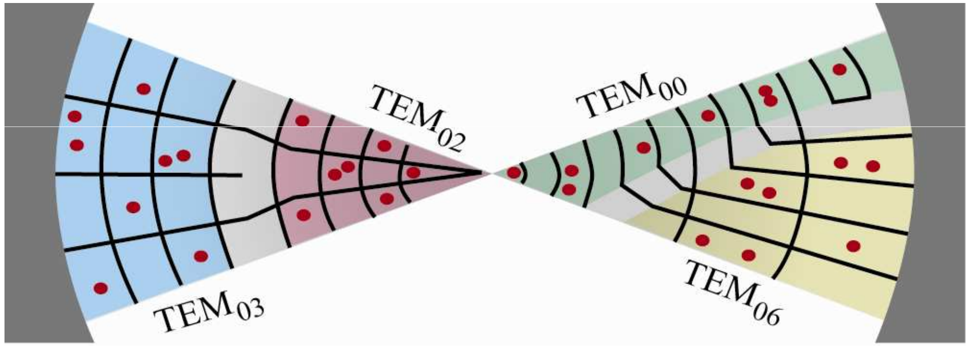
Photons as phonons:
from cavity QED to quantum solids



Sarang Gopalakrishnan
Benjamin Lev, Paul Goldbart
University of Illinois at Urbana-Champaign

Nature Physics **5**, 845 (2009); *PRA* **82**, 043612 (2010)

University of Virginia / Condensed matter seminar / Feb 3, 2011

Why cold atoms?

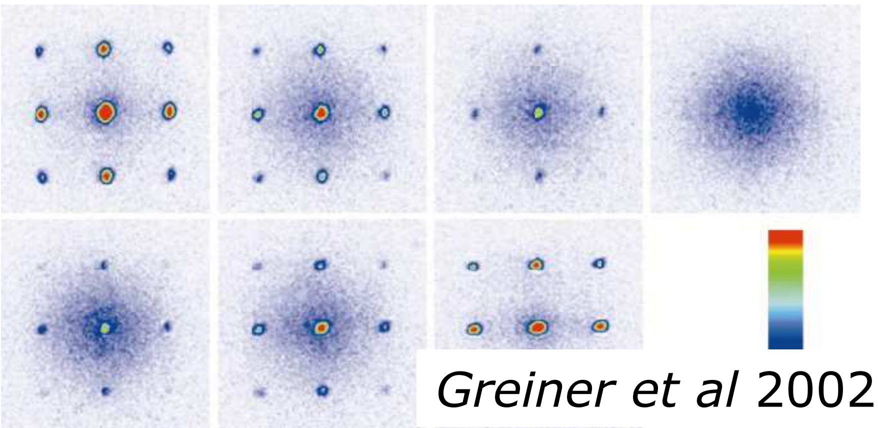
Advantages

- Control
  - Isolation and tunability
  - Well-understood Hamiltonian
  - Real-time dynamics accessible
- New probes (interference, single-site imaging…)
- New parameter space
  - Bosons, Bose-Fermi mixtures, unitary Fermi gases

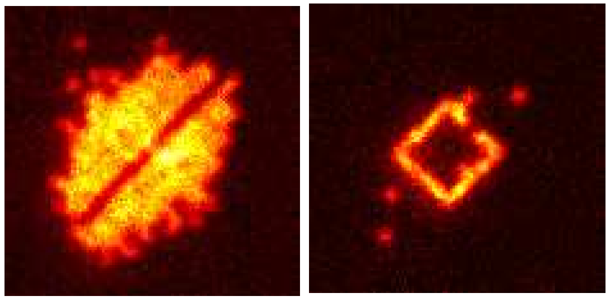
Drawbacks

- Short-lived systems
  - "Slow" dynamics harder
  - Destructive measurement (time-of-flight imaging)
- Fewer probes
  - Transport, spectroscopy hard
- System size, inhomogeneity

Nevertheless, worth seeing what one can realize…



Greiner et al 2002



Weitenberg et al. 2011

What Hamiltonians are realizable?

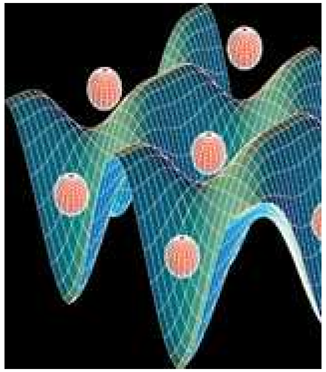
- Standard (effective) Hamiltonian:

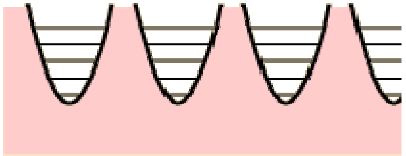
$$H = \mathrm{K.E.} + \sum_{\alpha\beta} \int d^d x V_{\alpha\beta} n_\alpha(\mathbf{x}) n_\beta(\mathbf{x})$$

- K.E. tunable via lattices, gauge fields, etc.
- V characterized by tunable scattering length(s)
  - Less tunable *structure*; always delta-function pseudopotential
  - Adequate as systems are dilute
  - Cf. structured potentials such as Lennard-Jones, RKKY, etc.
  - One consequence: no roton minimum in atomic BEC's
  - Generally: no tendency towards spatial ordering (except FFLO)
- How to move beyond this
  - **Intrinsic methods:** Use strong dipole-dipole interactions (e.g., Cr, Dy, Rydberg atoms, molecules…)
  - **Extrinsic methods:** *Light-mediated interactions (cavity QED)*, bipolarons with auxiliary atomic species, etc.

Optical vs. emergent lattices

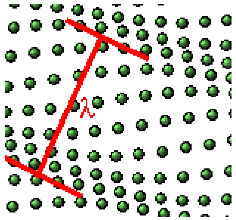
Optical lattices



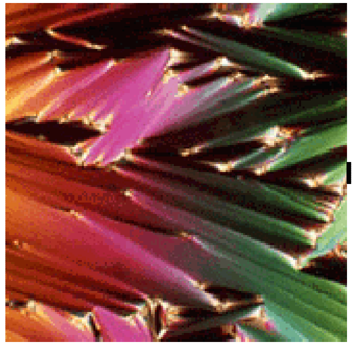


No lattice dynamics

Emergent lattices



Phonons, defects



The physics of emergent spatial ordering

- Liquid crystals, glasses, soft matter
- Supersolidity
- Topological defects, domains
- Electron-phonon coupling, polarons, superconductors

Our objective: to realize these phenomena in cold-atom physics

Outline

- Preliminaries
  - Atom-light interactions and cavity QED
  - Many atoms in a single-mode cavity
  - What are multimode cavities?
  - Single atom in a multimode cavity
- Crystallization of a BEC in a multimode cavity
  - Setup and approximations
  - Structure of effective low-energy theory
  - Character of phase transition
- Properties of ordered states
  - Excitations, supersolidity
- Extensions and prospects
  - Glassiness
  - Magnetism

Preliminaries

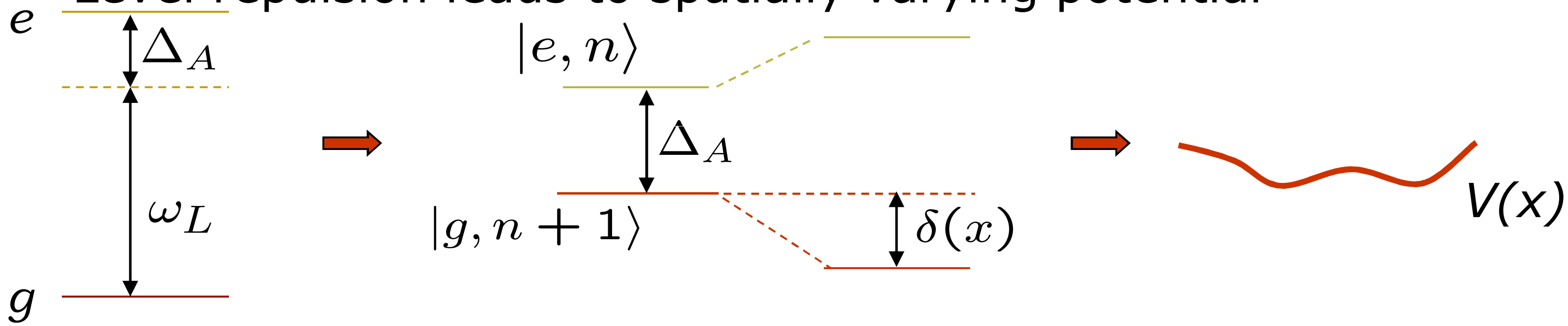
One atom, one mode

Many atoms, one mode

One atom, many modes

Atom-light interactions

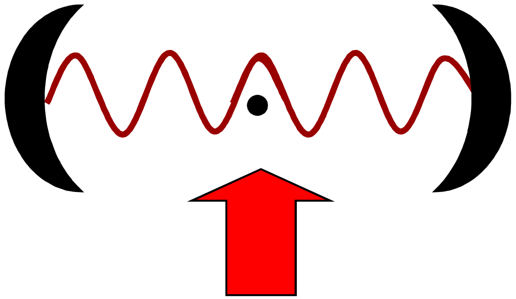
- a.c. Stark shift/dipole force [~ intensity / detuning]:
  - Assume two-level atoms, red-detuned laser
  - Rotating-wave approximation ($\omega_L >> \Delta_A$) , atom and field can be described as a two-level system in the "dressed-state" picture
  - Level repulsion leads to spatially varying potential

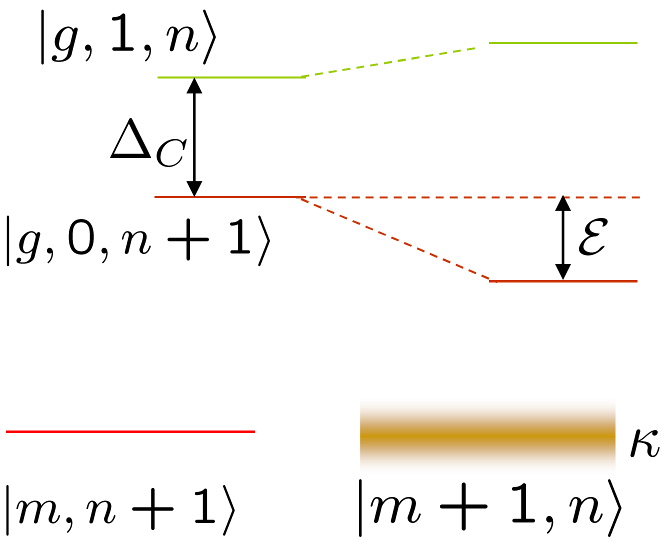


- Scattering force [~ linewidth x intensity / (detuning)^2]

  - Coupling to continuum leads to decoherence, heating (sets experiment lifetime)
  - Less important at large detunings

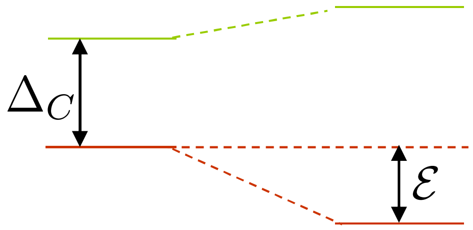
Laser-atom-cavity systems

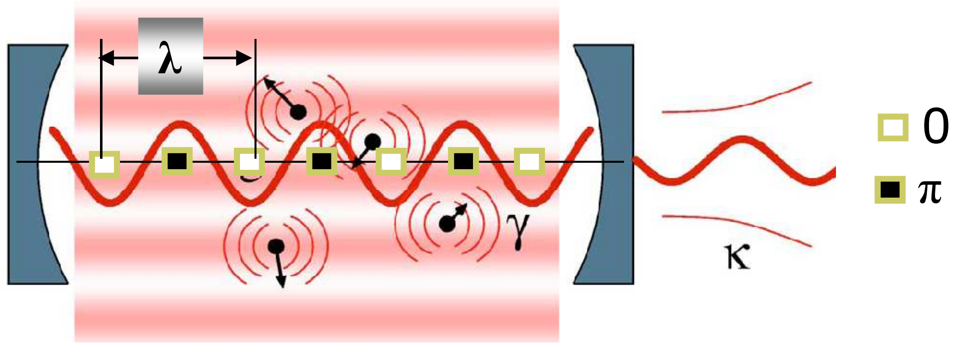




- Consider a high-finesse cavity
- Usually only one relevant mode
- Polarizable atom scatters light coherently, generates laser-cavity matrix element
- Make rotating-wave approximation in laser-cavity space
- Two aspects to resulting force
  1. Level repulsion (max. for atom at antinode) ~ 1 / Δ_C
  2. Scattering force due to cavity photon leakage ~ 1 / Δ_C^2
- Mobile atom (1) drawn to antinode, (2) given random kicks
- Reinforces high-field-seeking [total field intensity ~ $(E_L + E_C)^2$]
- At large Δ_C, ignore decay

Many atoms, self-organization





Domokos, Ritsch (PRL, 2002)

- Scattering from multiple atoms adds coherently [black – white]
- Hence, effective matrix element $\sim \int dx\, n(x)\cos(Kx), K \equiv 2\pi/\lambda$
- Optical energy gain: $\mathcal{E} \sim \text{const.} - \left[\int dx\, n(x)\cos(Kx)\right]^2$
- **Cavity mediates infinite-range interatomic interaction**
- Interaction tends to localize atoms at even/odd sites and break *discrete* even/odd symmetry
- Competes with atomic K.E., repulsion, etc. → phase transition

Phase transition for a BEC

- Consider the Hamiltonian:

$$H = \int dk\, k^2 \psi_k^\dagger \psi_k - \gamma \left[\int dk\, \psi_k^\dagger (\psi_{k+K} + \psi_{k-K}) \right]^2$$

- Can solve in Bogoliubov approximation for mode *K* (hybridized with –*K*)

$$E_K \sim \sqrt{K^2(K^2 - 2\gamma)}$$

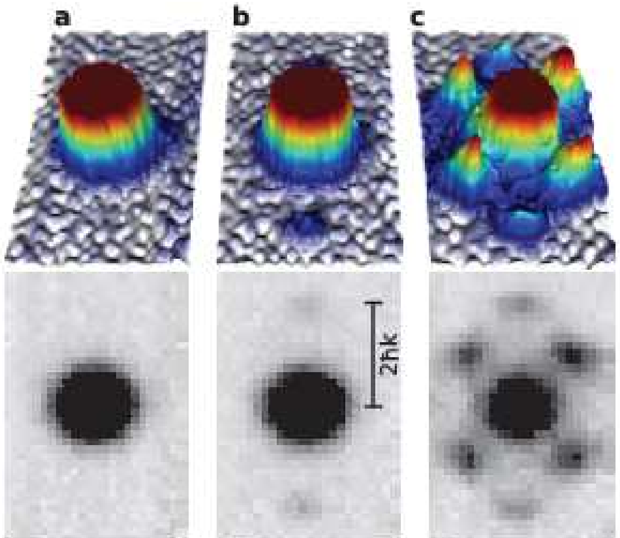
- Excitation of wavenumber *K* goes soft at sufficient γ; phase transition
- Spontaneous breaking of Z_2 symmetry
  - Inadequate if one wants to model crystallization
- Analogous to Dicke model, second-order quantum phase transition

Nonequilibrium aspects

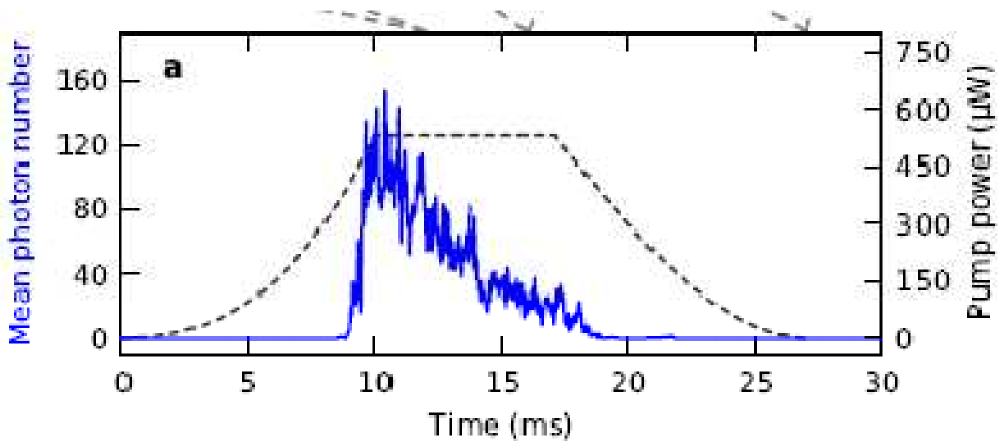
- *Prima facie* a damped driven system
- Appearances are deceptive:
  - If laser detuned from cavity and atoms, *and* loss rates low, few real transitions occur [need: $\gamma << \Delta_A$ $\kappa << \Delta_C$]
  - Adiabatic switching holds, equilibrium path integral works
  - This can be shown via Schwinger-Keldysh
  - More intuitively: cavity decay is analogous to spontaneous emission in standard AMO experiments
- Equivalent ground-state problem
  - Diagonalize H in manifold of fixed total photon number
  - For laser red-detuned from cavity, noninteracting ground state in this manifold is all photons in laser
  - Treat laser as a BEC of photons
- Analyze phase structure in this approximation, introduce departures from equilibrium afterwards
  - Cavity photon decay sets an effective temperature

Experimental realization

- Experimental realization: Baumann et al., *Nature* **464**, 1301 (2010) [see also: *Physics Today* (July 2010)]
- Satellite Bragg peaks in time-of-flight demonstrate: (a) emergence of lattice, (b) phase coherence
- Experimental lifetime **now** ~1 sec. (limited by atom loss)

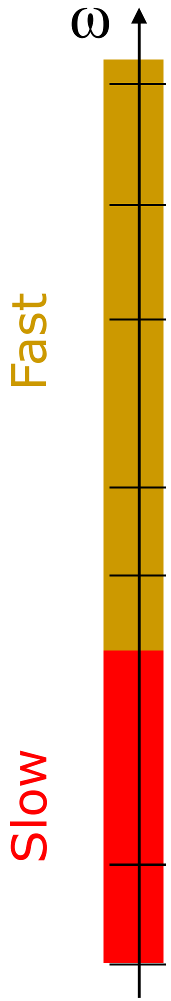


Momentum distributions (time-of-flight)



Baumann et al., *Nature* **464**, 1301 (2010)

Relevant energy scales



10^{15} Hz: optical frequencies (of order eV)

10^{11} – 10^{12} Hz: atom-laser detuning, cavity mode spacing

10^{9} Hz: atom-laser coupling Ω, laser-cavity detuning

10^{6} – 10^{7} Hz: atom-cavity coupling g

10^{5} – 10^{6} Hz: each of the competing energy scales involved in crystallization [recoil energy]

10^{3} Hz: system temperature (20-30 nK, < T_{BEC}.)

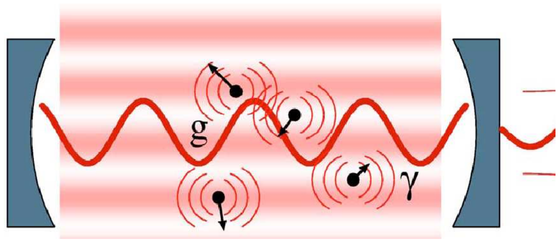
1-100 Hz: lifetime of experiment (set by loss/heating rates)

Self-organization (II)

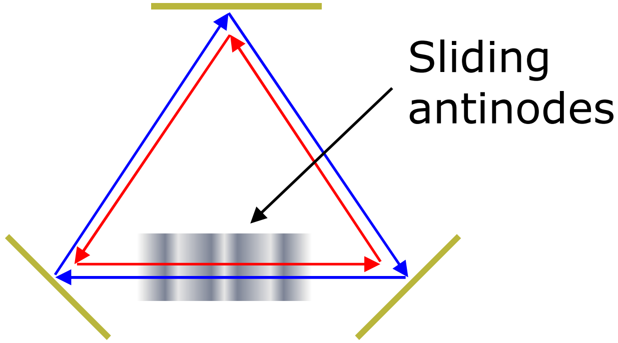
$$\text{P.E.} \sim \int dx\, V(x) n(x) \sim -\left[\int dx n(x) \cos(kx)\right]^2$$

- P.E. favors atoms being one wavelength apart, and "digging themselves into wells"
- Symmetry under $n(x) \rightarrow n(x + \frac{1}{2}\lambda)$
  - i.e., under moving atoms from even to odd antinodes
  - Thus, this term tends to spontaneously break even-odd symmetry
  - P.E. maximal when atoms localized precisely at antinodes
- Competes with thermal effects, kinetic energy, and/or interactions
  - All these effects favor spreading out of atoms
  - Thus, have phase transition as one increases laser strength
- Limitations: mean-field, infinite-range interaction
  - Also: discrete symmetry; would prefer continuous

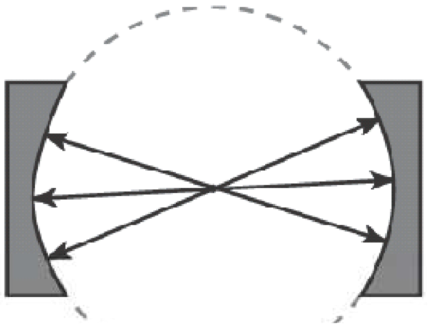
Single- and multi-mode cavities



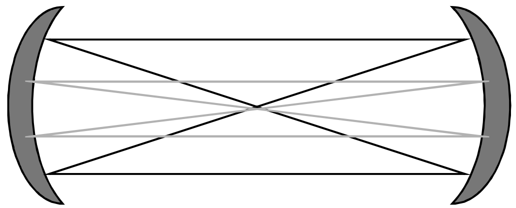
Single-mode cavity
Standing wave between two mirrors
All modes non-degenerate



Ring cavity
Two counter-propagating modes
Translational symmetry

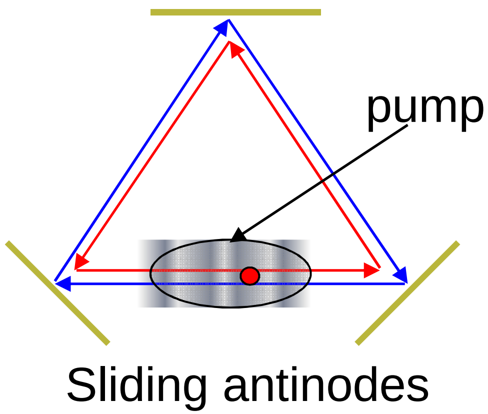


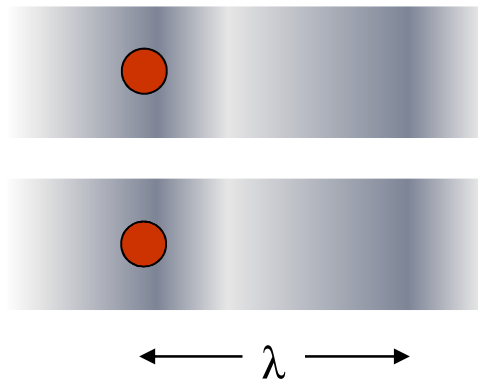
Concentric cavity
Rotational invariance
Many degenerate modes



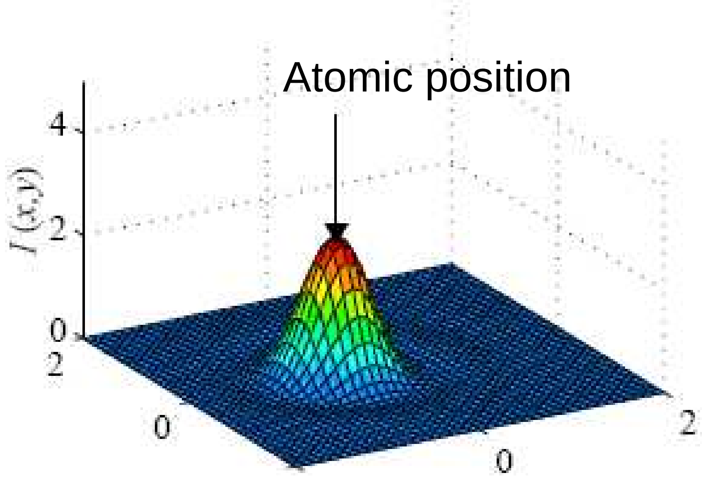
Confocal cavity
All even TEM modes degenerate

Atomic motion in multimode cavities





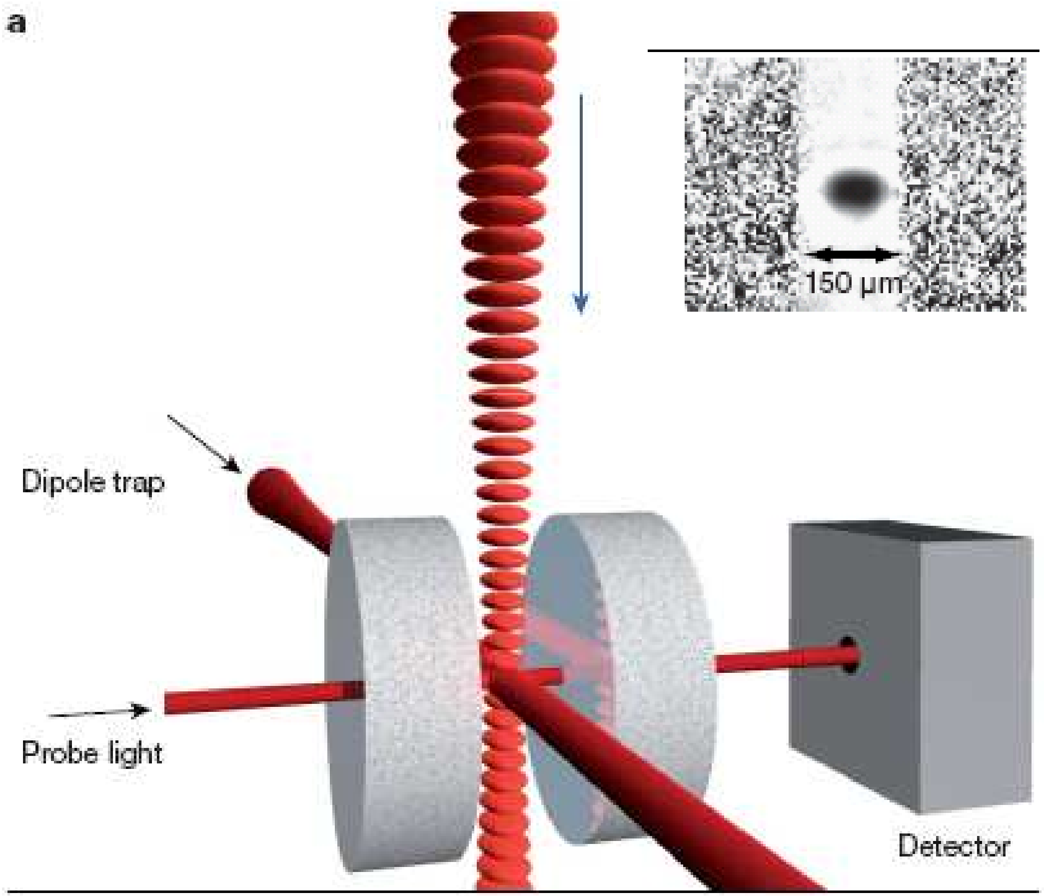
- Atom scatters light from pump to cavity
- High-field-seeking atom(s)
- Antinode forms near atom
- Atom drags antinode with it
- Different characteristic speeds
- More "local" coupling with many modes (i.e., can build wavepackets)
- Atom, intensity bump constitute "**polaron**"



Salzburger et al, 2002

Loading a BEC into a cavity

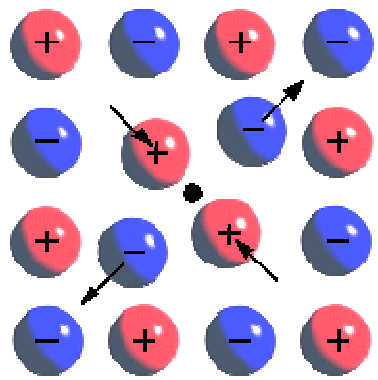
- Cool atoms elsewhere
- Optical lattice "conveyor belt"—two lasers at slightly different frequencies
- Potential maxima drift downwards
- Reflectivity for probe light changes when the BEC lands in the cavity
- Turn off conveyor belt at this point



Brennecke et al, Nature (2007)

Connection with polarons

- Polarons are slow electrons in ionic crystals
  - Electron dressed by lattice distortion
  - **Strong coupling**, perturbation theory inapplicable
  - Variational approaches (Feynman, Lee/Pines)



- Cavity QED problem maps onto polaron
  - Eliminate excited state; effective Hamiltonian:

$$H_{\text{eff}} = \frac{\mathbf{p}^2}{2m} + \hbar\omega_C \sum_\alpha a_\alpha^\dagger a_\alpha + \frac{\Omega e^{i\omega_L t}}{\Delta_A} \sum_\alpha \hbar g_\alpha(\mathbf{x}) a_\alpha + \text{h.c.} + \ldots$$

- Cross-fertilization between CM techniques, AMO systems
- E.g. variational effective masses at strong coupling:

Ring: $\frac{m^*}{m} \sim \Omega^4 g^4$ **Confocal:** $\frac{m^*}{m} \sim \Omega^8 g^8$

- Can also cor… …bility, terminal veloc… …pping etc.

Crystallization of a BEC in a multimode cavity

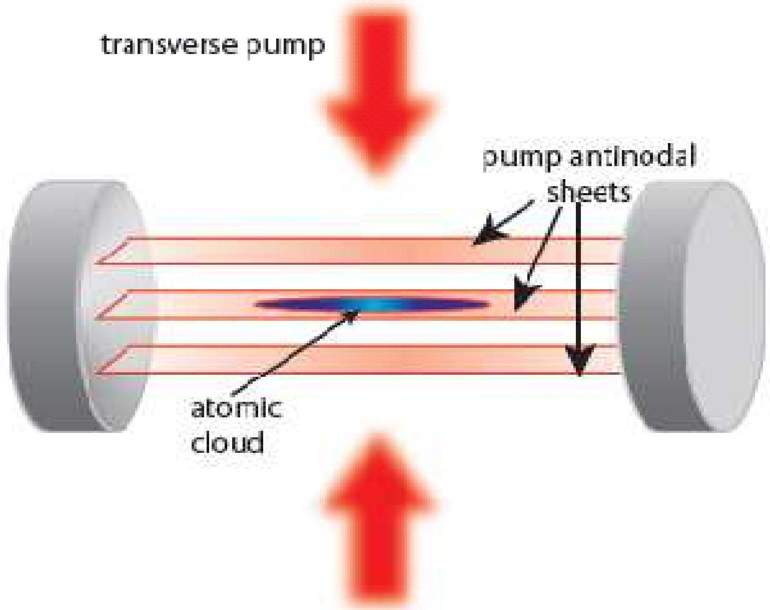
Microscopic model

Coarse-grained action

Theory of the crystallization transition

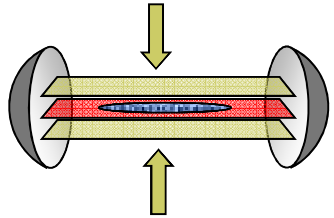
Model – Setup and Assumptions

- **$\omega_P << \omega_C << \omega_A$**
- Atoms strongly trapped in 2D layers at pump antinodes
- **First consider single layer at equator**



$$
\begin{aligned}
H &= \sum_{n=1}^{N}\left[\frac{\mathbf{p}_n^2}{2m} + V(\mathbf{x}_n) + \omega_A \sigma_n^z\right] + U\sum_{m<n}\delta(\mathbf{x}_m - \mathbf{x}_n) + \sum_{\mu=1}^{M}\omega_C a_\mu^\dagger a_\mu \\
&+ i\sum_{n,\mu}(g_\mu(\mathbf{x}_n)a_\mu^\dagger\sigma_n^- - h.c.) + i\Omega e^{i\omega_L t}\sum_n(\sigma_n^- - \sigma_n^+).
\end{aligned}
$$

+ coupling to environment



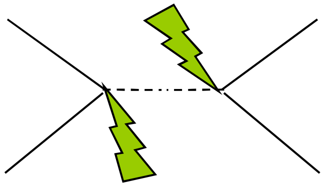
Effective action (I)

- Integrate out atomic excited state, all photon states
- Derive action in terms of atomic motional states

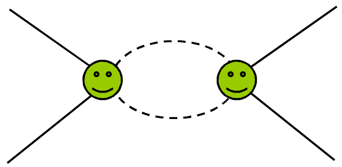
Atomic K.E. $\int d\omega d^2\mathbf{x}\psi^*(\omega,\mathbf{x})(i\omega-\nabla^2-\mu)\psi(\omega,\mathbf{x})$ *propagator*

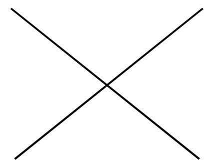
Laser-cavity-laser scattering

$$\frac{\Omega^2}{\Delta_A^2\Delta_C}\left(\int d\mathbf{x}g_\mu(\mathbf{x})|\psi(\mathbf{x})|^2\right)^2$$



Scattering via cavity photon exchange and contact repulsion

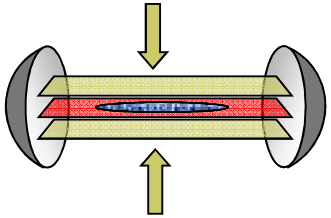




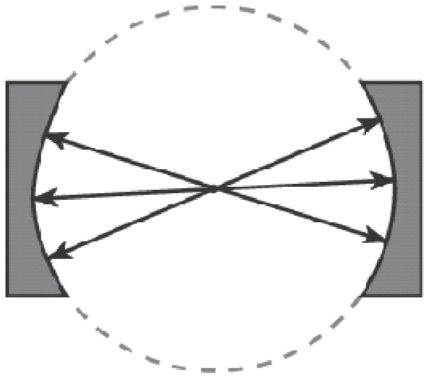
$$\frac{1}{\Delta_A^2\Delta_C}\left(\int d\mathbf{x}g_\mu(\mathbf{x})g_\nu(\mathbf{x})|\psi(\mathbf{x})|^2\right)^2 \qquad \lambda\int d\mathbf{x}|\psi(\mathbf{x})|^4$$

To proceed, must exploit cavity mode structure…

Cavity Mode Structure

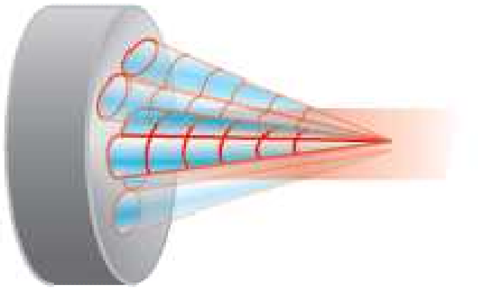


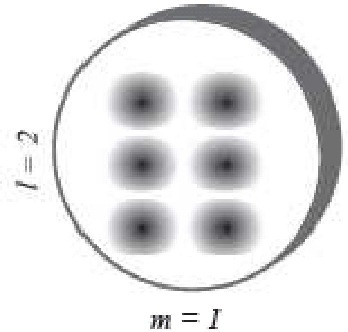
- Specialize to concentric cavity
- Laser breaks spherical symmetry
- 2D rotation symmetry in-plane
- Dirichlet b.c. on *r* and θ (approximate!)
- Mode functions [α = (m,n)]:

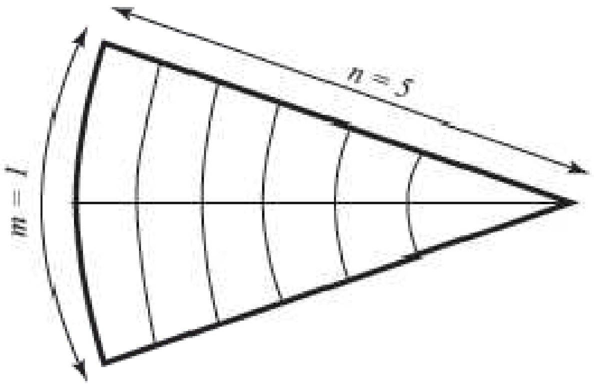


$$g_\alpha(r,\theta) \sim f(l)\,J_m(nr/R)\,\cos(m\theta), \omega_\alpha \sim (|m|+|n|)$$

- High-m modes have diffractive losses so truncate at (say) m < n/5
- Treat *f(l)* as peaked at some value of *l*

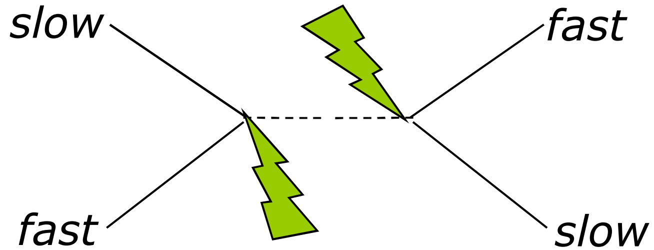






Induced Atomic Mode Structure

- Write action in (m,n) basis
- Primary diagram: L-C scattering
- Effective "momentum conservation"
- Laser has no momentum; cavity mode has momentum *K*

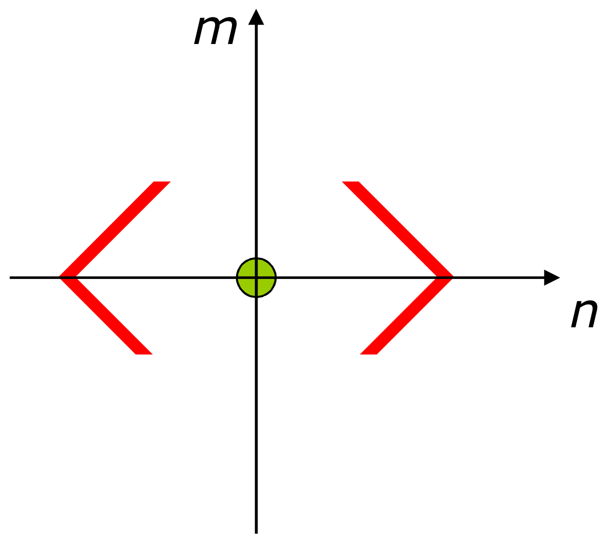


$$\frac{\Omega^2}{\Delta_C}\left(\int d\mathbf{x} g_\mu(\mathbf{x})|\psi(\mathbf{x})|^2\right)^2$$

- Two kinds of modes:
  - (m,n) small
  - |m| + |n| ≈ Λ
- Order parameter for crystallization (use ODLRO):

$$\rho_{mn} = \int d^d x|\psi(\mathbf{x})|^2 g_{mn}(\mathbf{x}) \approx (\psi_{mn} + \psi^*_{mn})\sqrt{N_0}$$

- Integrate out "green" modes
- Remaining modes analogous to nested Fermi surface



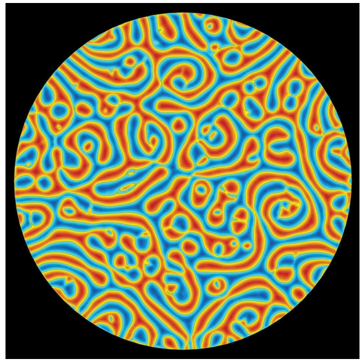
Effective Action

- Effective free energy/action (T > 0):

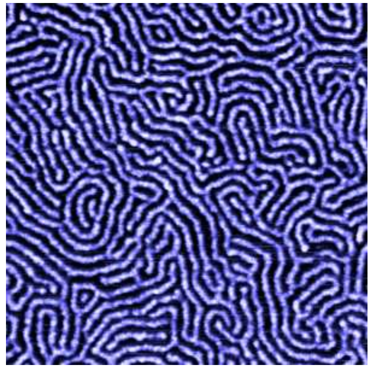
“dispersion” along z-axis

$$S_{\text{eff}} = \sum_{mn} \left[\mathcal{R} + \boxed{\chi}(m+n-K_0R)^2\right] \rho_{mn}\rho_{-mn} + \mathcal{U} \sum_{m_i,n_i} \rho_{m_1n_1}\rho_{m_2n_2}\rho_{m_3n_3}\rho_{m_4n_4}\,\delta_{\sum m_i}\delta_{\sum n_i},$$

- *m* < *n* constraint → **no cubic term**
- Realizes Brazovskii’s (1975) model (common in soft matter) [*modulo nesting-related subtleties*]
- MF: **2nd order** transition when τ = 0 (physically: K.E. + repulsion = optical potential energy)
- Fluctuation-driven **1st order** transition
- At T = 0: *z* = 1, action acquires ω^2 term
  - Physics qualitatively similar to *T* > 0 case

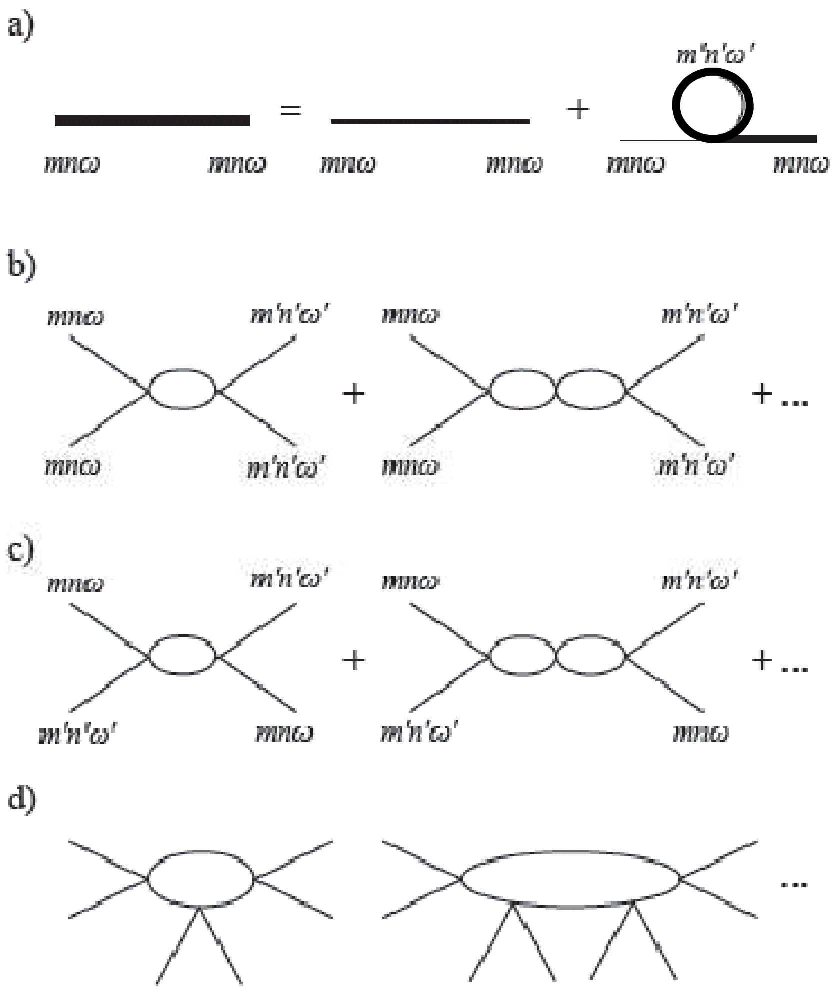


Convection



Diblock copolymers

Brazovskii's transition



Self-consistent 1-loop, T > 0

- Self-energy *r*:

$$r = \mathcal{R} + \alpha K_0 u / \sqrt{r}$$

renormalized

bare

- Always positive: no criticality
- Generic vertex correction (b)

$$u = \mathcal{U} / [1 + \alpha \mathcal{U} / r^{3/2}]$$

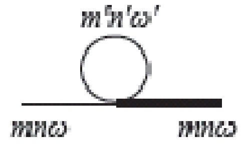
- Vertex correction when (mn) = (m'n') [sum both (b) and (c)]

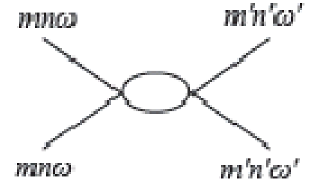
$$u' = \mathcal{U} \frac{1 - \alpha \mathcal{U} / r^{3/2}}{1 + \alpha \mathcal{U} / r^{3/2}}$$

- Changes sign: hence 1st order transition
- Stabilizing higher-order terms
- Consistent with 1-loop RG

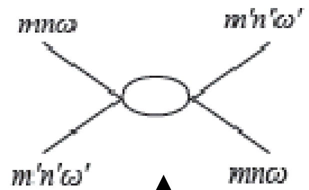
Brazovskii's transition (II)

- Ignore nesting
- Interactions only couple sets of opposite pairs of momenta
- Diagram A is O(n) invariant if n = number of Fermi surface directions (or modes)
- Analogy with O(n) model in 1 or (1+1) dimensions
- Renormalization of 2-point fn. by A prevents instability
- Diagram B breaks O(n) symmetry but does not renormalize 2-point fn. (only important in 1/n cases)
  - More generally, the O(n) symmetric part of the theory has a closed RG flow to leading order.
- However, the four-point fn changes sign when all four modes are the same
- Therefore, can have a first-order transition

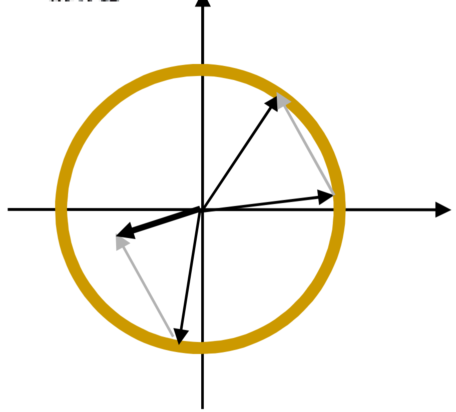




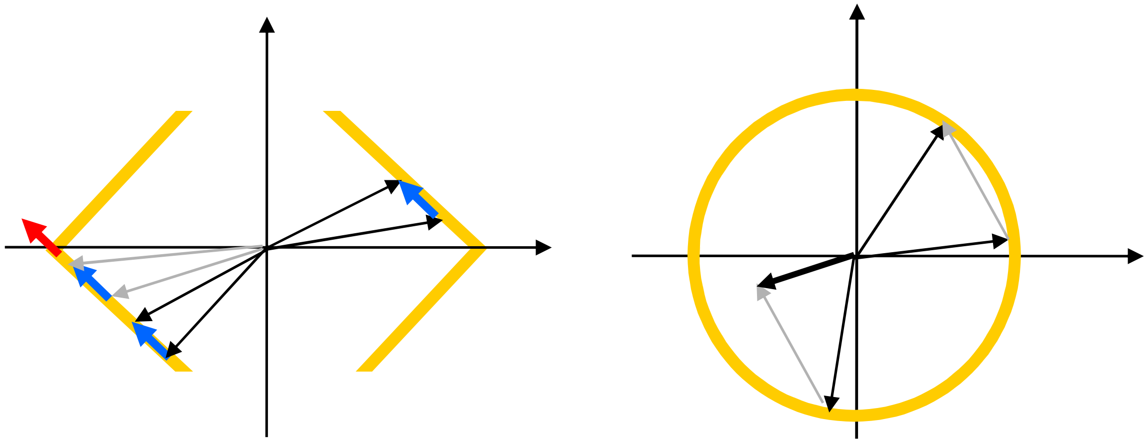
A



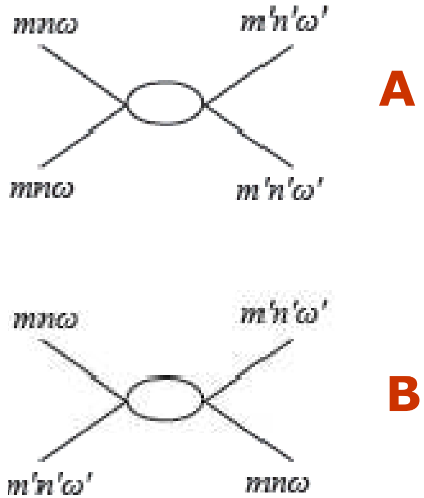
B



Significance of nesting



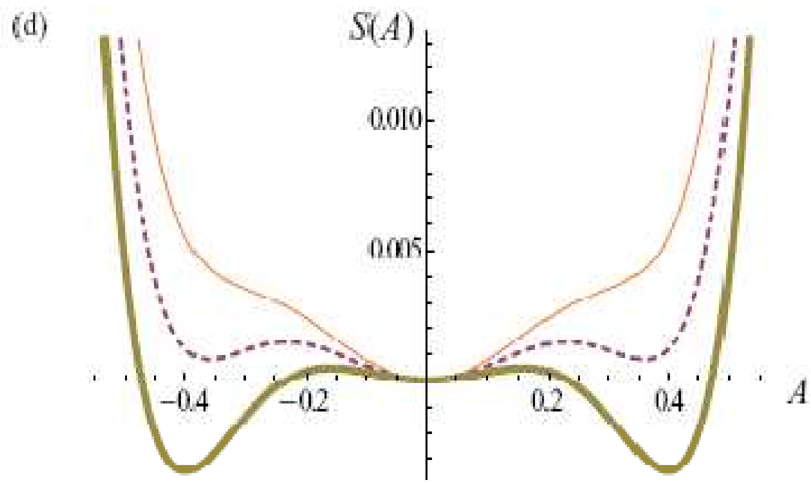
nested *non-nested*

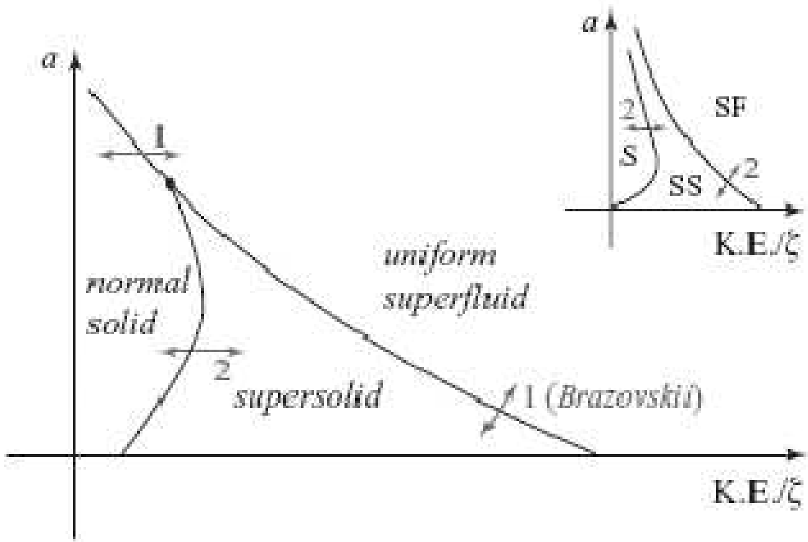


- Diagram B triggers first-order transition
- Without nesting (circle): momenta *must* be equal, opposite
  - Diagram A outweighs diagram B by a factor = # of modes
  - Diagram B only contributes when all four modes are the same
- With nesting (lines): all four modes must lie on manifold
  - Diagram B contributes less as length (m – m′, n – n′) increases
  - Diagram B always contributes, but *most* when all modes the same
- Instability still first arises in channel with four equal momenta

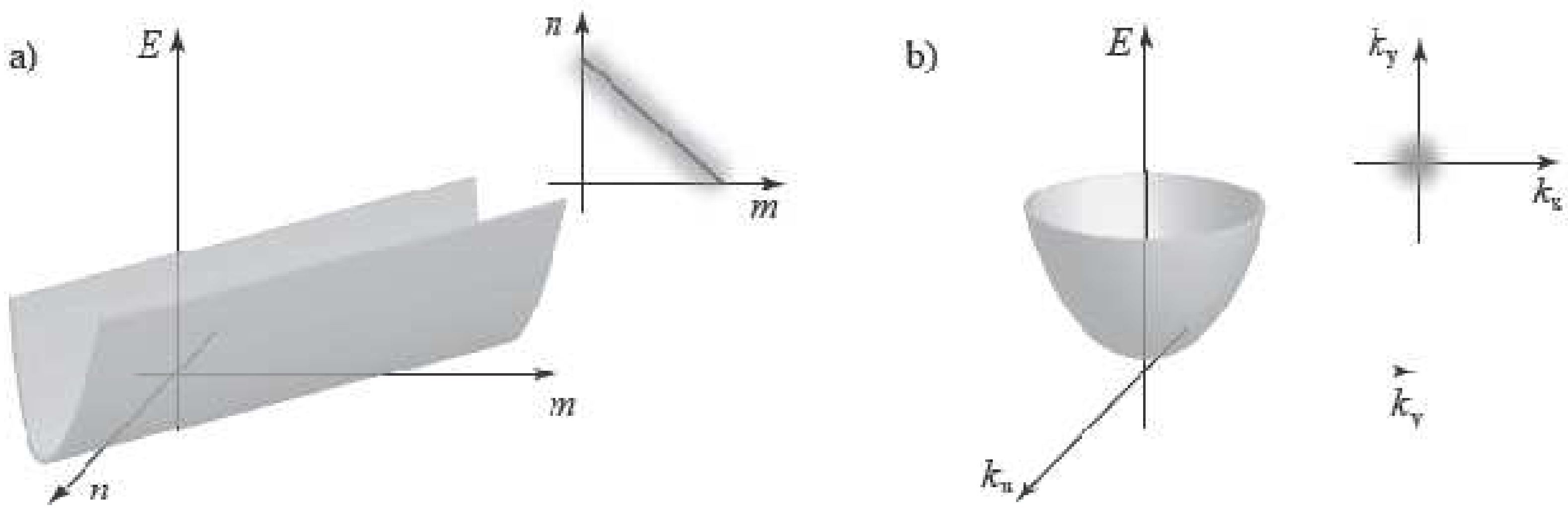
Structure of coarse-grained theory

- Integrating RG equations gives coarse-grained couplings
- Can use these to plot coarse-grained free-energy landscape as a function of bare R (i.e., laser strength)
- Overall phase structure
  - Near transition, lattice weak enough to preserve ODLRO
  - Deep in ordered state, transition into Mott state
- Discontinuous jump in lattice depth permits coincident Mott and Brazovskii transitions





Implications



- Large phase space for fluctuations
- Fluctuations change order of phase transition (2nd in MF, 1st with fluctuations) and threshold

$$\Omega_{\rm th}^2 - \Omega_{\rm mf}^2 \sim \left[\frac{g^2 \Delta_C}{\Delta_A^2(\Delta_C^2 + \kappa^2)} \frac{\Omega_{\rm th}^8 M R^2}{\hbar N \chi} \right]^{\frac{1}{3}}$$

- At T = 0: quantum Brazovskii transition

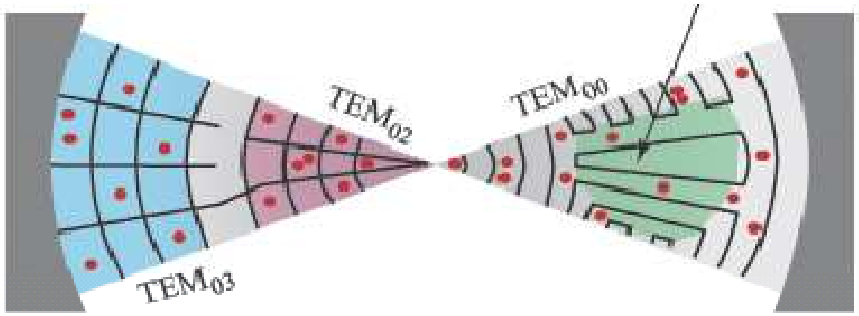
Properties of the ordered states

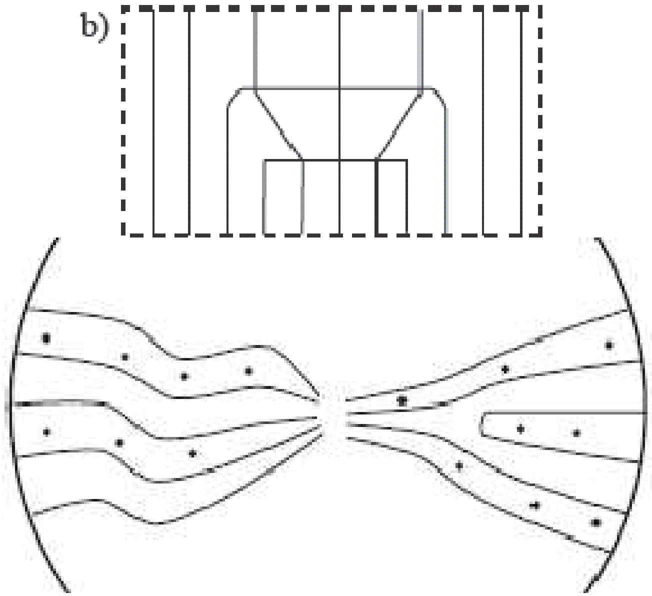
Excitations and defects

Supersolidity

Ripple and splay modes

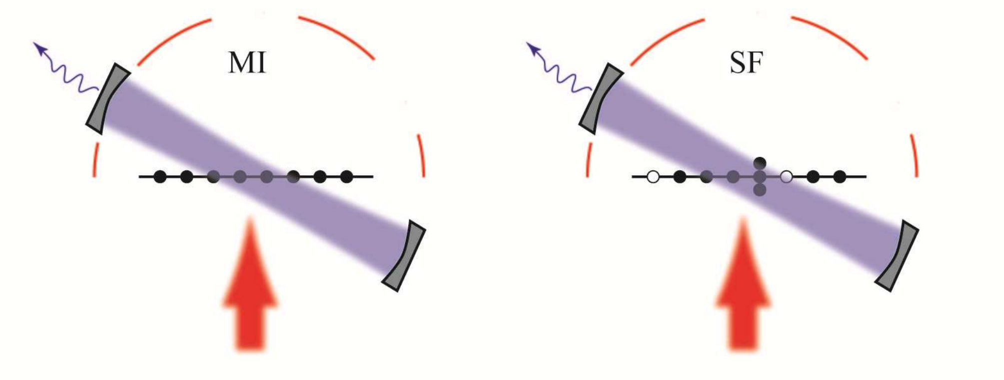
- Types of symmetry-breaking
  - Large mirrors: phase between ±m modes
  - Always: choice of modes (analogous to rotation)
- Two Goldstone-like modes
  - "Ripple" mode
  - "Splay" mode
- Two classes of defects
  - Edge dislocations
  - Closed lamellae
- Domain wall-like defects possible, not topological
- Hints of nucleation seen in simulations (Ritsch)
- Mermin-Wagner effects?

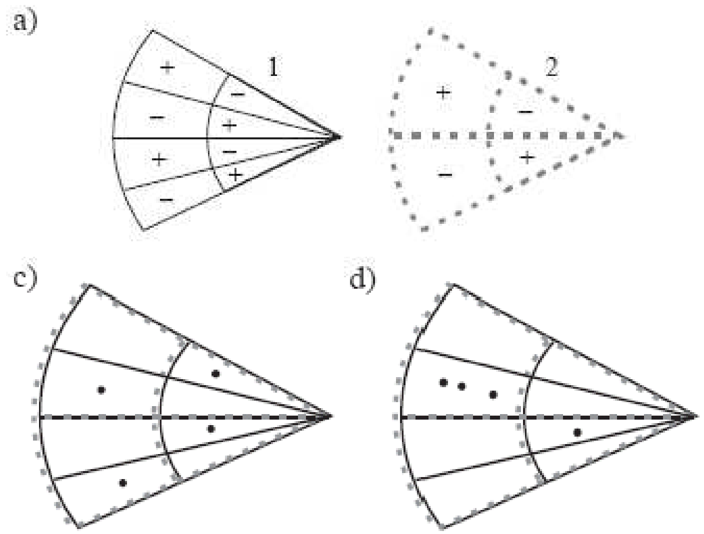




Detecting supersolidity

- Idea (Mekhov et al, 2007):
  - Consider Bragg scattering off "crystal"
  - Deep in MI: site occupation fixed, well-defined Bragg minima
  - Deep in SF: occupation fluctuates, transmission at minima
- Adaptation to present context:
  - Consider transmission into higher-order modes along z
  - Same principle applies
- Spatially resolved probe of supersolidity





Extensions

Glassiness

Magnetism

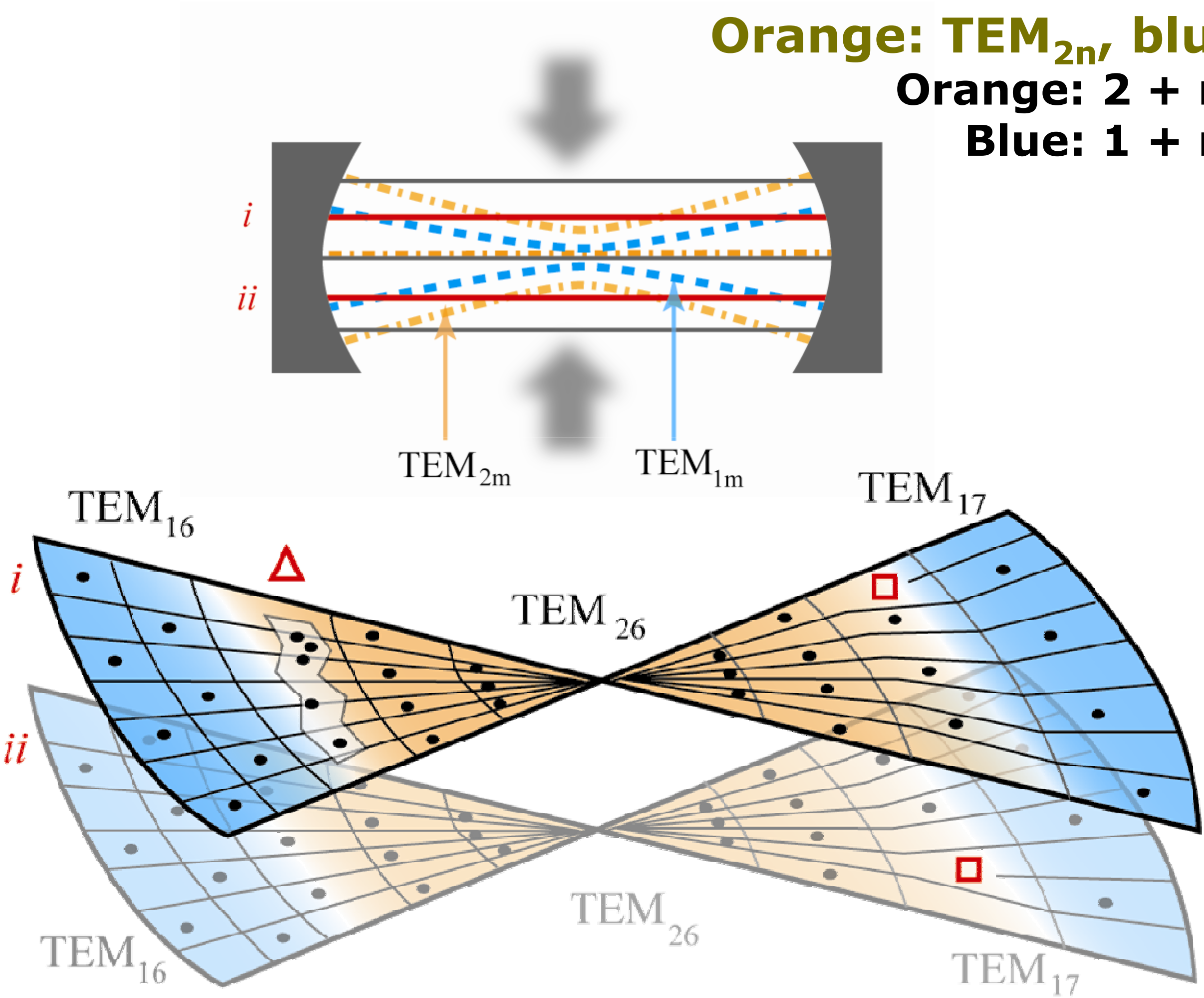
Multilayered systems, frustration

Orange: TEM_{2n}, blue: TEM_{1n}

Orange: 2 + m + n = K

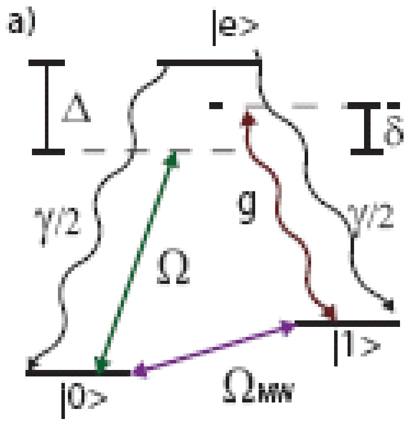
Blue: 1 + m + n = K

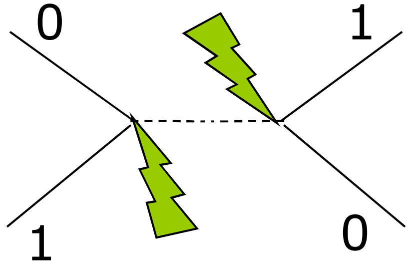
K fixed



Magnetism in cavities

- Idea: use 3-level atoms with two ground states, ring cavity
- State-selective dressing
  - Couple laser to 0 – e transition
  - Couple cavity to 1 – e transition
  - Scattering photons from laser to cavity changes internal state
- Integrating out excited state, cavity modes generates long-range state-dependent coupling
- In the spin language, effective Hamiltonian for stationary atoms:





$$H_{\text{spin}} \sim \sum_i \sigma_x^i + \sum_{ij} g_{ij}\sigma_+^i \sigma_-^j \qquad g_{ij} = g\cos[k(x_i - x_j)]$$

- Possible application: mean-field spin glasses, Sherrington-Kirkpatrick model

Summary

- Cavity photons can be used to mediate interatomic interactions
  - Infinite-ranged in the case of a single-mode cavity
  - More local for a multimode cavity
  - Interactions favor atoms integer cavity wavelengths apart, cause crystallization
- Crystallization occurs via a Brazovskii transition
  - Fluctuation-driven first-order transition both at $T = 0$ (quantum) and $T > 0$ (thermal)
  - Low-energy physics described by surface of excitations
- Prospects for further work
  - Structural glassiness
  - Magnetism via atoms with internal structure
  - Fermions (superconductivity?)

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