Measurement-Only Topological Quantum Computation

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work done in collaboration with:
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arXiv:0802.0279 (PRL '08) and arXiv:0808.1933

Introduction

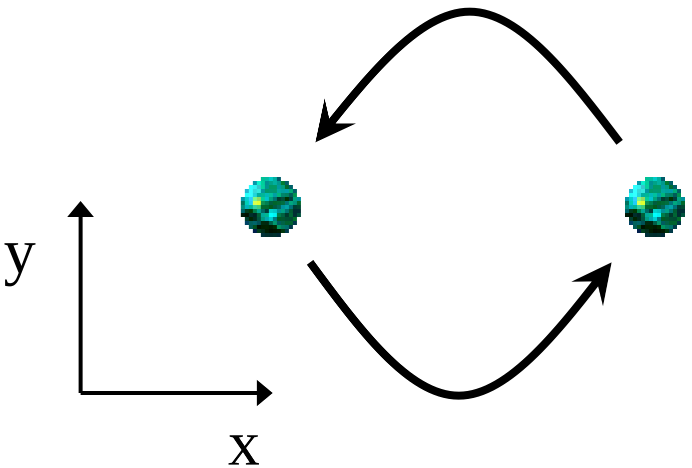
- Non-Abelian anyons are believed to exist in certain gapped two dimensional systems:

  - Fractional Quantum Hall Effect (ν=5/2, 12/5, …?)

  - ruthenates, topological insulators, rapidly rotating bose condensates, quantum loop gases/string nets?

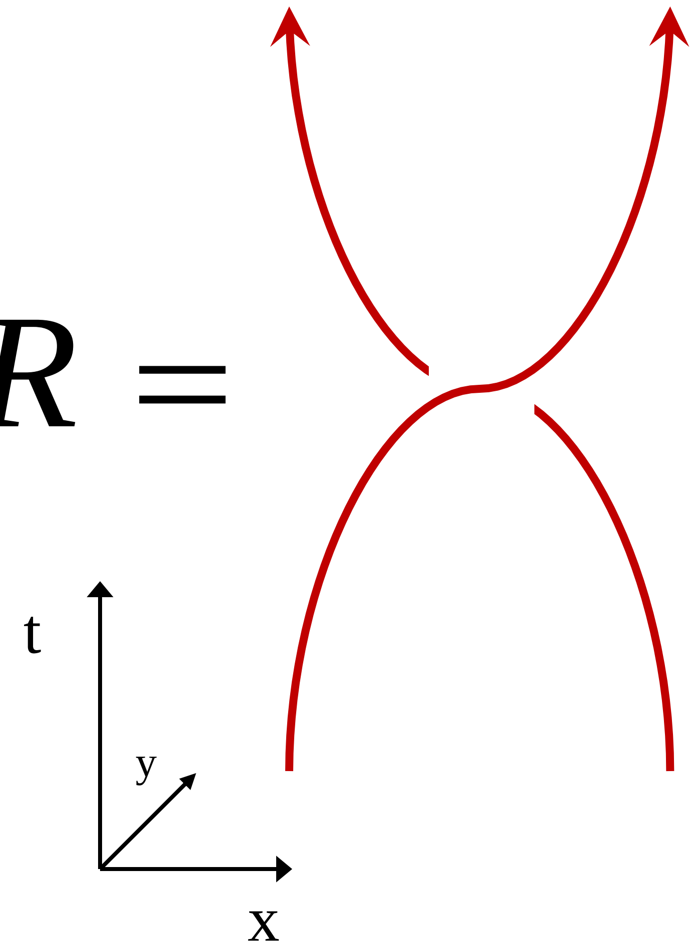
- If they exist, they could have application in quantum computation, providing naturally (“topologically protected”) fault-tolerant hardware.
- Assuming we have them at our disposal, what operations are necessary to implement topological quantum computation?

Particle Exchange "Statistics"

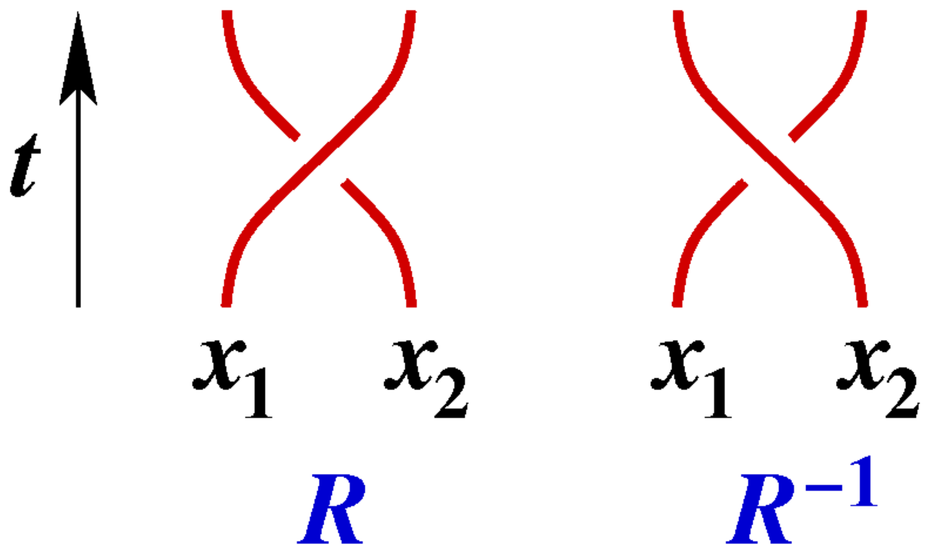


$$|\Psi\rangle \mapsto U[R]|\Psi\rangle$$

$R =$



Particle Exchange "Statistics"

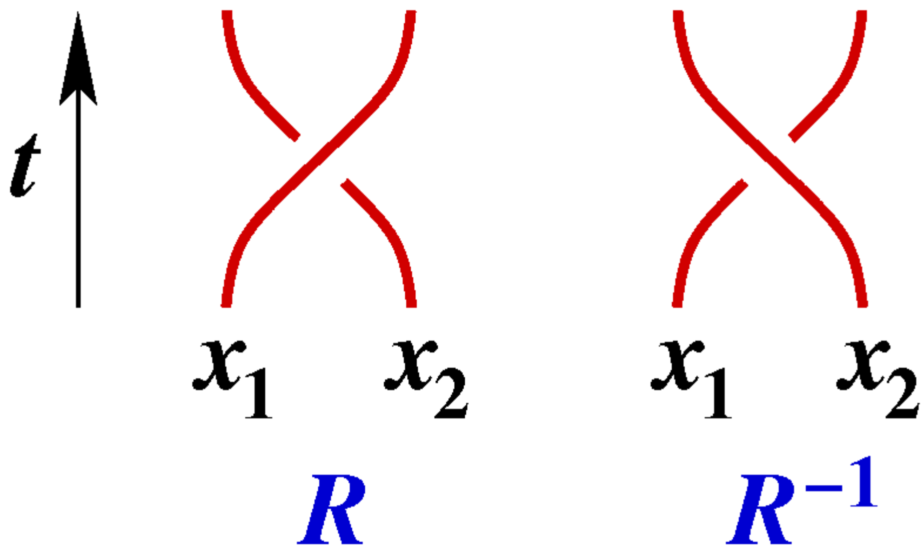


3 (and higher) spatial dimensions:

$$R = R^{-1} \quad \text{and} \quad R^2 = \mathbf{1}$$

- Only initial and final positions are topologically distinguished
- Statistics characterized by permutation group S_n
- Bosons and Fermions

Particle Exchange "Statistics"

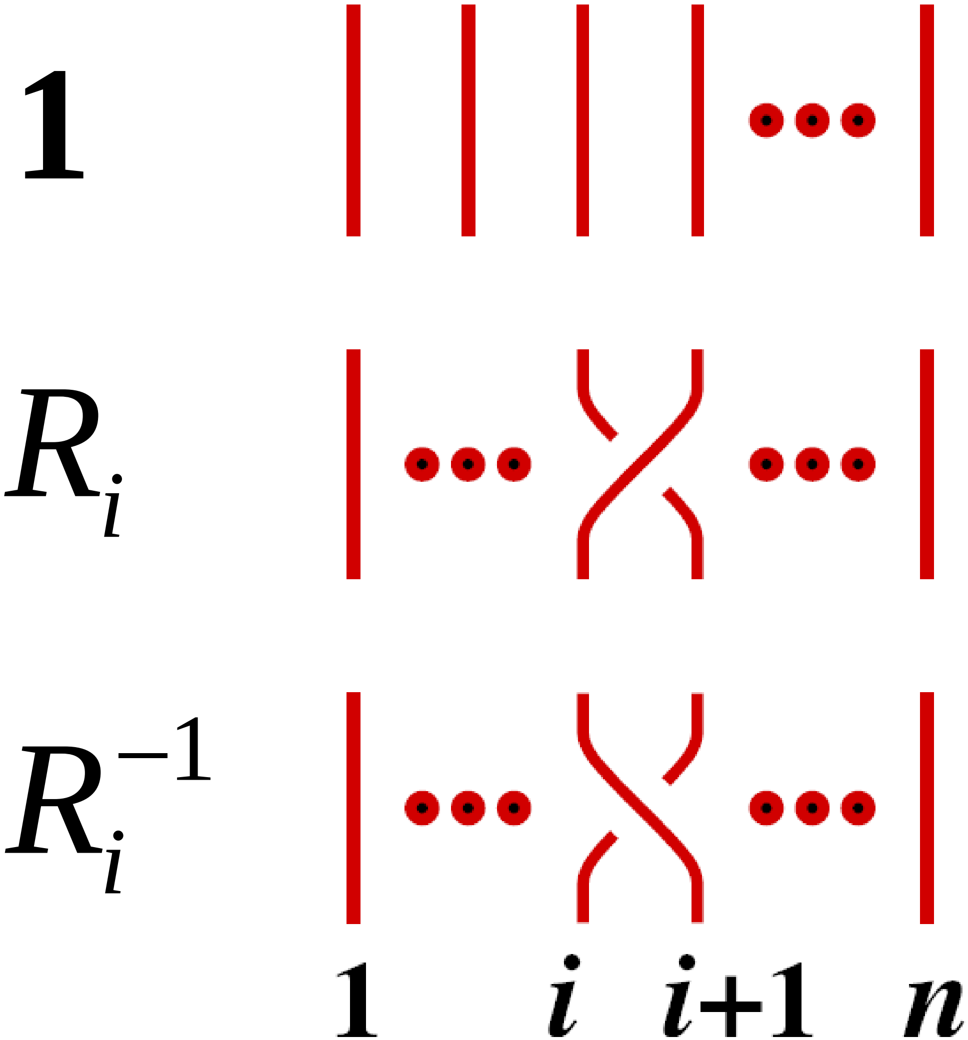


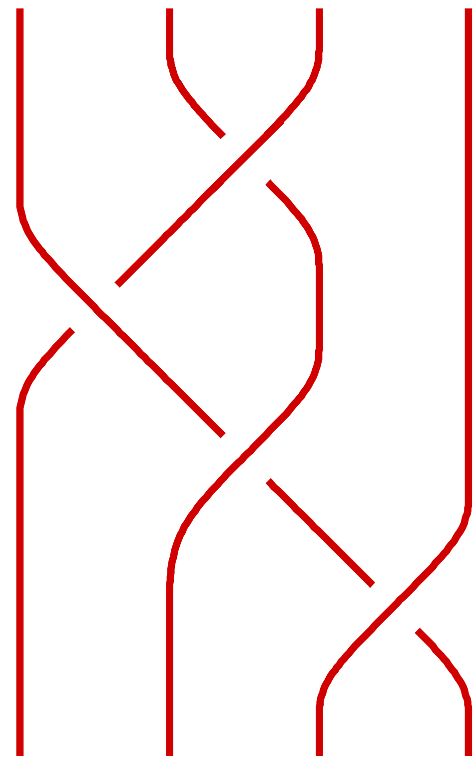
2 spatial dimensions:

$$R \neq R^{-1}$$

- Worldlines form topologically distinct braid configurations
- Statistics characterized by braid group B_n

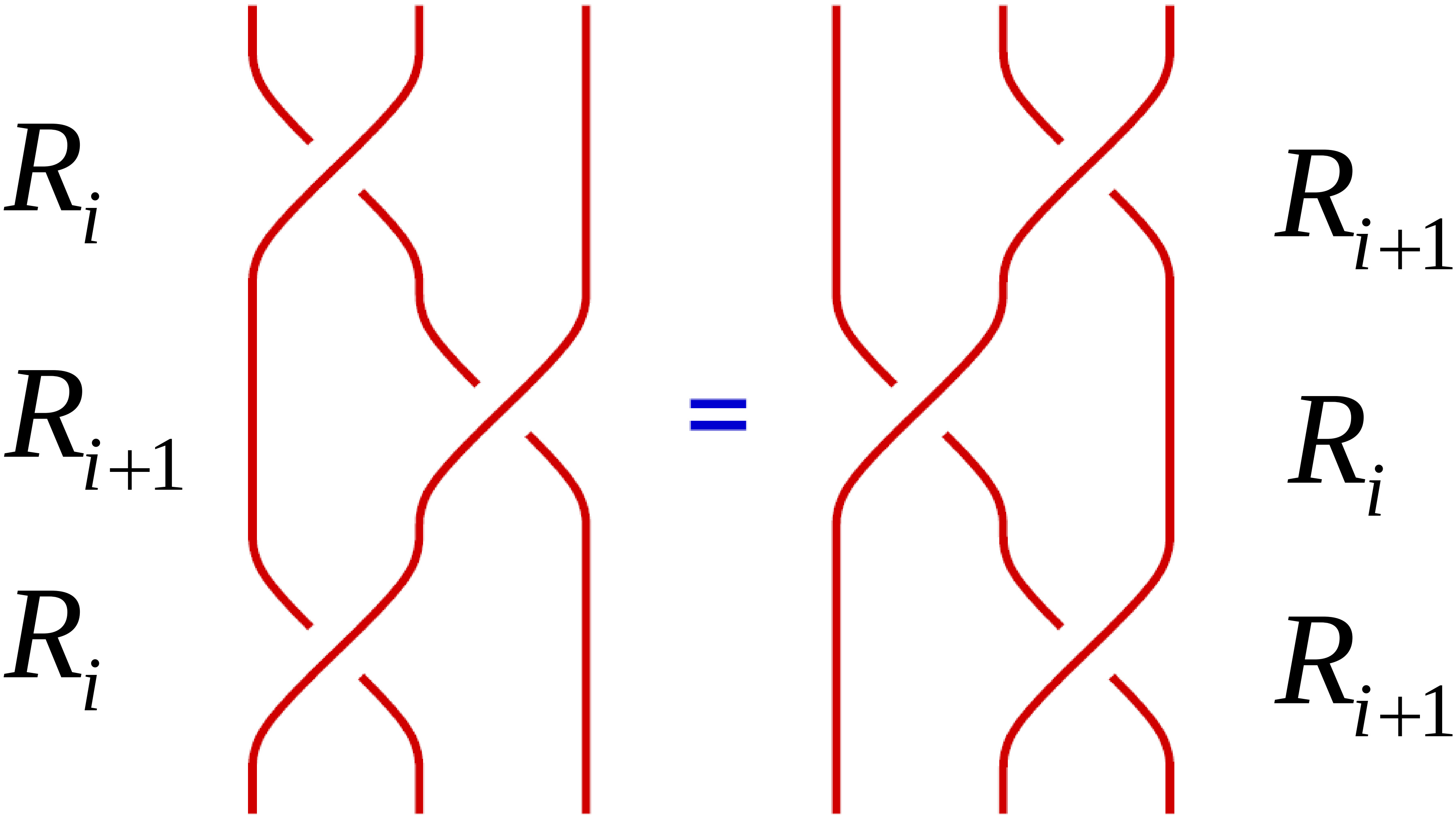
(n strand) braid group B_n



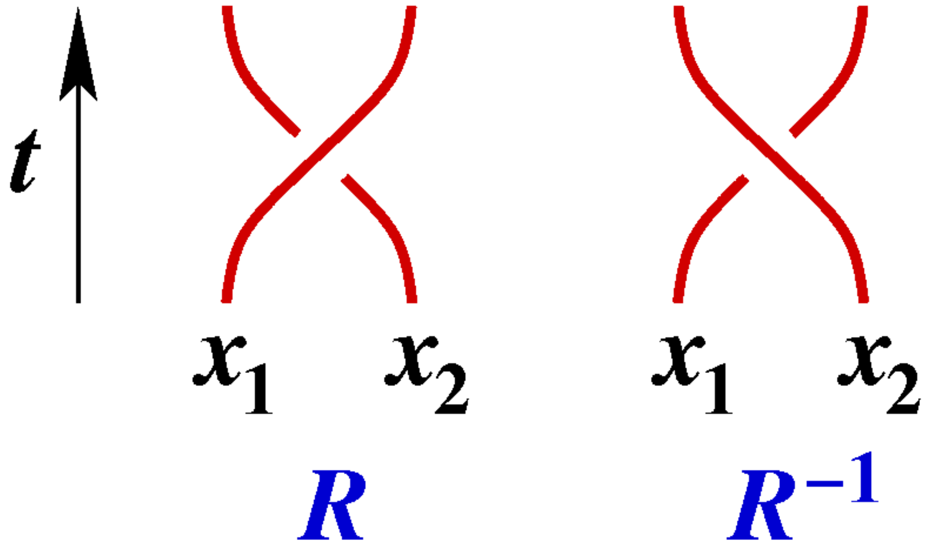


$R_2 R_1^{-1} R_2 R_3$

Yang - Baxter constraint : $R_i R_{i+1} R_i = R_{i+1} R_i R_{i+1}$



Particle Exchange "Statistics"



2 spatial dimensions:

$$R \neq R^{-1}$$

- Worldlines form topologically distinct braid configurations
- Statistics characterized by braid group B_n
- This gives…

Braiding "Statistics"

One dim unitary reps of B_n assign a phase to each braid generator:

$$U[R_i]|\Psi\rangle = e^{i\theta}|\Psi\rangle \quad \Rightarrow \quad \text{Abelian anyons}$$

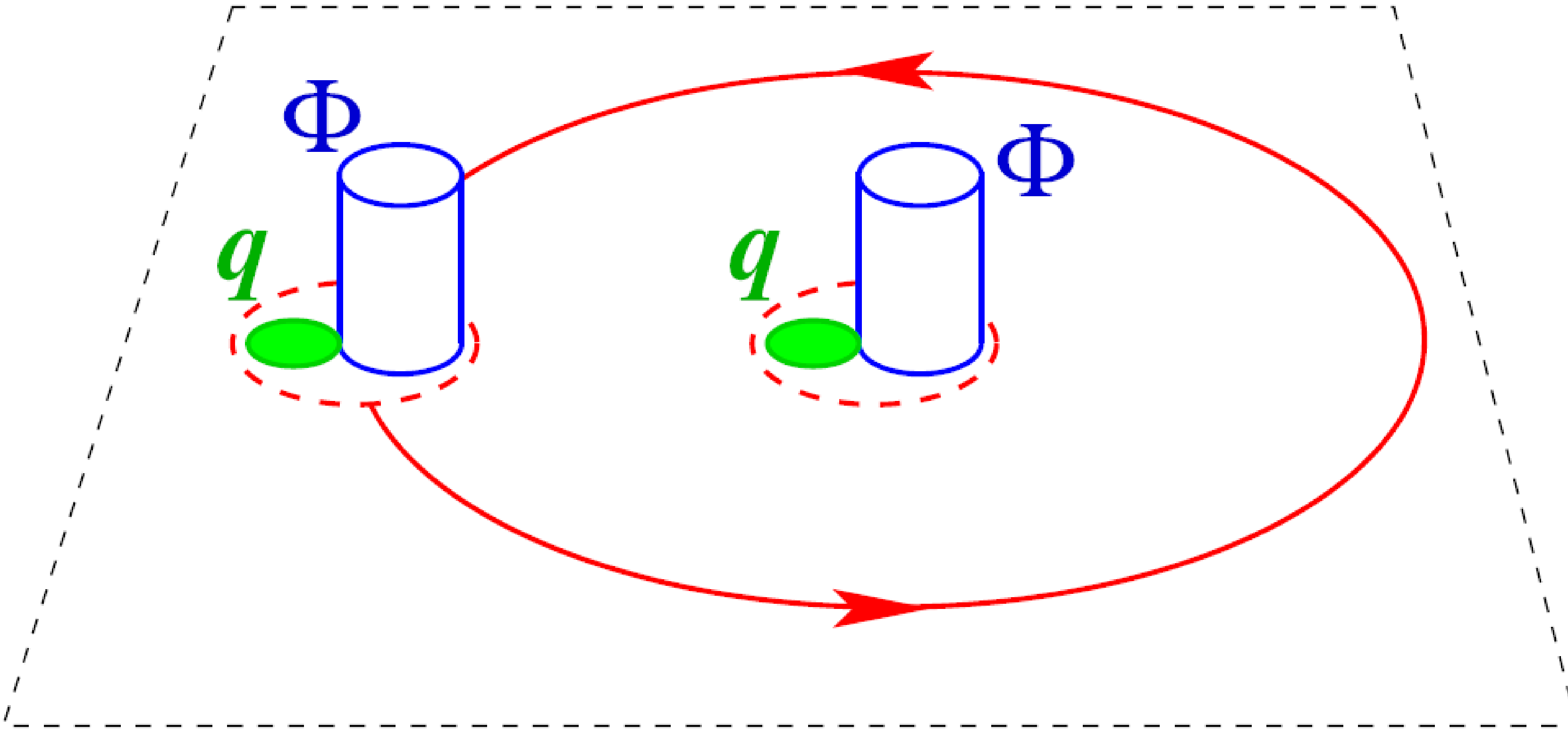
$$(\text{bosons}: \ \theta = 0, \ \text{fermions}: \ \theta = \pi)$$

Higher dim reps of B_n mean Hilbert space is multi-dimensional, and unitary **matrices** are assigned to braid generators:

$$U[R_i]|\Psi_\alpha\rangle = \sum_\beta U_{\alpha\beta}|\Psi_\beta\rangle \Rightarrow \text{non-Abelian anyons!}$$

Toy model of Abelian Anyons:

charge q - flux Φ composites

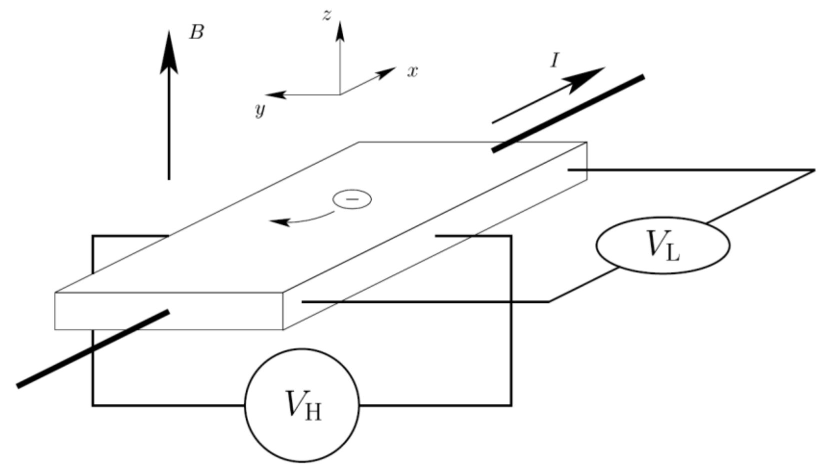


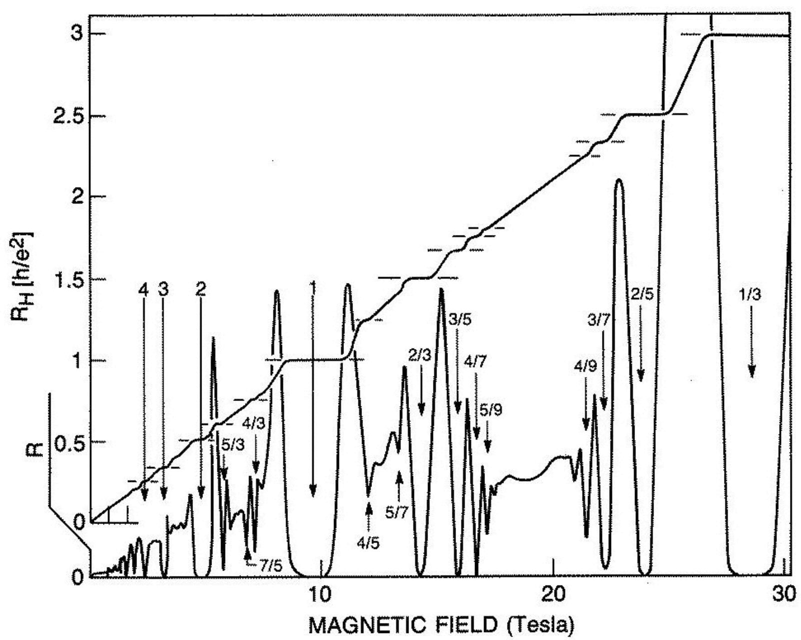
Aharonov - Bohm effect : $\theta = q\Phi$

Physical Anyons: Fractional Quantum Hall

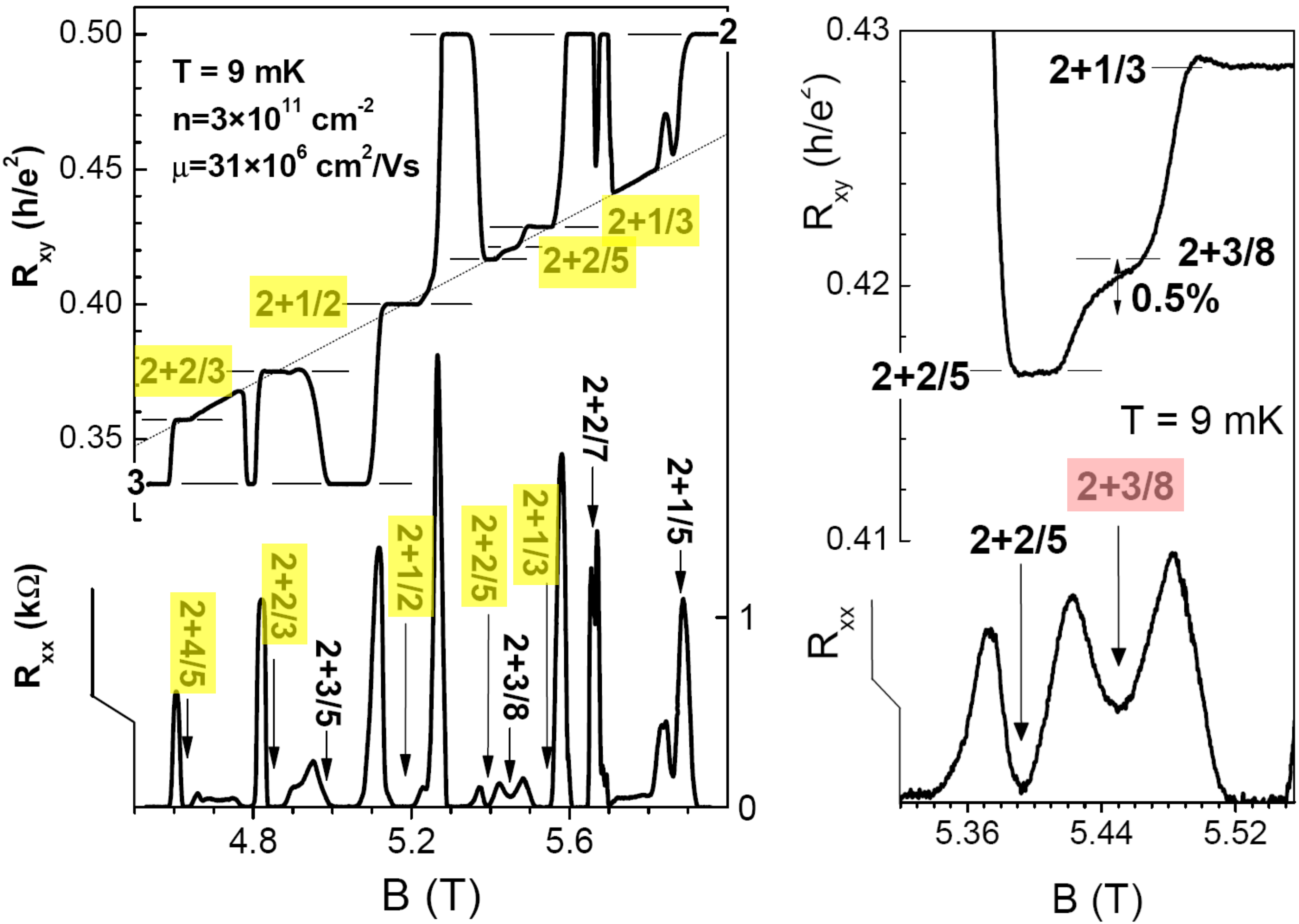
- 2DEG
- large B field (~ 10T)
- low temp (< 1K)
- gapped (incompressible)
- quantized filling fractions

$$\nu = \frac{n}{m}, \quad R_{xy} = \frac{1}{\nu}\frac{h}{e^2}, \quad R_{xx} = 0$$





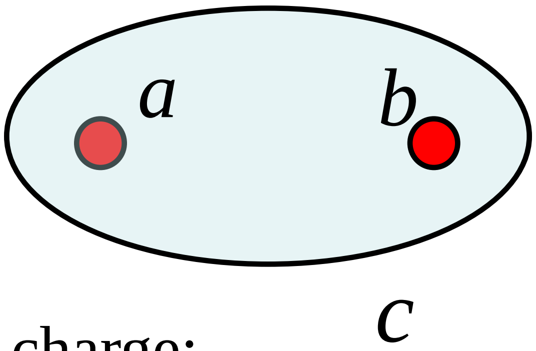
- fractionally charged quasiparticles
- Abelian anyons at most filling fractions $\theta = \pi \frac{p}{m}$
- non-Abelian anyons in 2^{nd} Landau level, e.g. ν= 5/2, 12/5, …



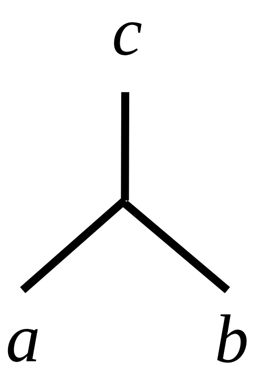
T = 9 mK
n=3×10^11 cm^-2
μ=31×10^6 cm^2/Vs
R_xy (h/e^2)
R_xx (kΩ)
0.50
0.45
0.40
0.35
2
3
2+2/3
2+1/2
2+2/5
2+1/3
2+4/5
2+2/3
2+3/5
2+1/2
2+2/5
2+3/8
2+1/3
2+2/7
2+1/5
1
0
4.8
5.2
5.6
B (T)
0.43
0.42
0.41
R_xy (h/e^2)
2+1/3
2+3/8
0.5%
2+2/5
T = 9 mK
2+3/8
2+2/5
R_xx
5.36
5.44
5.52
B (T)

non-Abelian anyons

Localized topological charge:



Non-local collective topological charge:
(multiple values are possible)



Fusion rules: $a \times b = \sum_c N_{ab}^c c$

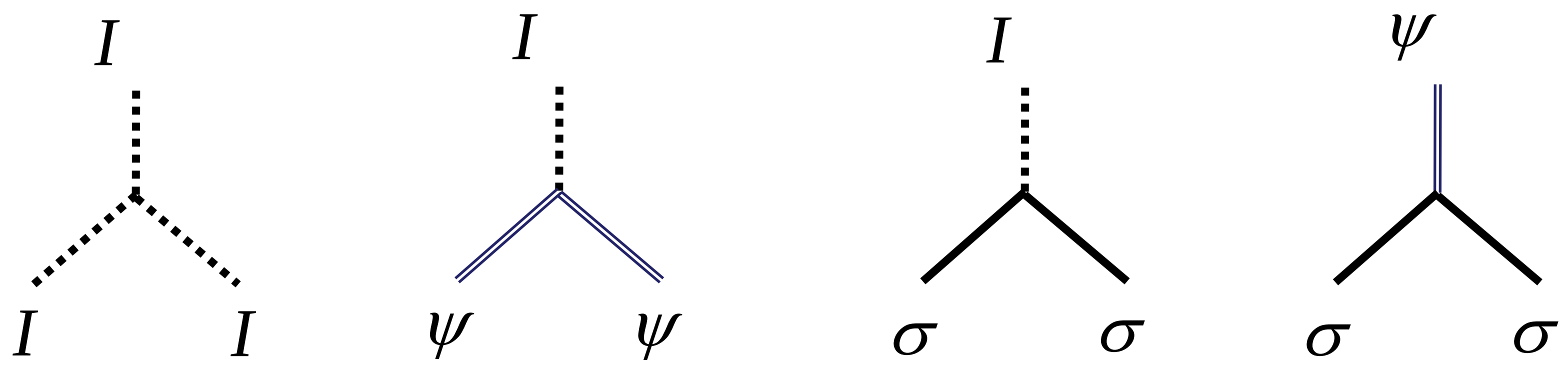
ang. mom. analog: $\frac{1}{2} \times \frac{1}{2} = 0 + 1$

Ising anyons

- $\nu = \frac{5}{2}$ FQH (Moore-Read `91)
- $\nu = \frac{12}{5}$ and other 2LL FQH? (PB and Slingerland `07)
- Kitaev honeycomb, topological insulators, ruthenates?

Topological charge types: $I,\ \sigma,\ \psi$

Fusion rules:



$$\psi \times \psi = I \qquad \sigma \times \sigma = I + \psi$$

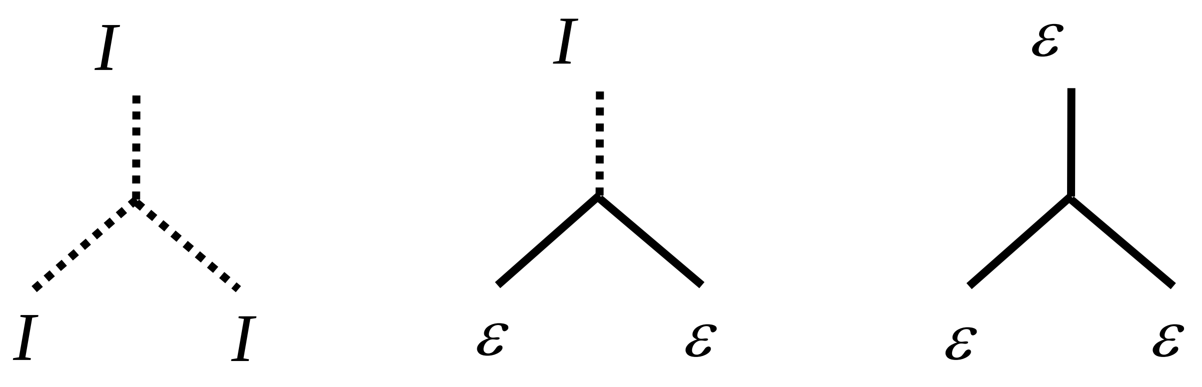
Fibonacci anyons

- $\nu = \frac{12}{5}$ FQH? (Read - Rezayi `98)

- string nets? (Levin - Wen `04, Fendley et. al. `08)

Particle types: $I, \ \varepsilon$

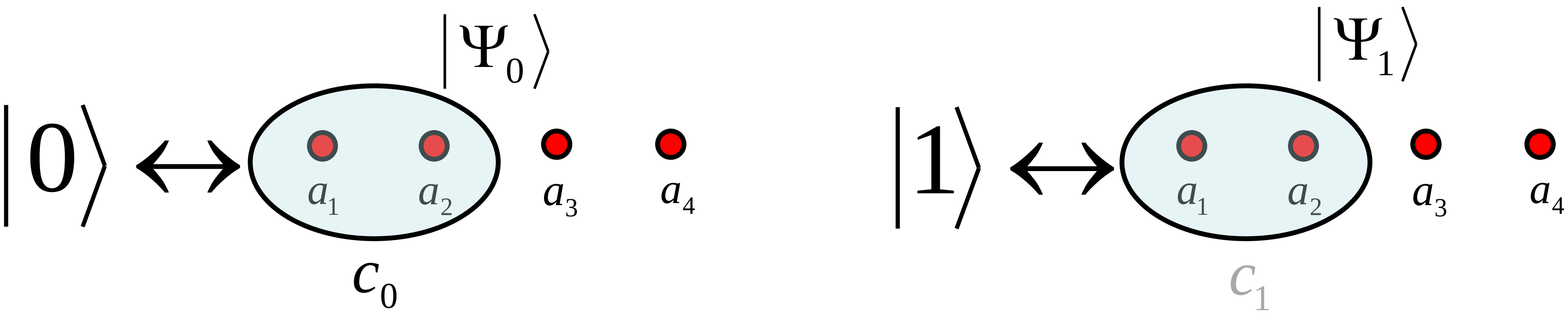
Fusion rules :



$$\varepsilon \times \varepsilon = I + \varepsilon$$

Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)



Topological Protection!

Ising: $a = \sigma,\ c_0 = I,\ c_1 = \psi$

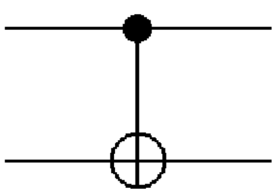
Fib: $a = \varepsilon,\ c_0 = I,\ c_1 = \varepsilon$

Topological Quantum Computation

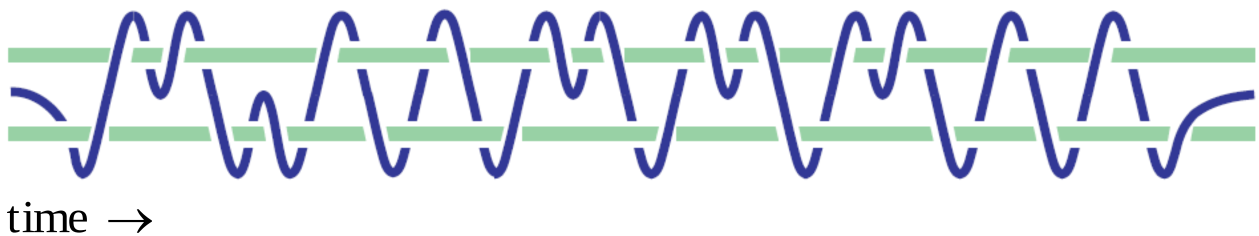
(Kitaev, Preskill, Freedman, Larsen, Wang)

$$|0\rangle \leftrightarrow |\Psi_0\rangle: \quad a_1 \quad a_2 \quad a_3 \quad a_4$$

$$|1\rangle \leftrightarrow |\Psi_1\rangle: \quad a_1 \quad a_2 \quad a_3 \quad a_4$$



$\leftrightarrow$



(Bonesteel, et. al.)

Is braiding computationally universal?

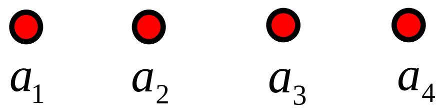
Ising: not quite
(must be supplemented)

Fib: yes!

Topological Quantum Computation

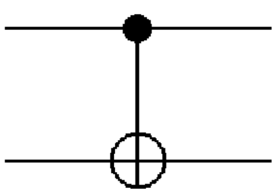
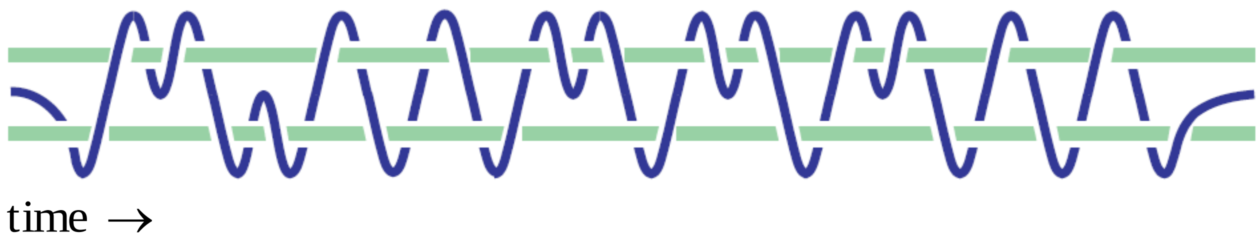
(Kitaev, Preskill, Freedman, Larsen, Wang)

$|\Psi_0\rangle$

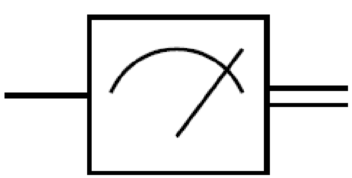
$|0\rangle \leftrightarrow$ 

$|\Psi_1\rangle$

$|1\rangle \leftrightarrow$ 

 $\leftrightarrow$ 

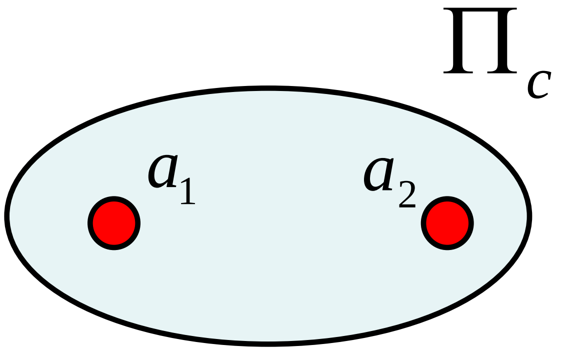
(Bonesteel, et. al.)

 $\leftrightarrow$ Topological Charge Measurement

Topological Charge Measurement
(measures anyonic state)

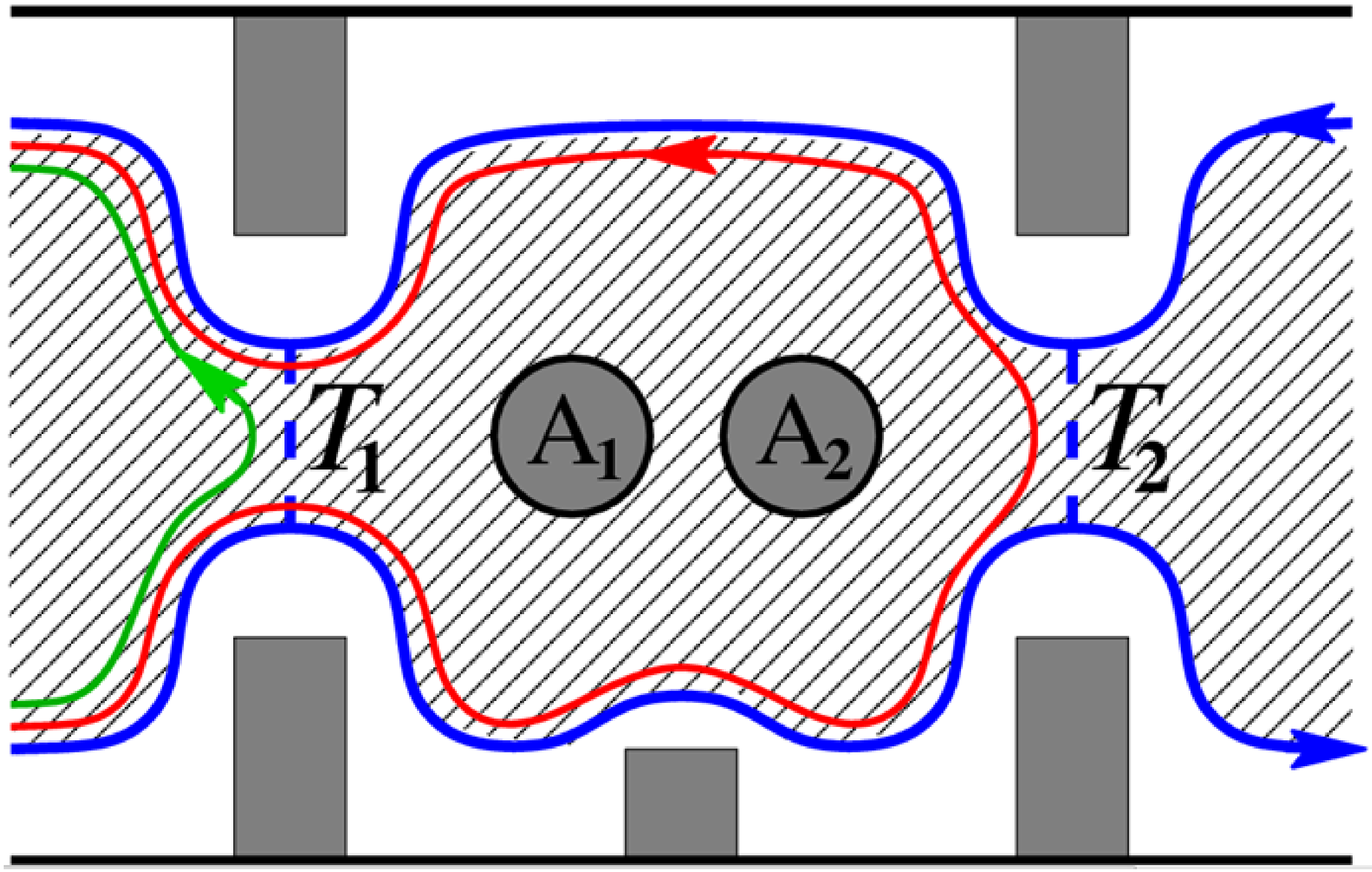
$$\Pi_c = |a_1, a_2; c\rangle\langle a_1, a_2; c|$$

$$|\Psi\rangle \mapsto \frac{\Pi_c|\Psi\rangle}{\langle\Psi|\Pi_c|\Psi\rangle}$$

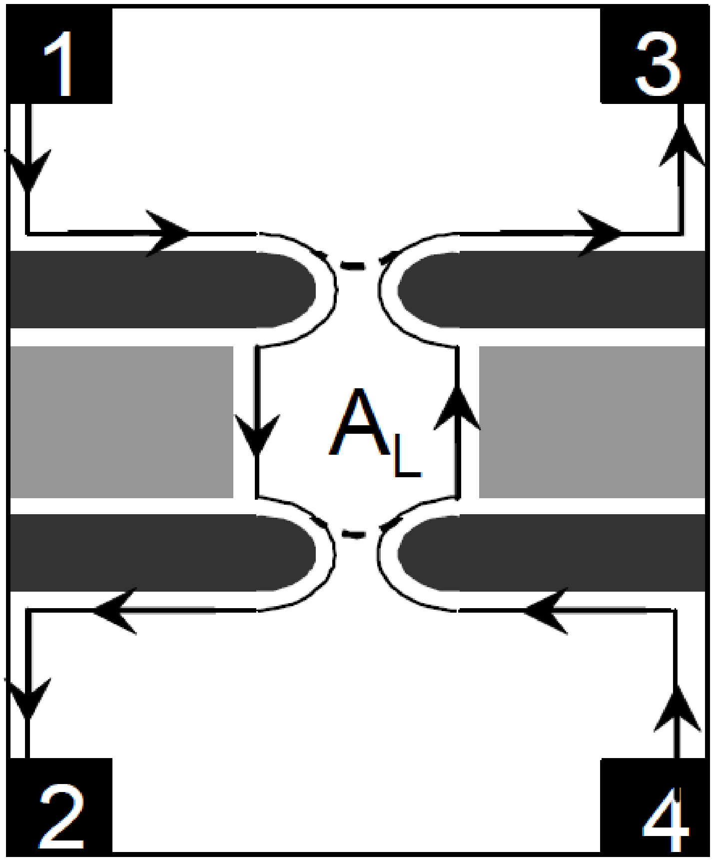


Topological Charge Measurement

e.g. FQH double point contact interferometer



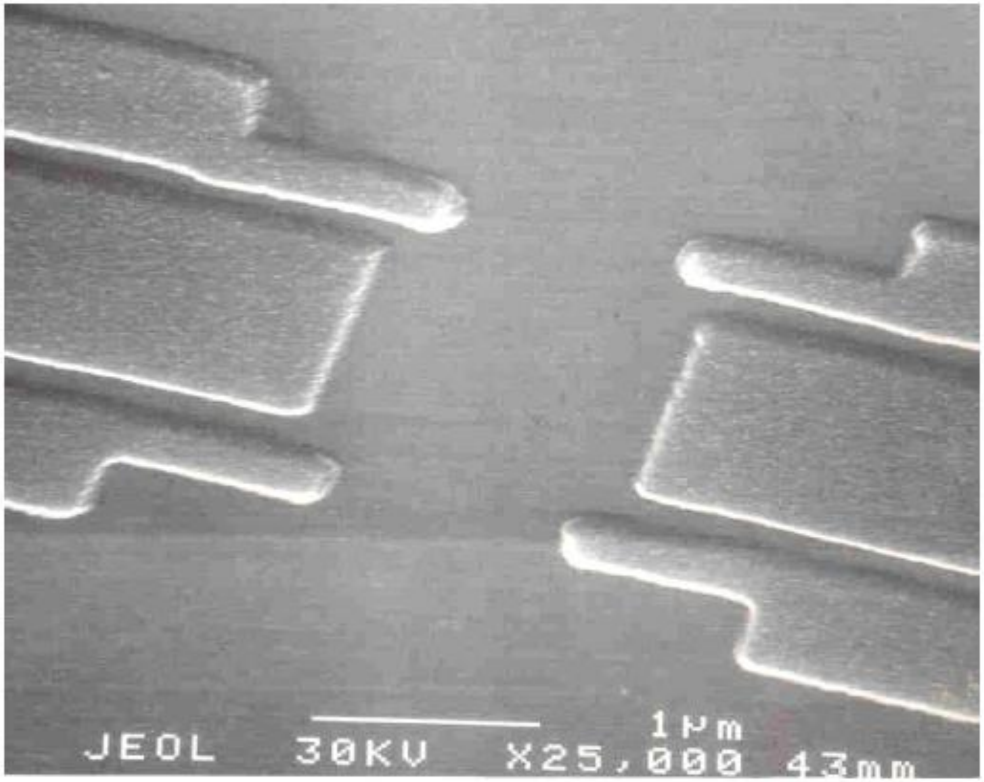
FQH interferometer



Willett, et. al. `08

for ν=5/2

(also progress by: Marcus, Eisenstein, Kang, Heiblum, Goldman, etc.)



Quantum State Teleportation

(for spin ½ systems)

Entanglement Resource: maximally entangled Bell states

$$\left|\Psi^-\right\rangle = \tfrac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big) = \mathbf{1}\otimes\sigma_0\left|\Psi^-\right\rangle$$

$$\left|\Phi^-\right\rangle = \tfrac{1}{\sqrt{2}}\Big(\left|\uparrow\uparrow\right\rangle - \left|\downarrow\downarrow\right\rangle\Big) = \mathbf{1}\otimes\sigma_1\left|\Psi^-\right\rangle$$

$$\left|\Phi^+\right\rangle = \tfrac{1}{\sqrt{2}}\Big(\left|\uparrow\uparrow\right\rangle + \left|\downarrow\downarrow\right\rangle\Big) = i\mathbf{1}\otimes\sigma_2\left|\Psi^-\right\rangle$$

$$\left|\Psi^+\right\rangle = \tfrac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle\Big) = \mathbf{1}\otimes\sigma_3\left|\Psi^-\right\rangle$$

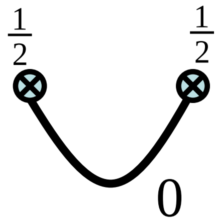
$$\left|\Phi_\mu\right\rangle = \mathbf{1}\otimes\sigma_\mu\left|\Psi^-\right\rangle \qquad \mu = 0,1,2,3$$

Quantum State Teleportation

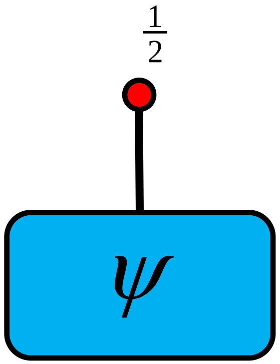
(for spin ½ systems)

Entanglement Resource: maximally entangled Bell pair

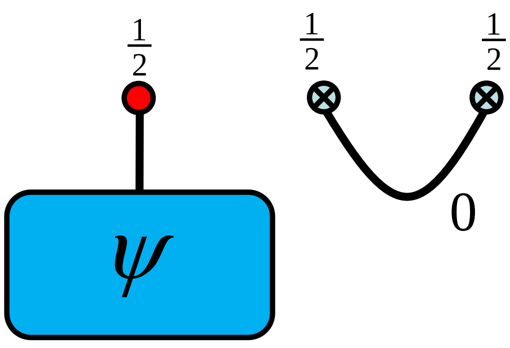
$$\left|\Phi_0\right\rangle = \left|\tfrac{1}{2},\tfrac{1}{2};0\right\rangle =$$



Want to teleport: $\left|\psi\right\rangle = \psi_\uparrow \left|\uparrow\right\rangle + \psi_\downarrow \left|\downarrow\right\rangle =$



Form: $\left|\psi\right\rangle_1 \left|\Phi_0\right\rangle_{23} =$



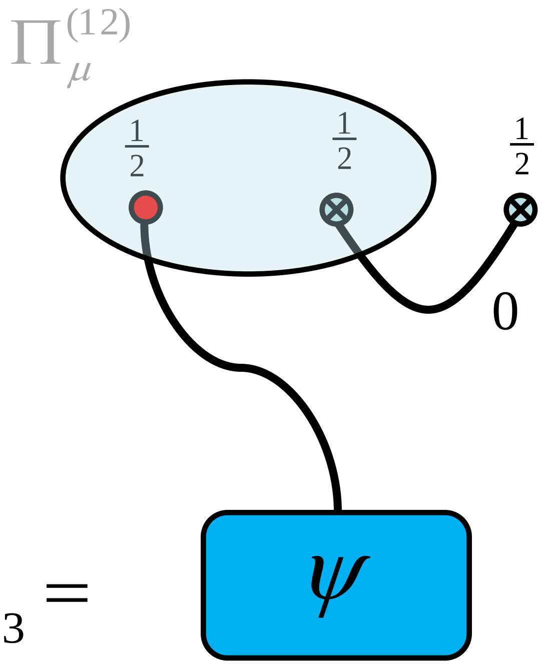
and perform a measurement on spins 12

Quantum State Teleportation

(for spin ½ systems)

Measurement

$$|\psi\rangle_1 |\Phi_0\rangle_{23} =$$



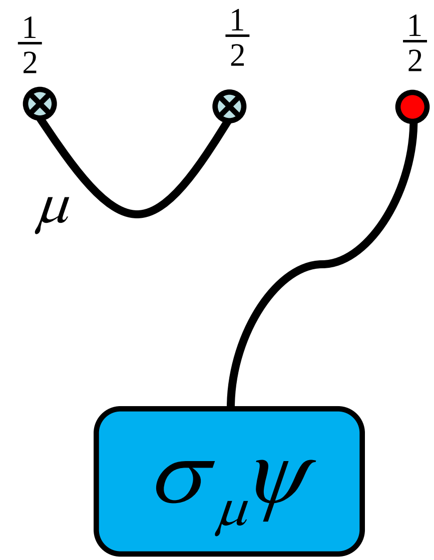
Quantum State Teleportation

(for spin ½ systems)

Measurement

$$\Pi_{\mu}^{(12)} : \left|\psi\right\rangle_1 \left|\Phi_0\right\rangle_{23}$$

$$\mapsto \left|\Phi_{\mu}\right\rangle_{12} \sigma_{\mu} \left|\psi\right\rangle_3 =$$



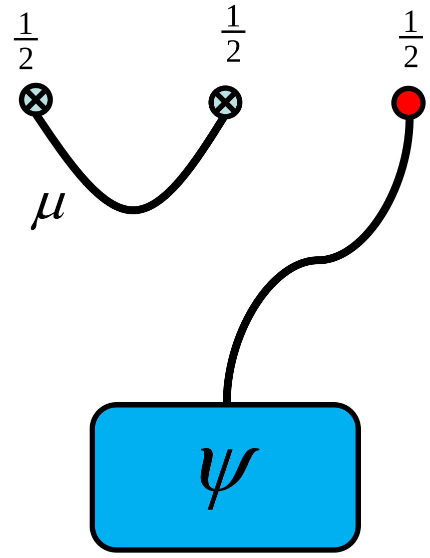
Now send two bits of classical info (the measurement result μ) from Alice to Bob and "fix" the state by applying the transformation σ_{μ} to spin 3

Quantum State Teleportation

(for spin ½ systems)

Measurement

$$\sigma_\mu^{(3)}\Pi_\mu^{(12)} : |\psi\rangle_1 |\Phi_0\rangle_{23} \mapsto |\Phi_\mu\rangle_{12} |\psi\rangle_3 =$$



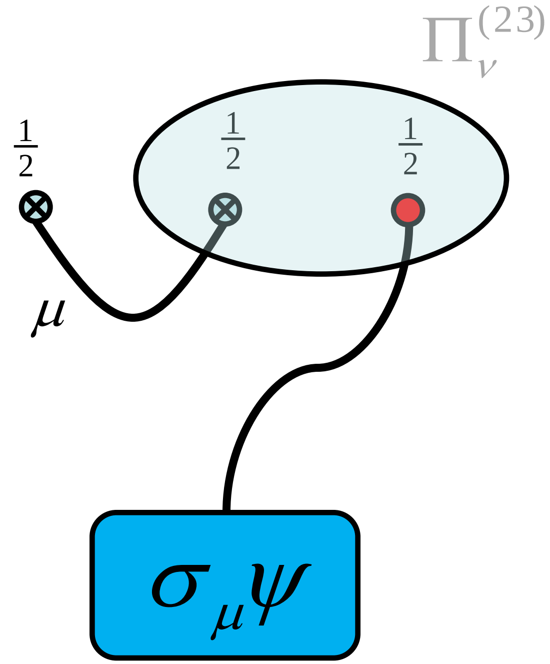
Now send two bits of classical info (the measurement result μ) from Alice to Bob and "fix" the state by applying the transformation σ_μ to spin 3

Quantum State Teleportation

(for spin ½ systems)

Alternative "fix":

Recombine and measure
the state of spins 23



$$\Pi_{\mu}^{(12)} : |\psi\rangle_{1} |\Phi_{0}\rangle_{23}$$

$$\mapsto |\Phi_{\mu}\rangle_{12} \sigma_{\mu} |\psi\rangle_{3} =$$

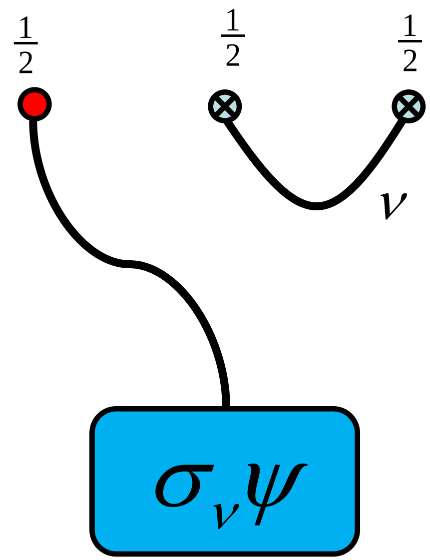
Quantum State Teleportation

(for spin ½ systems)

Alternative "fix":

Recombine and measure
the state of spins 23

Then try again:



$$\Pi_\nu^{(23)} \Pi_\mu^{(12)} : |\psi\rangle_1 |\Phi_0\rangle_{23}$$

$$\mapsto \sigma_\nu |\psi\rangle_1 |\Phi_\nu\rangle_{23} =$$

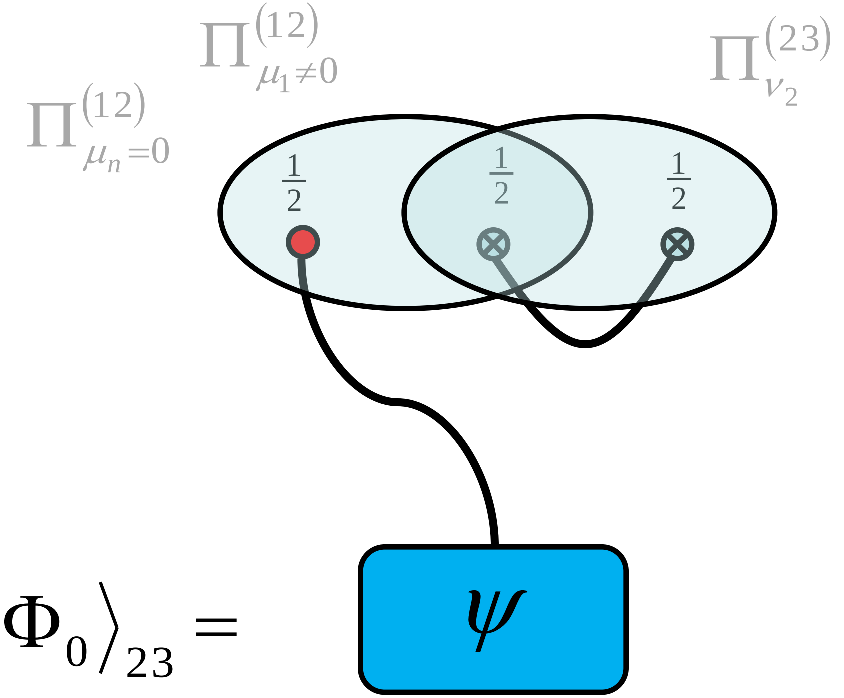
If measurement outcome is $\mu_n = 0$ then STOP! ("success")

If not REPEAT.

Quantum State Teleportation

(for spin ½ systems)

"Forced Measurement"



$$\left|\psi\right\rangle_1 \left|\Phi_0\right\rangle_{23} =$$

Quantum State Teleportation

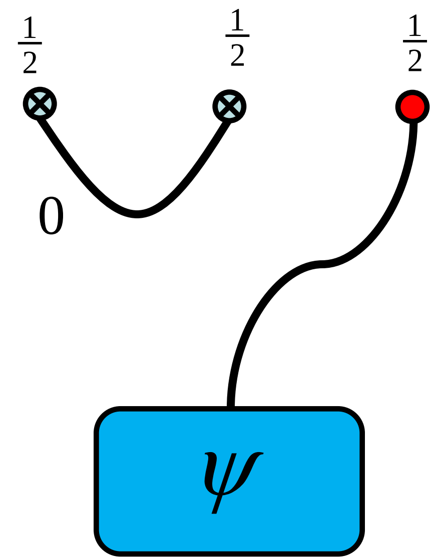
(for spin ½ systems)

“Forced Measurement”

$$\breve{\Pi}_0^{(12)} \equiv \Pi^{(12)}_{\mu_n=0}\Pi^{(23)}_{\nu_n}\ldots\Pi^{(23)}_{\nu_2}\Pi^{(12)}_{\mu_1}$$

$$\breve{\Pi}_0^{(12)} : |\psi\rangle_1 |\Phi_0\rangle_{23}$$

$$\mapsto |\Phi_0\rangle_{12} |\psi\rangle_3 =$$

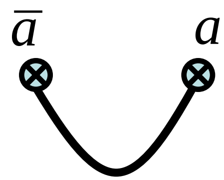


“Success” occurs with probability $= \frac{1}{4}$ for each repeat try.

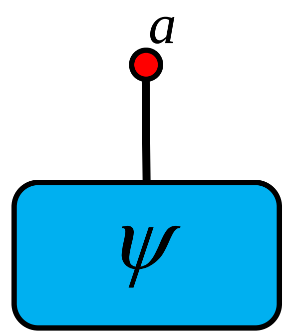
Anyonic State Teleportation

Entanglement Resource: maximally entangled anyon pair

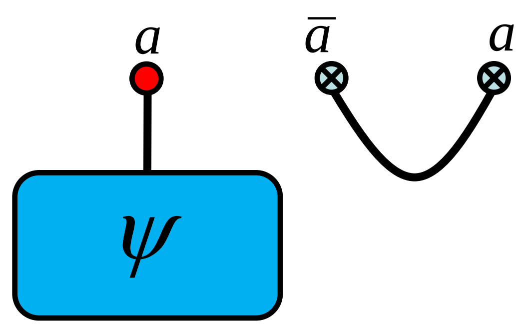
$$\left|\bar{a}, a; 0\right\rangle =$$



Want to teleport: $\left|\psi\right\rangle =$



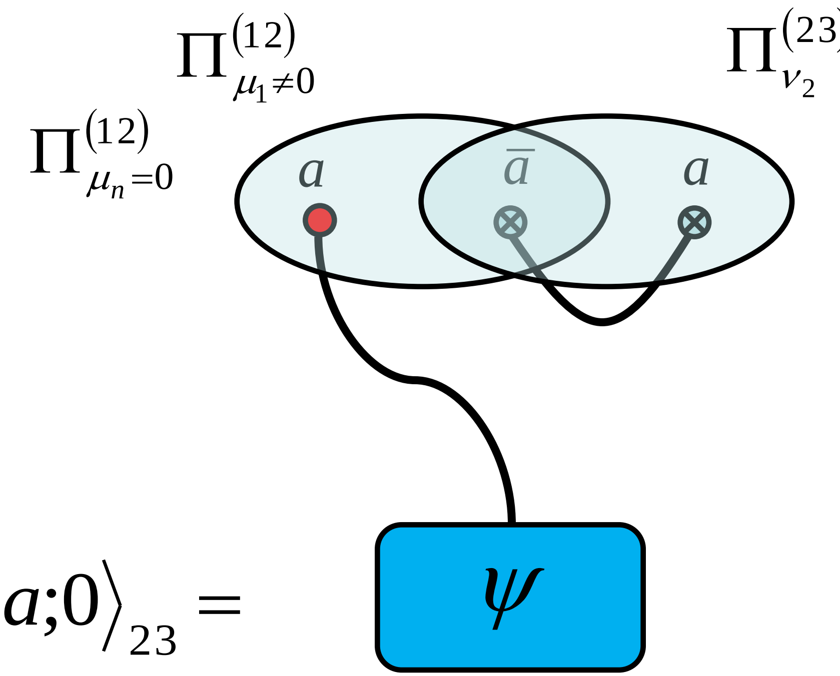
Form: $\left|\psi\right\rangle_1 \left|\bar{a}, a; 0\right\rangle_{23} =$



and perform Forced Measurement on anyons 12

Anyonic State Teleportation

Forced Measurement



$$\left|\psi\right\rangle_1 \left|\bar{a}, a; 0\right\rangle_{23} =$$

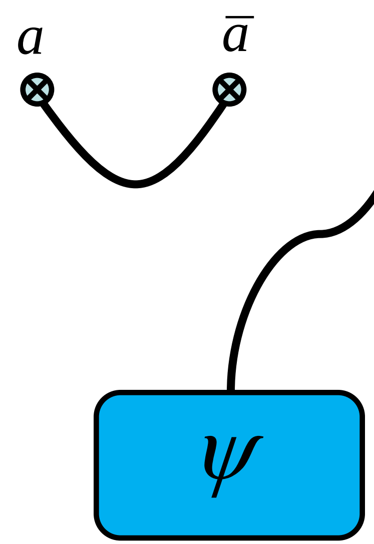
Anyonic State Teleportation

Forced Measurement

$$\breve{\Pi}_0^{(12)} \equiv \Pi_{\mu_n=0}^{(12)} \Pi_{\nu_n}^{(23)} \ldots \Pi_{\nu_2}^{(23)} \Pi_{\mu_1}^{(12)}$$

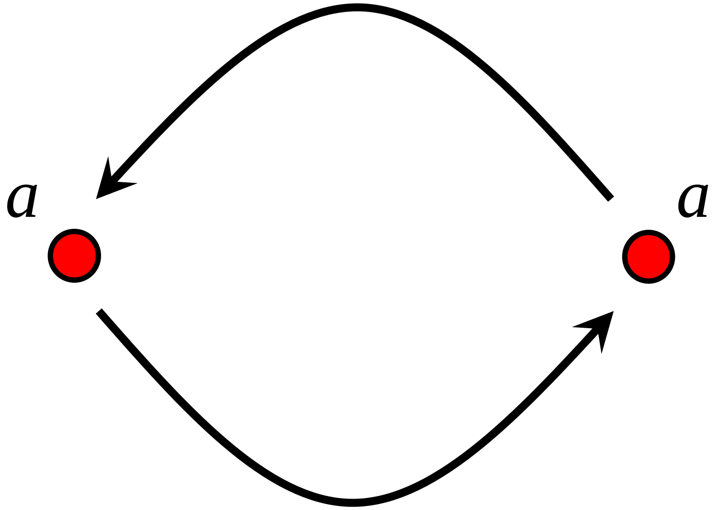
$$\breve{\Pi}_0^{(12)} : \left|\psi\right\rangle_1 \left|\bar{a}, a; 0\right\rangle_{23}$$

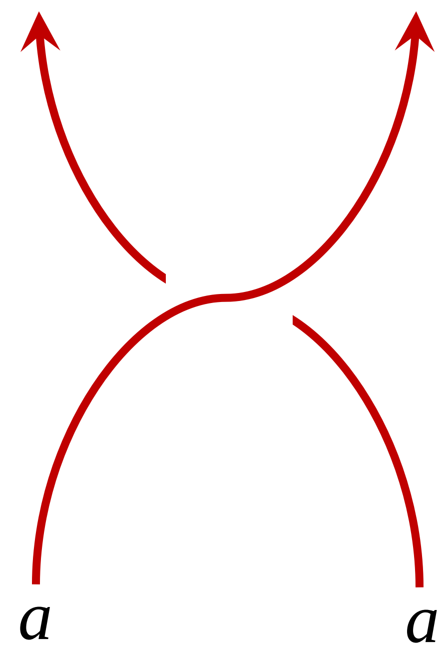
$$\mapsto \left|a, \bar{a}; 0\right\rangle_{12} \left|\psi\right\rangle_3 =$$



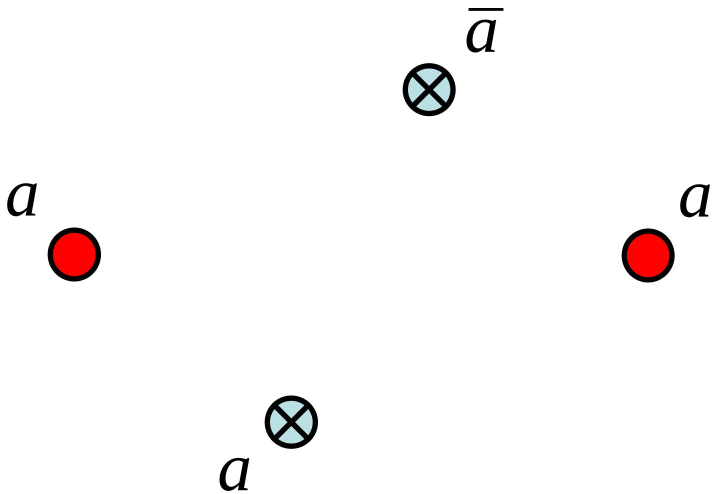
"Success" occurs with probability $\geq \frac{1}{d_a^2}$ for each repeat try.

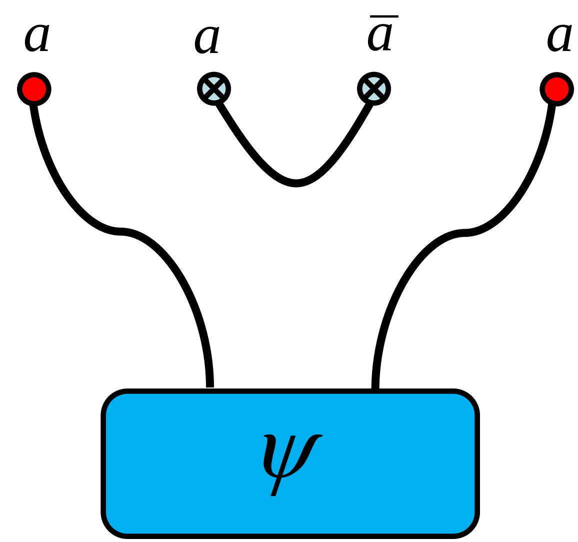
What good is this if we want to braid computational anyons?



$R =$ 

Use a maximally entangled pair and "forced measurements" for a series of teleportations





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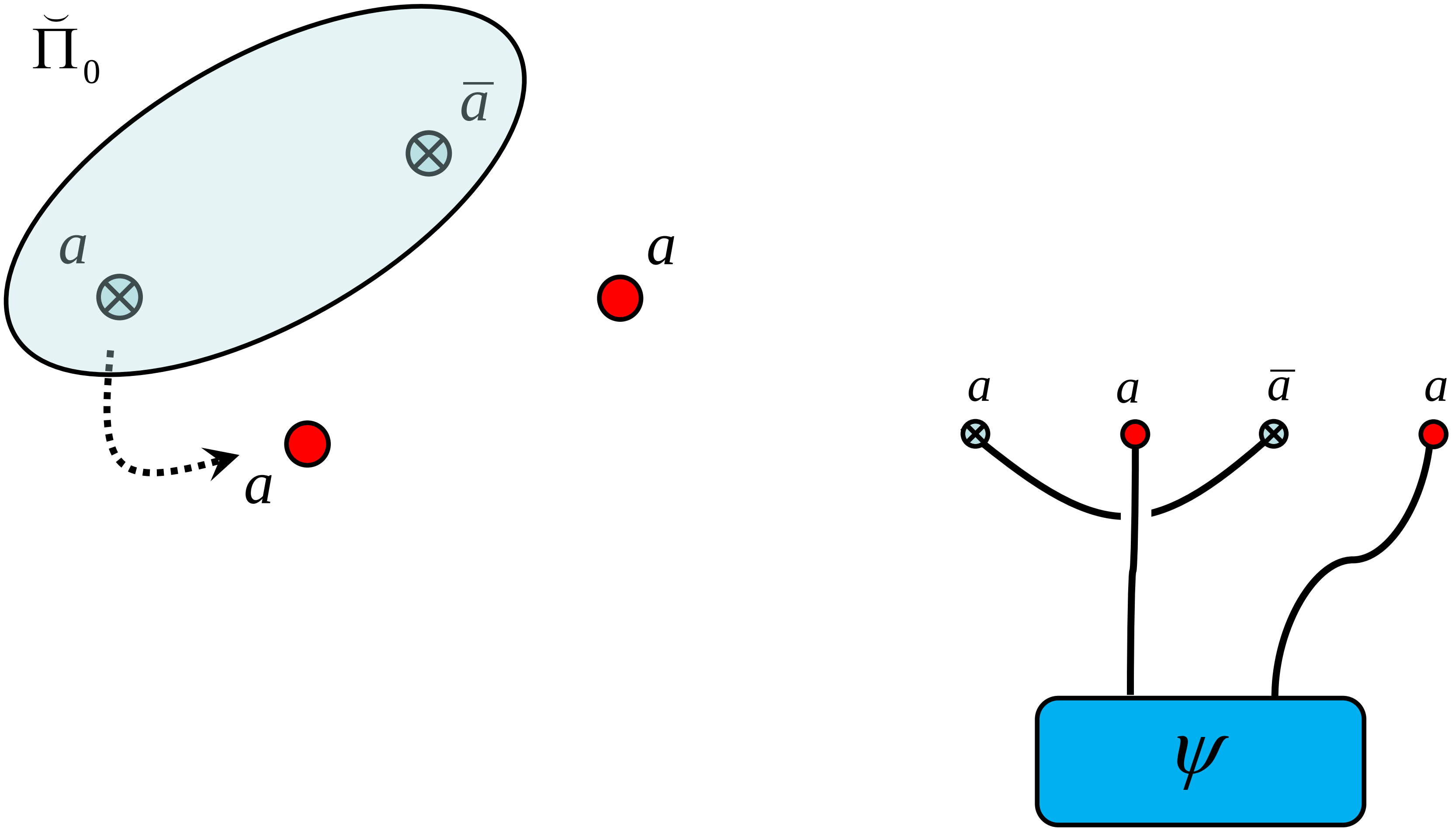

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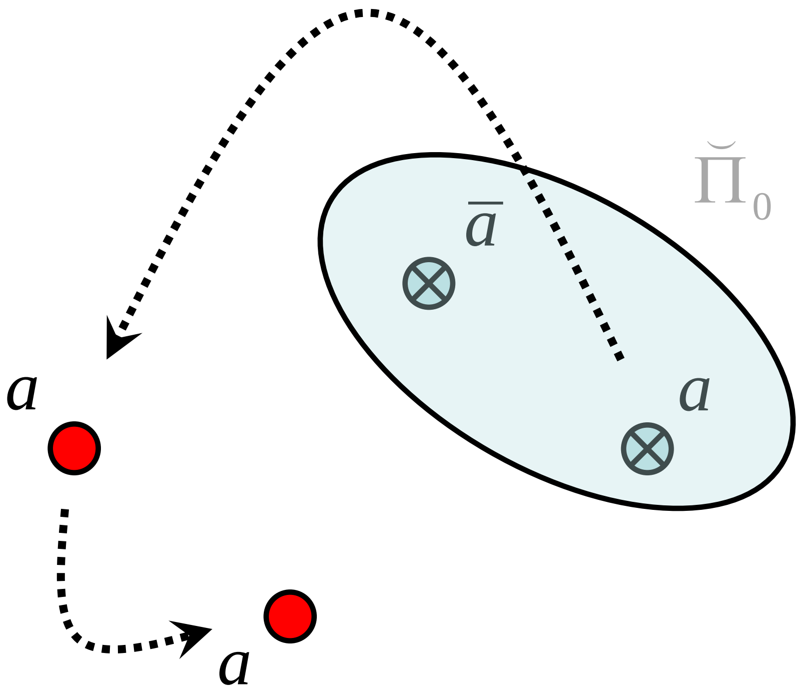

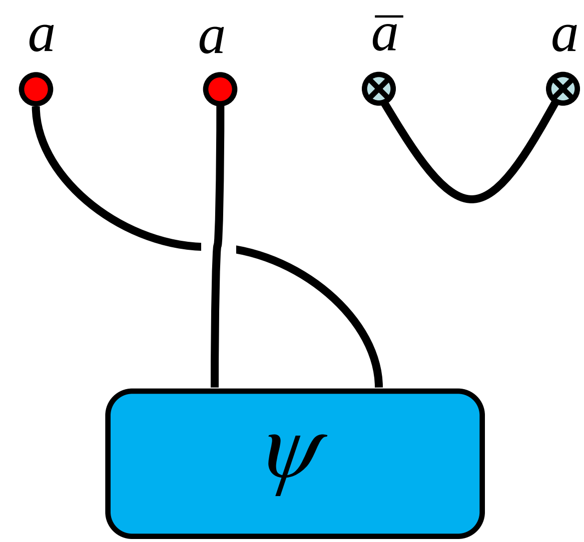

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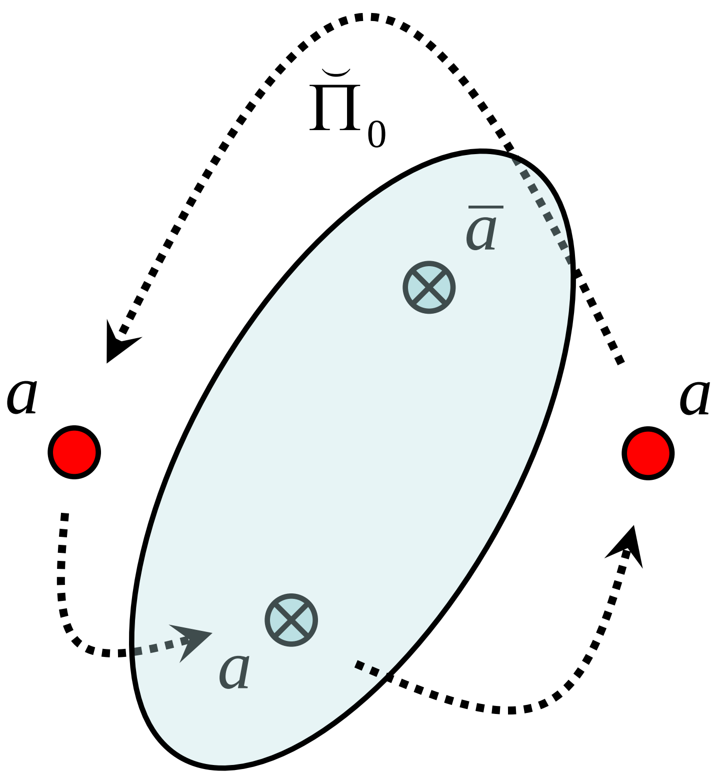
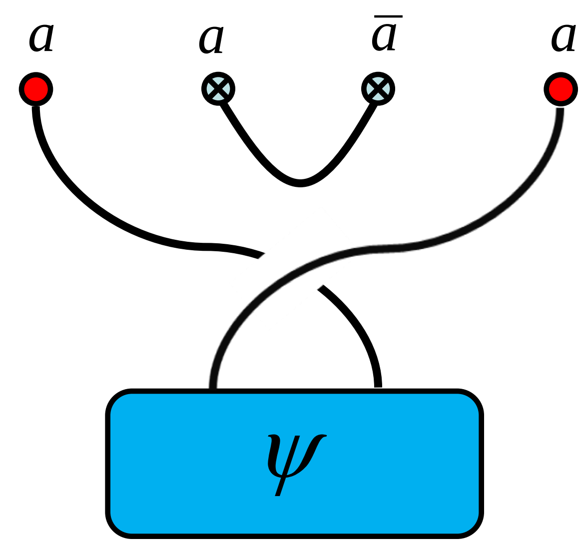
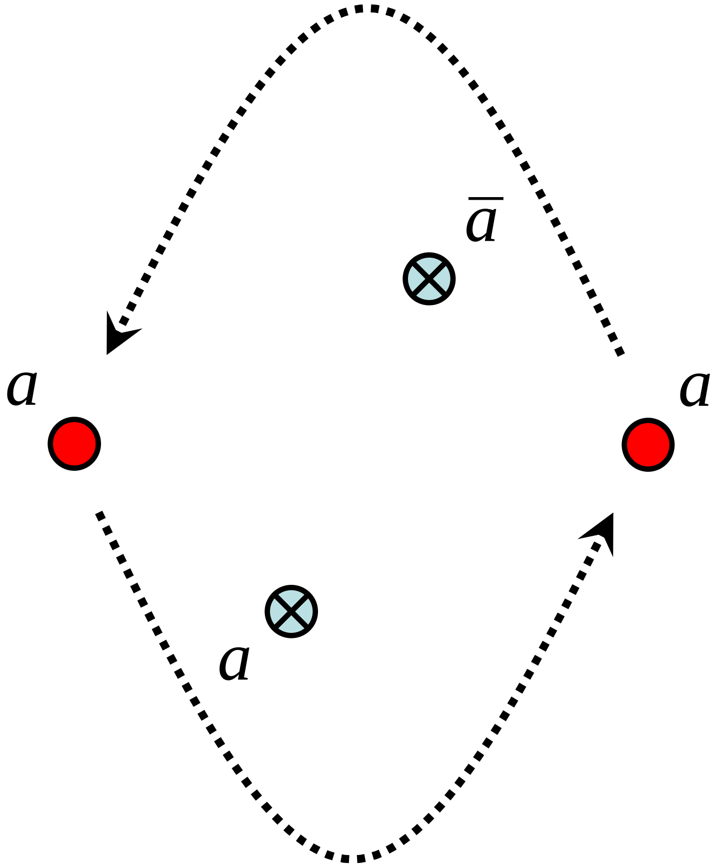
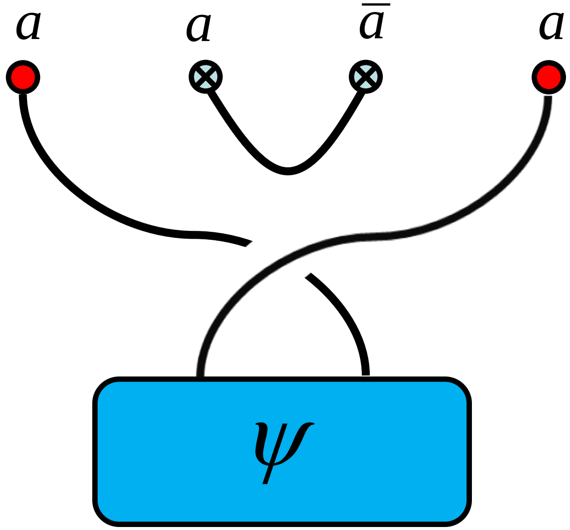




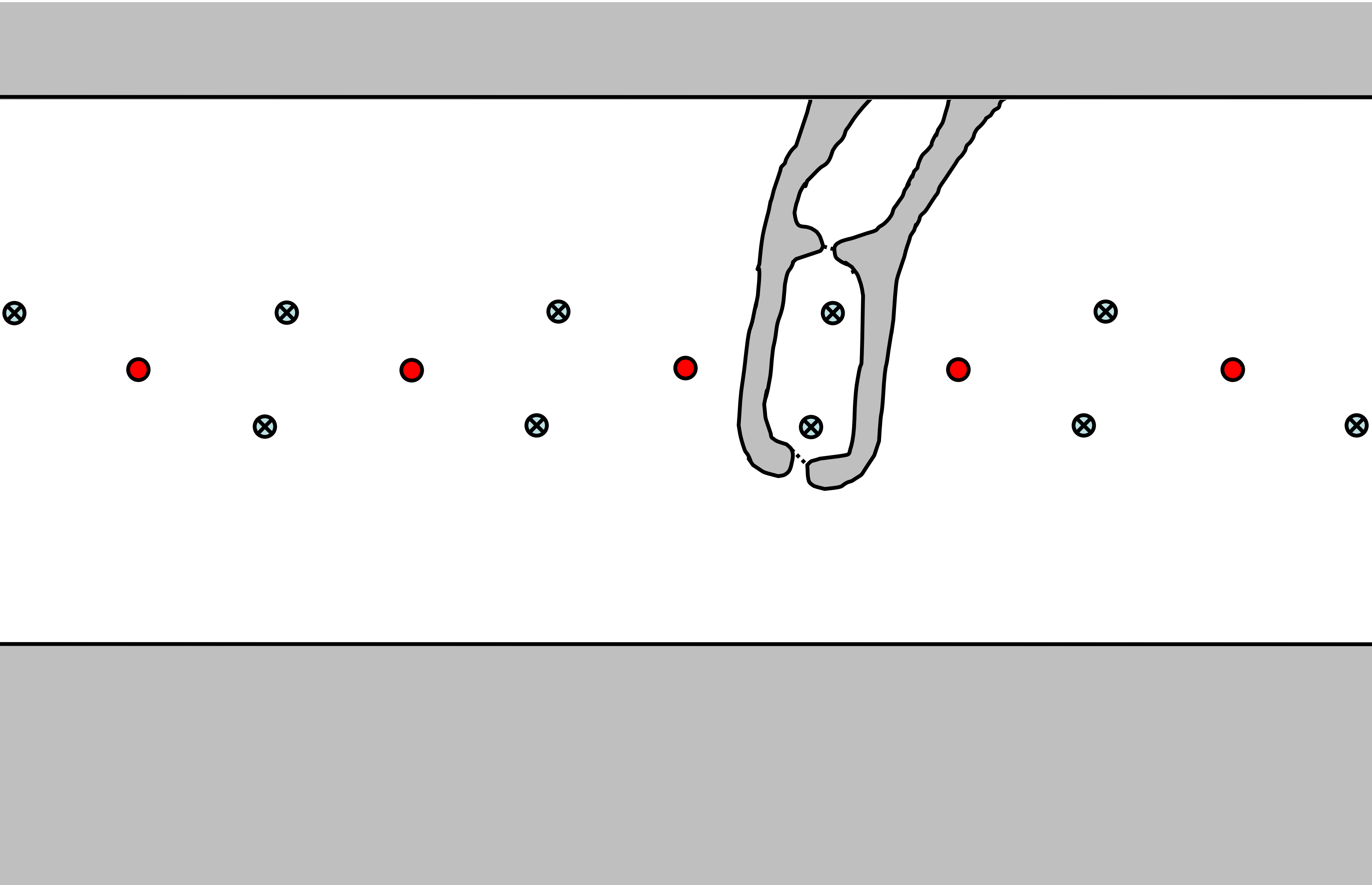
Measurement Simulated Braiding!



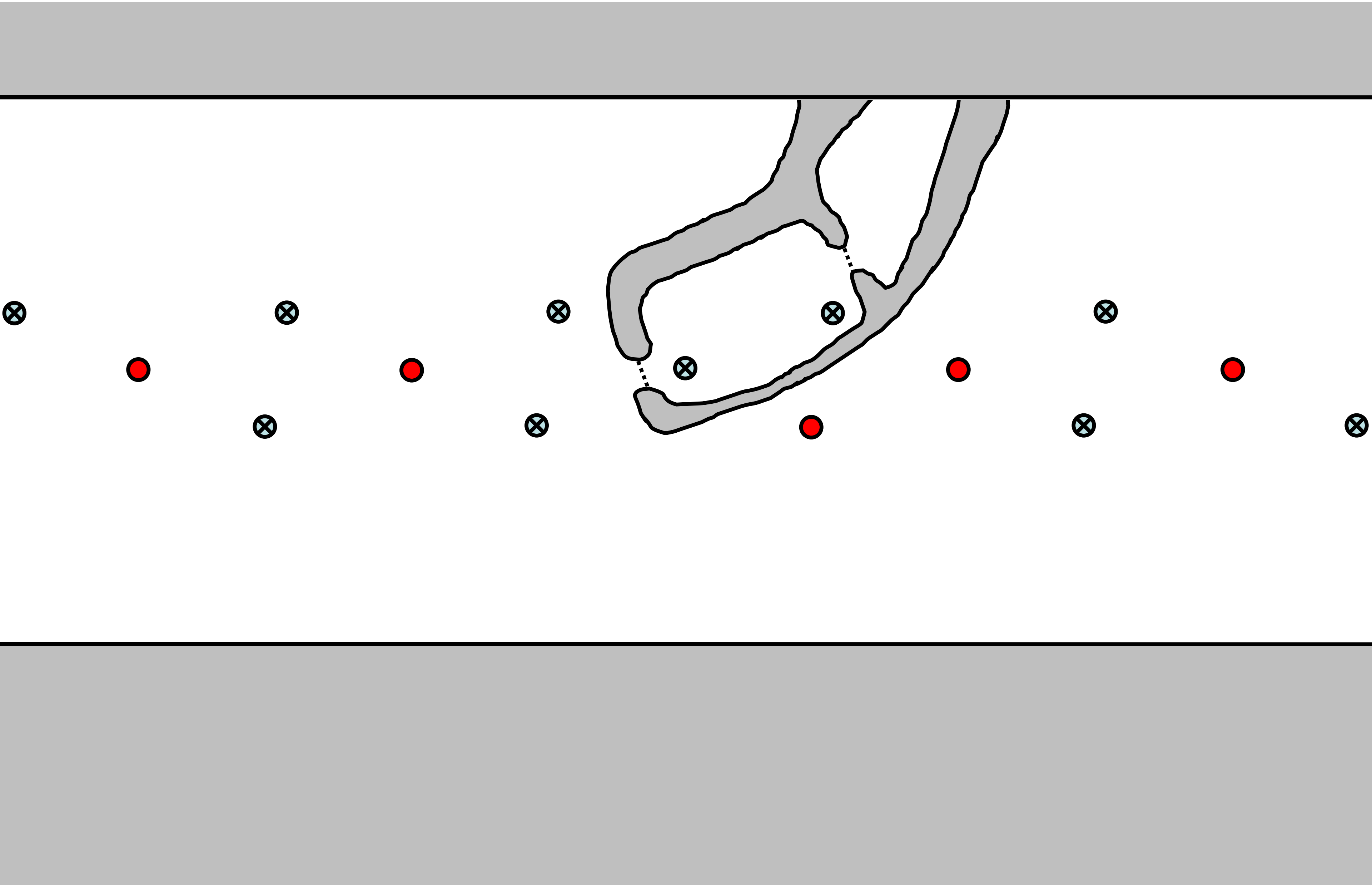


$$R^{(14)} \simeq \breve{\Pi}_0^{(23)} \breve{\Pi}_0^{(34)} \breve{\Pi}_0^{(13)}$$

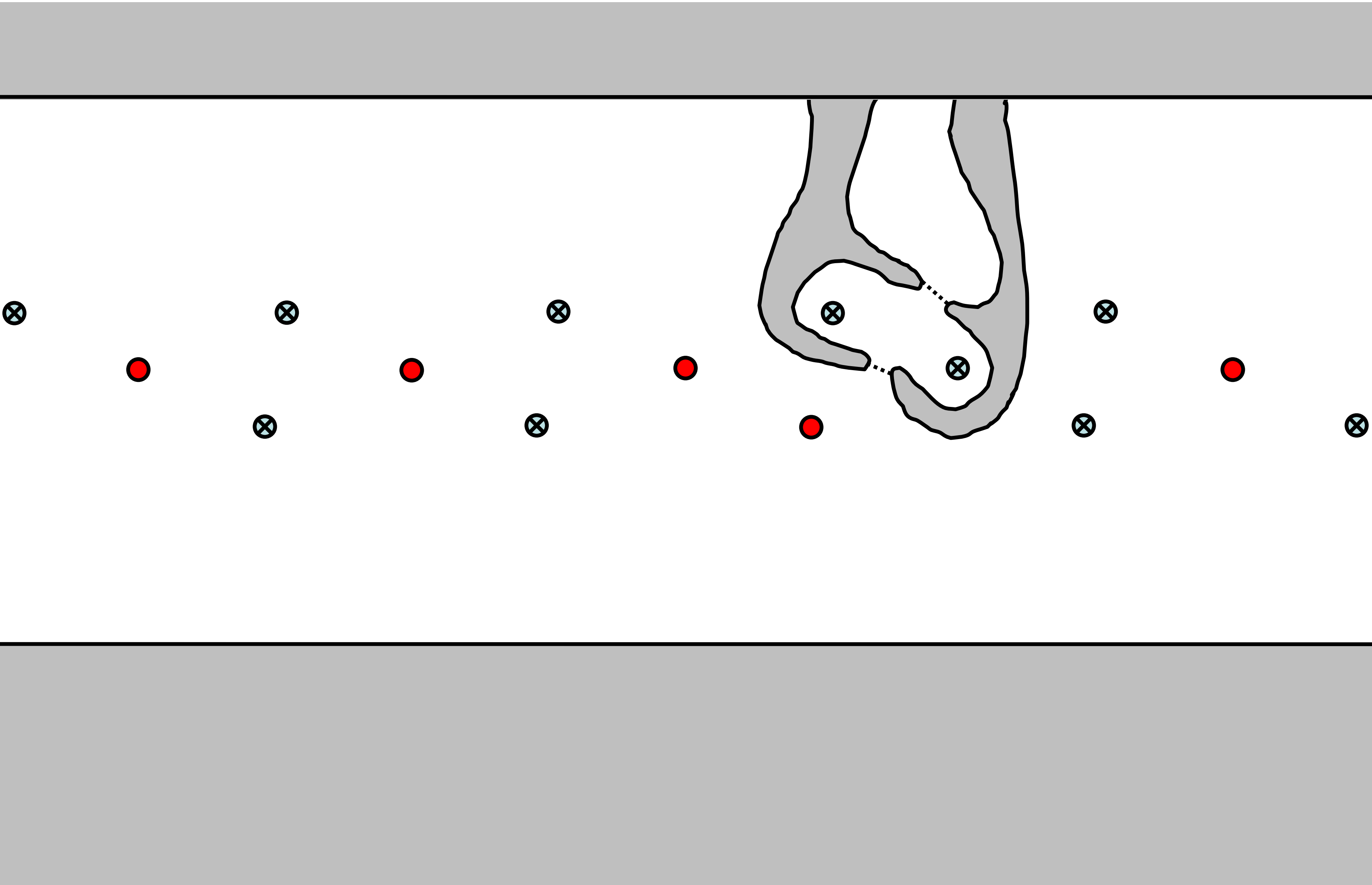
in FQH, for example



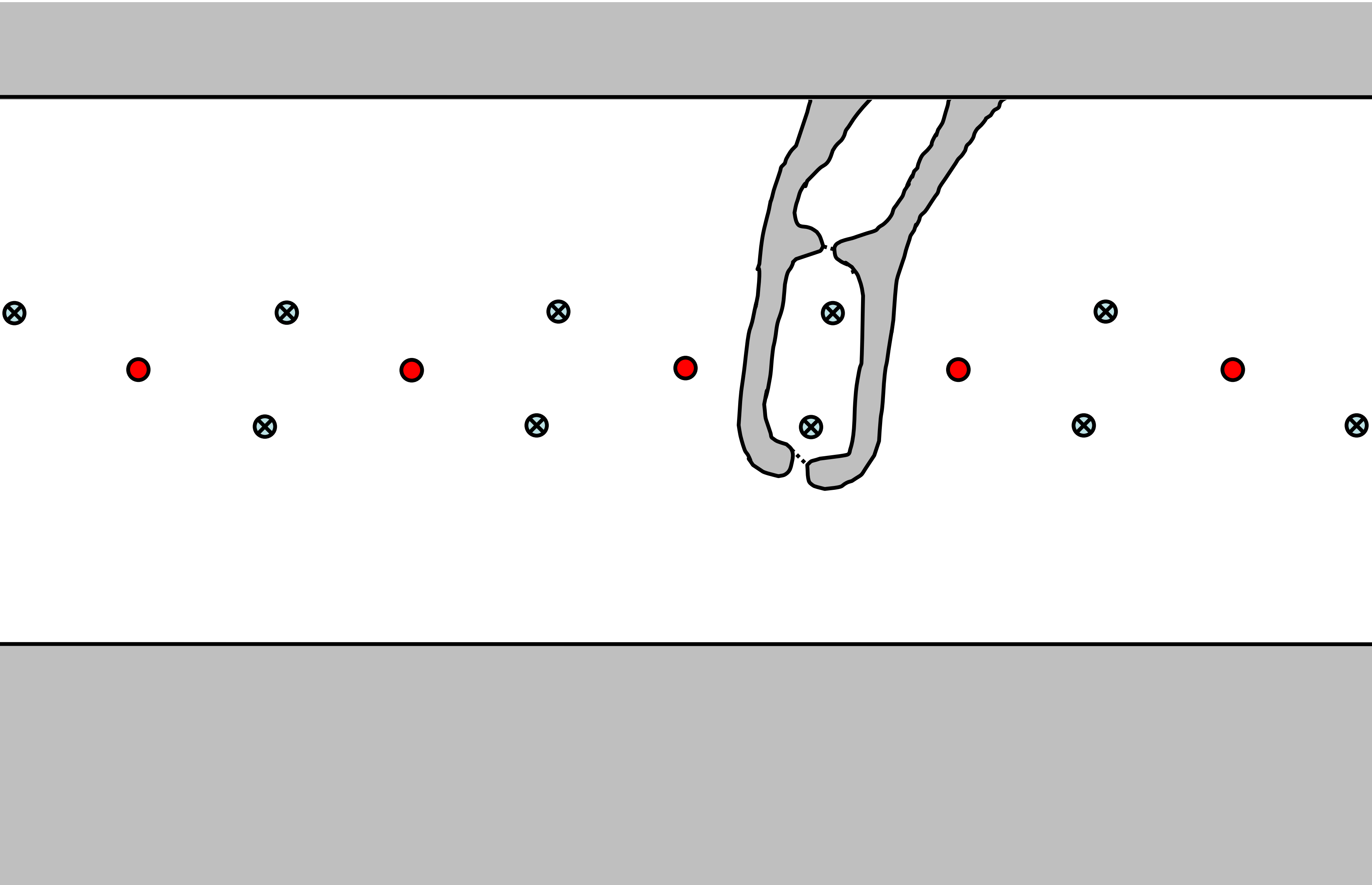
in FQH, for example



in FQH, for example

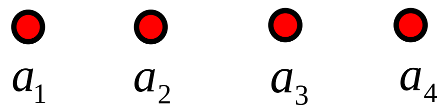


in FQH, for example

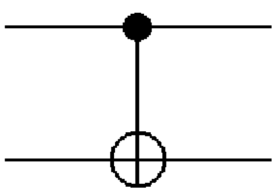


Topological Quantum Computation

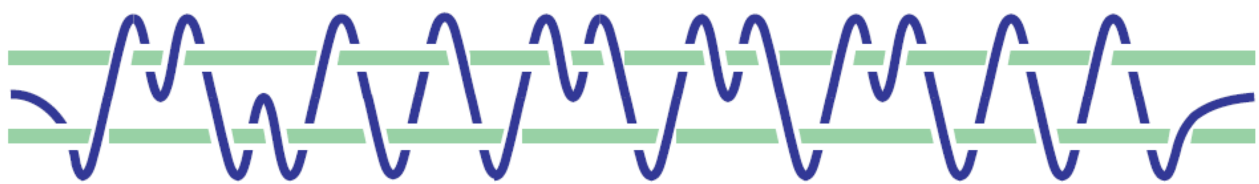
$|0\rangle \leftrightarrow$ $|\Psi_0\rangle$



$|1\rangle \leftrightarrow$ $|\Psi_1\rangle$

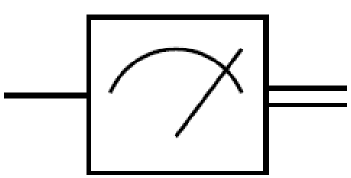


$\leftrightarrow$



time $\rightarrow$

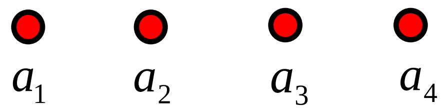
$\uparrow$ measurement simulated braiding



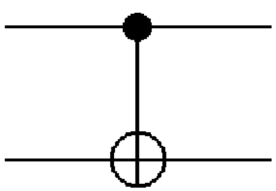
$\leftrightarrow$ Topological Charge Measurement

Measurement-Only Topological Quantum Computation

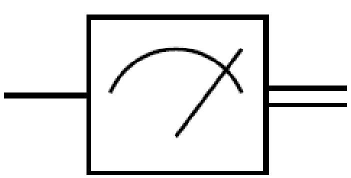
$|0\rangle \leftrightarrow |\Psi_0\rangle$



$|1\rangle \leftrightarrow |\Psi_1\rangle$



$\leftrightarrow$ Topological Charge Measurement



$\leftrightarrow$ Topological Charge Measurement

Conclusion

- Anyons could provide a quantum computer.
- Teleportation has anyonic counterpart.
- Bounded, adaptive, non-demolitional measurements can generate the braiding transformations used in TQC.
- Stationary anyons hopefully makes life easier for experimental realization.
- FQH interferometer technology is rapidly progressing.