Dynamical Electroweak Symmetry Breaking with a Heavy 4th Generation

Nov 10th, 2010, University of Virginia, Charlottesville

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(based on arXiv:0911.3890, 0911.3892, 1011.xxxx) with P.Q. Hung

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- Standard Model with four generations (SM4)

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- Schwinger-Dyson equation(SDE)

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- Strong Yukawa couplings and Quasi-fixed point
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- Schwinger-Dyson equation(SDE)
- Implications of RGE+SDE

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- A heavy 4th generation can alleviate the naturalness (hierarchy) problem of SM3

- Similar to the top-quark condensation models, the 4th generation might trigger the dynamical electroweak symmetry breaking.

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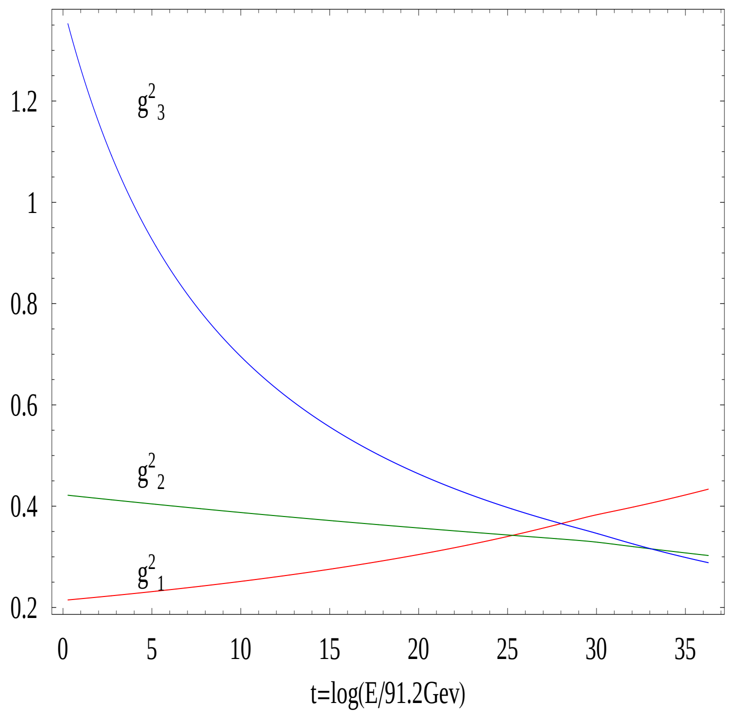
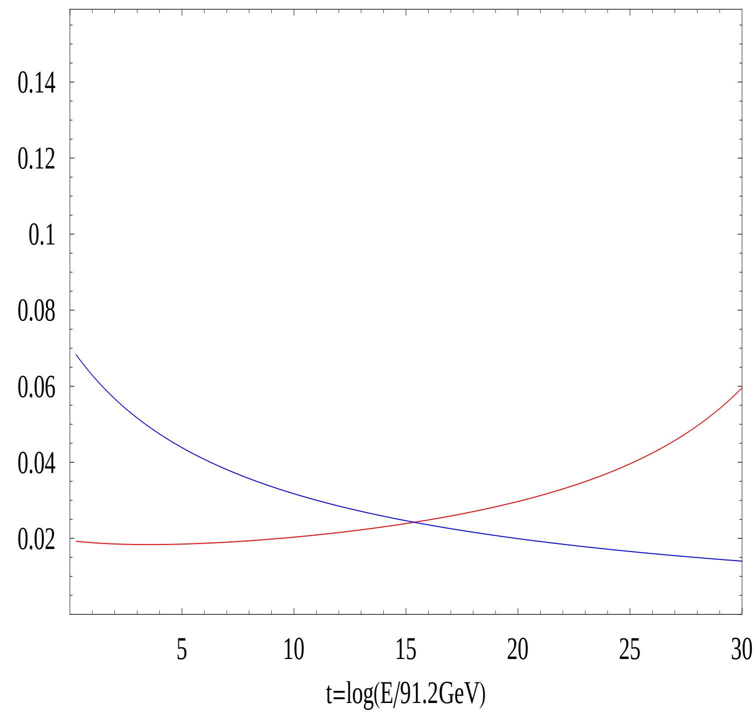
- suggests the restoration of scale invariance above $\Lambda \Rightarrow$ new conformal theories?

RG running of couplings in SM3

We begin with the RGE approach. The RG running of gauge couplings and Higgs couplings (quartic and Yukawa) in SM3 ($M_h = 120GeV \sim 180GeV$)





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- e.g. the Higgs quartic coupling

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\begin{aligned}
\beta_\lambda = \quad & 24\lambda^2 + 4\lambda(3g_t^2 + 6g_q^2 + 2g_l^2 - 2.25g_2^2 - 0.45g_1^2) \\
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& +288g_q^6 + 96g_l^6 - (3g_t^4 + 6g_q^4 + 2g_l^4 - 80g_3^2(g_t^2 \\
& +2g_q^2))\lambda - 6\lambda^2(24g_t^2 + 48g_q^2 + 16g_l^2) - 312\lambda^3 \\
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- other couplings $\beta_{y_q}, \beta_{y_l}, \beta_{y_t}, \beta_{g_i, i=1,2,3}$ ... (Machacek and Vaughn, 1983)

Zeros of β-functions

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g_3^2	g_2^2	g_1^2		λ	g_t^2	g_q^2	g_l^2
1.478	0.425	0.213		17.561	31.407	52.298	56.583
1.225	0.413	0.217		17.457	31.200	52.185	55.664
1.003	0.404	0.223		17.376	31.073	52.147	54.934
0.902	0.396	0.226		17.339	31.014	52.126	54.604
0.815	0.386	0.230		17.308	30.963	52.107	54.321
0.652	0.366	0.239		17.249	30.866	52.066	53.792
0.565	0.354	0.245		17.218	30.814	52.042	53.511
0.304	0.284	0.304		17.125	30.655	51.966	52.661
0.999	0.666	0.333		17.339	31.039	52.089	54.817
0.500	0.500	0.500		17.164	30.754	51.990	53.152
0.000	0.000	0.000		17.059	30.488	51.902	51.902

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The fixed point values of the quartic and Yukawa couplings are approximately

$$\boxed{\lambda^*/(4\pi)^2 \approx 0.11, \quad g_t^{2*}/(4\pi)^2 \approx 0.2, \quad g_q^{2*}/(4\pi)^2 \approx 0.33, \quad g_l^{2*}/(4\pi)^2 \approx 0.34}$$

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The gauge couplings contribute only small fluctuations. These correspond to naive $\overline{MS}$ masses (using $\overline{m}_H = v\sqrt{2\lambda}, \overline{m}_f = vg_f/\sqrt{2},\ v = 246$ GeV)

$$\overline{m}_H^* = 1.44 \text{ TeV}, \overline{m}_t^* = 0.97 \text{ TeV}, \overline{m}_q^* = 1.26 \text{ TeV}, \overline{m}_l^* = 1.28 \text{ TeV}.$$

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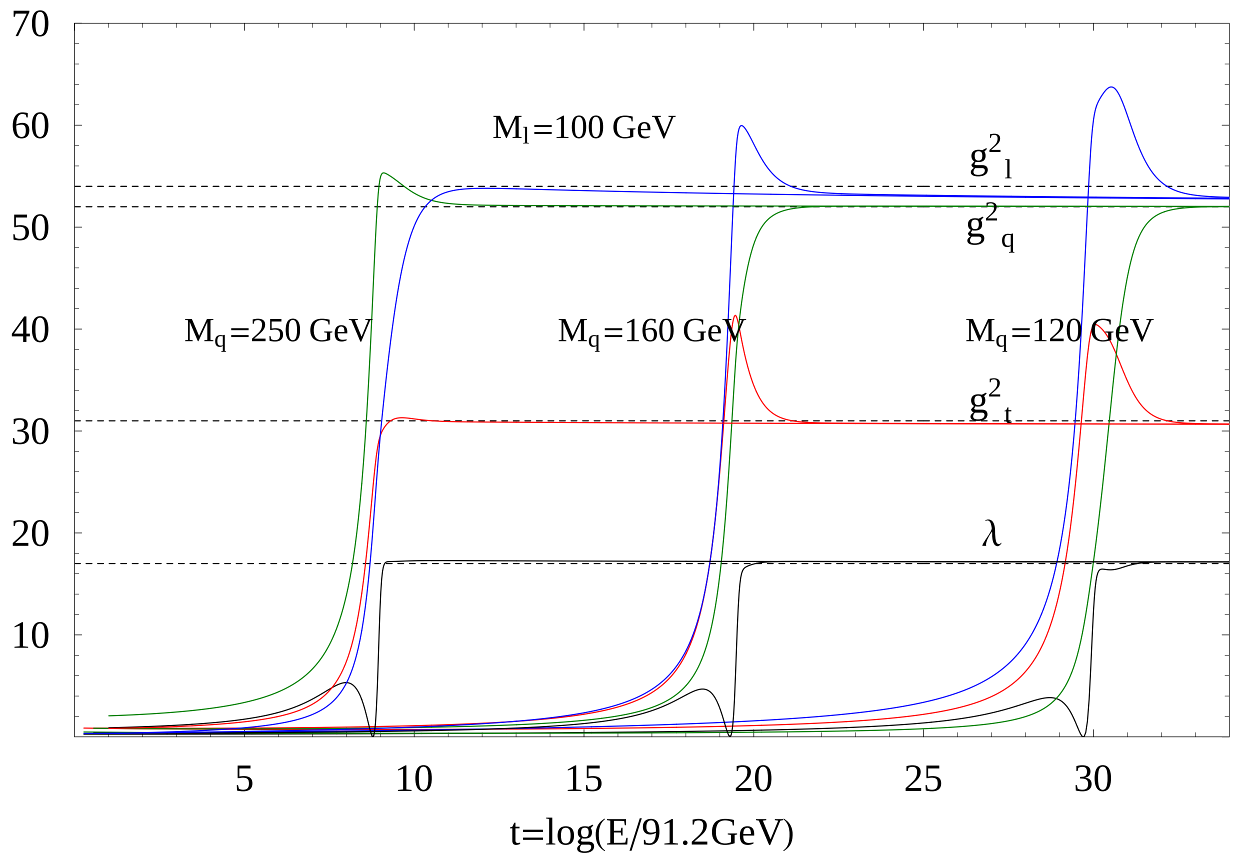
Questions:

- Can this (quasi)fixed point be reached?
- If yes, at what energy scale?

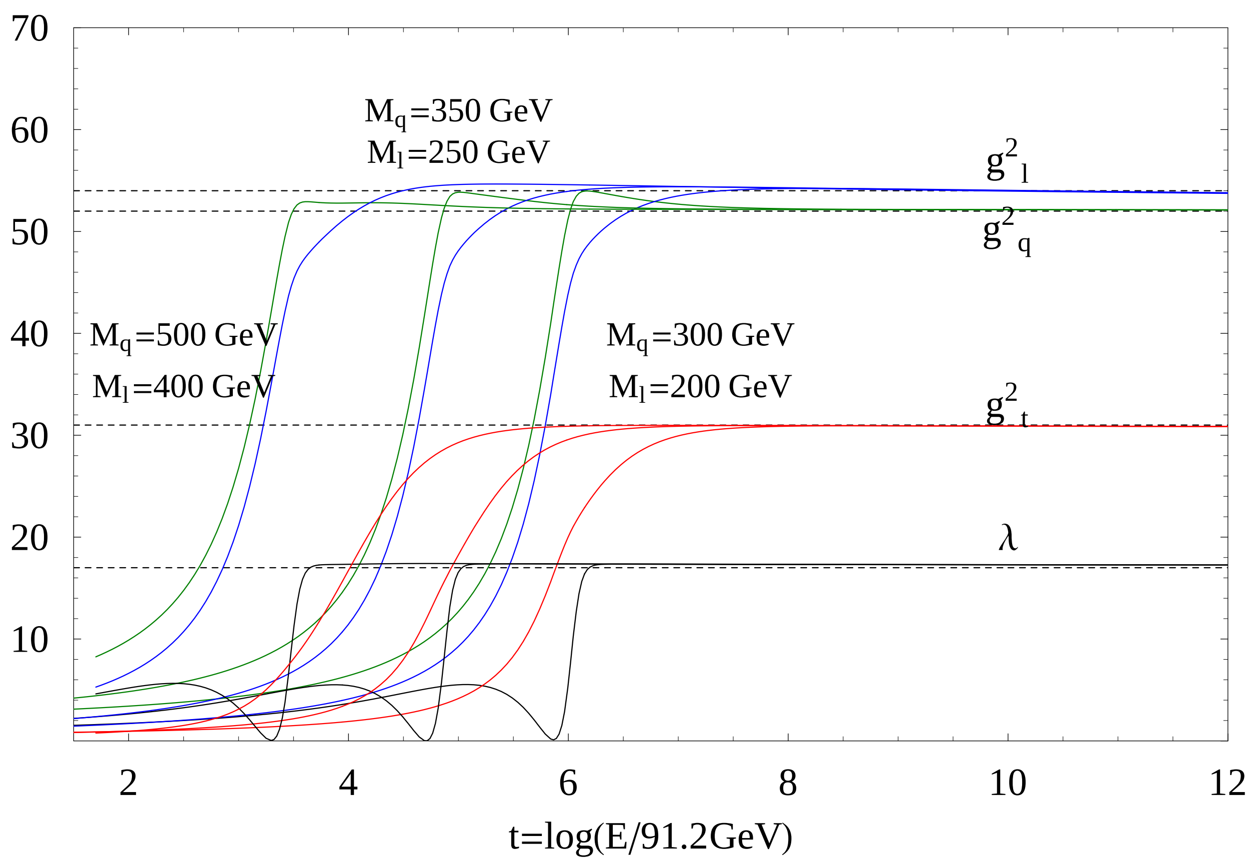
RG running of Higgs couplings

The RG running of Higgs couplings (quartic and Yukawa), light mass cases ($M_q = 120GeV \sim 250GeV$)



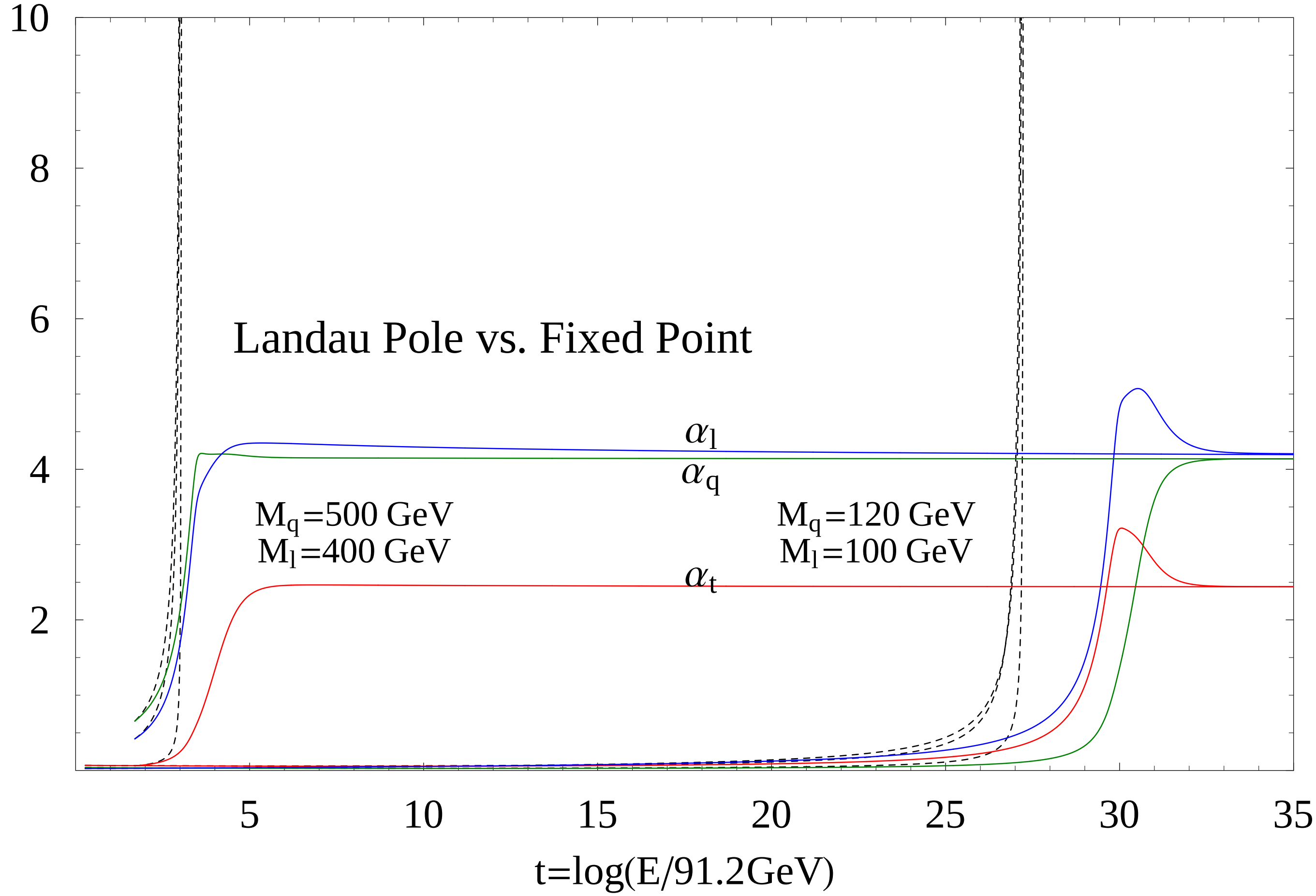
RG running of Higgs couplings

The RG running of Higgs couplings (quartic and Yukawa), heavy mass case ($M_q = 300GeV \sim 500GeV$)



Landau Pole vs. Fixed Point

Compare 1-loop and 2-loop results



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The existence of a quasi-fixed point $\Rightarrow$ the triviality problem;
The physical consequences of shifting the scale Λ_{FP} down to TeV level $\Rightarrow$ provide an alternative solution to the hierarchy problem.

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- For the RGEs, the expansion parameters are $g_t^{2*}/16\pi^2 \approx 0.2, \;\; g_q^{2*}/16\pi^2 \approx 0.33, \;\; g_q^{2*}/16\pi^2 \approx 0.34, \;\; \lambda^*/16\pi^2 \approx 0.11$.

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- $\frac{\alpha}{\pi}$ or $\frac{\alpha}{4\pi}$? An order of one expansion parameter? We have seen similar situations before, e.g., the Wilson-Fisher ϵ-expansion, $g_4 = 16\pi^2\epsilon/3$ for the physical value $\epsilon = 1, \epsilon = 4 - d$.

Wilson-Fisher ϵ-expansion

$$\mu \frac{d}{d\mu} g_4(\mu) = -\epsilon \, g_4(\mu) + \frac{3 g_4^2(\mu)}{16\pi^2} + \mathcal{O}(g_4^3(\mu))$$

$$\mu \frac{d}{d\mu} g_2(\mu) = g_2(\mu)[-2 + \frac{g_4(\mu)}{16\pi^2} + \mathcal{O}(g_4(\mu))]$$

where $\epsilon = 4 - d$ and g_2 and g_4 come from terms $g_2\phi^2/2, g_4\phi^4/4!$ respectively.

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For d=3, it corresponds to $g_4^* = 16\pi^2/3 \approx 52.64$ or $g_4^*/16\pi^2 = 1/3$

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The critical exponent ν is then given by the ϵ-expansion

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We do not expect such a precise calculation, but hopefully inclusion of higher order terms will not shift the location and the values of the quasi-fixed point by an order of magnitude.

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We perform a non-relativistic analysis at quantum mechanics level, with a Higgs-exchange potential

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where $\alpha_Y = m_1 m_2/4\pi v^2$.
The possibility of forming bound states is characterized by

$$K_f = \frac{g_f^3}{16\pi\sqrt{\lambda}}.$$

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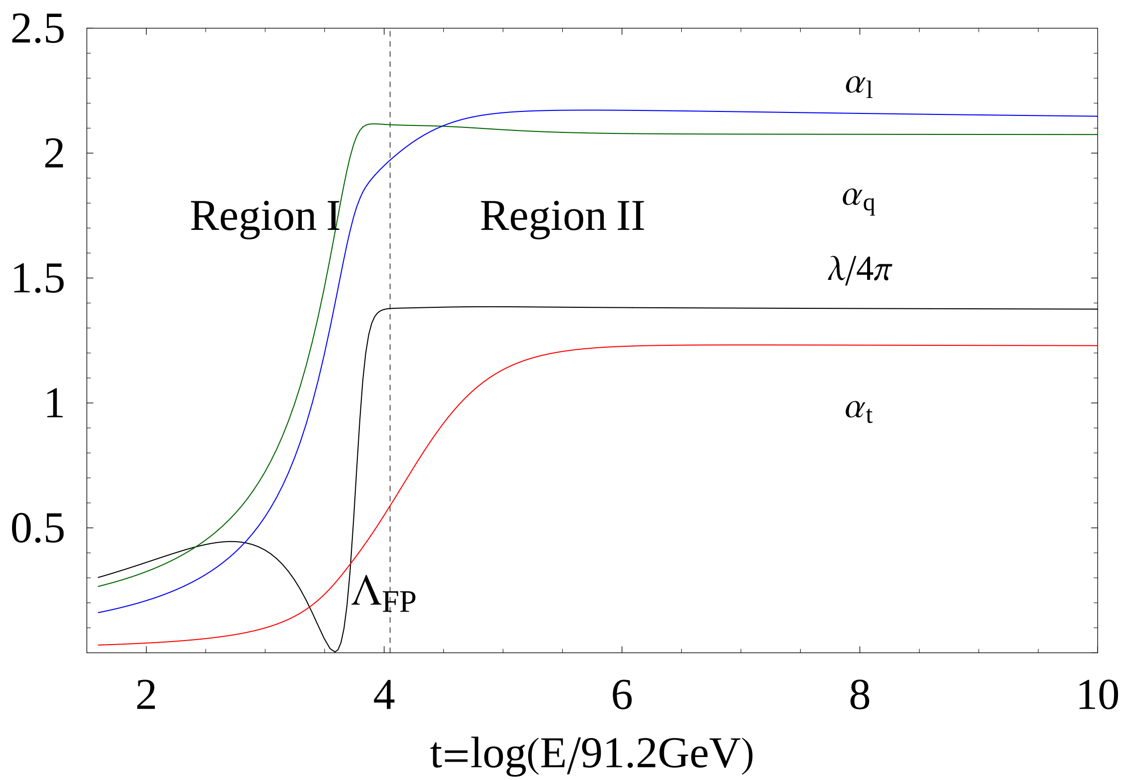
$K_q = 1.82, \quad K_l = 1.92, \quad K_t = 0.82$
An interesting region is around the “dip”, i.e. $\lambda \approx 0$, where the Yukawa potential becomes a strong Coulomb-like potential. $\longrightarrow$ formation of condensates (Rafelski, Fulcher and Klein, 1978).

Bound States/Condensates



Region I: Condensates v.s. Region II: Fixed Point

Note: Neither technicolor nor other unkown interactions are introduced for condensates.

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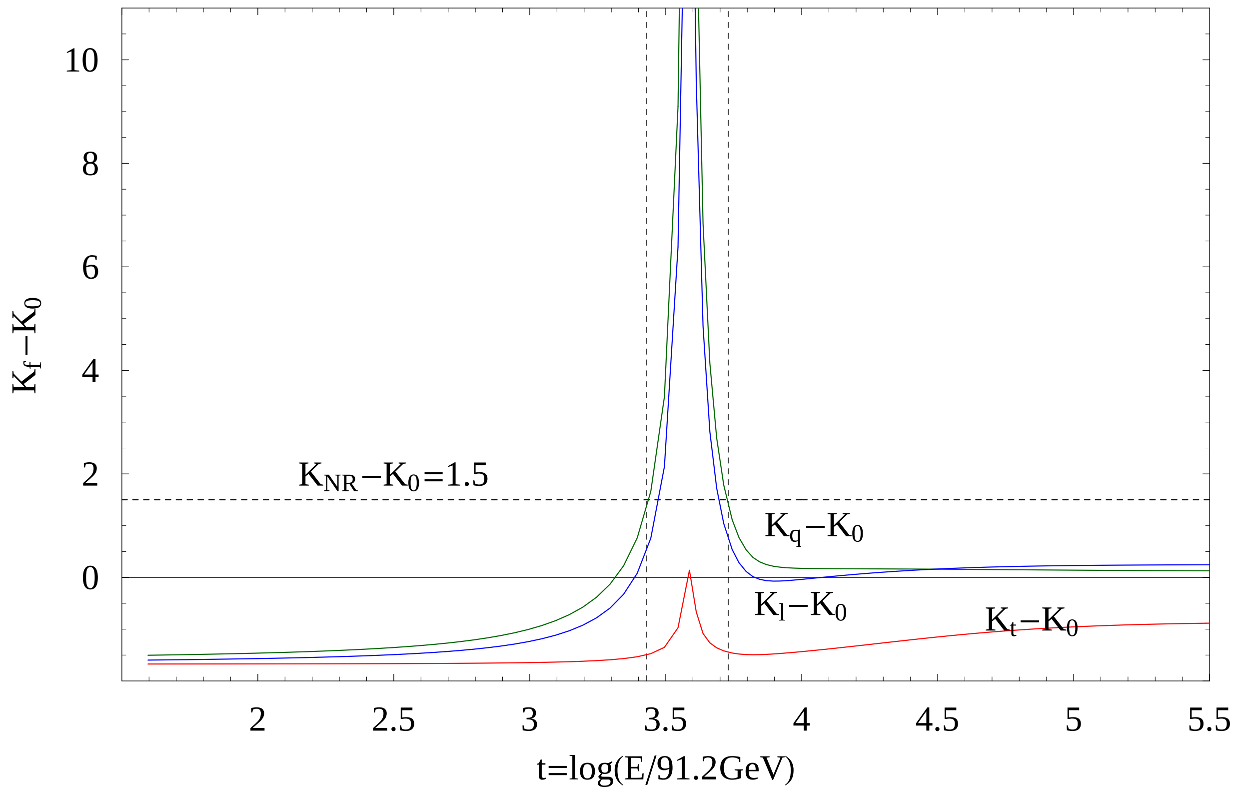

($m_q = 450$ GeV and $m_l = 350$ GeV) $K_f - K_0$ with $K_f = g_f^3/16\pi\sqrt{\lambda}$ and $K_0 = 1.68$. The horizontal dotted line indicates an estimate of K_f where the non-relativistic method is still applicable and the vertical dotted lines enclose the region where a fully relativistic approach is needed.

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- We use Schwinger-Dyson approach (Gap Equation, mean field theory, Hartree-Fock approximation,...)

Schwinger-Dyson Equation

Consider Yukawa couplings in SM4 (truncated to only the 4th generation)

$$\mathcal{L}_Y = -g_{b'}\, \bar{q}_L \Phi\, b'_R - g_{t'}\, \bar{q}_L \widetilde{\Phi}\, t'_R + h.c.$$

$\widetilde{\Phi} = i\tau_2 \Phi^*$, $q_L = (t', b')_L$ as usually defined in the SM.

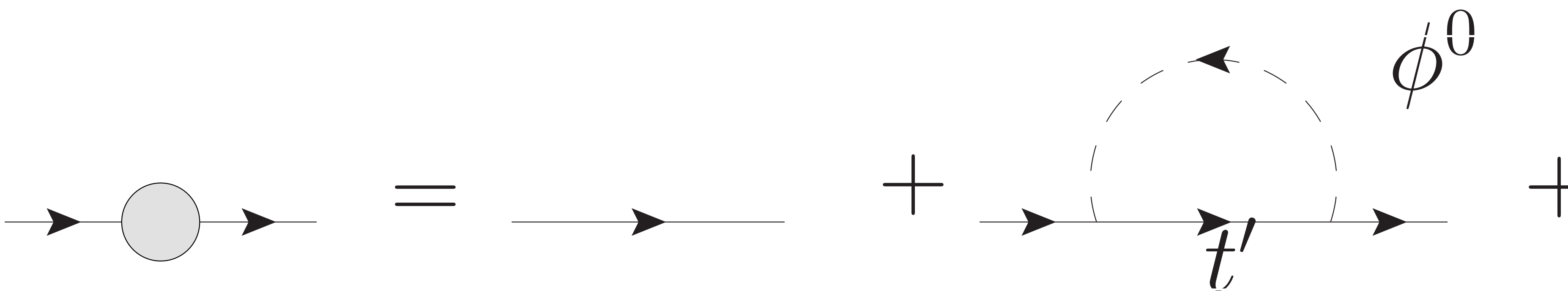

Figure 1: Graphic representation of the Schwinger-Dyson equation for the quark self-energy (quenched approximation)

Gap Equation

- For simplicity we only consider the 4th generation quarks. From the SDE the quark self energy satisfies

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- compared with $\alpha_c = \pi/3$ in strong QED (Fukuda & Kugo, Bardeen, Leung & Love)

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- The SDEs are similar, but the boundary conditions are different

$$\lim_{p\to 0} p^4 \frac{d\Sigma}{dp^2} = 0$$

$$\lim_{p\to \Lambda} p^2 \frac{d\Sigma}{dp^2} + \Sigma(p) = 0$$

solutions

- asymptotic solutions in the weak and strong coupling regions:

$$\Sigma(p) \sim p^{-1+\sqrt{1-\frac{\alpha}{\alpha_c}}}, \qquad \text{for } \alpha \leq \alpha_c$$

$$\Sigma(p) \sim p^{-1} \sin[\sqrt{\frac{\alpha}{\alpha_c} - 1}(\ln p + \delta)], \quad \text{for } \alpha > \alpha_c$$

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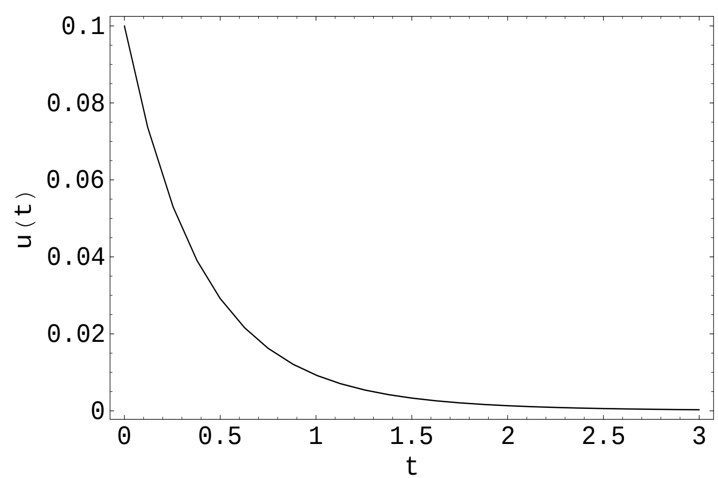
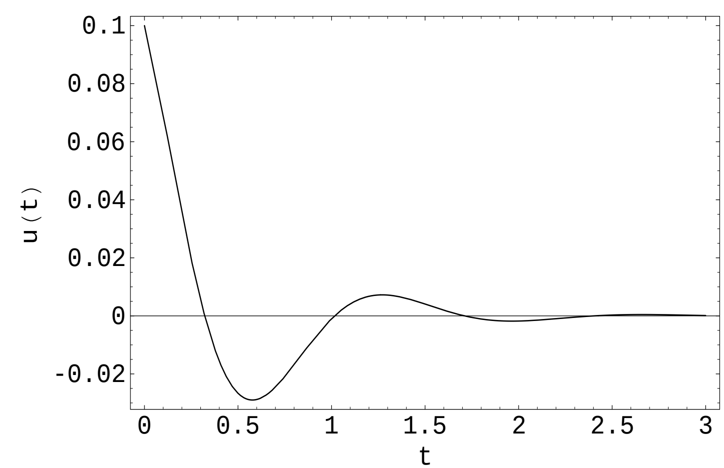
- Numerical Solutions





Condensates

- One can compute the condensates

$$\langle \bar{t}' t' \rangle = -\frac{1}{4\pi^4} \int d^4q \frac{\Sigma(q)}{q^2 + \Sigma^2(q)}$$

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$$\Sigma(0) \sim \Lambda\, e^{\frac{-\pi}{\sqrt{\alpha/\alpha_c - 1}} + 1}, \quad \langle \bar{t}' t' \rangle \sim -\Lambda^3\, e^{\frac{-2\pi}{\sqrt{\alpha/\alpha_c - 1}}}, \quad \delta m_\phi^2 \sim -\Lambda^2\, e^{\frac{-\pi}{\sqrt{\alpha/\alpha_c - 1}}}$$

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- Miransky fixed-point? In our case the exponential factors cannot suppress them simultaneously. To avoid fine-tuning, one has to choose a cutoff at TeV scale.

Cutoff vs. Yukawa couplings

$\frac{\alpha}{\alpha_c}$	1.0	1.1	1.2	1.4	1.8	2.2	2.6	3.0	3.4
$\frac{\Lambda}{\Sigma(0)}$	∞	7590	414	52.8	12.3	6.47	4.41	3.39	2.80
Λ (GeV)	∞	10^6	10^5	10^4	6167	3237	2205	1696	1398

Table 1: The relation between the cutoff scale and the Yukawa coupling.

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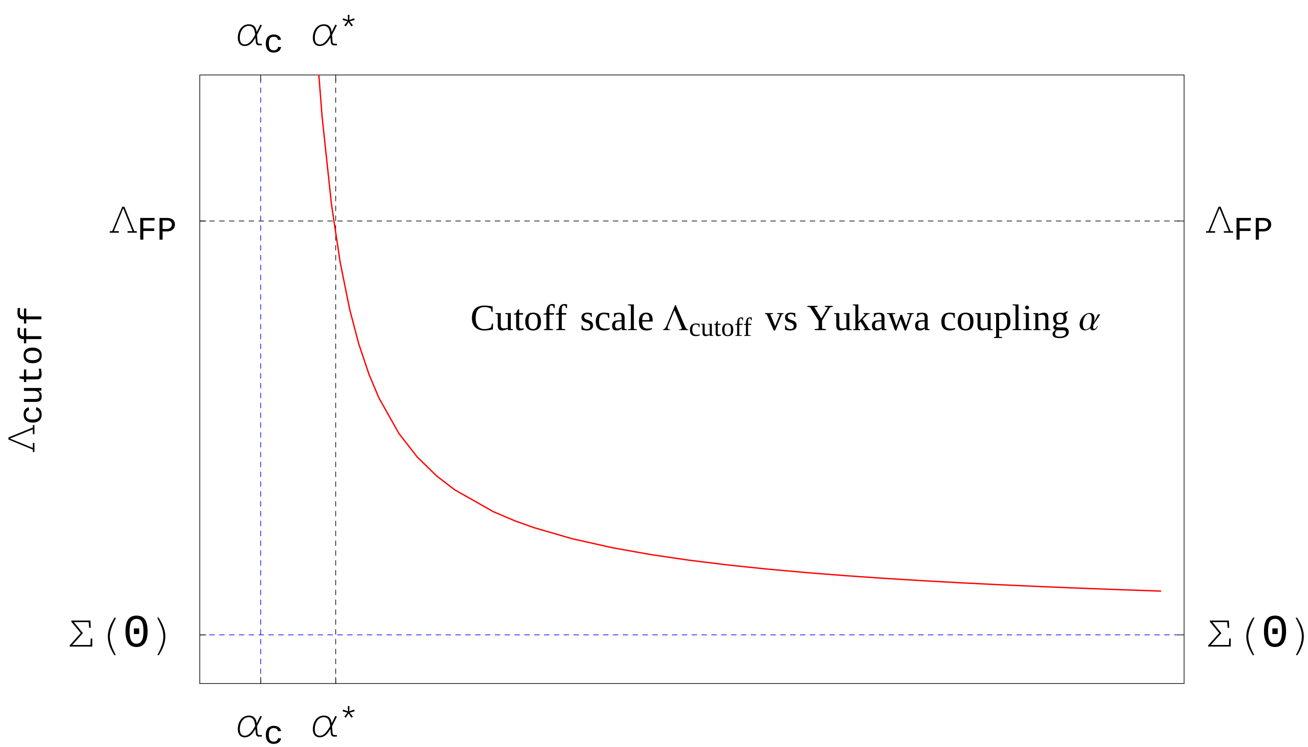
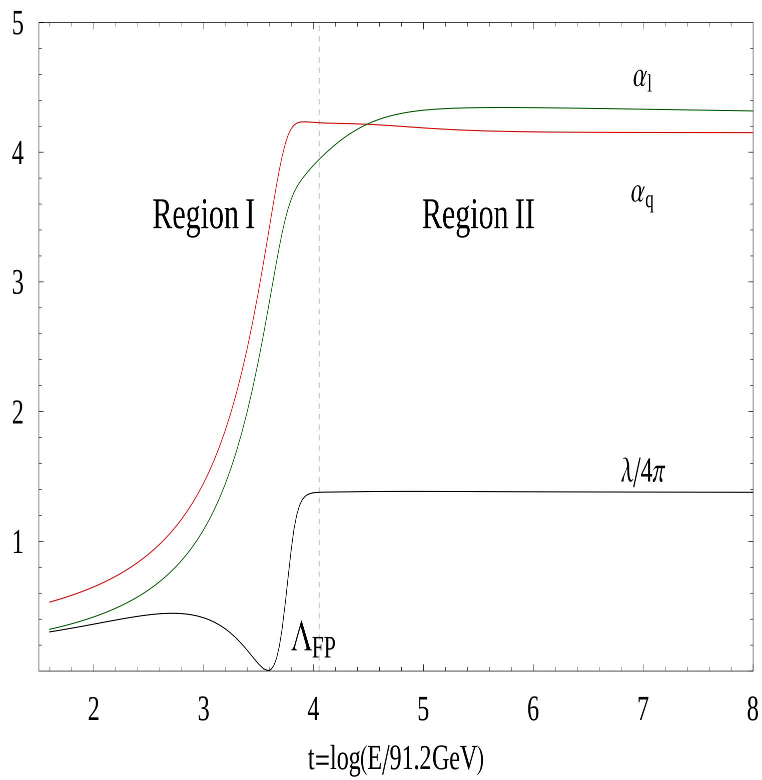
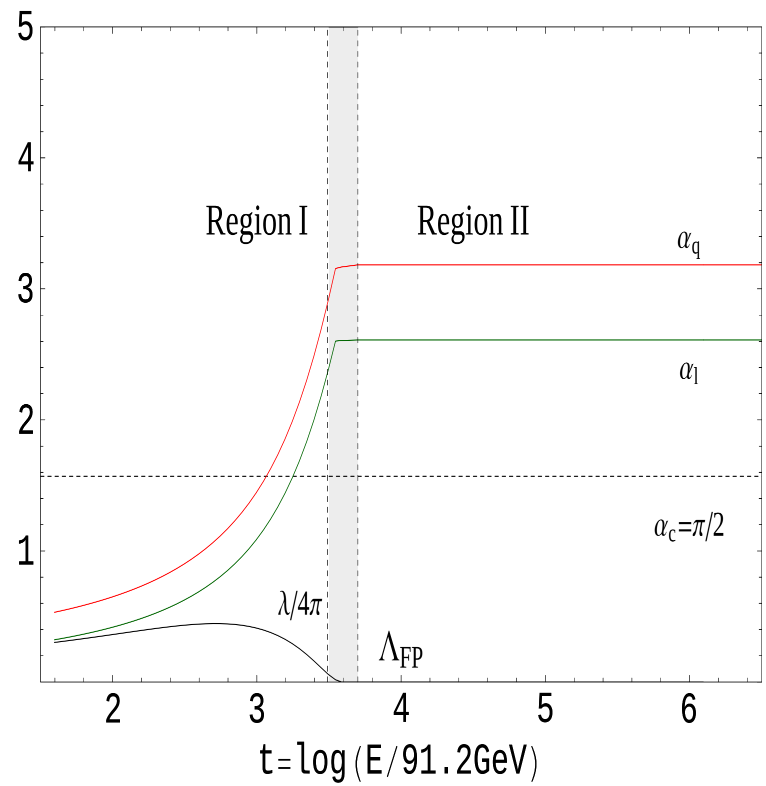


RGE vs. SDE

two figures from RGE and SDE respectively





Multiple Higgs doublets

- We might have three Higgs doublets: One fundamental, two composite

$$H_1 = (\pi^+, \pi^-, \pi^0, \sigma)$$

$$H_2 = (\bar{b}' t', \bar{t}' b', \bar{t}' t' - \bar{b}' b', \bar{t}' t' + \bar{b}' b')$$

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- The existence of the Nambu-Goldstone bosons leads to $\mathrm{det}\mathcal{M} = 0$ –modified gap equation

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- A heavy 4th generation also drives Yukawa couplings become strong at TeV scale
- Bound states/condensates of the 4th generation can be formed by exchanging Higgs bosons
- The perturbative (RGE) and non-perturbative (SDE) approaches lead to consistent results.