqrt{\frac{40}{3}}}{\pi\rho_1 r^2} \approx \frac{1.73\lambda}{r^2} \neq \frac{18}{r^2}$$

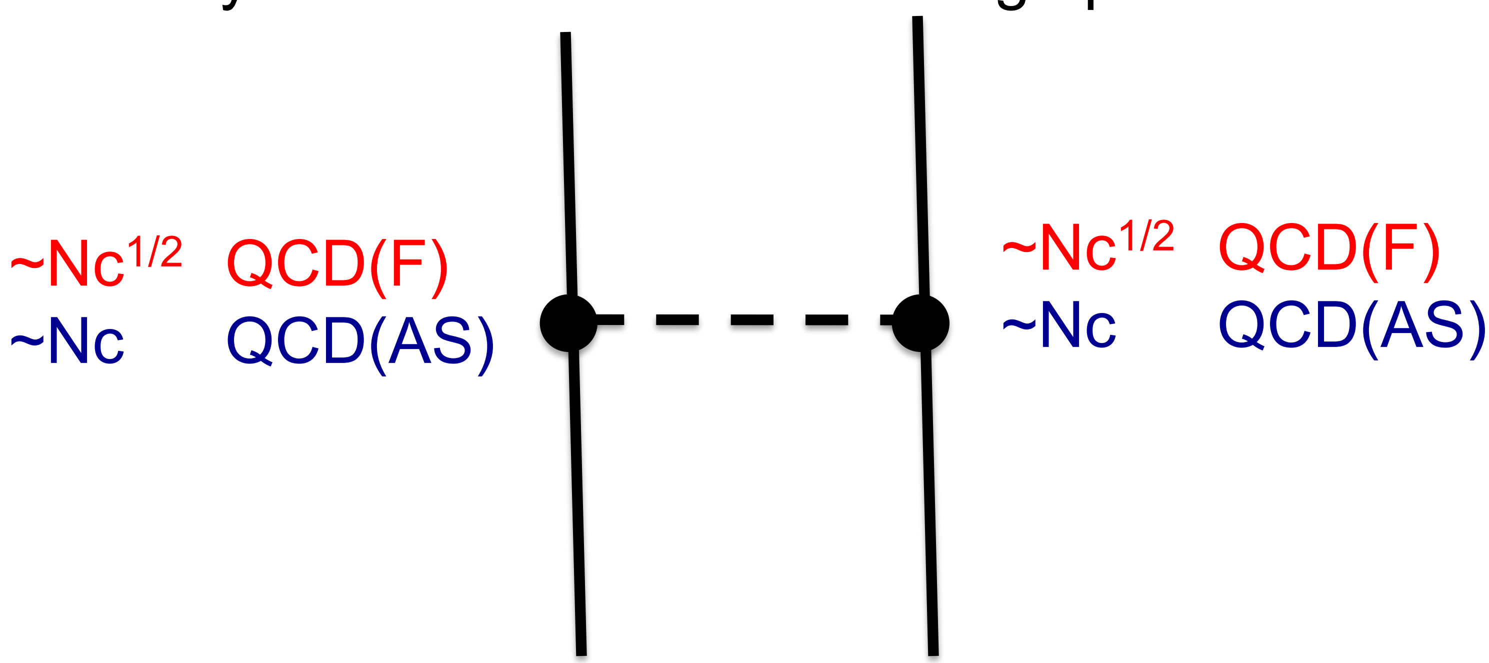
$\rho_1 \approx .669$ is a fixed numerical value associated with an eigenvalue in the theory

- Unfortunately, the model as implemented does not satisfy large N_c relation. Ratio depends on model parameter λ; as a model independent result it cannot. Note moreover that it diverges in the large λ limit.
- The model fails to correctly treat the long distance physics (which is supposed to be fixed by chiral symmetry). Apparently the large λ limit is implicitly being taken before the large *Nc* limit. **The implemetation of the model does not correctly encode large Nc and chiral physics of QCD.**

Implication for Nuclear Interactions and Dense Matter

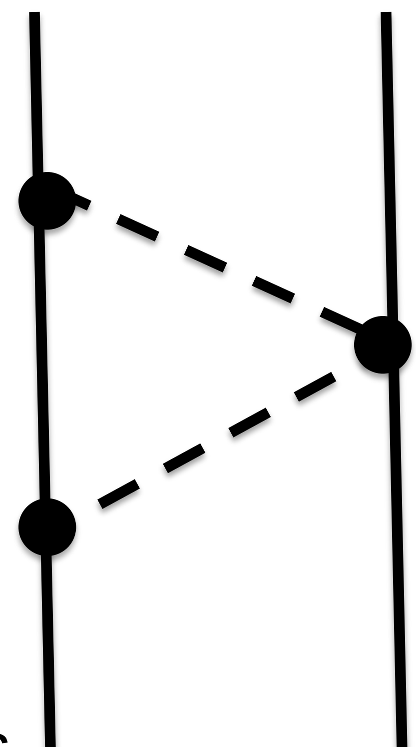
- May be of more theoretical then phenomenological importance as nucleon-nucleon forces are unnaturally strong in both large Nc limits
  - QCD(F) $V_{NN} \sim Nc$
  - QCD(AS) $V_{NN} \sim Nc^2$

Easily seen via a meson exchange picture



- Nucleon-Nucleon forces include dynamics of multi-meson exchanges at leading order in 1/Nc

~$Nc^{1/2}$ QCD(F)
~Nc   QCD(AS)



~Nc^0 QCD(F)
~Nc^0 QCD(AS)

~$Nc^{1/2}$ QCD(F)
~Nc   QCD(AS)

Overall contribiution is
QCD(F) V_{NN}~ Nc
QCD(AS) V_{NN}~ Nc^2
This is leading order scaling and is correctly captured by sensible large Nc model

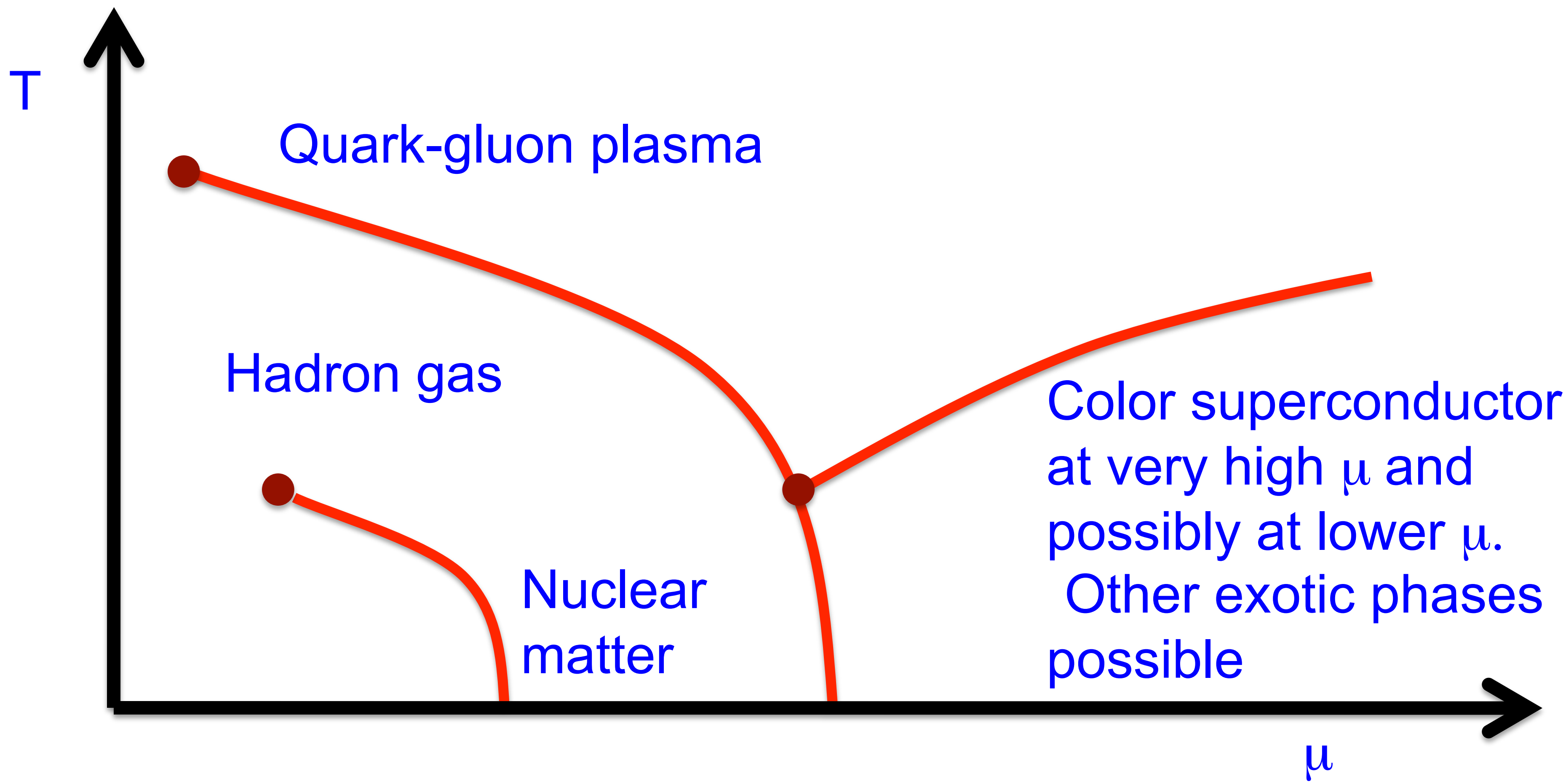
Note that this physics is absent in the SS treated as an instanton

- Nuclear matter is crystalline and saturates in both large Nc limits
  - QCD(F) $\rho_{sat} \sim Nc^0$ B$\sim Nc^1$
  - QCD(AS) $\rho_{sat} \sim Nc^0$ B$\sim Nc^2$

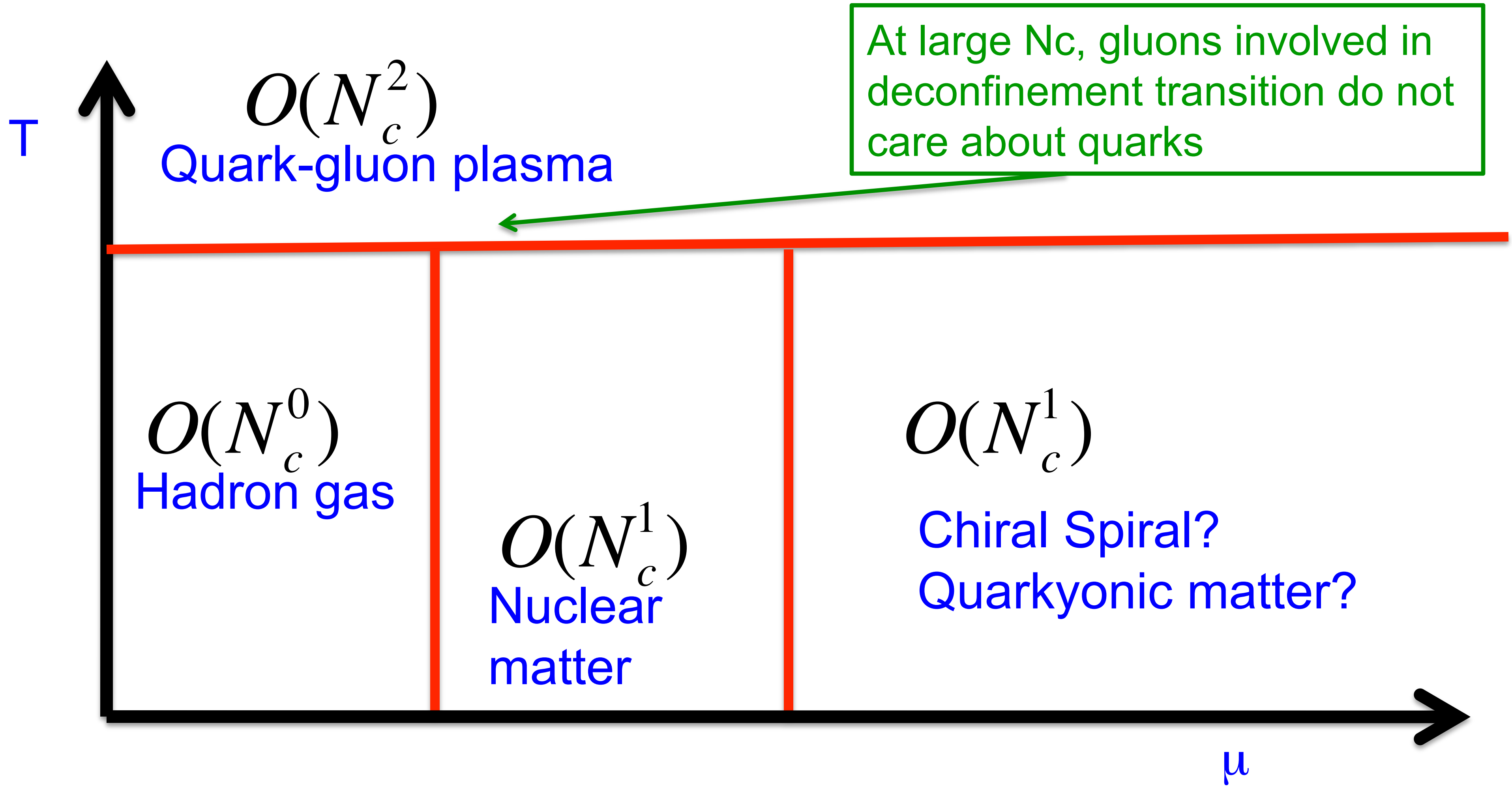
  - Pion exchange is dominant long range interaction and has an attractive channel. Any attractive quantum system with parametrically strong forces or heavy mass will become arbitrarily well localized around the classical minimum

- While both limits are similar in this respect there equations of state are expected to qualitatively different. Consider $T,\mu \sim Nc^0$

The Nc=3 QCD Phase Diagram:
A Cartoon

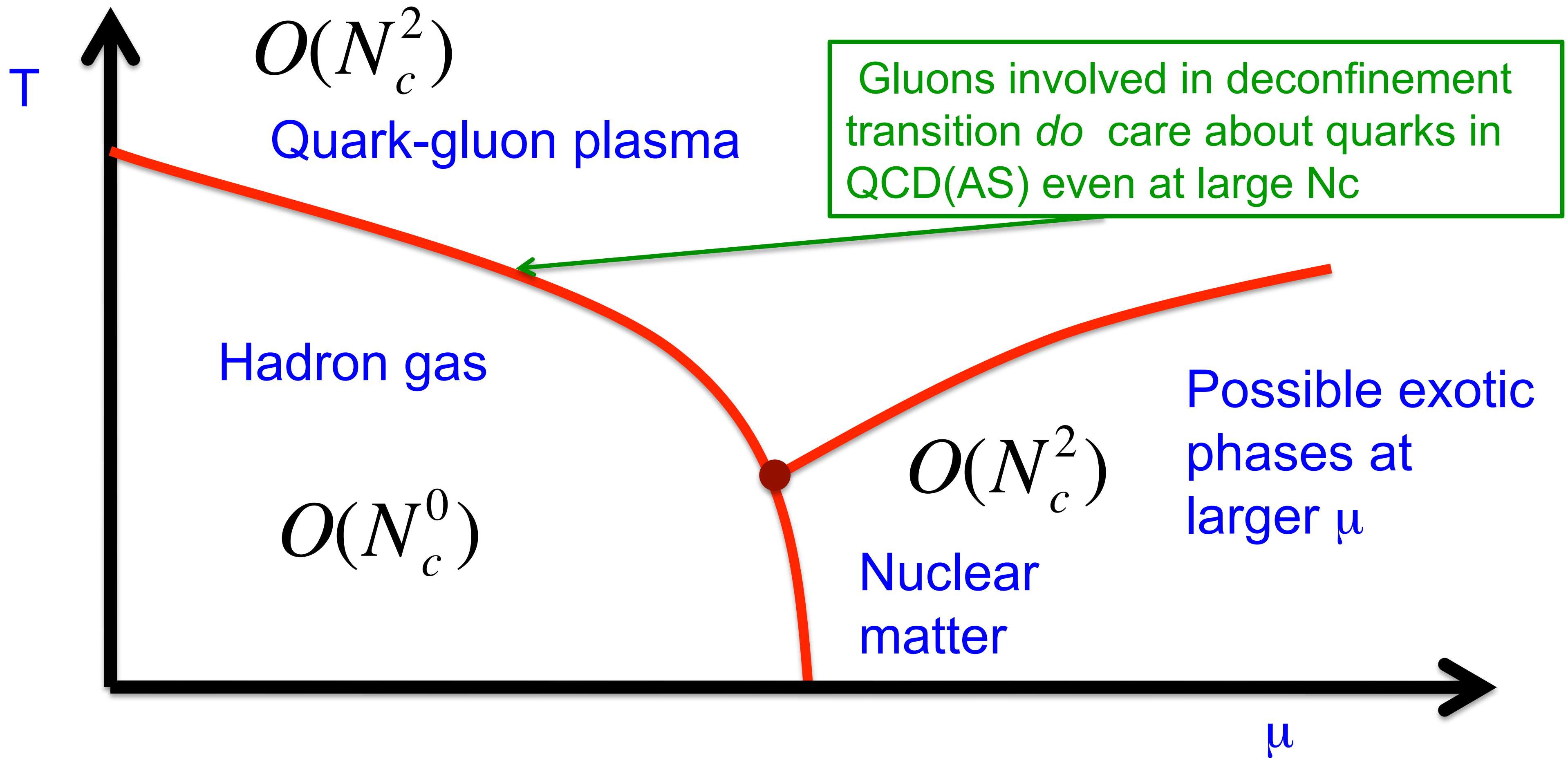


QCD (F) Phase Diagram at Large Nc : A Cartoon



Large Nc behavior for dense matter with μ~N_c^0 looks completely different from Nc=3!!!

QCD (AS) Phase Diagram at Large Nc : A Cartoon



Large Nc behavior for dense matter with *μ~N_c^0* in QCD(AS) looks qualitatively different from QCDS(F)

What about asymptotically high densities at low T?

- Characteristic momenta are small interactions via 1-gluon exchange; nonperturbative effects through infrared enhancement of effects with perturbative kernal.
- Nc=3 : As noted by Son (1999) there is Strong evidence for color superconductivity; BCS instability in RG flow; BCS gap given parametrically by

$$\Delta_{\mathrm{BCS}} \sim \mu g^5 \exp\left(\frac{-\sqrt{6}\pi^2}{g}\right)$$

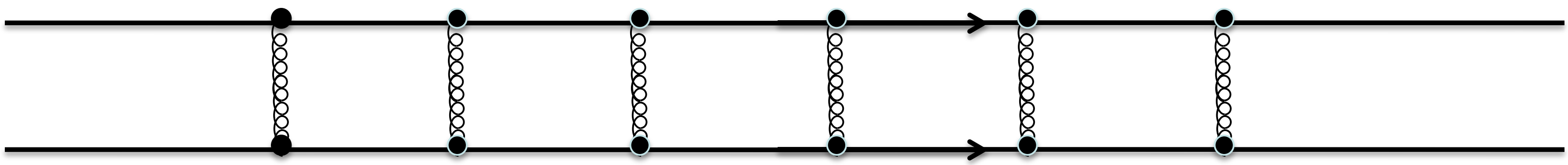
Note *1/g* not *1/g²* in exponential

- Nc $\rightarrow \infty$: $g = \sqrt{\frac{\lambda}{N_c}}$ where λ, the `t Hooft coupling, is independent of Nc

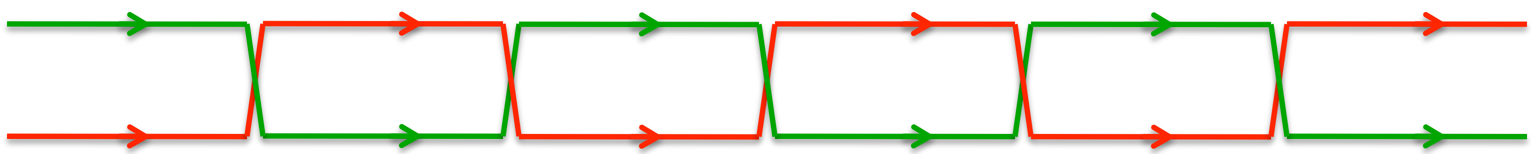
$$\Delta_{\rm BCS} \sim \mu \left(\frac{\lambda}{N_c} \right)^{\frac{5}{2}} \exp\left(-\sqrt{6}\pi^2 \sqrt{\frac{N_c}{\lambda}} \right)$$

  – **The gap is exponentially suppressed at large Nc!!**
- However this does not happen (at least in QCD(F)). The BCS calculation only shows that a Fermi gas is unstable against the BCS instability. If there are other instabilities to a different phase at a larger energy scale they will dominate.
  – Note that $\langle qq \rangle$ type condensates such as BCS depend on g^2 not $N_c\, g^2$. This is why the effect is exponentially small.

Ladders are key ingredient

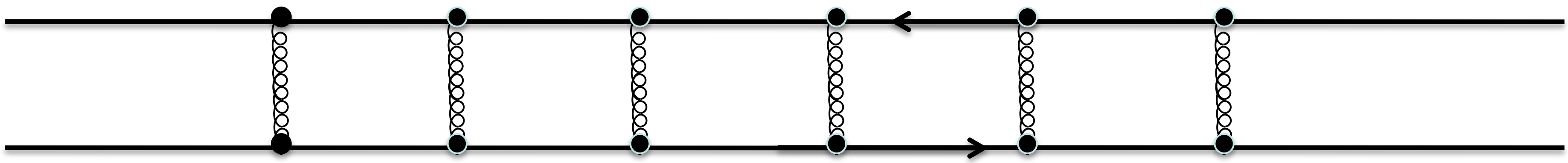


Look at color flow ('t Hooft diagrams with gluons carrying color-anticolor)

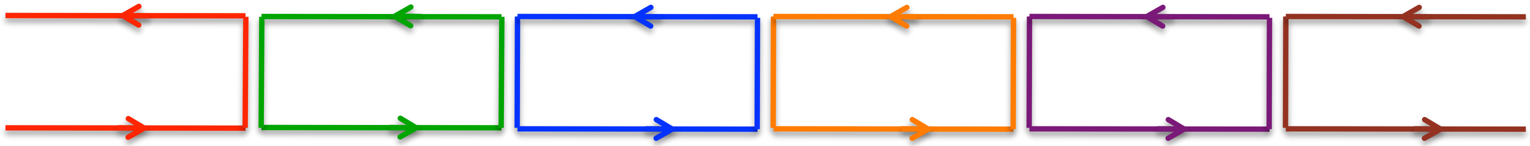


Note factors of couplings cost 1/Nc but no loop factors counteract it. The color just bounces back and forth.

The situation is quite different with instabilities towards condensates which are color singlets (although not necessarily gauge invariant), *eg*. some type of possibly nonlocal $\langle \bar{q} q \rangle$ condensate.



Look at color flow ('t Hooft diagrams with gluons carrying color-anticolor)



Note factors of couplings cost 1/Nc but are compensated by color loop factors. The relevant combination is $N_c\, g^2 = \lambda$. Thus, effects should not be exponentially down in N_c.

Thus ***IF*** an instability towards a color-singlet condensate exists at large *Nc* it will occur rather than the BCS phase.

Son and Shuster (1999) showed that that such a condensate exists in standard ‘t Hooft-Witten large Nc limit.

It is a spatially varying chiral condensate of the Deryagin, Grigoriev, and Rubakov (DGR) type:

$$\langle \overline{q}(x')q(x) \rangle = e^{i\vec{P}\cdot(\vec{x}'+\vec{x})} \int d^4q\, e^{-iq(\vec{x}-\vec{x}')} f(q) \qquad |\vec{P}| = \mu$$

The DGR instability can only be reliably computed for $\mu >> \Lambda_{QCD}$ (perturbatively large) and only occurs for $\mu < \mu_{\text{crit}}$.

The reason that μ_{crit} exists is that at sufficiently high values of μ, the Debye mass cuts off the RG running before the instability sets in.

$$\mu_{\text{crtit}} \sim \Lambda_{QCD} \exp\left(\gamma \log^2(N_c)\right) \qquad \gamma \approx .02173$$

As $Nc \rightarrow \infty$, $\mu_{\text{crit}} \rightarrow \infty$ and the DGR instability exists for all pertubative values of μ.

Moreover as expected its scale is NOT exponentially down in *Nc*

$$\Delta_{\mathrm{DGR}} \sim \mu \exp\left(- \underbrace{\frac{4\pi^3}{g^2 N_c}}_{\lambda} \right)$$

Thus, the DGR instability is much stronger than the BCS instability. The system will form a DGR phase rather than a BCS phase when possible and at large *Nc* it is always possible.

However it is only possible when $\mu < \mu_{\mathrm{crit}}$ where

$$\mu_{\mathrm{crtit}} \sim \Lambda_{QCD} \exp\left(\gamma \log^2 (N_c)\right) \qquad \gamma \approx .02173$$

For moderate *Nc*, μ_{crit} is small enough so that DGR instability does not occur---at least not in the perturbative regime where it is computable. One needs Nc~1000 to have a DGR phase (in perturbative regime)

The bottom line: the DGR phase will not occur at Nc=3 and color superconductivity will occur. At large Nc the DGR phase exists. **The large Nc world for QCD(F) at high density is qualitatively different from Nc=3**

- However QCD(AS) and QCD(F) are qualitatively different.
- Recall that for QCD(F) at asymptotically high chemical potentials color superconductivity lose to a DGR instability if the DGR instability occurs.

$$\Delta_{\mathrm{BCS}}^{(F)} \sim \mu\left(\frac{N_c}{\lambda}\right)^{\frac{5}{2}} \exp\left(-\sqrt{6}\pi^2\sqrt{\frac{N_c}{\lambda}}\right) \qquad \Delta_{\mathrm{DGR}}^{(F)} \sim \mu\exp\left(-\frac{4\pi^3}{\lambda}\right)$$

- DGR won because it is a color singlet (although not gauge invariant.

Recall that in QCD(F) the DGR phase is only possible when $\mu < \mu_{crit}$ where

$$\mu_{\mathrm{crtit}} \sim \Lambda_{QCD} \exp\left(\gamma \log^2(N_c)\right) \qquad \gamma \approx .02173$$

But for large Nc $\mu_{crit} \rightarrow \infty$.

What happens in QCD(AS)?

Both the BCS and DGR instabilities using were studied by standard means Buchoff, Cherman, TDC (2010) :

An RG equation was set up for excitations near the Fermi surface. Now if the Fermi surface is unstable the coupling strength will diverge as one integrates out the contributions of everything except a small shell near the Fermi surface.

The gap is determined qualitatively from the position at which the divergence occurs.

For QCD(AS) we found that

$$\Delta_{\mathrm{BCS}}^{(AS)} \sim \mu \frac{\lambda^{5/2}}{N_c^3} \exp\left(-\pi^2 \sqrt{\frac{3N_c}{2\lambda}} \right)$$

As compared to

$$\Delta_{\mathrm{BCS}}^{(F)} \sim \mu \left(\frac{N_c}{\lambda} \right)^{\frac{5}{2}} \exp\left(-\sqrt{6}\pi^2 \sqrt{\frac{N_c}{\lambda}} \right)$$

Note that the dependence is not just $N_c^{1/2} \rightarrow N_c$. The RG equations depend explicitly on the representation of the quark field and are non-linear. As with QCD(F) the gap is exponentially down in Nc.

Thus we again expect that the DGR instability will win as it is a color singlet, **provided that it occurs.**

Does it?

NO!!

The RG analysis is done using the same effective 1-d theory near the Fermi surface as was done for QCD(F). However, in QCD(AS) the RG running is affected by quark loops. These serve to screen the gluons and cutoff the RG flow before the instability is reached.

Thus QCD(AS) at very high densities is qualitative different QCD(F) at large Nc. As for the case of Nc=3 it is likely to be in a BCS phase and is certainly not in a DGR

An optimist *might* take this to mean that QCD(AS) is more likely than QCD(F) to be qualitatively similar to QCD at Nc=3 than QCD(F) even at smaller densities and might serve as a useful first step for modeling in that region.

Perhaps with enough good wine I could be convinced of this

But it would take **a lot** of good wine

Summary

- QCD(AS) is an alternative way to extrapolate to large Nc.
- Typical models of the baryon capture the leading Nc behavior of QCD for both limits but baryons in the SS model (treated as an instanton) do not.
- At very high density QCD(AS) does not undergo a DGR transition at large Nc while QCD(F) does.