Analysis on the Ammonia Target Polarization for the Study of the GDH Integral

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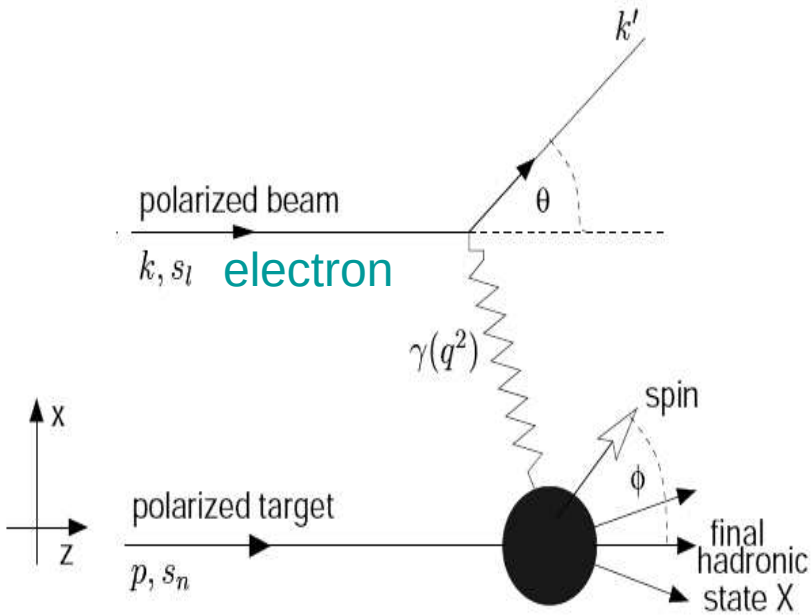


$NH_3(ND_3)$

Incident electrons Scattering off Target Nucleons

$$Q^2 = 2EE'(1 - cos\theta) = 4EE'sin^2(\tfrac{\theta}{2})$$

$$\nu = E - E' = \tfrac{p \cdot q}{M}$$

energy of the virtual photon

$$x = \frac{Q^2}{2M\nu}$$

Bjorken dimensionless variable

My goal in this experiment:

1. Study the GDH Integral

2. Calculate the first moment of g_1 at low Q^2 regions.

GDH Sum Rule

In the real photon limit, the GDH (Gerasimov-Drell-Hearn) Sum Rule relates the difference of the two photoabsorption cross-sections to the anomalous magnetic moment of the nucleon κ. M is the mass of the nucleon, $\sigma_T^{1/2}$ and $\sigma_T^{3/2}$ are the cross sections. ν_{th} is the one pion photoproduction threshold.

$$-\frac{\kappa^2}{4} = \frac{M}{8\pi^2\alpha}\int_{\nu_{th}}^{\infty}\frac{\sigma_T^{1/2}-\sigma_T^{3/2}}{\nu}d\nu,$$

Strictly speaking, the GDH Sum Rule is derived from the real photon limit, (Q^2=0) but assuming that the real photon cross sections are connected smoothly to the virtual photon ones, the Sum Rules can be generalized for the case of the virtual photons. Expressing the cross sections in terms of the spin structure functions gives the generalized Sum Rule:

$$I_{GDH}(Q^2) = \frac{2M}{Q^2}\int_0^{x_0}\left[g_1(x,Q^2) - \frac{Q^2}{\nu^2}g_2(x,Q^2)\right]dx$$

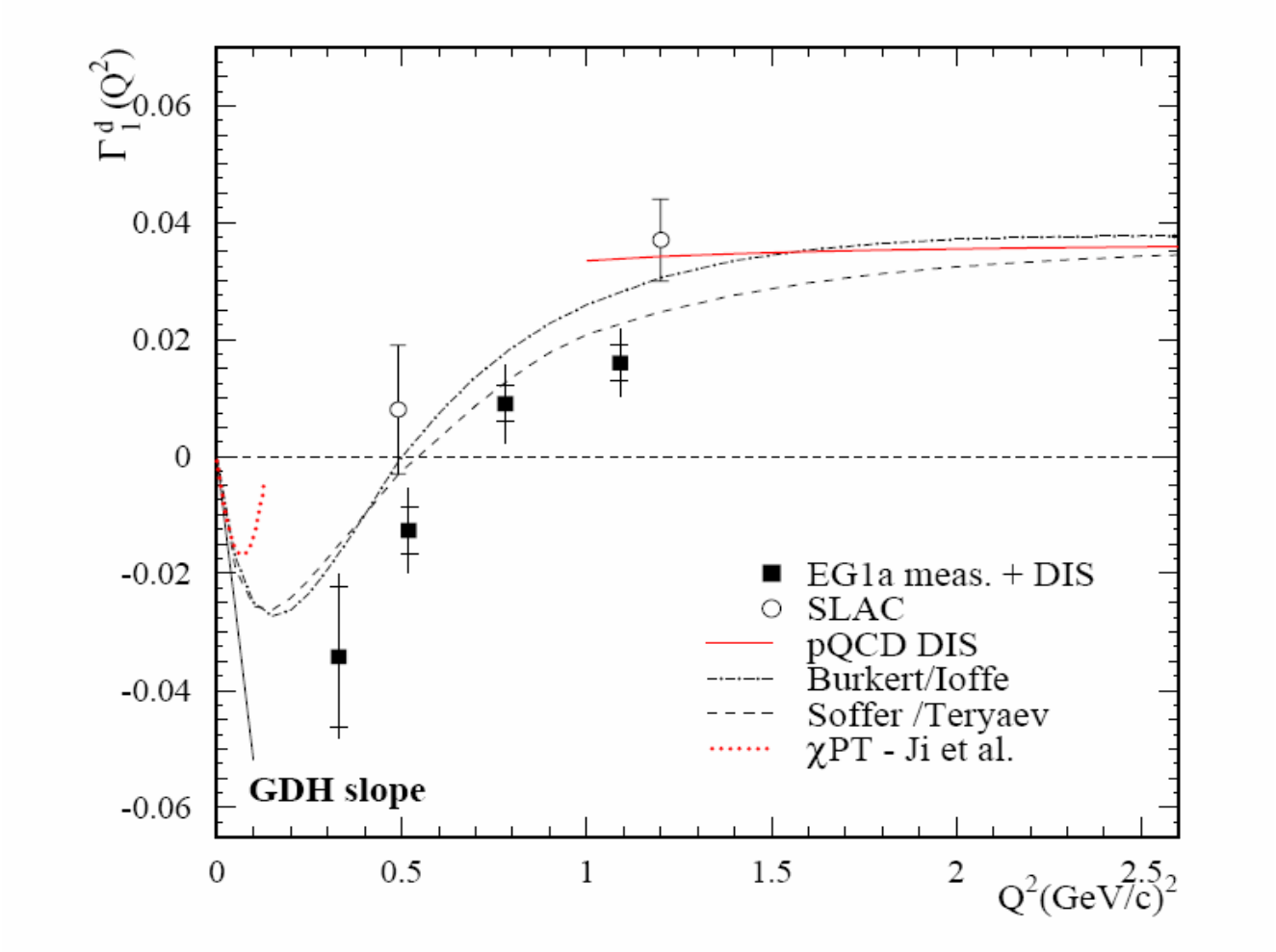
$$= \frac{M^2}{8\pi^2\alpha}\int_{\nu_{th}}^{\infty}(1 - \frac{Q^2}{2M\nu}(\sigma_{1/2}(\nu,Q^2) - \sigma_{3/2}(\nu,Q^2))\frac{d\nu}{\nu}.$$

As $Q^2 \to 0$:

$$\lim_{Q^2\to 0} I_1(Q^2) \Longrightarrow \frac{M^2}{8\pi^2\alpha}\int_{\nu_{th}}^{\infty}(\sigma_{1/2}(\nu,Q^2) - \sigma_{3/2}(\nu,Q^2))\frac{d\nu}{\nu},$$

$$\Gamma_1 = \frac{Q^2}{2M^2}I_1(Q^2) \Longrightarrow \frac{Q^2}{16\pi^2\alpha}(-\frac{2\pi^2\alpha\kappa^2}{M^2}) \Longrightarrow -\frac{Q^2\kappa^2}{8M^2}.$$

The conclusion that can be drawn here is that Γ_1 approaches 0 with a negative slope, and since it is positive in the high Q^2 region, it should rapidly change its sign somewhere in the resonance region $0 < Q^2 < 1$ GeV.


$\Gamma_1^d(Q^2)$
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
0.5
1
1.5
2
2.5
$Q^2(GeV/c)^2$
EG1a meas. + DIS
SLAC
pQCD DIS
Burkert/Ioffe
Soffer /Teryaev
χPT - Ji et al.
GDH slope

$\sigma_T^{1/2}$ and $\sigma_T^{3/2}$, the transverse cross sections for the photons, can be expressed in terms of the unpolarized structural functions F1, F2 and the spin dependent structural functions g1 and g2:

$$\sigma_{\frac{3}{2}} \propto W_1 - M\nu G_1 - q^2 G_2 \propto F_1(\xi, Q^2) - g_1(\xi, Q^2) \quad = \quad \sum_i e_i^2 q_i^{\downarrow}(\xi, Q^2),$$

$$\sigma_{\frac{1}{2}} \propto W_1 + M\nu G_1 + q^2 G_2 \propto F_1(\xi, Q^2) + g_1(\xi, Q^2) \quad = \quad \sum_i e_i^2 q_i^{\uparrow}(\xi, Q^2).$$

Where

$$F_1(\xi) \quad = \quad \frac{1}{2}[\frac{4}{9}(u(\xi) + \bar{u}(\xi)) + \frac{1}{9}(d(\xi) + \bar{d}(\xi) + s(\xi) + \bar{s}(\xi)],$$

$$g_1(\xi, Q^2) = \frac{1}{2} \sum_i e_i^2 [q_i^{\uparrow}(\xi, Q^2) - q_i^{\downarrow}(\xi, Q^2)].$$

The total cross section at a given resonance can be expressed in terms of helicity amplitudes A_1 and A_2:

$$A_1 = \frac{\sigma^T_{\frac{1}{2}} - \sigma^T_{\frac{3}{2}}}{\sigma^T_{\frac{1}{2}} + \sigma^T_{\frac{3}{2}}}. \qquad A_2 = \frac{2\sigma_{LT}}{\sigma^T_{\frac{1}{2}} + \sigma^T_{\frac{3}{2}}}.$$

In our experiment, we use longitudinally polarized electron as the incident beam particle. The measured electron cross section asymmetry is as follows:

$$A_{||} = \frac{A_{exp}}{P_B P_T F} + \Delta_{RC}, \qquad A_{exp} = \frac{N^{\uparrow\downarrow}/N_e^{\uparrow\downarrow} - N^{\uparrow\uparrow}/N_e^{\uparrow\uparrow}}{N^{\uparrow\downarrow}/N_e^{\uparrow\downarrow} + N^{\uparrow\uparrow}/N_e^{\uparrow\uparrow}}.$$

This can be related to the desired virtual photon asymmetries:

$$A_{||} = D(A_1 + \eta A_2),$$

$$A_{\perp} = d(A_2 - \zeta A_1),$$

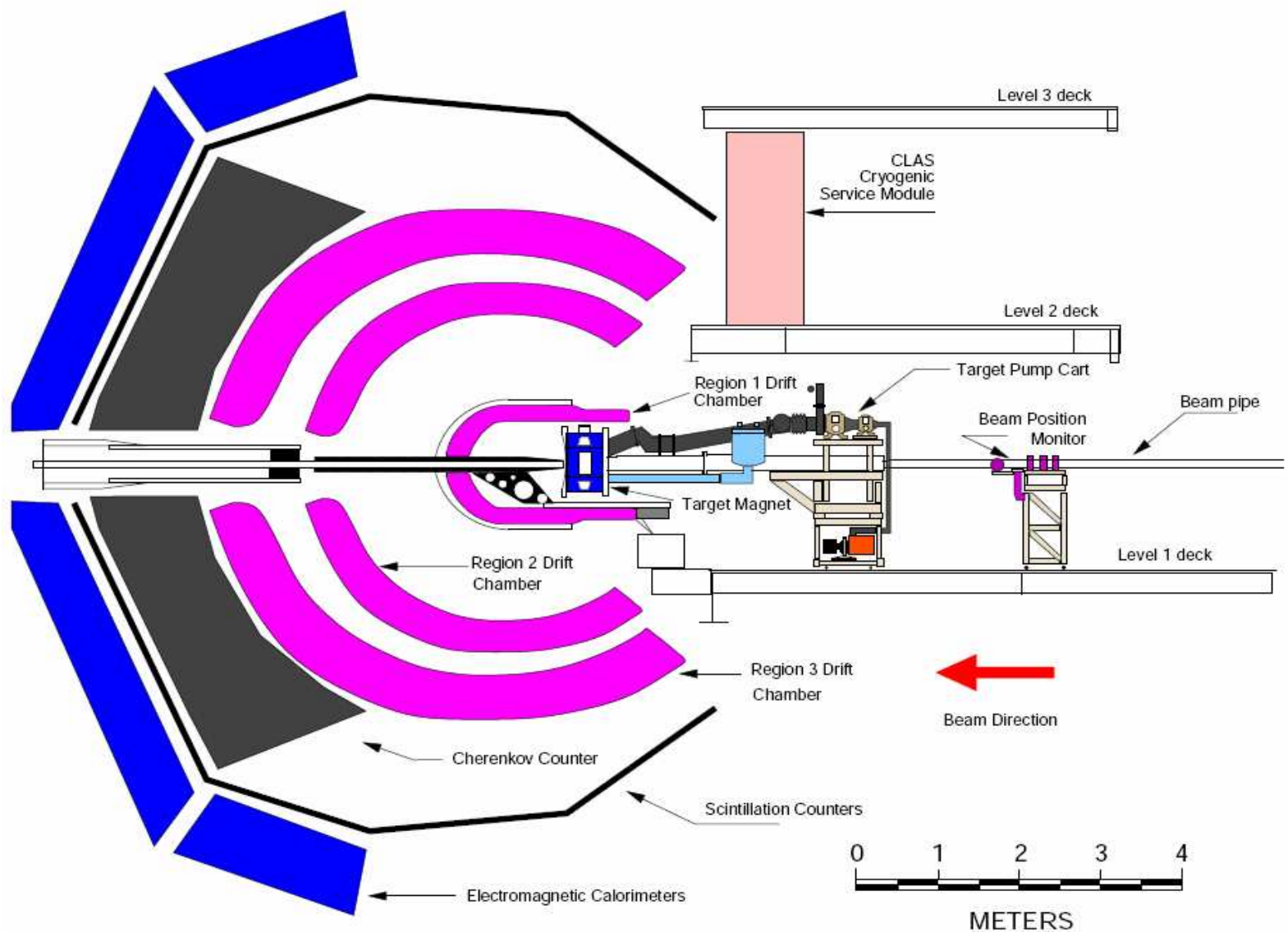
The spin structure function g_1 can be calculated:

$$g_1 = \frac{F_1}{1+\gamma^2}\left[A_{||}/D + (\gamma - \eta)A_2\right],$$

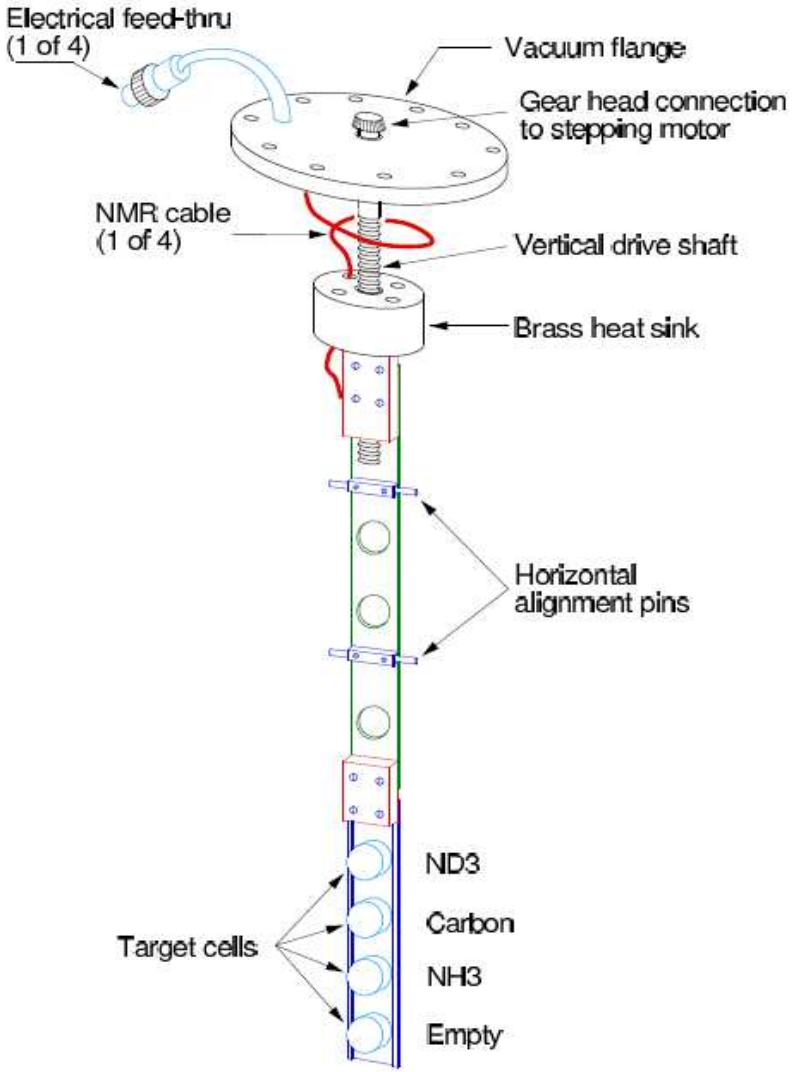
The first moment of g_1 :

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)dx,$$

EG4 Experiment Setup

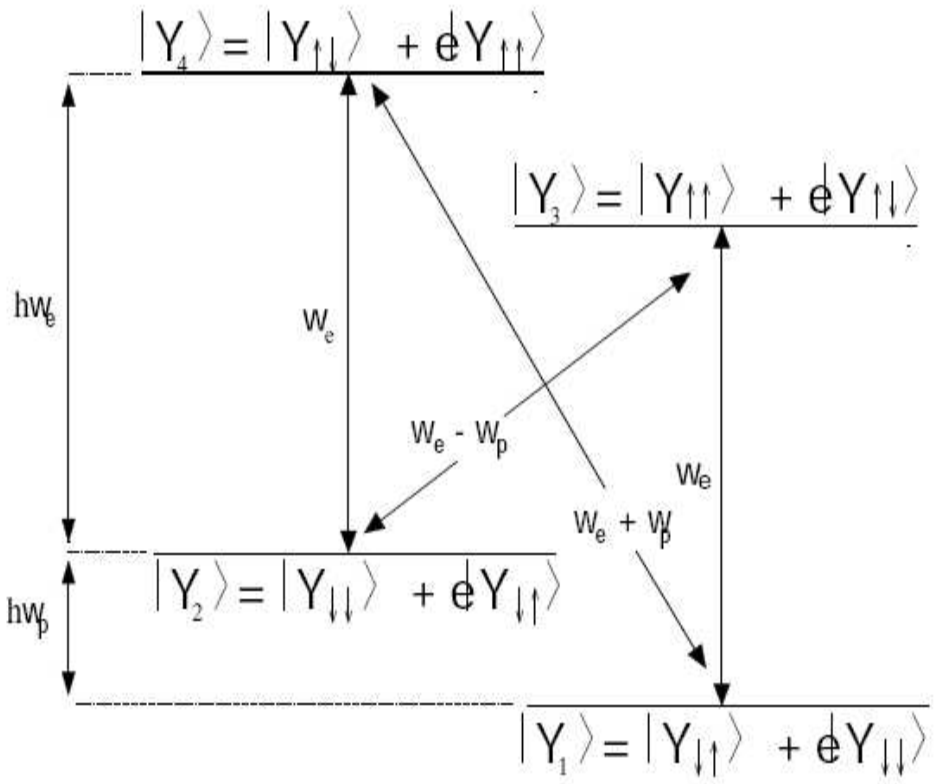


Target Insert:



The upper end of the insert is connected with NMR cables and microwave input. The lower end of the target insert has 4 cells that hold ND_3 , NH_3 and the carbon target respectively. The carbon target and empty cell are used as the calibration.

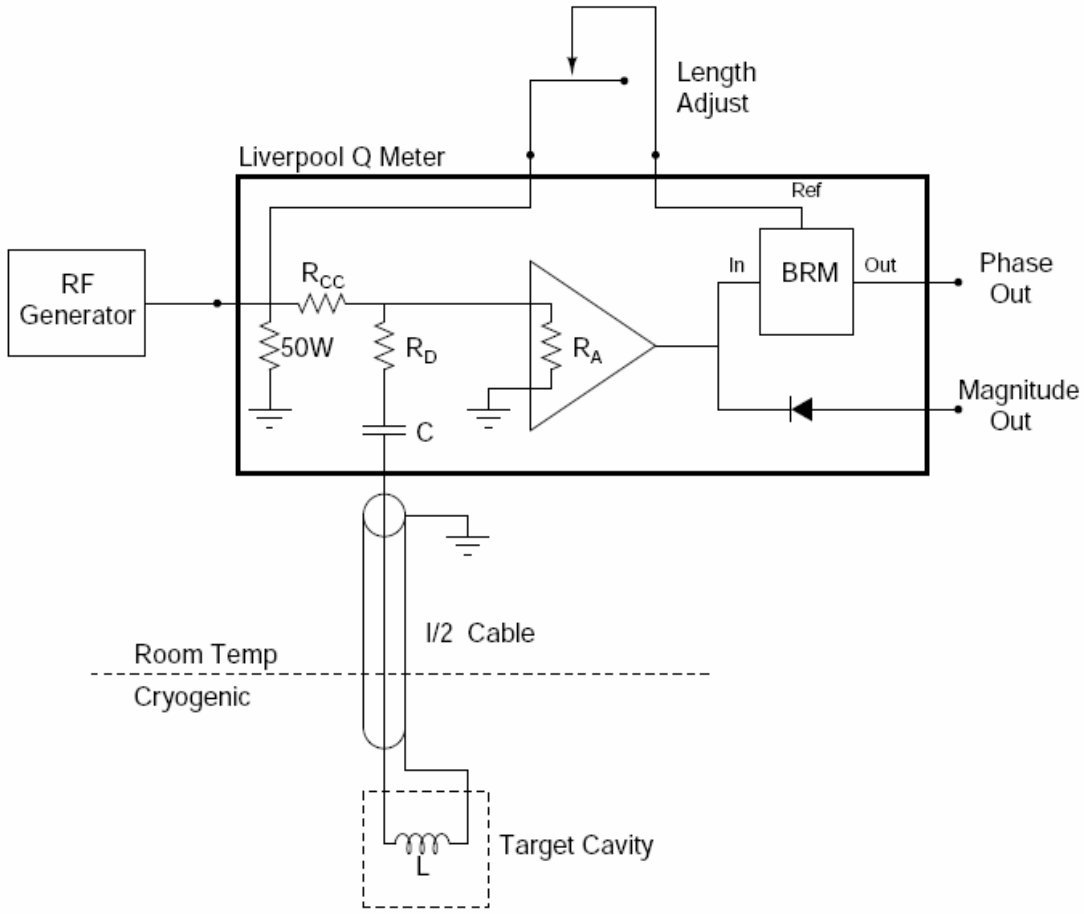
Target preparation --- the DNP method:



- Electrons are much easier to polarize than nucleons.
- The dipole interaction between the nucleon and electron magnetic moment induces the simultaneous spin flips of an electron and a nucleon.
- The electron spin transitions (interaction between spin and lattice) takes place at a much faster rate than the relaxation time of the nucleons, thus the polarization can be transferred from the electrons to the nucleons.

A photo of the target cell after the beam exposure. The beads in the center turned purple from the exposure while the beads at the edges did not receive much beam. Size: 1-2 mm

NMR System:

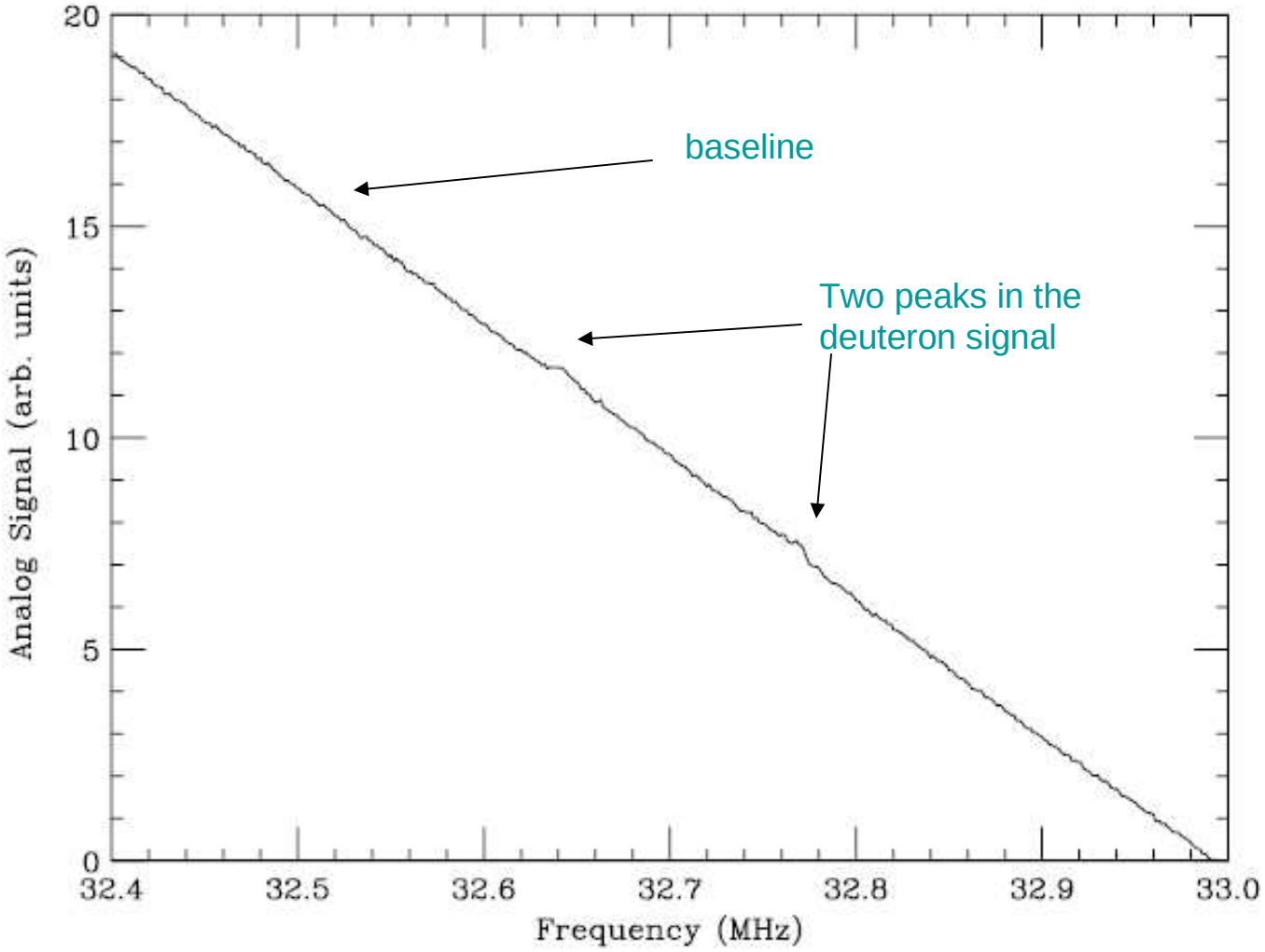


Circuit diagram of a Liverpool Q-meter

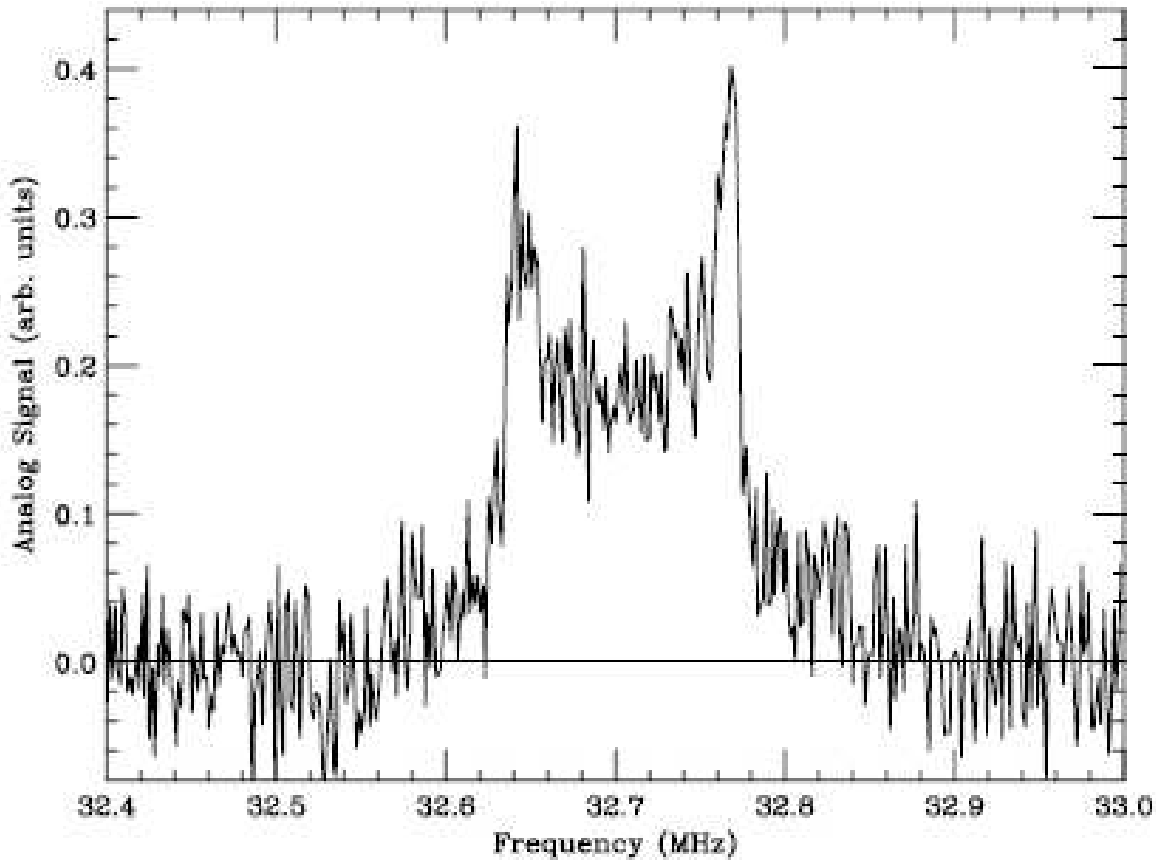
The Mechanism of the Q-meter:

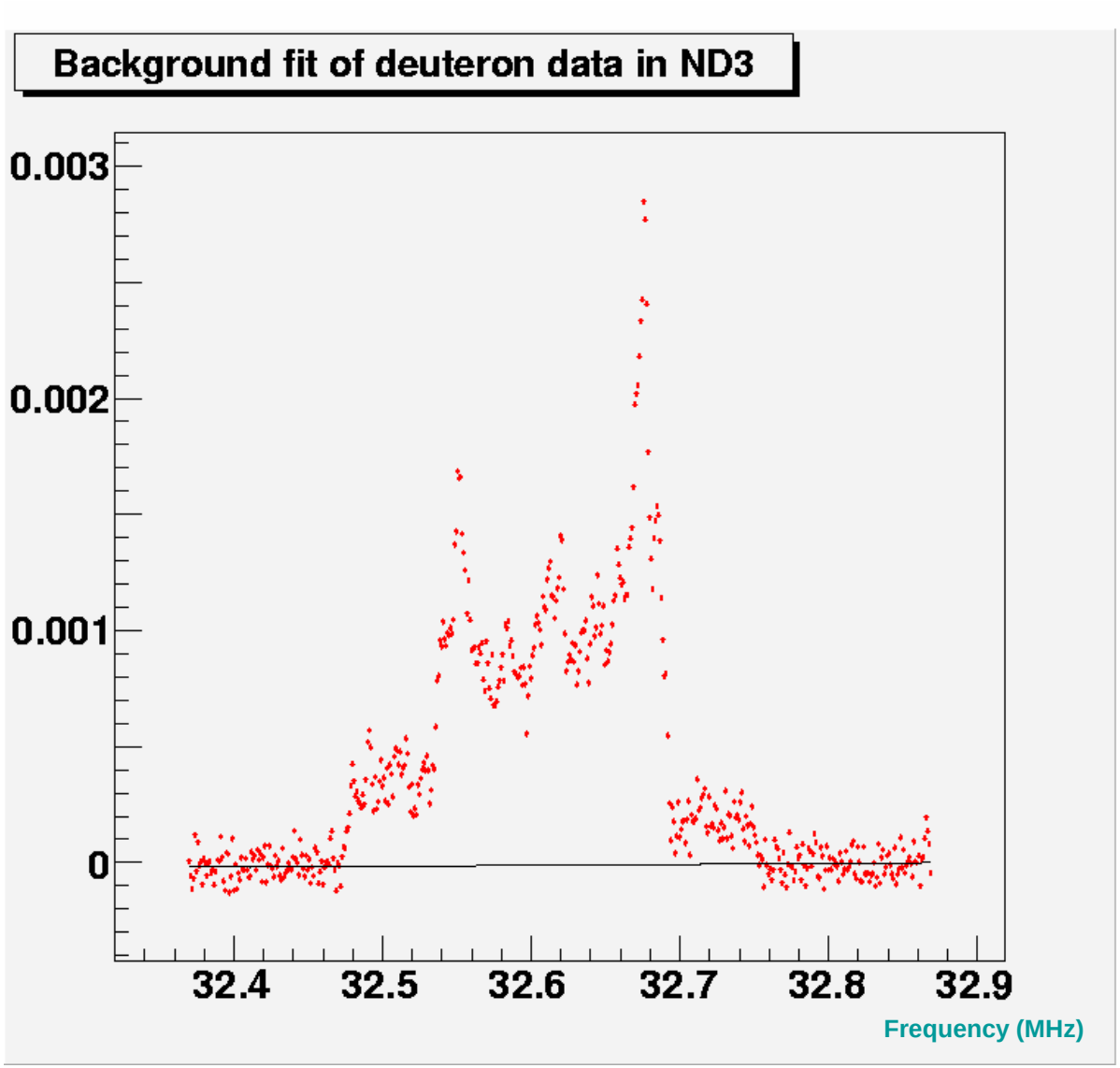
- Input: rf - frequency sweeping through the Larmor frequency of the transition.
- Output: V vs. f.

V proportional to energy gain or loss during the transition



Deuteron target polarization before the baseline subtraction


0.4
0.3
0.2
0.1
0.0
Analog Signal (arb. units)
32.4
32.5
32.6
32.7
32.8
32.9
33.0
Frequency (MHz)



Deuteron target signal after the baseline subtraction.

Target Polarization Analysis:

1. Area Method:

$P_{enh}/P_{TE} = Area_{ehn}/Area_{TE}$

$$P_{TE} = \frac{4\tanh(\frac{\hbar\omega_0}{2kT})}{3+\tanh^2(\frac{\hbar\omega_0}{2kT})}$$

However, in the EG4 experiment, the TE signals of deuteron target is very small, which significantly limits the accuracy of the area measurement for the TE signal. Therefore, we adopted the ratio method to determine the deuteron polarization. The target we used is ND_3.

2. Ratio Method:

$$P(r) = \frac{r^2 - 1}{r^2 + r + 1}$$

When the quadruple coupling is weak, as in the case for the deuteron signals in this experiment, this equation is a good approximation of the relation between the deuteron polarization and the peak heights ratio.

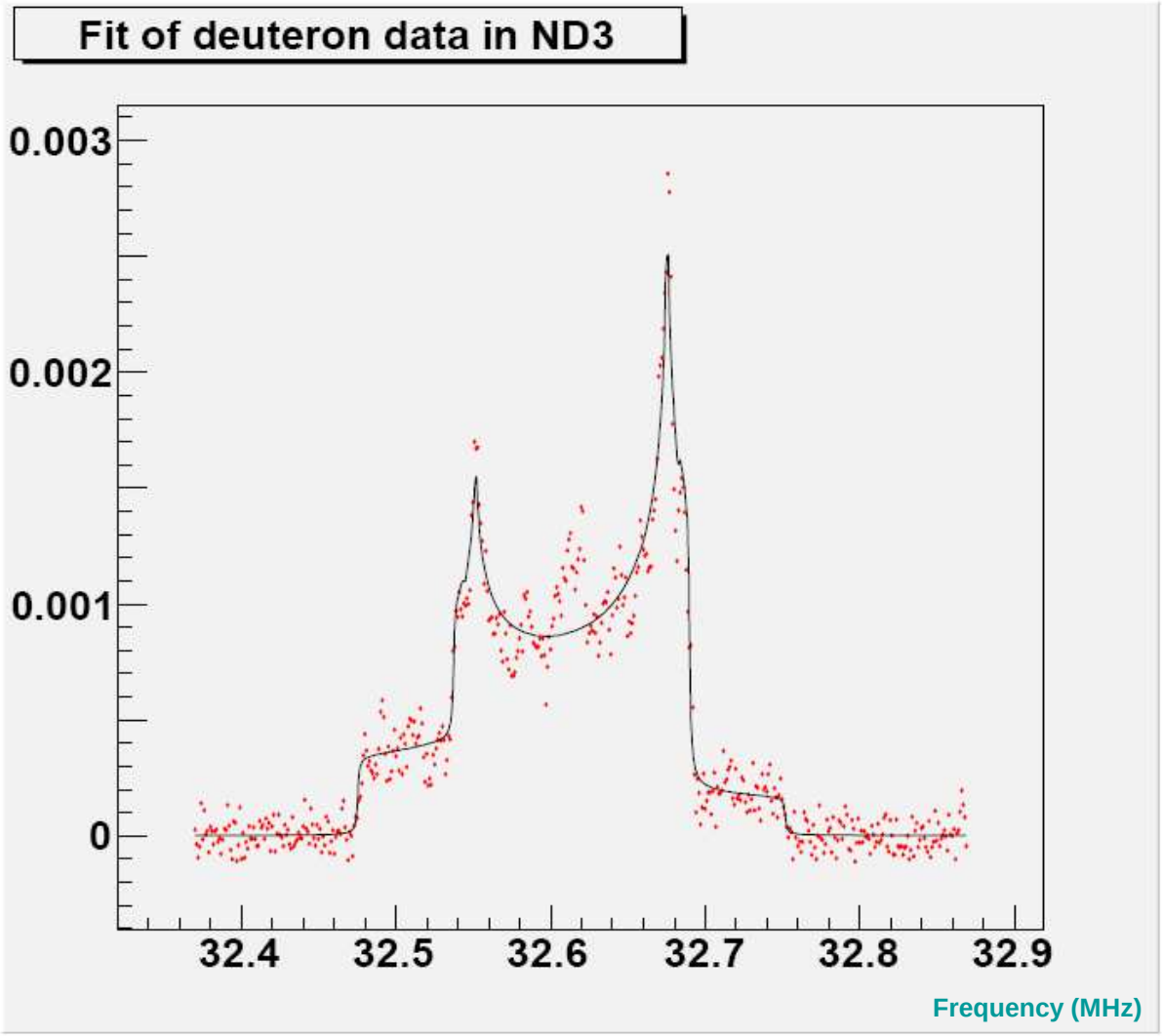


Fig 9: Fitting the NMR signal by the theoretical Lineshape function. Black line is the fitting lineshape and red dots are the NMR data

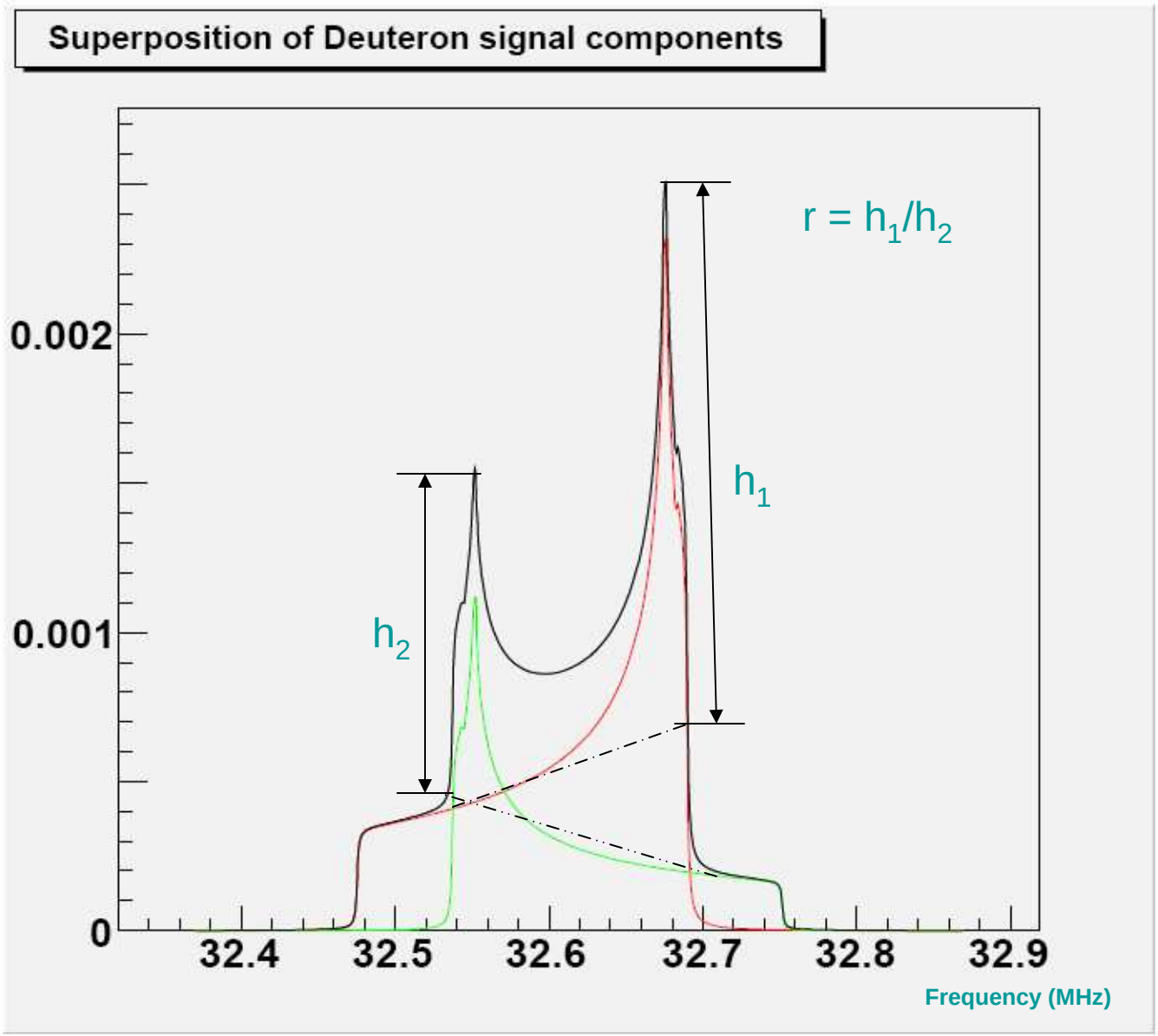


Fig 10: Peaking fitting process done by the program

- CC = P (r) / A

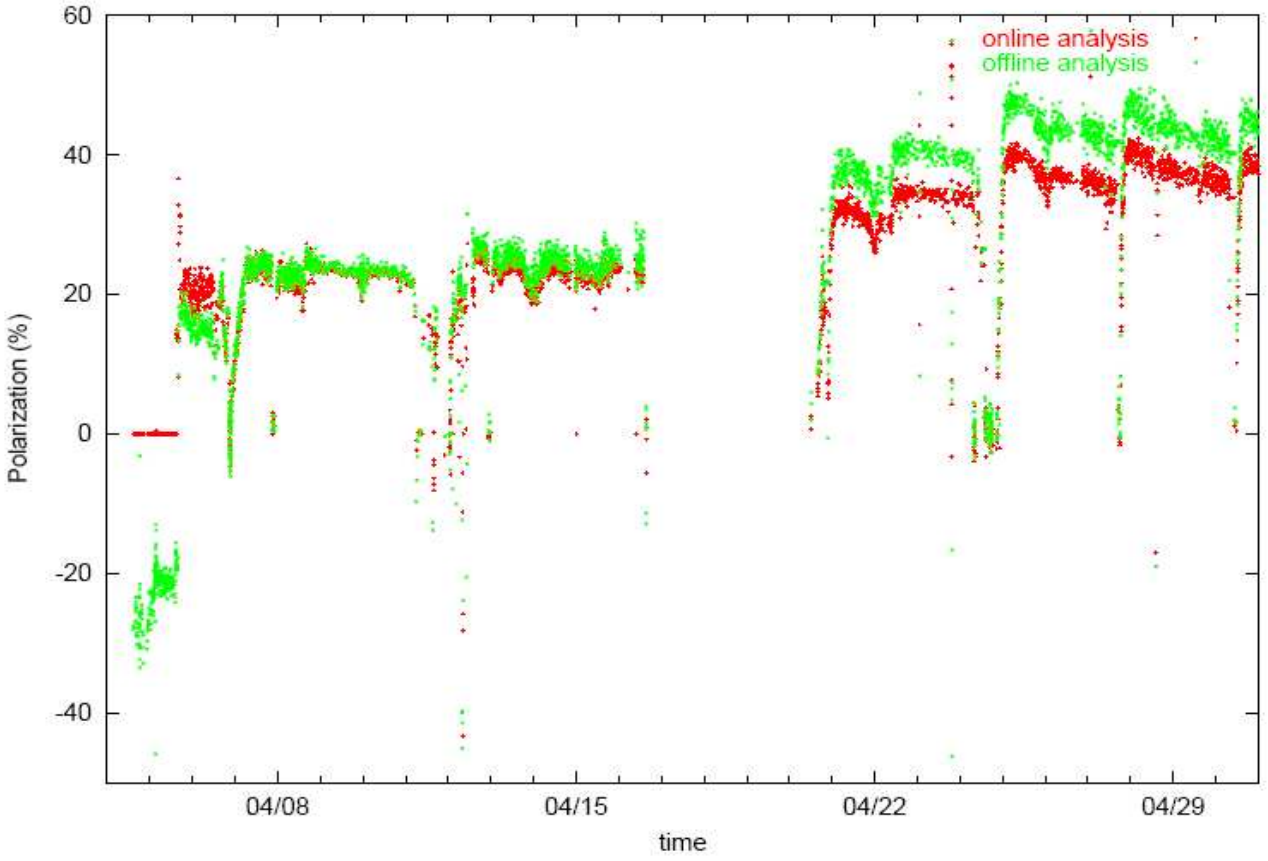
- We classified all the runs in April into two periods, the first one from April 5th to April 19th, the second one from April 20th to April 30th. Each polarization period has its own value of CC. The reason for this is because our ND_3 target material was taken out from the chamber once and we should expect some small changes in CC value associated with this change.

$$AverageCC = 219.874 \pm 10.78$$

For the period between April 20th to April 30th, and

$$AverageCC = 193.245 \pm 9.56$$

For the period between April 5th to April 19th.



ND_3 target polarization timeline during April, 2006. The red dots are online polarization values and the green dots are offline analysis polarization values.

Error Analysis:

1. Baseline Adjustment

Baseline	CC	Difference
04-27-06T09:29AM	219.874	
04-24-06T11:26AM	218.984	0.4%
04-24-06T08:35PM	223.797	1.8%
04-24-06T03:59PM	213.773	2.7%

As is shown the value of the CC decreased as the less recently updated baselines were used. The baselines that date even further back were not used as the CC obtained using those baselines deviate too much from the value 219.874. Varying baselines that were taken around the time the most recent one was taken gives us an idea about the stability of the fitting program.

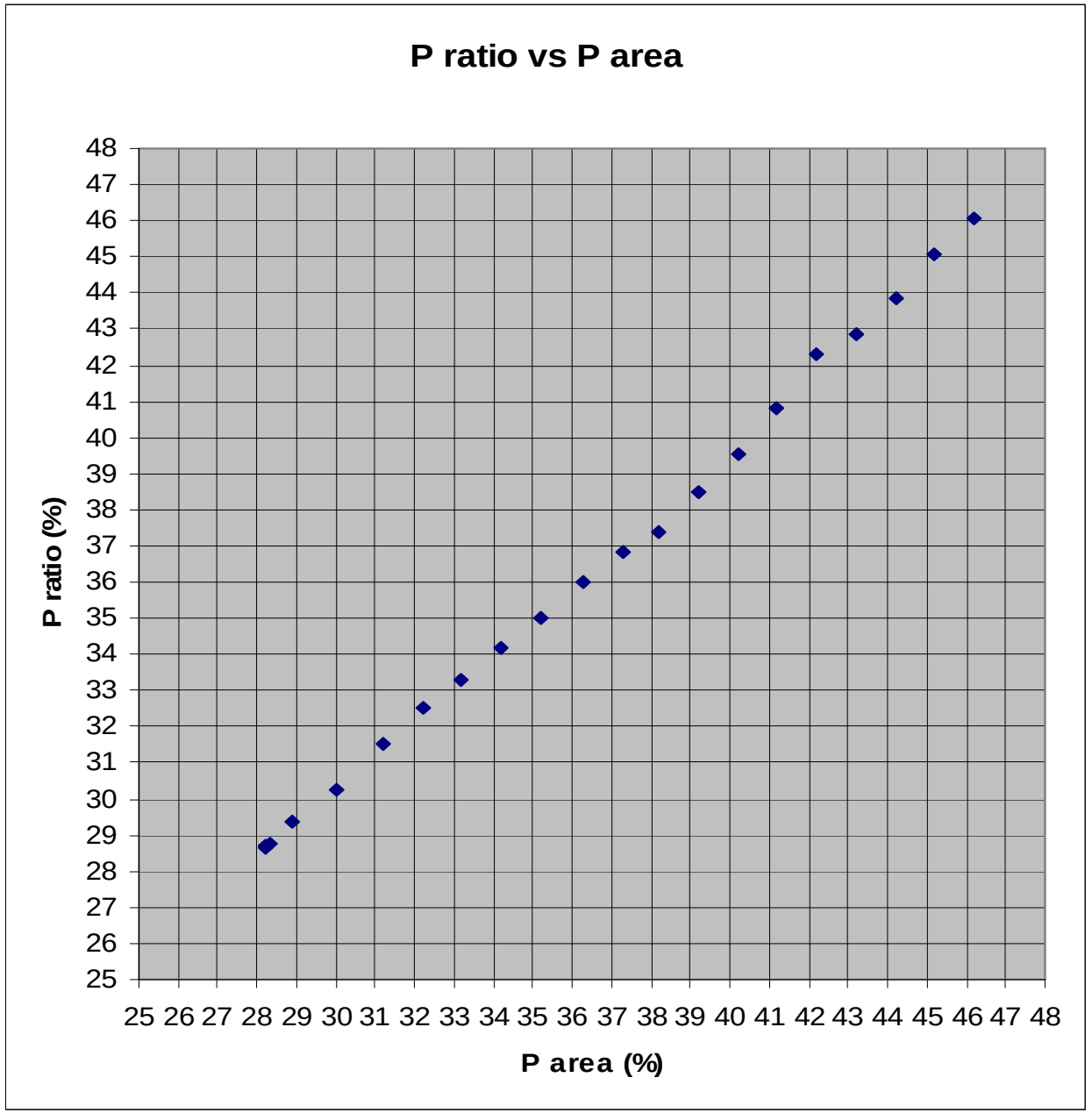
2. Comparison with the Area Method Results using the GEN Data

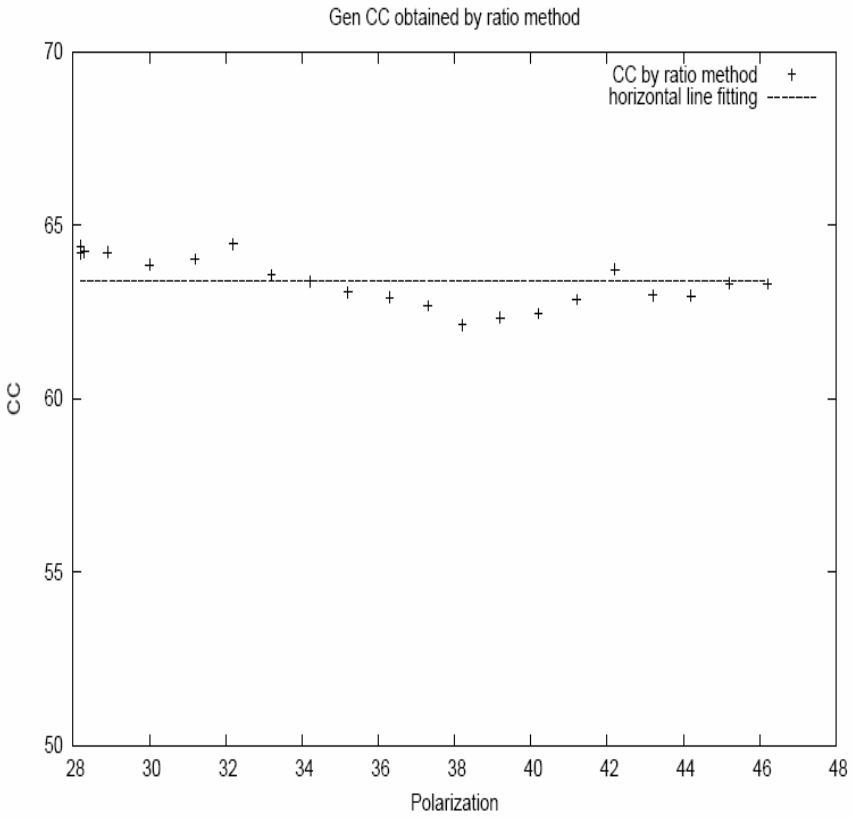
The reasons we chose to use the GEN data are as follows:

- Big NMR signals were available in the GEN experiment, which makes the polarization values obtained from the area method more accurate and reliable;
- The baselines were updated frequently during the polarization measurements to minimize the error associated with the baseline subtraction.

The following table shows the comparison between the two deuteron polarization results:

P_A	P_r	A1	A2	CC_r	CC_A	error
28.3	28.7374	0.4473	0.4473	64.24637	63.26850	1.52206
28.2	28.6458	0.4462	0.4461	64.19946	63.21453	1.53418
28.2	28.6923	0.4455	0.4453	64.40471	63.32809	1.67165
28.9	29.3618	0.4572	0.4572	64.22091	63.21085	1.57279
30.0	30.2509	0.4737	0.4736	63.86088	63.34459	0.80846
31.2	31.5377	0.4925	0.4923	64.03594	63.37599	1.03059
32.2	32.5362	0.5047	0.5077	64.46642	63.42328	1.61810
33.2	33.3072	0.5239	0.5237	63.57549	63.39507	0.28379
34.2	34.1944	0.5394	0.5392	63.39340	63.42730	0.05348
35.2	34.9877	0.5547	0.554	63.07500	63.53791	0.73391
36.3	35.9823	0.5719	0.571	62.91712	63.57268	1.04194
37.3	36.8196	0.5874	0.5865	62.68233	63.59761	1.46019
38.2	37.4032	0.602	0.6017	62.13156	63.48679	2.18122
39.2	38.499	0.6176	0.6173	62.33646	63.50235	1.87031
40.2	39.5605	0.6334	0.633	62.45737	63.50711	1.68072
41.2	40.813	0.6493	0.6489	62.85692	63.49206	1.01045
42.2	42.2913	0.6636	0.6619	63.73011	63.75585	0.04040
43.2	42.873	0.6807	0.68	62.98369	63.52941	0.86644
44.2	43.8665	0.6967	0.6959	62.96326	63.51487	0.87609
45.2	45.0867	0.712	0.711	63.32402	63.57243	0.39229
46.2	46.0833	0.7279	0.727	63.30993	63.54883	0.37735


P ratio vs P area
P ratio (%)
48
47
46
45
44
43
42
41
40
39
38
37
36
35
34
33
32
31
30
29
28
27
26
25
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48
P area (%)



CC values obtained via ratio method using the GEN NMR data over different polarization values.

A plot of the CC values obtained using the ratio method. We fitted those values to a horizontal line of CC = 63.3891 and got chi^2/D.F = 0.5042, which indicates that the CC values obtained by applying this program to the GEN experiment data are stable for different time periods and measurements. The deuteron target polarization results obtained via the ratio method agreed well with that obtained via the area method during the GEN data analysis.

Determining the Target Polarization from the Scattering Asymmetry:

1. Drawbacks of the NMR Signals:

- The signal size affected the accuracy of the fitting and peak ratio (or area) calculation;
- The target material in the center of the cell is likely to have radiation damage due to the exposure to the electron beam. The radiation damage could cause a depolarized region towards the center. But the NMR wires are located outside of the cell, therefore it mostly measured the outer polarization.

$$A_{\parallel} = \frac{2\tau r * \left[\frac{m_p}{e} + r(\tau\frac{m_p}{e} + (1+\tau)\tan^2(\frac{\theta}{2}))\right]}{1+\tau\frac{r^2}{\epsilon}},$$

$$r = \frac{G_{M_p}}{G_{E_p}} \approx 2.79. \qquad \tau = \frac{Q^2}{4m_p^2}$$

Theoretical elastic asymmetry evaluated for every beam energy setting and every Q^2 bin

$$A_{el} = \frac{nh_3^{\uparrow} - nh_3^{\downarrow}}{nh_3^{\uparrow} + nh_3^{\downarrow} - background}.$$

Experimental elastic asymmetry

$$P_b P_t = \frac{A_{measured}}{A_{predicted}}.$$

Conclusion

My analysis work on the EG4 experiment has focused on using the ratio method and applying the revised program of Chris Dulya's to analyze the ND_3 target polarization. It is shown by analyzing the error, applying the method on the previous GEN experiment data to compare the results with that obtained via the area method, that the ratio method is both reliable and accurate in measuring the deuteron NMR polarization. In the case of the EG4 experiment, when taking the TE signals and using the area method for the deuteron target signals are not practical due to the large error, the ratio method becomes even more important.

Next I will analyze the deuteron target polarization from the cross section asymmetry and compare the results with the polarization values I obtained from the NMR signals using the ratio method for a more precise measurement of the deuteron target polarization. .

References

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2. Measurement of the Spin Structure Function $g_1(x,Q^2)$ of the Proton in the Resonance Region, Yelena A. Prok, Department of Physics, University of Virginia, May, 2004
3. The SMC Polarized Target, The Spin Muon Collaboration, SMC/97/XX, September 22, 1997 .
4. C. Dulya et al., Nucl. Instr. and Meth. in Phys. Res. A 398 109-125 (1997)