Observation of Shock waves and beyond Luttinger liquid physics in cold atoms

Manas Kulkarni
(Stony Brook and Brookhaven)

- **arXiv: 1012.2885 (submitted PRL)**
- **Nucl. Phys.B, 846, 122 (2011)**
- **Phys. Rev. B 80, 165105 (2009)**
- **Nucl. Phys. B, 825, 320 (2010)**
- **A. G Abanov, A. Gromov, M. K (in preparation)**

STONY BROOK STATE UNIVERSITY OF NEW YORK
BROOKHAVEN NATIONAL LABORATORY
Duke Physics Atom Cooling and Trapping
ICTP

Collaboration:

Theory:

A. G. Abanov (Stony Brook)
F. Franchini (ICTP/SISSA)
A. Gromov (Stony Brook)

Experiment:

John Thomas (Duke)
James Joseph (Duke)

Acknowledgements:

Theory:

C. N. Yang and Zhong-Qi Ma (Tsinghua)
P. Wiegmann (Chicago)
A. Polychronakos (CUNY)
Y. Kato (Kyoto)
H. Katsura (KITP/RIKEN)
E. Shuryak (Stony Brook)

Experiment:

Peter. van der Straten (Utrecht)
D. Schneble (Stony Brook)

Contents

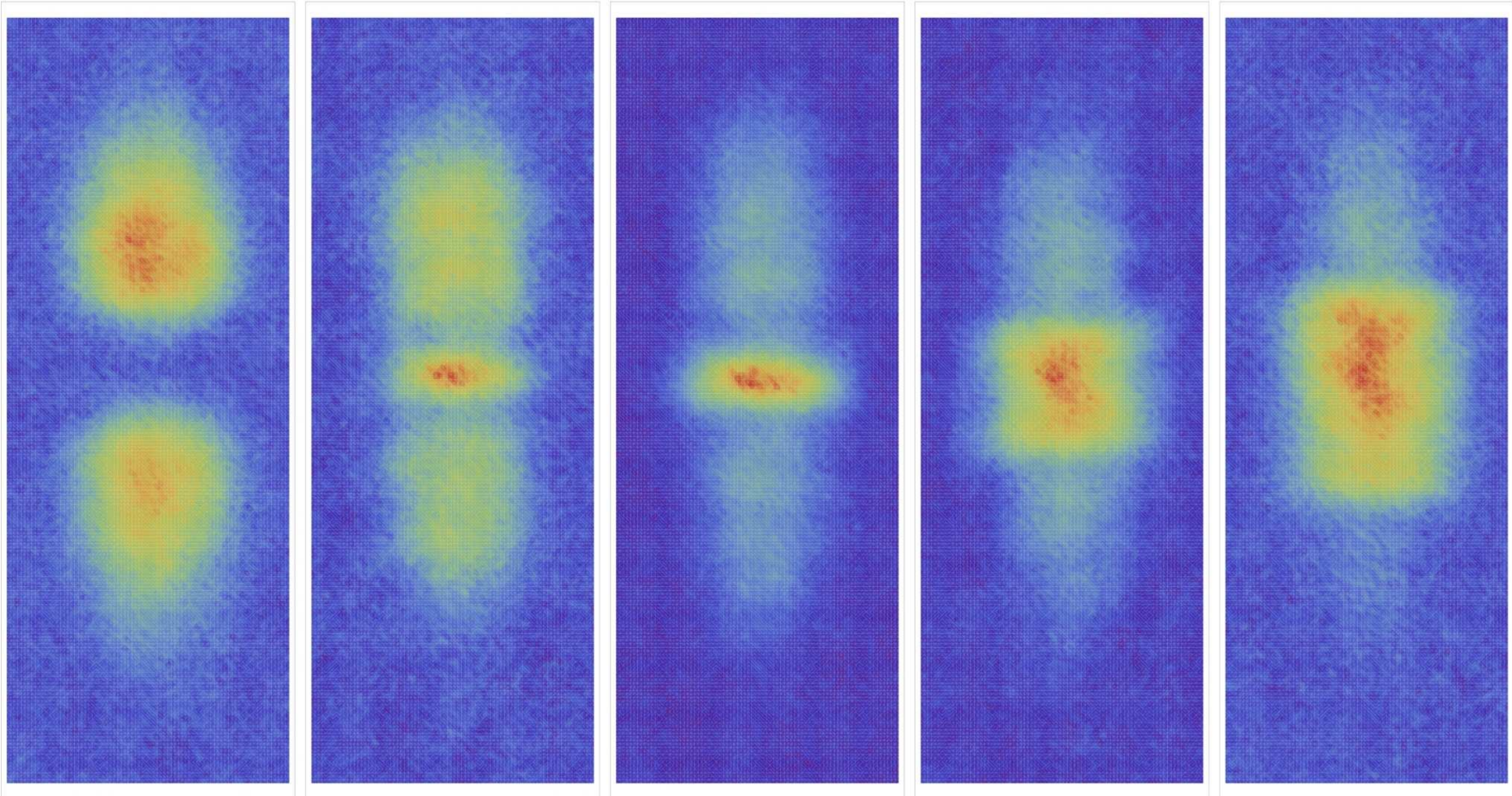
- Nonlinearity, Dissipation and Dispersion
- Unitary gas and Duke experiment
- 1D reduced hydrodynamics

Part-1

- Beyond luttinger liquid theory for a harmonically trapped integrable system

Part-2

First Observation of Shock Waves



Absorption images: Collision of atomic clouds

John Thomas group (Duke)

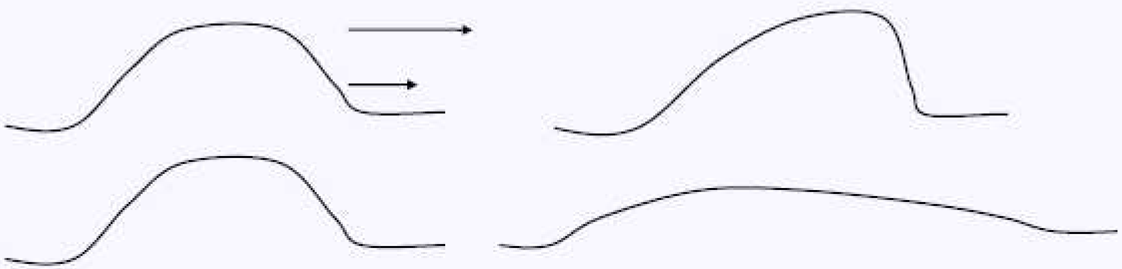
Nonlinearity, dissipation, dispersion

$$\boxed{u_t + c u_x = 0}$$

$$u_t + c u_x + \underbrace{\alpha\, u u_x}_{\text{nonlinearity}} + \underbrace{\beta\, u_{xx}}_{\text{dissipation}} + \underbrace{\gamma\, u_{xxx}}_{\text{dispersion}} = 0$$

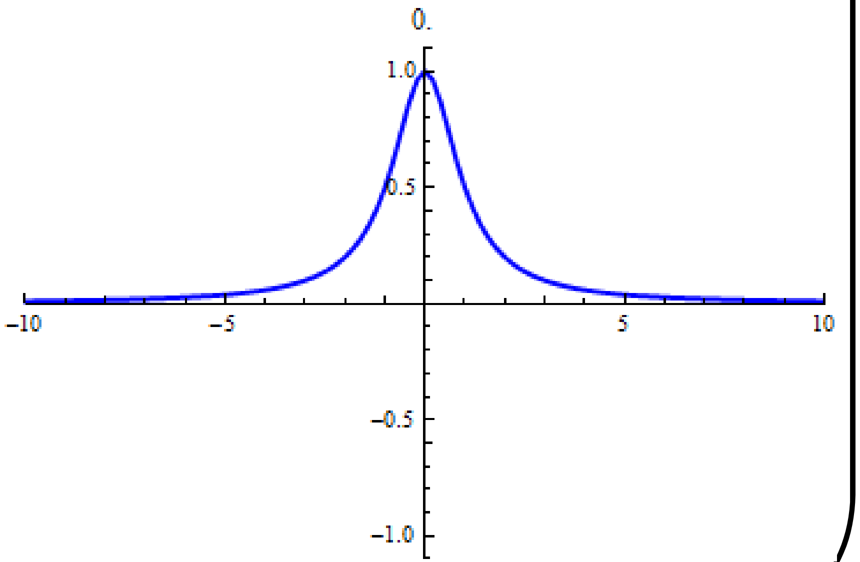
$$\omega = ck - i\beta\, k^2 - \gamma\, k^3$$

- Nonlinearity
- Dispersion



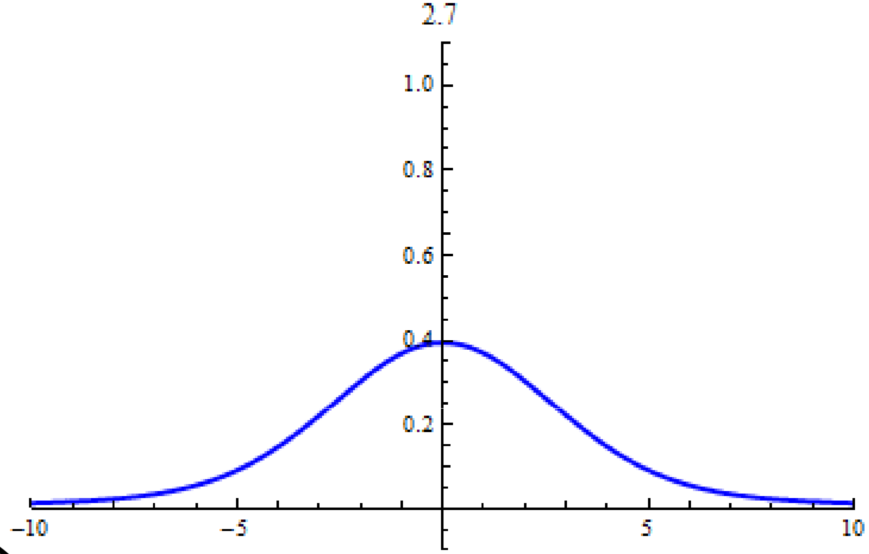
Nonlinearity

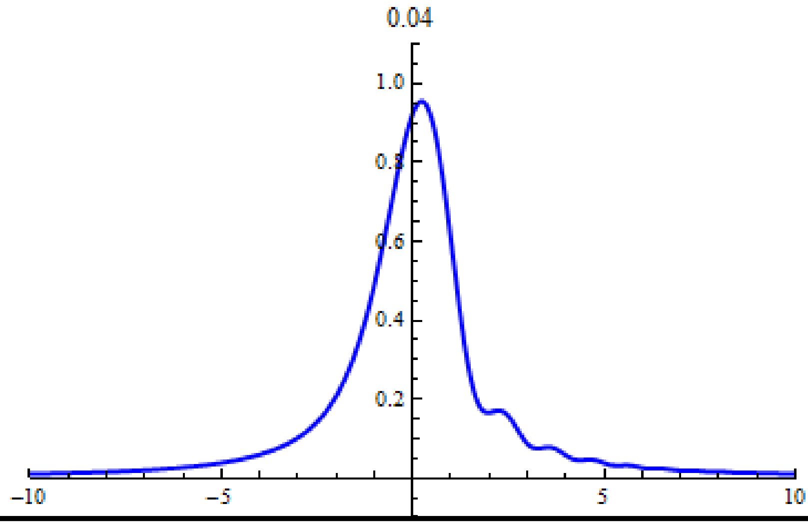
$$u_t + uu_x = 0$$



Dissipation

$$u_t + u_{xx} = 0$$

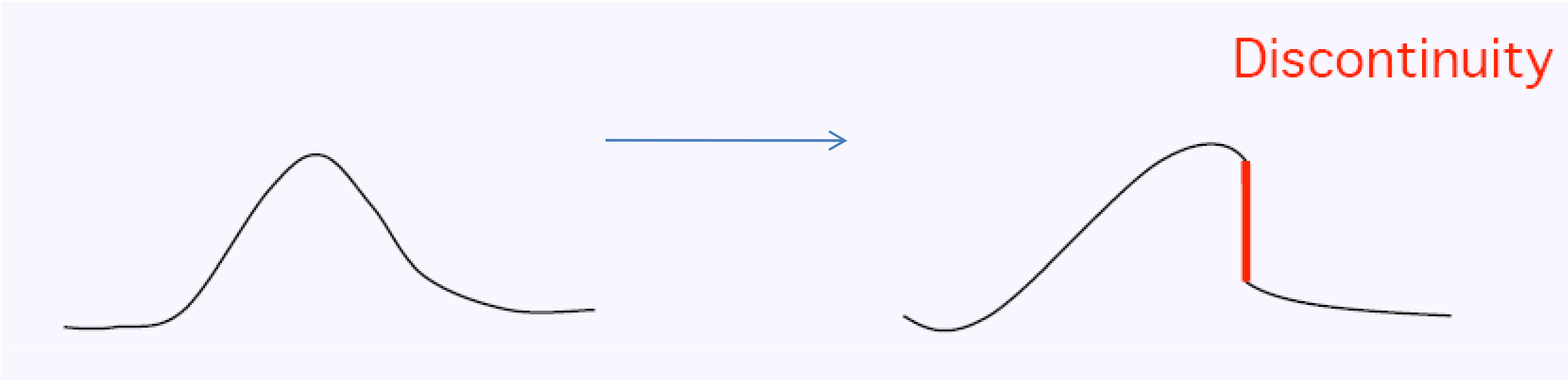




Dispersion

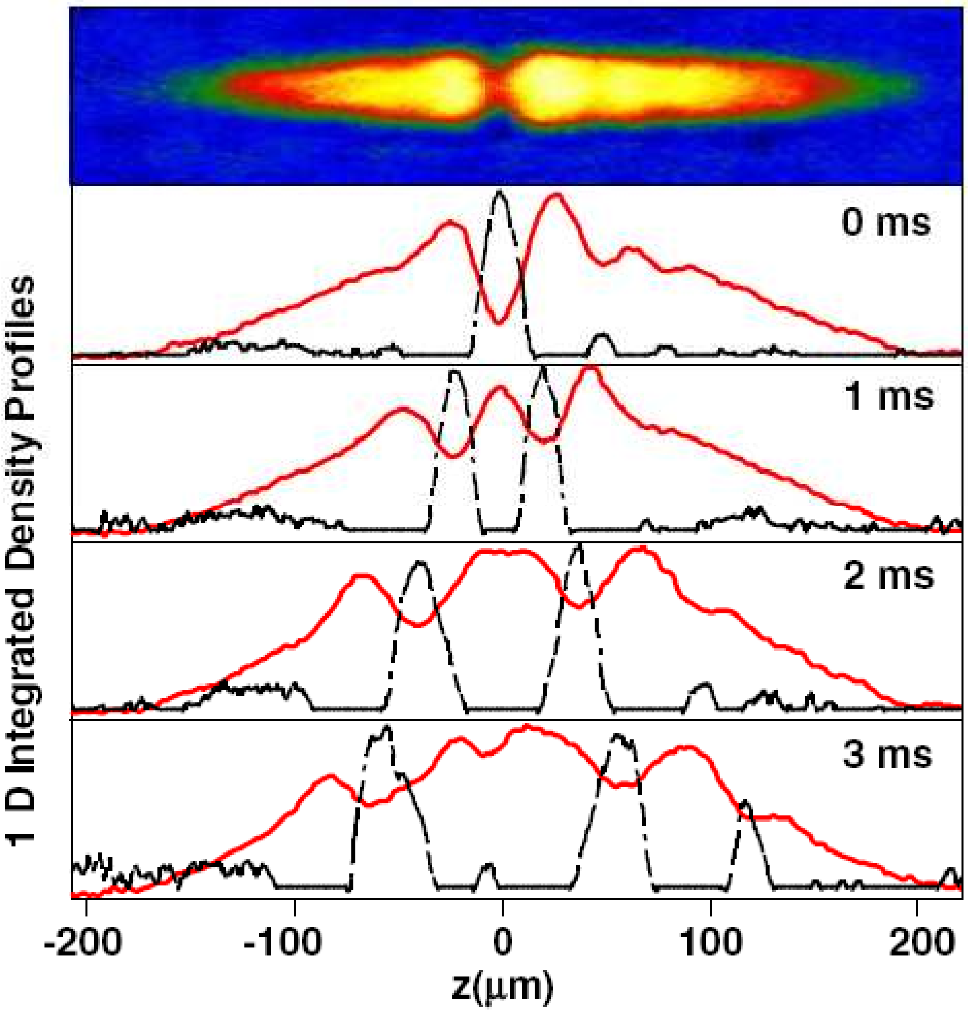
$$u_t + u_{xxx} = 0$$

Shock Waves

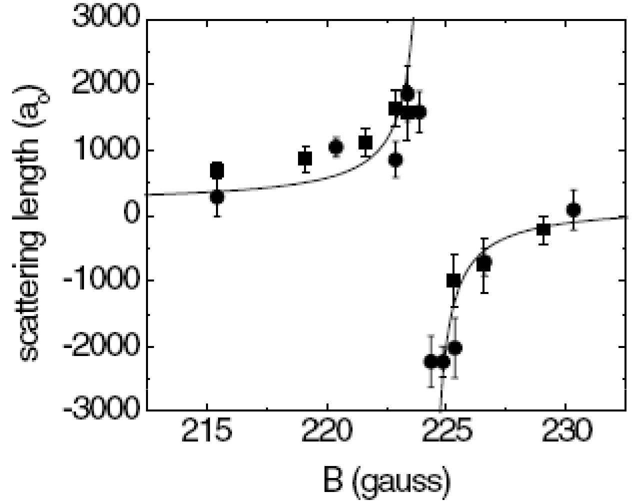


Dissipative or dispersive shock waves

Cold-Fermi Experiments (Duke)



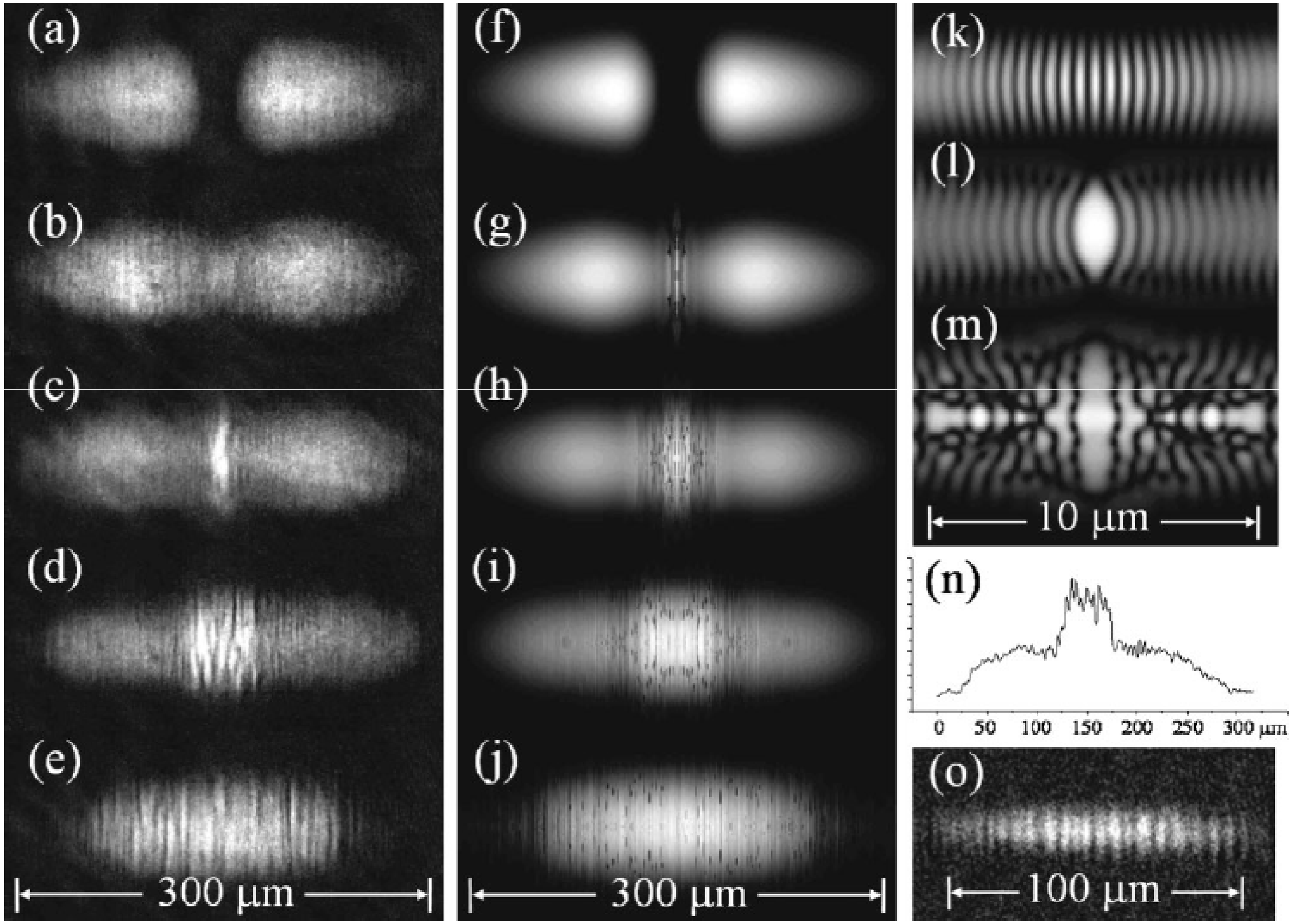
Courtesy : J. Joseph et al, PRL **98,** 170401 (2007)



Courtesy : Regal and Jin,
PRL **90,** 230404 (2003)

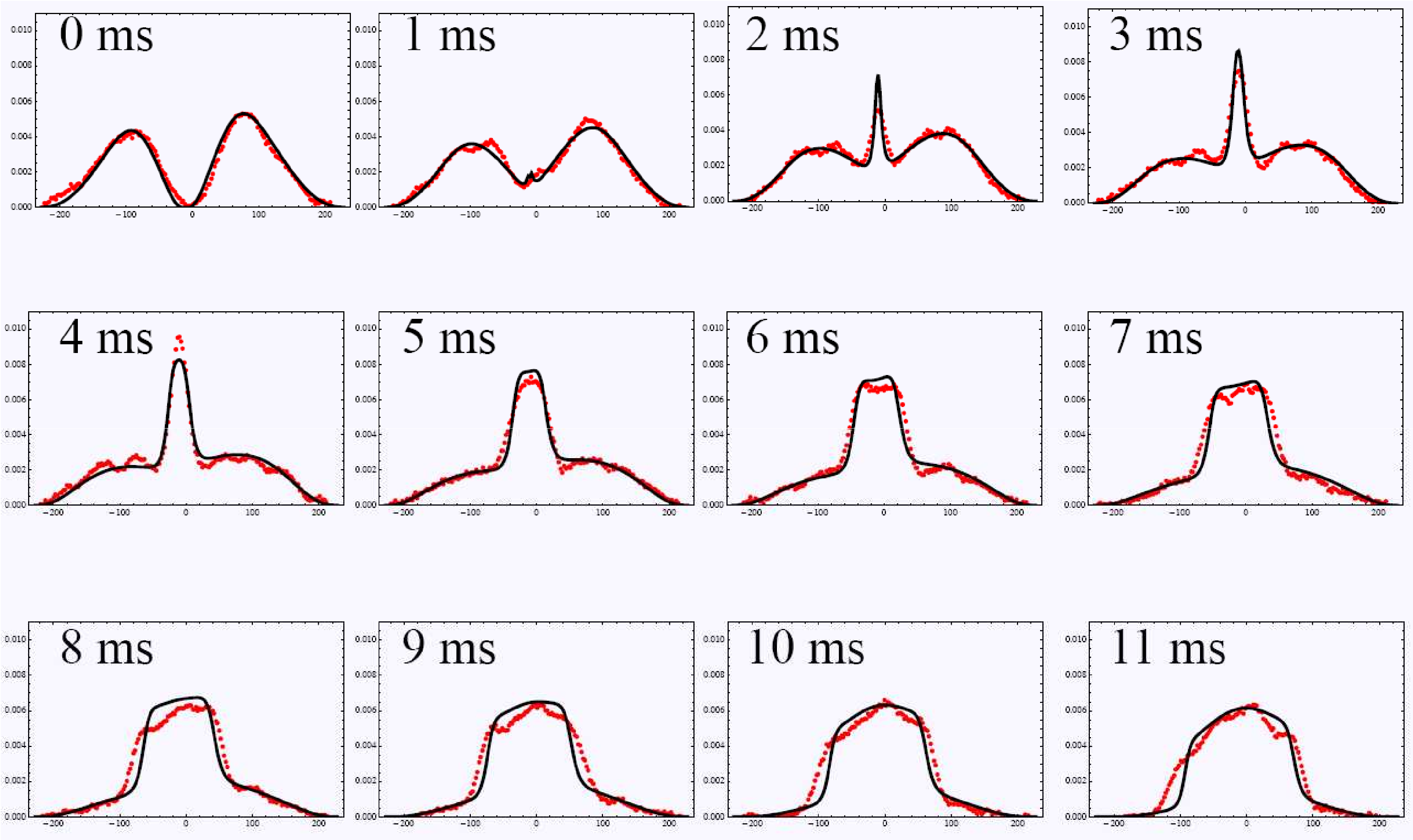
No Shock waves

Dispersive Shock Waves by Merging and Splitting Bose-Einstein Condensates



Chang et al, PRL **101**, 170404 (2008)

John Thomas Experiment & Theory

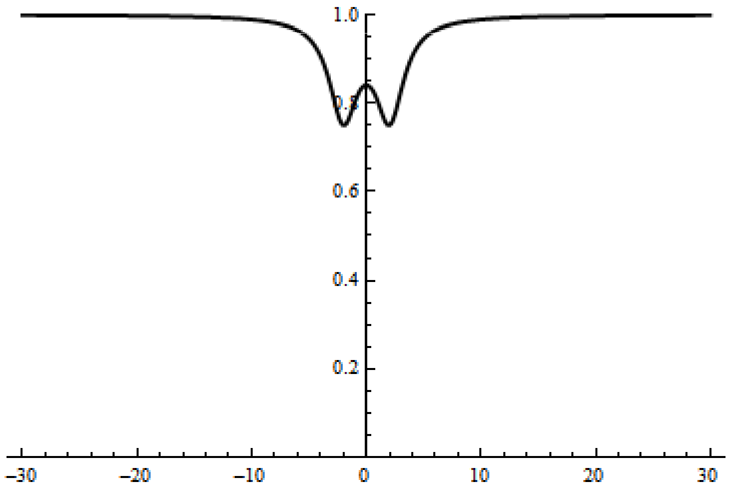


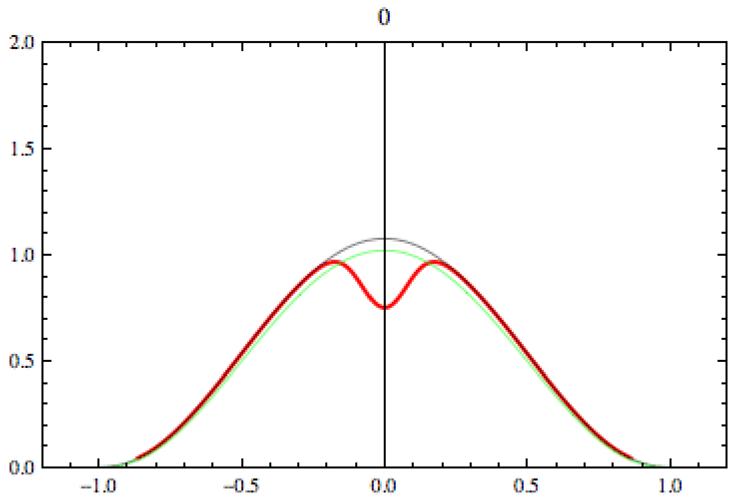
Dip splitting picture

Colliding clouds: peak formation

Change of “paradigm”: colliding clouds ⟶ dip splitting

Wave equation With background





Peak is just a “left over” background!

Fermions at Unitarity

$$f_0(k) = \frac{i}{k}$$

No scale except k_F

$$k_F = \hbar\left(3\pi^2 n\right)^{\frac{1}{3}}$$

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$$

$$\mu = (1+\beta)E_F \qquad \beta = -0.564$$

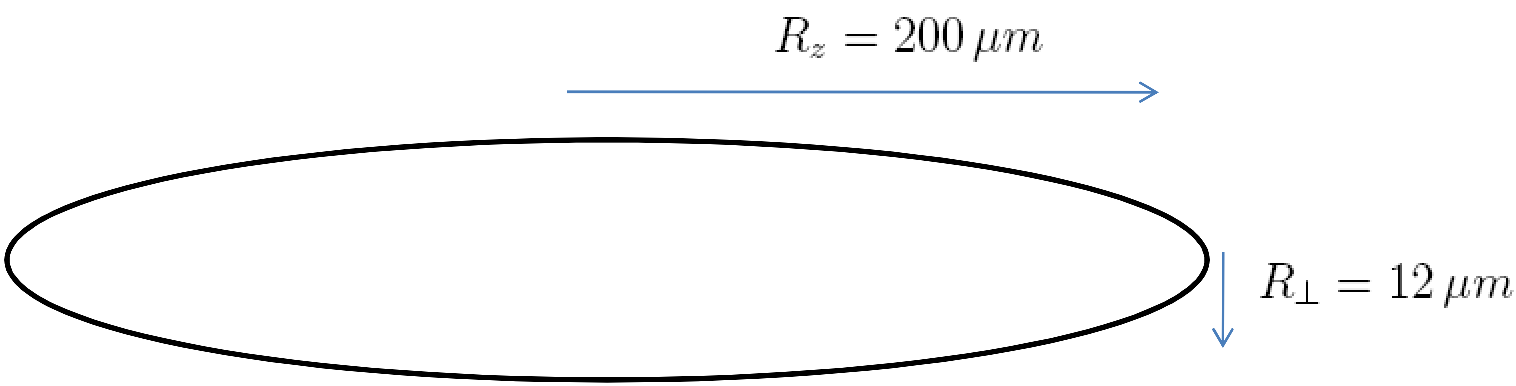
$$\tfrac{1}{2}m\omega_\perp^2 r^2 + \tfrac{1}{2}m\omega_z^2 z^2$$

$$V_0 \exp\left(-\frac{z^2}{z_0^2}\right)$$

$$\mu(n) + V_{\mathrm{harm}} + V_{\mathrm{rep}} = \mu_0$$

$$n_{3D}(r,z) = \bar{n}\left(1 - \frac{r^2}{R_\perp^2} - \frac{z^2}{R_z^2} + \frac{V_{rep}(z)}{\mu_G}\right)^{\frac{3}{2}}$$

Duke Geometry and Numbers

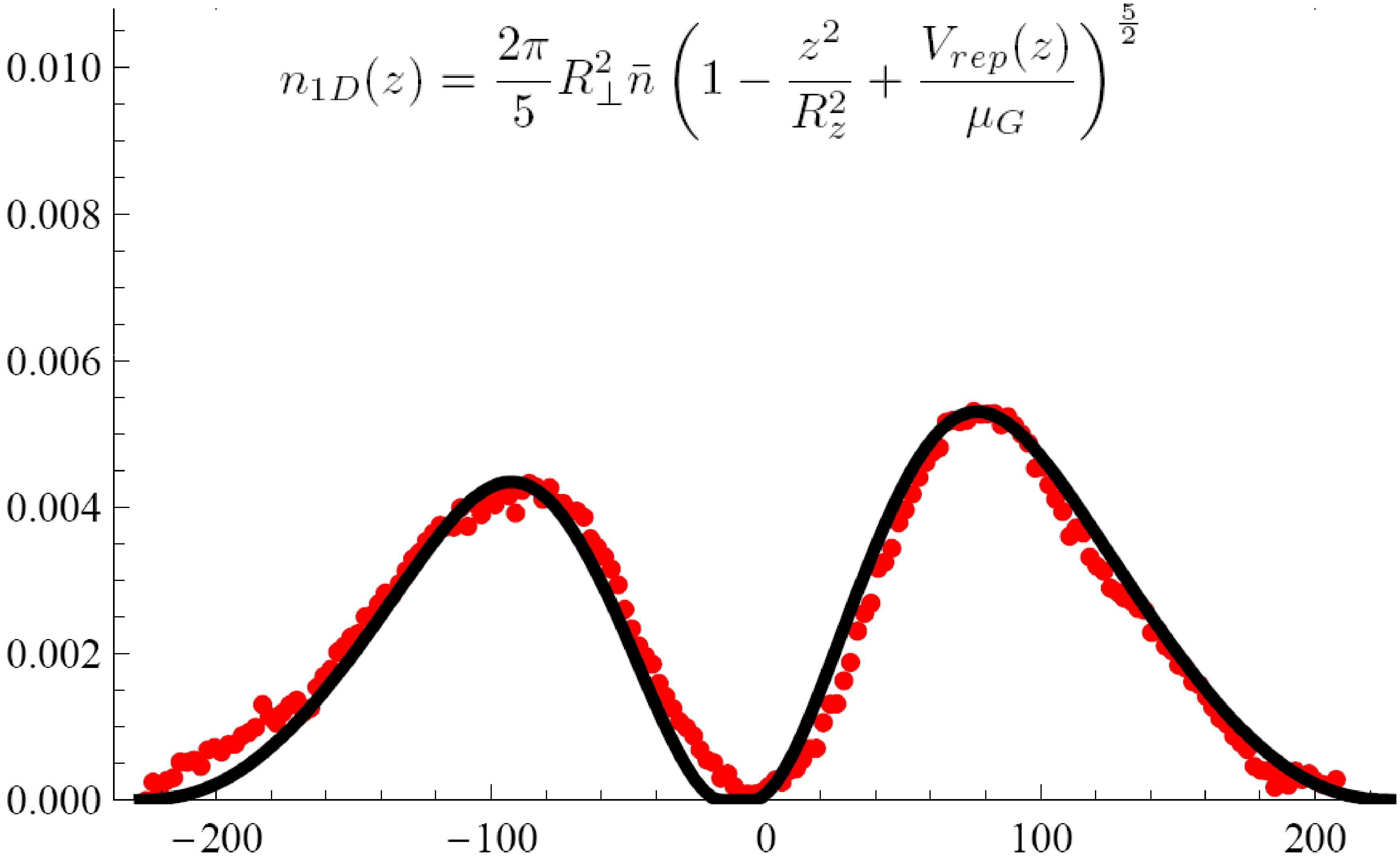


$$\omega_z = 2\pi \times 27.7\ Hz$$
$$\omega_\perp = 2\pi \times 437\ Hz$$

$$N_P = 2 \times 10^5$$
$$\bar{n} = 4.6\,\mu m^{-3}$$

$$\mu_G = 465\,nK$$

Integrated 1D Density versus position (**Theory** & **Experiment**)



1D reduced hydrodynamics

(i) Equilibrium profile in r direction
(ii) Slow dynamics in z direction

$$n_{3D} = \bar{n}\left[\left(\frac{n_{1D}(z,t)}{\frac{2\pi}{5}R_\perp^2\bar{n}}\right)^{2/5} - \frac{r^2}{R_\perp^2}\right]^{3/2}$$

$$\boldsymbol{v}_{3D} = v_{1D}(z,t)\hat{\boldsymbol{z}}$$

$$\partial_t n + \partial_z(nv) = 0$$

1D hydro

$$\partial_t v + \partial_z\left(\frac{v^2}{2} + Cn^{2/5} + \frac{\omega_z^2}{2}z^2\right) = 0$$

$$C = \frac{1}{2}\omega_\perp^2 l_\perp^2\left(\frac{15\pi}{2}l_\perp\right)^{2/5}(1+\beta)^{3/5}$$

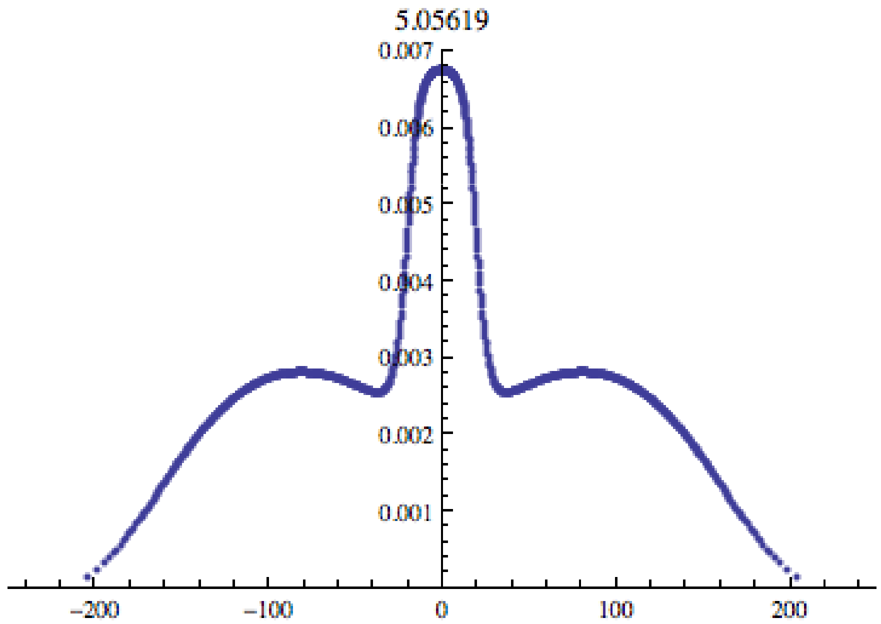
Initial Density Profile with Repulsive Knife

Dimensionally Reduced Hydrodynamics

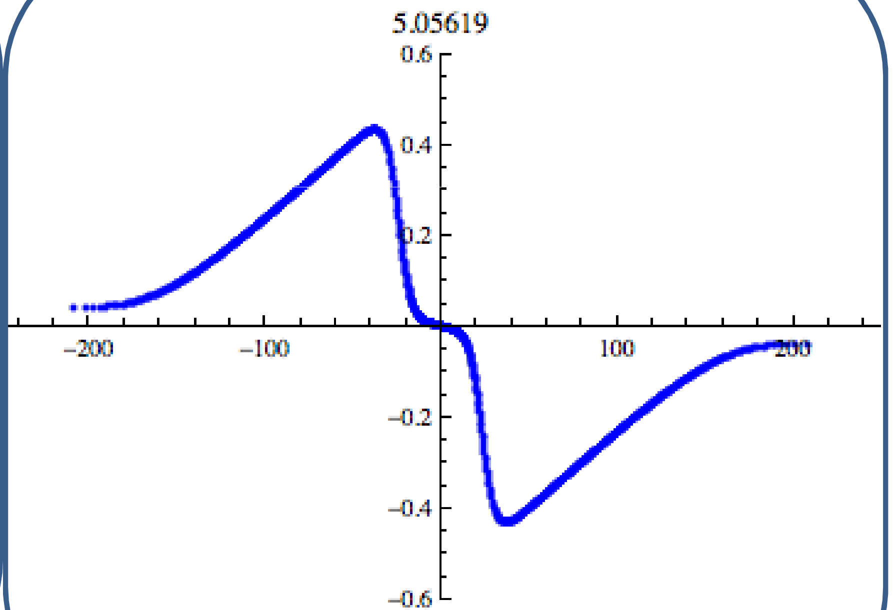
$$\partial_t n = -\partial_z (nv)$$
$$\partial_t v = -\partial_z \left(\frac{v^2}{2} + C n^{\frac{2}{5}} + \frac{1}{2} \omega_z^2 z^2 \right) + \eta \frac{1}{\rho} \partial_x (\rho v_x)$$

$n_{1D}(z,t) \quad v_{1D}(z,t)$

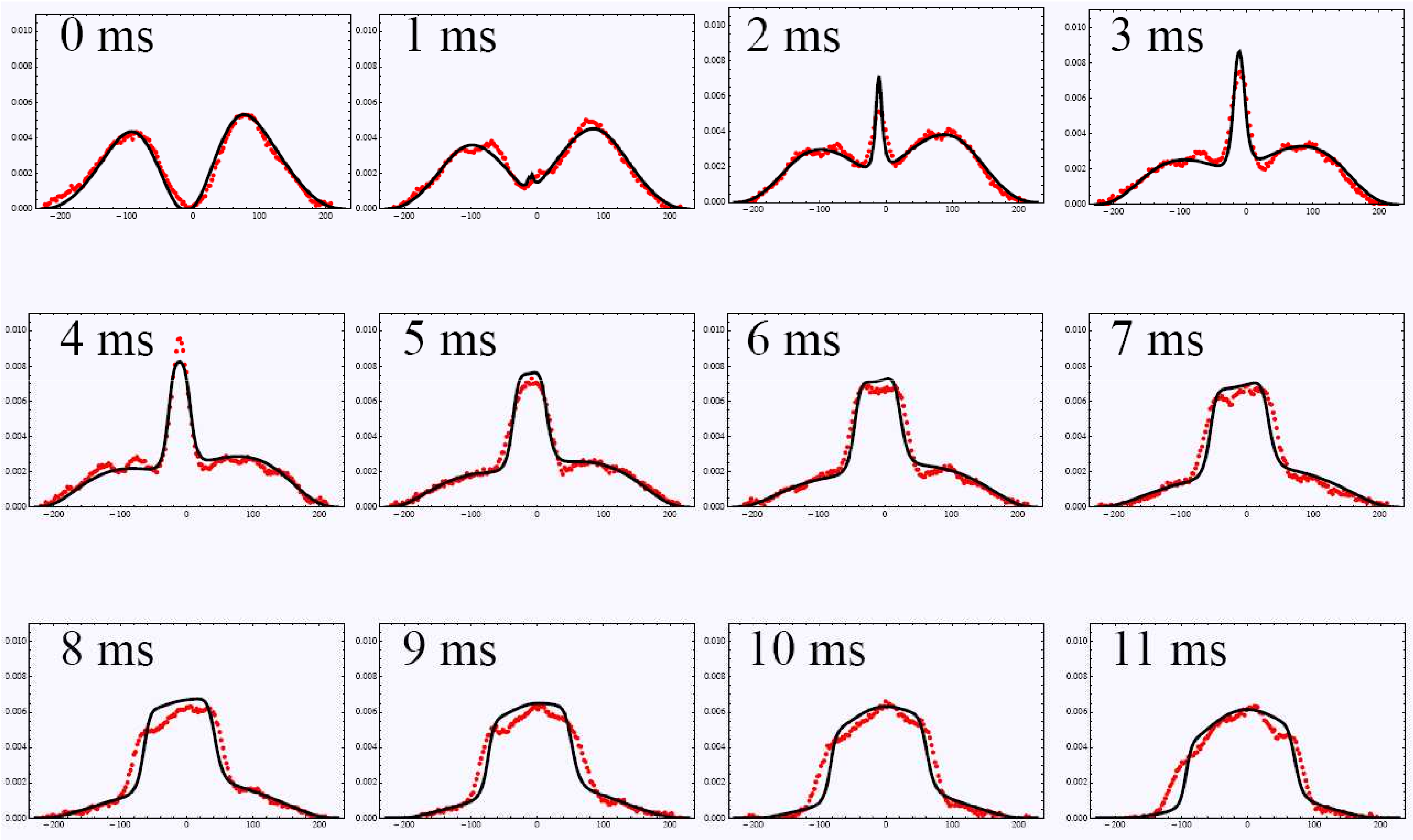
Density Evolution



Velocity Evolution



John Thomas Experiment & Theory



Conclusions for Part 1

- First observation of shock waves in Unitary Fermi gas.

- Hydrodynamics of unitary gas gives a good description of collision of atomic clouds even deep in nonlinear regime.

- Near perfect quantitative agreement with experiment without any fitting parameter except the phenomenologically introduced viscosity term.

- Additional experiments necessary to clarify the nature of shock waves.

- Effects of moving away from unitarity and finite temperature effects remain an open question.

- The experiments on strongly interacting Fermi gases form an ideal playground for studying out of equilibrium nonlinear hydrodynamics beyond the Luttinger liquid paradigm.

Harmonically trapped integrable model of cold atoms

SU(2) Spin Calogero Model in Harmonic trap:

$$\frac{1}{2}\left[\sigma_i \cdot \sigma_j + 1\right]$$

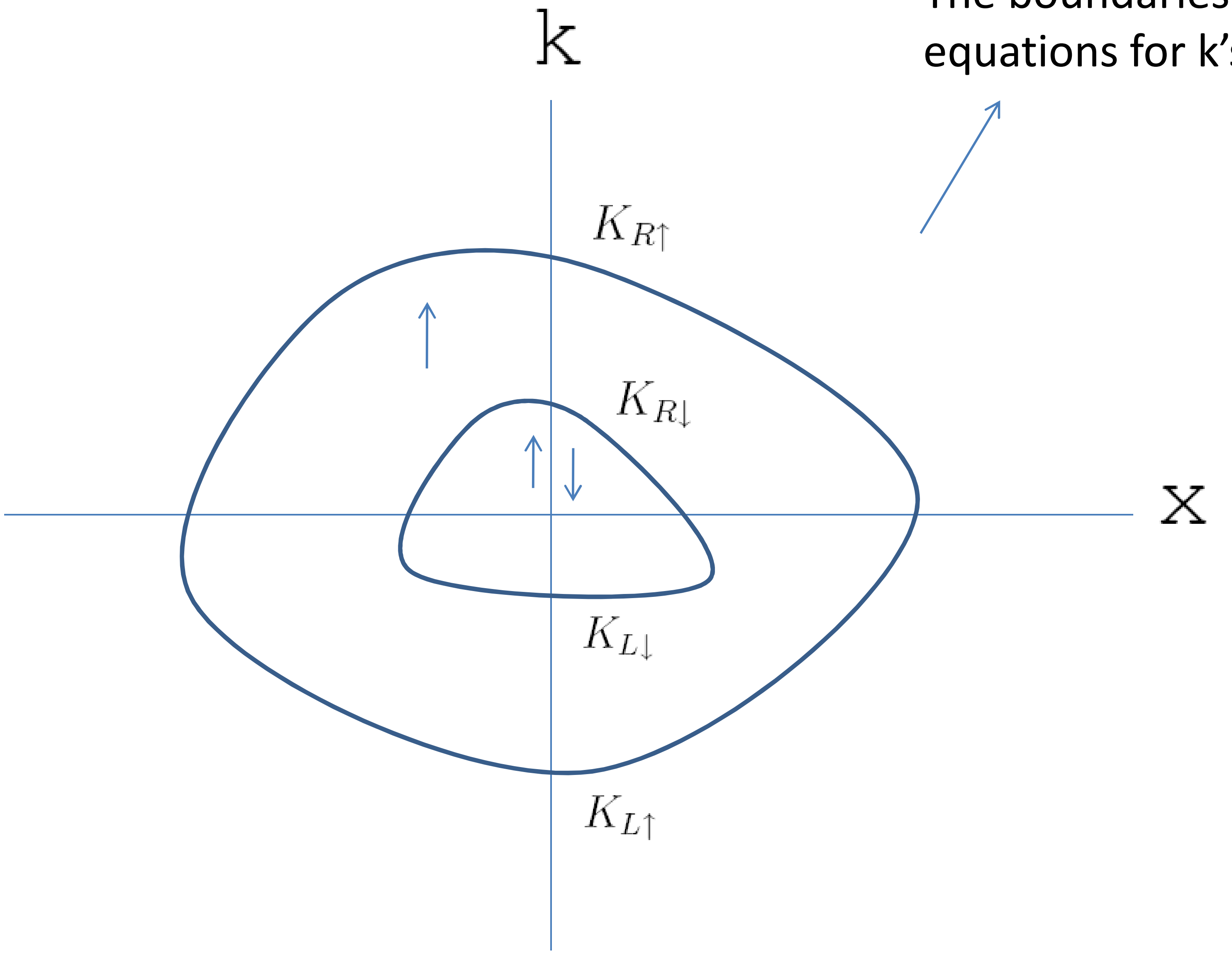
$$H = -\frac{\hbar^2}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \frac{\hbar^2}{2}\sum_{i\neq j}\frac{\lambda(\lambda - P_{ij})}{(x_i - x_j)^2} + \frac{m\omega^2}{2}\sum_{i=1}^{N} x_i^2$$

Important limits of the model are:

a) Free Fermions with spin
b) Splinless Calogero model
c) Haldane-Shastry spin chain (similar to Heisenberg chain)

Phase Space Picture

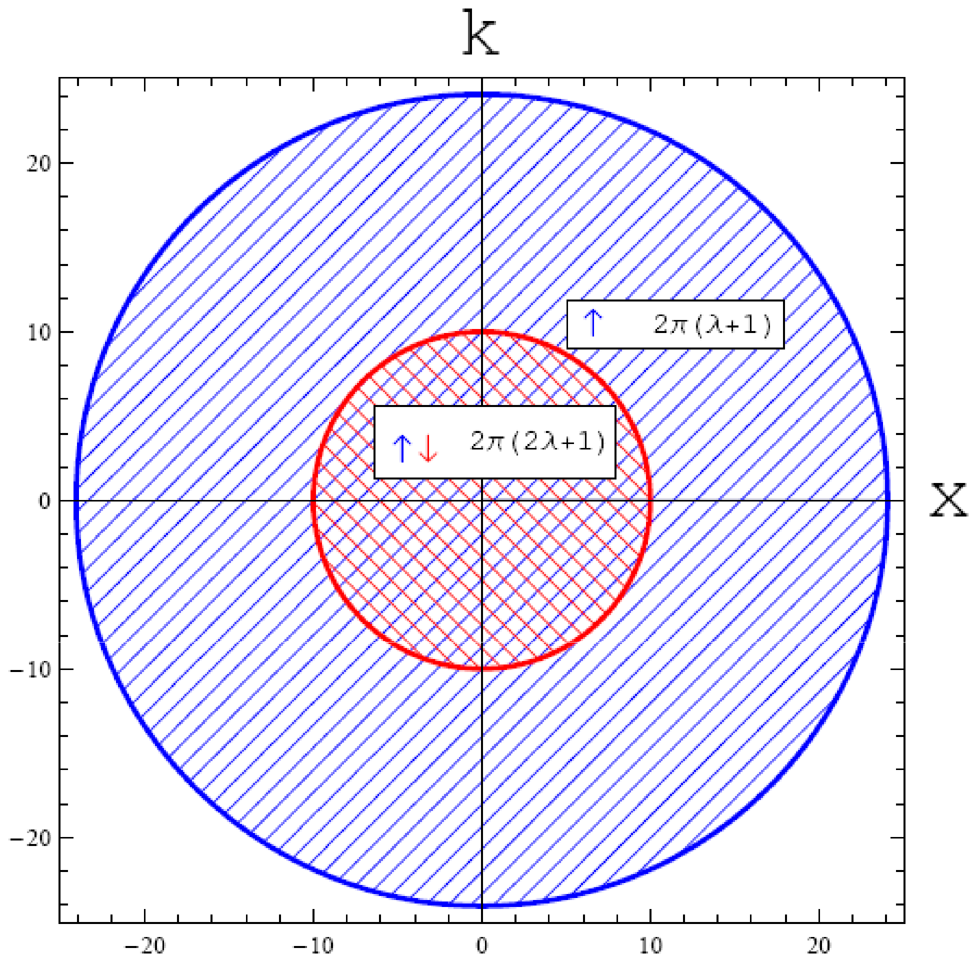
The boundaries give the equations for k's



Static Configuration:

- Two circles with different radii

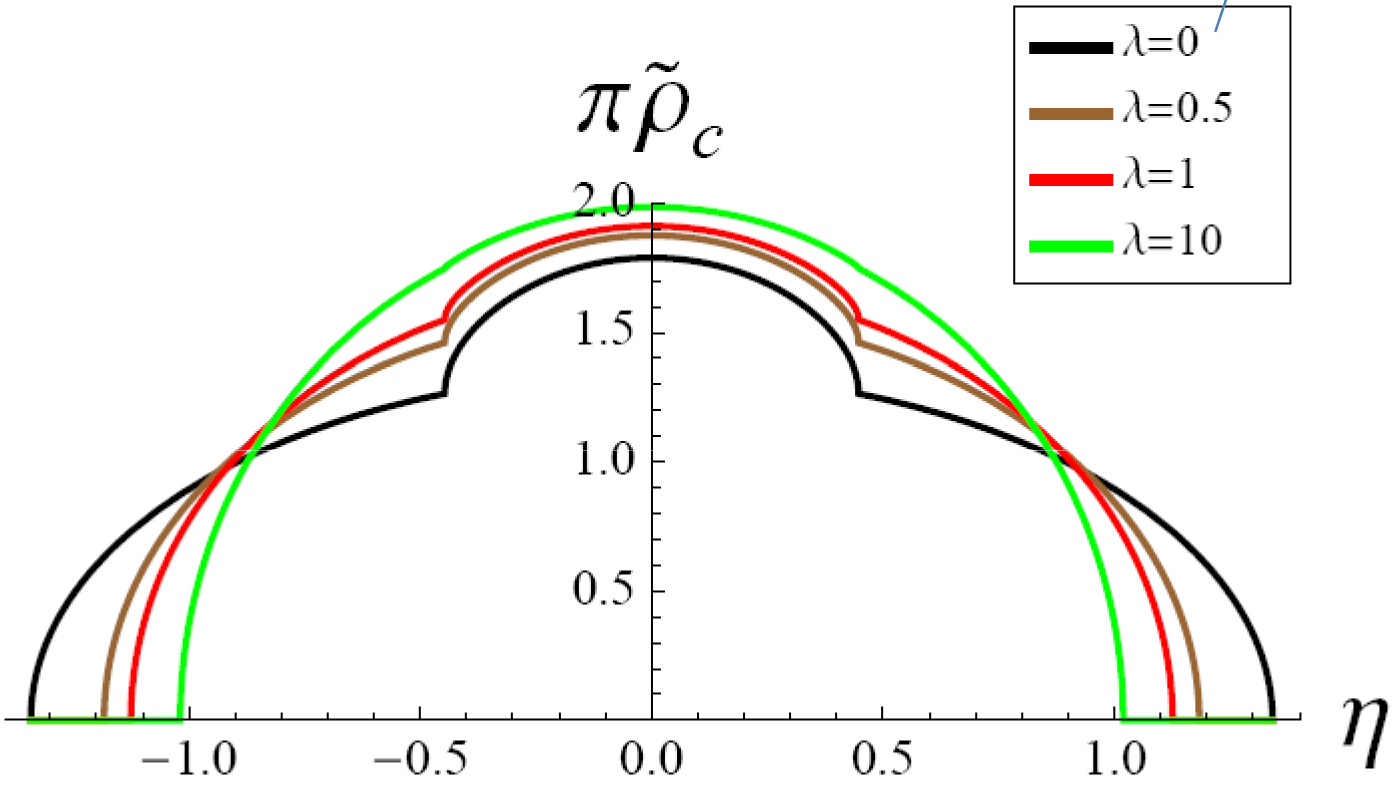
- Radii depends on coupling and number of spin up & spin down particles.



$$
\begin{aligned}
k_1^2 + x^2 &= 2(\lambda+1)M_1 + 2\lambda M_2 \\
k_2^2 + x^2 &= 2(2\lambda+1)M_2.
\end{aligned}
$$

Equilibrium (static) charge density profile:

Free Fermions with spin



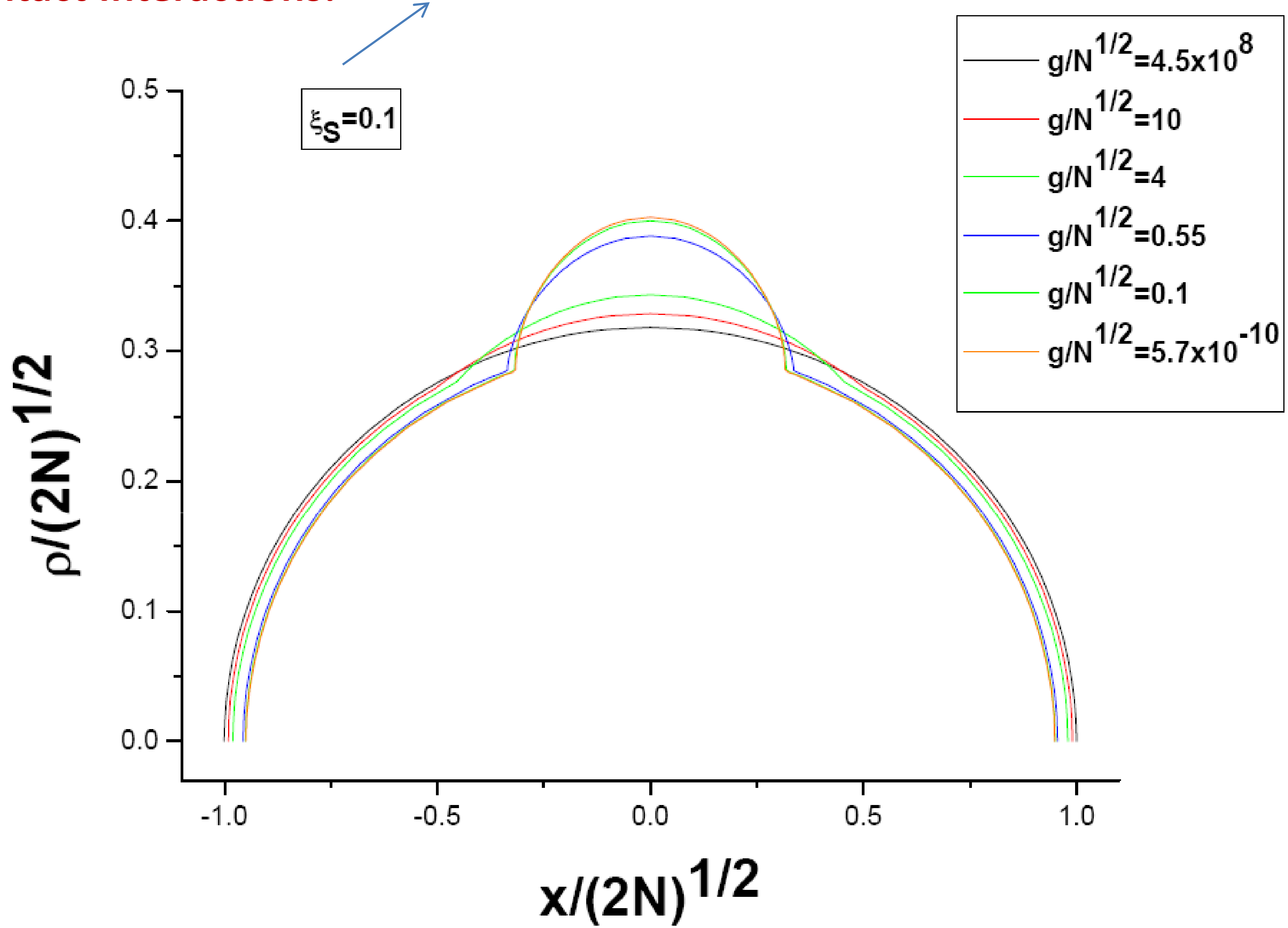
$$L_{cigar} = 2\sqrt{N(2\lambda + 1 + \nu)}$$

(M. K. , A. G. Abanov, Nucl. Phys.B, 846, 122 (2011))

Striking similarity with Ma and Yang for fermions with contact interactions

Contact Interactions:

10 % are spin-down



Courtesy : Ma and Yang (private communication, (CPL, **27**, 080501 (2010))
(similar behaviour with spin profile)

Dynamics Configuration

Remarkably simple and exactly like free fermions

$$k_t + k k_x = -\omega^2 x$$

Parameter

Exact parametric solutions:

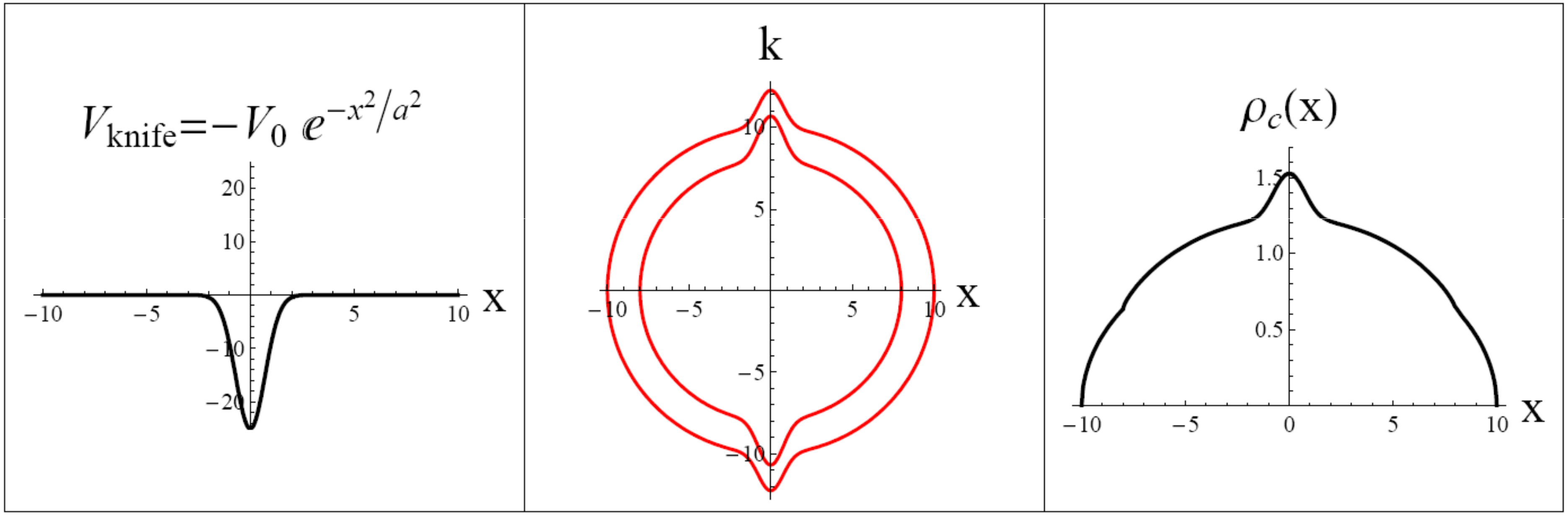
$$\begin{aligned} \alpha(s) &= \tan^{-1}\left(\frac{s}{k_0(s)}\right) \\ R(s) &= \sqrt{s^2 + k_0(s)^2} \end{aligned}$$

$$\begin{aligned} x(s;t) &= R(s)\sin\left[t + \alpha(s)\right] \\ k(s;t) &= R(s)\cos\left[t + \alpha(s)\right] \end{aligned}$$

Initial profile

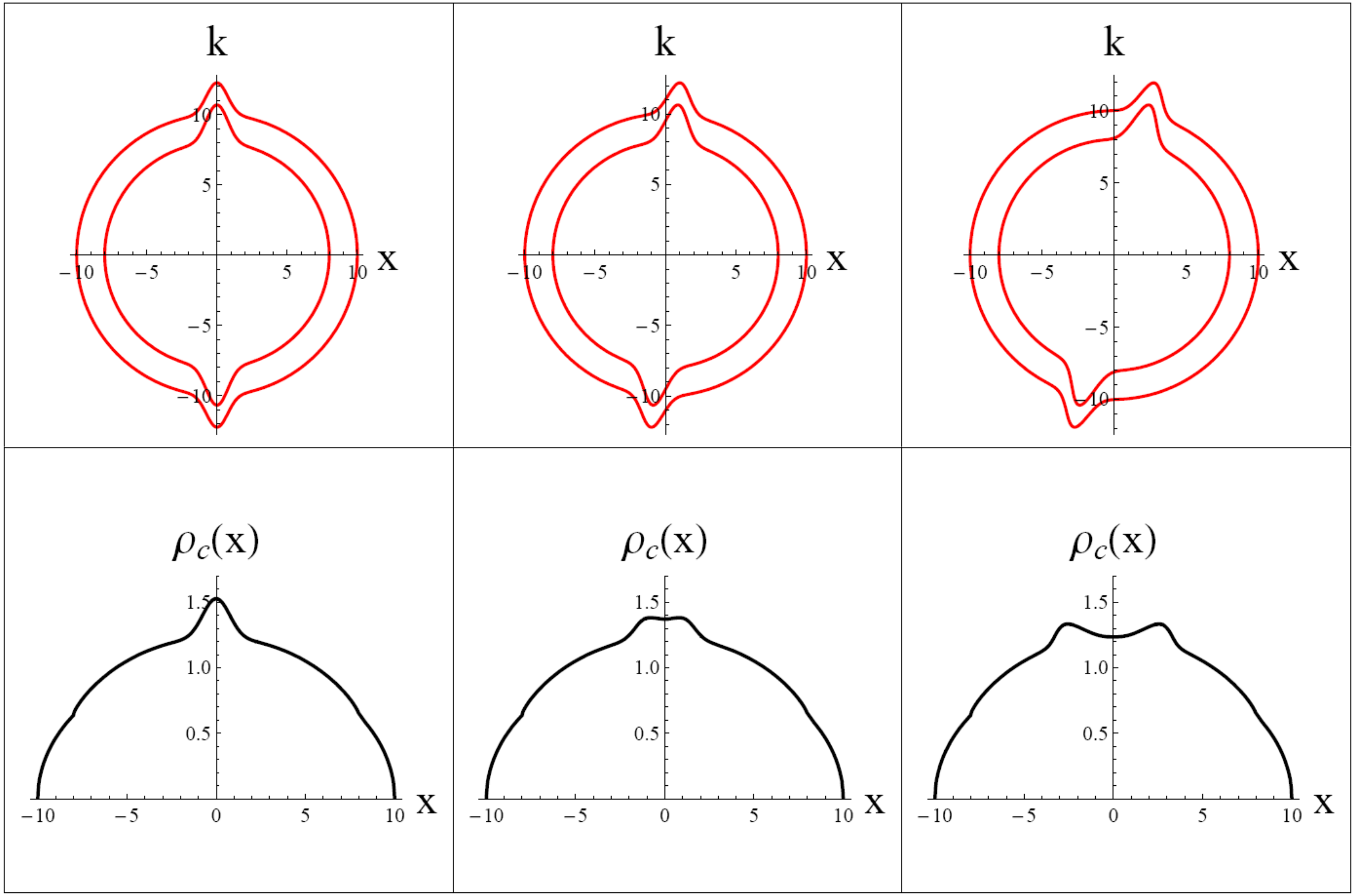
(M. K. , A. G. Abanov, Nucl. Phys.B, 846, 122 (2011))

Cooling with additional potential – “knife” in place



(M. K. , A. G. Abanov, Nucl. Phys.B, 846, 122 (2011))

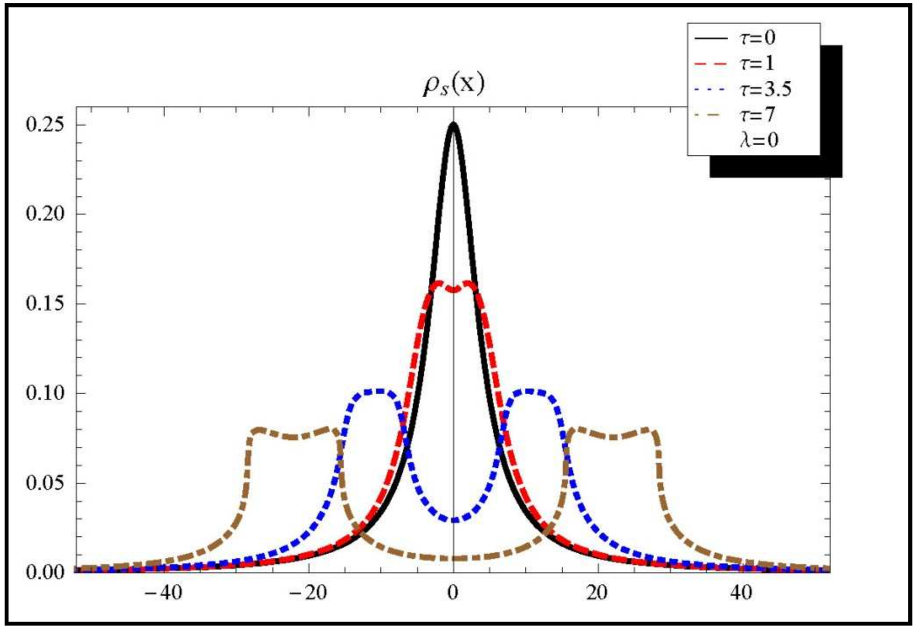
Dynamics



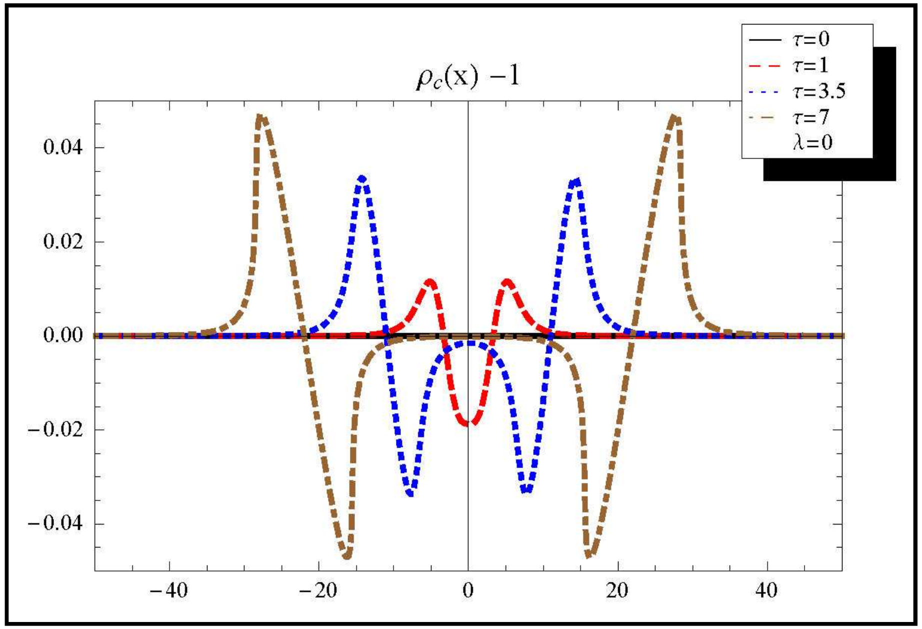
This “peak” to “box” transition has been observed in Duke for quasi-1D unitary Fermi gas

Dynamics of a polarized center: FF

$$\rho_s = \rho_\uparrow - \rho_\downarrow$$



$$\rho_c = \rho_\uparrow + \rho_\downarrow$$

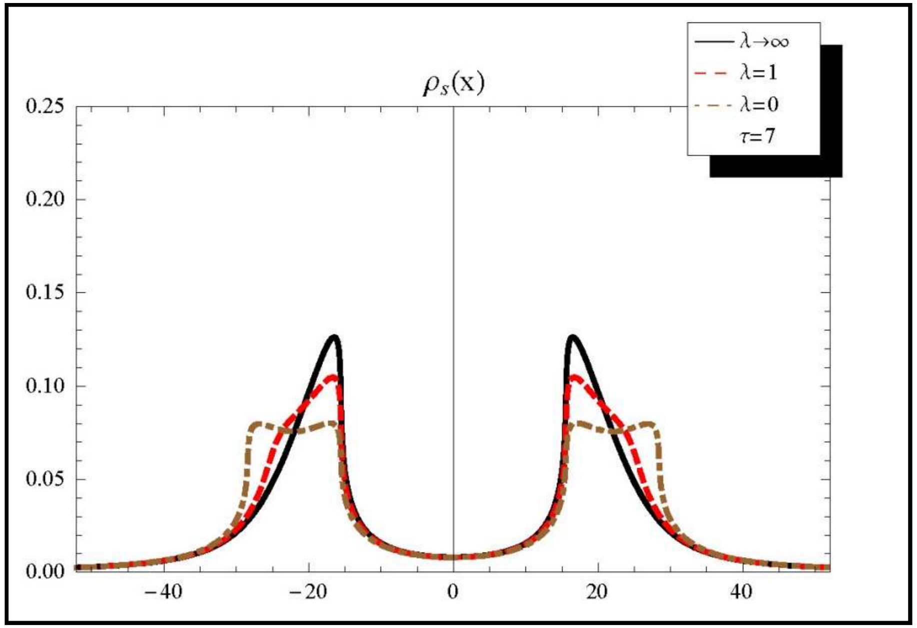


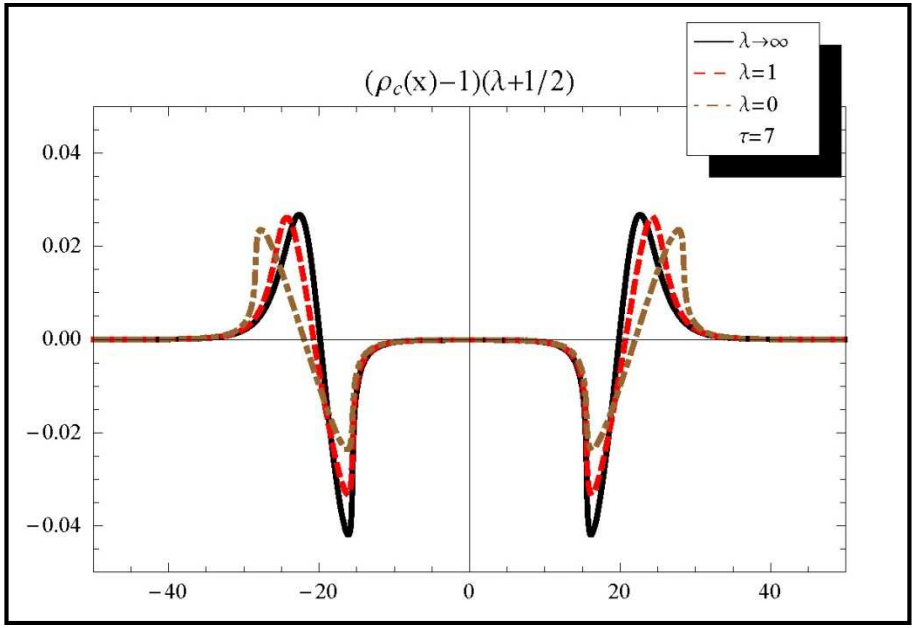
M. K. , F. Franchini, A. G. Abanov (PRB 2009)

- Spin drags charge
- Profiles exhibit steepening

Dynamics of a polarized center: sCM

$$\rho_s = \rho_\uparrow - \rho_\downarrow \qquad\qquad \rho_c = \rho_\uparrow + \rho_\downarrow$$



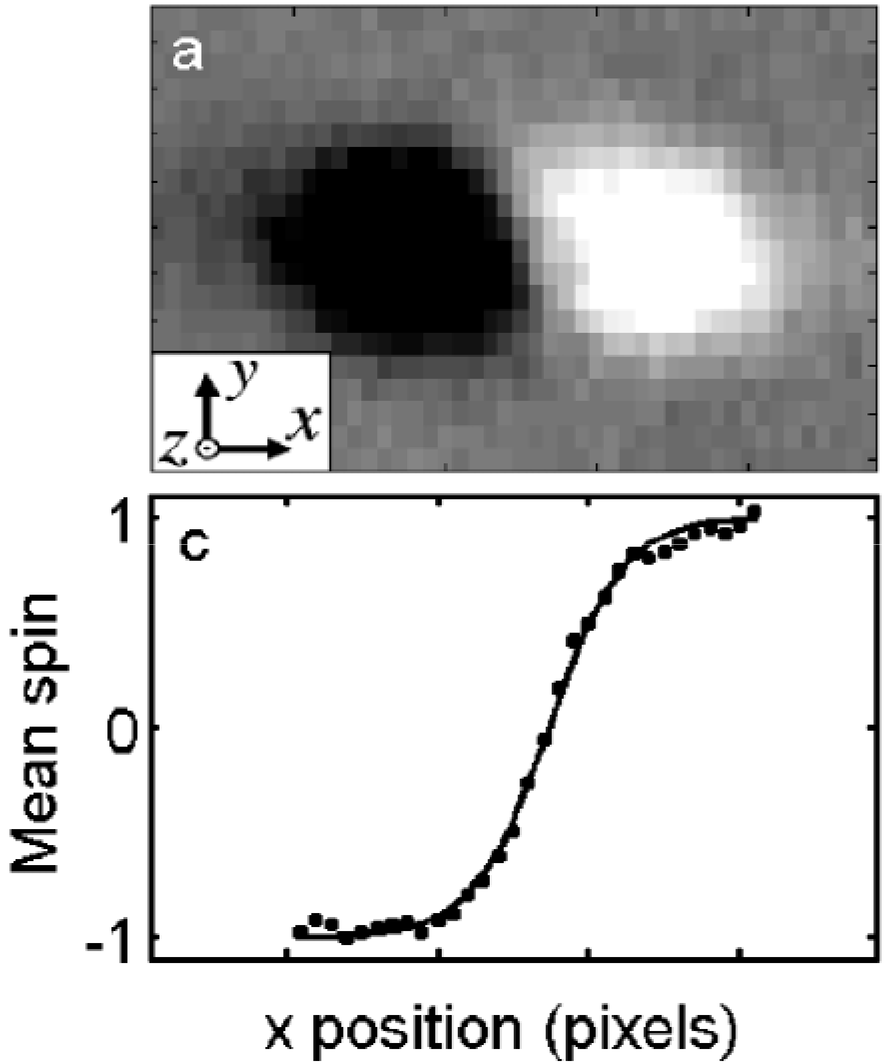


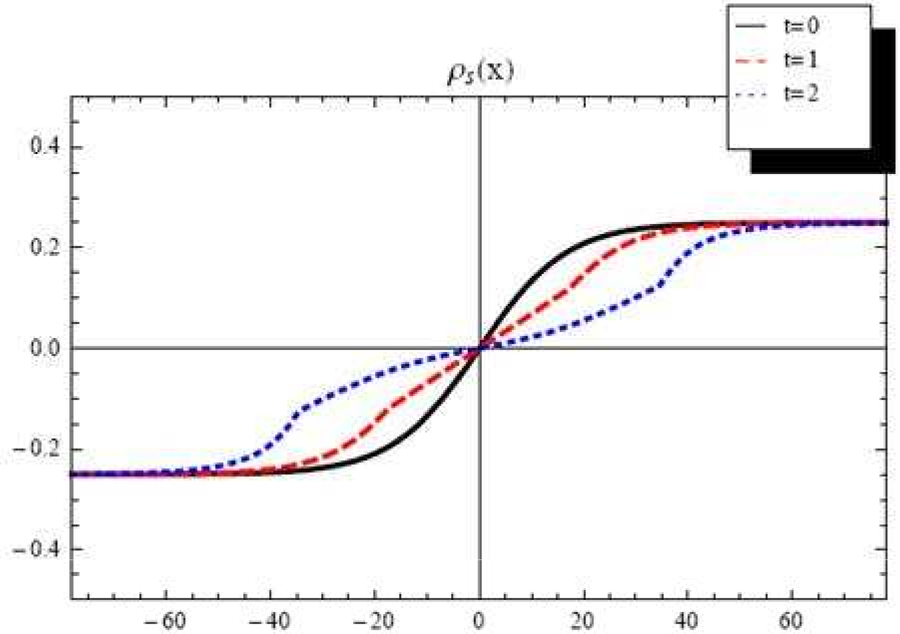
- Qualitatively similar behaviors for rescaled quantities (t = (λ+1/2)t)
- $\lambda \longrightarrow \infty$ → freezing of charge → Haldane-Shastry Model

(Polychronakos, PRL, 1993)

Field Theory Perspective → **M. K. , F. Franchini, A. G. Abanov (PRB 2009)**

Spin Chains and spin dynamics





Domain wall dynamics for the Haldane-Shastry model which is similar to XXX Heisenberg chain.

Courtesy : D. Weld et al, PRL **103,** 245301 (2009)
(Heisenberg-like interaction)

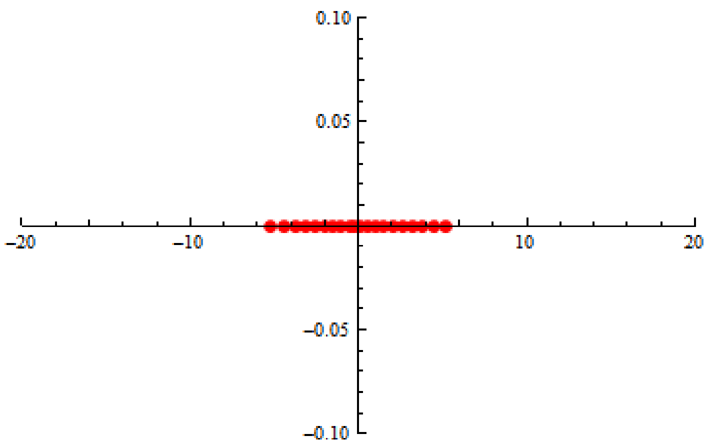
Solitons on a curved background

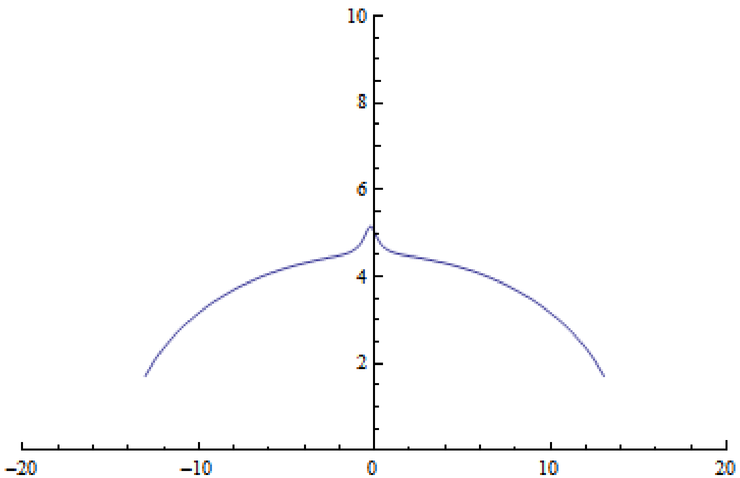
•Conventional definition of a soliton as "a pulse that maintains its shape while it travels at a constant speed" doesnot make sense on a curved background

•Soliton is a finite dimensional reduction of N-dimensional model and its infinite dimensional field theory limit

$$\rho(x,t) = \rho(x; \{z_j(t)\})$$
$$v(x,t) = v(x; \{z_j(t)\})$$

$$z_j(t) \rightarrow M \text{ complex time dependent parameters}$$





A. G. Abanov, A. Gromov, M. Kulkarni (in preparation)

Conclusions for Part 2

- Nonlinear Collective Field Theory starting from a microscopic model to capture collective physics in the hydrodynamic limit
- Static and dynamic features of fermions with inverse square range interactions in trap
- Steepening of profiles
- Spin drags charge
- n-point correlators (Emptiness Formation Probability) as an instanton approach to field theory
- Soliton solutions of spinless Calogero in harmonic trap as finite dimensional reductions