Hamiltonian approach to Yang-Mills Theories in 2+1 Dimensions: Glueball and Meson Mass Spectra

Aleksandr Yelnikov
Virginia Tech

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with **Rob Leigh**
and **Djordje Minic**

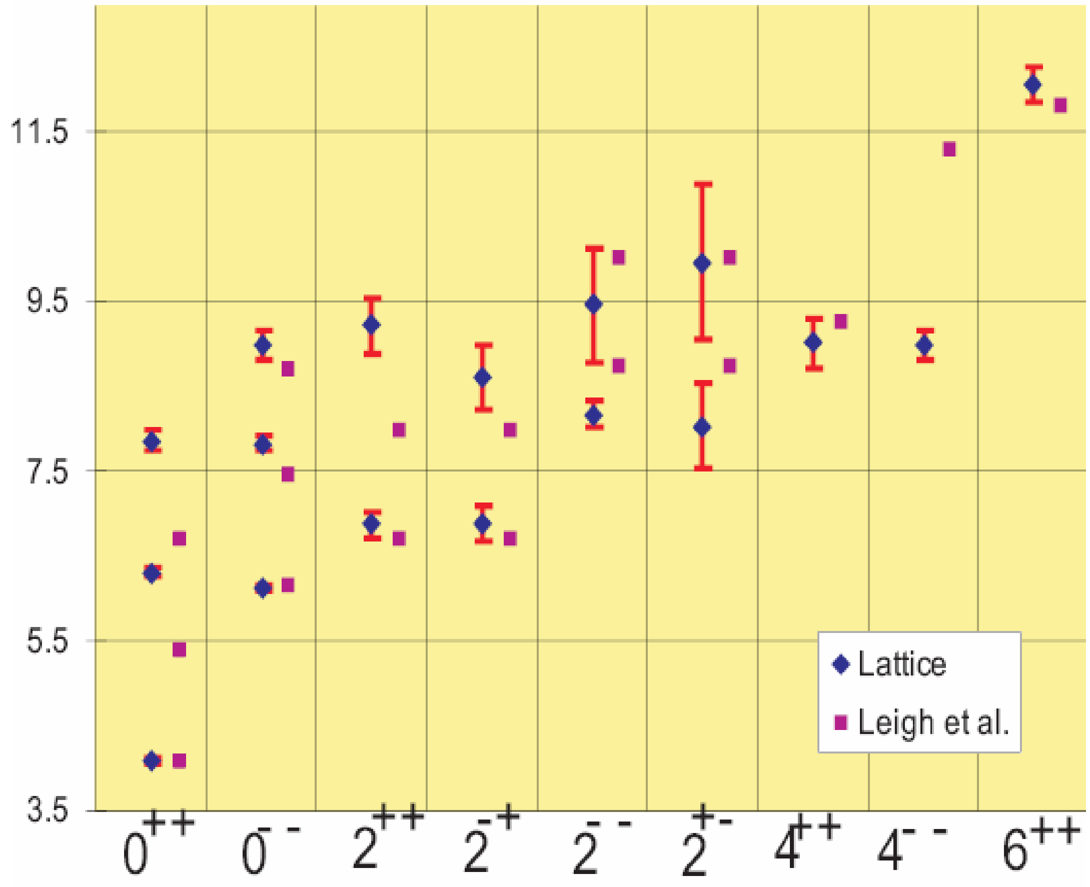
Preliminaries and Summary

- We work in the Hamiltonian formalism
- The new ingredient is provided by the computation of the new nontrivial form of the ground state wave-functional. This wave-functional seems to correctly interpolate between asymptotically free regime and low energy confining physics
- With this vacuum state it is possible to (quantitatively) demonstrate important observable features of the theory:
  - Signals of confinement: area law, string tension, mass gap
  - Compute the spectrum of glueball states
- Very good agreement with available lattice data:
  - Large-N string tension is agrees to 1% with lattice data (Bringoltz and Teper, hep-lat/0611286)

$$\sqrt{\sigma} \simeq \sqrt{\frac{\pi}{2}}\, m \approx 1.2533\, m \qquad \sqrt{\sigma}_{lattice} \approx (1.2409 \pm 0.0013)\, m$$

Summary of glueball mass spectrum

- All masses are in units of string tension (courtesy of Barak Bringoltz)



Motivation (Why is $YM_{(2+1)}$ interesting?)

$YM_{(1+1)}$	$YM_{(2+1)}$	$YM_{(3+1)}$
No propagating degrees of freedom; exactly solvable (‘t Hooft ’74)	Propagating degrees of freedom, nontrivial. Exactly solvable? (Polyakov ’80)	Highly nontrivial; difficult

- $YM_{(2+1)}$ is *superrenormalizable*
- Coupling constant is dimensionful: $[g^2_{YM}] = \mathrm{mass}$
- It is convenient to introduce new massive parameter

$$m = \frac{g^2_{YM} N}{2\pi} \qquad \sim \ '\mathrm{t\,Hooft\ coupling}$$

Motivation (cont'd)

- Feynman (1981) argued that (2+1)YM should confine, with mass gap generated because configuration space is compact
- Lattice compact QED in (2+1)D (Polyakov '75) – explicit demonstration of confinement, condensation of magnetic monopoles

- Gauge invariant variables:
  - Nonlocal – Wilson loops
    - Equation of motion ⟷ loop equation (Makeenko & Migdal) ⟷ hard to proceed
  - Local – many proposals (Bars, Halpern, Freedman&Khuri, etc)
    - *Karabali, Kim and Nair formalism* (hep-th/9705087, hep-th/9804132, hep-th/0007188)

Hamiltonian Analysis

- Inner product
  - Matrix variables, calculation of gauge-invariant volume
- Hamiltonian $\mathcal{H}$ in the new wariables
- Solve Schrodinger equation $\mathcal{H}\,\Psi = E\,\Psi$

$YM_{(2+1)}$ in the Hamiltonian Formalism

- We consider (2+1)D SU(N) pure YM theory with the Hamiltonian

$$\mathcal{H}_{YM} \equiv T + V = \int \mathrm{Tr}\left(g_{YM}^2 {E_i}^2 + \frac{1}{g_{YM}^2} B^2 \right)$$

- We choose the temporal gauge $A_0=0$
- E^a_i is the momentum conjugate to A^a_i ; i=1,2, a=1,2,…,N^2-1
  - Quantize: $E_i^a \rightarrow -i\frac{\delta}{\delta A_i^a}$
- Time-independent gauge transformations preserve $A_0=0$ gauge condition and gauge fields A_i transform as

$$A_i \rightarrow gAg^{-1} - \partial_i g g^{-1}, \qquad g \in SU(N)$$

- Gauss' law implies that observables and physical states are gauge invariant

Matrix parameterization

- Choose complex coordinates $z = x_1 - ix_2, \quad \bar{z} = x_1 + ix_2$

$$A = A_z = \frac{1}{2}(A_1 + iA_2), \qquad \bar{A} = A_{\bar{z}} = \frac{1}{2}(A_1 - iA_2),$$

- Parameterize $A, \ \bar{A}$ as

$$A = -\partial M M^{-1}, \quad \bar{A} = M^{\dagger -1} \bar{\partial} M^{\dagger}$$

This parameterization is well-known in 2-dimensional YM context, gauged WZW models, etc.

- Here M is a complex invertible unimodular matrix $A \ traceless \leftrightarrow \det M = 1$, $M \in SL(N, \mathbb{C})$

- The basic advantage of this parameterization is behavior under gauge transformations

$$M \rightarrow M^g = g \ M$$

Matrix parameterization (cont'd)

- Gauge invariant variables may be written simply as

$$H = M^\dagger M \qquad\qquad H \;-\; \text{Hermitian}$$

  Note that H is a *local field*. Roughly, M may be thought of as analogous to an open Wilson line, and H as a closed loop

- The Wilson loop evaluates to

$$\Phi(C) = TrPe^{i\oint_C \left(Adz+\bar{A}d\bar{z}\right)} = TrPe^{-i\oint_C dz\ \partial H H^{-1}}$$

$$J = \frac{c_A}{\pi}\partial H H^{-1}$$

- There is an ambiguity in parameterization

$$M(z,\bar{z}) \mapsto M(z,\bar{z})h^\dagger(\bar{z}) \qquad M^\dagger(z,\bar{z}) \mapsto h(z)M^\dagger(z,\bar{z})$$

$$H(z,\bar{z}) \mapsto h(z)H(z,\bar{z})h^\dagger(\bar{z})$$

- One must ensure that all results are holomorphic invariant

Volume element

- The inner product of states in position representation

$$\langle 1|2\rangle \;=\; \int [dAd\bar{A}] \;\; \Psi_1^* \Psi_2$$

- Change of variables gives

$$[dAd\bar{A}] = \det(-D\bar{D}) \qquad \underbrace{d\mu(M, M^\dagger)}_{\text{Haar measure for SL}(N,\mathbf{C})}$$

- We can split the $\mathrm{SL}(N, \mathbf{C})$ volume element as

$$d\mu(M, M^\dagger) \;=\; \underbrace{d\mu(H)}_{\text{Haar for SL}(N,\mathbf{C})/\text{SU}(N)} \qquad \underbrace{d\mu(U)}_{\text{Haar for SU}(N)}$$

- Also the computation of $\det(-\bar{D}D)$ presents no difficulties

$$\det(-\bar{D}D) \;\sim\; e^{2c_A S_{WZW}(H)}$$

$$S_{WZW}[H] = \frac{1}{2\pi}\int d^2z\ \mathrm{Tr}\left(\partial H \bar{\partial} H^{-1}\right) + \frac{i}{12\pi}\int d^3x\ \epsilon^{\mu\nu\lambda}\mathrm{Tr}(H^{-1}\partial_\mu H H^{-1}\partial_\nu H H^{-1}\partial_\lambda H)$$

Volume element (cont'd)

- Finally the inner product of states can be written as overlap integral with nontrivial measure

$$\langle 1|2\rangle \;=\; \int d\mu(H)\; e^{2c_A S_{WZW}(H)}\; \Psi_1^* \Psi_2$$

- Comments:
  - Matrix elements in YM(2+1) = correlators of a hermitian WZW model;
  - Volume of configuration space is finite!

$$\int d\mu(H)\, e^{2c_A S_W ZW(H)} \;<\; \infty$$

    this means, in particular, that $\Psi \equiv 1$ is normalizable
  - Note that nontrivial measure factor disappears for U(1) theory. This is the basic difference in configuration space geometry for Abelian and non-Abelian theories

Intuitive argument for mass gap

- The Hamiltonian has the form

$$\mathcal{H} = \frac{1}{2}\int \left[e^2 E^2 + B^2/e^2\right]$$

$$[E, B] \sim p \text{ (in momentum space)} \implies \Delta E\, \Delta B \sim p, \text{ or } \Delta E \sim p/\Delta B$$

$$\mathcal{E} \;=\; \langle \mathcal{H} \rangle \;\approx\; \frac{1}{2}\int \left[e^2 \frac{p^2}{(\Delta B)^2} + \frac{(\Delta B)^2}{e^2}\right]$$

- Minimize with respect to $\Delta B \implies (\Delta B)^2 \sim p \implies \mathcal{E} \sim p$. This is the photon.
- For non-Abelian theory

$$\langle \mathcal{H} \rangle \;=\; \int d\mu(H)\; \underbrace{e^{2c_A S_{WZW}(H)}}_{e^{-\frac{c_A}{2\pi}\int B \frac{1}{p^2} B + \dots}} \;\frac{1}{2}\left[e^2 \frac{p^2}{(\Delta B)^2} + \frac{(\Delta B)^2}{e^2}\right]$$

$$\text{Gaussian} \implies (\Delta B)^2 \sim \frac{\pi p^2}{c_A} \implies \mathcal{E} \sim m + \frac{p^2}{2m} \qquad m = \frac{c_A e^2}{2\pi} \;-\; \text{mass gap}$$

The Hamiltonian

- It is natural to introduce the WZW current

$$J = \frac{c_A}{\pi}\partial H H^{-1}$$

J is a connection for holomorphic invariance: $J \mapsto hJh^{-1} + \frac{\pi}{c_A}\partial h h^{-1}$

- The YM Hamiltonian can then be rewritten in terms of J

$$\mathcal{H}_{KN}[J] = \underbrace{m\left(\int_x J^a(x)\frac{\delta}{\delta J^a(x)} + \int_{x,y}\Omega_{ab}(x,y)\frac{\delta}{\delta J^a(x)}\frac{\delta}{\delta J^b(y)}\right)}_{T} + \underbrace{\frac{\pi}{mc_A}\int_x \bar\partial J^a \bar\partial J^a}_{V}$$

- This has the collective field form (Jevicki & Sakita '81) and

$$\Omega_{ab}(x,y) = \frac{c_A}{\pi^2}\frac{\delta_{ab}}{(x-y)^2} - \frac{i}{\pi}\frac{f_{abc}J^c(x)}{(x-y)}$$

- This can be rechecked by self-adjointness of $\mathcal{H}_{KN}(J)$

The Hamiltonian (cont'd)

- Ignore V for the moment. Then we can take $\Psi_0 \equiv 1$ as a ground state for T. This is OK since $T\,\Psi_0 \equiv 0$ and since $\Psi_0 \equiv 1$ is normalizable
- As a first excited state (of T) we may try $\Psi_1 \;=\; J^a$

$$T\,J^a \;=\; mJ^a$$

Note, however, that this is not holomorphically invariant

- Therefore we may try $\Psi_2 \;= :\bar{\partial}J\bar{\partial}J: \sim\; V$

$$T\;:\bar{\partial}J\bar{\partial}J: =\; 2m\;:\bar{\partial}J\bar{\partial}J:$$

- Higher excitations of T can (in principle) be constructed using higher (mass) dimension operators
  - Assume that $\quad T:\bar{\partial}J(\Delta)^n\bar{\partial}J: \;=\; (2+n)m:\bar{\partial}J(\Delta)^n\bar{\partial}J: + \ldots$
  - Here $\quad \Delta \;=\; \frac{\bar{\partial}D+D\bar{\partial}}{2} - \text{covariant Laplacian}$

Vacuum wave functional

- Include V perturbatively, $\Psi_{VAC} = e^P$

$$P = -\frac{\pi}{m^2 c_A} \int :\bar{\partial} J \bar{\partial} J: \; - \frac{2}{3}\left(\frac{\pi}{m^2 c_A}\right)^2 \int :\bar{\partial} J (\Delta) \bar{\partial} J: \; + \mathcal{O}\left(\frac{1}{m^6}\right)$$

- In principle vacuum state can be constructed order-by-order in strong coupling expansion
- We want to sum up all terms quadratic in J (B)

$$\bar{\partial} J \bar{\partial} J \sim B^2$$

$$\bar{\partial} J (\Delta)^n \bar{\partial} J \sim B (D^2)^n B$$

Vacuum Wave-Functional

- We want to solve Schrödinger equation to *quadratic order* in J^a, therefore we take the most general *gauge invariant* ansatz which contains *all* terms quadratic in J^a

$$\Psi_0 = \exp\left(-\frac{\pi}{2c_A m^2}\int tr\ \bar{\partial}JK(L)\bar{\partial}J + \dots\right). \qquad L = \Delta/m^2$$

- The (quasi)-Gaussian part of the vacuum wave functional contains a (non-trivial) kernel $K(L)$ which will be determined by the solution of Schrödinger equation
  - Thinking of $K(L)$ as Taylor-expandable function shows that introducing $K(L)$ is just a convenient way to parameterize summed up perturbation series

$$K\left(\frac{\Delta}{m^2}\right) = \sum c_n \left(\frac{\Delta}{m^2}\right)^n$$

$$tr(\bar{\partial}JK(L)\bar{\partial}J) = c_1\, tr(\bar{\partial}J\bar{\partial}J) + c_2\frac{tr(\bar{\partial}J\Delta\bar{\partial}J)}{m^2} + c_3\frac{tr(\bar{\partial}J\Delta^2\bar{\partial}J)}{m^4} + \dots$$

Vacuum Wave-Functional (cont'd)

$$\Psi_0 = \exp\left(-\frac{1}{2g_{YM}^2 m} \int tr\, B\, K\left(\frac{D^2}{4m^2} \right) B + \ldots \right)$$

Vacuum Wave-Functional (cont'd.)

- Asymptotic behavior of the vacuum state:
  - In the UV we expect to recover the standard perturbative result

$$\Psi_0^{UV} \mapsto \exp\left(-\frac{1}{2g_{YM}^2}\int B^a \frac{1}{|p|} B^a\right)$$

$$K \rightarrow \frac{2m}{p} \qquad \text{as} \qquad p \rightarrow \infty$$

  - In the IR we expect

$$\Psi_0^{IR} \mapsto \exp\left(-\frac{1}{2g_{YM}^2 m}\int \mathrm{Tr}\, B^2\right)$$

$$K \rightarrow 1 \qquad \text{as} \qquad p \rightarrow 0$$

Schrödinger Equation

- The Schrödinger equation takes the form

$$\mathcal{H}_{YM}\Psi_0 = E_0\Psi_0 = \left[E_0 + \int tr\, B(\mathcal{R})B + \dots\right]\Psi_0$$

- By careful computation we find the differential equation for the kernel *K(L)*

$$\mathcal{R} = -K(L) - \frac{L}{2}\frac{d}{dL}[K(L)] + LK(L)^2 + 1 = 0$$

- This may be compared to *U(1)* theory without matter in which case we obtain an algebraic equation describing free photons

$$LK^2(L) + 1 = 0$$

$$K(L) = \pm\frac{1}{\sqrt{-L}} = \frac{2m}{p}$$

Vacuum Solution

- The differential equation for kernel is of Riccati type and, by a series of redefinitions, it can be recast as a Bessel equation.

$$K(L) = \frac{1}{\sqrt{L}} \frac{C J_2(4\sqrt{L}) + Y_2(4\sqrt{L})}{C J_1(4\sqrt{L}) + Y_1(4\sqrt{L})}$$

- *The only normalizable wave functional is obtained for* $C \to \infty$ *, which is also the only case that has both the correct UV behavior appropriate to asymptotic freedom as well as the correct IR behavior appropriate to confinement and mass gap!*

- This solution is of the form

$$K(L) = \frac{1}{\sqrt{L}} \frac{J_2(4\sqrt{L})}{J_1(4\sqrt{L})}$$

String tension

- The expectation value of the large spatial Wilson loop can be calculated using IR asymptotic form of vacuum state

$$\langle W_R(\mathcal{C})\rangle = \int [dA]\, W_R(\mathcal{C}) \ \exp\left(-\frac{1}{g_{YM}^2 m}\int Tr B^2\right)$$

- This is equivalent to 2d Euclidean YM theory with 2d coupling $g_{2D}^2 \equiv m g_{YM}^2$

- This means, in particular, that large spatial Wilson loops obey area law with string tension (Karabali, Kim and Nair, hep-th/9804132)

$$\sigma_R = g_{YM}^4 \frac{C_A C_R}{4\pi} \qquad\qquad C_R \ - \ \text{Casimir for representation } R$$

- Agrees to 1% with large-N lattice string tension (Bringoltz and Teper, hep-lat/0611286)

$$\sqrt{\sigma} \ \simeq \ \sqrt{\frac{\pi}{2}}\, m \ \approx \ 1.2533\, m \qquad\qquad \sqrt{\sigma_{lattice}} \ \approx (1.2409 \pm 0.0013)\, m$$

Inverse Kernel

- Elementary $\langle B^a(x)\, B^b(y)\rangle$ correlator is

$$\langle B^a(x)\, B^b(y)\rangle \;\sim\; \delta^{ab} K^{-1}(|x-y|)$$

- Using the standard Bessel function identities we may expand where the $\gamma_{2,n}$ are the ordered zeros of $J_2(u)$.

$$\frac{J_1(u)}{J_2(u)} = \frac{4}{u} + 2u \sum_{n=1}^{\infty} \frac{1}{u^2 - \gamma_{2,n}^2}$$

- Inverse kernel is thus ($L \cong p^2/4m^2$)

$$K^{-1}(p) = 1 + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\vec{p}^2}{\vec{p}^2 + M_n^2} \qquad\qquad M_n = \frac{\gamma_{2,n} m}{2}$$

Inverse Kernel (cont'd.)

- M_n can be interpreted as constituents out of which glueball masses are constructed

$$M_1 = 2.568m \qquad M_2 = 4.209m \qquad M_3 = 5.810m$$

- At asymptotically large spatial separations $|x-y| \to \infty$ inverse kernel takes the form

$$K^{-1}(|x-y|) \approx -\frac{1}{4\sqrt{2\pi|x-y|}} \sum_{n=1}^{\infty} (M_n)^{\frac{3}{2}} e^{-M_n|x-y|}$$

Glueball masses

- To find glueball states of given space-time quantum numbers, we compute equal-time correlators of invariant probe operators with appropriate J^{PC}
- For example, for 0^{++} states we take $tr(B^2)$ as a probe operator and compute

$$\langle tr(B^2)_x \, tr(B^2)_y \rangle \;\sim\; K^{-2}(|x-y|)$$

- At large distance, we will find contributions of *single particle poles*

$$\langle tr(B^2)_x \, tr(B^2)_y \rangle \;\sim\; \frac{1}{|x-y|} \sum_{n,m=1}^{\infty} (M_n M_m)^{3/2} \, e^{-(M_n+M_m)|x-y|}$$

$$\begin{aligned}
M_{0^{++}} &= M_1 + M_1 = 5.14m \\
M_{0^{++*}} &= M_1 + M_2 = 6.78m \\
M_{0^{++**}} &= M_1 + M_3 = 8.38m \\
M_{0^{++***}} &= M_1 + M_4 = 9.97m
\end{aligned}$$

0^{++} Glueballs

- For 2+1 Yang-Mills, the "experimental data" consists of a number of lattice simulations, largely by M. Teper et al (hep-lat/9804008, hep-lat/0206027)

- The following table compares lattice results for 0^{++} glueball states with analytic predictions. All masses are in units of the square root of string tension

State	Lattice, $N \to \infty$	Sugra	Our prediction	Diff, %
0^{++}	4.065 ± 0.055	4.07(input)	4.098	0.8
0^{++*}	6.18 ± 0.13	7.02	5.407	12.5
0^{++**}	7.99 ± 0.22	9.92	6.716	16
0^{++***}	9.44 ± 0.38 [5]	12.80	7.994	15
0^{++****}	$--$	15.67	9.214	$--$

0^{++} Glueballs (cont'd.)

- There are no adjustable parameters in the theory; the ratios of $M_{0^{++}}$ to $\sqrt{\sigma}$ are *pure numbers*
- We are able to predict masses of 0^{++} resonances, as well as the mass of the lowest lying member
- Results for excited state masses differ at the 10-15% level from lattice simulations.
- The table below gives an updated comparison with relabeled lattice data

State	Lattice, $N \to \infty$	Our prediction	Diff, %
0^{++}	4.065 ± 0.055	4.098	0.8
0^{++*}	6.18 ± 0.13	5.407	$--$
0^{++**}	6.18 ± 0.13	6.716	$--$
0^{++***}	7.99 ± 0.22	7.994	0.05
0^{++****}	9.44 ± 0.38	9.214	2.4

0^{--} Glueballs

- For 0^{--} glueballs we compute

$$\langle \mathrm{Tr}\,(\bar\partial J \bar\partial J \bar\partial J)_x\, \mathrm{Tr}\,(\bar\partial J \bar\partial J \bar\partial J)_y \rangle \sim \frac{1}{64(2\pi|x-y|)^{\frac{3}{2}}} \sum_{n,\,m,\,k=1}^{\infty} (M_n M_m M_k)^{3/2} e^{-(M_n+M_m+M_k)|x-y|}$$

- Masses of 0^{--} resonances are the sum of three constituents : $M_n+M_m+M_k$

- The following table compares analytic predictions with available lattice data. All masses are in units of the $\sqrt{\sigma}$

State	Lattice, $N \to \infty$	Sugra	Our prediction	Diff,%
0^{--}	5.91 ± 0.25	6.10	6.15	4
0^{--*}	7.63 ± 0.37	9.34	7.46	2.3
0^{--**}	8.96 ± 0.65	12.37	8.73	2.5

Spin–2 States

- Similarly, analytic predictions for $2^{\pm+}$ states are compared with existing lattice data in the table above
- By *parity doubling*, masses of J^{++} and J^{-+} resonances should be the same which is not the case with lattice values for 2^{++*} and 2^{-+*}. This indicates that apparent 7-14% discrepancy may be illusory.
- An updated comparison with relabeled lattice data is given in the table below

State	Lattice, $N \to \infty$	Our prediction	Difference, %
2^{++}	6.88 ± 0.16	6.72	2.4
2^{-+}	6.89 ± 0.21	6.72	2.5
2^{++*}	8.62 ± 0.38	7.99	7.6
2^{-+*}	9.22 ± 0.32	7.99	14
2^{++**}	10.6 ± 0.7 [6]	9.26	13
2^{++***}	$--$	10.52	$--$

State	Lattice, $N \to \infty$	Our prediction	Difference, %
2^{++}	6.88 ± 0.16	6.72	2.4
2^{++*}	8.62 ± 0.38	7.99	7.6
2^{++**}	9.22 ± 0.32	9.26	0.4
2^{++***}	10.6 ± 0.7	10.52	0.8

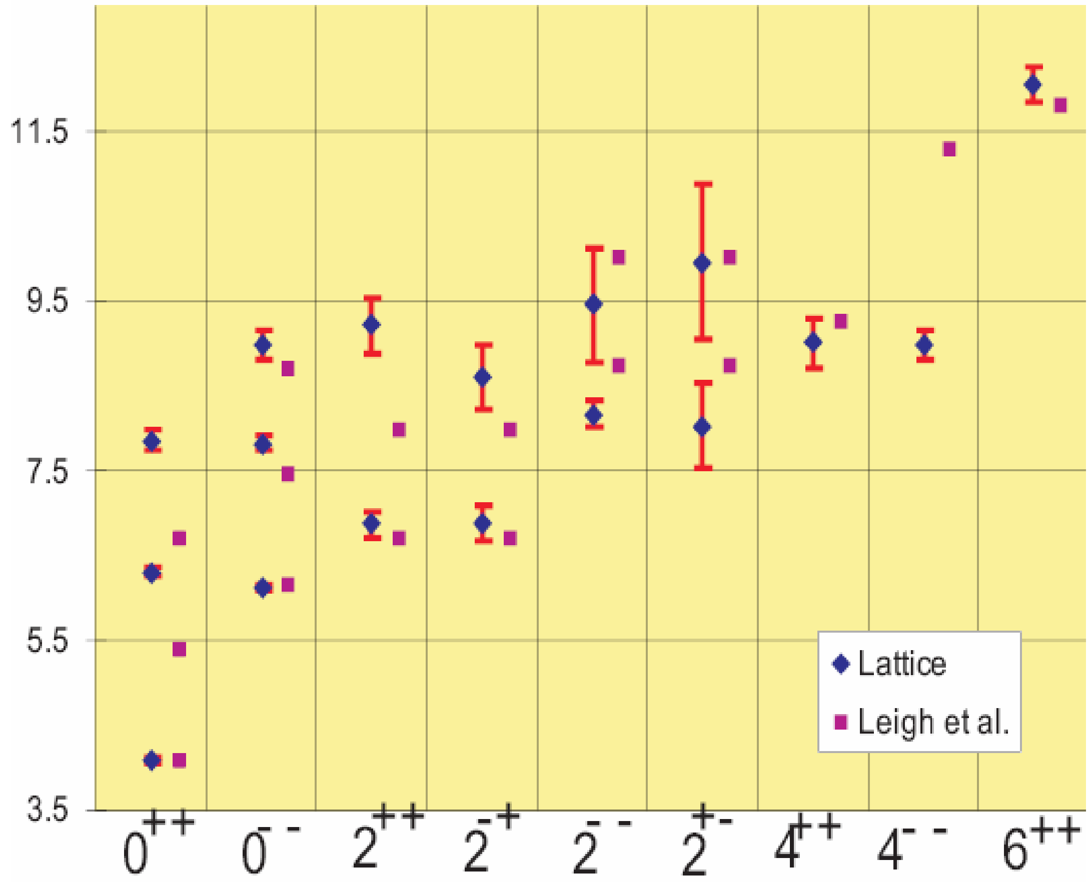
Spin–2 States (cont'd.)

- Finally, the table below summarizes available lattice data for $2^{\pm-}$ states and compares it to analytic predictions

State	Lattice, $N \to \infty$	Our prediction	Difference, %
2^{+-}	8.04 ± 0.50	8.76	8.6
2^{--}	7.89 ± 0.35	8.76	10.4
2^{+-*}	9.97 ± 0.91	10.04	0.7
2^{--*}	9.46 ± 0.66	10.04	5.6

Summary of glueball mass spectrum

- All masses are in units of string tension (courtesy of Barak Bringoltz)



Higher Spin States and Regge Trajectories

- It is possible to generalize our results for higher spin states

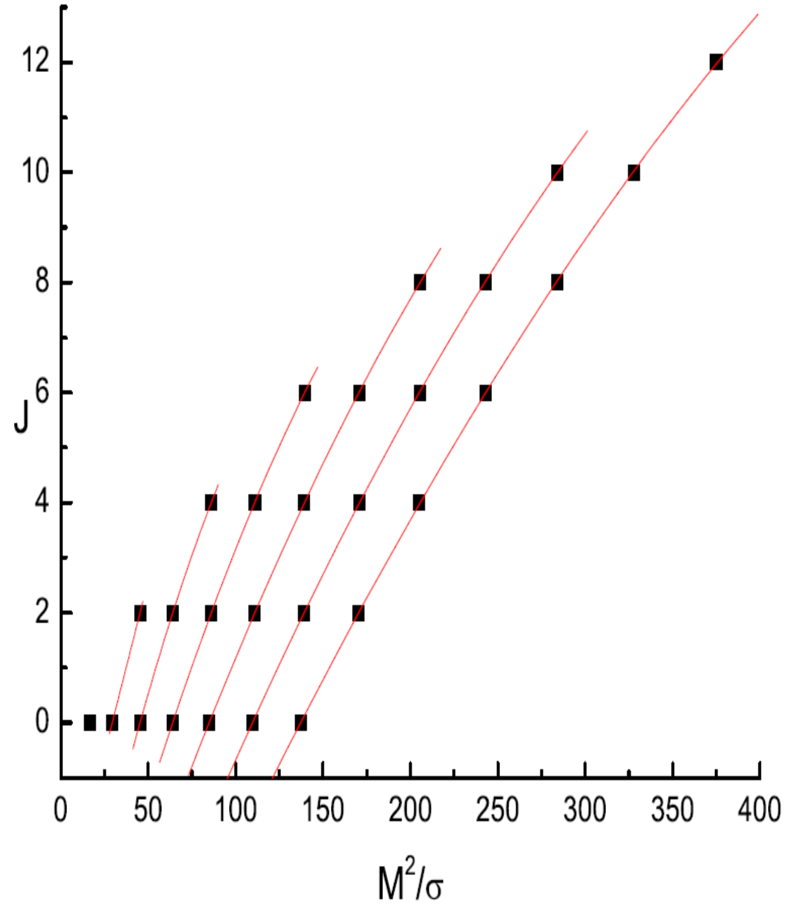
- For example, the masses of J^{++} resonances with even J are

$$M_{J^{++*n}} = M_{J/2+1} + M_{J/2+1+n}$$

- Similarly, the masses of J^{--} resonances with even J are

$$M_{J^{--*n}} = M_1 + M_{J/2+1} + M_{J/2+1+n}$$

- It is possible to draw nearly linear Regge trajectories.
  - Graph on the right represents a Chew-Frautschi plot of large N glueball spectrum. Black boxes correspond to J^{++} resonances with even spins up to J=12

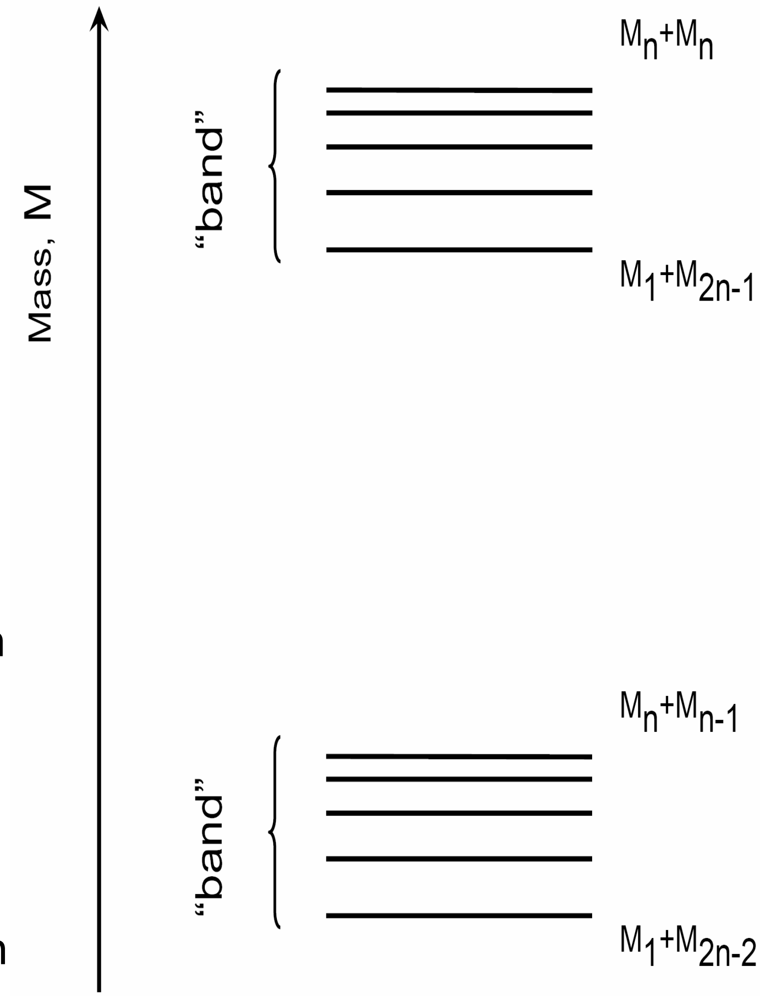


Approximate Degeneracy of Mass Spectrum

- The Bessel function is essentially sinusoidal and so its zeros are approximately evenly spaced (better for large n)
- Thus, the predicted spectrum has approximate degeneracies, e.g.

$$\begin{aligned} M_{0^{++**}} &= M_1 + M_3 = 8.38m \\ M_{2^{++}} &= M_2 + M_2 = 8.42m \end{aligned}$$

- The spectrum is organized into “bands” concentrated around a given level (which are well separated)
- At each level one finds more and more spin states
- We believe this is the basic manifestation of QCD string



Quarks

$$H = -\frac{g^2}{2}\int \frac{\delta^2}{\delta A_i^{a2}} + \frac{1}{2g^2}\int B^a B^a + \int \bar{\psi}(-i\gamma^i D_i + M)\psi \equiv H_g + H_f$$

- Schrodinger representation for fermions (Floreanini and Jackiw, 88)

$$\psi(\vec{x}) = \frac{1}{\sqrt{2}}[\theta(\vec{x}) + \frac{\delta}{\delta\theta^\dagger(\vec{x})}] \qquad \psi^\dagger(\vec{x}) = \frac{1}{\sqrt{2}}[\theta^\dagger(\vec{x}) + \frac{\delta}{\delta\theta(\vec{x})}]$$

- Vacuum ansatz

$$\Psi_v = \Psi_g\Psi_f = \Psi_0 = \exp(-\frac{1}{2g^2 m}\int Tr[B\,(K(L))\,B)]\exp(\int_{x,y}\theta^\dagger(y)[K_f(D_i)]_{y-x}\theta(x))$$

Quarks (cont'd)

- Free fermions

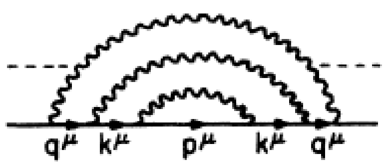
$$K(p) = -\frac{\gamma^0 M + \gamma^0\gamma^i p_i}{\sqrt{p_i^2 + M^2}}$$

- Physical interpretation

$$K(k) = v(-k)v^\dagger(-k) - u(k)u^\dagger(k)$$

$$\psi(x) = \int \frac{dp}{\sqrt{2\pi}}[b(p)u(p) + d^\dagger(-p)v(-p)]e^{ipx}$$

- QCD in (1+1)D ('t Hooft model, 74)
  - Rainbow diagrams
  - it is possible to reproduce
    Bars-Green equation (1977)



$$\left\{p\gamma_5 + m\gamma_0 + \frac{\gamma}{2}\int\frac{dk}{(p-k)^2}[u(k)u^\dagger(k) - v(-k)v^\dagger(-k)]\right\}u(p) = E(p)u(p)$$

Outlook

- Results are very encouraging but many open questions remain
- Extensions in (2+1)D:
  - Meson and baryon spectrum*
  - Finite temperature*
  - Scattering amplitudes
- Extension to (3+1)-dimensional YM*
  - It is possible to generalize KKN (I. Bars) formalism to 3+1 dimensions: L. Freidel, hep-th/0604185.