PLAYING WITH QUANTUM MODES OF LIGHT

AN INSIGHT INTO HIGHLY MULTIMODE QUANTUM PHYSICS

C. Fabre

IRCOQ

ACQAO

Laboratoire Kastler Brossel
Physique quantique et applications

HiDEaS

UPMC PARIS UNIVERSITAS

IUF

INTRODUCTION

WHAT IS A LIGHT MODE ?

INTRINSIC PROPERTIES OF LIGHT STATES

« SUPERMODES »

TAILORING HAMILTONIANS

MODES AND MEASUREMENTS

MEASUREMENTS IN IMAGES

MEASUREMENTS WITH FREQUENCY COMBS

CONCLUSION

Quantum computing now: a few qubits

useful quantum computing in the future:

many qubits

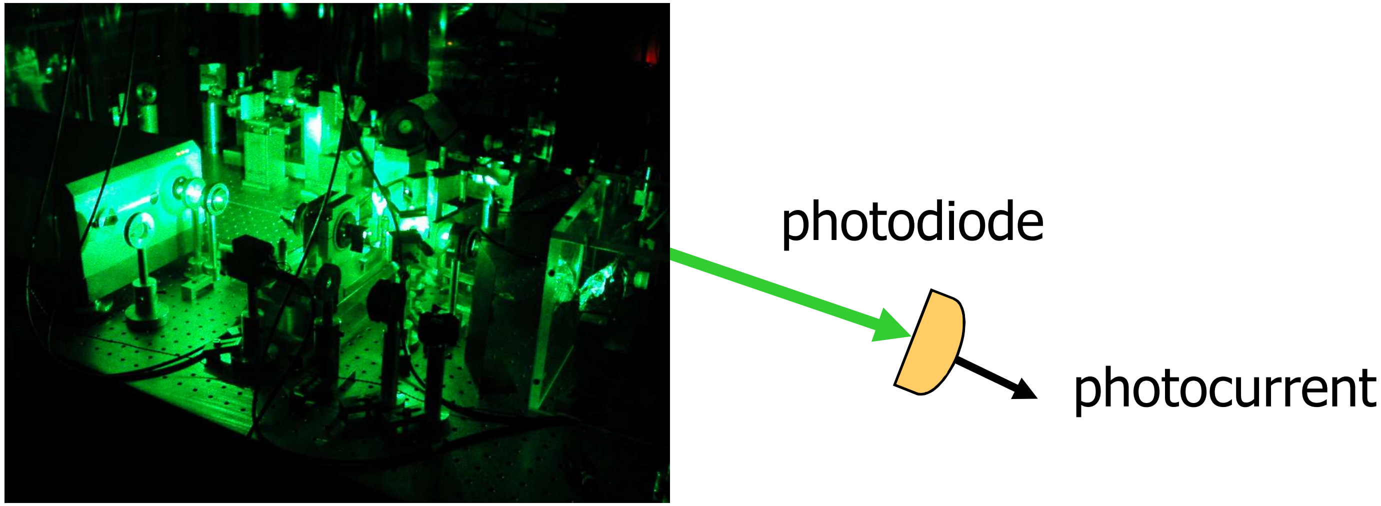
7 qubits to factorize 15 !

Necessity of investigations on quantum properties of systems with many degrees of freedom

This talk : multimode quantum states of **light**

Quantum light 1

continuous variable regime



single mode, many photons

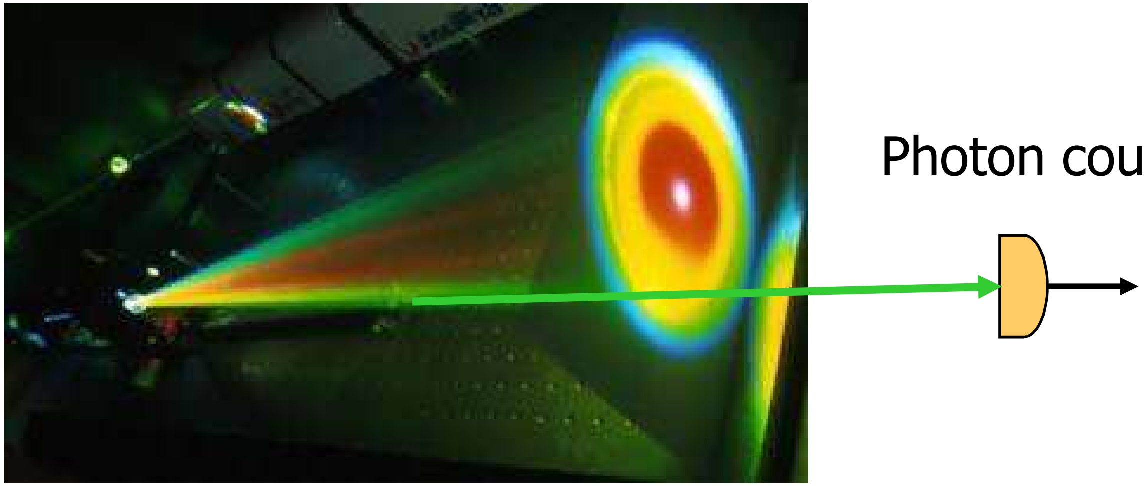
$$|\Psi\rangle = \sum_n c_n |n : \ell = \ell_0\rangle$$

Photon number

mode label

Quantum light 2

photon counting regime



single photon, many modes

$$\left|\Psi\right\rangle = \sum_{\ell} c_{\ell} \left| n = 1 : \ell \right\rangle$$

Photon number

mode label

Multimode quantum light

$$\left|\Psi\right\rangle = \sum_{n_1} .. \sum_{n_\ell} .. c_{n_1,..,n_\ell,..} \left| n_1 : 1,.., n_\ell : \ell,.. \right\rangle$$

many photons, many modes …

…and an enormous amount of entanglement !

INTRODUCTION

WHAT IS A LIGHT MODE ?

INTRINSIC PROPERTIES OF LIGHT STATES

« SUPERMODES »

TAILORING HAMILTONIANS

MODES AND MEASUREMENTS

MEASUREMENTS IN IMAGES

MEASUREMENTS WITH FREQUENCY COMBS

CONCLUSION

Positive electric field operator:

$$\hat{\vec{E}}^{(+)}(\vec{r},t) = \sum_{\ell} \mathcal{E}_{\ell} \hat{a}_{\ell} \vec{u}_{\ell}(\vec{r},t)$$

Scaling factor

Annihilation operator

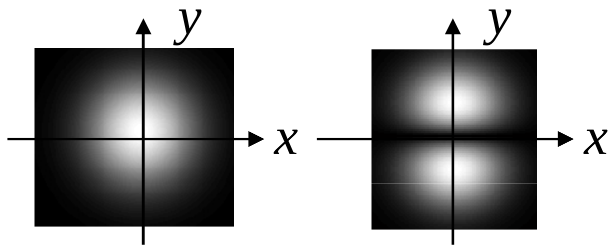
Mode amplitude

$\{\vec{u}_{\ell}\}$ Orthonormal and complete set of solutions of Maxwell equations

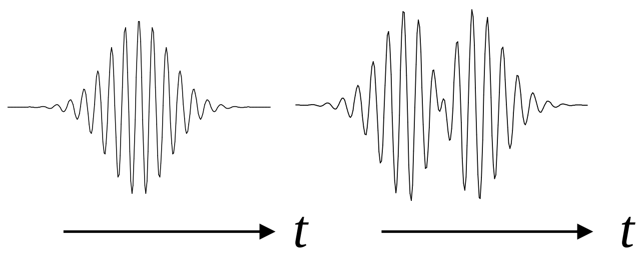
Example of modes

$$\vec{u}_\ell(\vec{r},t)=\frac{1}{\sqrt{V}}\vec{\varepsilon}_\ell\; e^{i\left(\vec{k}_\ell\cdot\vec{r}-\omega_\ell t\right)}$$ Travelling plane wave

Spatial Hermite-Gauss modes



Temporal Hermite-Gauss modes



.....

Freedom of choice of modal basis

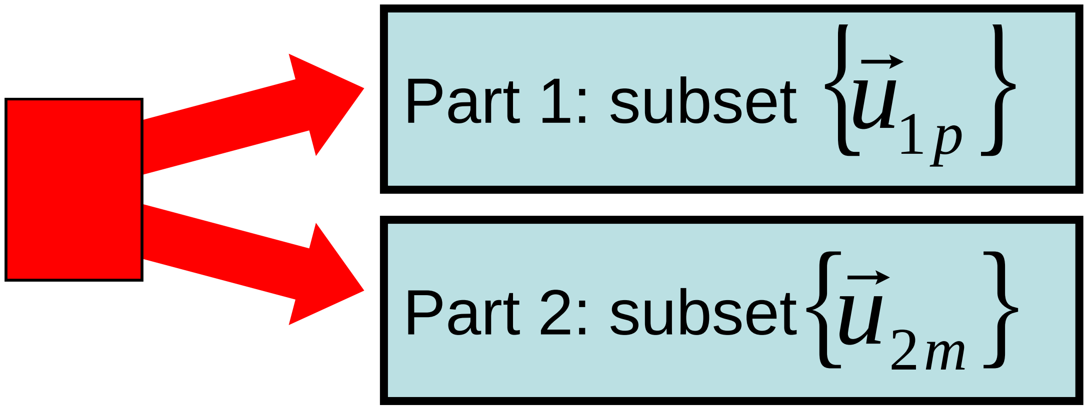
The same quantum state $|\Psi\rangle$

may have quite different forms

when expressed in different modal bases

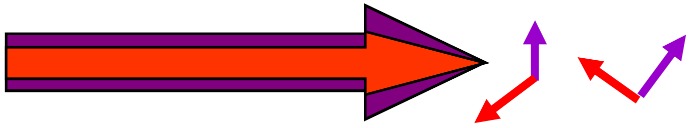
In this talk : **total freedom** in change of mode basis

A related problem: bipartite entanglement



Only
« local operations »
allowed

ex1: polarization modes



$$|\Psi\rangle = |\psi_x^{squeezed}\rangle \otimes |\varphi_y^{squeezed}\rangle$$

Separable state in the basis of Ox,Oy polarizations

$$|\Psi\rangle = \alpha|\varphi_{45}\rangle \otimes |\psi_{-45}\rangle + \beta|\chi_{45}\rangle \otimes |\xi_{-45}\rangle$$

Entangled state on the basis of $O_{+45}O_{-45}$ polarizations

ex2: multimode single photon state

$$\left|\Psi\right\rangle = \sum_{\ell} c_{\ell} \left| n = 1 : \ell \right\rangle$$

One defines $\vec{v}_1(\vec{r},t) = \sum_{\ell} c_{\ell} \vec{u}_{\ell}(\vec{r},t)$

Completed basis: $\{\vec{v}_j\}$

$$\left|\Psi\right\rangle = \left|1 : j = 1\right\rangle \otimes \left|0 : j \neq 1\right\rangle$$

Multimode on one basis, single mode in another

ex3: multimode coherent state

$$\left|\Psi\right\rangle = \left|\alpha_0 : 0, \alpha_1 : 1, \ldots, \ \alpha_\ell : \ell, \ldots\right\rangle$$

One defines $\vec{w}_1(\vec{r},t) = \frac{1}{\beta} \sum_\ell \alpha_\ell \vec{u}_\ell(\vec{r},t)$

with: $|\beta|^2 = \sum_\ell |\alpha_\ell|^2$

Completed basis: $\{\vec{w}_k\}$

$$\left|\Psi\right\rangle = \left|\beta : k=1\right\rangle \otimes \left|0 : k \neq 1\right\rangle$$

Multimode on one basis, single mode in another

Invariants
with change of modal bases

- The vacuum state $|0\rangle = |0,..,0,..\rangle$

- The total number of photons

$$\hat{N} = \sum_{\ell} \hat{a}_{\ell}^{+} \hat{a}_{\ell} = \sum_{j} \hat{b}_{j}^{+} \hat{b}_{j}$$

Definition of an intrinsic single mode state

For an **intrinsic single mode state**,

there exists a mode basis $\{\vec{u}_i(\vec{r})\}$

where the quantum state is written as:

$$\left|\Psi\right\rangle = \left|\varphi : \vec{u}_1\right\rangle \otimes \left|0,\ldots 0,0,\ldots\right\rangle$$

$$\rho = \left(\sigma : \vec{u}_1\right) \otimes \left|0,\ldots 0,0,\ldots\right\rangle\left\langle 0,\ldots, 0,\ldots\right|$$

For an **intrinsic multimode state**,
there is no such basis

Criterion for a single mode state

N. Treps, V. Delaubert, A. Maître, J.M. Courty, C. Fabre Phys. Rev A**71** 013820 (2005)

$\forall \ell \quad \hat{a}_\ell \left| \Psi \right\rangle$ (or $\hat{a}_\ell \rho$) are proportional

$$\Leftrightarrow \qquad \left| g^{(1)}(\vec{r},\vec{r}',t,t') \right| = 1$$

where

$$g^{(1)}(\vec{r},\vec{r}',t,t') = \frac{\left\langle \hat{E}^{(-)}(\vec{r},t)\hat{E}^{(+)}(\vec{r}',t') \right\rangle}{\sqrt{\left\langle \hat{E}^{(-)}(\vec{r},t)\hat{E}^{(+)}(\vec{r},t) \right\rangle \left\langle \hat{E}^{(-)}(\vec{r}',t')\hat{E}^{(+)}(\vec{r}',t') \right\rangle}}$$

Whatever the light state, whatever the mode shape,
one always has perfect first order coherence

Intrinsic number of modes

dimension of space spanned by $\{\hat{a}_\ell |\Psi\rangle\}$ $\{\hat{a}_\ell \rho\}$

Example $|\Psi\rangle = |1,1\rangle$ is an intrinsic two-mode state

- cannot be written as $|2,0\rangle$

-will not produce perfect first order interference fringes

How to count intrinsic modes ?

Difficult task experimentally

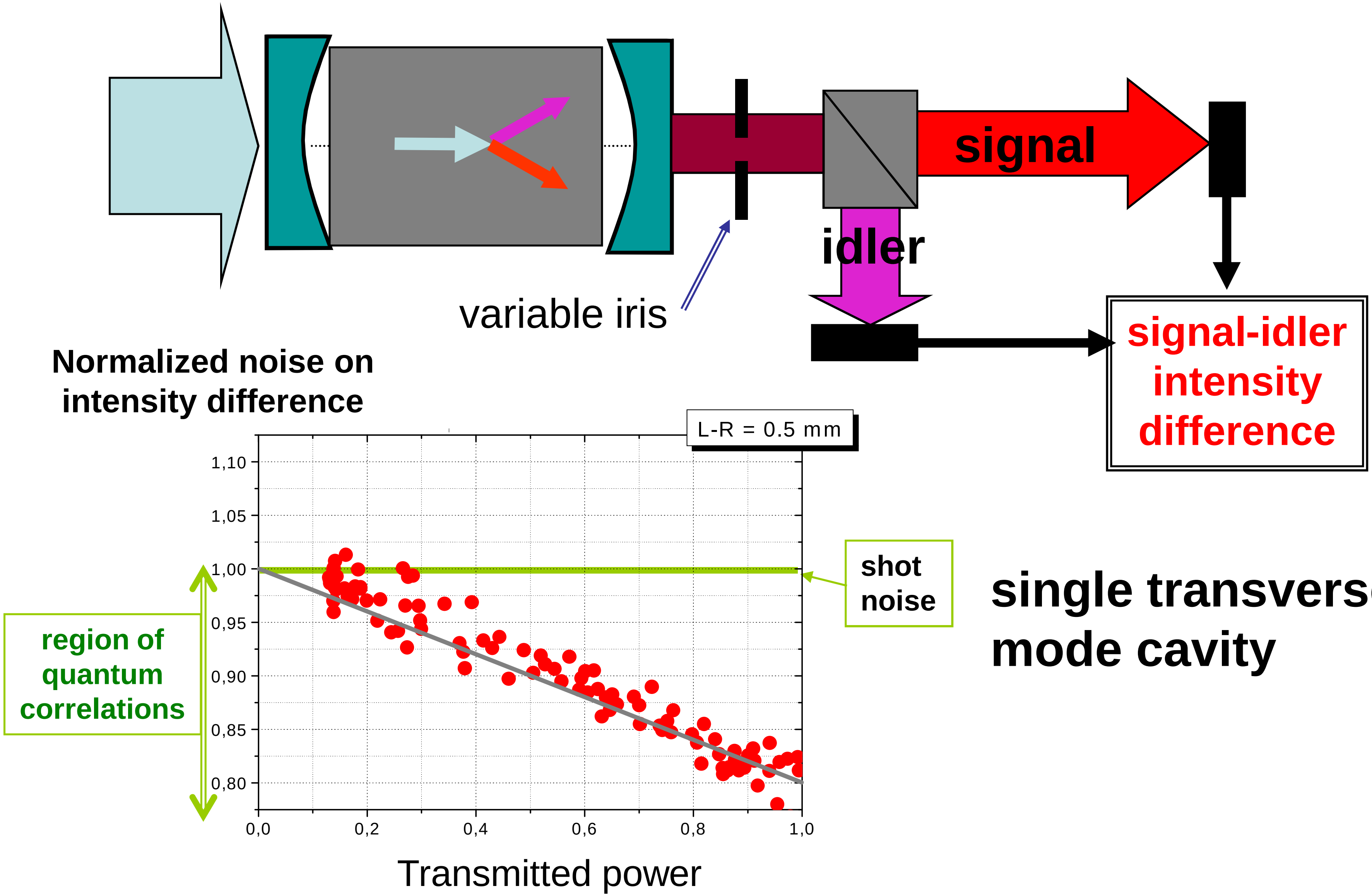
A sufficient criterion for a non-single mode state:

In a single mode field, all the observables
have the same spatio-temporal variation

In a non-single mode field
the noise and the mean have **not** the same variation

Non single mode non-classical light

M. Martinelli, et al Phys. Rev. ***A67****, 023808 (2003)*

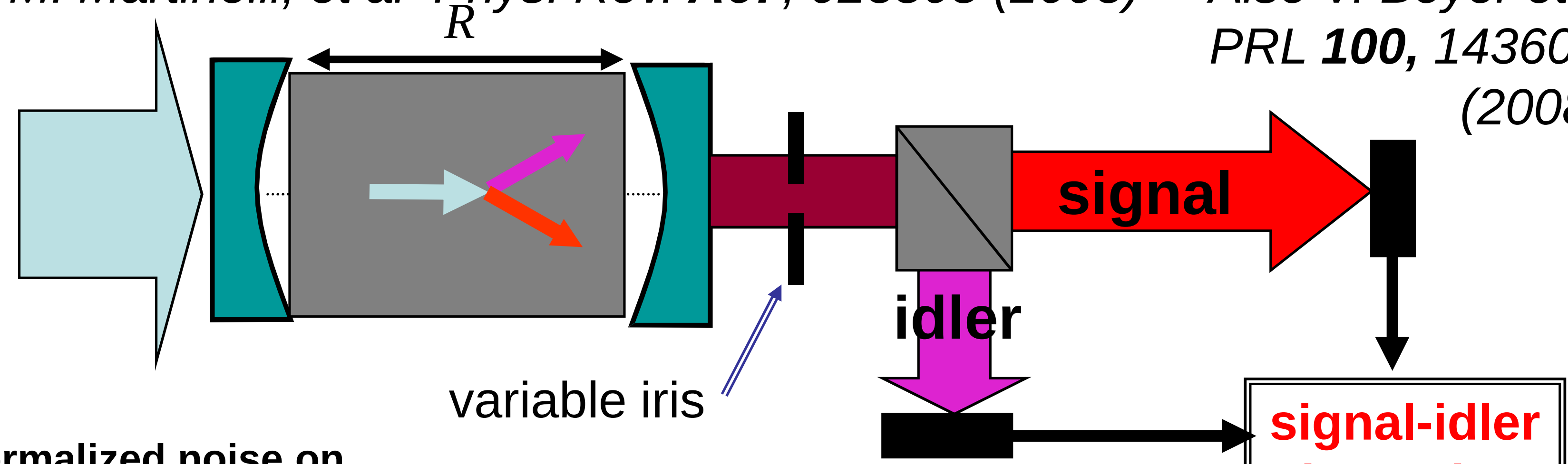


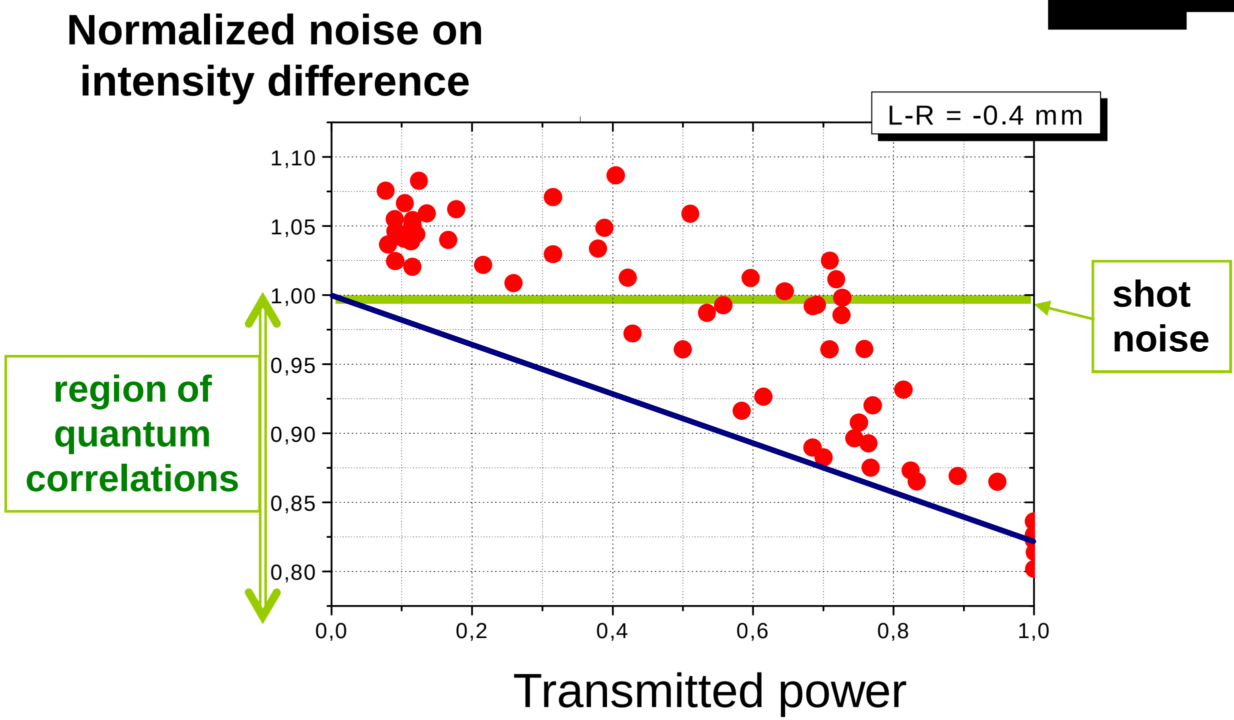
single transverse mode cavity

Non single mode non-classical light

M. Martinelli, et al Phys. Rev. ***A67****, 023808 (2003)*

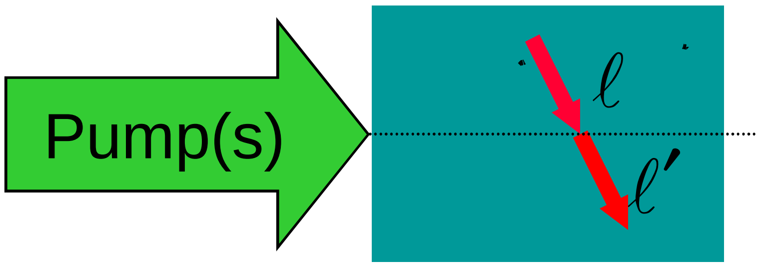
Also V. Boyer et al PRL ***100,*** *143601 (2008)*





confocal cavity

Twin photons



$$\hat{H} = \sum_{\ell,\ell'} \left(G_{\ell,\ell'} \hat{a}_\ell \hat{a}_{\ell'} + G^*_{\ell,\ell'} \hat{a}_\ell^+ \hat{a}_{\ell'}^+ \right)$$

Diagonalization of matrix G:

$$\hat{H} = \sum_{m=1}^{N_m} \left(\Lambda_m \left(\hat{b}_m\right)^2 + \Lambda^*_m \left(\hat{b}_m^+\right)^2 \right)$$

$$\hat{b}_m = \sum_\ell U_m^\ell \hat{a}_\ell$$

$\hat{b}_m^+ |0\rangle = |1 : m\rangle$ photon in « supermode » $\vec{w}_m = \sum_\ell U_m^\ell \vec{u}_\ell$

Generated quantum state

$$\hat{H} = \sum_{m=1}^{N_m} \left(\Lambda_m \left(\hat{b}_m\right)^2 + \Lambda^*{}_m \left(\hat{b}_m^+\right)^2 \right)$$

$|\Psi\rangle$ tensor product of squeezed states

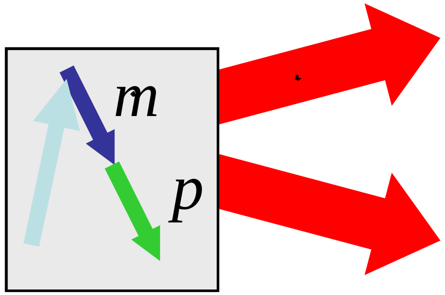
Intrinsic number of modes :

= dimension of space spanned by $\left\{ \hat{b}_m \,|\Psi\rangle \right\}$

= number N_m of non zero Λ_m

Link with Schmidt modes

Case of the bipartite system



Part 1: modes $\left\{\vec{u}_{1p}\right\}$

Part 2: modes $\left\{\vec{u}_{2m}\right\}$

$$\hat{H} = \sum_{m,p}\left(G_{m,p}\hat{a}_{1m}\hat{a}_{2p} + G^{*}{}_{m,p}\hat{a}_{1m}^{+}\hat{a}_{2p}^{+}\right)$$

Bloch-Messiah decomposition

Parker et al PRA **61**, 032305 (2000) Law Eberly, PRL **92**, 127903 (2004)

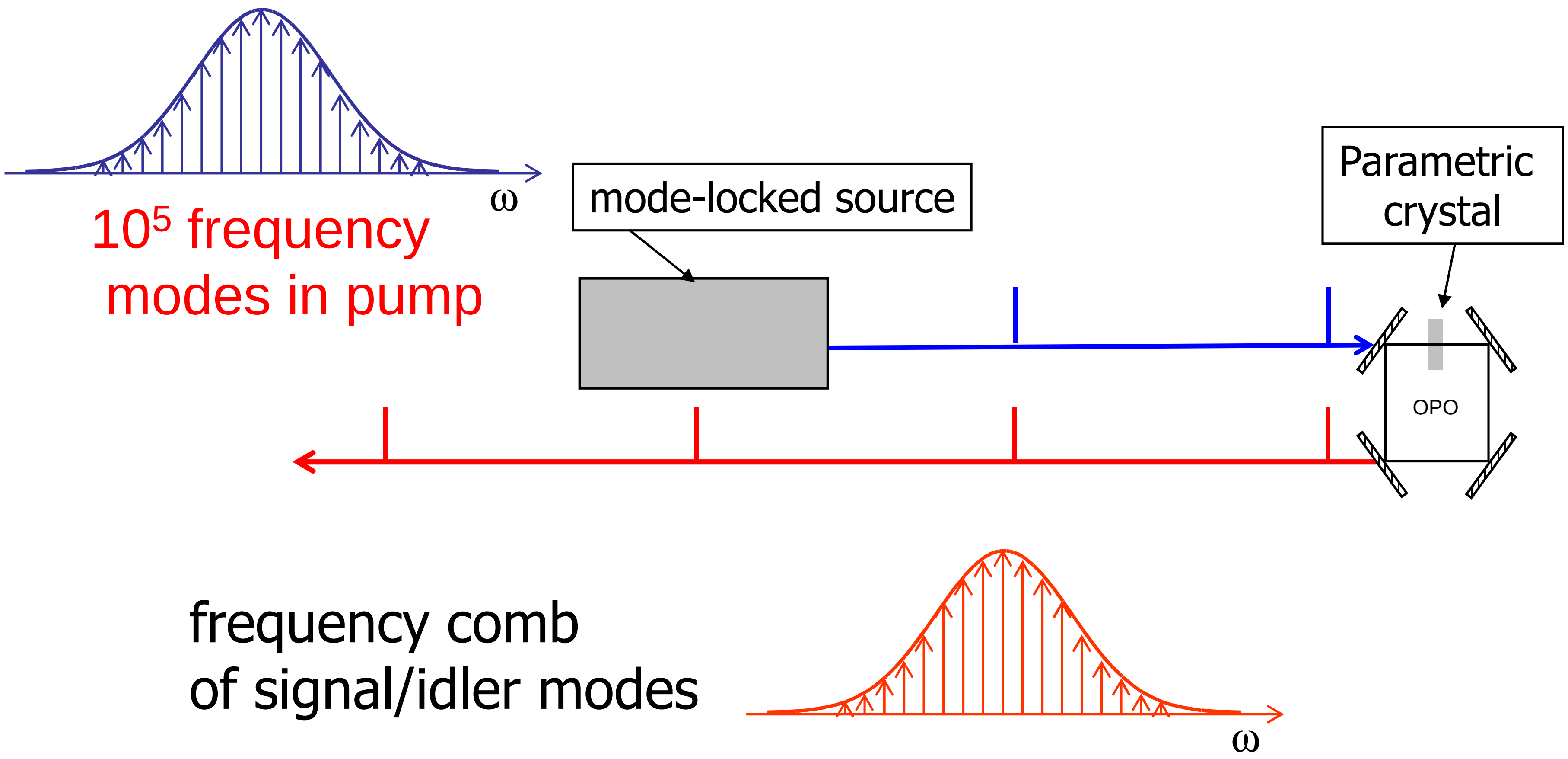
$$\hat{H} = C\sum_{i}\lambda_i\left(\hat{b}_{1i}\hat{b}_{2i} + H.c.\right)$$

Schmidt modes defined in each part

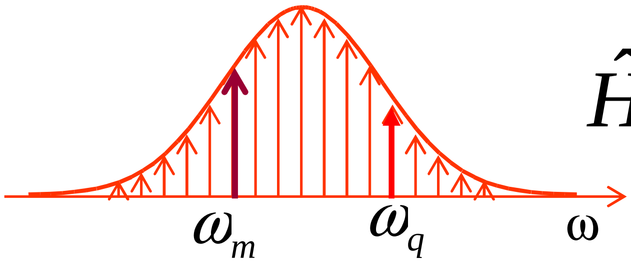
Degree of entanglement : Schmidt number

Example: the « SPOPO »

Synchronously Pumped type I Optical Parametric Oscillator



Hamiltonian of the system



$$\hat{H} = \sum_{m,q} \left(G_{m,q} \hat{a}_{\omega_m} \hat{a}_{\omega_q} + G^{*}{}_{m,q} \hat{a}^{+}_{\omega_m} \hat{a}^{+}_{\omega_q} \right)$$

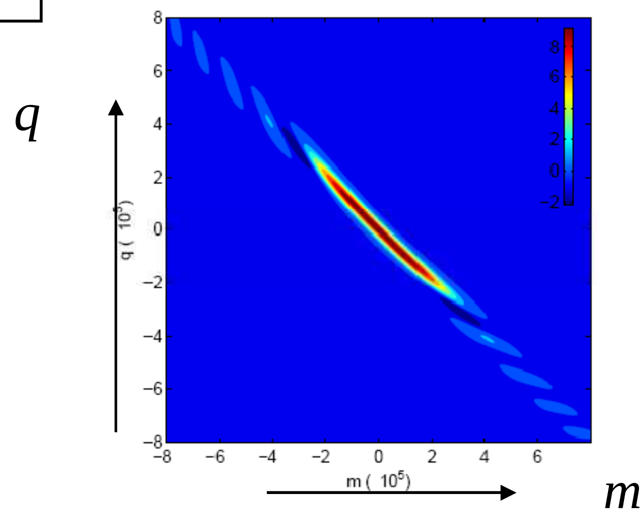
$$G_{mq} = \chi\left(\omega_m, \omega_q\right) \; \alpha_{pump}\left(\omega_m + \omega_q\right)$$

Crystal phase matching coefficient

pump spectral amplitude

G_{mq}

($10^5 \times 10^5$ matrix)



INTRODUCTION

WHAT IS A LIGHT MODE ?

INTRINSIC PROPERTIES OF LIGHT STATES

« SUPERMODES »

TAILORING HAMILTONIANS

MODES AND MEASUREMENTS

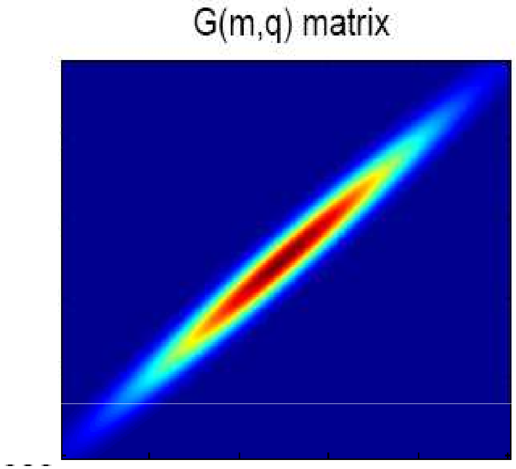
MEASUREMENTS IN IMAGES

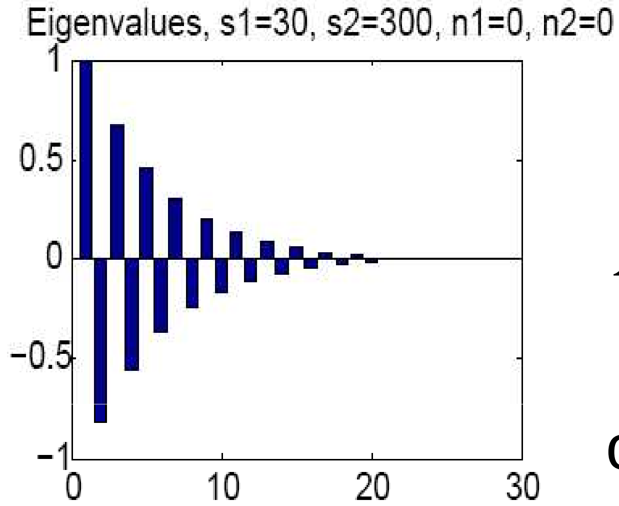
MEASUREMENTS WITH FREQUENCY COMBS

CONCLUSION

Eigenmodes, Gaussian case

Gaussian variation of G_{mq} : analytical solution





$$\Lambda_k = \Lambda_0 (-r)^k$$

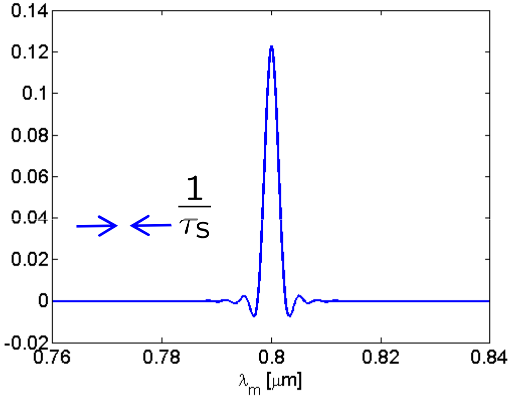
out of 10^5 ,
only roughly 30
non zero eigenvalues

Hermite-Gauss
supermodes

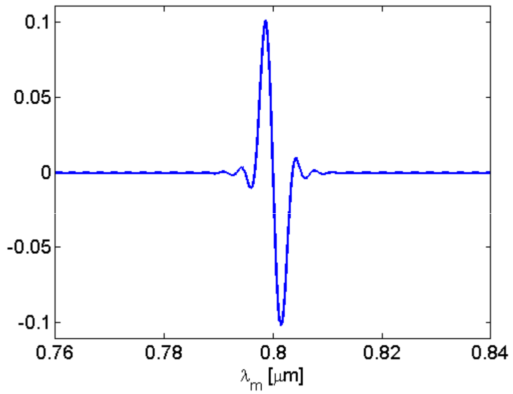
Eigenmodes, non-Gaussian case

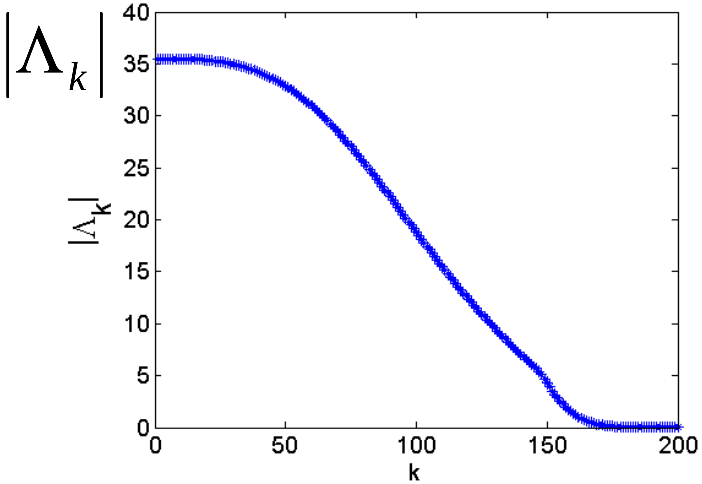
Degenerate collinear type I phase-matched BIBO
at 0.4µm with t_p=100fs and l=5mm (long crystal)

supermode 1



supermode 2

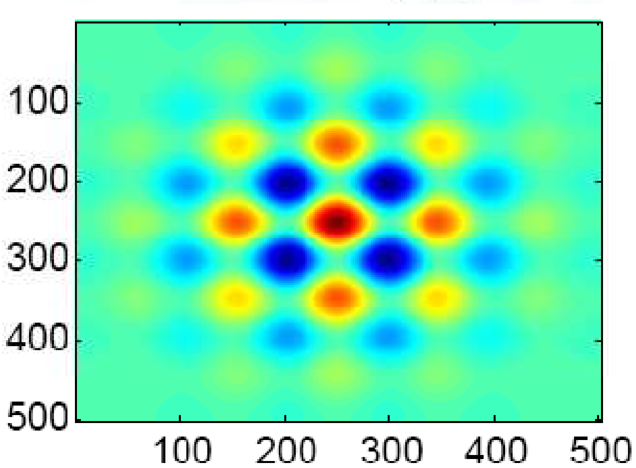




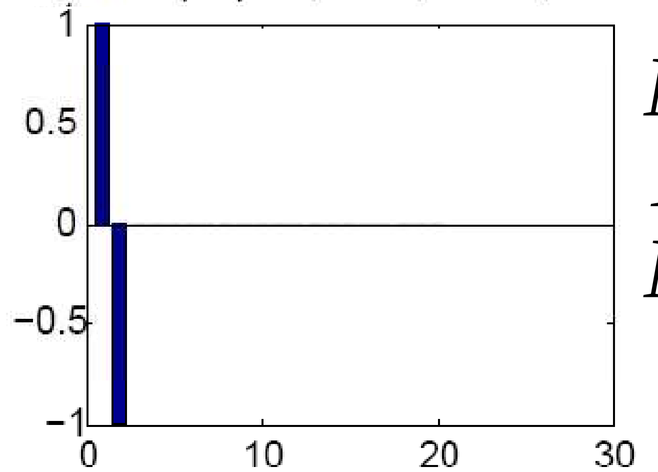
150 non zero eigenvalues

Tailoring supermodes and Hamiltonians

By changing pump and/or nonlinear medium shape
It is possible to taylor at will the number and the spectrum of eigenvalues



G_{mq} matrix



eigenvalues Λ_k

$$N_m = 2$$
$$N_m = 3$$
$$N_m = 4$$

$$H = \hbar g \left(\hat{b}_1^2 - \frac{1}{2}\hat{b}_2^2 + \frac{1}{2}\left(\hat{b}_3^2 - \hat{b}_4^2 \right) \right) + h.c.$$

What about entanglement ?

Any mixing of n up to N_S squeezed supermodes yields entanglement

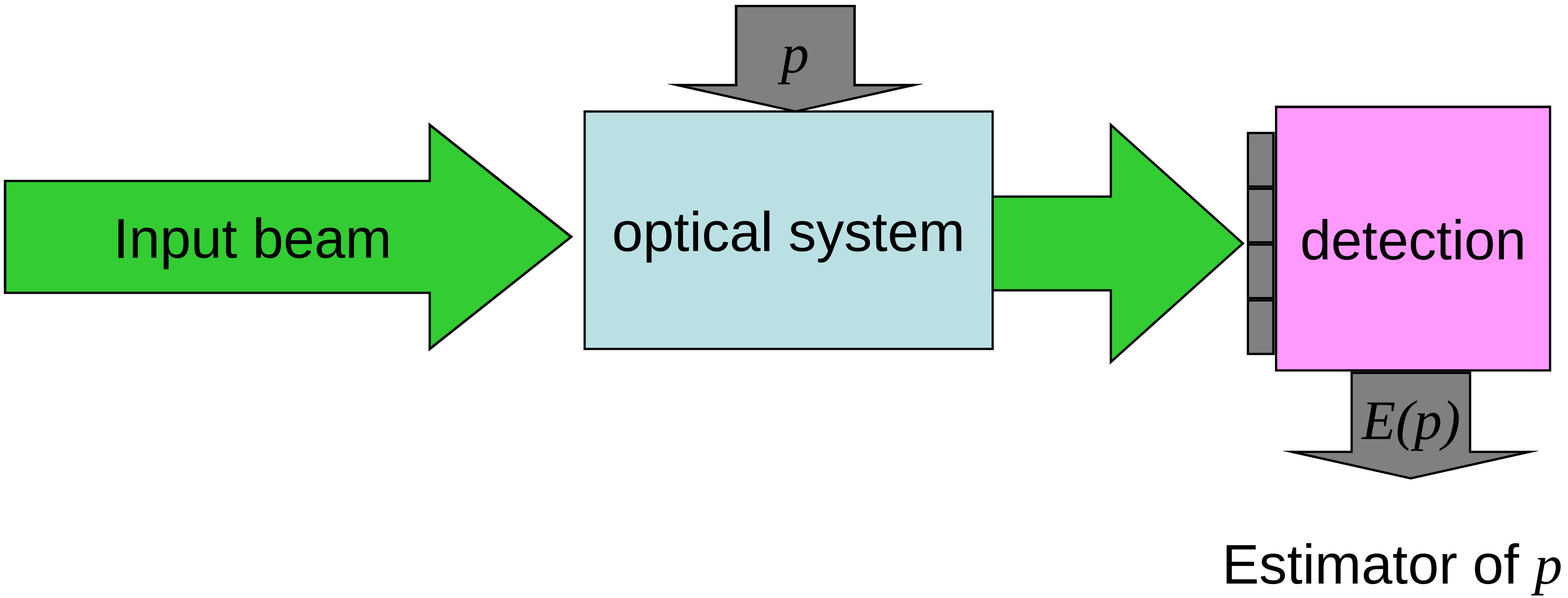
Mixed modes $\vec{u}_+ = \sum_i \alpha_i \vec{u}_1$ and $\vec{u}_- = \sum_i (\operatorname{sgn} \Lambda_i) \alpha_i \vec{u}_1$

are EPR entangled

Yields also multipartite entanglement

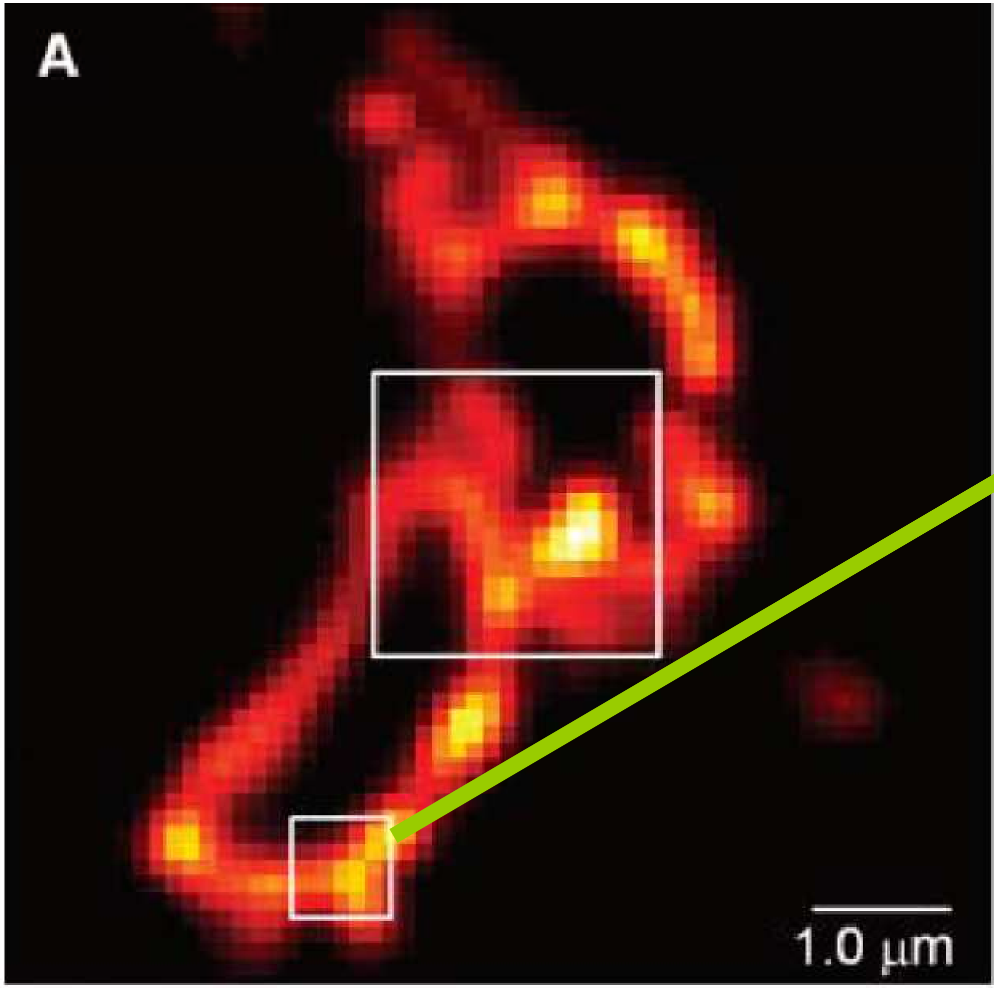
Another way to produce CV cluster states ?

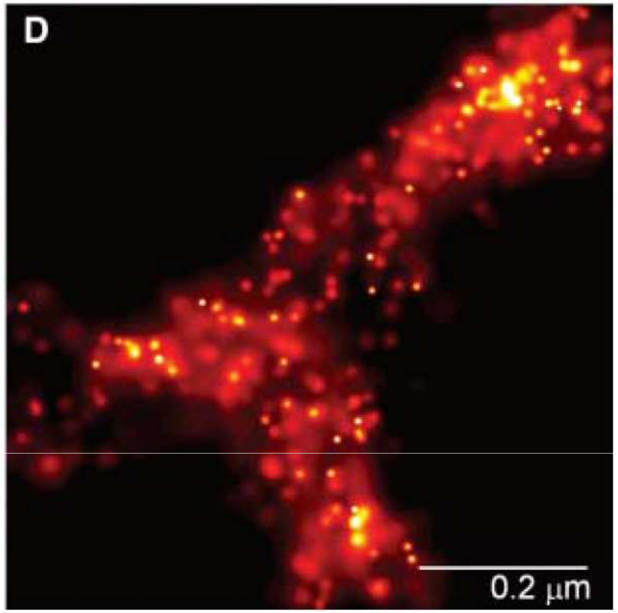
generic optical measurement



An example: super-resolution

Image of fluorescent proteins by
conventional high resolution microscopy

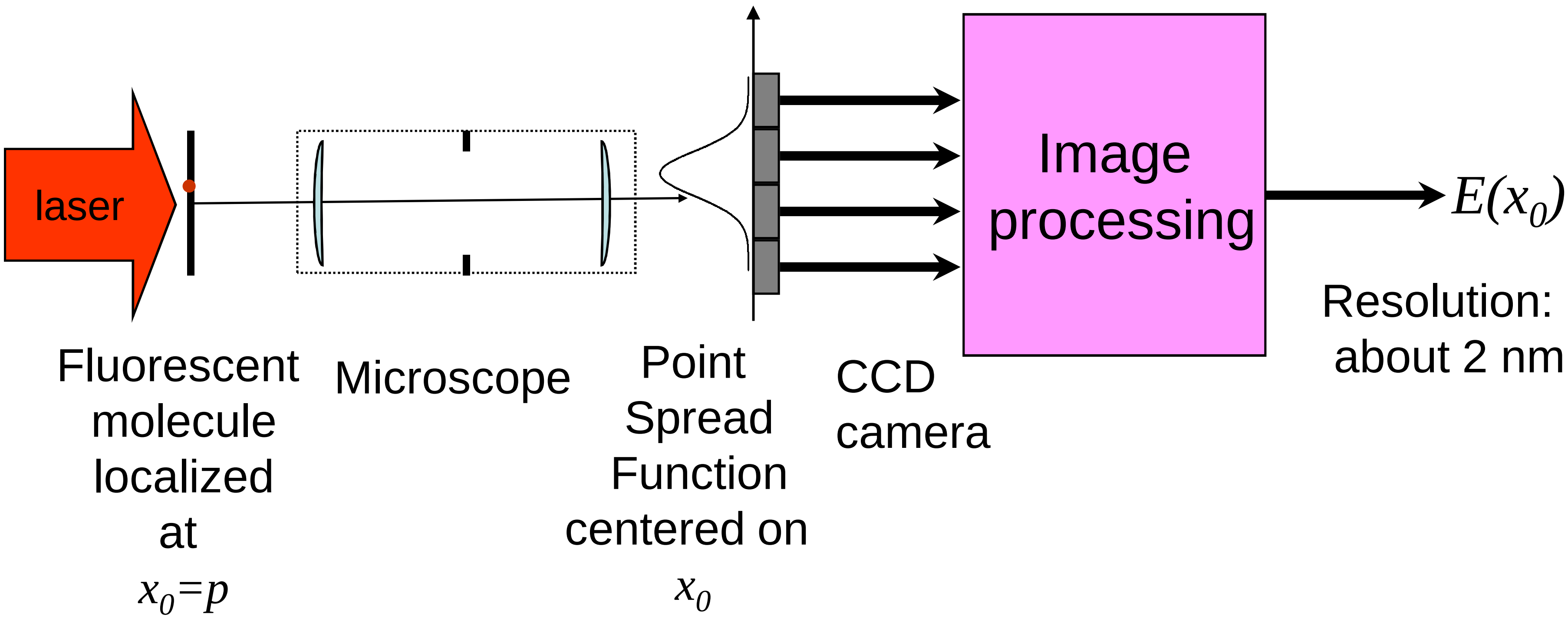




Super-resolution image

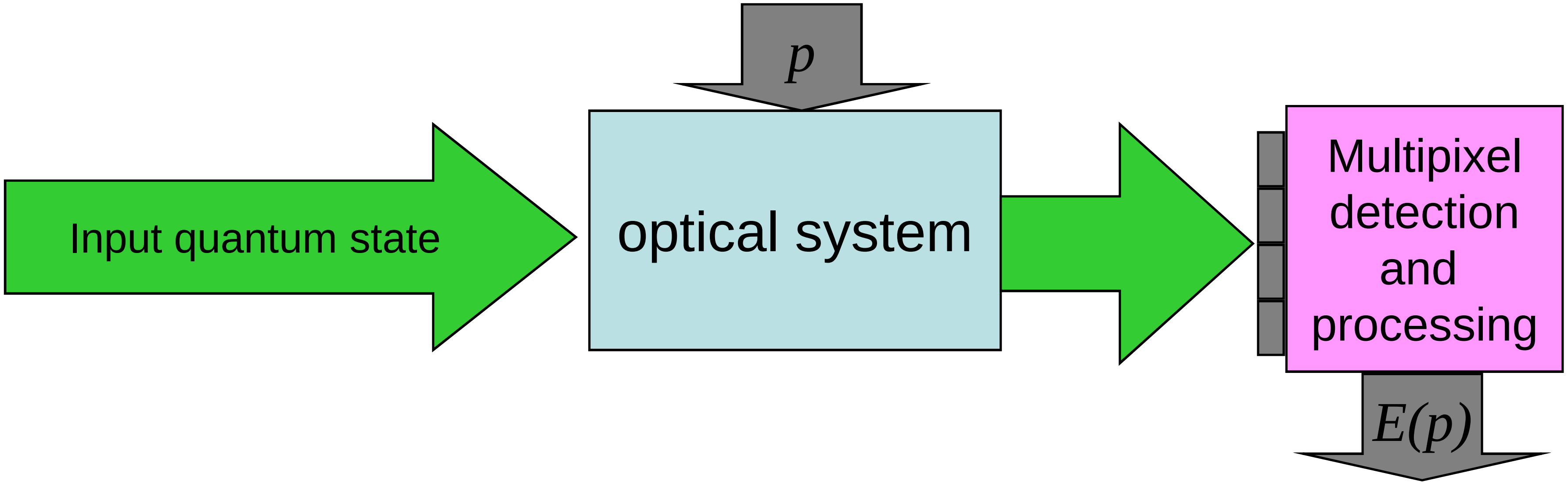
Imaging Intracellular Fluorescent
Proteins at Nanometer Resolution
E. Betzig et al Science **313** 1642 (2006)

Measurement strategy



Relevant information is distributed over many pixels

Best possible accuracy on p ?



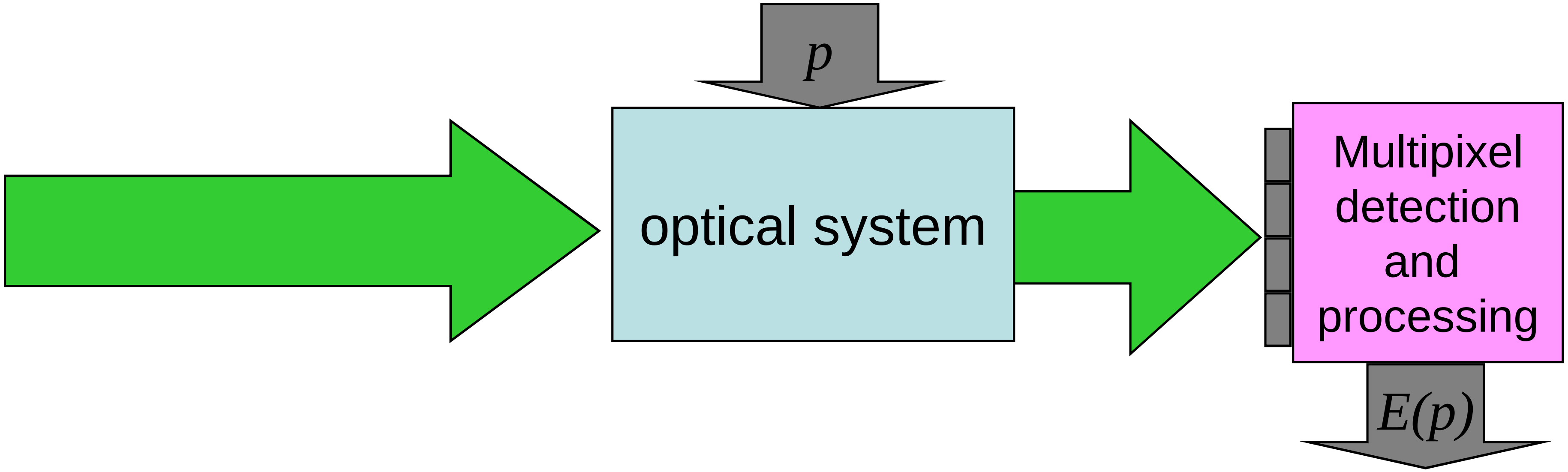
The Cramer Rao bound gives the minimum variance on p

-given the input light state

-independently of measurement strategy

When input state is a coherent state, one can determine the «Standard Cramer Rao bound » (SCRb)

The « noise mode »



The quantum noise on the estimator $E(p)$ comes

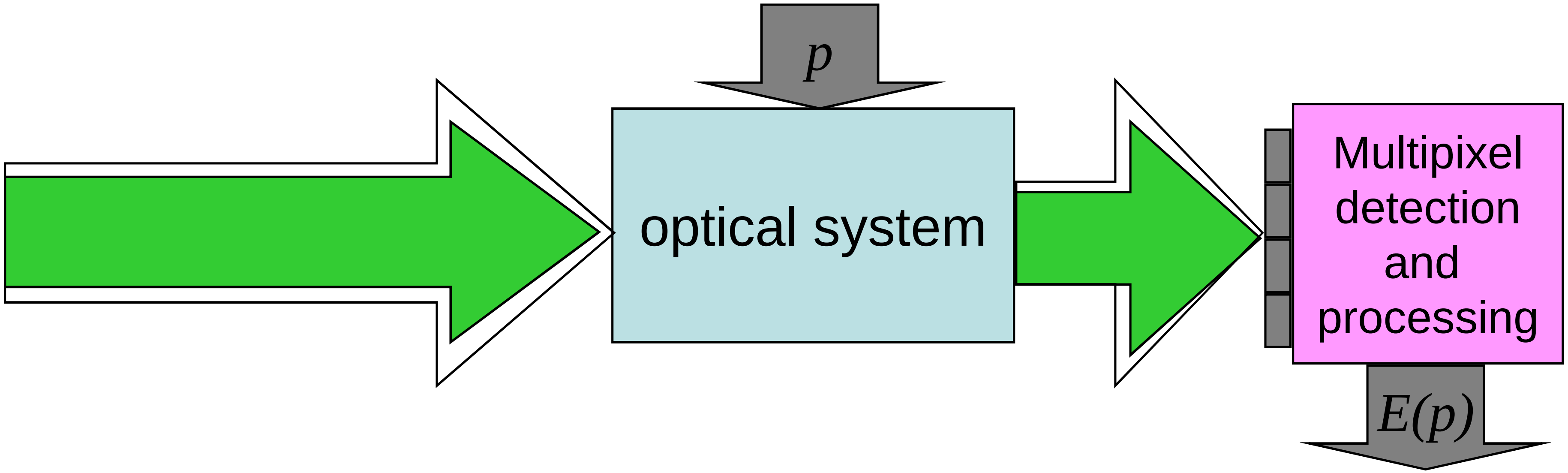
from a single « **noise mode** » $u_1(x,y)$

N. Treps, V. Delaubert, A. Maître, J.M. Courty, C. Fabre Phys. Rev A**71** 013820 (2005)

One can build a basis of transverse functions starting with $u_1(x,y)$

Quantum fluctuations on $E(p)$ come only from mode $u_1(x,y)$

Beyond the SCRb



Solution:
superimpose to the input light beam
a squeezed vacuum in mode $u_1(x,y)$

Two-mode state: - one for « illumination »
- one for noise reduction

Back to the intrinsic number of modes

N_m: intrinsic number of modes
defined in twin-photon generation

allows us to improve simultaneously
N_m independent measurements,
and never more

INTRODUCTION

WHAT IS A LIGHT MODE ?

INTRINSIC PROPERTIES OF LIGHT STATES

« SUPERMODES »

TAILORING HAMILTONIANS

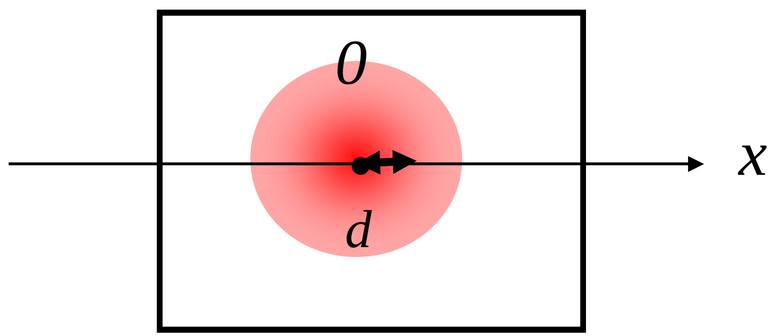
MODES AND MEASUREMENTS

MEASUREMENTS IN IMAGES

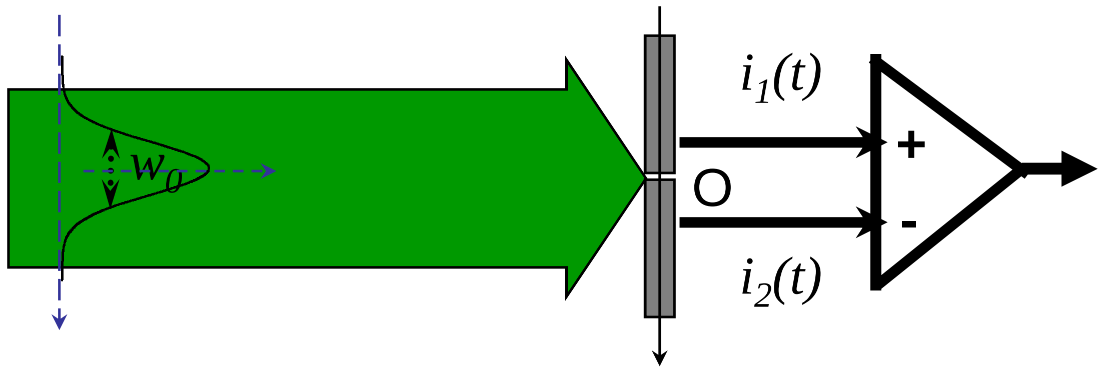
MEASUREMENTS WITH FREQUENCY COMBS

CONCLUSION

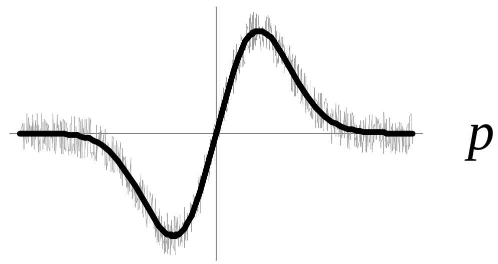
A simple example : spatial nanopositioning in transverse plane



Usual technique using split detector



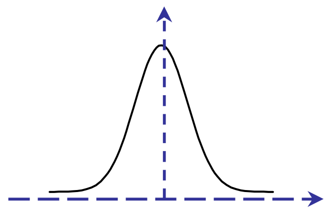
$i_1(t)$- $i_2(t)$=$E(p)$



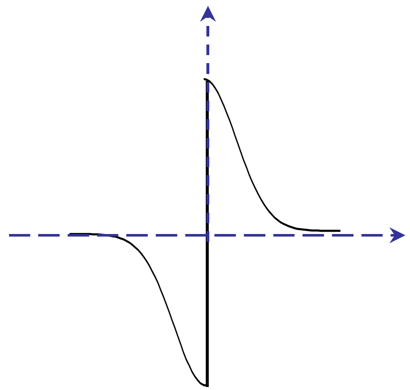
shot noise limit in split detector technique:

$$\Delta p = \frac{\sqrt{8}}{\pi}\frac{w_0}{\sqrt{N}}$$

Beyond the standard quantum limit by squeezing the noise mode



Gaussian beamTEM$_{00}$

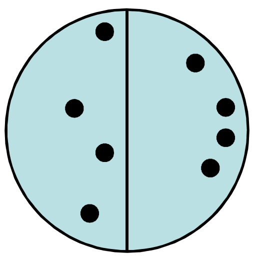


Noise mode

C. Fabre, J.-B. Fouet, A. Maître, Optics Letters 25, 76 (2000)

to go beyond the shot noise limit:

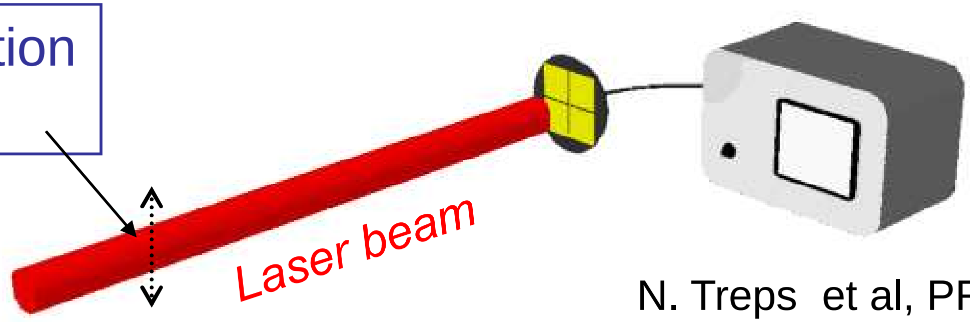
TEM$_{00}$ beam $\otimes$ squeezed vacuum in noise mode



Photons spatially ordered two by two

Experimental implementation

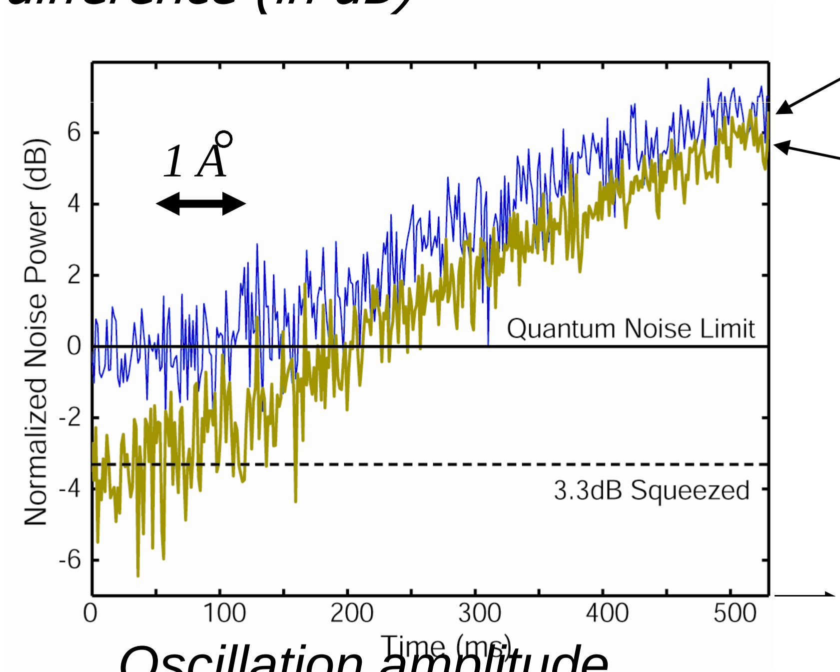
Very small oscillation at 5 MHz



Collaboration with ANU Australia

N. Treps et al, PRL **88**, 203601 (2002)
, Science, **301**, 940 (2003)

Intensity difference (in dB)



Oscillation amplitude

TEM$_{00}$ beam

TEM$_{00}$ + squeezed noise mode

What about the SCRb ?

V. Delaubert, N. Treps, C. Fabre, H. Bachor, P. Réfrégier,
"*Quantum limits in image processing*" Europhys. Letters **81** 44001 (2008)

$$\boxed{(\Delta p)_{S-CRb} = \frac{w_0}{2\sqrt{N}}}$$

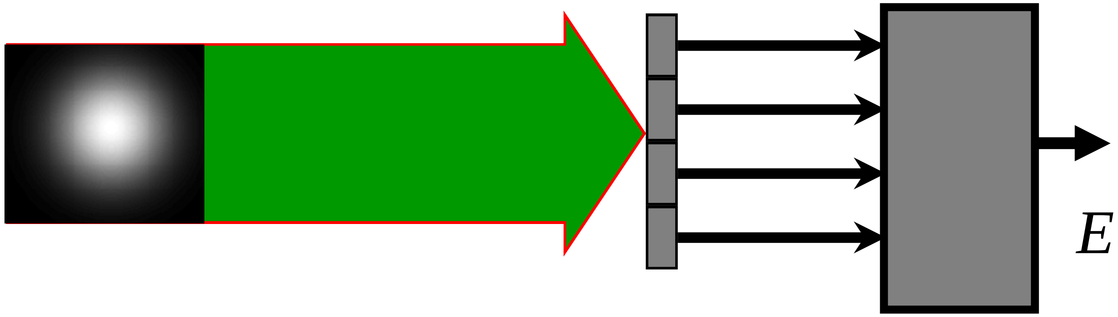
N : total number of photons measured

w_0 : beam waist

$$(\Delta p)_{\text{split}} = \frac{\sqrt{8}}{\pi}\frac{w_0}{\sqrt{N}} = 1.22(\Delta p)_{\text{S-CRb}}$$

The split detector method is not the best technique !

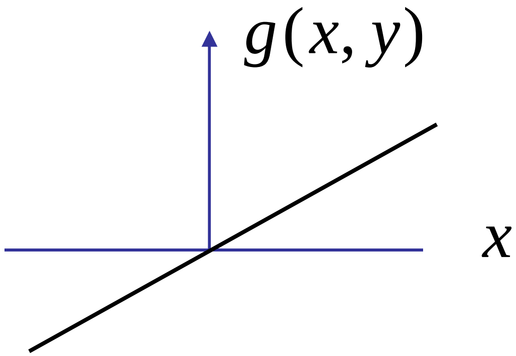
Optimal technique 1



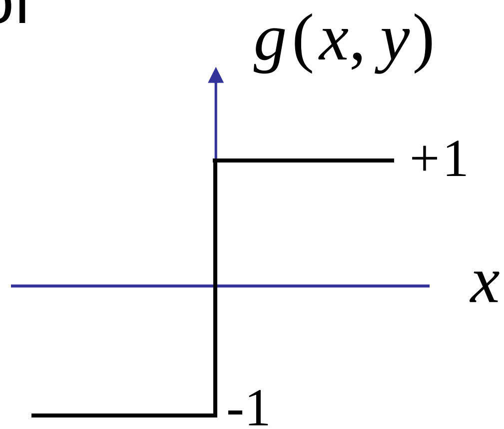
$$E(p) = \iint dxdy\, i(x,y,p)\, g(x,y)$$

optimized choice of $g(x,y)$ for a TEM_{00} beam:

$$\boxed{g(x,y) = x}$$

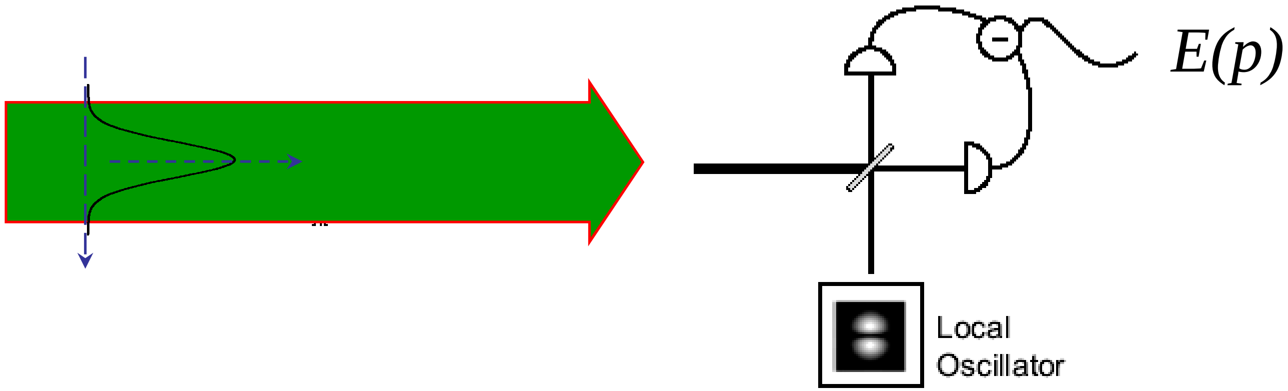


instead of



Standard Cramer Rao Bound reached

Optimal technique 2



optimized choice of local oscillator for a TEM_{00} beam:

$$u_{LO}(x, y) = \mathrm{TEM}_{10}$$

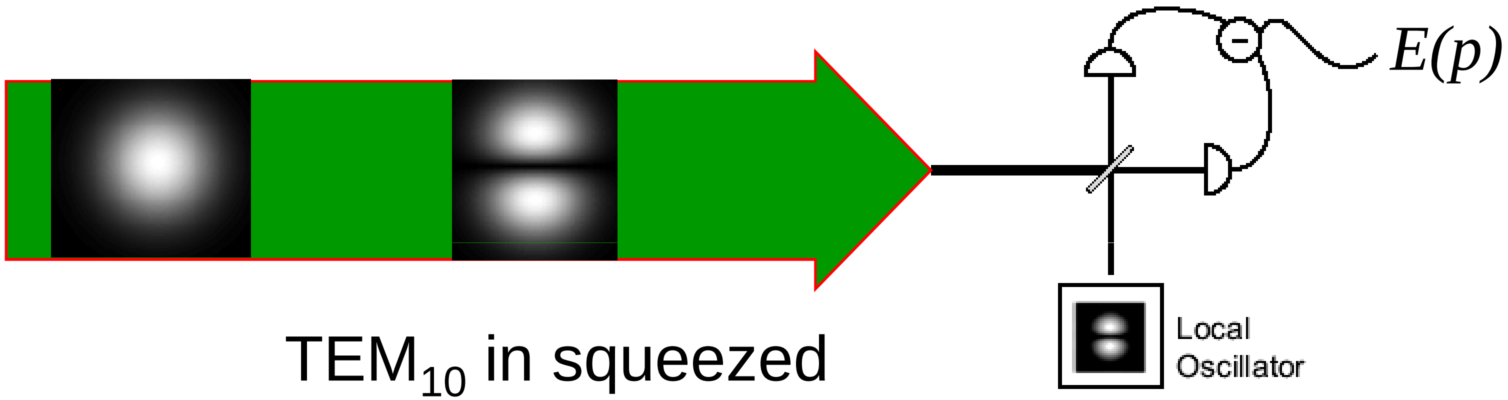
V. Delaubert et al Phys. Rev A **74** 053823 (2006)

Standard Cramer Rao Bound reached again

In both cases, no other shot noise limited measurement strategy can do better !

Beyond the SCRb

Noise mode : the TEM_{10} mode



TEM_{10} in squeezed vacuum state

The upper and lower parts of the beams are **entangled**

Experimental implementation:

M. Lassen et al. Phys. Rev Letters **98**, 083602 (2007)

INTRODUCTION

WHAT IS A LIGHT MODE ?

INTRINSIC PROPERTIES OF LIGHT STATES

« SUPERMODES »

TAILORING HAMILTONIANS

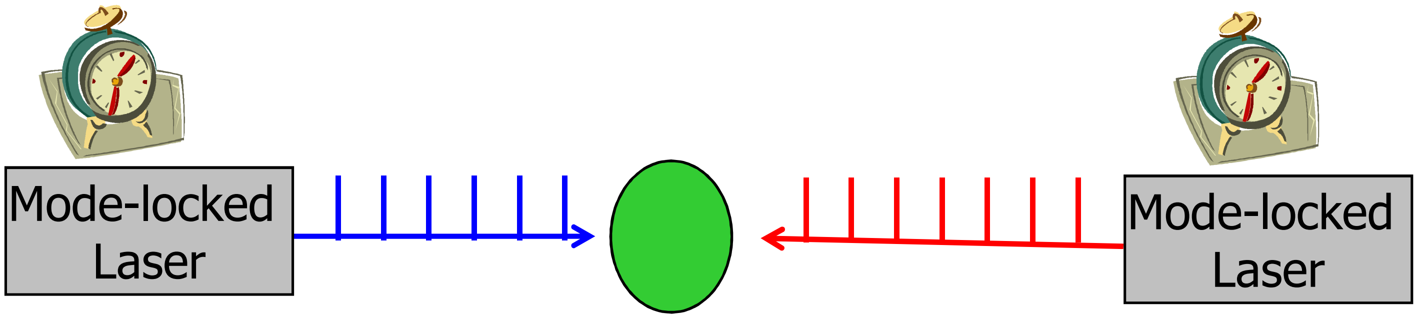
MODES AND MEASUREMENTS

MEASUREMENTS IN IMAGES

MEASUREMENTS WITH FREQUENCY COMBS

CONCLUSION

A related measurement: clock synchronization



time transfer problem

Implementation of Einstein's protocol for clock synchronization

SCRb in clock synchronization ?

B. Lamine, C. Fabre, N. Treps, Phys. Rev. Letters **101** 123601 (2008)

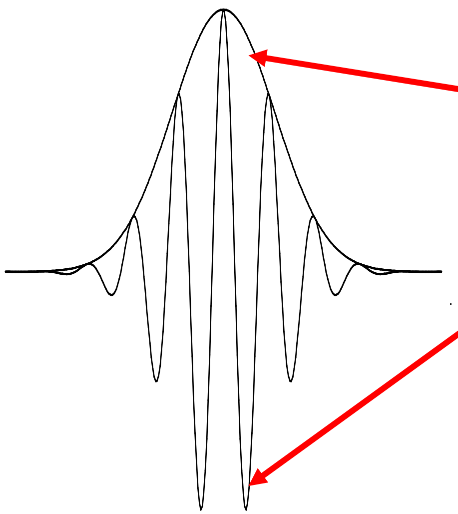
in the case of a Gaussian coherent pulse:

$$\boxed{(\Delta t)_{S-CRb} = \frac{1}{\sqrt{N}} \frac{1}{2\sqrt{\omega_0^2 + \Delta\omega^2}}}$$

N : total number of photons

ω_0 : mean frequency

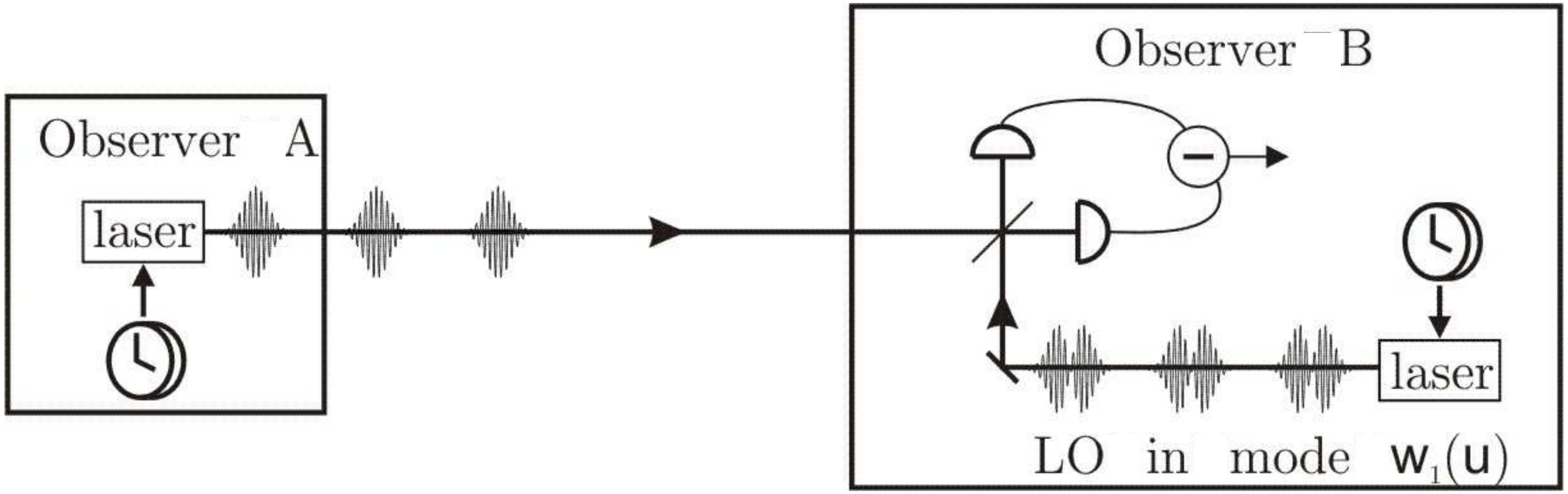
$\Delta\omega$: frequency spread



shift of pulse enveloppe maximum

shift of oscillation at optical frequency

Optimal measurement



Local Oscillator of optimized temporal shape

Standard Cramer Rao bound reached

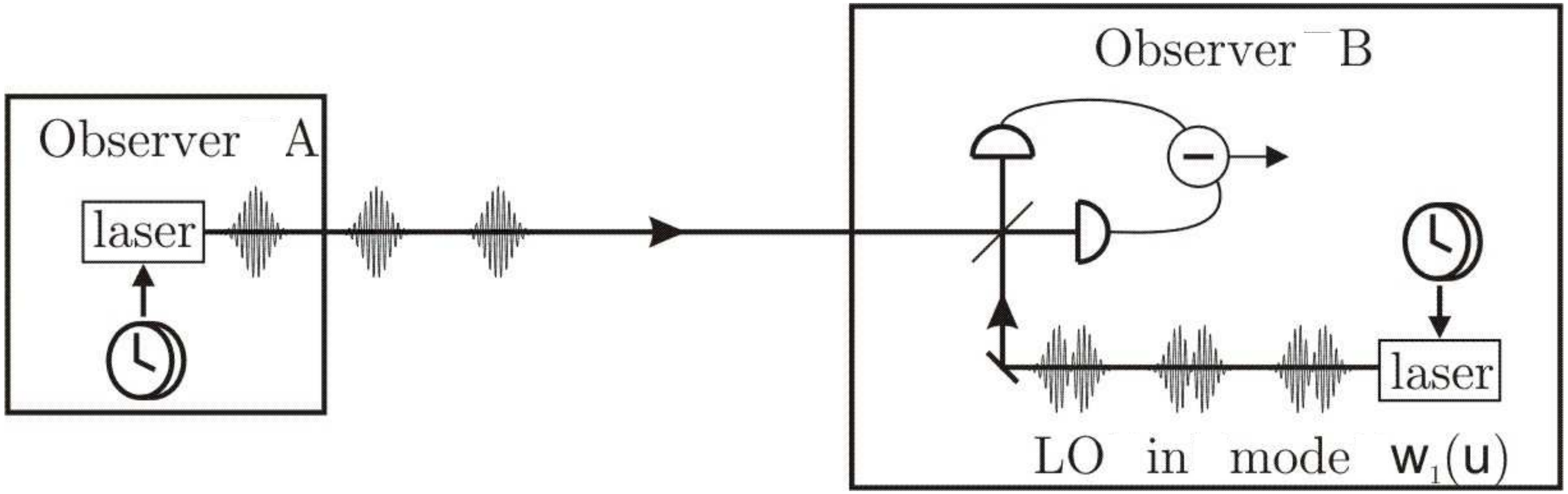
no other measurement can do better on a shot noise limited pulse

Ultimate sensitivity ?

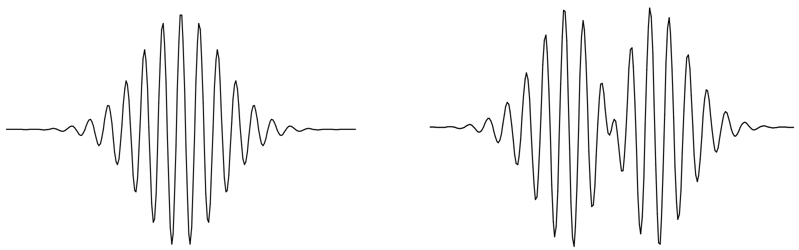
10mW, 10 fs, 1s integration time

SCRb = **20 yoctoseconds**

Beyond the SCRb



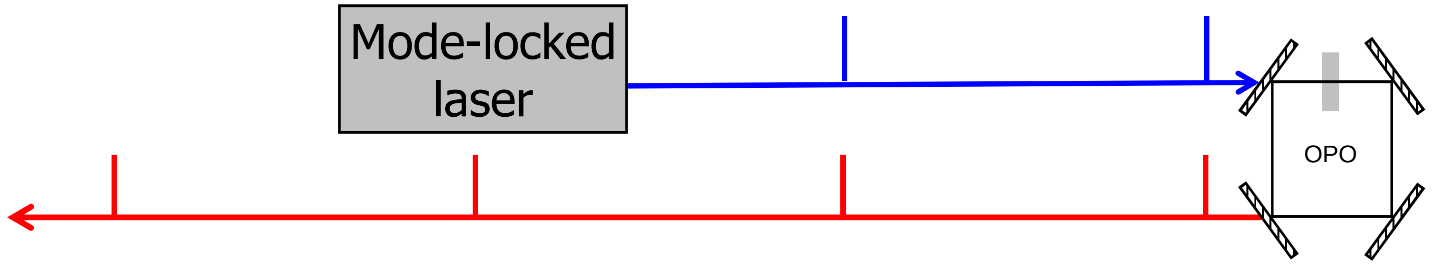
Observer A sends a squeezed vacuum state in noise mode:

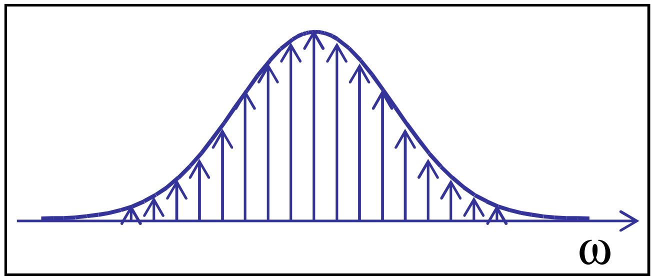


Much better sensitivity
than by sharing entangled light between A and B

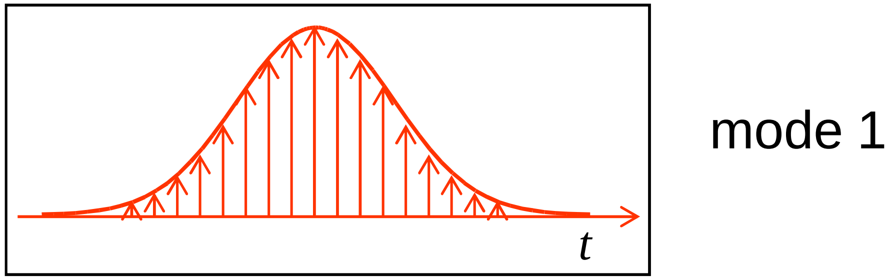
Generation of required squeezed states

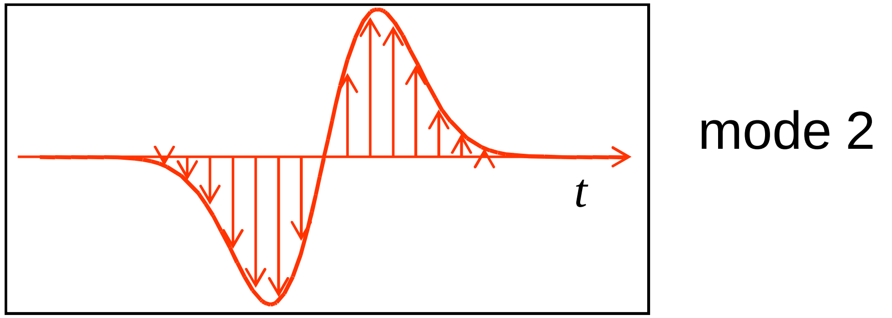
Use the Synchronously pumped OPO





Gaussian variation
of pump*phase matching





Mode-independent properties of quantum states
have been defined

Extract « eigenmodes » of a problem is always fruitful

Possibility of generating at will
interesting multimode quantum states states
by tailoring the pump shape

Whatever its shape a given mode can be measured
destructively by homodyne technique

How to isolate physically a given « supermode » ?


C.F.
N. Treps,
B. Lamine
V. Delaubert
G.Patera
B. Chalopin
O. Pinel
J.-F.Morizur
G. de Valcarcel (Valencia, Spain)
Collaborations with
H. Bachor ANU Canberra Australia
P. Réfrégier I. Fresnel Marseille France