Flavor-branes in gauge/string duality and M-theory

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Historical introduction

- AdS/CFT correspondence

(Maldacena ’97; Gubser, Klebanov, Polyakov; Witten, ’98)

- Extensions to NonAdS/NonCFT and reduced supersymmetry

(Itzhaki, Maldacena, Sonnenschein, Yankielowicz ’98; Girardello, Petrini, Porrati, Zafferoni ’98; Klebanov, Witten/Strassler/Tseytlin ’98 and ’00; Polchinski, Strassler ’00; Maldacena, Núñez ‘01)

- Flavor-branes

(Karch, Katz ’02)

- Backreacting flavors

(Klebanov, Maldacena ’04)

- Smearing

(Bigazzi, Casero, Cotrone, Kiritsis, Paredes and Casero, Núñez, Paredes ‘06)

- Flavor-branes and generalized calibrations

(Gaillard, Schmude ’08)

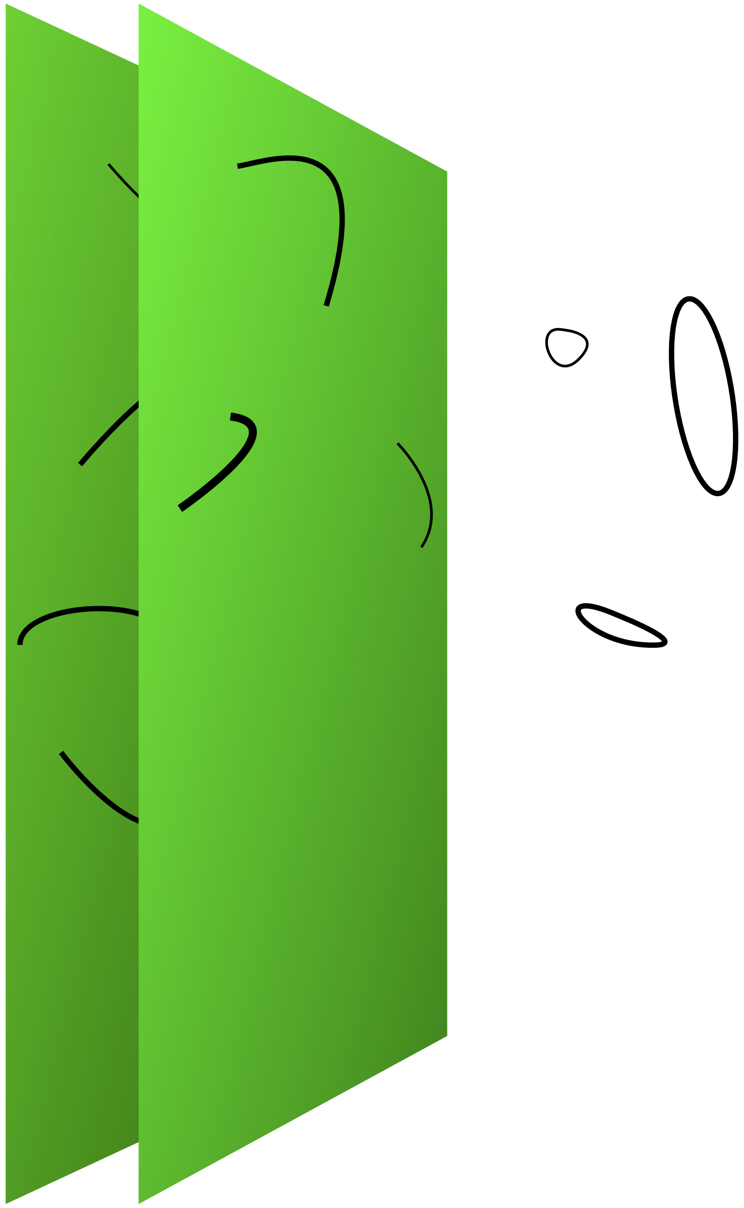
- Kaluza-Klein monopole condensation and M-theory with torsion

(Gaillard, Schmude ’09)

Outline

Introduction

Essential string theory

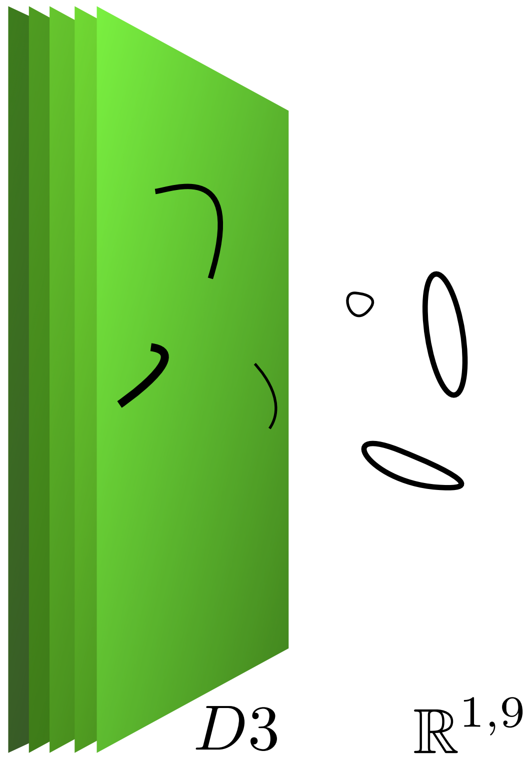


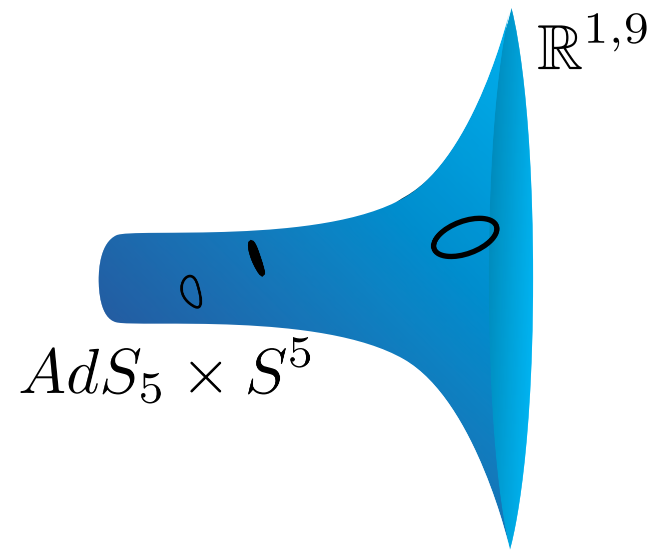
Two perspectives on string theory:
- Open & closed strings, D-branes
- Supergravity and brane actions

In type IIA/B:
- There are always closed strings (gravity)
- D-branes add an open string sector (Yang-Mills)

$$S_{\mathrm{IIA}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g}[e^{-2\Phi}(R + 4\partial_\mu \Phi \partial^\mu \Phi)] - \frac{1}{4} F_{(2)} \wedge *F_{(2)} + \mathcal{O}(H_{(3)}, F_{(4)}, \Psi)$$

Gauge/string duality





- Duality between

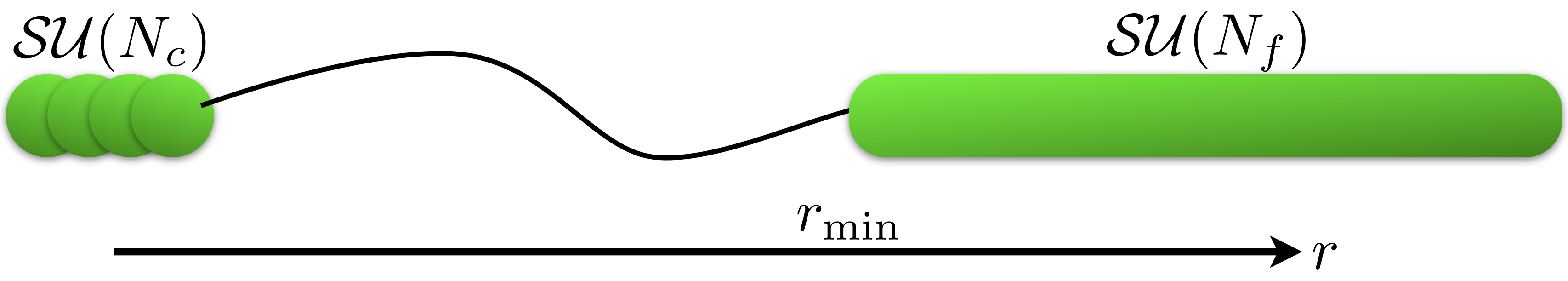
$$\mathcal{N} = 4, SU(N_c) \leftrightarrow \text{IIB } AdS_5 \times S^5$$

- Extended to less SUSY or no conformal symmetry:
  - Deformations

$$\mathcal{N} = 0^*, \mathcal{N} = 1^*, \mathcal{N} = 2^*$$

  - Branes at singularities
  - Wrapped branes

The flavoring problem



- Local symmetry in the bulk provides global symmetry in the gauge theory.
- For the 'flavor gauge group' to decouple, flavor branes must wrap non-compact cycles.

$$S_p = -T_p \int_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}\xi e^{-\Phi}\sqrt{-\det(g_{\mathrm{ind}} + 2\pi\alpha'\mathcal{F})} + T_p \int_{\mathcal{M}_{p+1}} (X^* C) \wedge e^{2\pi\alpha'\mathcal{F}}$$

- Quark mass related to string length

$$r_{\mathrm{min}} \sim m_f$$

- Probe approximation

$$N_c \gg N_f$$

- Need to consider backreaction to go beyond that limit

$$S = S_{\mathrm{IIA/B}} + S_{\mathrm{flav}}$$

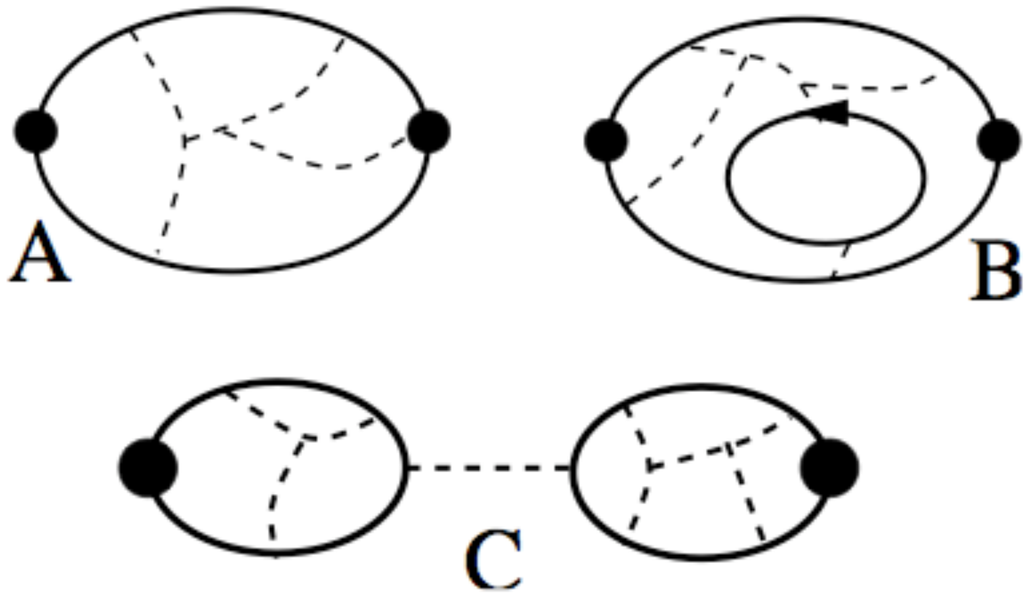
Backreaction - The physics

•'t Hooft limit

$$\lambda = g_{\rm YM}^2 N_c = \text{const.} \quad g_{\rm YM}^2 \to 0 \quad N_c \to \infty \quad N_f = \text{const.}$$

•Veneziano limit

$$\lambda = g_{\rm YM}^2 N_c = \text{const.} \quad g_{\rm YM}^2 \to 0 \quad N_c \to \infty \quad N_f \to \infty \quad N_c/N_f = \text{const.}$$



$$A \sim 1 \quad B \sim N_f/N_c \quad C \sim 1/N_c$$

Hoyos, Núñez, Papadimitriou

The geometry of backreacting flavors

Brane actions

- The standard brane action

$$S_{\text{flavor}} = \sum_{N_f} -T_6 \int_{\mathcal{M}_7} \mathrm{d}^{6+1}\xi e^{-\Phi}\sqrt{-g_{\text{ind}}} + T_6 \int_{\mathcal{M}_7} X^* C_7$$

- Supersymmetric branes satisfy a calibration condition (equivalent to kappa symmetry)

$$X^*\phi_{\text{D6}} = \mathrm{d}^{6+1}\xi\sqrt{-g_{\text{ind}}}$$

$$\phi_{\text{D6}} = (\bar{\epsilon}\Gamma_{a_0\ldots a_6}\epsilon)e^{a_0\ldots e_6}$$

- We can express the action using forms defined on space time

$$S_{\text{flavor}} = \sum_{N_f} -T_6 \int_{\mathcal{M}_7} X^*(e^{-\Phi}\phi_{\text{D6}} - C_7)$$

- And finally as a ten-dim integral

$$S_{\text{flavor}} = -T_6 \int_{\mathcal{M}_{10}} (e^{-\Phi}\phi_{\text{D6}} - C_7) \wedge \Xi$$

- Where we have defined a distribution density (smearing form)

$$\Xi$$

- Supersymmetry requires

$$*\mathrm{d}(e^{-\Phi}\phi_{\text{D6}}) = F$$

Backreaction - The mathematics

$$S = S_{IIA} + S_{\text{flav}} = \int \cdots - \frac{1}{8\kappa_{10}^2} F_{(2)} \wedge * F_{(2)} + T_p C_{(7)} \wedge \Xi + \ldots$$

- Modified type IIA/B equations of motion (D6-branes in type IIA)

$$\begin{aligned} \mathrm{d}F_{(2)} &= -(2\kappa_{10}^2 T_6)\Xi_{(3)} \\ 0 &= \mathrm{d} *_{10} F_{(2)} \\ 0 &= D + \mathcal{O}(\Xi) \\ 0 &= E_{\mu\nu} + \mathcal{O}(\Xi) \end{aligned}$$

Integrability

$$\delta_\epsilon \psi_\mu = D_\mu \epsilon + \frac{1}{64} e^{\frac{3}{4}\Phi} F_{\mu_1\mu_2} ({\Gamma_\mu}^{\mu_1\mu_2} - 14 \delta_\mu^{\mu_1} \Gamma^{\mu_2}) \Gamma^{11} \epsilon = \mathcal{D}_\mu \epsilon$$

$$\delta_\epsilon \lambda = \frac{\sqrt{2}}{4} \partial_\mu \Phi \Gamma^\mu \Gamma^{11} + \frac{3}{16} \frac{1}{\sqrt{2}} e^{\frac{3}{4}\Phi} F_{\mu_1\mu_2} \Gamma^{\mu_1\mu_2} \epsilon$$

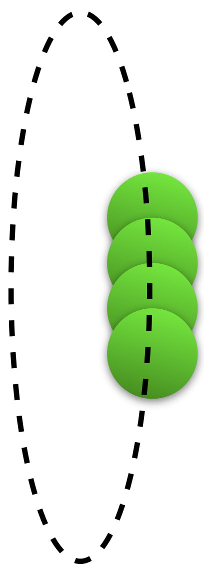
•Supersymmetry together with Bianchi identities and form e.o.m. implies Einstein and Dilaton e.o.m. (under certain, mild assumptions)

$$2[\mathcal{D}_\mu, \mathcal{D}_\nu] \epsilon = (\frac{1}{4} R_{\mu\nu\rho\sigma} \Gamma^{\rho\sigma} + \dots) \epsilon$$

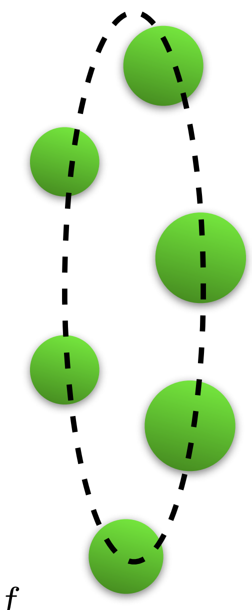
•Modified Bianchi identities and form e.o.m. are the signature of the presence of brane sources.

IIA: Lüst, Tsimpis - IIB: Gauntlett, Martelli, Sparks, Waldram - With sources: Koerber, Tsimpis

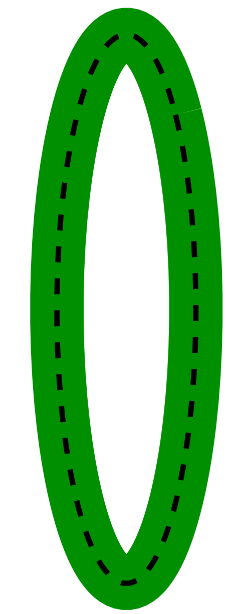
Smearing I



$$\Xi \sim N_f \delta^3(y) \mathrm{d}^3 y$$



$$\Xi \sim \sum_{i=1}^{N_f} \delta^3(y - y_m) \mathrm{d}^3 y$$



$$\Xi \sim N_f \mathrm{vol}_3$$

- Smearing approximates the brane density as continuous.
- Valid for sufficiently large number of branes

$$\frac{N_f}{\mathrm{VOL}_3} \geq \frac{1}{l_s^3}$$

- Smearing restores R-symmetries associated with transverse cycles.

Smearing II - Distribution densities and the smearing form

- Supersymmetry and equations of motion conspire...

$$*\mathrm{d}(e^{-\Phi}\phi_{\mathrm{D6}}) = F \quad \mathrm{d}F = -(2\kappa_{10}^2 T_6)\Xi$$

- ...to fix the most general distribution density, the smearing form.

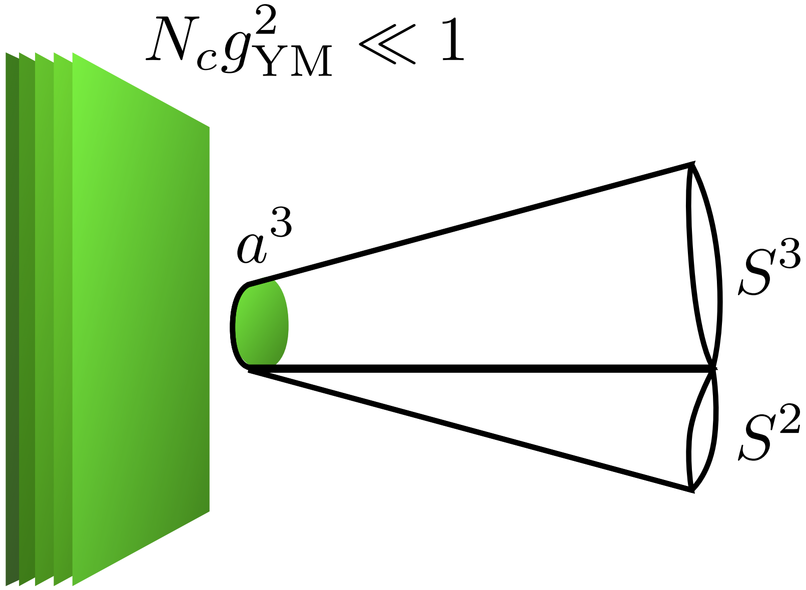
$$\mathrm{d} * \mathrm{d}(e^{-\Phi}\phi_{\mathrm{D6}}) = -(2\kappa_{10}^2 T_6)\Xi$$

- The smearing form depends on the ansatz for the background.

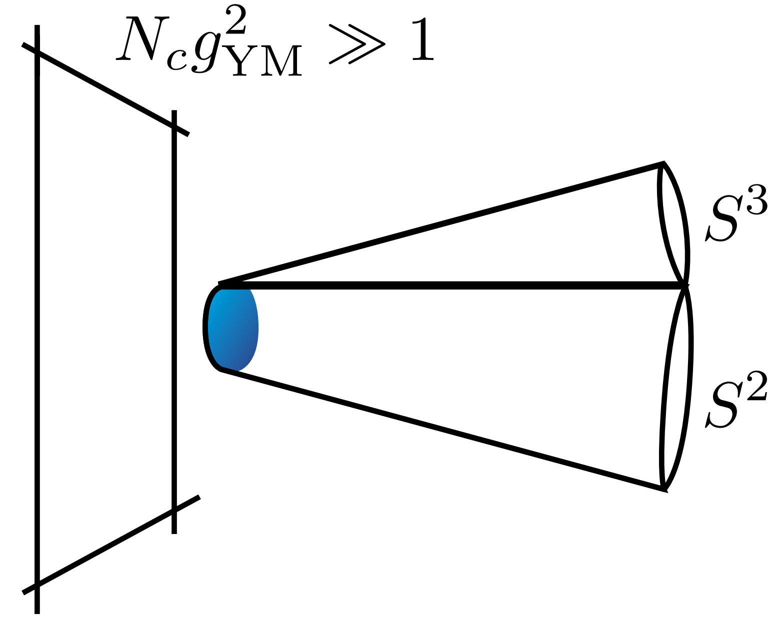
- It often imposes symmetries of the original background onto the additional flavor branes.

- Technically it is not necessary to search for flavor brane embeddings.

An example - D6-branes on the conifold



D6 branes wrapping S^3
Deformed conifold



Flux on S^2
Resolved conifold

$$\mathrm{d}s^2 = e^{\frac{2}{3}\Phi}\{\mathrm{d}x^2_{1,3} + E^2\mathrm{d}\rho^2 + A^2(\sigma_1^2+\sigma_2^2) + C^2[(\Sigma_1 - f\sigma_1)^2 + \Sigma_2 - f\sigma_2)^2]$$
$$+ D^2\sin^2\alpha(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi - \cos\tilde{\theta}\mathrm{d}\tilde{\phi})^2\}$$
$$A^2 = B^2 = \rho^2 \qquad C^2 = D^2 = \frac{\rho^3}{9}(1-\frac{a^3}{\rho^3}) \quad f = g = \frac{1}{2}$$
$$E^2 = 12(1-\frac{a^3}{\rho^3})^{-1} \quad \sin^2\alpha = \frac{B^2}{B^2+(1-g)^2D^2}$$

Gopakumar, Vafa

An example - D6-branes on the conifold

- After a choice of frame

$$\Xi = e^{-5\Phi/3}[\Xi_1 e^{\rho 34} + \Xi_2(e^{\rho 23} + e^{\rho 14}) + \Xi_3 e^{\rho 12} + \Xi_4(e^{135} + e^{245})]$$
$$\Xi_i = \Xi_i[A, B, C, D, E, f, g](\rho)$$

- Subject to

$$\Xi_3 = -\Xi_1 - \frac{2\Xi_2}{\tan\alpha} \quad \Xi_4 = \frac{F_{34}}{2\kappa_{10}^2 T_6 D \sin^2\alpha} \quad \Xi_1 = \Xi_2 \tan\alpha$$

- Color-flux quantization

$$\int_{S^2} F = 2\pi N_c$$

An example - D6-branes on the conifold

- Assuming

$$\Xi_1 = \Xi_2 = \Xi_3 = 0 \qquad \partial_\rho F_{\mu\nu} = 0$$

- We have

$$\Xi = \Xi_4(e^{135} + e^{245})$$

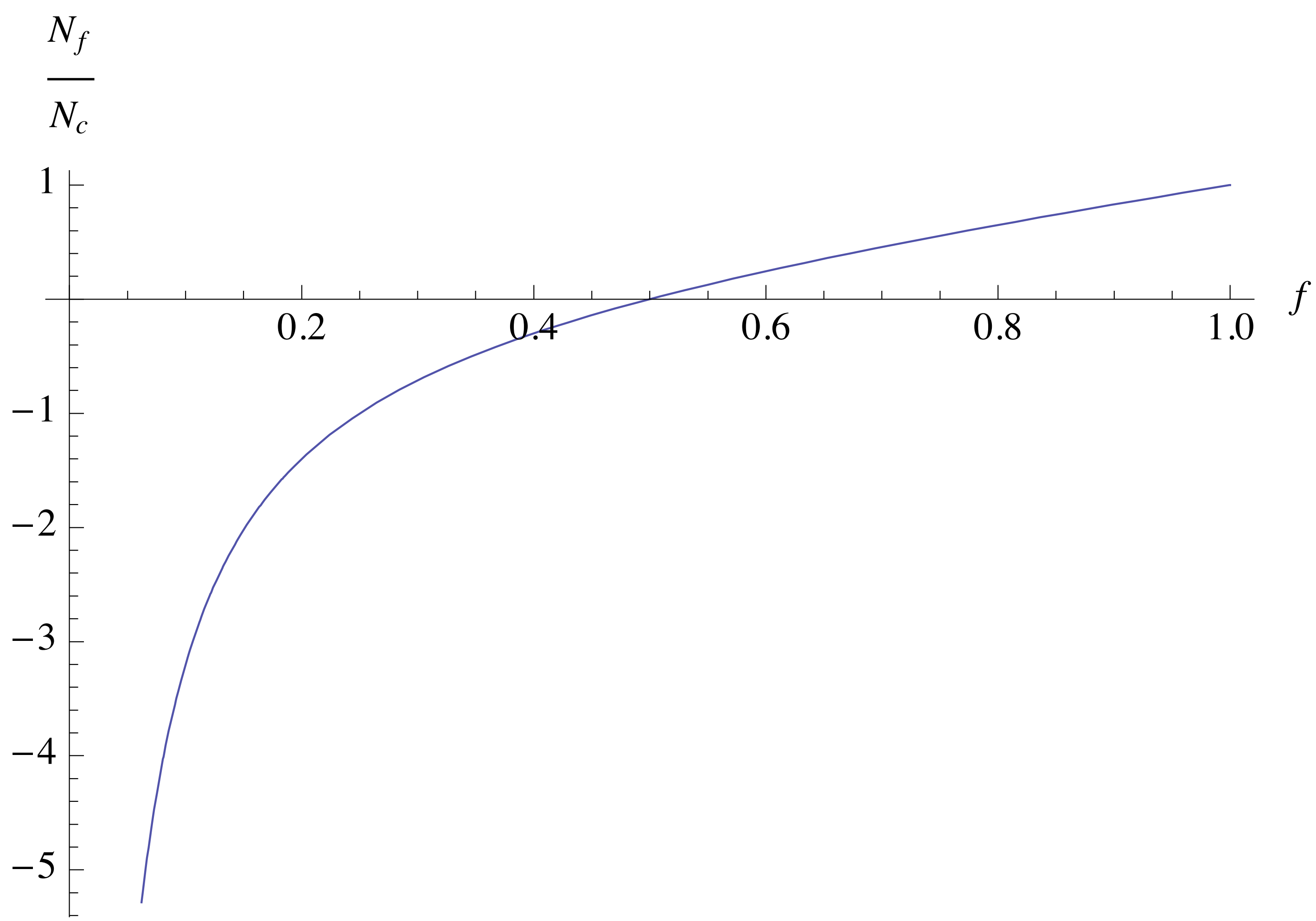
- And can solve the first-order system

$$\begin{aligned}
F &= N_f[\sin\psi(\mathrm{d}\theta \wedge \mathrm{d}\tilde{\theta} + \sin\theta \sin\tilde{\theta}\mathrm{d}\phi \wedge \mathrm{d}\tilde{\phi}) + \cos\psi(\sin\tilde{\theta}\mathrm{d}\theta \wedge \mathrm{d}\tilde{\phi} + \sin\theta\mathrm{d}\tilde{\theta} \wedge \mathrm{d}\phi)] \\
&\quad - N_c(\sin\theta\mathrm{d}\theta \wedge \mathrm{d}\phi + \sin\tilde{\theta}\mathrm{d}\tilde{\theta} \wedge \mathrm{d}\tilde{\phi}) \\
\mathrm{d}s^2 &= e^{2\Phi/3}(\mathrm{d}x_{1,3}^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega_{\mathrm{int}}^2) \\
\mathrm{d}\Omega_{\mathrm{int}}^2 &= \frac{1}{12}(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) + \frac{1}{12(1-f^2)}[(\omega_1 - f\mathrm{d}\theta)^2 + (\omega_2 - f\sin\theta\mathrm{d}\phi)^2] \\
&\quad + \frac{1}{16(1-f^2)}(\omega_3 - \cos\theta\mathrm{d}\phi)^2 \\
\Phi(r) &= \frac{3}{2}\log(\frac{f}{4N_c(1-f^2)}r)
\end{aligned}$$

An example - D6-branes on the conifold

- We find that the additional sources modify the fibration between the spheres

$$N_f = \pm N_c \frac{4f^2 - 1}{3f}$$



$$\begin{array}{lcl} f \to 0 & \Leftrightarrow & \frac{N_f}{N_c} \to -\infty \\ f = \frac{1}{2} & \Leftrightarrow & N_f = 0 \\ f = 1 & \Leftrightarrow & N_f = N_c \end{array}$$

D6-sources in M-theory

M-theory and type IIA supergravity

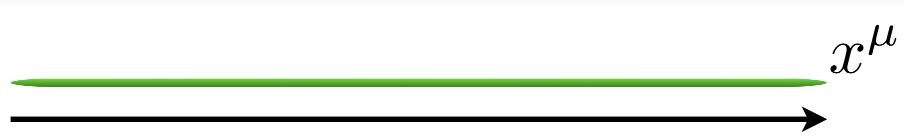
d=11 M-theory - M2, M5

$$S_{\mathrm{M}} = \frac{1}{2\kappa_{11}^2}\int \mathrm{d}^{11}x\sqrt{g}R + \mathcal{O}(G,\Psi)$$

$$\mathrm{d}s_{11}^2 = e^{-\frac{2}{3}\Phi}\mathrm{d}s_{10}^2 + e^{\frac{4}{3}\Phi}(C_\mu \mathrm{d}x^\mu + \mathrm{d}\psi)^2$$

d=10 Type IIA - D0, F1, D2, D4, NS5, D6

$$S_{\mathrm{IIA}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}[e^{-2\Phi}(R + 4\partial_\mu\Phi\partial^\mu\Phi)] - \frac{1}{4}\mathrm{d}C\wedge *\mathrm{d}C + \mathcal{O}(H_{(3)},F_{(4)},\Psi)$$



Kaluza-Klein monopoles

$$ds^2 = \underbrace{dx_{1,6}^2}_{\text{Brane world-volume}} + \overbrace{\underbrace{H}_{\text{Size of M-theory circle}} dy_3^2 + H^{-1}(d\psi + A_i dy^i)^2}^{\text{Euclidean Taub-NUT}}, \quad \nabla \times A = -\nabla H, \quad H = 1 + \sum_{i=1}^{n+1} \frac{2|N_i|}{|r - r_i|}$$

NUT charge

$A \mapsto A + d\Lambda \leftrightarrow \psi \mapsto \psi + \Lambda$

Position of branes

- M-theory circle quenches at D6 positions
- Periodicity related to charge
- Coincident branes give modding by

$$\mathbb{Z}_n \Rightarrow A_{n-1}$$

- In general, D6-branes lift to pure gravity

$$R_{\mu\nu} = 0 \qquad \delta_\epsilon \psi_\mu = D_\mu \epsilon$$

M-theory on G_2 holonomy manifolds

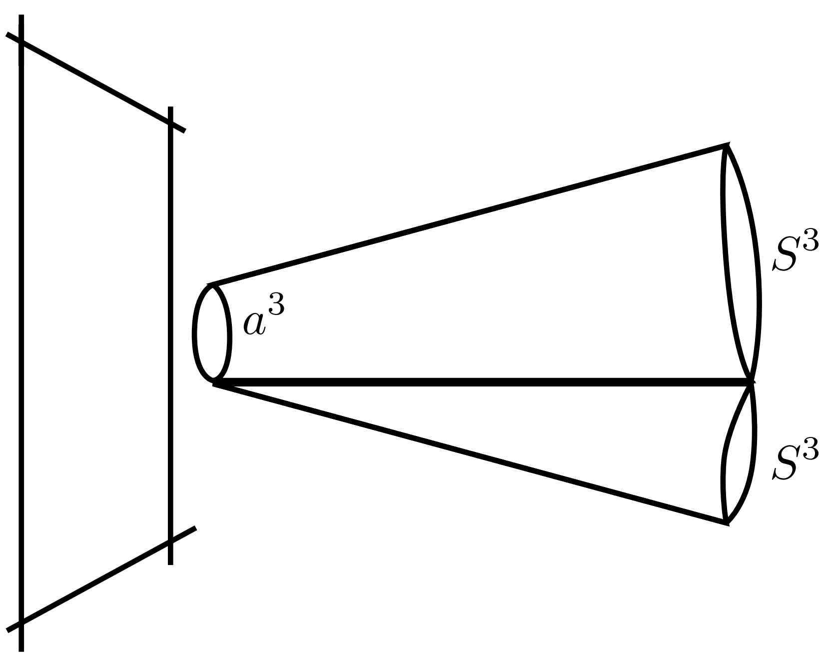
World-volume

$S^3 \geq a^3$

$\tilde{S}^3 \geq 0$

$$ds^2 = dx_{1,3}^2 + \left(1 - \frac{a^3}{\rho^3}\right)^{-1} d\rho^2 + \frac{\rho^2}{12}\tilde{w}^2 + \frac{\rho^2}{9}\left(1 - \frac{a^3}{\rho^3}\right)\left(w - \frac{1}{2}\tilde{w}\right)^2$$

$SU(2)_L \times \widetilde{SU}(2)_L \times SU(2)_D$



- Flop transition
- Cone over $S^3 \times \tilde{S}^3 \sim T^* S^3$
- Holonomy/supersymmetry

$G_2 \Leftrightarrow \mathcal{N} = 1 \qquad \mathbf{1} \oplus \mathbf{7} = \mathbf{8}$

Atiyah, Maldacena, Vafa
Edelstein, Núñez
Brandhuber, Gomis, Gubser, Gukov
Cvetic, Gibbons, Lu, Pope

Arbitrary D6-sources in M-theory

The problem

- Kaluza-Klein reduction works in terms of

$$\mathrm{d}s_{11}^2 = e^{-\frac{2}{3}\Phi}\mathrm{d}s_{10}^2 + e^{\frac{4}{3}\Phi}(C_\mu \mathrm{d}x^\mu + \mathrm{d}\psi)^2 \quad \Rightarrow \quad \mathrm{d}F = \mathrm{d}^2 C = 0$$

- Yet the signature for D6 sources is

$$\mathrm{d}F = -(2\kappa_{10}^2 T_6)\Xi$$

- SUSY transformations and e.o.m.s, when derived from M-theory, are in terms of

$$\mathrm{d}C$$

- We will partially solve this problem at the level of the supergravities.

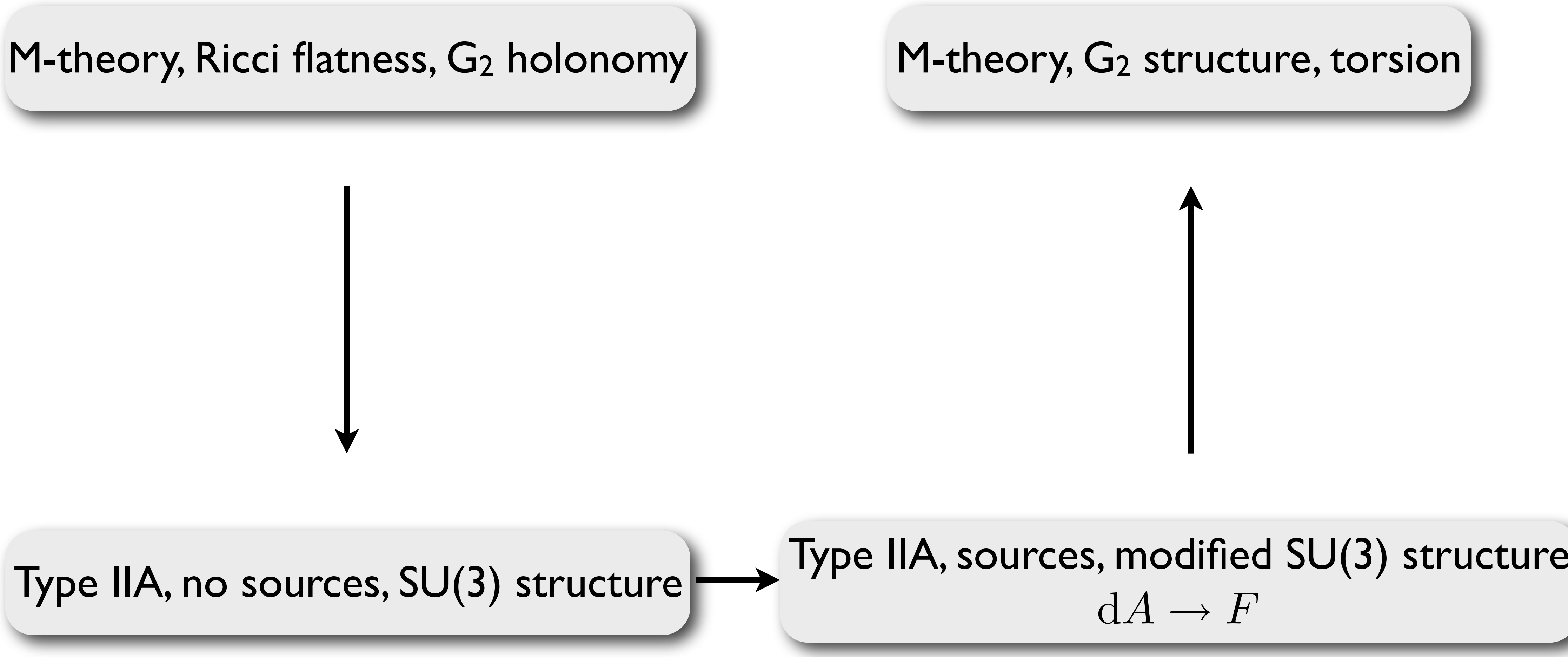
- Possible solutions:
  - Kaluza-Klein vortices, Kaluza-Klein domain walls
  - Additional M-theory fields (Four-form field strength)
  - Kaluza-Klein monopole actions

$$S_{\mathrm{KK7}} = -T_{KK7}\int_{\mathcal{M}_7} \mathrm{d}^7\xi K^2 \sqrt{-\det \partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu}}$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - K^{-1}K_\mu K_\nu$$

Acharya, Witten
Bergshoeff, Jansen, Ortín

The argument



Initial considerations

- Prior to flavoring: Ricci flatness and supersymmetry caused special holonomy

$$\mathrm{d}\phi_{G_2} = \mathrm{d}(e^{\Phi}\Psi) + \mathrm{d}J \wedge (C + \mathrm{d}\psi) + J \wedge \mathrm{d}C = 0$$

$$\mathrm{d} *_7 \phi_{G_2} = -\frac{1}{2}\mathrm{d}(e^{-\frac{4}{3}\Phi} J \wedge J) + \mathrm{d}(e^{-\frac{\Phi}{3}} *_6 \Psi) \wedge (C + \mathrm{d}\psi) - e^{-\frac{\Phi}{3}}(*_6\Psi) \wedge \mathrm{d}C = 0$$

- The SU(3) structure is intimately linked to the presence of calibrated sources

$$\phi_{D6} = -e^{x^0 x^1 x^2 x^3} \wedge \Psi$$

Kaste, Minasian, Petrini, Tomasiello

- Now we expect:
  - Sources carry energy-momentum and cause loss of Ricci flatness
  - Therefore G_2 structure instead of G_2 holonomy

$$\mathrm{d}\phi_{G_2} = -J \wedge (F - \mathrm{d}C)$$

$$\mathrm{d} *_7 \phi_{G_2} = e^{-\frac{\Phi}{3}}(*_6\Psi) \wedge (F - \mathrm{d}C)$$

- We are led to consider geometries with torsion.

Supergravity variations

- SUSY variations in M-theory (with torsion)

$$\delta_{\hat\epsilon}\hat\psi_M = \partial_M\hat\epsilon + \frac{1}{4}\hat\omega_{MAB}\hat\Gamma^{AB}\hat\epsilon + \frac{1}{4}\hat\tau_{MAB}\hat\Gamma^{AB}\hat\epsilon$$

- Reduction gives the desired type IIA equations

$$\delta_\epsilon\psi_\mu = D_\mu\epsilon + \frac{1}{64}e^{\frac{3}{4}\Phi}F_{\mu_1\mu_2}(\Gamma_\mu{}^{\mu_1\mu_2} - 14\delta_\mu^{\mu_1}\Gamma^{\mu_2})\Gamma^{11}\epsilon = \mathcal{D}_\mu\epsilon$$

$$\delta_\epsilon\lambda = \frac{\sqrt{2}}{4}\partial_\mu\Phi\Gamma^\mu\Gamma^{11} + \frac{3}{16}\frac{1}{\sqrt{2}}e^{\frac{3}{4}\Phi}F_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2}\epsilon$$

- If and only if we make a suitable choice for the torsion

$$\hat\tau \sim F - \mathrm{d}C$$

- We obtain a torsion modified covariant derivative

$$\nabla_M^{(\tau)}\phi_{G_2} = 0 \qquad \nabla_M^{(\tau)}(*_7\phi_{G_2}) = 0$$

Integrability and equations of motion

- To find the relevant equations of motion, we reverse the integrability argument.

$$0 = [\nabla_K^{(\tau)}, \nabla_L^{(\tau)}]\phi_{G_2 MNP}$$

- So supersymmetry implies the following second order equation

$$0 = 2R_{KL}^{(\tau)} + R_{MNPL}^{(\tau)}(*_7\phi_{G_2})_K{}^{MNP}$$

The argument

$$
\begin{tikzcd}
\text{M-theory, Ricci flatness, } G_2 \text{ holonomy} \arrow[d] & \text{M-theory, } G_2 \text{ structure, torsion} \\
\text{Type IIA, no sources, SU(3) structure} \arrow[r] & \begin{array}{c}\text{Type IIA, sources, modified SU(3) structure} \\ \mathrm{d}A \to F\end{array} \arrow[u]
\end{tikzcd}
$$

Conclusions

- Calibrated geometry provides a powerful tool in constructing flavored string duals.
- Use of calibrations makes the search for embeddings technically redundant.
- The more formal perspective allows explanations of characteristics of brane embeddings.

- Smeared D6-sources cannot be accommodated for by standard Kaluza-Klein mechanism.
- Issue can be resolve by adding torsion terms to eleven-dimensional supergravity.
- Present explanation is not fully general. It relies on supersymmetry and topology.
- Possible implications for monopole condensation.

- There is need to further study the physics of flavored solutions.
- Up to now that is mainly done on a case-by-case basis.
- Calibrated flavors may provide unifying formalism to do so.

Thank you for your attention.

Additional material

The modified SU(3) structure

$$
\begin{aligned}
0 &= \mathrm{d}J \\
0 &= \mathrm{d}(e^{-\Phi/3} *_6 \Psi) \\
0 &= \mathrm{d}(e^{\Phi}\Psi) + J \wedge \mathrm{d}C \\
0 &= -\frac{1}{2}\mathrm{d}(e^{-4\Phi/3} J \wedge J) - e^{-\Phi/3}(*_6\Psi) \wedge \mathrm{d}C
\end{aligned}
$$

$$
\begin{aligned}
0 &= \mathrm{d}J \\
0 &= \mathrm{d}(e^{-\Phi/3} *_6 \Psi) \\
0 &= \mathrm{d}(e^{\Phi}\Psi) + J \wedge F \\
0 &= -\frac{1}{2}\mathrm{d}(e^{-4\Phi/3} J \wedge J) - e^{-\Phi/3}(*_6\Psi) \wedge F
\end{aligned}
$$

Torsion

$$\delta_{\hat{\epsilon}}\hat{\psi}_M = \partial_M\hat{\epsilon} + \frac{1}{4}\hat{\omega}_{MAB}\hat{\Gamma}^{AB}\hat{\epsilon} + \frac{1}{4}\hat{\tau}_{MAB}\hat{\Gamma}^{AB}\hat{\epsilon}$$

$$\hat{\tau}_{\psi a\psi} = 0 \qquad \hat{\tau}_{\psi bc} = \frac{e^{\frac{3}{2}\Phi}}{2T_6\kappa_{10}^2}(F-\mathrm{d}C)_{bc}$$

$$\hat{\tau}_{\mu bc} = \frac{e^{\frac{3}{2}\Phi}}{2T_6\kappa_{10}^2}C_\mu(F-\mathrm{d}C)_{bc} \qquad \hat{\tau}_{\mu bc} = -\frac{e^{\frac{3}{4}\Phi}}{2T_6\kappa_{10}^2}(F-\mathrm{d}C)_{\mu b}$$

$$\delta_\epsilon\psi_\mu = D_\mu\epsilon + \frac{1}{64}e^{\frac{3}{4}\Phi}F_{\mu_1\mu_2}({\Gamma_\mu}^{\mu_1\mu_2} - 14\delta_\mu^{\mu_1}\Gamma^{\mu_2})\Gamma^{11}\epsilon = \mathcal{D}_\mu\epsilon$$

$$\delta_\epsilon\lambda = \frac{\sqrt{2}}{4}\partial_\mu\Phi\Gamma^\mu\Gamma^{11} + \frac{3}{16}\frac{1}{\sqrt{2}}e^{\frac{3}{4}\Phi}F_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2}\epsilon$$

The Wilson loop

- Graphic shows the quark-antiquark potential calculated with the "standard methods".
- There is actually need for a cutoff-brane as the length of the string is otherwise divergent.