Time-Reversal Symmetry Breaking and Spontaneous Anomalous Hall Effect in Fermi Fluids

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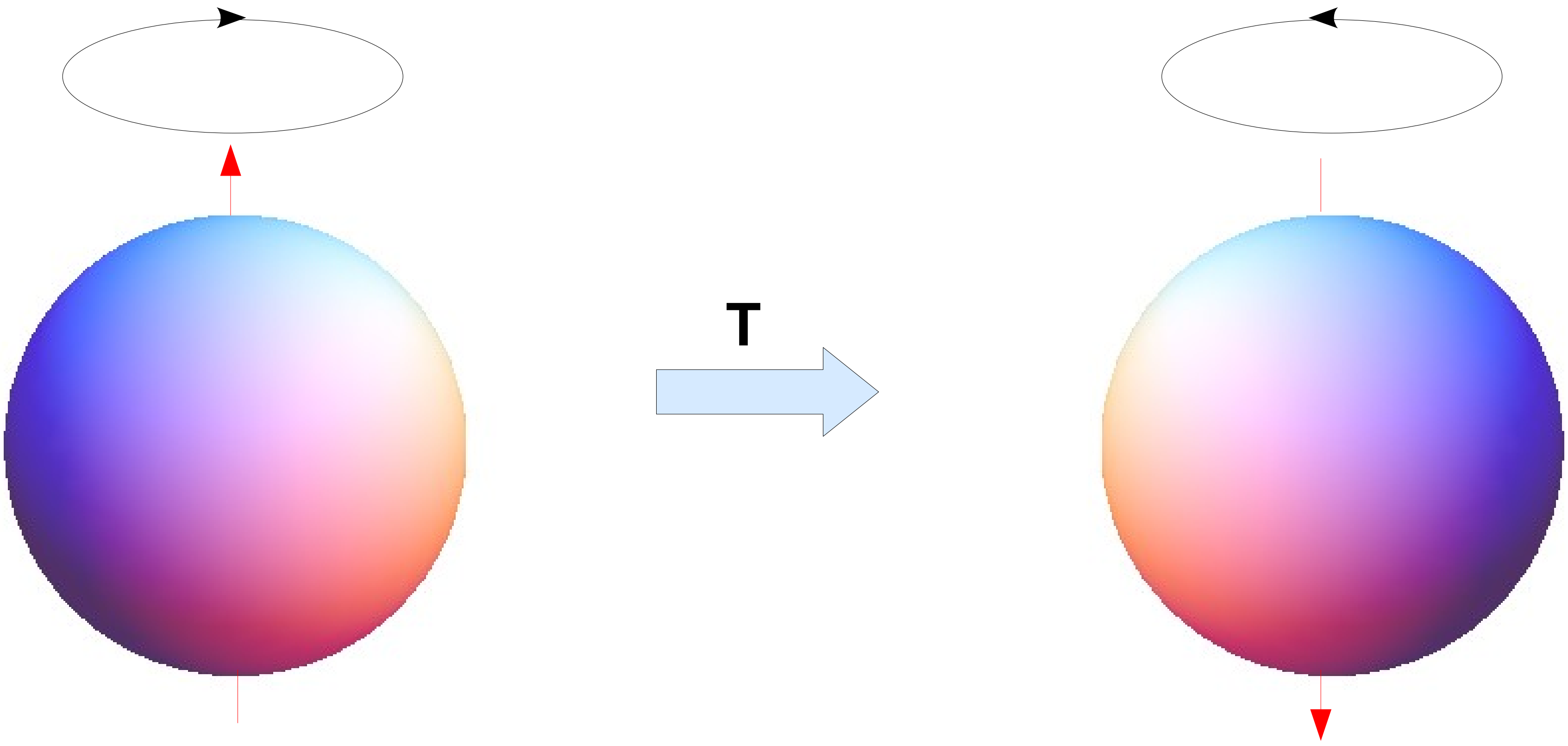
Spontaneous Time-Reversal Symmetry Breaking

- Ferromagnetic



- Non-ferromagnetic?

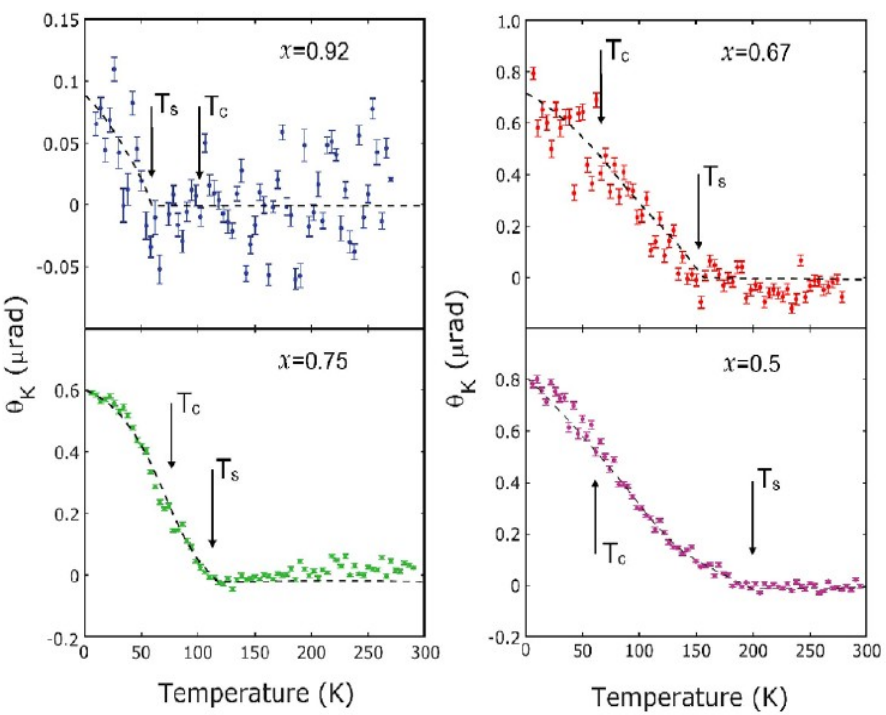
Spontaneous **T** symmetry breaking without ferromagnetism

- p+i p superconductor (**T**, chirality, and gauge)
- Chiral spin liquid (**T**, chirality, and possibly translation)
- Varma loop (**T** and rotation or 2D inversion)
- DDW (**T**, 2D inversion and translation)
- d+i d DDW (**T**, translation, and chirality)

Experimental candidates

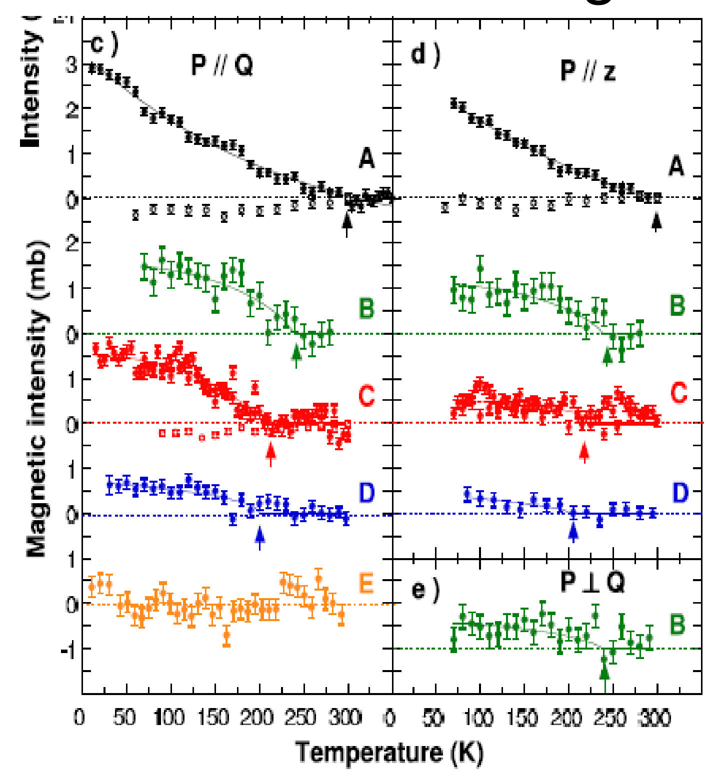
- Sr_2RuO_4 (p+ip superconductor?)
- High T_c ($YBa_2Cu_3O_{6+y}$ and $HgBa_2CuO_{4+d}$)

Kerr rotation



J. Xia, *et al*, PRL, **100**, 127002 (2008)

neutron scattering



B. Fauque, *et al*, PRL, **96,** 197001 (2006)

Questions and Strategy

Q: mechanism

Q: classification

Q: strong coupling vs. weak coupling

Q: is lattice necessary? (flux)

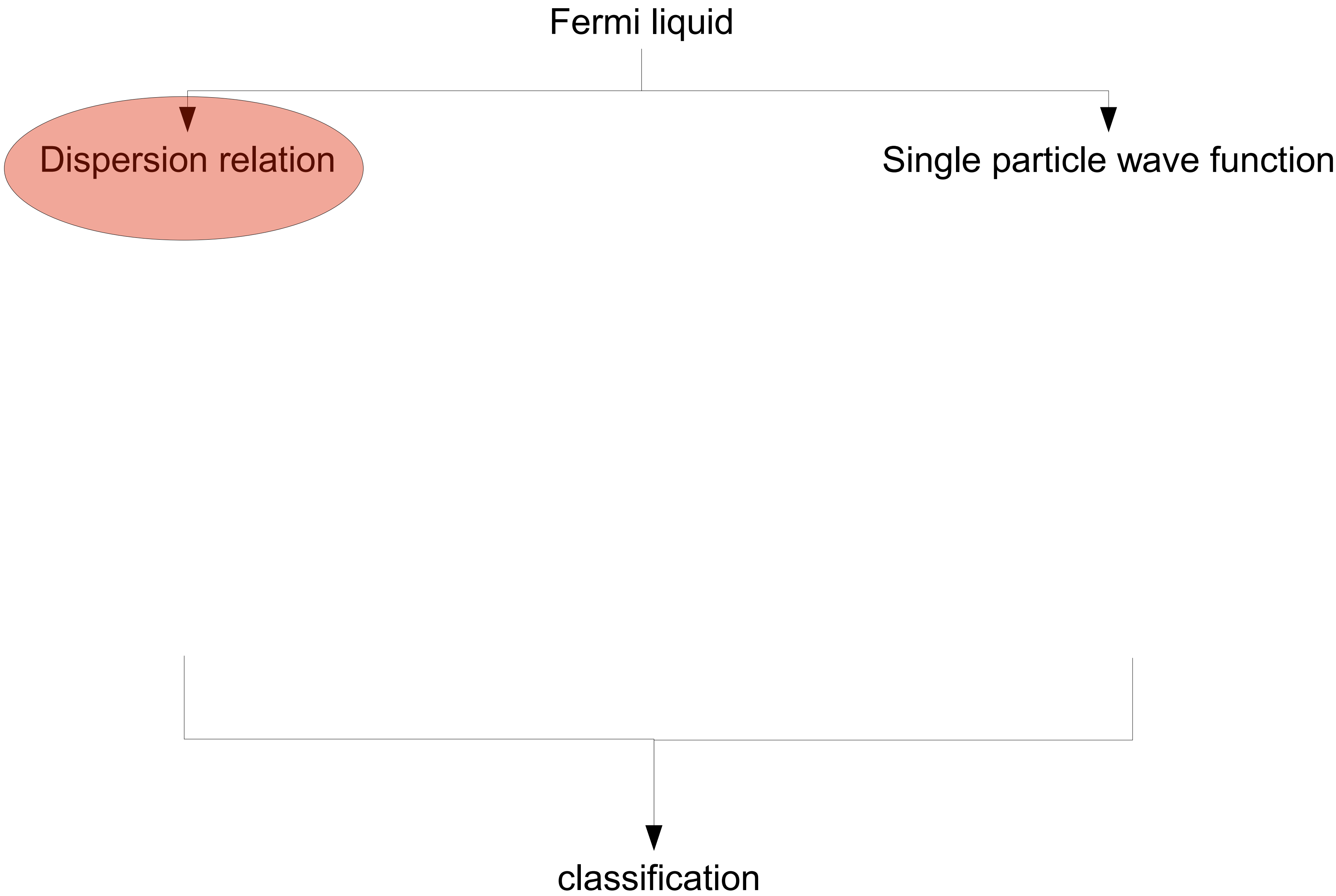
Q: experimental signatures

S: 2D Fermi liquid (weak coupling)

S: general theory (classification)

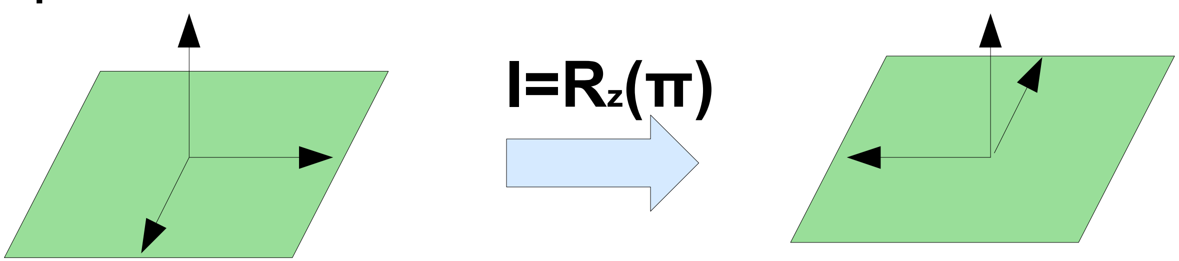
S: realization in specific models

Road map

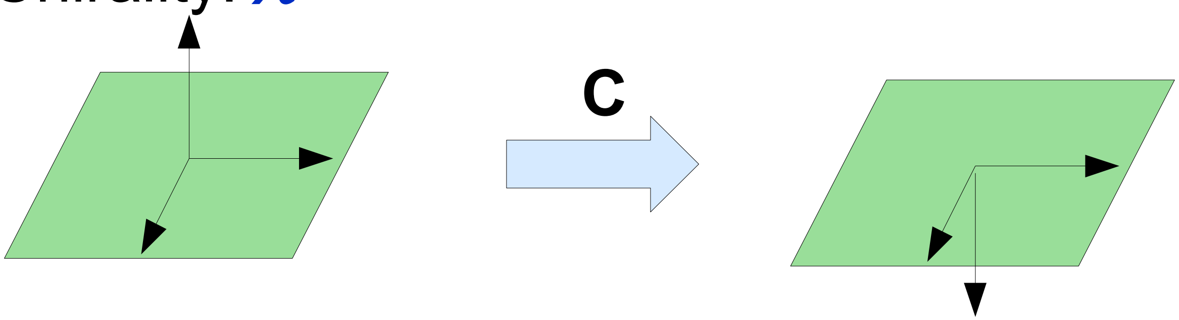

Fermi liquid
Dispersion relation
Single particle wave function
classification

Symmetry properties of the dispersion relation

- Time reversal: **T** $$\mathrm{T}\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k})$$
- Space inversion: **I** $$\mathrm{I}\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k})$$



- Chirality: χ = **Kai** ? $$\mathrm{C}\epsilon(\mathbf{k}) = \epsilon(\mathbf{k})$$



- If ϵ(**k**) is not even, breaks **T** and **I**. (Type I)
- **C** invariant (no Hall effect)
- **CIT** invariant

Type I **T**-symmetry breaking

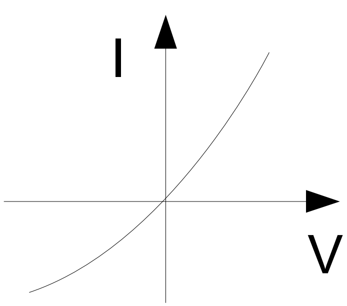
- Breaks **I** and **T**
- Stabilized by forward scattering

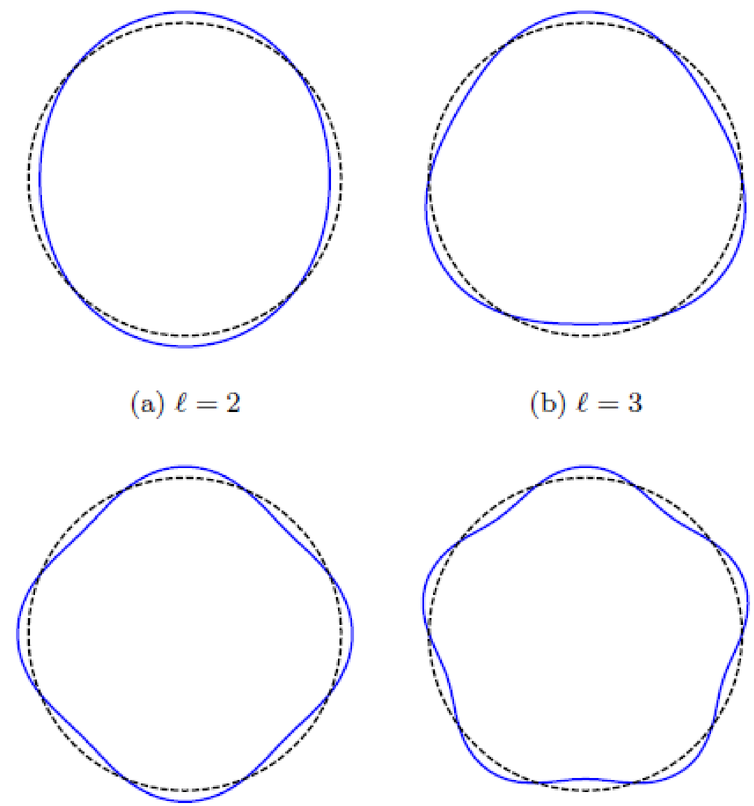
  Pomeranchuk instability

- Preserve **C**

  no Hall effect at B=0

- Not a lattice effect
- Asymmetric I-V curve





(a) $\ell = 2$ (b) $\ell = 3$

(c) $\ell = 4$ (d) $\ell = 5$

Road map

Fermi liquid

Dispersion relation

Single particle wave function

- any number of bands
- breaks **T** and **I**
- preserve **C** and **IT**
- Type I
- **CIT** invariant

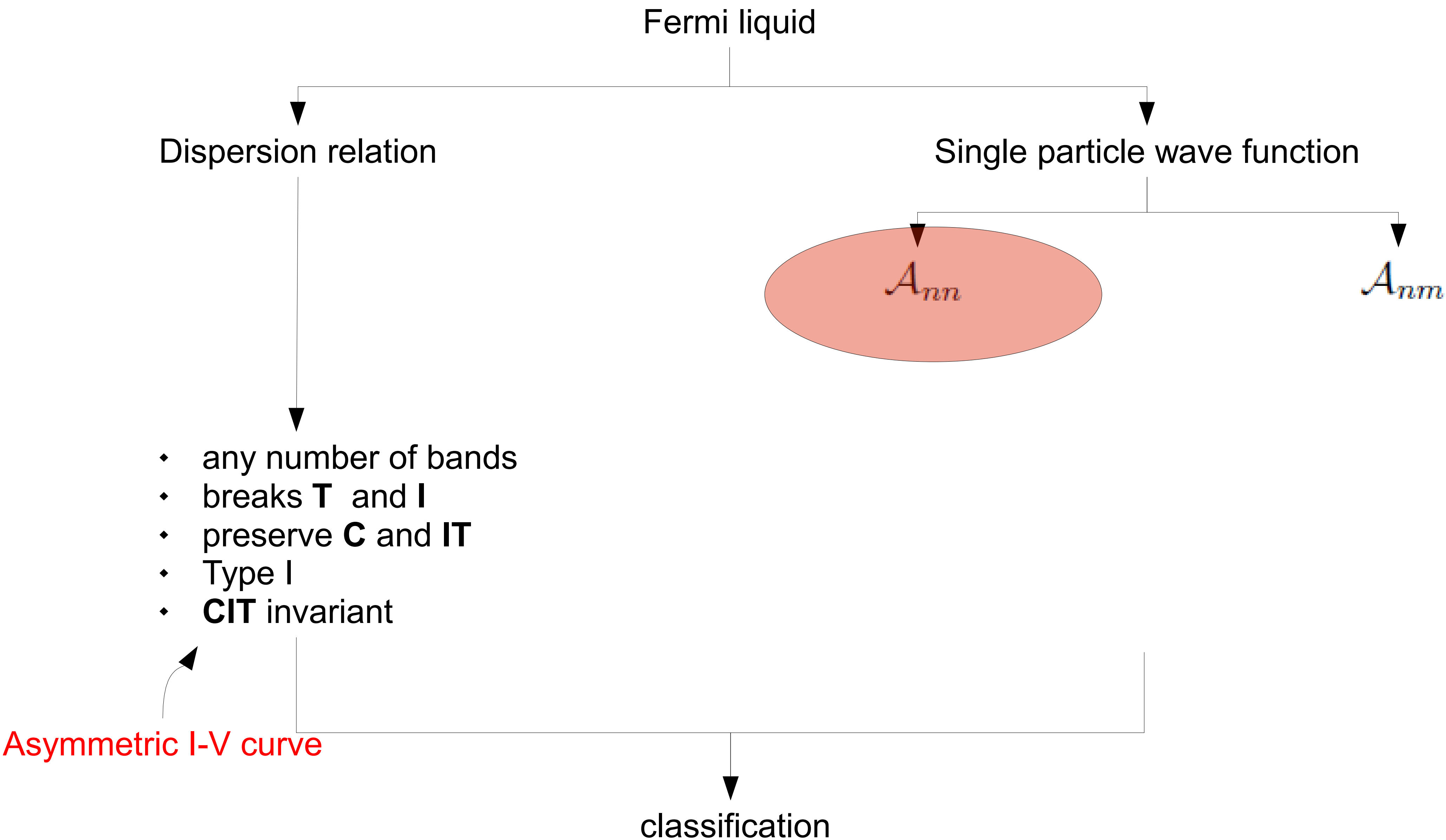
Asymmetric I-V curve

classification

Wave function and overlap matrix

- Wave function: $|\psi_n(\mathbf{k})\rangle$
- Overlap matrix: $\mathcal{A}^a_{nm} = -i\langle\psi_n(\mathbf{k})|\nabla^a_{\mathbf{k}}|\psi_m(\mathbf{k})\rangle$
  - Position operator: $\mathbf{x} = i\nabla_{\mathbf{k}} + \mathcal{A}_{nm}$
  - Diagonal terms $\mathcal{A}^a_{nn} \rightarrow \mathcal{A}^a_{nn} + \nabla^a_{\mathbf{k}}\varphi_n$
    - $U(1)^N$ gauge
    - Berry connection
  - Off-diagonal terms: $\mathcal{A}^a_{nm} \rightarrow e^{i\varphi_n}\mathcal{A}^a_{nm}e^{-i\varphi_m}$

Road map


Fermi liquid
Dispersion relation
Single particle wave function
$\mathcal{A}_{nn}$
$\mathcal{A}_{nm}$
• any number of bands
• breaks **T** and **I**
• preserve **C** and **IT**
• Type I
• **CIT** invariant
Asymmetric I-V curve
classification

Diagonal terms (Berry phase)

- Requires at least two bands to break **T** if B=0

$$\nabla_{\mathbf{k}} \times \mathrm{tr}\mathcal{A} = 0$$ (E. I. Blount, 1962)

- Wilson loops in k space

$$W_\Gamma^n = \exp(i\Phi_\Gamma^n)$$

- Berry phase and anomalous Hall

Haldane, PRL 93, 206602 (2004).

- Half of WZ term (S to S)

$$\Phi_\Gamma^n = \oint_\Gamma \sum_a \mathcal{A}_{nn}^a dk^a$$

- Hall conductivity(if the contour is FS)
  - break C and T if berry phase nontrivial

$$\sigma_{xy} = \frac{\Phi_\Gamma}{2\pi}$$

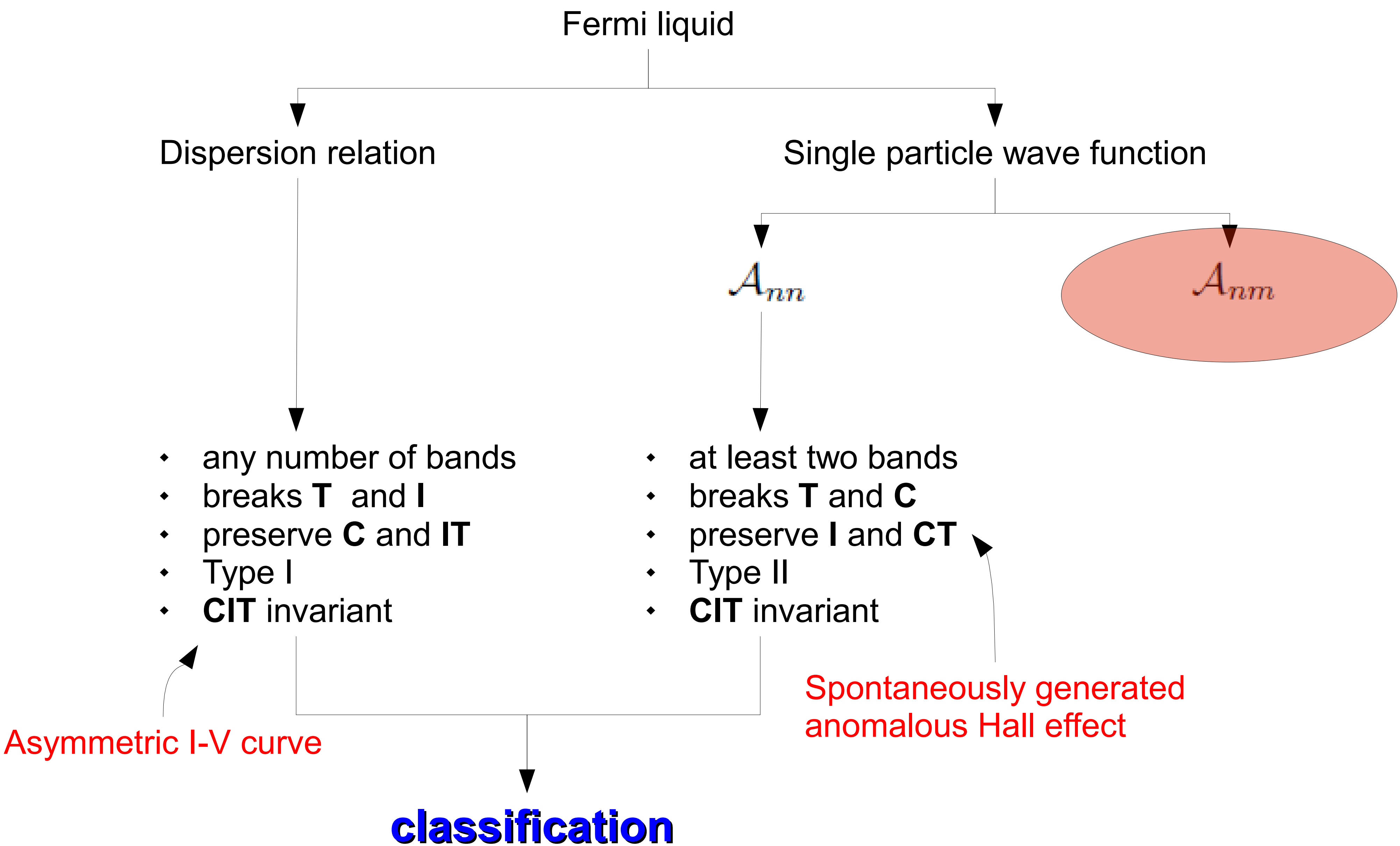
- **CIT** symmetry

$$\mathrm{C}W_\Gamma^n = (W_\Gamma^n)^*$$

$$\mathrm{I}W_\Gamma^n = W_{\mathbf{I}\Gamma}^n$$

$$\mathrm{T}W_\Gamma^n = (W_{\mathbf{I}\Gamma}^n)^*$$

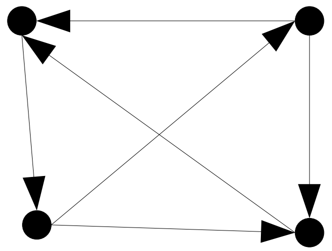
Road map


Fermi liquid
Dispersion relation
Single particle wave function
$\mathcal{A}_{nn}$
$\mathcal{A}_{nm}$
• any number of bands
• breaks **T** and **I**
• preserve **C** and **IT**
• Type I
• **CIT** invariant
• at least two bands
• breaks **T** and **C**
• preserve **I** and **CT**
• Type II
• **CIT** invariant
Asymmetric I-V curve
Spontaneously generated anomalous Hall effect
classification

Off-diagonal terms

- Requires at least three bands to break **T**
- Carton picture

$$|\psi_n(\mathbf{k})\rangle \to e^{i\varphi_n(\mathbf{k})}|\psi_n(\mathbf{k})\rangle \qquad \mathcal{A}^a_{nm} = e^{-i\varphi_n}\mathcal{A}^a_{nm}e^{i\varphi_m}$$



- Rigorous proof

(E. I. Blount, 1962)

$$\nabla^a_{\mathbf{k}}\mathcal{A}^b_{nn} - \nabla^b_{\mathbf{k}}\mathcal{A}^a_{nn} = -i\sum_m(\mathcal{A}^a_{nm}\mathcal{A}^b_{mn} - \mathcal{A}^b_{nm}\mathcal{A}^a_{mn})$$

- Preserve **C** symmetry
  - no Hall effect

Road map

Fermi liquid

Dispersion relation

- any number of bands
- breaks **T** and **I**
- preserve **C** and **IT**
- Type I
- **CIT** invariant

Asymmetric I-V curve

Single particle wave function

$\mathcal{A}_{nn}$

- at least two bands
- breaks **T** and **C**
- preserve **I** and **CT**
- Type II
- **CIT** invariant

Spontaneously generated anomalous Hall effect

$\mathcal{A}_{nm}$

- at least three bands
- breaks **T**
- **C** invariant

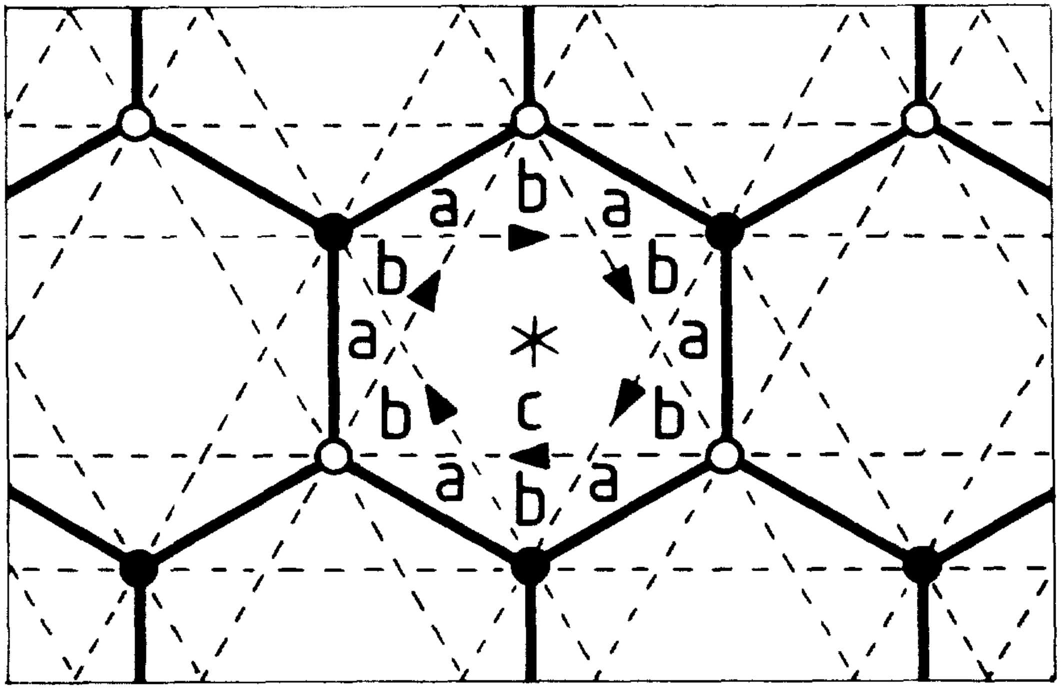
not considered

- **CIT** invariance for one and two-band model
- Classification of type I and II
- Type I and II mixing: 2D multiferroics

Example I

Flux state in honeycomb lattice

F. D. M. Haldane, PRL 61, 2015 (1988).

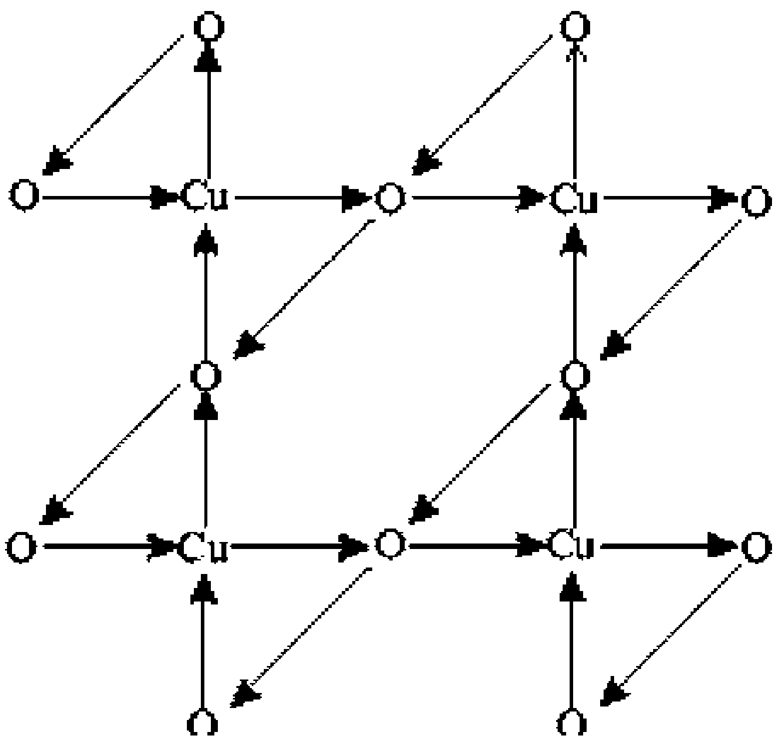


- Break **T** and **C**
- Type II
- Insulator
- Quantized Hall conductivity at B=0

Example II

Varma loop state

C.M. Varma, PRB **73**, 155113 (2006).

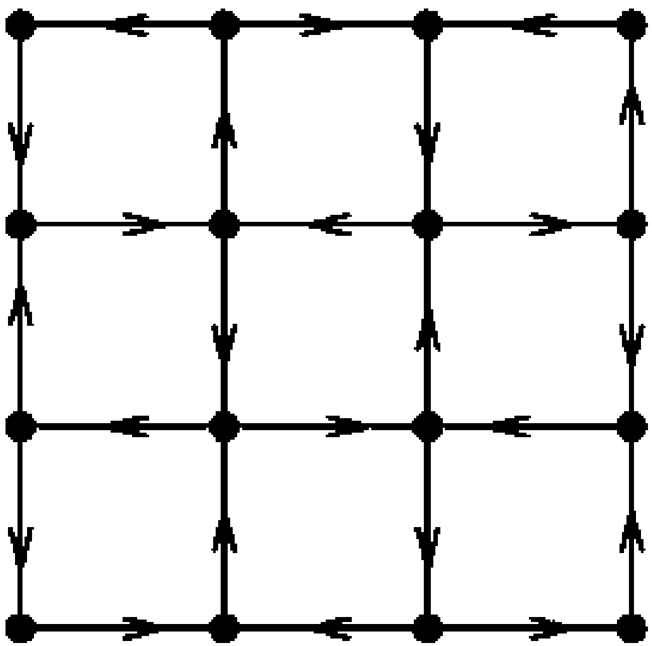


- Break **T** and **I**
- Type I
- no Hall effect if B=0

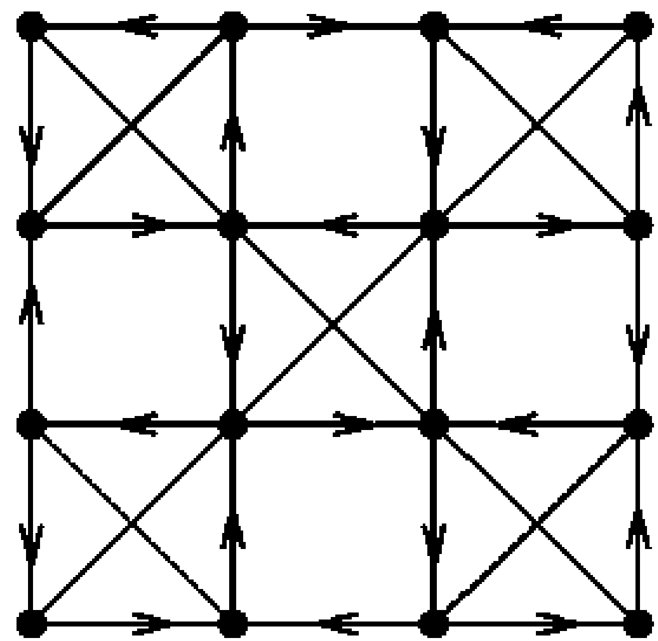
Example III

DDW and d+id DDW

S. Chakravarty, *et. al.*, PRB **63**, 094503 (2001).
C. Nayak, PRB **62**, 4880 (2000).



DDW (type I)

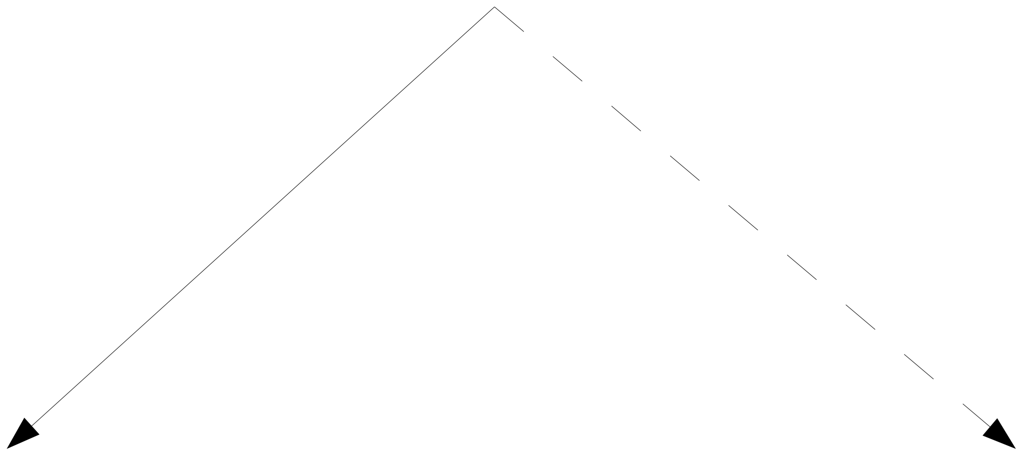


d+id DDW (type II)

S. Tewari, et. al., PRL 100, 217004 (2008).
Kerr effect in YBCO

- Effective two-band model due to TSB
- This is the reason why TSB is needed here

Realization

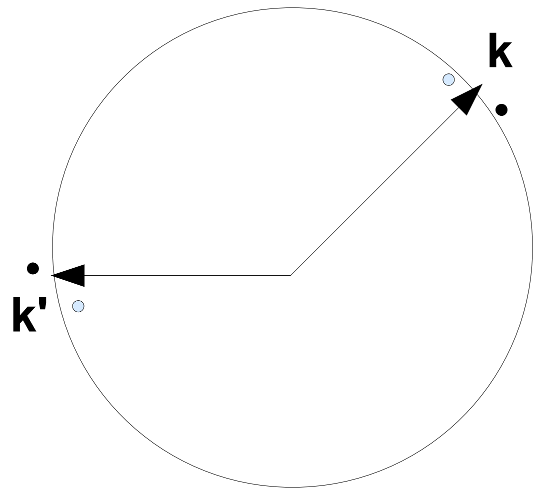


- Continuous Model
- Lattice Model

Pomeranchuk instability in a single-band model

- Model: $$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} f_{\mathbf{k},\mathbf{k}',\mathbf{q}} \psi^\dagger_{\mathbf{k}+\mathbf{q}} \psi_{\mathbf{k}} \psi^\dagger_{\mathbf{k}'} \psi_{\mathbf{k}'-\mathbf{q}}$$
- Forward scattering:

  $q << k_F$

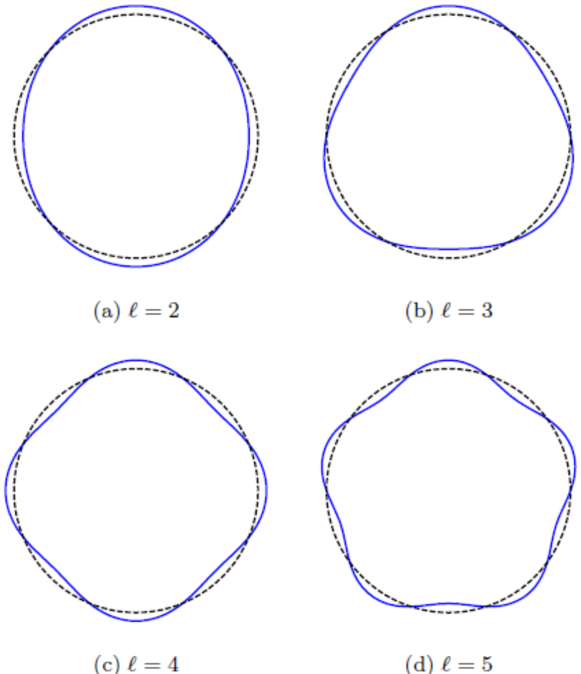


- Fermion multipoles:

$$\phi_{\ell,1}(\mathbf{q}) = \sum_{\mathbf{k}} \psi^\dagger_{\mathbf{k}+\mathbf{q}/2} \cos(\ell\theta_{\mathbf{k}}) \psi_{\mathbf{k}-\mathbf{q}/2}$$

$$\phi_{\ell,2}(\mathbf{q}) = \sum_{\mathbf{k}} \psi^\dagger_{\mathbf{k}+\mathbf{q}/2} \sin(\ell\theta_{\mathbf{k}}) \psi_{\mathbf{k}-\mathbf{q}/2}$$

- $$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \sum_{\ell,\mathbf{q},i} \frac{f_\ell}{2} \phi_{\ell,i}(\mathbf{q}) \phi_{\ell,i}(-\mathbf{q})$$



Two-band model

- Two bands: pseudospin S=1/2

$$\phi_{\ell,1,\mu}(\mathbf{q}) = \sum_{\mathbf{k}} \psi^{\dagger}_{n,\mathbf{k}+\mathbf{q}/2} \cos(\ell\theta_{\mathbf{k}}) \sigma^{n,m}_{\mu} \psi_{m,\mathbf{k}-\mathbf{q}/2}$$

$$\phi_{\ell,2,\mu}(\mathbf{q}) = \sum_{\mathbf{k}} \psi^{\dagger}_{n,\mathbf{k}+\mathbf{q}/2} \sin(\ell\theta_{\mathbf{k}}) \sigma^{n,m}_{\mu} \psi_{m,\mathbf{k}-\mathbf{q}/2}$$

- $\sigma_\mu=(I,\sigma_x,\sigma_y,\sigma_z)$
- I and σ_z:
  - intraband
  - Condensation implies FS distortion
- σ_x and σ_y:
  - interband
  - Condensation implies FS distortion+Locking phase

Free energy

$$H=\sum_{\mathbf{k},n}\epsilon_n(\mathbf{k})\psi^\dagger_{n,\mathbf{k}}\psi_{n,\mathbf{k}}+\sum_{\mathbf{q},i}\frac{f_\ell(q)}{2}\left(\phi_{\ell,i,x}(\mathbf{q})\phi_{\ell,i,x}(-\mathbf{q})+\phi_{\ell,i,y}(\mathbf{q})\phi_{\ell,i,y}(-\mathbf{q})\right)+\text{other interactions}$$

- Order parameters

$$\vec{\phi}_1=(<\phi_{1,x}>,<\phi_{1,y}>)_{q=0}\qquad \vec{\phi}_2=(<\phi_{2,x}>,<\phi_{2,y}>)_{q=0}$$

- Free energy

$$F=(m+\delta/2)(\phi_{1,x}^2+\phi_{2,x}^2)+(m-\delta/2)(\phi_{1,y}^2+\phi_{2,y}^2)$$
$$+u(|\vec{\phi}_1|^2+|\vec{\phi}_2|^2)^2+4v(|\vec{\phi}_1\times\vec{\phi}_2|)^2$$
$$+\text{higher ordered terms}$$

$$m=-\left(\frac{N(0)}{4}+\frac{1}{4\left(f_l(0)+|g_l(0)|\right)}+\frac{1}{4\left(f_l(0)-|g_l(0)|\right)}\right)\ +\Delta^2\frac{N(0)}{96}\left[3\left(\frac{N'(0)}{N(0)}\right)^2-\frac{N''(0)}{N(0)}\right]$$

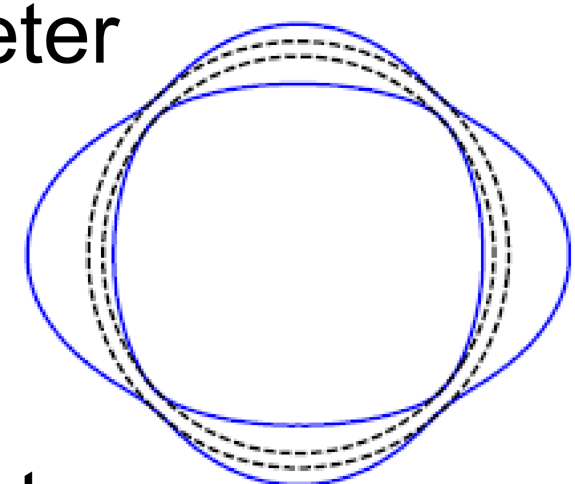
$$\delta=-\frac{1}{4\left(f_l(0)+|g_l(0)|\right)}+\frac{1}{4\left(f_l(0)-|g_l(0)|\right)}.$$

$$u=\frac{N(0)}{64}\left[2\left(\frac{N'(0)}{N(0)}\right)^2-\frac{N''(0)}{N(0)}\right]\qquad v=\frac{N''(0)}{48}$$

α and β phases

$$\vec{\phi}_1 = (< \phi_{1,x} >, < \phi_{1,y} >)_{q=0} \qquad \vec{\phi}_2 = (< \phi_{2,x} >, < \phi_{2,y} >)_{q=0}$$

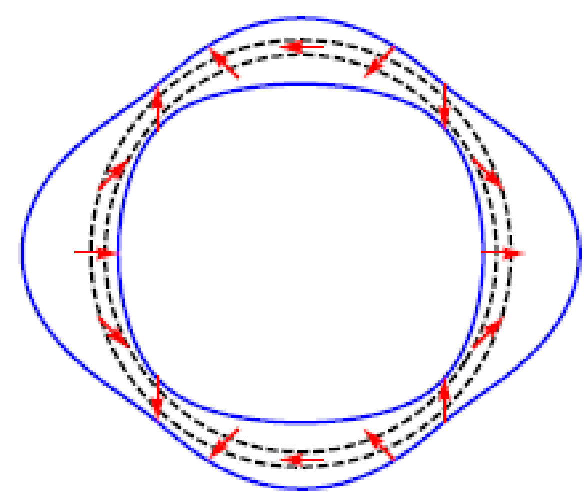
- α phase: one nonzero order parameter
  - ✭ Rotational symmetry breaking
  - ✭ **T** invariant



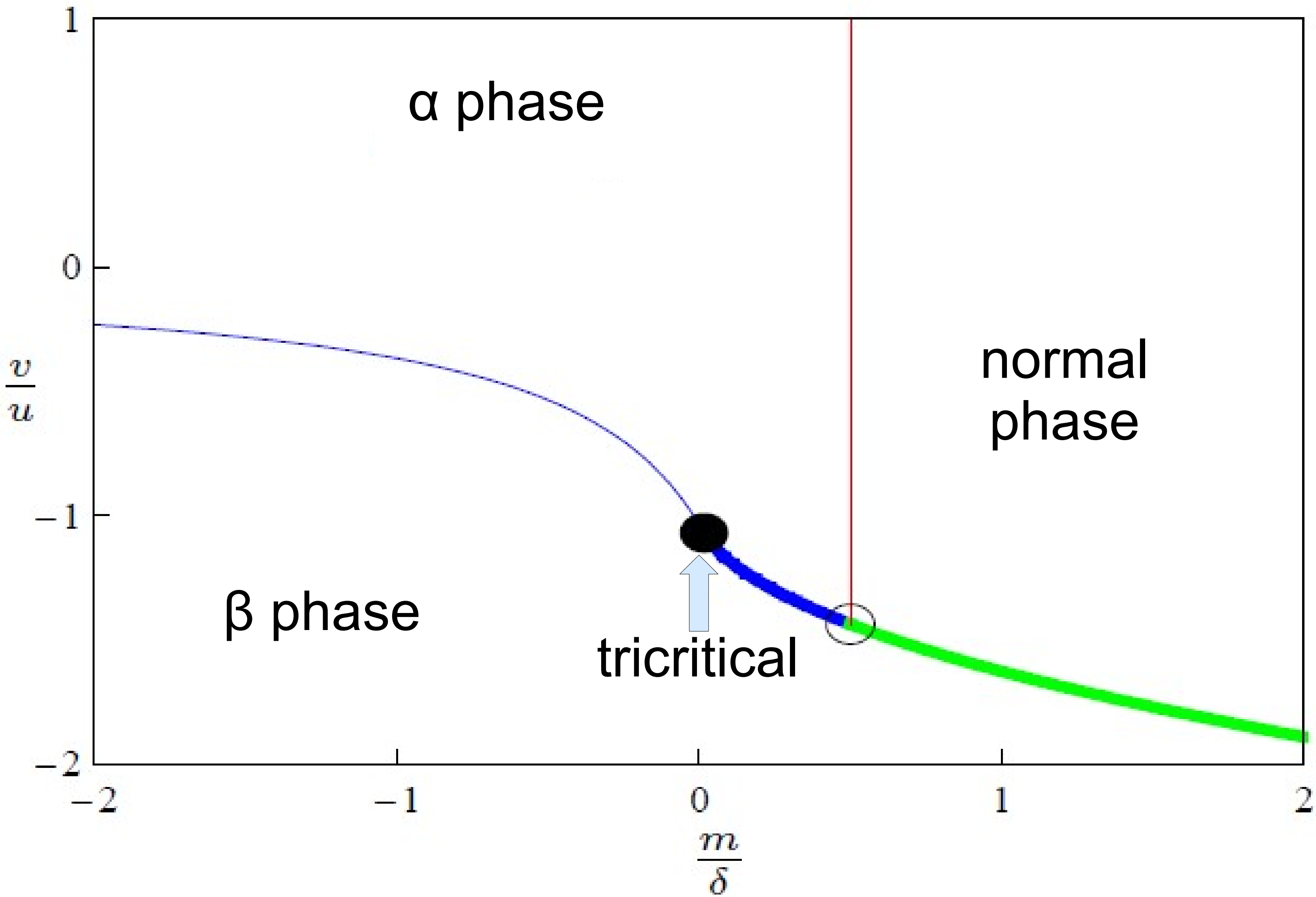
- β phase: two nonzero order parameters
  - ✭ →: the relative phase between the two band

    $$\arg\langle \psi_1^\dagger(\mathbf{k})\psi_2(\mathbf{k})\rangle$$

  - ✭ Winding number: proportional to Berry phase
  - ✭ Breaks **T** and rotation
  - ✭ Breaks **C** Anomalous Hall effect (metal)



Phase Diagram at T=0

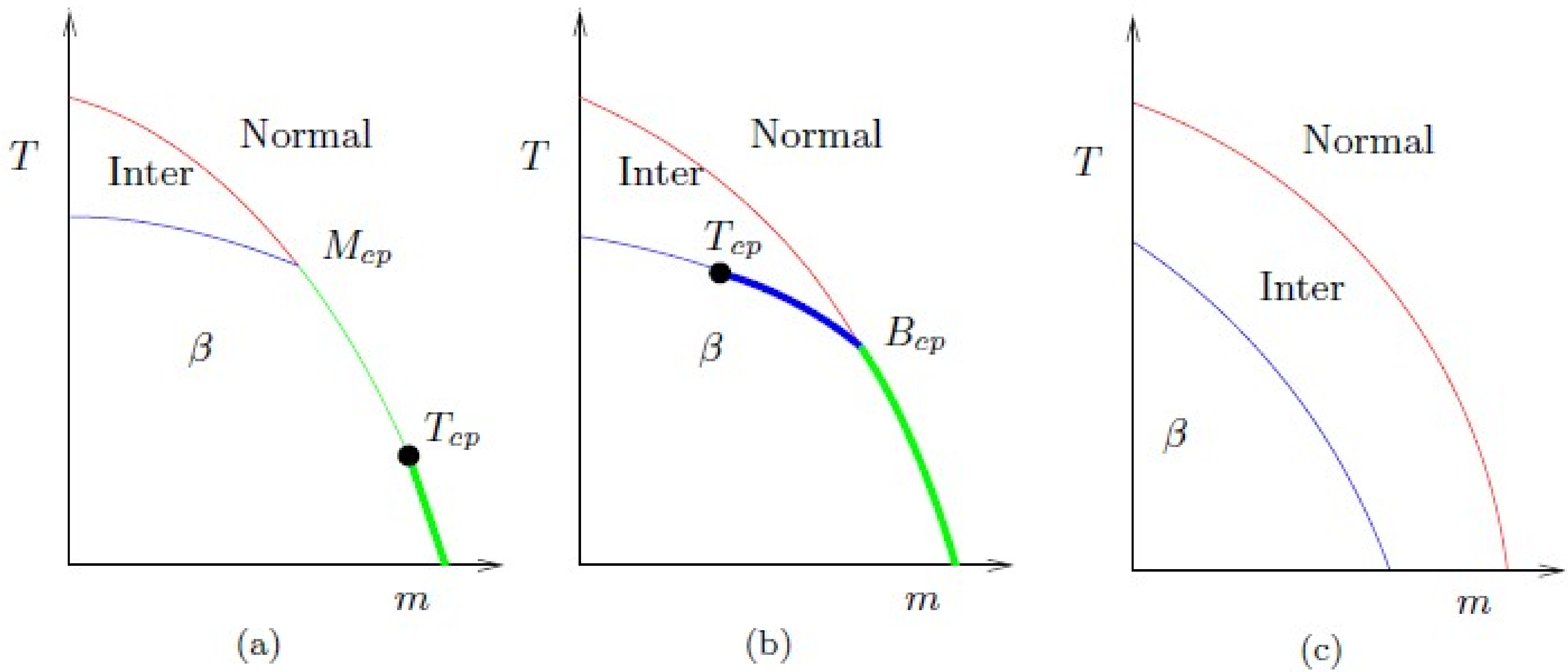


Lattice effect part I
without band crossing

- α and β phases can be generalized
- Rotational symmetry breaking is discrete
- Square lattice:
  - Z_4 rotational symmetry breaking to Z_2
  - β phase: Z_2 X Z_2

    rotational ↑ ↑ time-reversal
- Triangular lattice and Honeycomb lattice
  - Z_6 rotational symmetry breaking to Z_3 or Z_2
  - β phase: Z_2 X Z_2 or Z_3 X Z_2

Z_2 X Z_2 thermal transition



Z_3 X Z_2 thermal transition

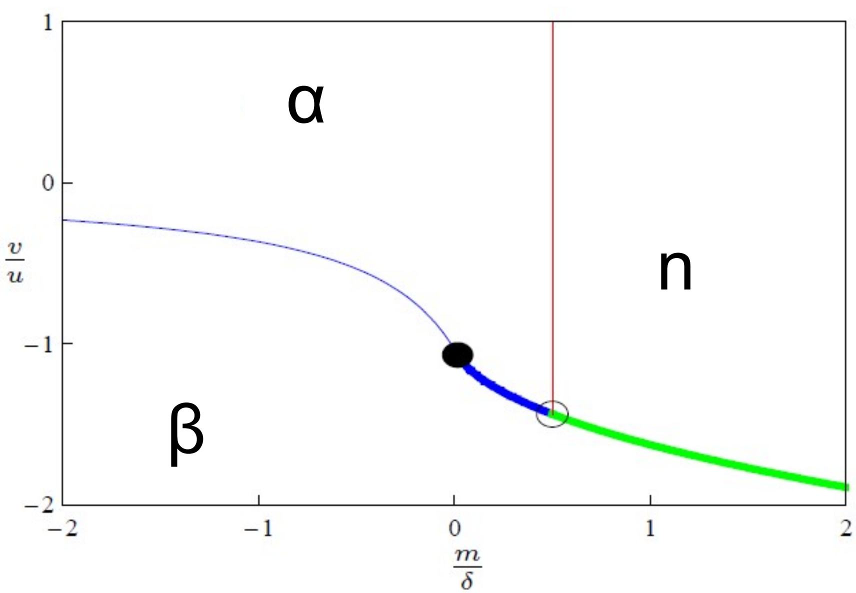
- two transitions (Ising and 3-states Potts)
- a first order transition
- a critical region

Lattice effect part II with band crossing

- Example: Graphene
- Type I: no fundamental difference
- Type II: only need one second order transition
  - Normal phase: crossing point has Berry flux n**π**
  - Example: flux state in honeycomb lattice (graphene)

F. D. M Haldane, PRL 61, 2015 (1988).

S. Raghu, et. al., PRL 100, 156401 (2008).

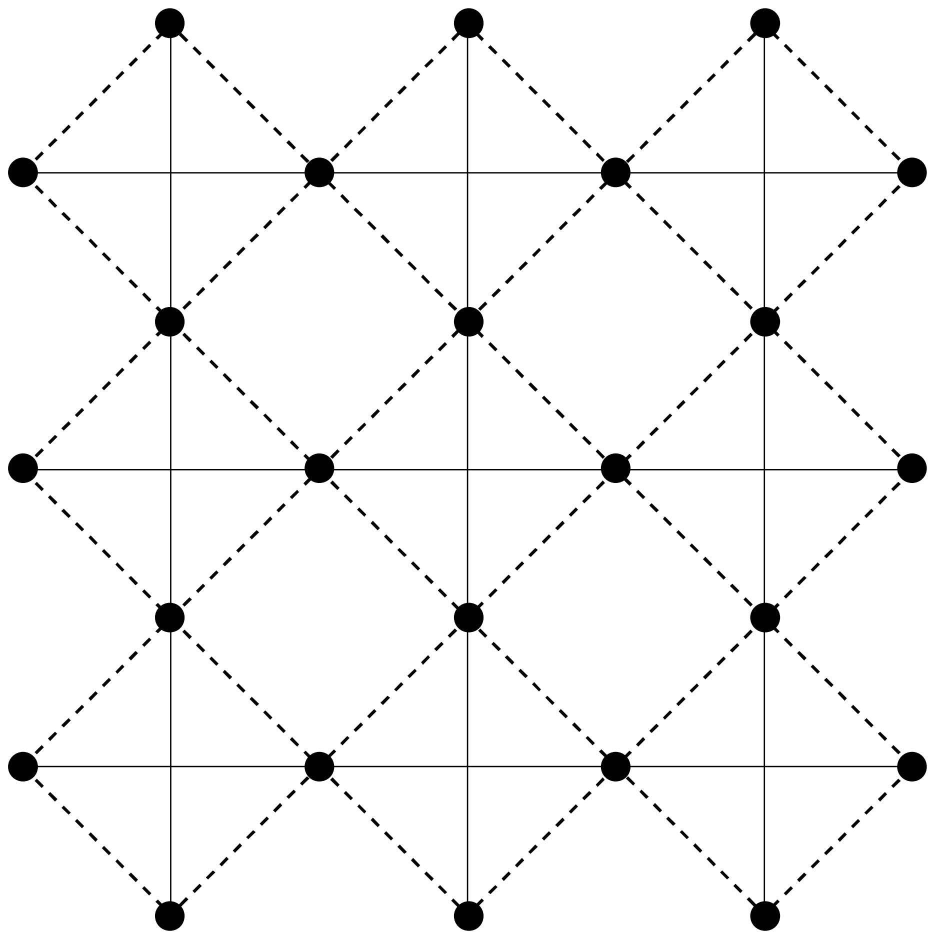


Crossed-Chain Lattice

Emery lattice
(CuO_2 Plane)

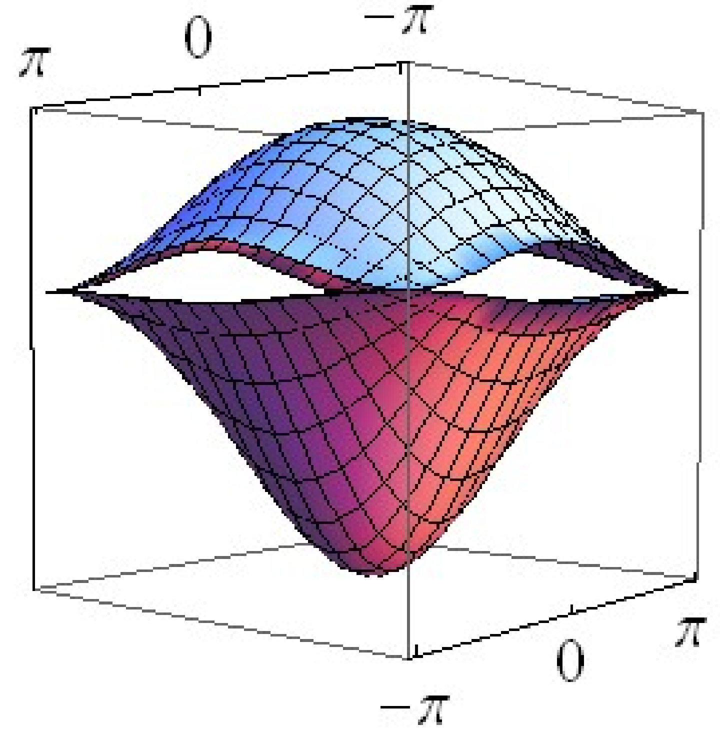
$u \to \infty$
$\varepsilon >> t$

Crossed-chain lattice



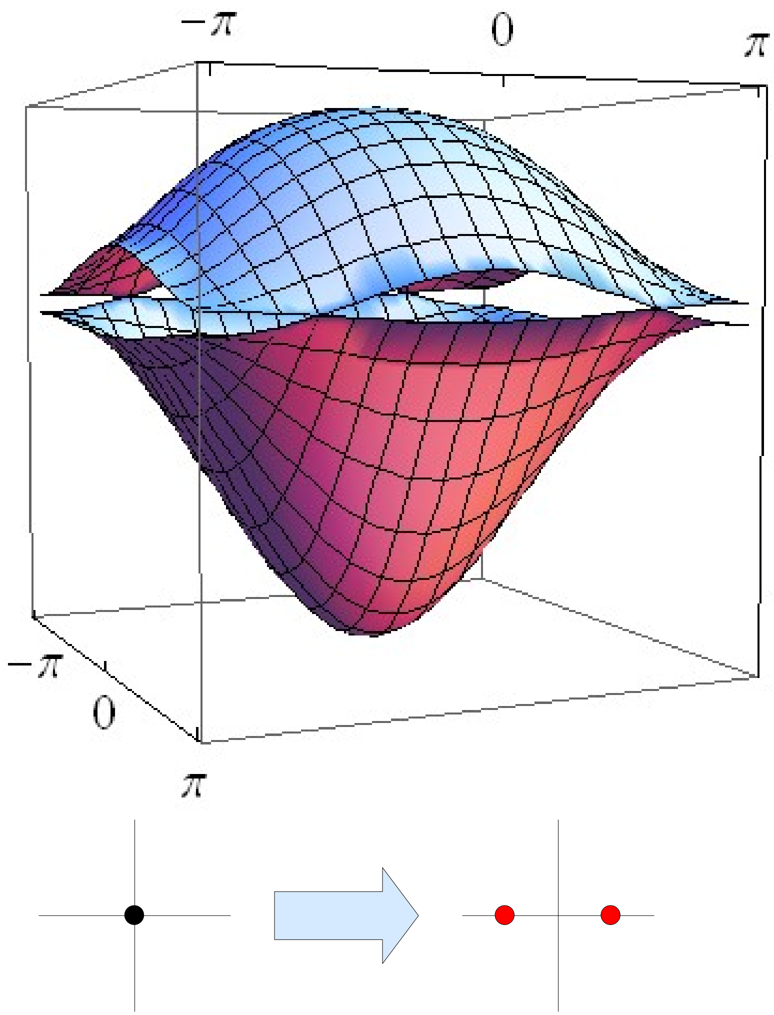
Degenerate Points

- band touching at half-filling (NOT Dirac point)
  - quadratic dispersion
  - “two Dirac points”
- protected by
  - 4-fold rotational symmetry
  - **T** symmetry
- unstable under interaction
  - Due to the finite DOS

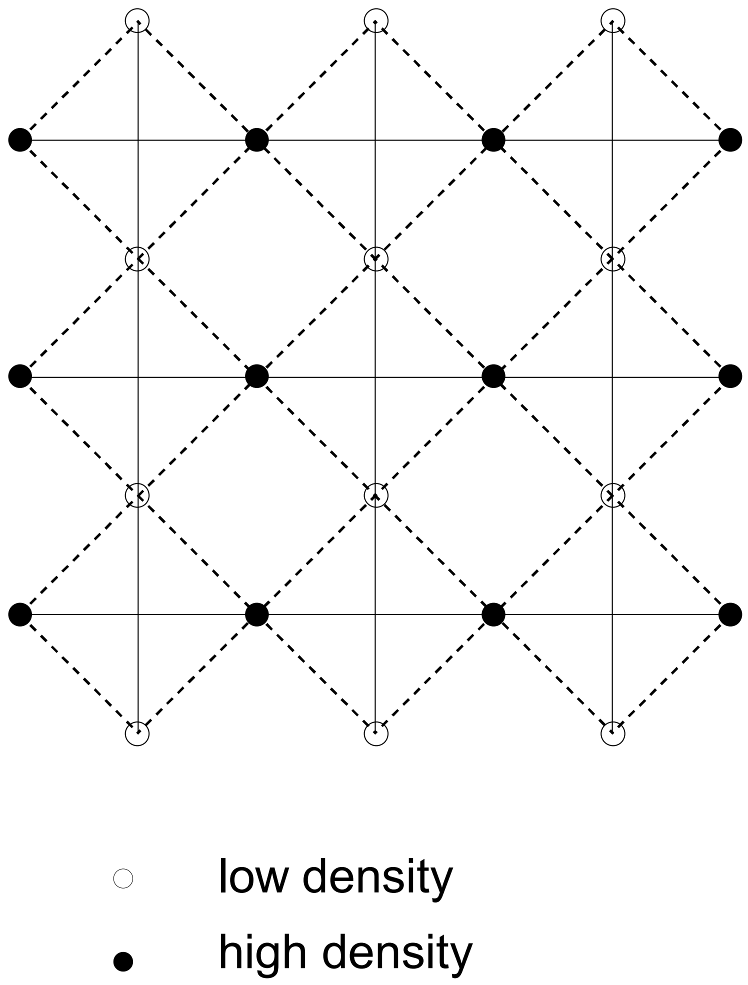


Nematic instability

Momentum space

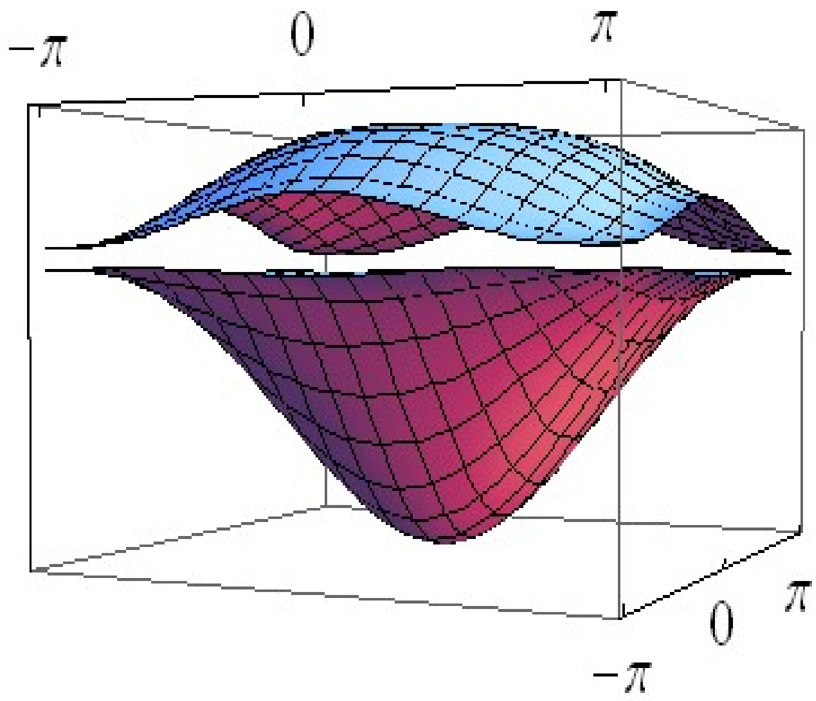


Real space

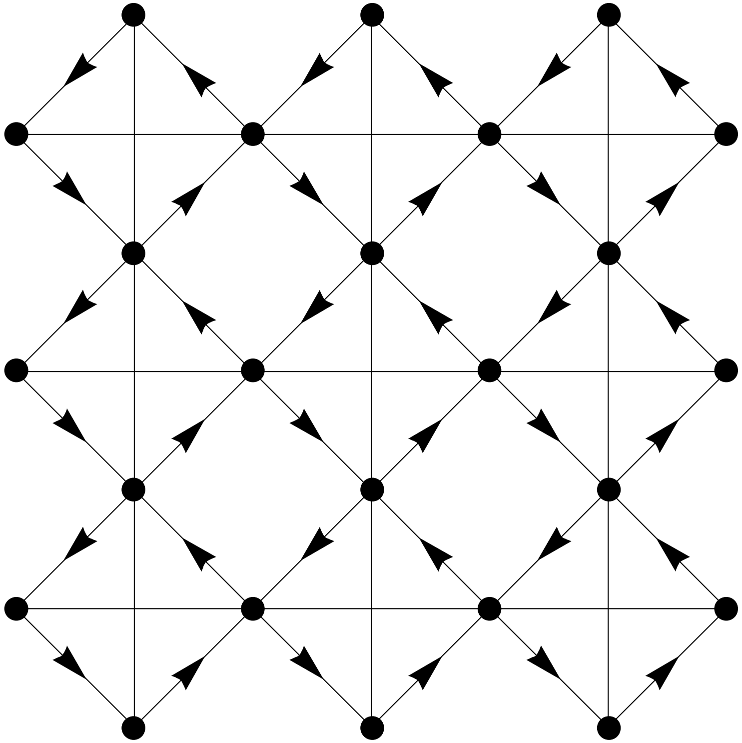


T and **C** symmetry breaking (type II)

Momentum space



Real space



Half filled crossed-chain lattice

- x=1 “cuprates”
- Infinitesimal instability
- Charge
  - Nematic semi-metal
  - **T** breaking insulator (energetically favored in weak-coupling limit)
- Spin interactions
  - Ferromagnetic: **T** breaking insulator
  - Anti-ferromagnetic:Spin Hall insulator

Conclusions

- General theory based on Berry phase (classification)
  - Type I (**T** and **I**) and II (**T** and **C**)
  - One-band model: type I only
  - Two-band model: type I or II
  - Multi-band models: type I, II or other
  - Type II states have spontaneous anomalous Hall effect
- Without band touching, Pomeranchuk type of interactions stabilize **T** symmetry breaking phases
  - Type I: Intra-band interactions + one transition
  - Type II: Inter-band interactions + two transitions
- With band touching,
  - May break **T** and **C** by a second order transition
  - Crossed-chain lattice has infinitesimal instability

T breaking and nematic

Coincidence?

Theory:

- β phase with L=2: nematic+T breaking
- Half-filled crossed-chain lattice

  nematic and T breaking phase competing

Experiments:

- underdoped $YBa_2Cu_3O_{6+y}$

  nematic+T symmetry breaking

- SrRuO

  Sr_2RuO_4 (p+ip superconductor) and $Sr_3Ru_2O_7$ (nematic)

- Quantum Hall fluid

  T breaking by B fields and nematic at 9/2 filling

Thank you