Physically Motivated Generalized Parton Distributions

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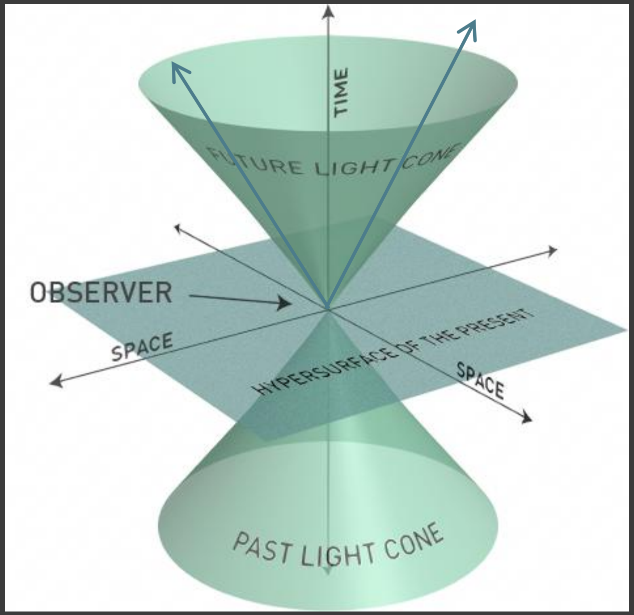
Fourth Year Seminar

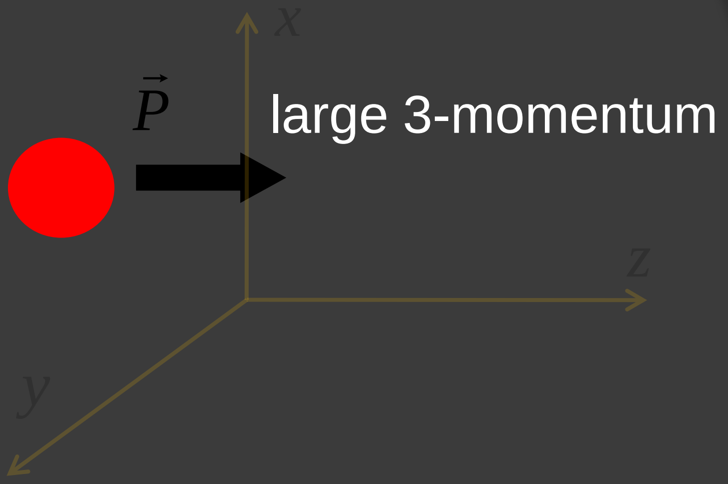
OUTLINE

- Motivation: DVCS
- Observables and GPD’s
- How to Build a Parametrization?
- Possible applications
- Ongoing and Future Projects

DVCS

- Light cone coordinates





$$P^+ = \sqrt{M^2 + \vec{P}^2} + P^3 \quad \rightarrow \quad \infty$$

$$P^- = \frac{M^2 + \vec{P}_\perp^2}{P^+} \quad \rightarrow \quad 0$$

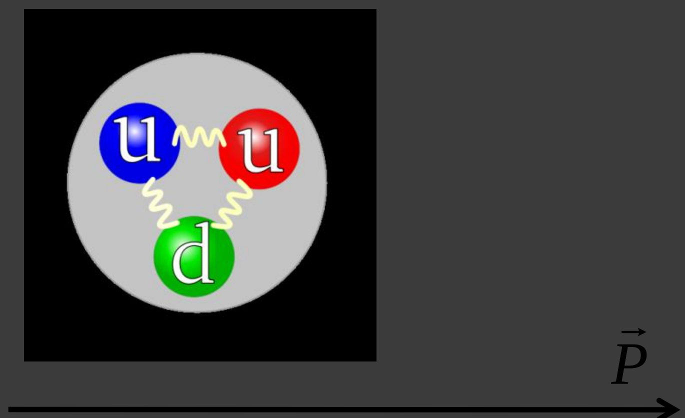
$$P = (P^+, P^-, \vec{P}_\perp) \quad \rightarrow \quad (P^+, 0, \vec{0}_\perp)$$

$$(e_0)^\mu = (1,0,0,0) \rightarrow (n_-)^\mu = (1,0,0,1)$$

$$(e_3)^\mu = (1,0,0,0) \rightarrow (n_+)^\mu = (1,0,0,-1)$$

DVCS

- Light cone coordinates

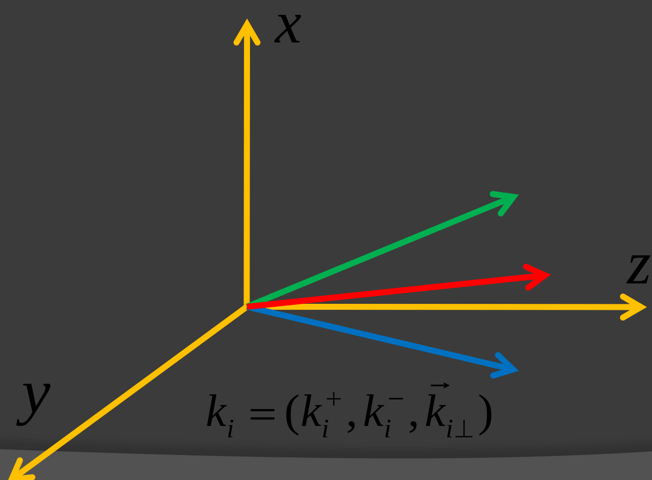


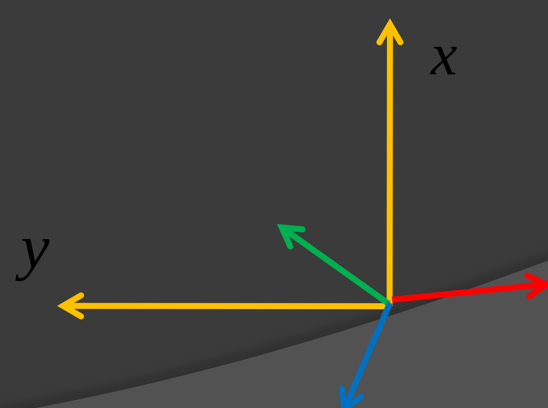
$$x_i = \frac{k_i^+}{P^+} \qquad \vec{k}_{i\perp} \text{ (relative to Proton)}$$

$$\sum_i x_i = 1 \qquad \sum_i k_{i\perp} = 0 \qquad \text{Momentum Conservation}$$

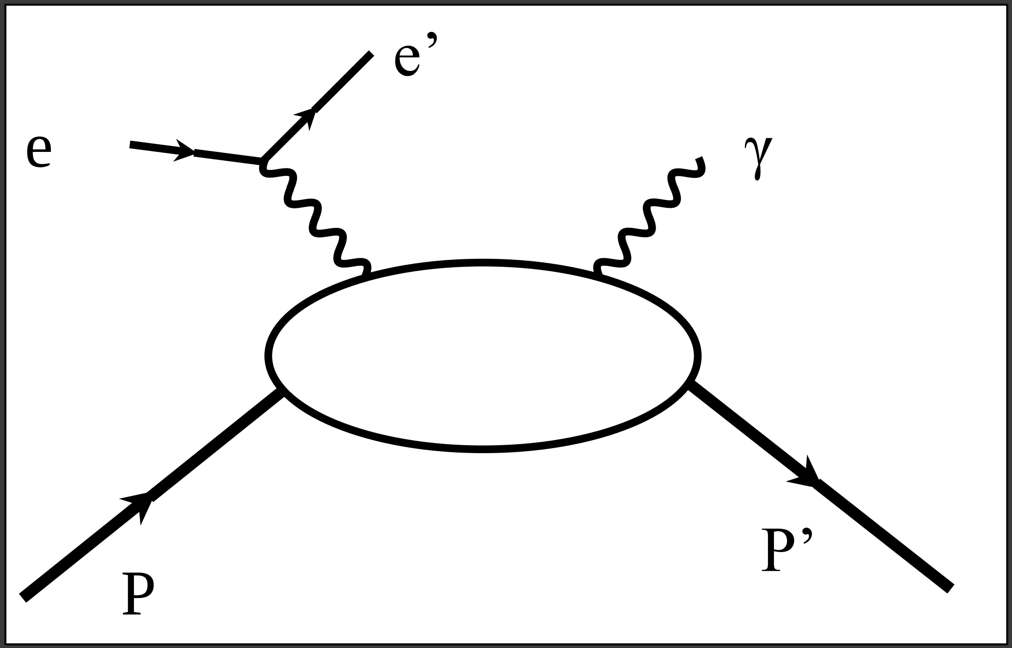
$$\sum_i k_i^- = P^- \qquad \text{Energy Conservation (covariant)}$$

$$k_i^- = \frac{m_i^2 + \vec{k}_\perp^{\,2}}{k^+} \qquad \text{On-shell Partons (time ordered)}$$



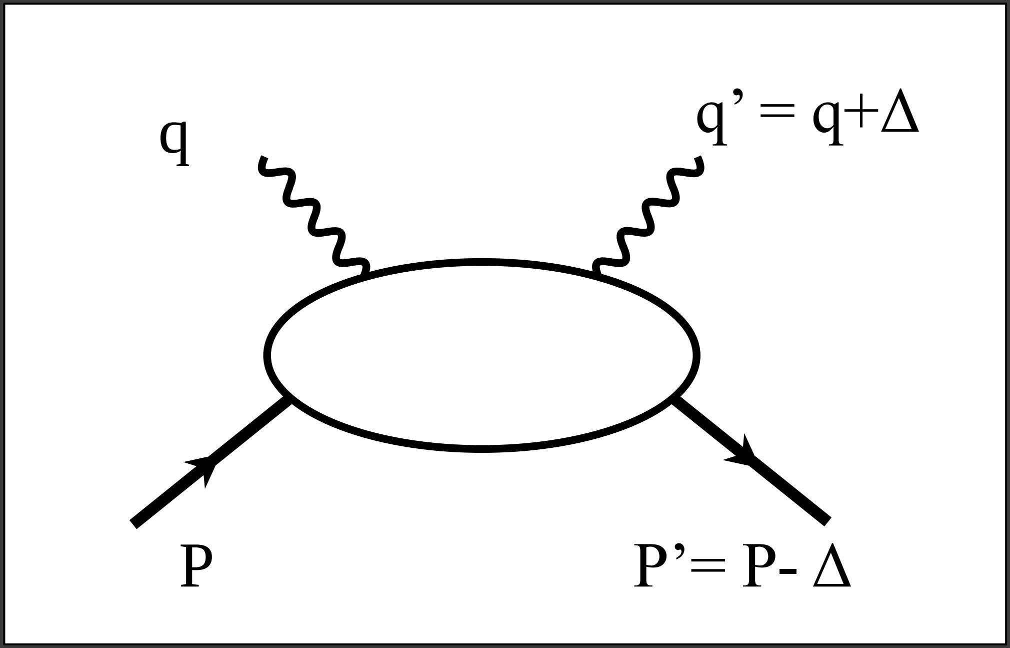


DVCS



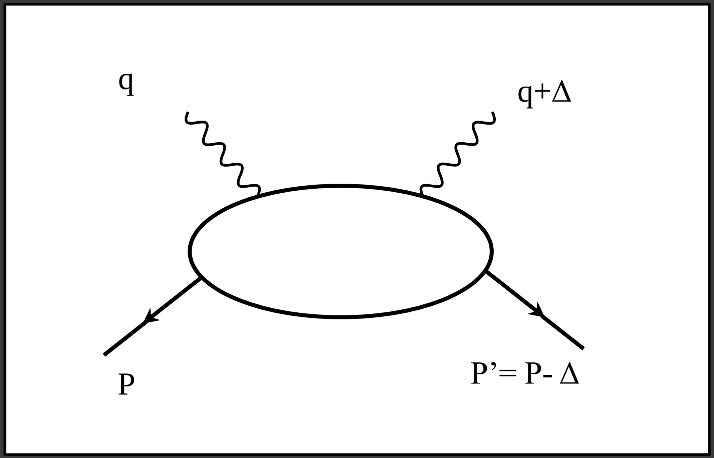
$$e\,P \longrightarrow e'\,P'\,\gamma$$

DVCS



$$\gamma^* P \longrightarrow P' \gamma$$

DVCS



- Jμ is the interacting current.
- $M^{\mu\nu}\ (\varepsilon^{(I)*}\varepsilon^{(J)})_{\mu\nu}$ is a Lorentz invariant.

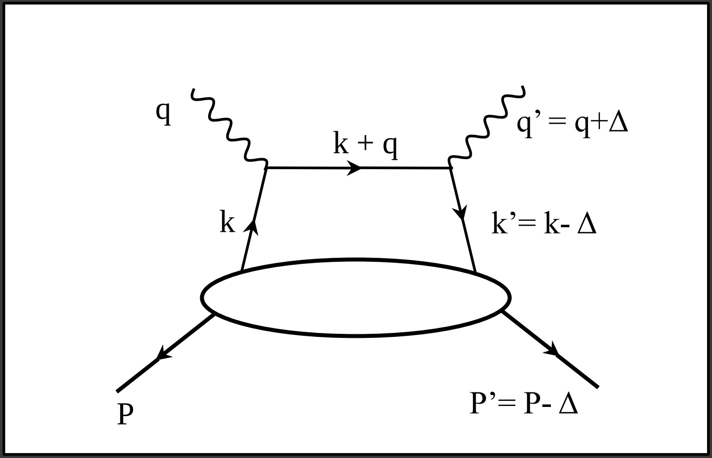
$$M^{\mu\nu} = ie^2 \int \frac{dx}{(2\pi)^4} e^{ik\cdot x} \langle P' | T[J^\mu(x) J^\nu(0)] | P \rangle$$

- Amplitude depends on three Lorentz invariants

$$\varepsilon_\mu^{*(I)} \varepsilon_\nu^{(J)} M^{\mu\nu} = M^{IJ}(Q^2, p\cdot q, \Delta^2) \quad where$$

$$Q^2 = -q^2, \quad \Delta^2 = (P - P')^2$$

DVCS



- Asymptotic freedom in QCD observable in Bjorken limit.

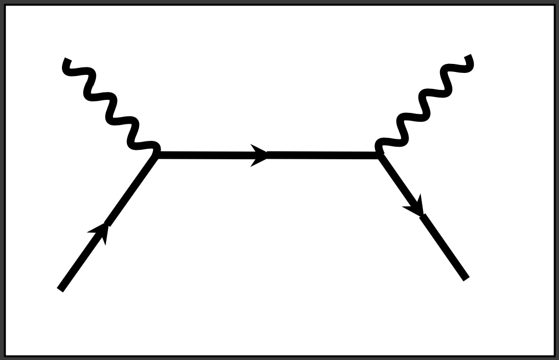
$$Q^2 \to \infty, \quad P \cdot q \to \infty \quad \frac{Q^2}{2P \cdot q} \equiv x_B \;\; finite$$

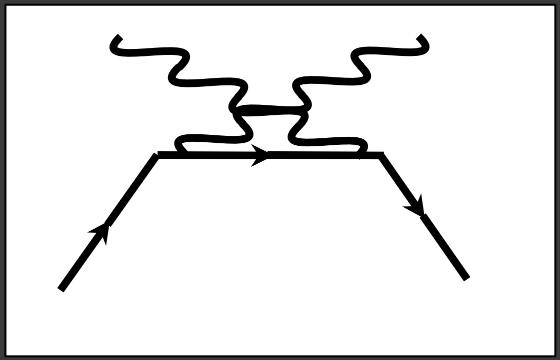
$$M^{\mu\nu} = ie^2 \int \frac{dx}{(2\pi)^4} e^{ik \cdot x} \langle P' | T[j^\mu(x) j^\nu(0)] | P \rangle; \quad j^\mu = \overline{\Psi} \gamma^\mu \Psi$$

- Relevant invariants :

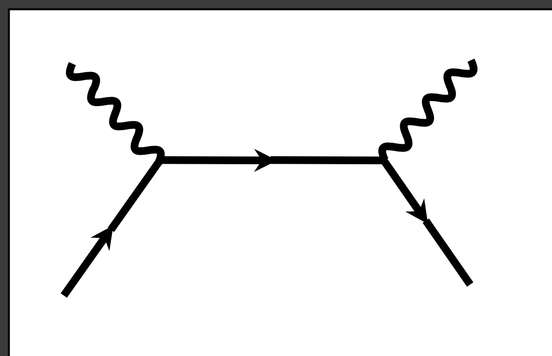
$$\zeta = \frac{\Delta^+}{P^+}, \quad \Delta^2 = t$$

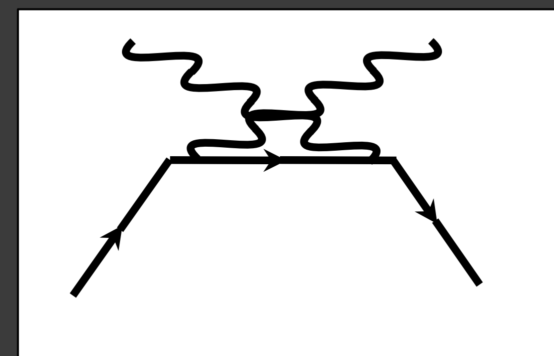
DVCS





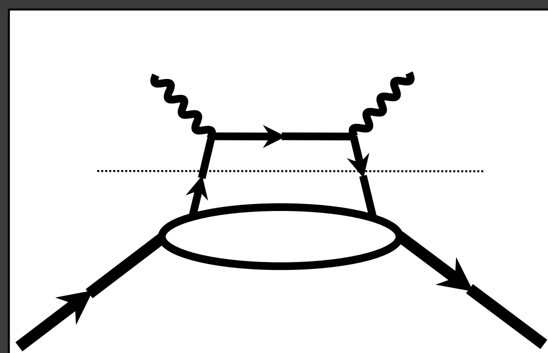
DVCS

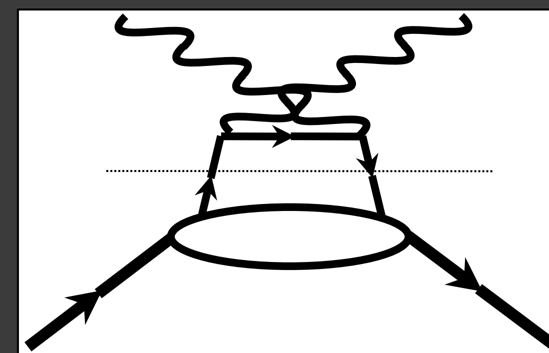




$$M^{\mu\nu} = ie^2 Tr\left\{\left(\gamma^\mu \frac{i}{\not{k} + \not{q} - m + i\varepsilon}\gamma^\nu + \gamma^\nu \frac{i}{\not{k} - \not{q}' - m + i\varepsilon}\gamma^\mu\right) U_k^s \overline{U}_{k'}^{s'}\right\}$$

DVCS





$$M^{\mu\nu} = ie^2 \int \frac{d^4k}{(2\pi)^4} Tr\left\{\left(\gamma^\mu \frac{i}{\not{k} + \not{q} - m + i\varepsilon}\gamma^\nu + \gamma^\nu \frac{i}{\not{k} - \not{q}' - m + i\varepsilon}\gamma^\mu\right) M(k)\right\}$$

$$M(k) = \int d^4z \, e^{ikz} \left\langle P' \middle| \overline{\Psi}(0)\Psi(z) \middle| P \right\rangle$$

DVCS

$$M^{\mu\nu} = ie^2 \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} Tr\left\{\left(\gamma^\mu \frac{(\not{k}+\not{q}+m)}{(k+q)^2-m^2+i\varepsilon}\gamma^\nu + \gamma^\nu \frac{(\not{k}-\not{q}'+m)}{(k-q')^2-m^2+i\varepsilon}\gamma^\mu\right) M(k)\right\}$$

Bjorken Limit

$$(M^{\mu\nu})_S = ie^2\left(g^{\mu\nu} - n^\mu_{(+)}n^\nu_{(-)} - n^\mu_{(-)}n^\nu_{(+)}\right)\int \frac{dx}{2\pi}\left(\frac{1}{x-\zeta+i\varepsilon} + \frac{1}{x+\zeta+i\varepsilon}\right) F(x,\zeta,\Delta^2)$$

$$F(x,\zeta,\Delta^2) = \int dz^- e^{i\frac{1}{2}xP^+z^-} \langle P'|\overline{\Psi}(0)\gamma^+\Psi(z)|P\rangle \qquad at \quad z^+ = 0, \ \ \vec{z}_\perp = 0.$$

$$F(x,\zeta,\Delta^2) \equiv \frac{1}{\overline{P}^+}\overline{U}^{s'}_{P'}\left[H(x,\zeta,\Delta^2)\,\gamma^+ + E(x,\zeta,\Delta^2)\,\frac{i}{2M}\sigma^{+\alpha}(-\Delta_\alpha)\right]U^s_P$$

DVCS

$$M^{\mu\nu} = ie^2 \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} Tr\left\{ \left(\gamma^\mu \frac{(\not{k} + \not{q} + m)}{(k+q)^2 - m^2 + i\varepsilon} \gamma^\nu + \gamma^\nu \frac{(\not{k} - \not{q}' + m)}{(k-q')^2 - m^2 + i\varepsilon} \gamma^\mu \right) M(k) \right\}$$

Bjorken Limit

$$(M^{\mu\nu})_A = ie^2 \left(g^{\mu\nu} - n^\mu_{(+)} n^\nu_{(-)} - n^\mu_{(-)} n^\nu_{(+)} \right) \int \frac{dx}{2\pi} \left(\frac{1}{x - \zeta + i\varepsilon} - \frac{1}{x + \zeta + i\varepsilon} \right) \tilde{F}(x, \zeta, \Delta^2)$$

$$\tilde{F}(x, \zeta, \Delta^2) = \int dz^- e^{i\frac{1}{2} x P^+ z^-} \langle P' | \overline{\Psi}(0) \gamma^+ \gamma^5 \Psi(z) | P \rangle \qquad at \quad z^+ = 0, \ \vec{z}_\perp = 0.$$

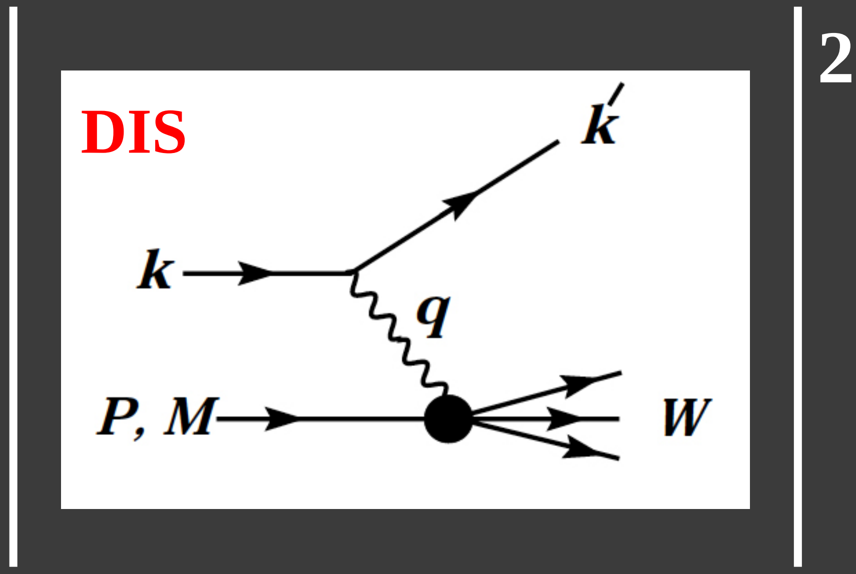
$$\tilde{F}(x, \zeta, \Delta^2) \equiv \frac{1}{\overline{P}^+} \overline{U}^{s'}_{P'} \left[\tilde{H}(x, \zeta, \Delta^2) \gamma^+ \gamma^5 + \tilde{E}(x, \zeta, \Delta^2) \frac{1}{2M} \gamma^5 (-\Delta^+) \right] U^s_P$$

Observables and GPD's

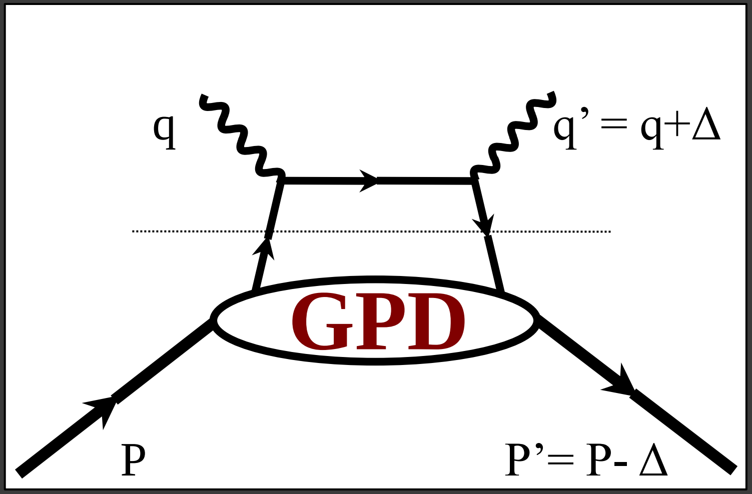
	Helicity non-flip	Helicity flip
Unpolarized Protons	$H(x,\zeta,\Delta^2)$	$E(x,\zeta,\Delta^2)$
Polarized Protons	$\widetilde{H}(x,\zeta,\Delta^2)$	$\widetilde{E}(x,\zeta,\Delta^2)$

- Real functions
- Lorentz Invariant
- Reduce to ordinary PDF's (forward limit)

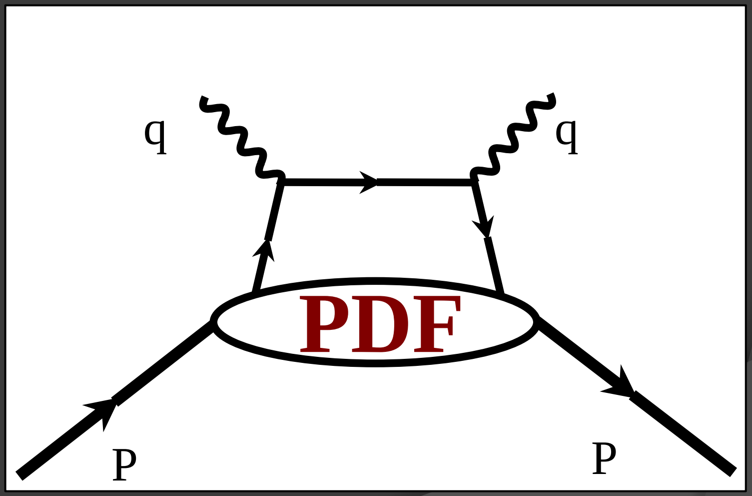
Observables and GPD's



Optical Theorem



Δ → 0



Observables and GPD's

Polynomiality:

$$\int_0^1 dx\, X^n H(X,\xi,\Delta^2) = \sum_{i=even}^{n} \xi^i A_{n+1,i}(t) + \mathrm{mod}(n,2)\xi^{n+1} C_{n+1}(t)$$

$$\int_0^1 dx\, X^n E(X,\xi,\Delta^2) = \sum_{i=even}^{n} \xi^i B_{n+1,i}(t) - \mathrm{mod}(n,2)\xi^{n+1} C_{n+1}(t)$$

$$x \to X - \xi/2$$

$$x - \zeta \to X + \xi/2$$

Observables and GPD's

Polynomiality:

$$\int_0^1 dx\, X^n H(X,\xi,\Delta^2) = \sum_{i=even}^{n} \xi^i A_{n+1,i}(t) + \mathrm{mod}(n,2)\xi^{n+1} C_{n+1}(t)$$

$$\int_0^1 dx\, X^n E(X,\xi,\Delta^2) = \sum_{i=even}^{n} \xi^i B_{n+1,i}(t) - \mathrm{mod}(n,2)\xi^{n+1} C_{n+1}(t)$$

$$x \rightarrow X - \xi/2$$

$$x - \zeta \rightarrow X + \xi/2$$

$$F_1 = \int_0^1 dx\, \frac{H(x,\zeta,\Delta^2)}{1-\zeta/2} \qquad F_2 = \int_0^1 dx\, \frac{E(x,\zeta,\Delta^2)}{1-\zeta/2}$$

$$J^\mu = \left[F_1(\Delta^2)\,\gamma^\mu + F_2(x,\zeta,\Delta^2)\,\frac{i}{2M}\,\sigma^{\mu\alpha}(-\Delta_\alpha) \right]$$

Observables and GPD's

Polynomiality:

$$\int_0^1 dX\; X^n \widetilde{H}(X,\xi,\Delta^2) = \sum_{i=even}^{n} \xi^i \widetilde{A}_{n+1,i}(t)$$

$$\int_0^1 dX\; X^n \widetilde{E}(X,\xi,\Delta^2) = \sum_{i=even}^{n} \xi^i \widetilde{B}_{n+1,i}(t)$$

$$x \rightarrow X - \xi/2$$

$$x - \zeta \rightarrow X + \xi/2$$

$$g_A = \int_0^1 dx\; \frac{\widetilde{H}(x,\zeta,\Delta^2)}{1-\zeta/2} \qquad g_P = \int_0^1 dx\; \frac{\widetilde{E}(x,\zeta,\Delta^2)}{1-\zeta/2}$$

$$J_5^\mu = \left[g_A(\Delta^2)\,\gamma^\mu\gamma_5 + g_P(x,\zeta,\Delta^2)\,\frac{1}{2M}\,\gamma_5(-\Delta^\mu) \right]$$

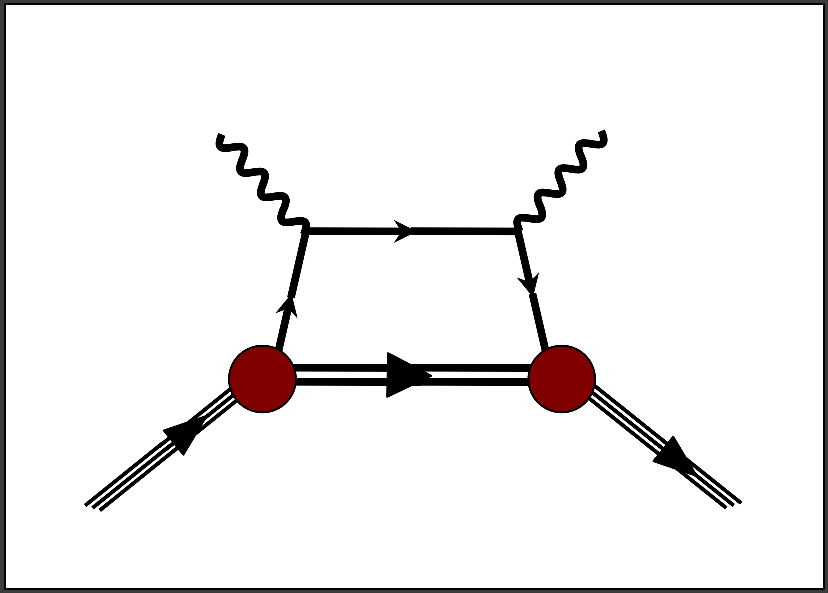
- It is not possible to calculate GPD’s by elementary means (nonperturbative nature of QCD)

- Some models have been built:

  - M. Burkardt (2002)
  - M.Diehl, T.Feldman, Jakob, Kroll (2005)
  - M. Guidal, Polyakov, Radyushkyn,Vanderhaegen(2005)

- Can we incorporate some Physics into a parametrization?

Diquark Model



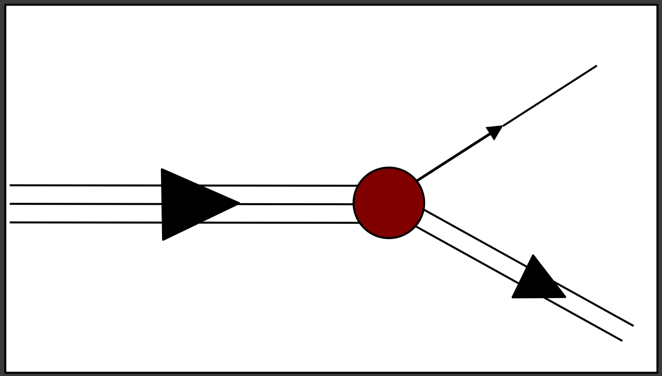
- Proton is thought as being composed by a quark a *diquark* of mass Mx

Must consider:

- **Lorentz Invariance**
- **Parity**
- **Spin properties**

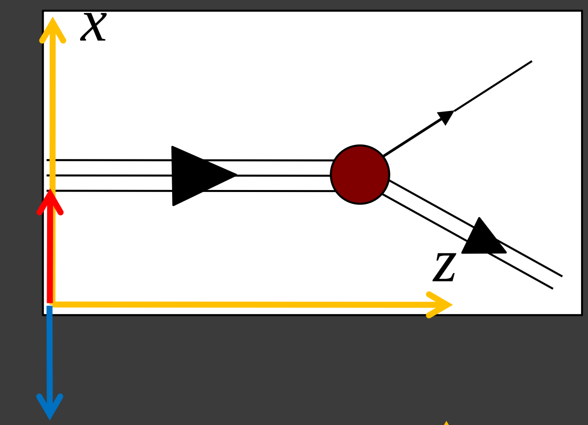
Diquark S= 0,1
Diquark S=1 must be an axial vector
Constrains Dirac Structure

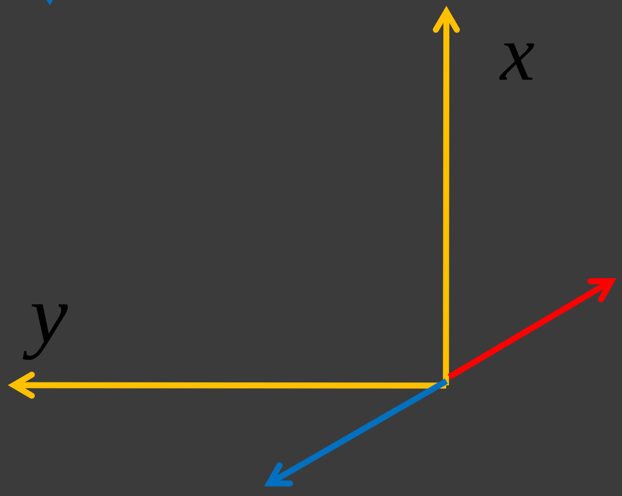
Diquark Model



What is this vertex?

Diquark Model





Vertex function can be written as:

$$\Upsilon^{\mu}_{ax}(N) = \frac{g_{ax}(p^2)}{\sqrt{3}} \gamma_5 \left[\gamma^{\mu} - R_g \frac{P^{\mu}}{M} \right].$$

Axial coupling

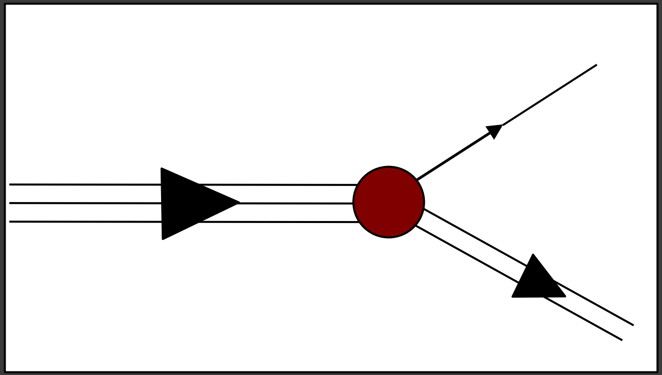
$$\Upsilon_s(N) = g_{sc}(p^2);$$

Scalar coupling

$\mathbf{g_{ax,sc}}$ guarantees small perpendicular components of momentum

Parametrization worked out using scalar coupling. (Similar to Meyer,Mulders)

Diquark Model



What is this vertex?

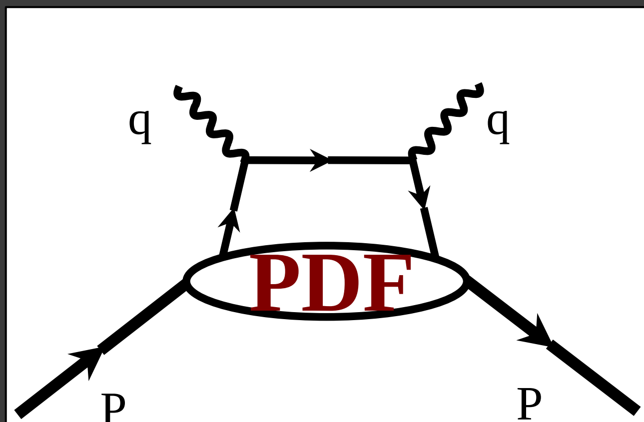
Spin 1 Case:

$$d^{\mu\nu}(P-p) = \begin{cases} -g^{\mu\nu} + \dfrac{(P-p)^\mu n_-^\nu + (P-p)^\nu n_-^\mu}{(P-p)\cdot n_-} - \dfrac{M_a^2}{[(P-p)\cdot n_-]^2}\, n_-^\mu\, n_-^\nu \\ -g^{\mu\nu} + \dfrac{(P-p)^\mu\,(P-p)^\nu}{M_a^2} \\ -g^{\mu\nu} + \dfrac{P^\mu\, P^\nu}{M_a^2} \\ -g^{\mu\nu} \end{cases}$$

- Some of these choices produce cumbersome expressions.

Diquark Model

Gamberg, Goldstein, Schlegel (2008)



$$f_1^{ax}(x, \vec{p}_T^{\,2}) = \frac{1}{6(2\pi)^3} \frac{|g_{ax}(p^2)|^2}{M^2 m_s^2 (1-x)[\vec{p}_T^{\,2} + \tilde{m}^2]^2} \times \mathcal{R}_1^{ax}(x, \vec{p}_T^{\,2}; R_g, \{\mathcal{M}\}),$$

$$\begin{aligned}
&\mathcal{R}_1^{ax}(x, \vec{p}_T^{\,2}; R_g, \{\mathcal{M}\}) \\
&= M^2[(\vec{p}_T^{\,2})^2 + 2(1 - x(1-x)) m_s^2 \vec{p}_T^{\,2} + x^2 m_s^4 + 6x(1-x)^2 m_q M m_s^2 + (1-x)^2 M^2 (\vec{p}_T^{\,2} + 2x^2 m_s^2 + (1-x)^2 m_q^2) \\
&\quad + m_q^2 (1-x)^2 (\vec{p}_T^{\,2} + 2m_s^2)] + R_g[M^2(-(\vec{p}_T^{\,2})^2 - x^2 m_s^4 + (1 - (4-x)x) m_s^2 \vec{p}_T^{\,2} - (1-x)^2 m_q^2 (\vec{p}_T^{\,2} - m_s^2) \\
&\quad - M^2 (1-x)^2 (\vec{p}_T^{\,2} - x^2 m_s^2 + (1-x)^2 m_q^2) + m_q M (x(\vec{p}_T^{\,2} + m_s^2)^2 + 2x(1-x)^2 M^2 (\vec{p}_T^{\,2} - m_s^2 + x(1-x)^4 M^2))] \\
&\quad + R_g^2 \Big[\frac{1}{4} (\vec{p}_T^{\,2} + (m_s - (1-x)M)^2)(\vec{p}_T^{\,2} + (xM - m_q)^2)(\vec{p}_T^{\,2} + (m_s + (1-x)M)^2)\Big].
\end{aligned}$$

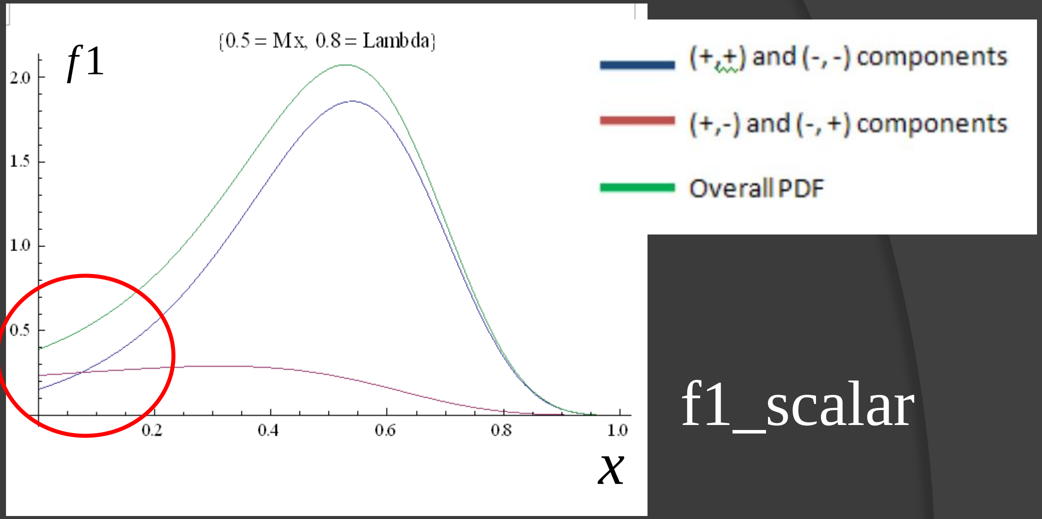
- An expression for a GPD would not be as simple.

Diquark Model

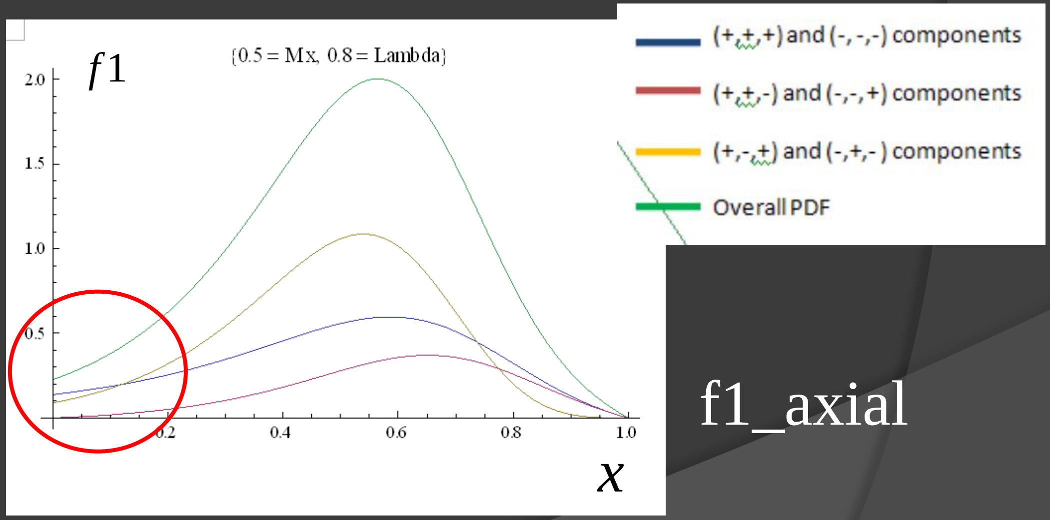
$$\sum \varepsilon_{\mu}^{*(I)} \varepsilon_{\nu}^{(J)} = -g^{\nu\nu}$$

$$g(k^2) = \frac{k^2 - m^2 + i\varepsilon}{(k^2 - \Lambda^2 + i\varepsilon)^2}$$

Regge Behavior??



f1_scalar



f1_axial

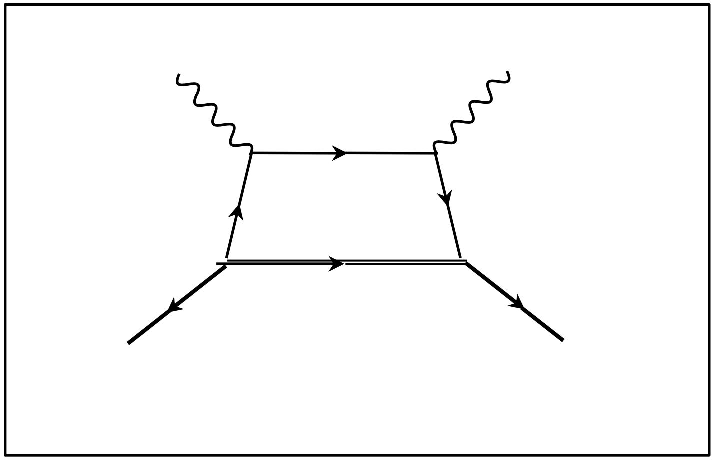
How to Build a Parametrization?

The recipe…

- Take Scalar diquark model.
- Calculate GPD's :
    - Covariant calculation.
    - Light Cone Fock Expansion.
- Reggeization.
- Fix Parameters:
    - Altarelli-Parisi equations.
    - $\Delta = 0$ case: GPD's → PDF's.
    - $\zeta = 0$ case: Fit to Form Factors.

Covariant vs Time Ordered

Covariant | **Light Cone Fock Expansion**



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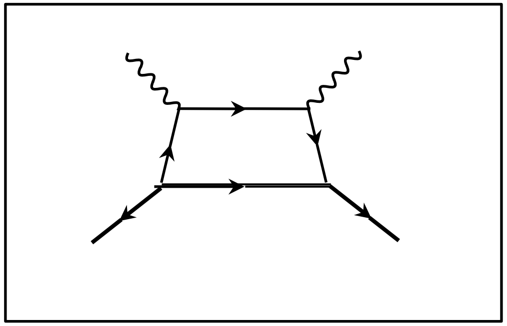

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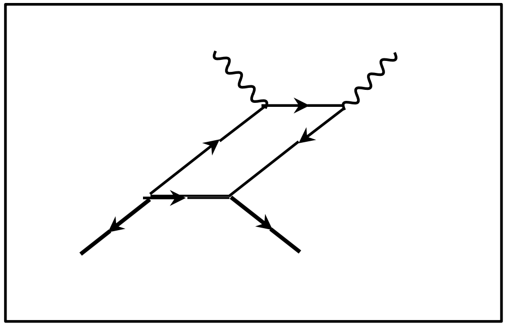


$$M^{\mu\nu} = ie^2\left(g^{\mu\nu} - n^{\mu}_{(+)}n^{\nu}_{(-)} - n^{\mu}_{(-)}n^{\nu}_{(+)}\right)\int \frac{dx}{2\pi}\left(\frac{1}{x - i\varepsilon} + \frac{1}{x - \zeta + i\varepsilon}\right)F(x,\zeta,\Delta^2)$$

$$F(x,\zeta,\Delta^2) \equiv \frac{1}{\overline{P}^+}\overline{U}^{s'}_{P'}\left[H(x,\zeta,\Delta^2)\,\gamma^+ + E(x,\zeta,\Delta^2)\,\frac{i}{2M}\sigma^{+\alpha}(-\Delta_\alpha)\right]U^s_P$$

Covariant vs Time Ordered

Covariant

$$\int \frac{dk^-}{(2\pi)} \frac{Tr\{...\}}{(P-k)^2 - m^2 + i\varepsilon} \cdot \frac{g(k^2)}{k^2 - m^2 + i\varepsilon} \cdot \frac{g((k-\Delta)^2)}{(k-\Delta)^2 - m^2 + i\varepsilon}$$

$$g(k^2) = \frac{k^2 - m^2 + i\varepsilon}{(k^2 - \Lambda^2 + i\varepsilon)^2}$$

Calculate residues ➡ Get result!!!

Covariant vs Time Ordered

Light Cone Fock Expansion

Start from matrix element:

$$F(x,\zeta,\Delta^2) = \int dz^- e^{i\frac{1}{2}xP^+z^-} \langle P' | \overline{\Psi}(0)\gamma^+ \Psi(z) | P \rangle \qquad at \quad z^+ = 0,\ \vec{z}_\perp = 0.$$

$$\left|\psi_p(P^+,\vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^n \frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\,16\pi^3} 16\pi^3\delta\left(1-\sum_{i=1}^n x_i\right)\delta^{(2)}\left(\sum_{i=1}^n \vec{k}_{\perp i}\right) \times \psi_n(x_i,\vec{k}_{\perp i},\lambda_i)\left|n;\ x_iP^+, x_i\vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\right\rangle.$$

$\langle n;\ x_i, \vec{k}_{\perp i}, \lambda_i | \psi_p \rangle \propto \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ LC Wave Functions

Calculate LCWF → Get result!!!

Covariant vs Time Ordered

Covariant

Light Cone Fock Expansion

Not Clear how to treat vertex:

$$g(k^2) = \frac{k^2 - m^2 + i\varepsilon}{(k^2 - \Lambda^2 + i\varepsilon)^2}$$

- Vertex function seems to change pole structure

- What is on-shell?

Covariant vs Time Ordered

Covariant

• Relates On-shell conditions to TOPT

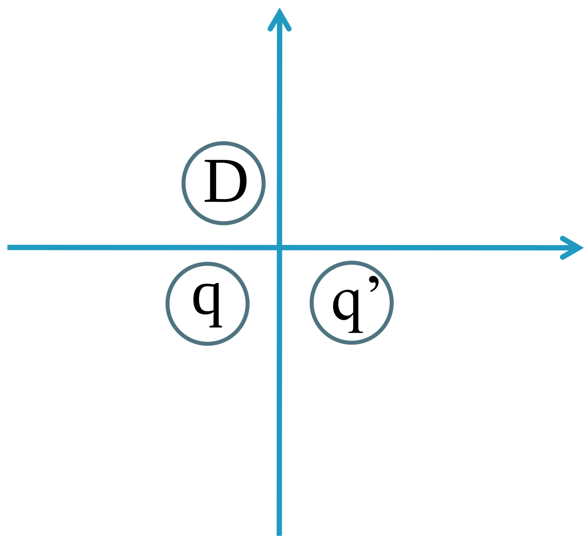
Light Cone Fock Expansion

• Kinematic regions are easily interpreted (partonic picture)

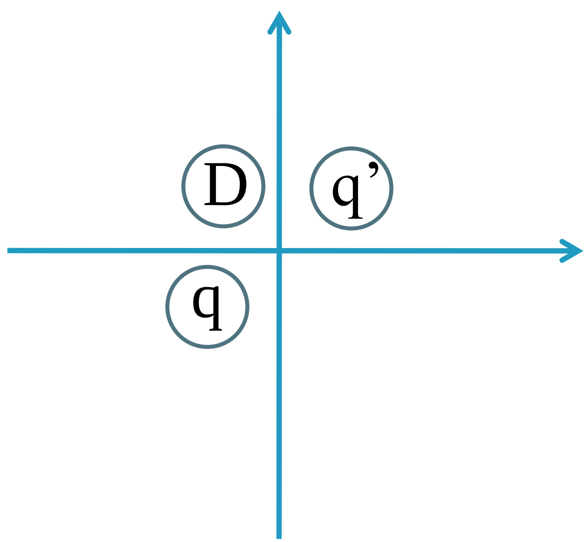
Covariant vs Time Ordered

Covariant

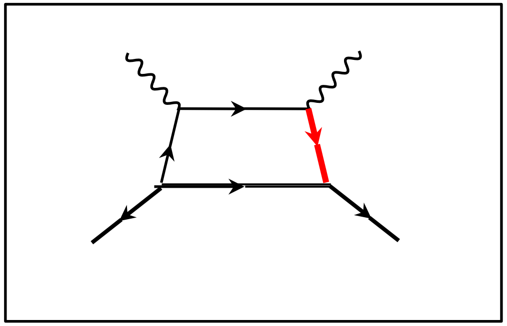
$x > \zeta$



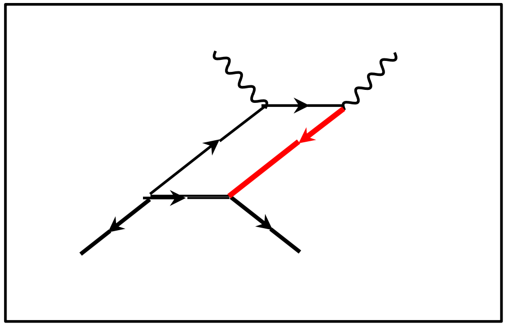
$x < \zeta$



Light Cone Fock Expansion



DGLAP



ERBL

antiquark with

$$\frac{k'^{+}}{P^{+}} = \zeta - x$$

Covariant vs Time Ordered

Covariant

Light Cone Fock Expansion

Check pole structure →

Use to determine form of vertex in TOPT

↓

Use Fock expansion to calculate Matrix element F

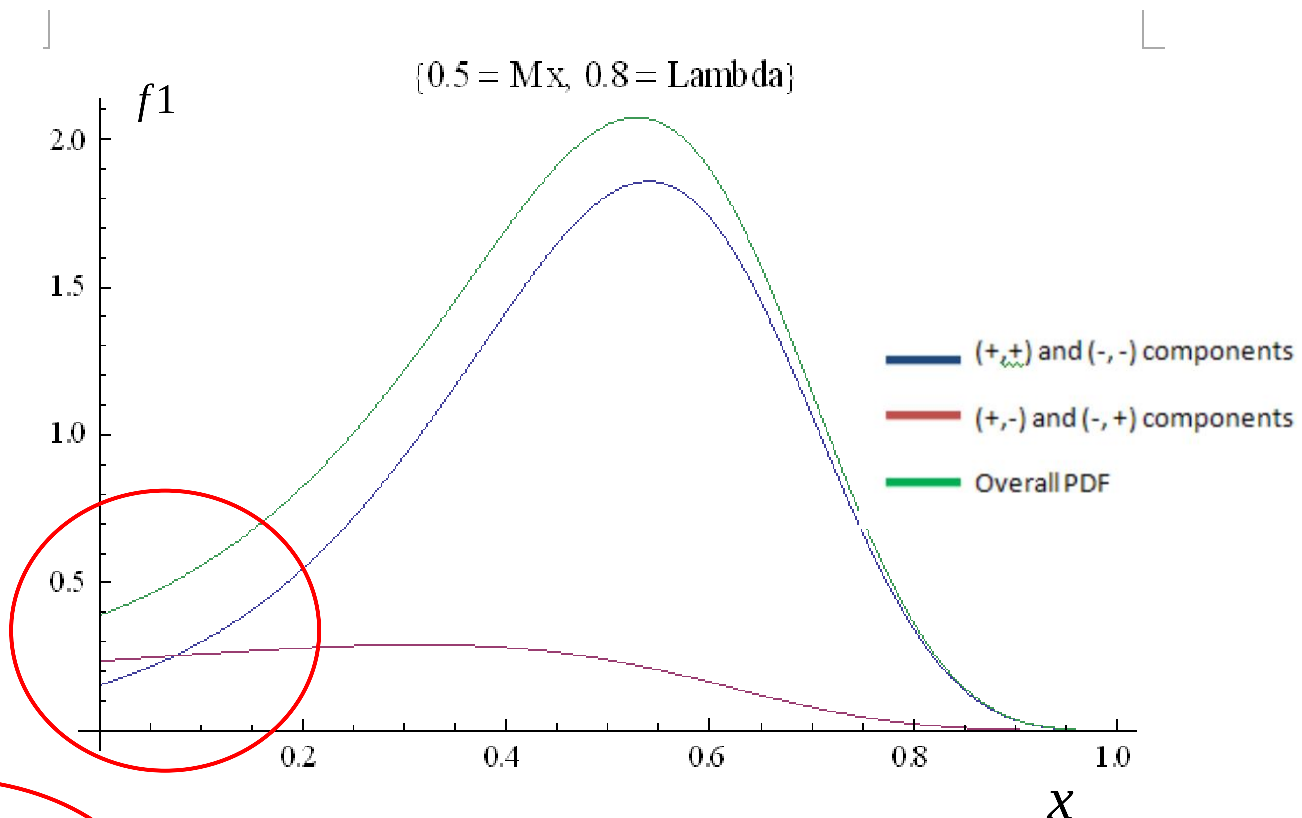
$$M^{\mu\nu} = ie^2\left(g^{\mu\nu} - n^{\mu}_{(+)}n^{\nu}_{(-)} - n^{\mu}_{(-)}n^{\nu}_{(+)}\right)\int \frac{dx}{2\pi}\left(\frac{1}{x-\zeta+i\varepsilon} + \frac{1}{x+\zeta+i\varepsilon}\right)F(x,\zeta,\Delta^2)$$

$$F(x,\zeta,\Delta^2) \equiv \frac{1}{P^+}\overline{U}^{s'}_{P'}\left[H(x,\zeta,\Delta^2)\,\gamma^+ + E(x,\zeta,\Delta^2)\,\sigma^{+\alpha}\Delta_\alpha\right]U^{s}_{P}.$$

Regge Behavior?

- From Regge Theory

$$s^{\alpha+\alpha' t}$$



$$x^{-\alpha-\beta(1-x)^p t}$$

(Burkardt)

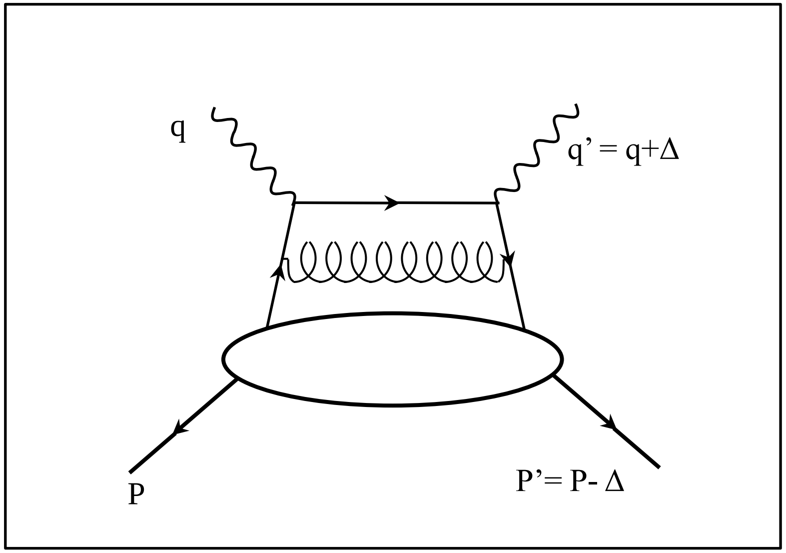
Regge Behavior?

- Have six paramers…

$$m, M_x, \Lambda, \alpha, \beta, p$$

Fix Parameters

- Take into account Altarelli-Parisi equations



Fix Parameters

- $\Delta = 0$ case: GPD's → PDF's.

$$H(x,0,0) = f1(x)$$

$$\tilde{H}(x,0,0) = g1(x)$$

→ Fix m, Mx, Λ, α

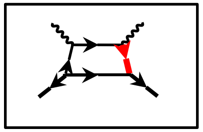
- $\zeta = 0$ case: Fit to Form Factors.

$$F_1 = \int_0^1 dx\, H(x,\zeta,\Delta^2) \quad F_2 = \int_0^1 dx\, E(x,\zeta,\Delta^2)$$

$$g_A = \int_0^1 dx\, \tilde{H}(x,\zeta,\Delta^2) \quad g_P = \int_0^1 dx\, \tilde{E}(x,\zeta,\Delta^2)$$

→ Fix β,p

Results



DGLAP

$$H = \int \frac{d^2\vec{k}_\perp}{16\pi^3}\varphi_D \left\{ \left(\frac{1-\zeta/2}{\sqrt{1-\zeta}}\right)\left((m+xM)(m+x'M)+\vec{k}_\perp\cdot\vec{k}'_\perp\right) + \left(\frac{\zeta^2 M}{2\vec{\Delta}_\perp^2\sqrt{1-\zeta}}\right)\left(\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)\right)\right\}$$

$$E = \left(\frac{2M\sqrt{1-\zeta}}{\vec{\Delta}_\perp^2}\right)\int \frac{d^2\vec{k}_\perp}{16\pi^3}\varphi_D \left\{\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)\right\}$$

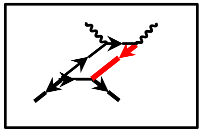
$$\varphi_D = \frac{(1-x)^{3/2}(1-x')^{3/2}}{[\vec{k}_\perp^2+L(x)^2]^2[\vec{k}'^2_\perp+L(x')^2]^2}$$

$$L(x)^2 = xM_x^2+(1-x)\Lambda^2-x(1-x)M^2$$

$$x' = \frac{x-\zeta}{1-\zeta}$$

$$\vec{k}'_\perp = \vec{k}_\perp - \frac{1-x}{1-\zeta}\vec{\Delta}_\perp$$

Results



ERBL

$$H = \int \frac{d^2\vec{k}_\perp}{16\pi^3}\varphi_E \left\{ \begin{aligned} &\left(\frac{1-\zeta/2}{\sqrt{1-\zeta}}\right)\left((m+xM)(m+x'M)+\vec{k}_\perp\cdot\vec{k}'_\perp\right) \\ &+\left(\frac{\zeta^2 M}{2\vec{\Delta}_\perp^2\sqrt{1-\zeta}}\right)\left(\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)\right) \end{aligned} \right\}$$

$$E = \left(\frac{2M\sqrt{1-\zeta}}{\vec{\Delta}_\perp^2}\right)\int \frac{d^2\vec{k}_\perp}{16\pi^3}\varphi_E \left\{\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)\right\}$$

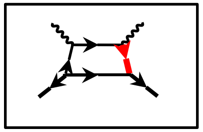
$$\varphi_E = \frac{x^2(1-x)\sqrt{1-\zeta}}{\zeta^2[\vec{k}_\perp^2+L(x)^2]^2[(\vec{k}_\perp-\frac{x}{\zeta}\vec{\Delta}_\perp)^2+\tilde{L}^2]^2}$$

$$\tilde{L}^2 = \Lambda^2 + \frac{x}{\zeta}\frac{\zeta-x}{1-\zeta}\left(\zeta M^2+\frac{\vec{\Delta}_\perp^2}{\zeta}\right)$$

$$x' = \frac{x-\zeta}{1-\zeta}$$

$$\vec{k}'_\perp = \vec{k}_\perp - \frac{1-x}{1-\zeta}\vec{\Delta}_\perp$$

Results



DGLAP

$$\tilde{H} = \int \frac{d^2\vec{k}_\perp}{16\pi^3}\varphi_D \left\{ \begin{array}{l} \left(\frac{1-\zeta/2}{\sqrt{1-\zeta}}\right)\left((m+xM)(m+x'M)-\vec{k}_\perp\cdot\vec{k}'_\perp\right) \\ +\left(\frac{\zeta(1-\zeta/2)M}{2\vec{\Delta}_\perp^2\sqrt{1-\zeta}}\right)\left(-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)-\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)\right) \end{array} \right\}$$

$$\tilde{E} = \left(\frac{4M(1-\zeta/2)\sqrt{1-\zeta}}{\vec{\Delta}_\perp^2\zeta}\right)\int \frac{d^2\vec{k}_\perp}{16\pi^3}\varphi_D\left\{-\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)\right\}$$

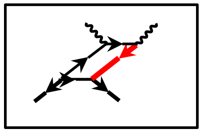
$$\varphi_D = \frac{(1-x)^{3/2}(1-x')^{3/2}}{[\vec{k}_\perp^2+L(x)^2]^2[\vec{k}'^2_\perp+L(x')^2]^2}$$

$$L(x)^2 = xM_x^2+(1-x)\Lambda^2-x(1-x)M^2$$

$$x' = \frac{x-\zeta}{1-\zeta}$$

$$\vec{k}'_\perp = \vec{k}_\perp - \frac{1-x}{1-\zeta}\vec{\Delta}_\perp$$

Results



ERBL

$$\tilde{H} = \int \frac{d^2\vec{k}_\perp}{16\pi^3}\,\varphi_E \left\{ \left(\frac{1-\zeta/2}{\sqrt{1-\zeta}}\right)\left((m+xM)(m+x'M)-\vec{k}_\perp\cdot\vec{k}'_\perp\right) + \left(\frac{\zeta(1-\zeta/2)M}{2\vec{\Delta}_\perp^2\sqrt{1-\zeta}}\right)\left(-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)-\vec{\Delta}_\perp\cdot\vec{k}_\perp(m+x'M)\right) \right\}$$

$$\tilde{E} = \left(\frac{4M(1-\zeta/2)\sqrt{1-\zeta}}{\vec{\Delta}_\perp^2\zeta}\right)\int \frac{d^2\vec{k}_\perp}{16\pi^3}\,\varphi_E \left\{-\vec{\Delta}_\perp\cdot\vec{k}'_\perp(m+xM)-\vec{\Delta}_\perp\cdot\vec{k}_\perp(m-x'M)\right\}$$

$$\varphi_E = \frac{x^2(1-x)\sqrt{1-\zeta}}{\zeta^2[\vec{k}_\perp^2+L(x)^2]^2[(\vec{k}_\perp-\frac{x}{\zeta}\vec{\Delta}_\perp)^2+\tilde{L}^2]^2}$$

$$\tilde{L}^2 = \Lambda^2 + \frac{x}{\zeta}\frac{\zeta-x}{1-\zeta}\left(\zeta M^2+\frac{\vec{\Delta}_\perp^2}{\zeta}\right)$$

$$x' = \frac{x-\zeta}{1-\zeta}$$

$$\vec{k}'_\perp = \vec{k}_\perp - \frac{1-x}{1-\zeta}\vec{\Delta}_\perp$$

Results

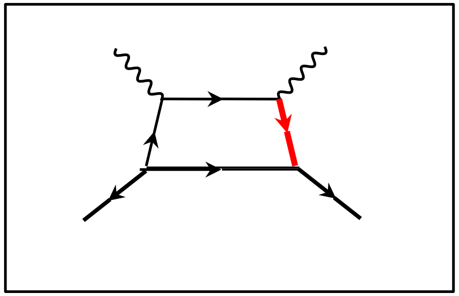
Some properties:

✓Continuity of DGLAP and ERBL

$$at \quad \zeta = x \qquad \varphi_D = \varphi_E = \frac{(1-x)^{3/2}}{[\vec{k}_\perp^2 + L(x)^2]^2[(\vec{k}_\perp - \vec{\Delta}_\perp)^2 + \Lambda^2]^2}$$

✓Helicity structure.

DGLAP



$$(M^{\mu\nu})_S \Rightarrow \quad \gamma^+$$

$$F(x,\zeta,\Delta^2) \propto \sum_{\lambda\lambda'} \delta_{\lambda\lambda'} \Psi^{*p'}_{s'\lambda'}(x',\vec{k}'_\perp)\Psi^{p}_{s\lambda}(x,\vec{k}_\perp)$$

$$(M^{\mu\nu})_A \Rightarrow \quad \gamma^+\gamma^5$$

$$\tilde{F}(x,\zeta,\Delta^2) \propto \sum_{\lambda\lambda'} \lambda\delta_{\lambda\lambda'} \Psi^{*p'}_{s'\lambda'}(x',\vec{k}'_\perp)\Psi^{p}_{s\lambda}(x,\vec{k}_\perp)$$

Results

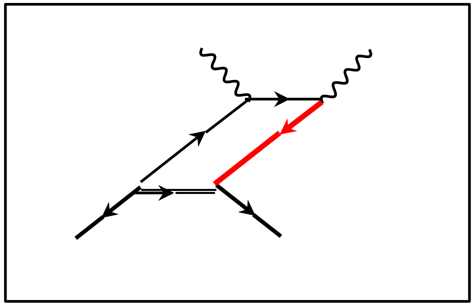
Some properties:

✓Continuity of DGLAP and ERBL

$$at \quad \zeta = x \qquad \varphi_D = \varphi_E = \frac{(1-x)^{3/2}}{[\vec{k}_\perp^2 + L(x)^2]^2[(\vec{k}_\perp - \vec{\Delta}_\perp)^2 + \Lambda^2]^2}$$

✓Helicity structure.

ERBL



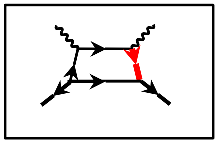
$$(M^{\mu\nu})_S \Rightarrow \quad \gamma^+$$

$$F(x,\zeta,\Delta^2) \propto \sum_{\lambda\lambda'} \delta_{-\lambda\lambda'} \Phi^{*p'}_{s'\lambda'}(x', \vec{k}'_\perp) \Psi^p_{s\lambda}(x, \vec{k}_\perp)$$

$$(M^{\mu\nu})_A \Rightarrow \quad \gamma^+\gamma^5$$

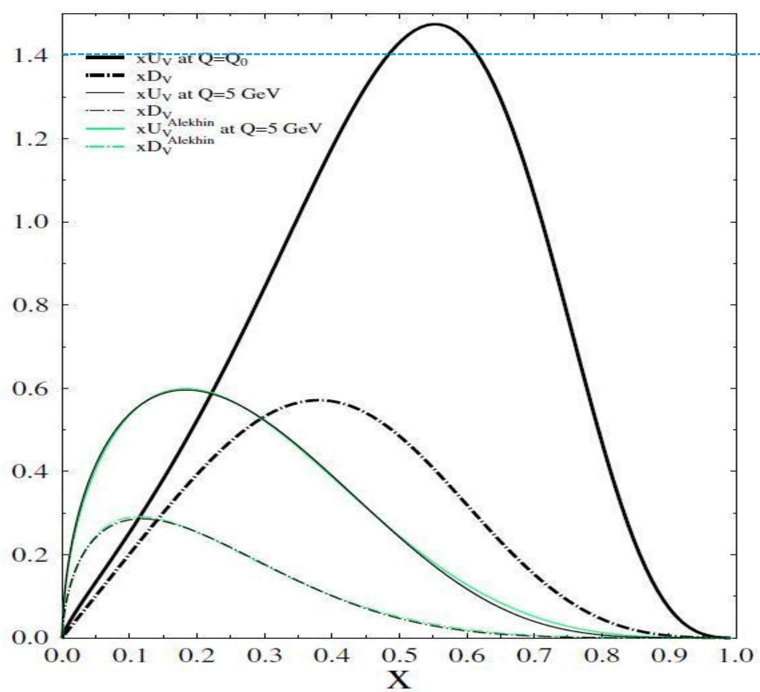
$$\widetilde{F}(x,\zeta,\Delta^2) \propto \sum_{\lambda\lambda'} \lambda\delta_{-\lambda\lambda'} \Phi^{*p'}_{s'\lambda'}(x', \vec{k}'_\perp) \Psi^p_{s\lambda}(x, \vec{k}_\perp)$$

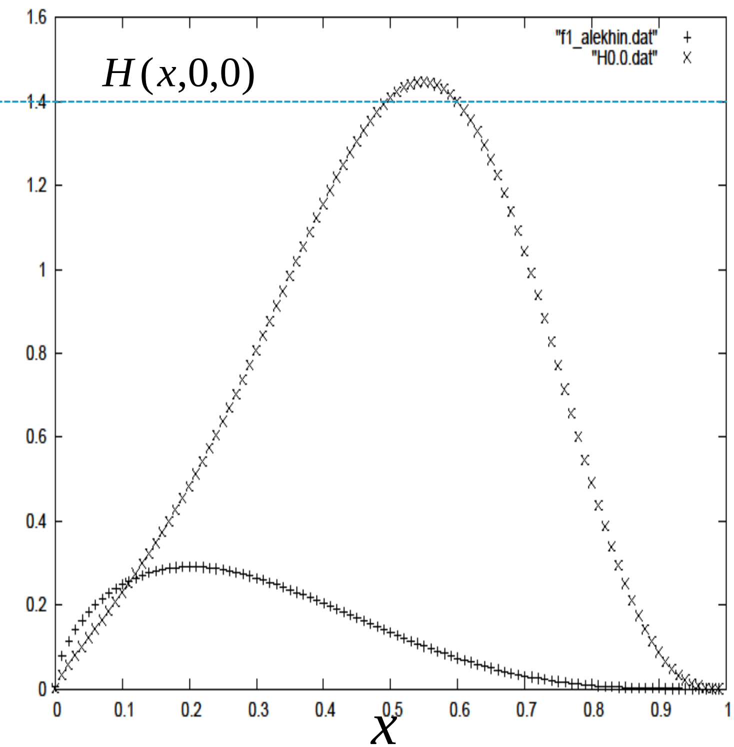
Preliminary Numerical Results



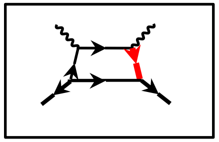
DGLAP

Forward Limit for H





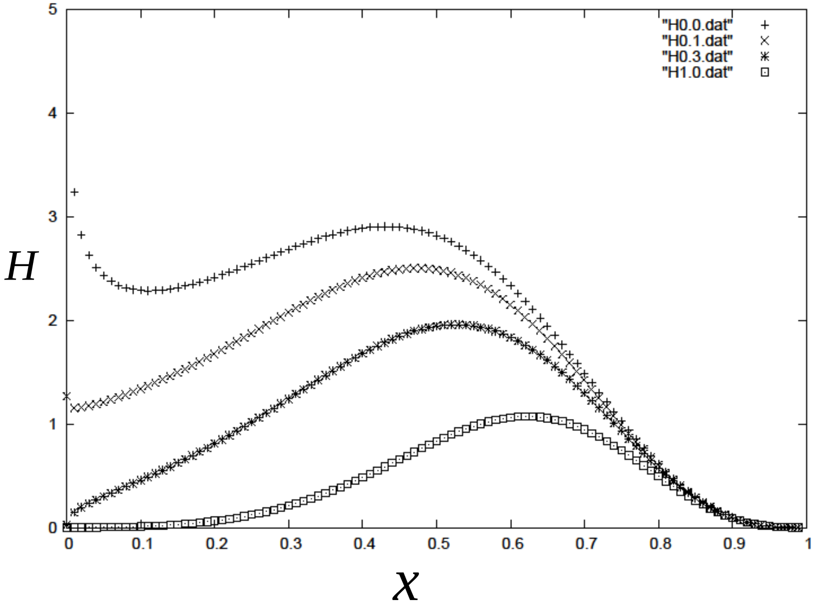
Preliminary Numerical Results



DGLAP

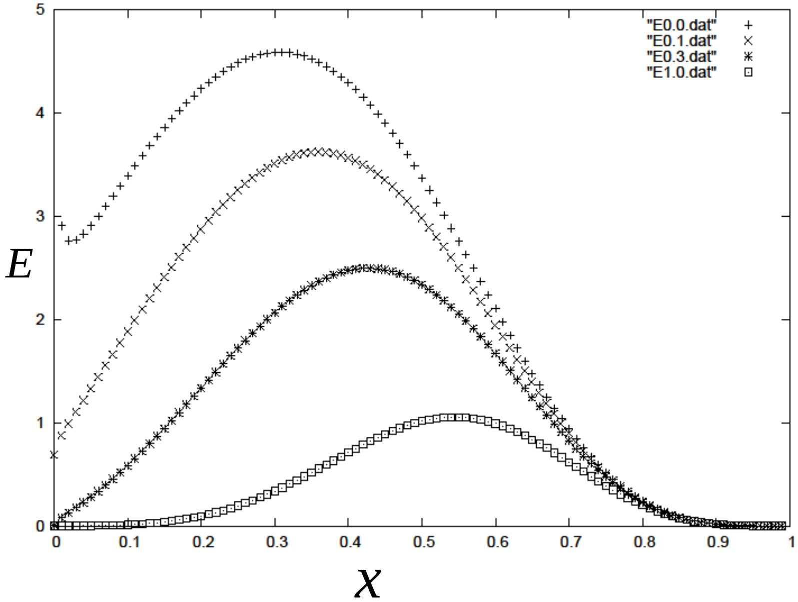
- For ζ = 0

$H(x,\zeta,\Delta^2)$

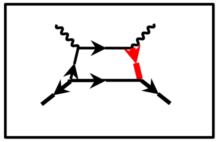


- For ζ = 0

$E(x,\zeta,\Delta^2)$



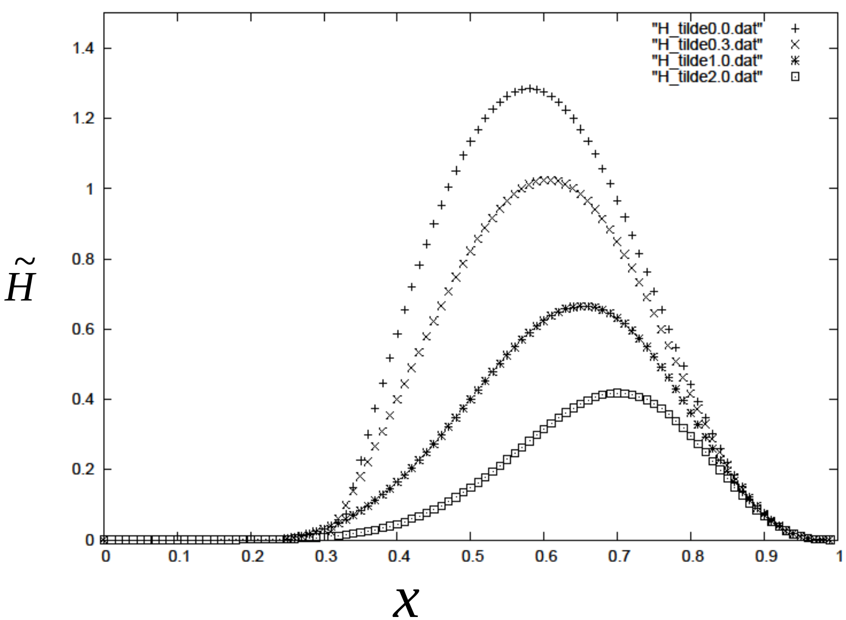
Preliminary Numerical Results



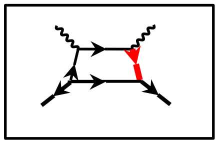
DGLAP

- For $\zeta = 0$

$\tilde{H}(x,\zeta,\Delta^2)$



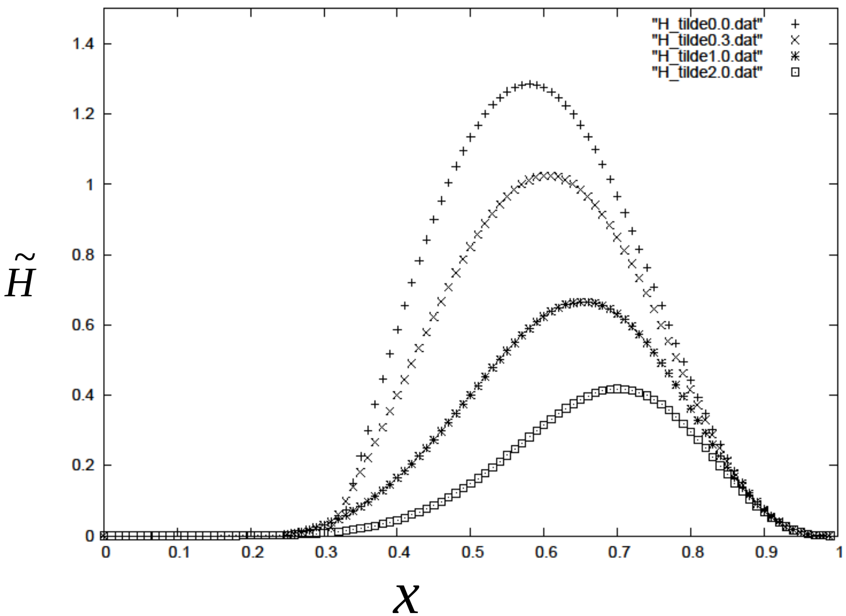
Preliminary Numerical Results



DGLAP

- For $\zeta = 0$

$\tilde{H}(x,\zeta,\Delta^2)$

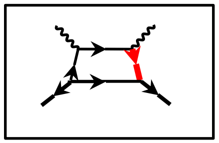


$\tilde{E}(x,\zeta,\Delta^2)$

- Diverges at $\zeta = 0$ and at $\Delta^2 = 0$

$$0 < \zeta < -\frac{\Delta^2}{2M}\left(\sqrt{1+\frac{4M^2}{-\Delta^2}}-1\right)$$

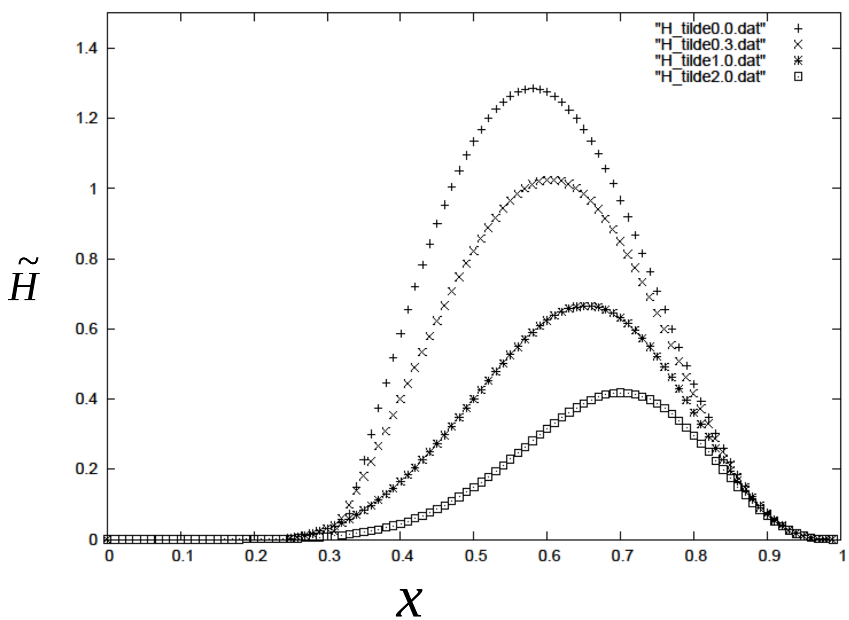
Preliminary Numerical Results



DGLAP

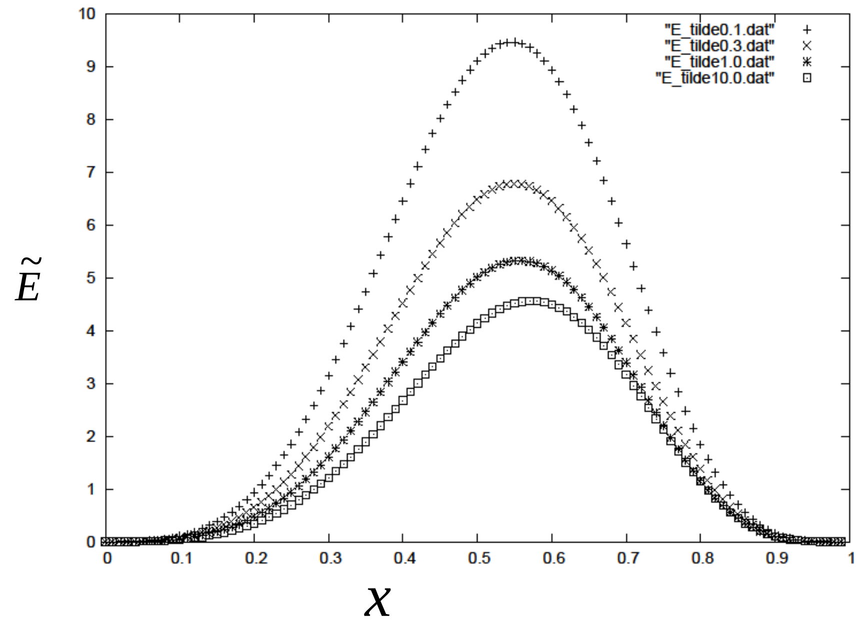
- For $\zeta = 0$

$\tilde{H}(x, \zeta, \Delta^2)$



- For $\zeta = \zeta_{max}$

$\tilde{E}(x, \zeta, \Delta^2)$

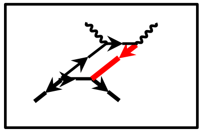


Possible Aplications

- Data Analysis for DVCS
  - Jefferson Lab

- Neutrino Pion-production
  - MINERvA (Fermilab)

Ongoing and Future Projects

-  ERBL
- Implementation of our parametrization in Neutrino Pion-production data analysis.
- Regge Behavior.

GRACIAS

GRACIAS

GRACIAS

GRACIAS

More diagrams…

Radiative Corrections

Final State Interactions