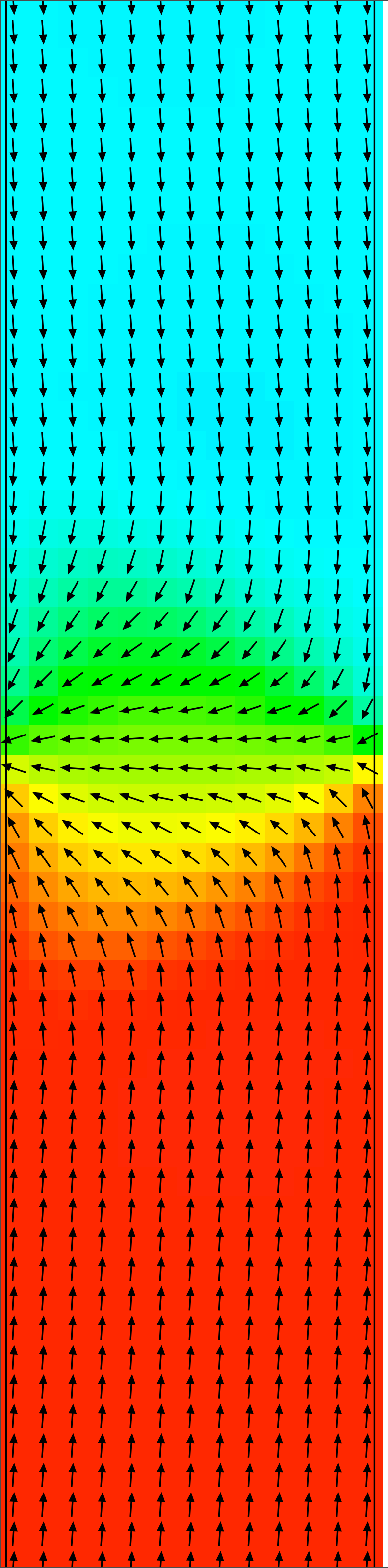
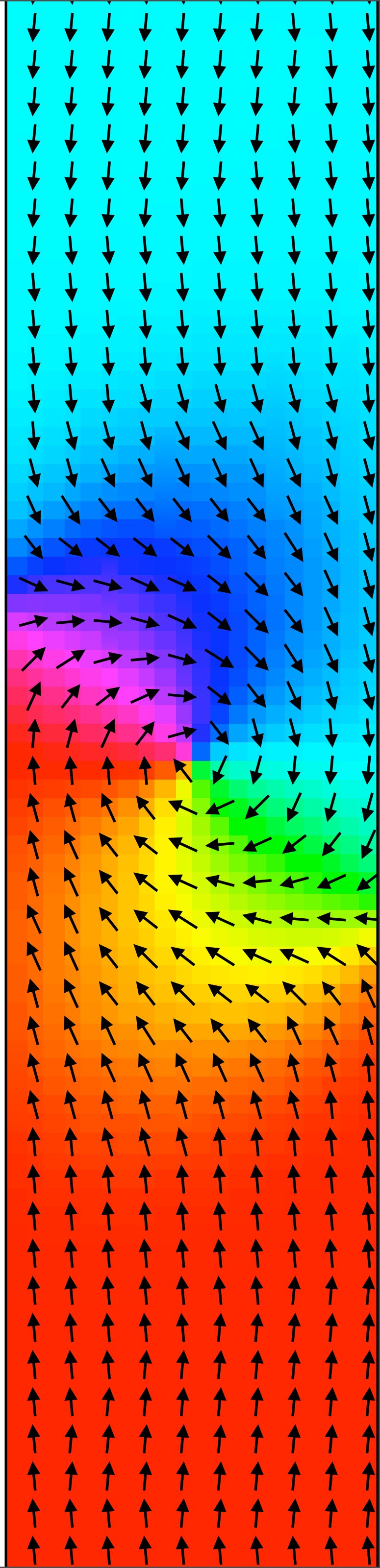


Topological defects in nanomagnets

Oleg Tchernyshyov

JOHNS HOPKINS UNIVERSITY



Acknowledgments

- Ya. Bazaliy (IBM Almaden → Leiden → USC).
- G.-W. Chern, D. Clarke, O. Tretiakov (JHU).
- C.-L. Chien, N. Markovic (JHU).

NSF

MRSEC Grant DMR-0520491.

Size does matter

- A macroscopic magnet has two states:
  - magnetized (uniform magnetization **M**),
  - demagnetized (domains with different **M**).
  - Domains separated by sharp domain walls.
- New physics on the nanoscale:
  - domain walls don’t fit inside a small magnet,
  - intricate continuous textures form instead.
- We’ll discuss properties of these textures.
- Fractional vortices, skyrmions, monopoles ahead!

Ferromagnetism basics

- Quantum exchange interaction:
  - short-range,
  - lines up spins parallel to each other.
- Crystalline anisotropies:
  - short-range,
  - lines up spins with crystalline axes.
- Dipolar interaction:
  - long-range,
  - discourages formation of magnetic charges.

Exchange

$$E = A \int_{\text{sample}} d^3r \, |\nabla \hat{\mathbf{m}}|^2, \quad \hat{\mathbf{m}} = \mathbf{M}/M.$$

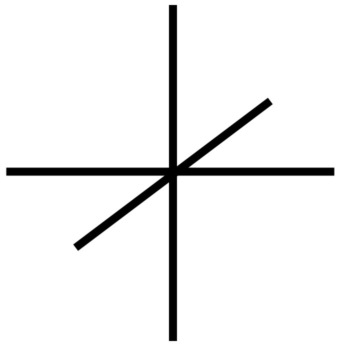
Exchange constant.

Ground state: $\hat{\mathbf{m}} = \text{const}.$

Spontaneously breaks the O(3) symmetry.

Scales as the linear size of the system.

Anisotropies



Crystal anisotropy + spin-orbit interaction.

Example: crystal with a cubic symmetry.

$$E = -K \int_{\text{sample}} d^3r \, (m_x^4 + m_y^4 + m_z^4).$$

Rotational symmetry is explicitly broken.

Ground state: $\hat{\mathbf{m}} = (\pm 1, 0, 0), \ (0, \pm 1, 0), \ (0, 0, \pm 1).$

Breaks the residual discrete group D_2.

Dipolar

$$E = \frac{\mu_0}{2} \int_{\text{all space}} d^3r\, H^2.$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \nabla \cdot (\mathbf{H} + \mathbf{M}) = 0.$$

$$\nabla \cdot \mathbf{H} = 4\pi\rho, \quad \rho = -\nabla \cdot \mathbf{M}/4\pi.$$

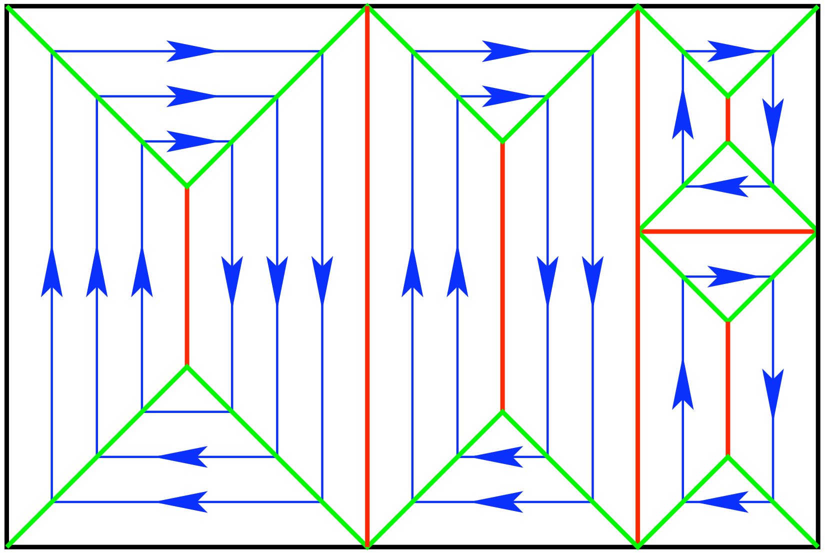
$$E = \frac{\mu_0}{8\pi} \int_{\text{sample}} d^3r\, d^3r' \, \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Scales as the volume of the sample.

Comparing the energies

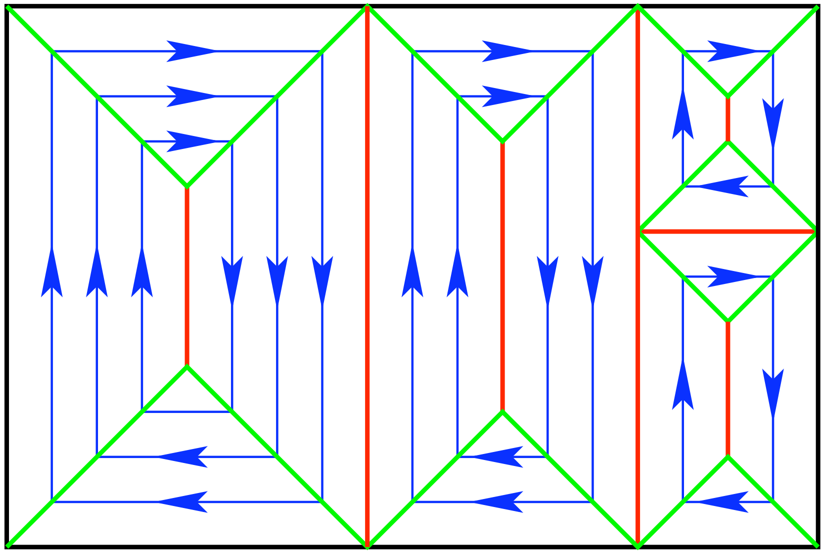
- Exchange: scales as sample length.
- Dipolar: scales as system volume.
- Hence **H**=0 (and ρ=0) in a large sample.
- Lines of **M** do not originate or terminate.
- Cost of domain walls less than cost of **H**.

Magnetic domains in Fe



Domains of uniform **M** separated by 90° and 180° walls.

Magnetic domains in Fe



Domains of uniform **M** separated by 90° and 180° walls.

$$\lambda_1 = \sqrt{2A/\mu_0 M^2} \approx 10 \text{ nm}, \quad \lambda_2 = \sqrt{A/K} \gtrsim 100 \text{ nm}.$$

Exchange vs dipolar

Exchange vs anisotropy

Real Fe sample

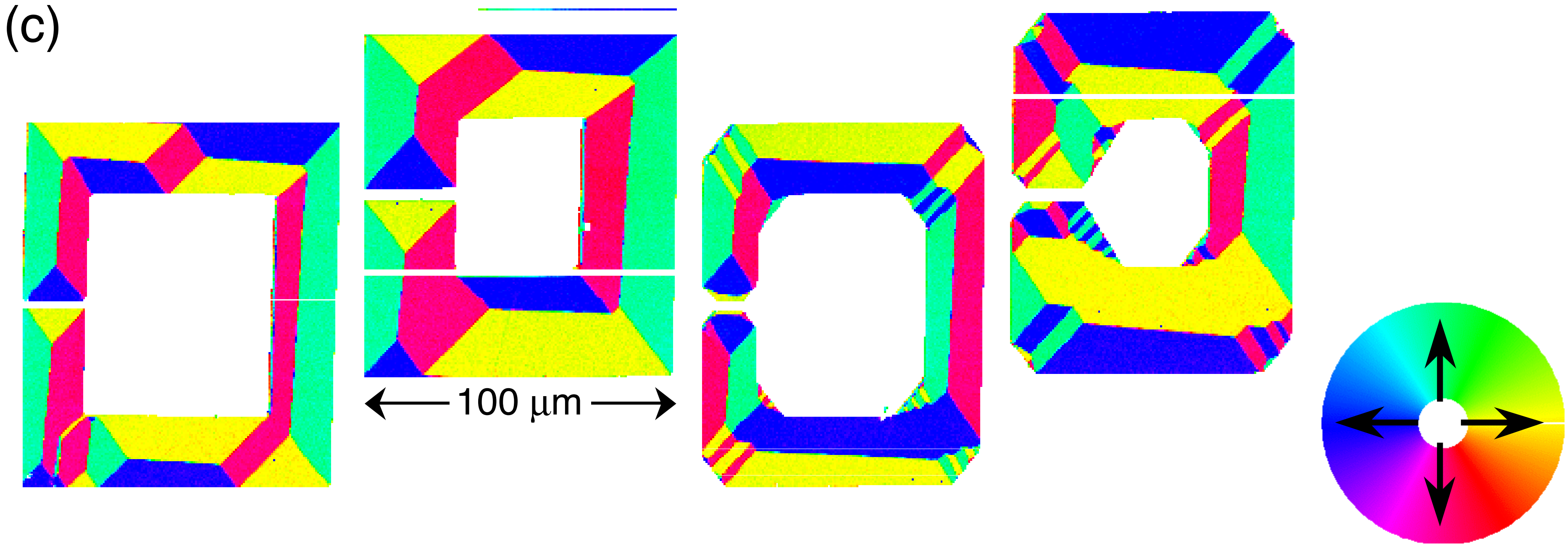
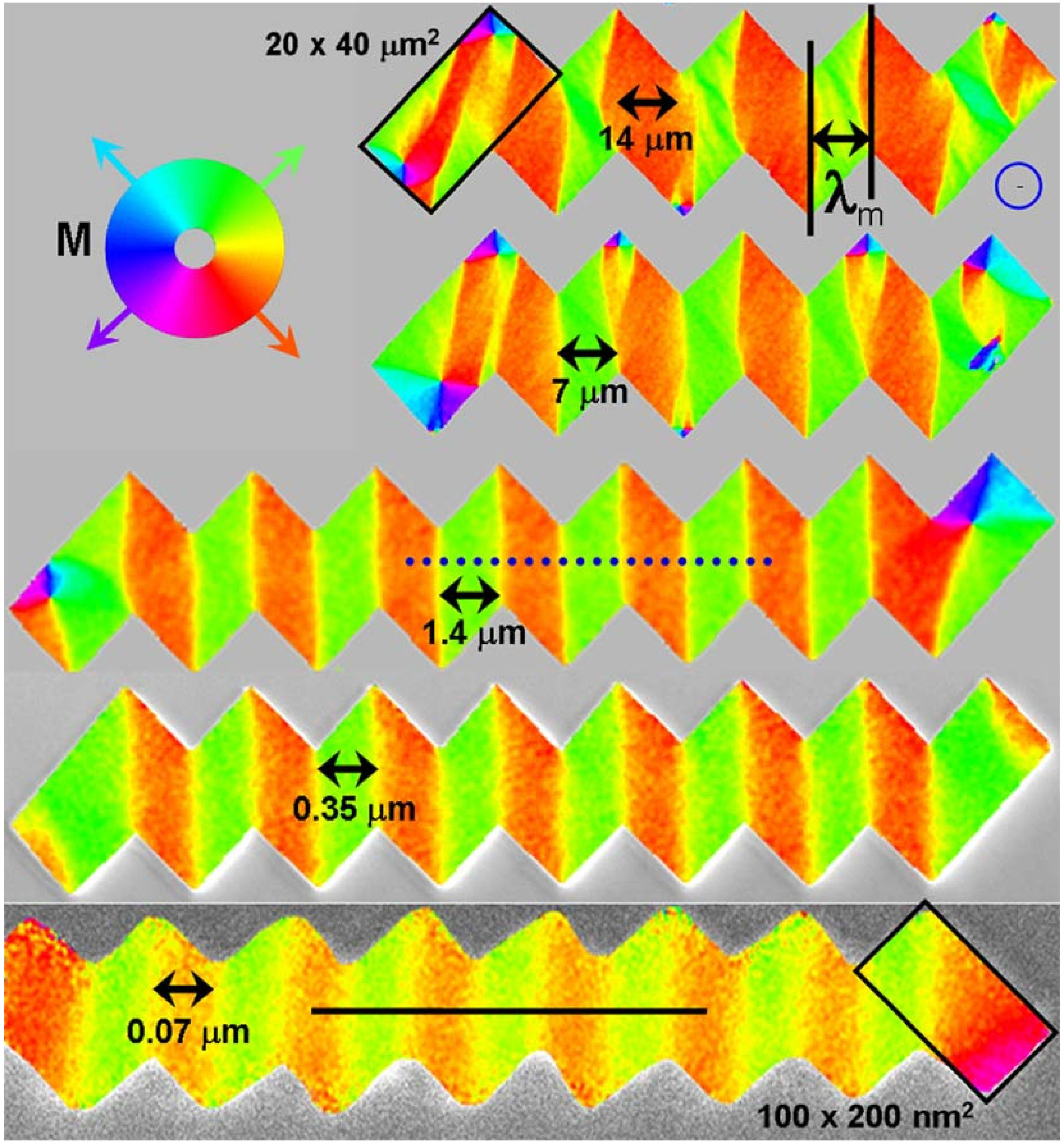


Fig. 6.17 SEMPA images of magnetization direction in (a) an amorphous ribbon, (b) a Co/Cu mulitlayer, and (c) patterned Fe films. Relationship between color and direction is given by colorwheel.

Scanning electron microscopy, J. Unguris (2000).

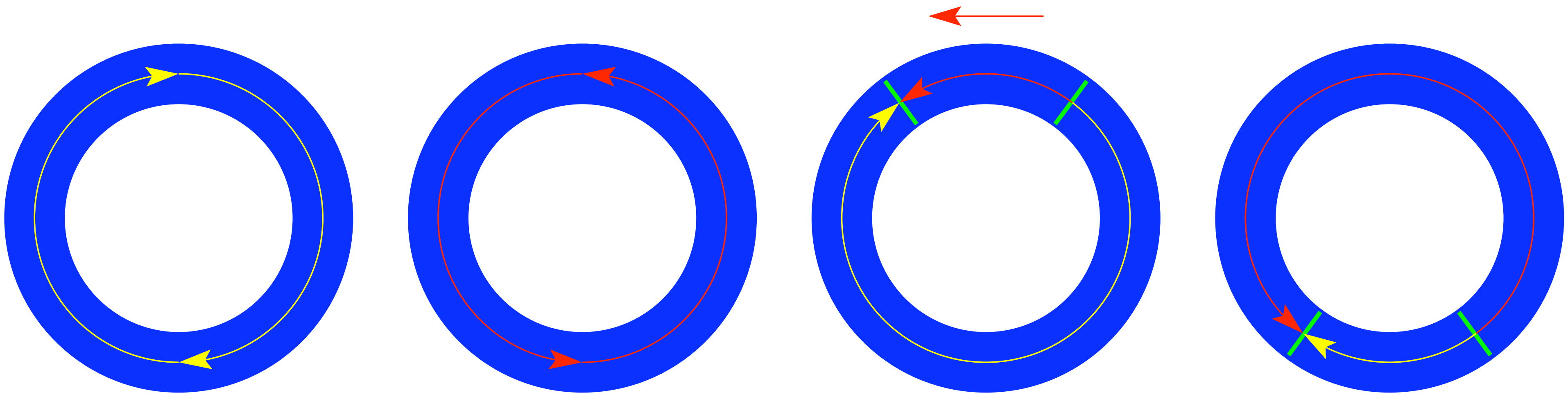
New physics on the nanoscale

- In submicron samples $E_{\text{exchange}} \sim E_{\text{dipolar}}$.
- Anisotropy is small in amorphous alloys.
- Domain walls have internal structure,
- exhibit nontrivial dynamic behavior.



Scanning electron microscopy, W. C. Uhlig and J. Unguris, JAP (2006).

Memory based on magnetic nanorings



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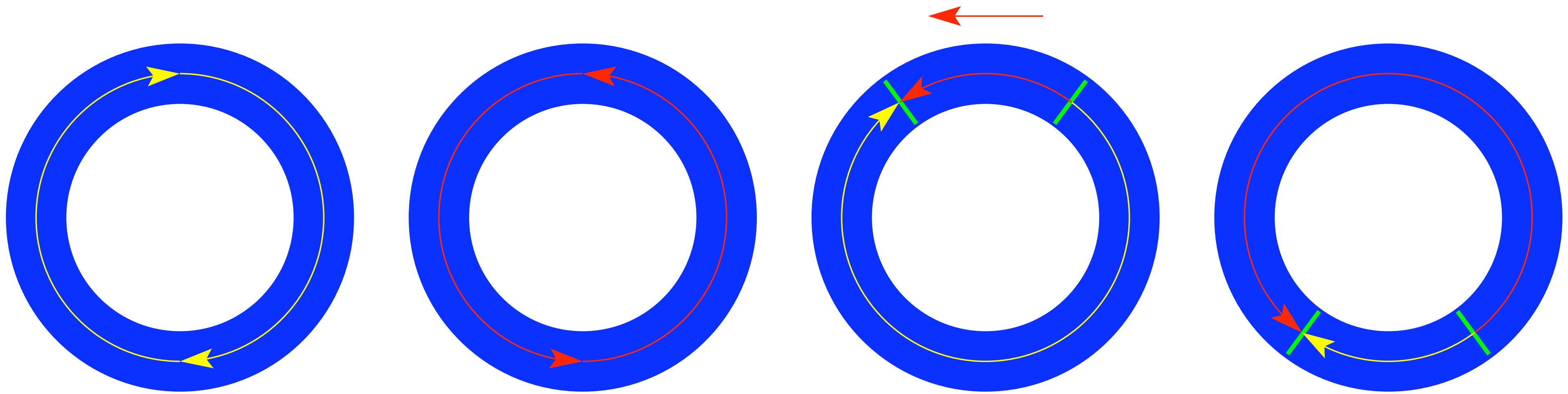
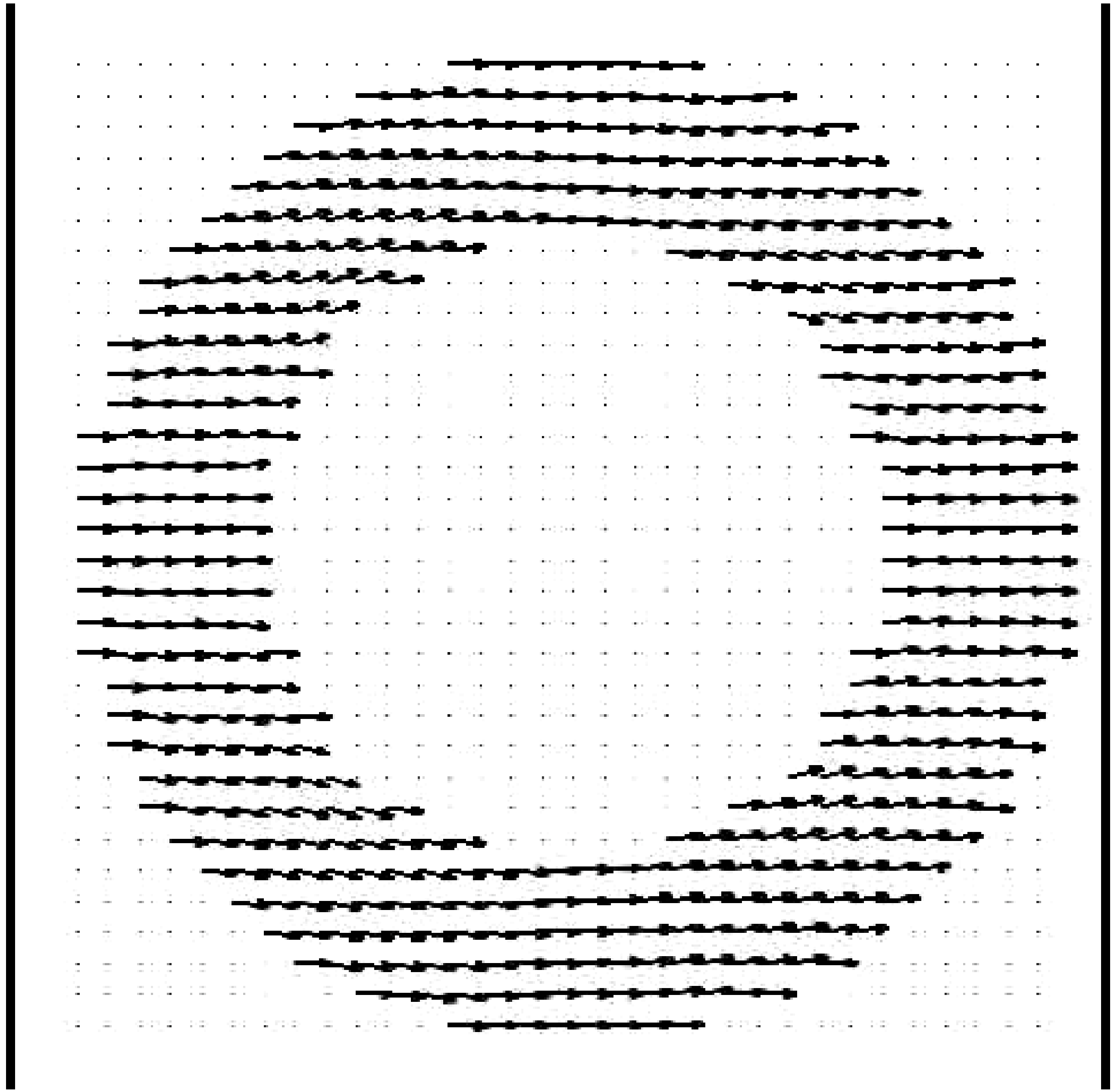
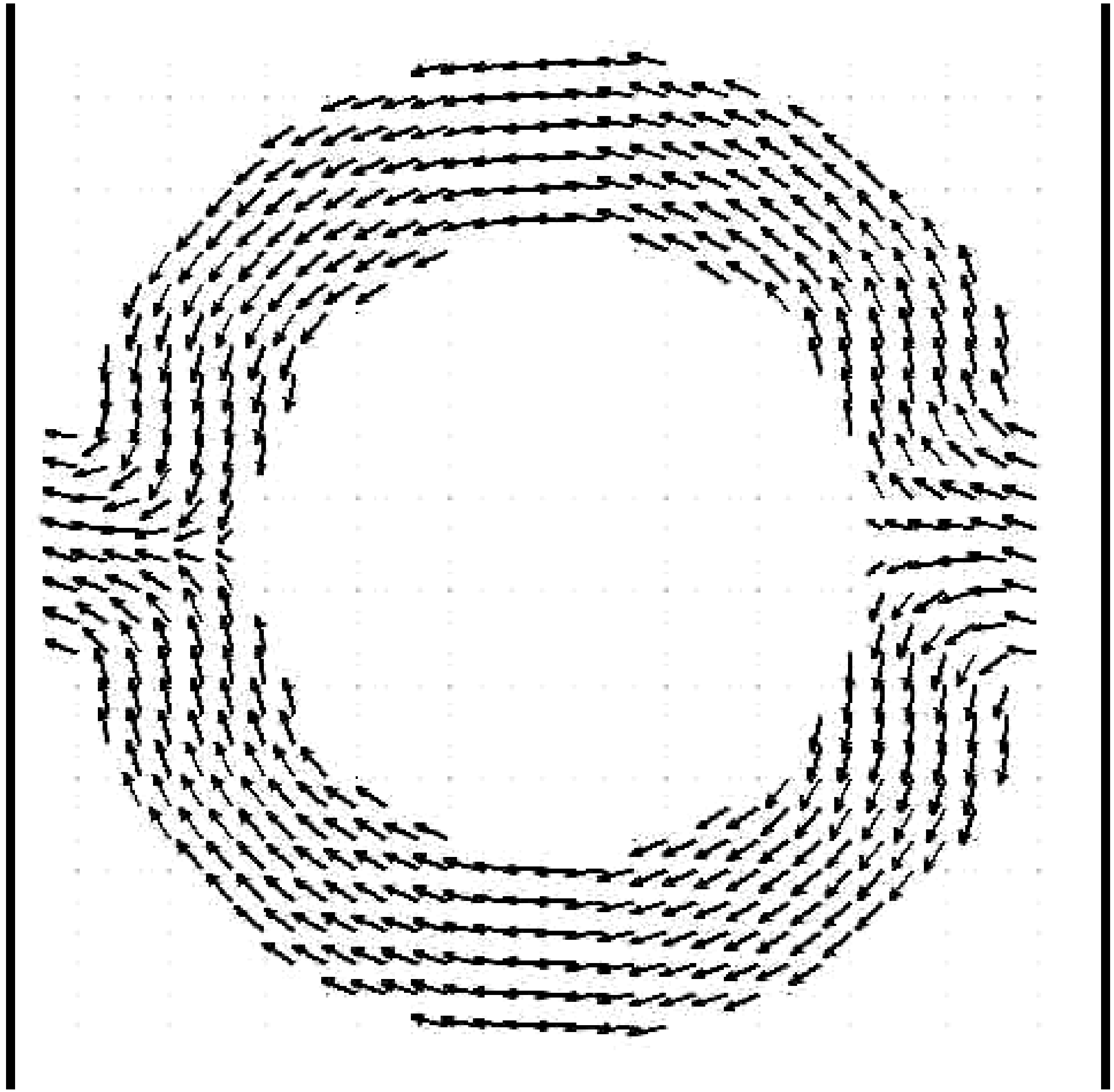


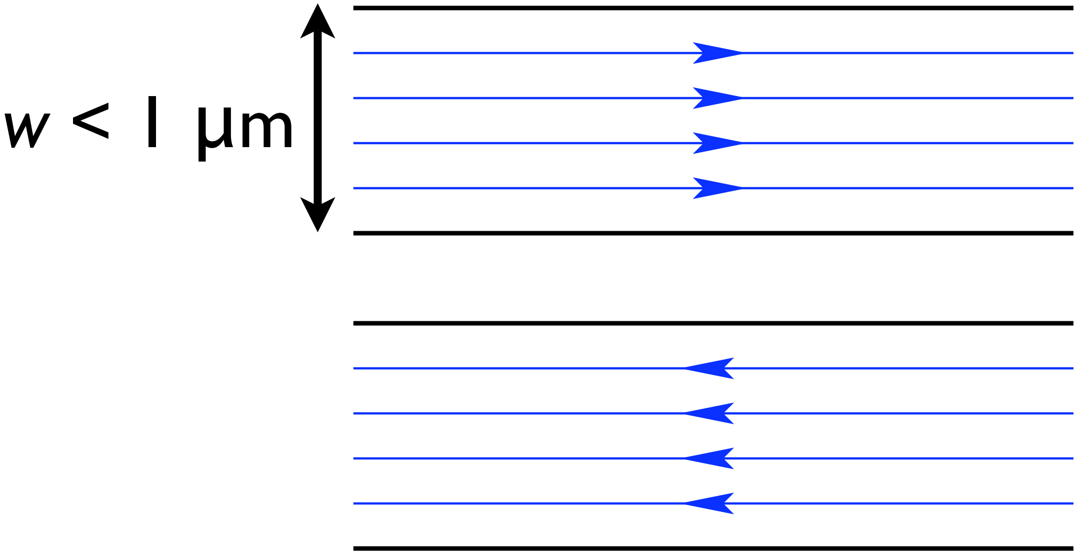
The switching involves
creation, propagation, and annihilation
of 2 domain walls



Numerical simulation. J. G. Zhu *et al.* (2004).



DWs in a submicron strip



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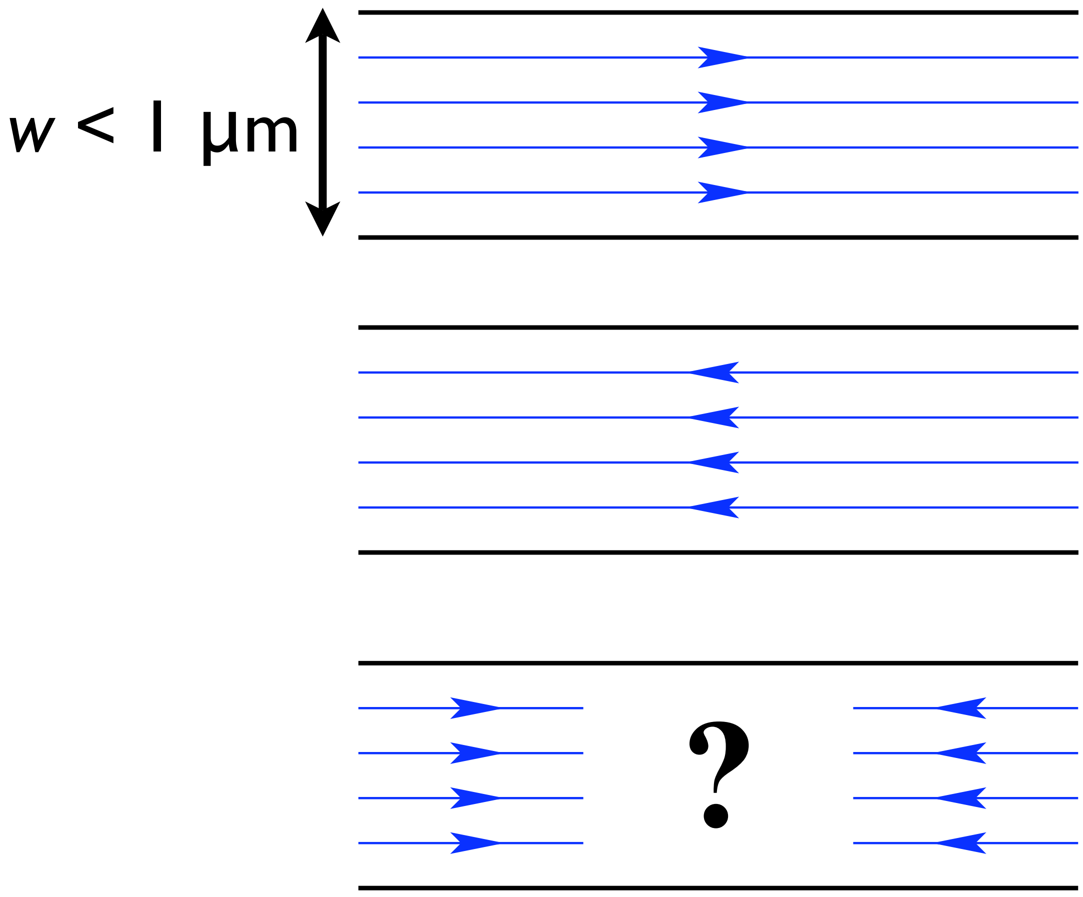

What does a domain wall in a nanostrip look like?

Educated guess

Educated guess

- If a strip is narrow enough, it is a 1d object:
  - $\mathbf{M} = \mathbf{M}(x)$, rather than $\mathbf{M}(x,y)$.

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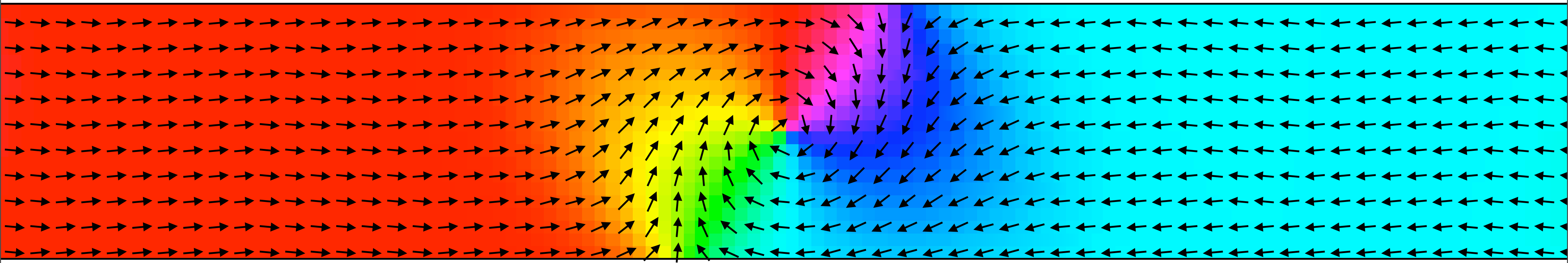
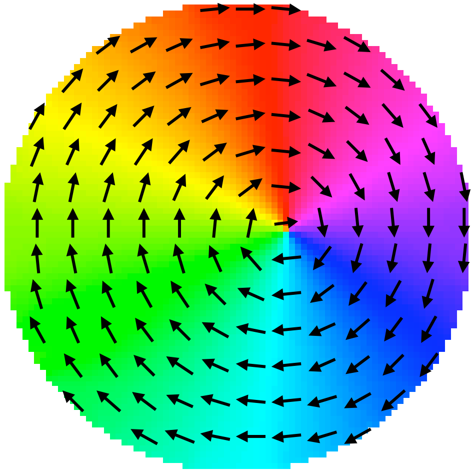


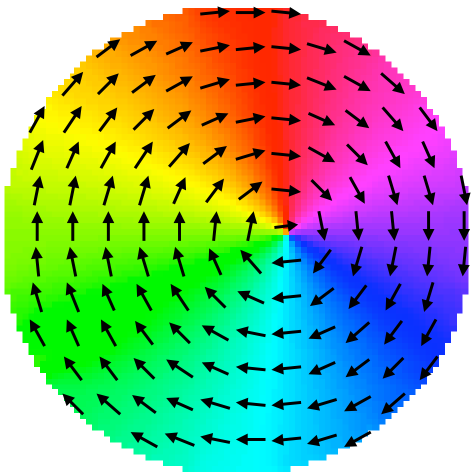
Numerical simulation. McMichael and Donahue (1997).

Topological defects in thin magnetic films



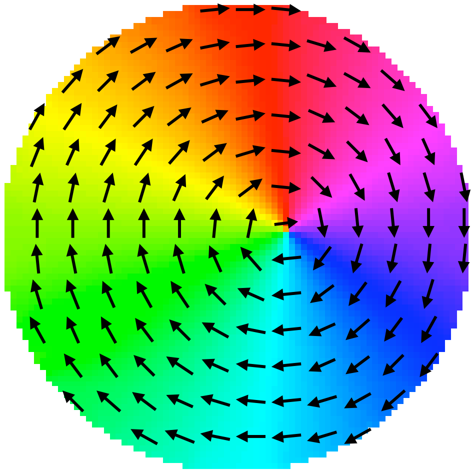
Topological defects in thin magnetic films

- Thickness is the shortest geometrical size.



Topological defects in thin magnetic films

- Thickness is the shortest geometrical size.
- Dipolar forces tie **M** to the geometry:
  - **M** prefers to stay in the plane of the film,
  - **M** also tends to be parallel to the edge.



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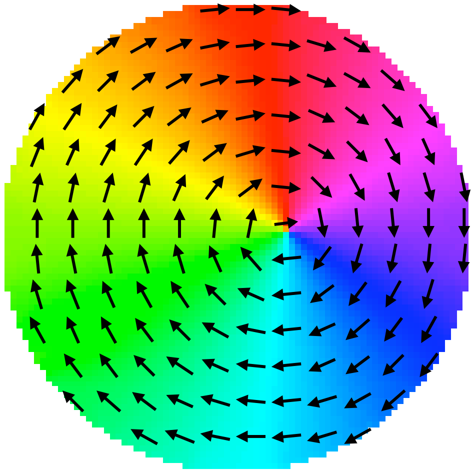
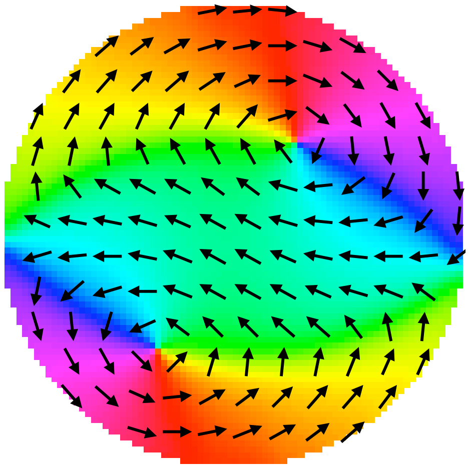
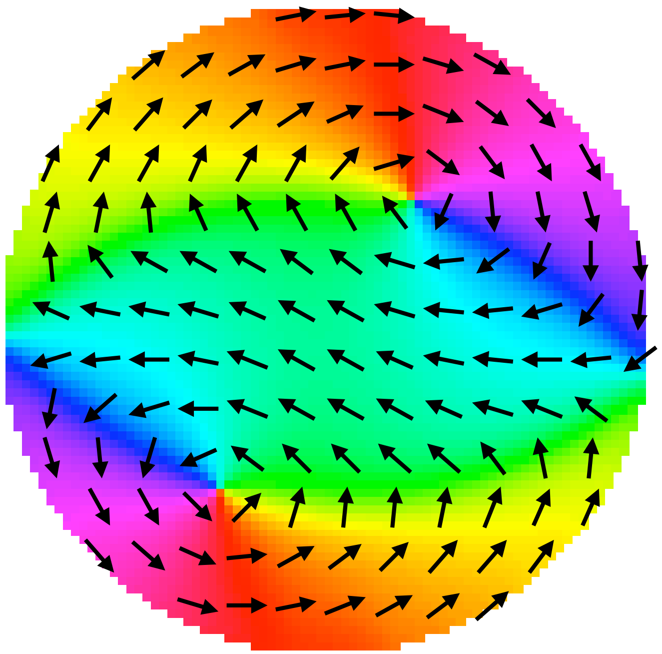




Topological defects

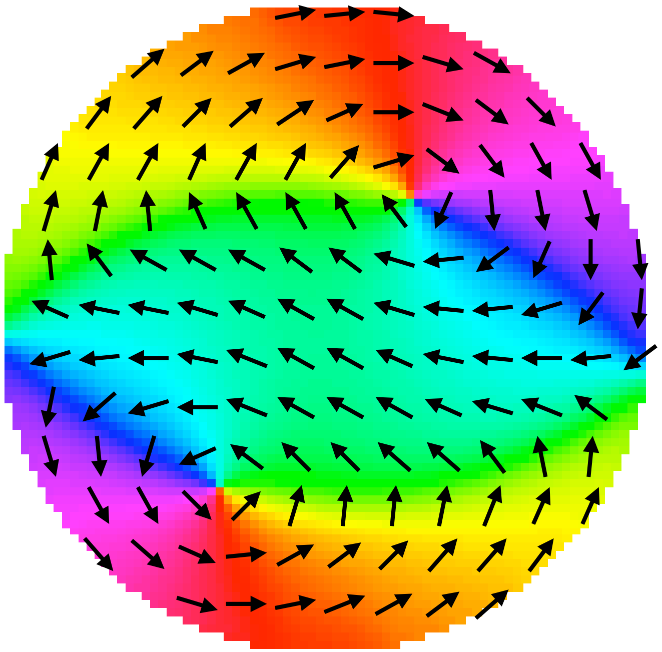


*R. Moser (2004); M. Kurzke (2004).

†O.T. and G.-W. Chern, PRL (2005).

Topological defects

- Planar (XY) model $\Rightarrow$ O(2) winding numbers.



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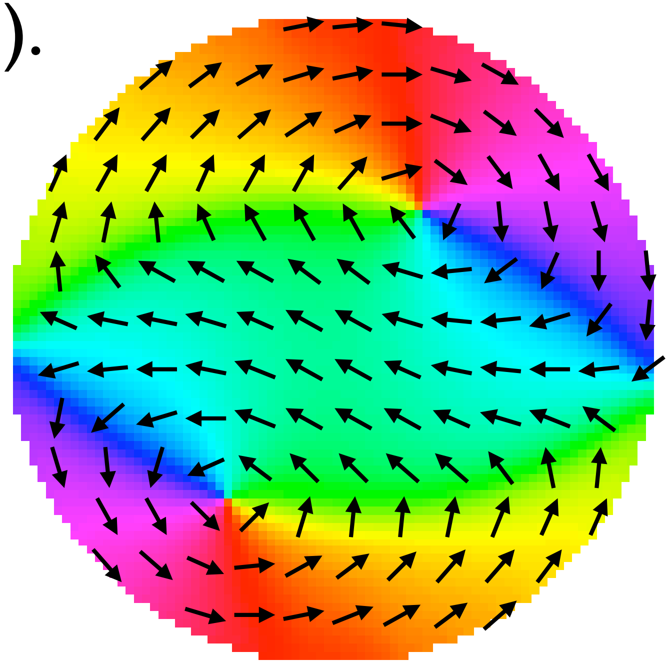

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Topological defects

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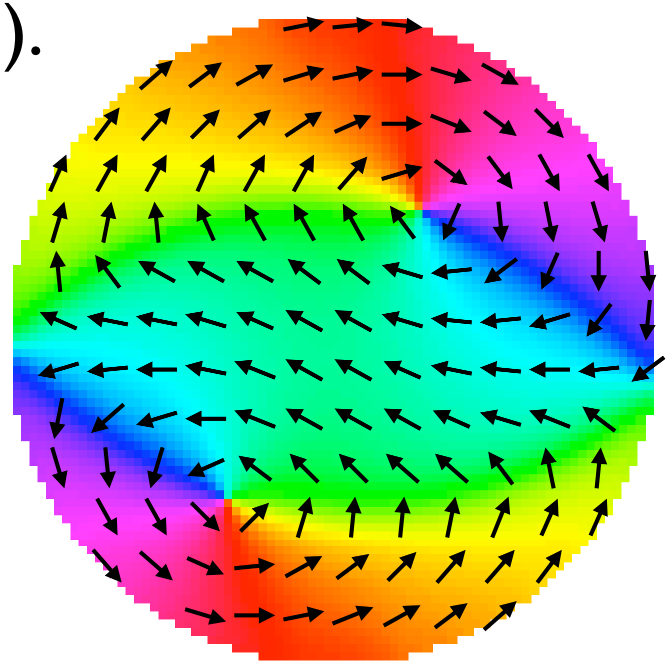
- Edge:
  - boundary vortices* ($\pm 1/2$)†.



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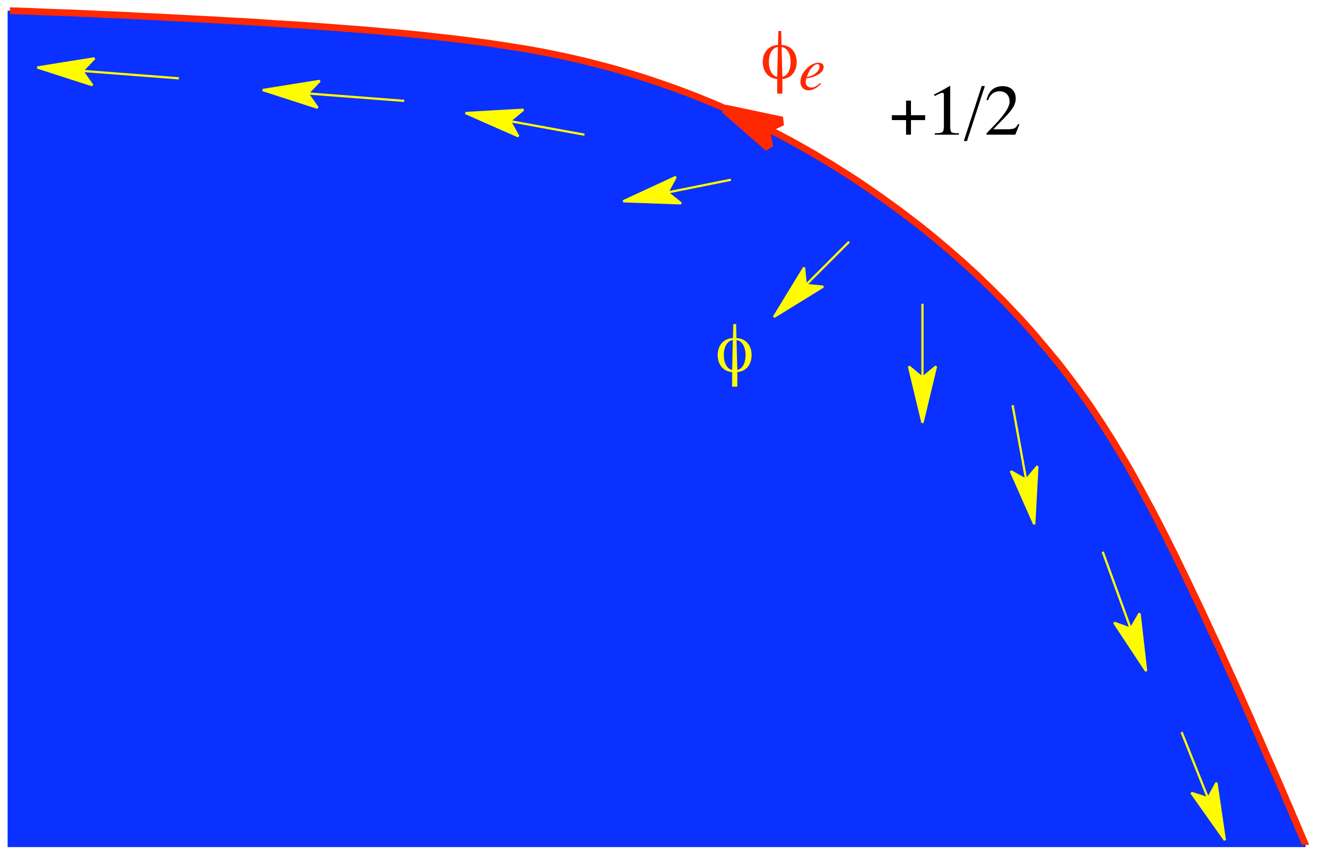
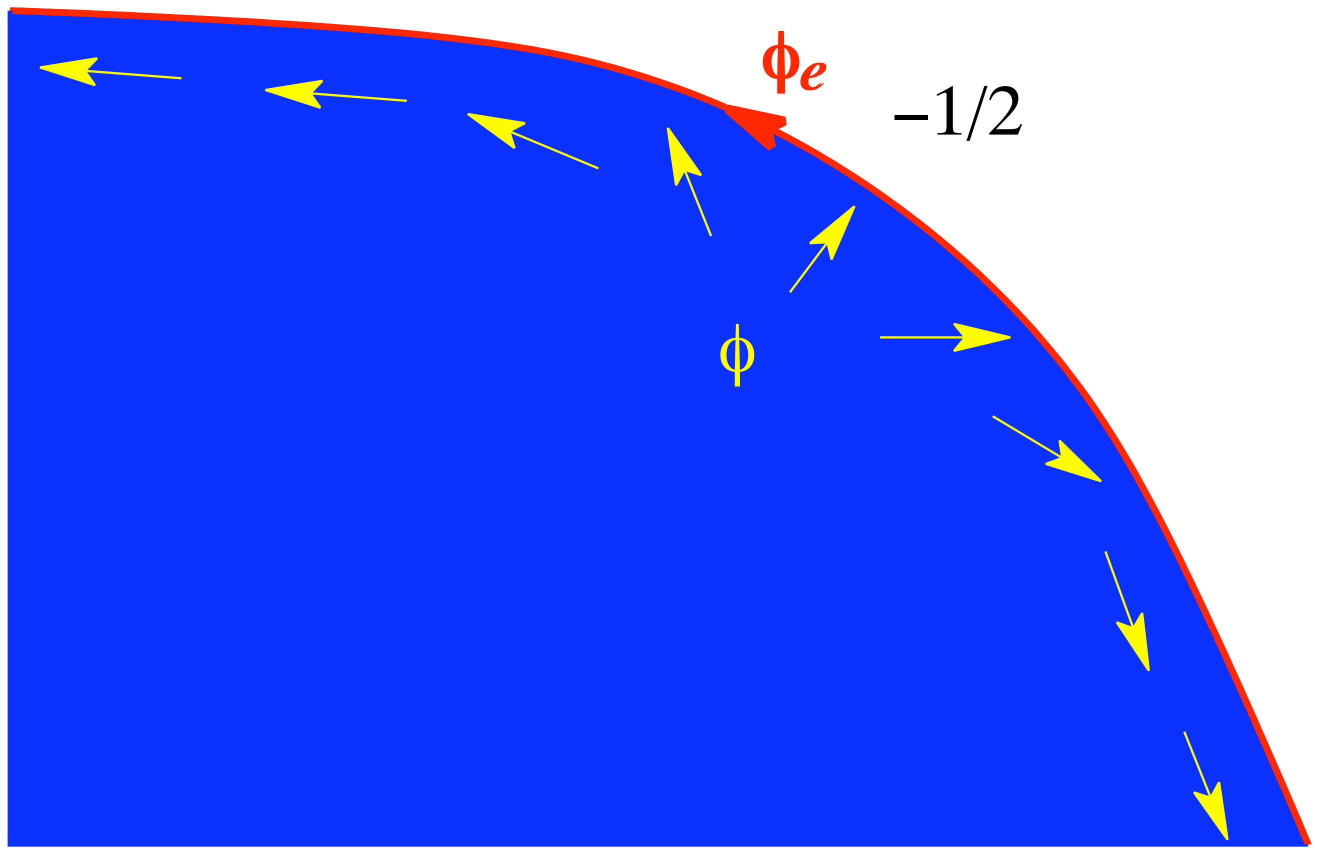
Winding numbers of edge defects



$$n = -\frac{1}{2\pi}\int_{\text{edge}} \nabla(\phi - \phi_e)\cdot d\mathbf{r} = \pm\frac{1}{2}.$$

Winding numbers of edge defects



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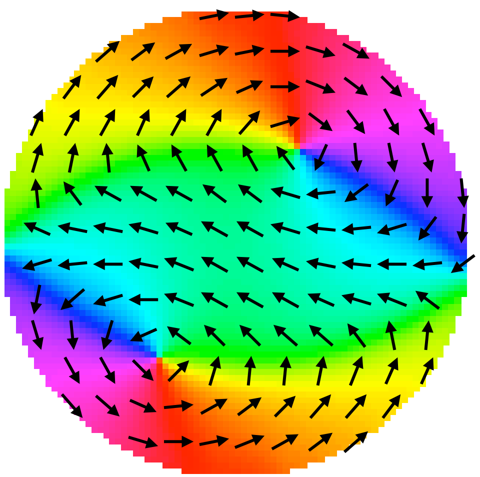
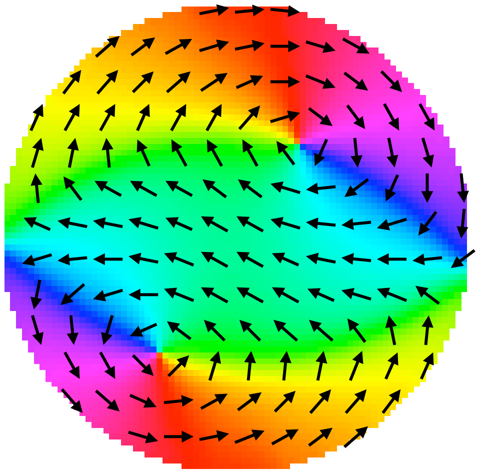
Are they really halfvortices?



$$+1 + 1 - 1/2 - 1/2 = +1.$$

Are they really halfvortices?

- Take a vortex (+1) and 2 edge defects.



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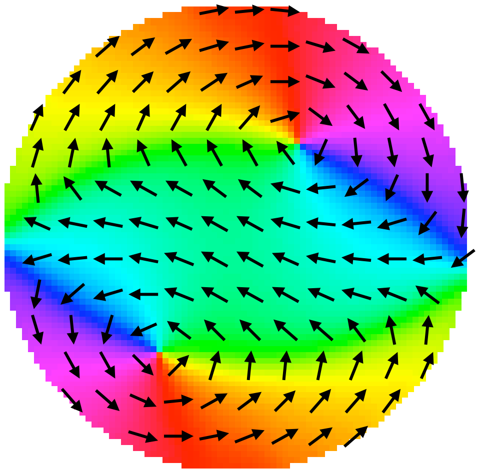

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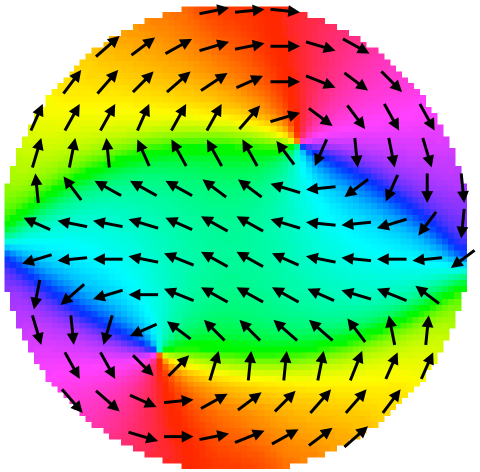

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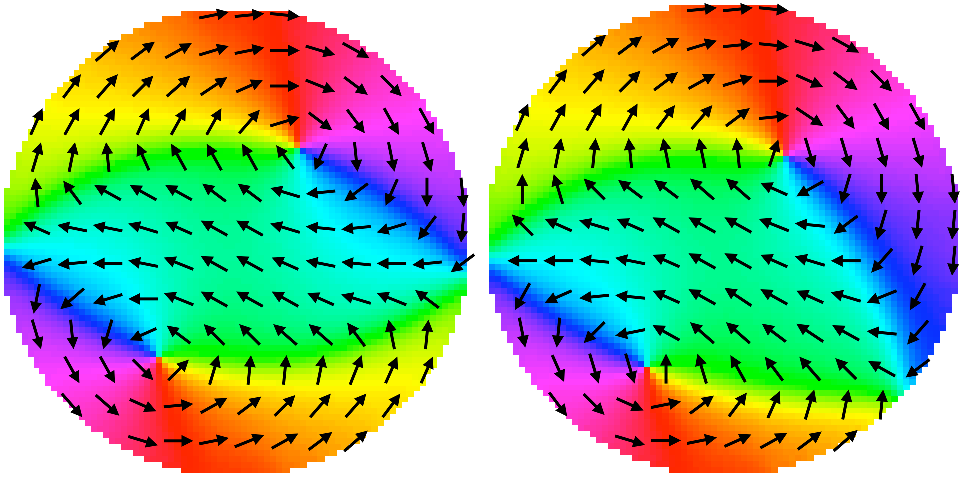

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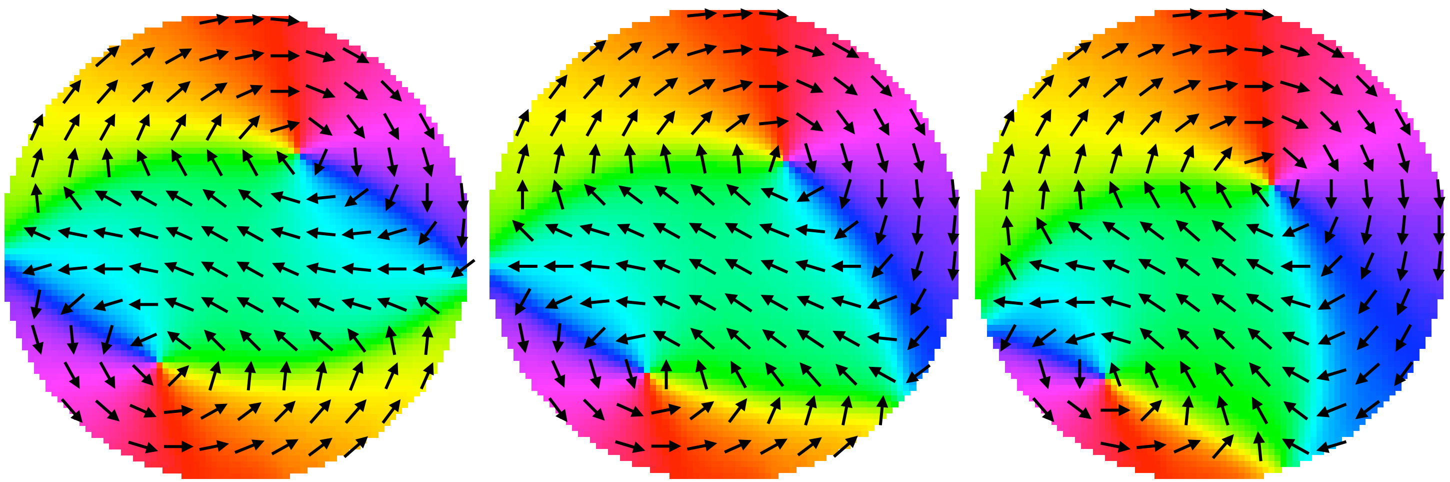

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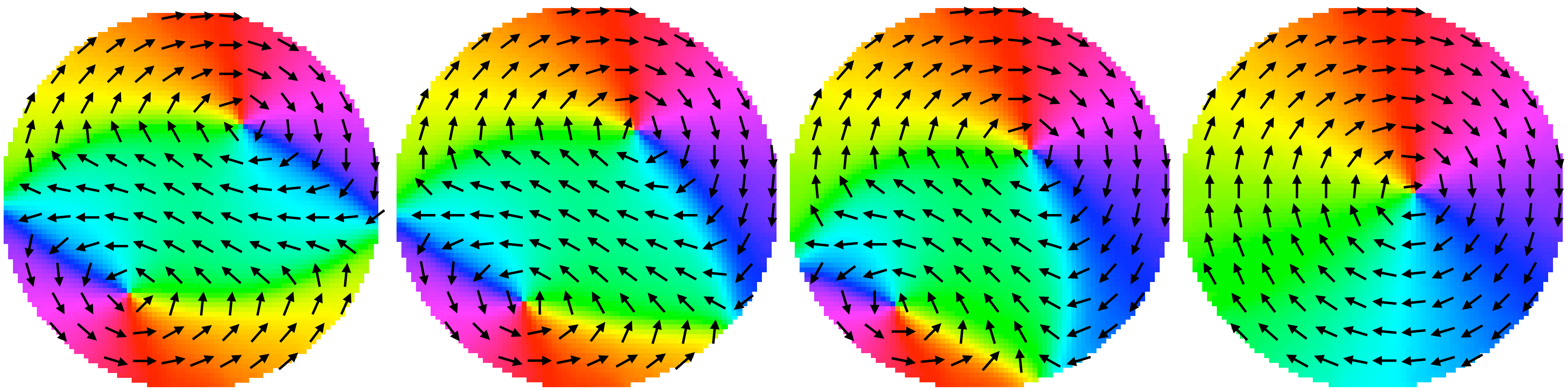
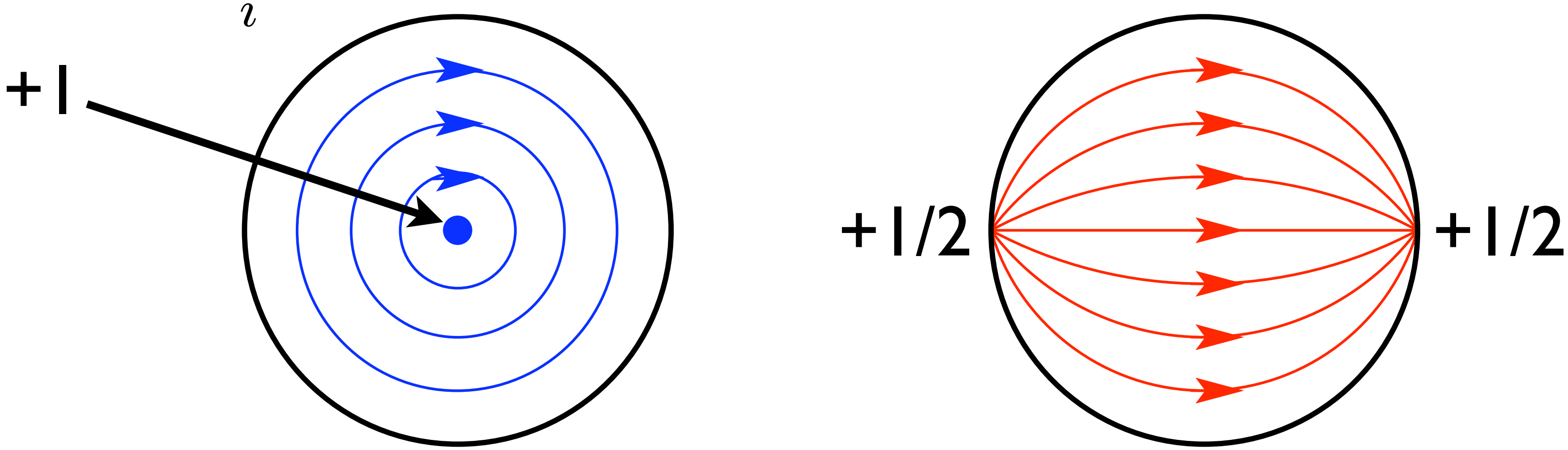


$$+1 + 1 - 1/2 - 1/2 = +1.$$

Mathematically speaking...

Sum of the winding numbers (incl. bulk and edge) is a constant:

$$\sum_i n_i = 1 \quad \text{(topology of a disk)}$$



Note that there are no distinct topological sectors!

Mathematically speaking...

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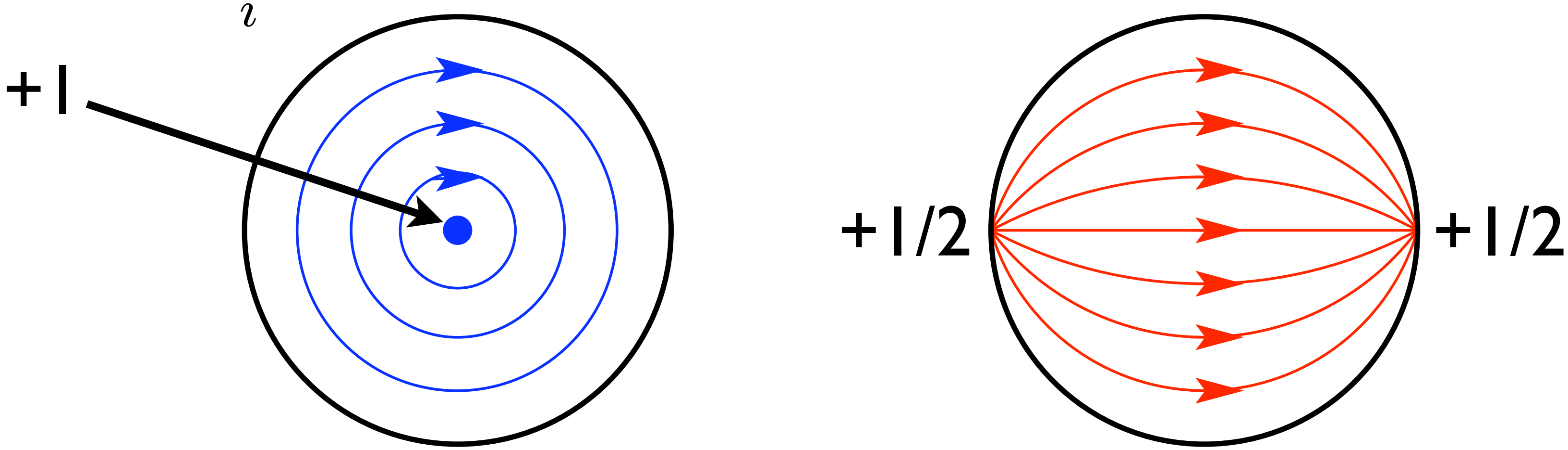
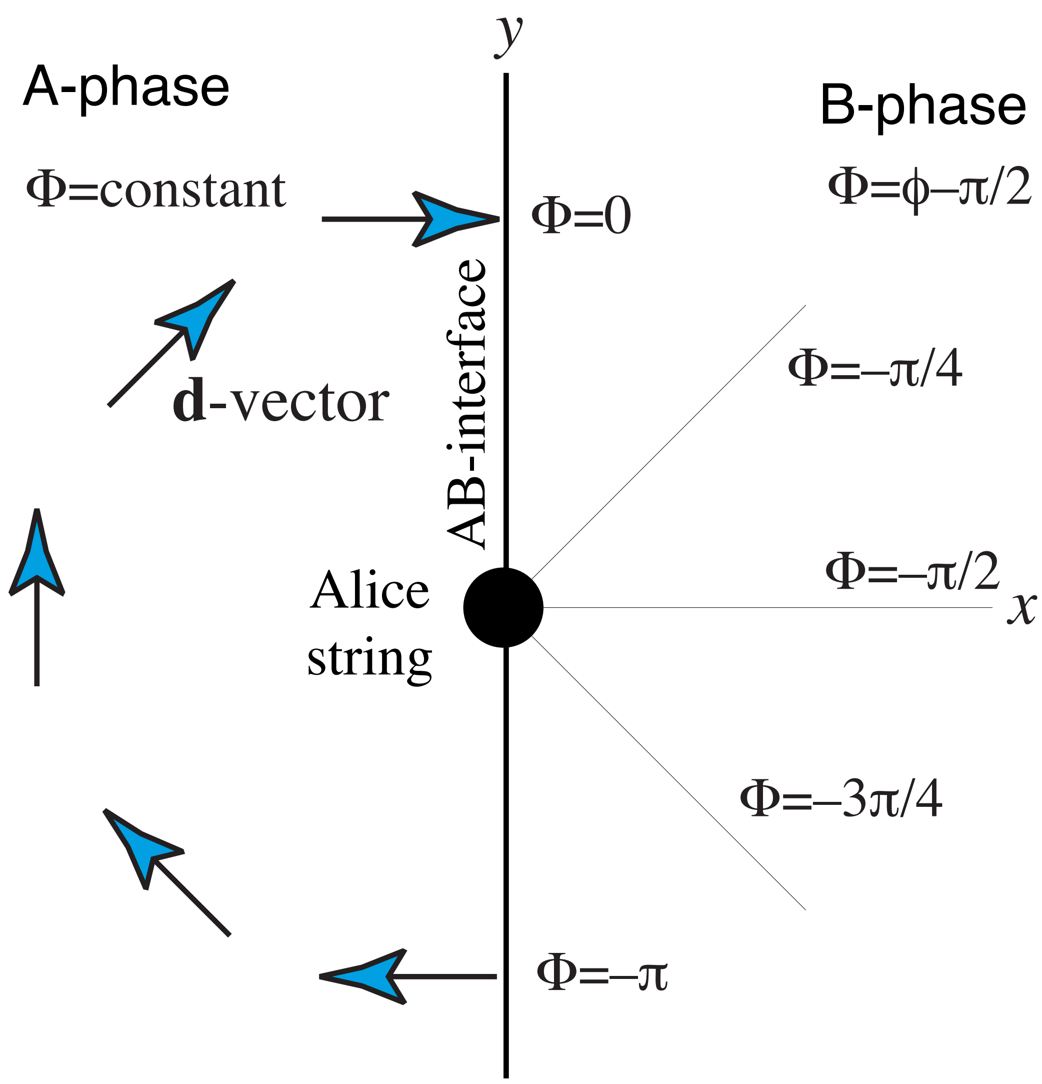


$$\sum_i n_i = 1 - g \quad \text{(disk with g holes)}$$

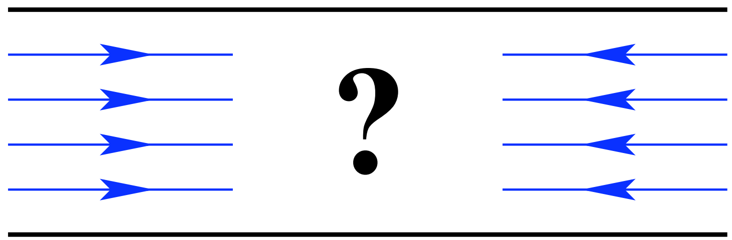
Note that there are no distinct topological sectors!

Analogs of boojums in ^{3}He.



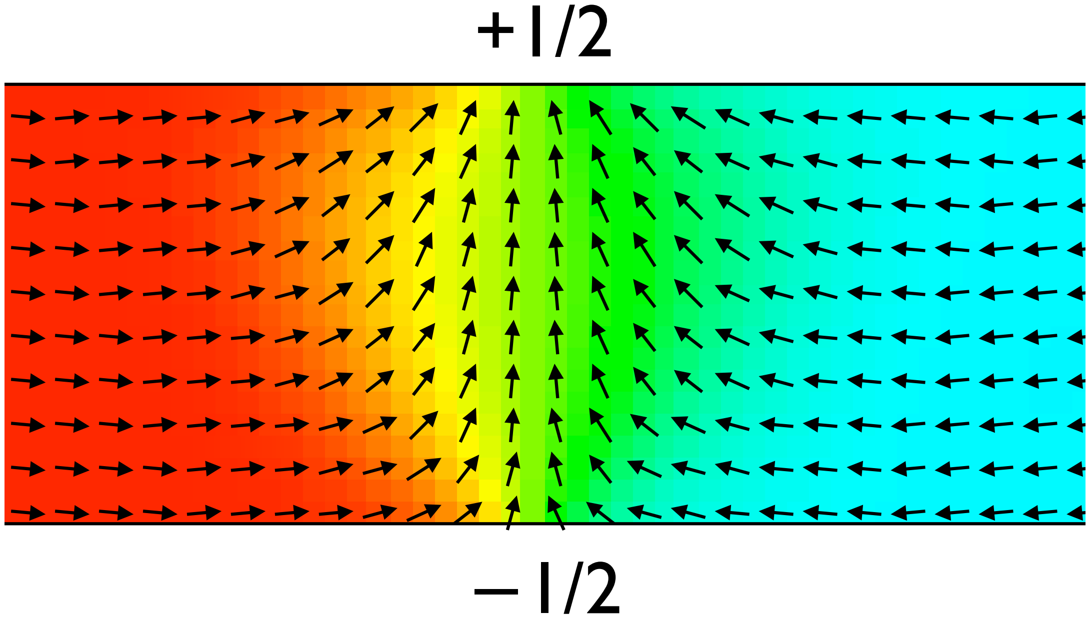
G. E. Volovik, *Universe in a Helium Droplet* (2003).

What about domain walls?



- DWs in nanostrips are composite objects.
- Basic rules for putting together a DW:
    - one halfvortex at each edge (or an odd number),
    - total winding number of a wall is 0.
- Examples:
    - $+1/2 - 1/2 = 0.$
    - $-1/2 - 1/2 + 1 = 0.$

"Transverse" wall



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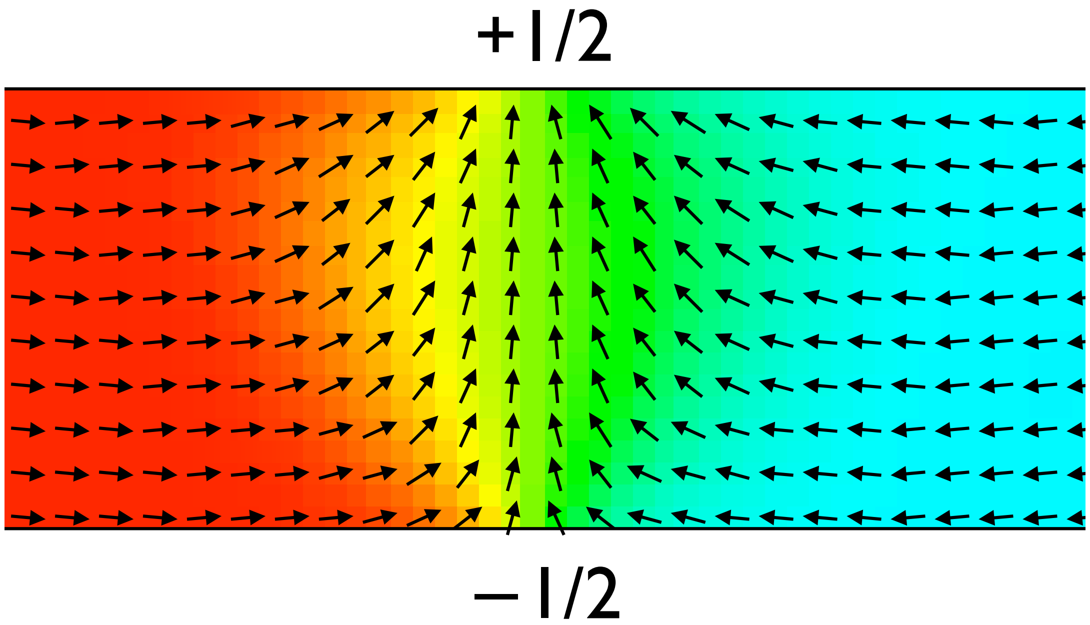
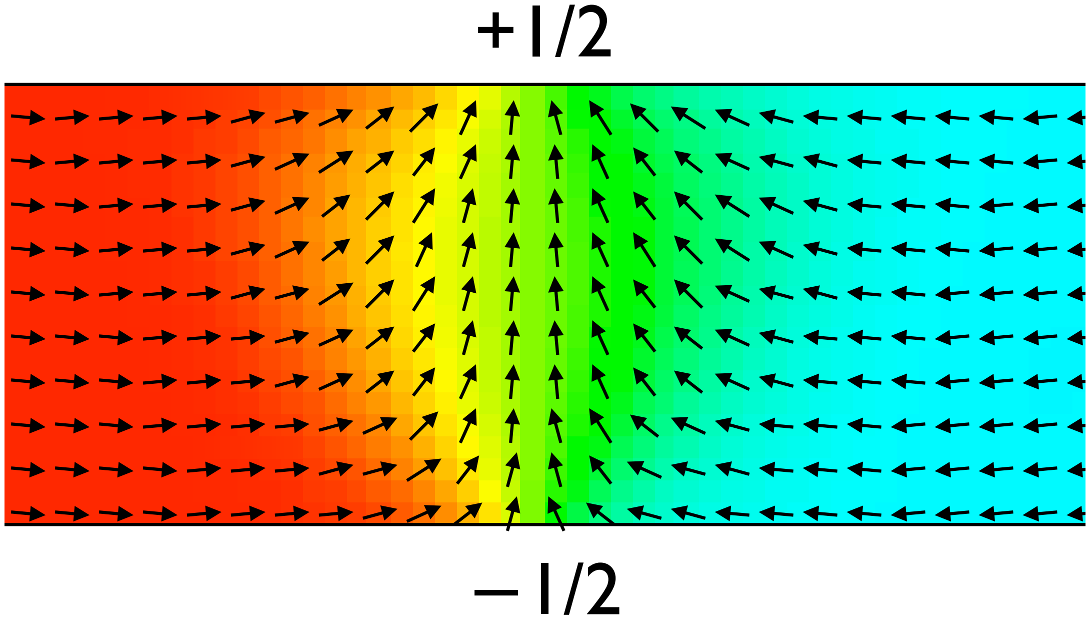


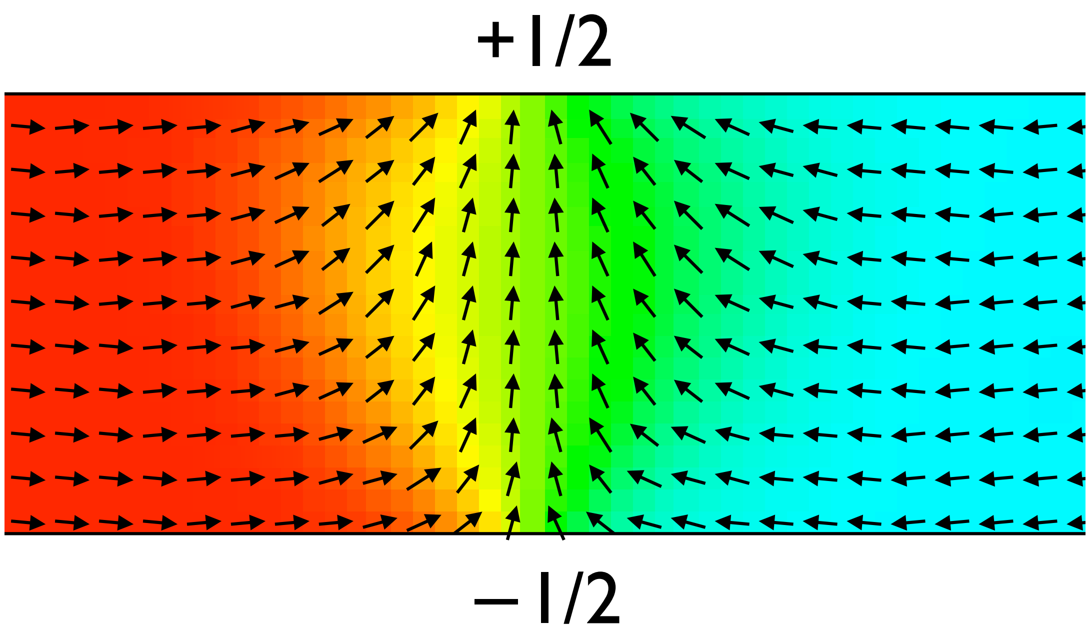
- Most economical (fewest defects).

“Transverse” wall



- Most economical (fewest defects).
- Seen in thin and narrow strips (< 50 nm).

“Transverse” wall



- Most economical (fewest defects).
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- Becomes 1d kink as strip width → 0:

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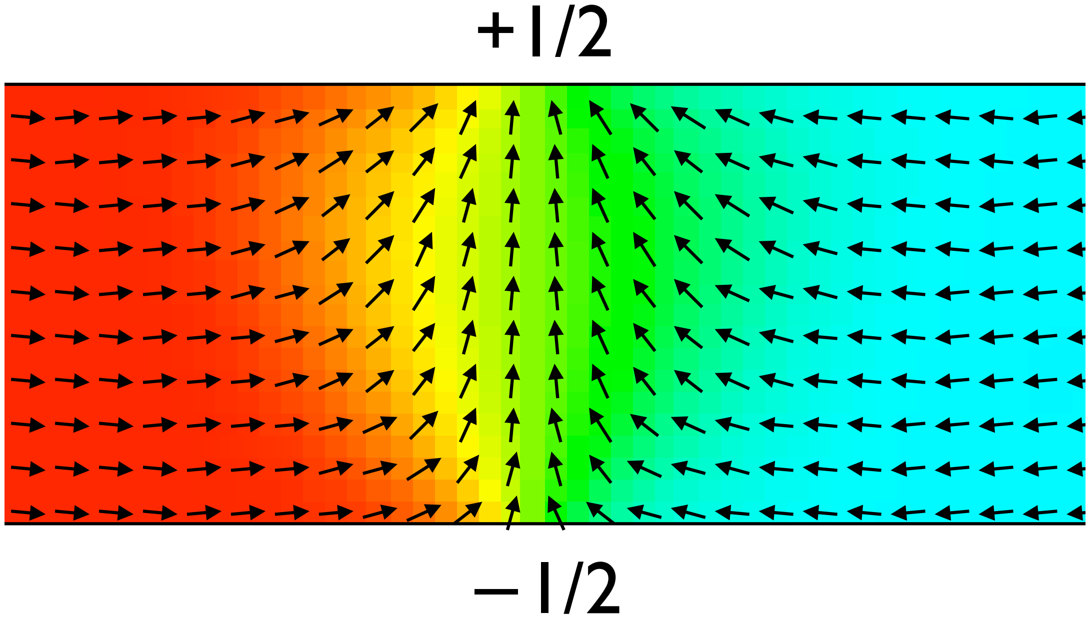

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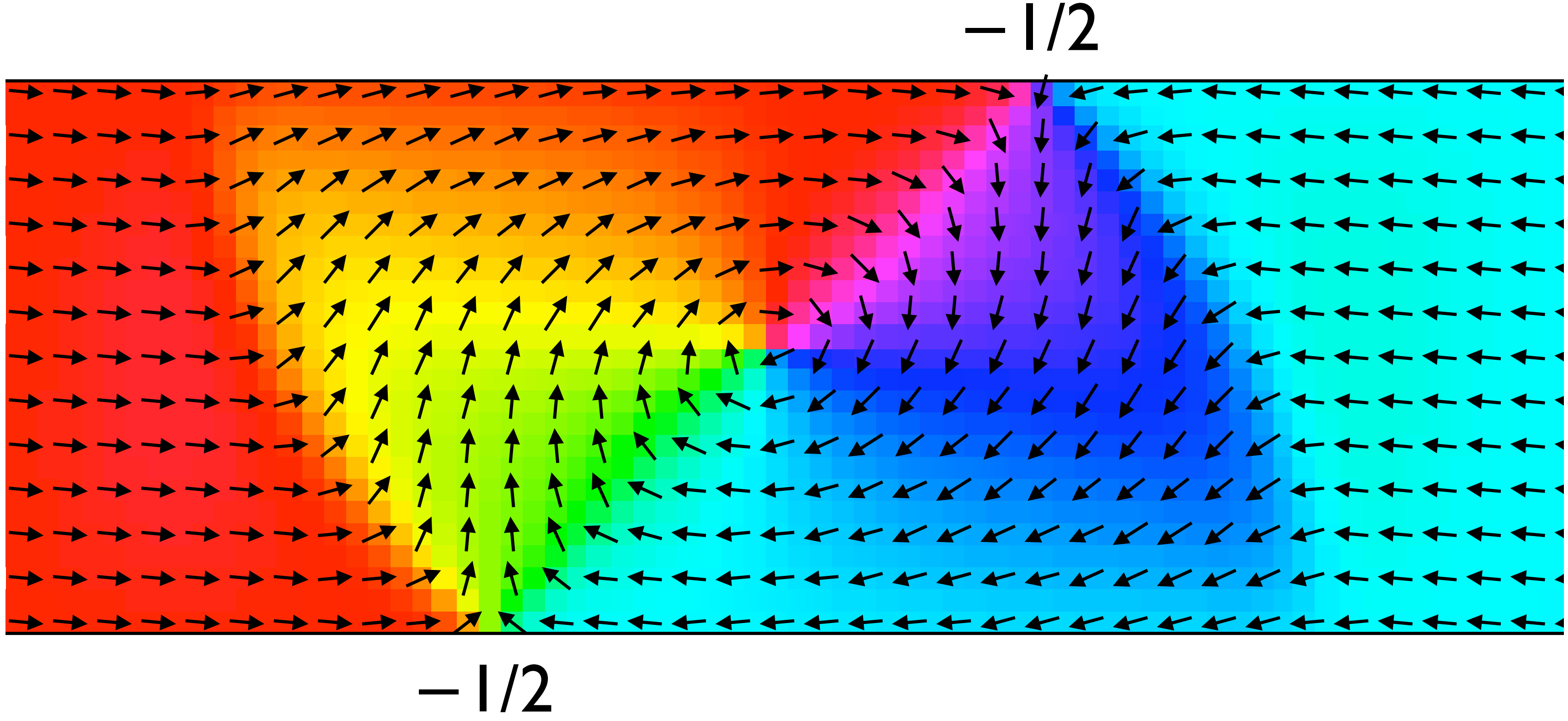
$$\cos\phi(x, y) = \tanh x/\lambda.$$

“Vortex” wall



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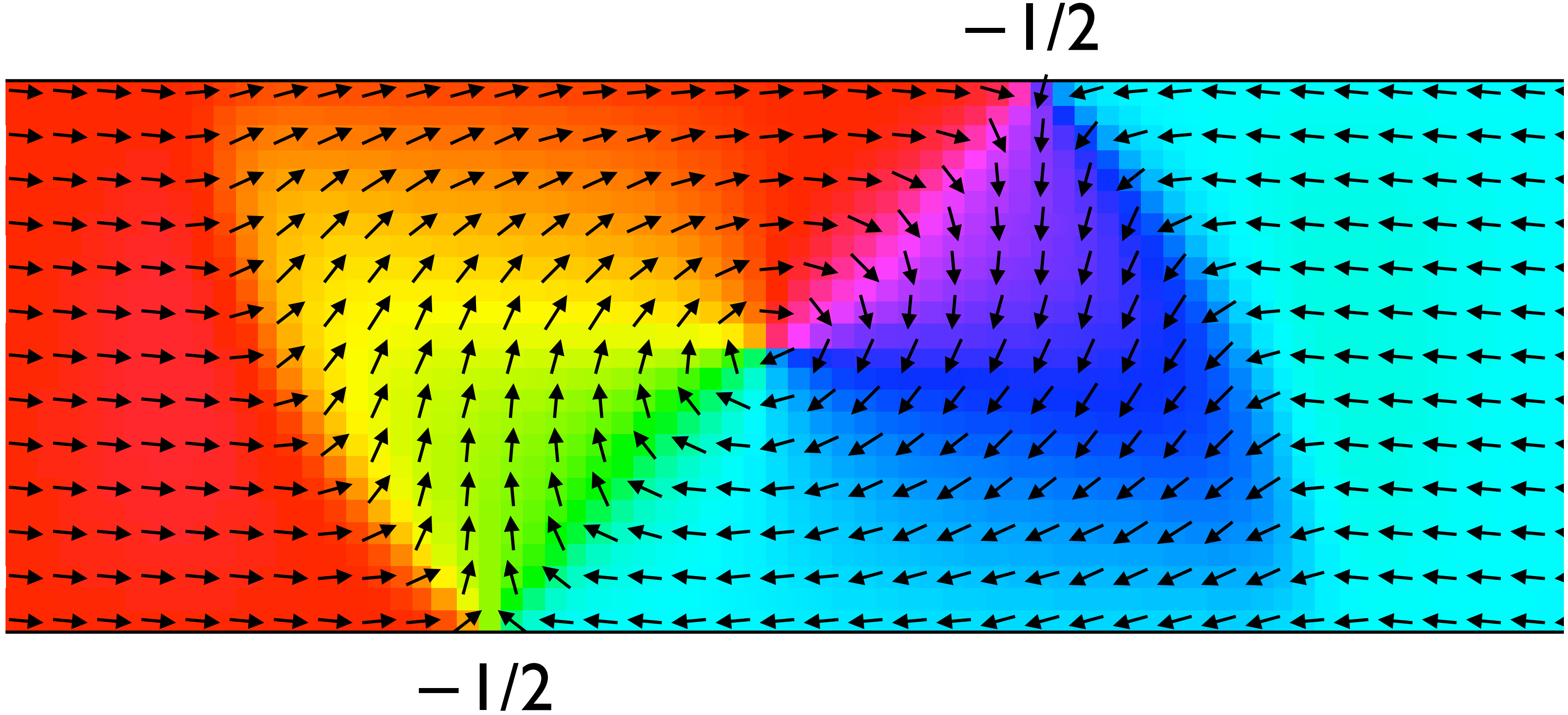
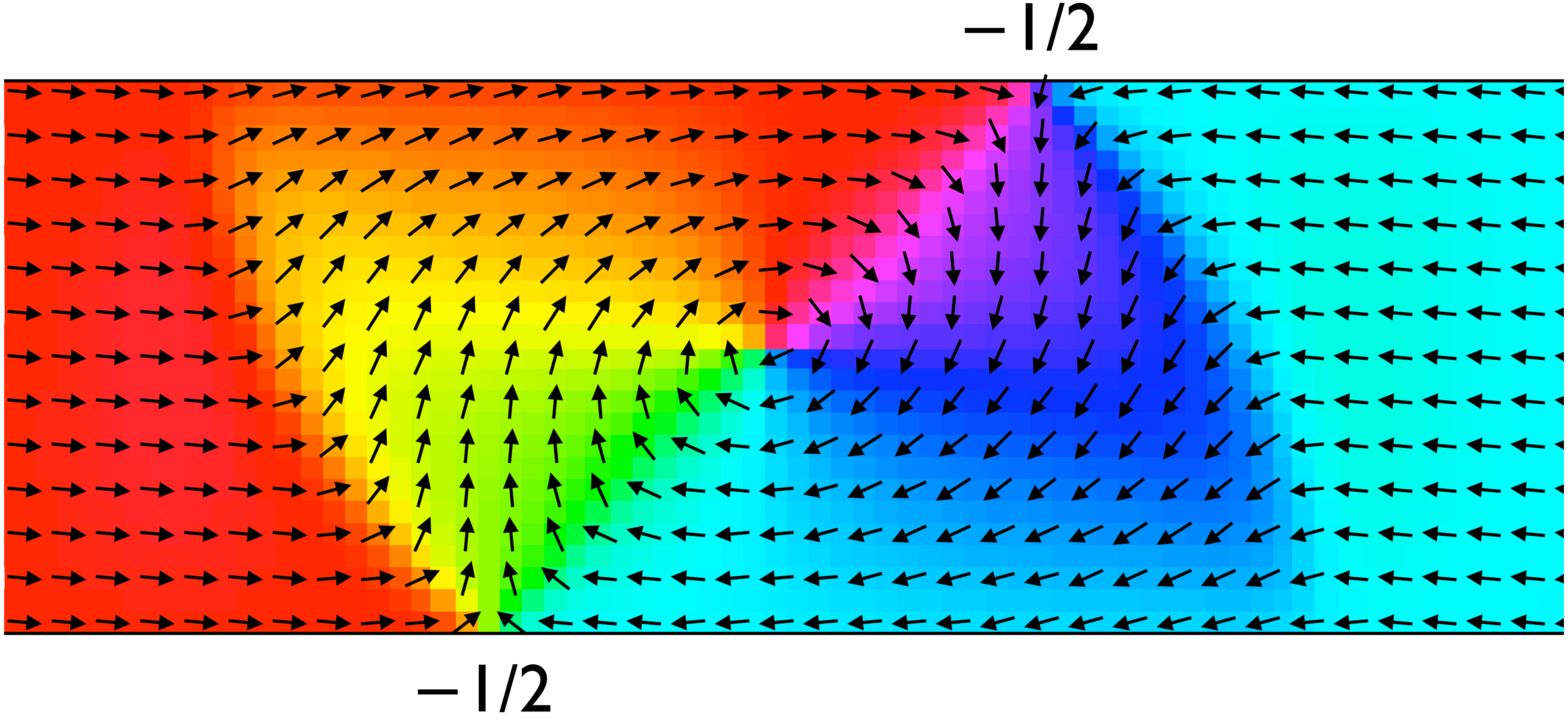


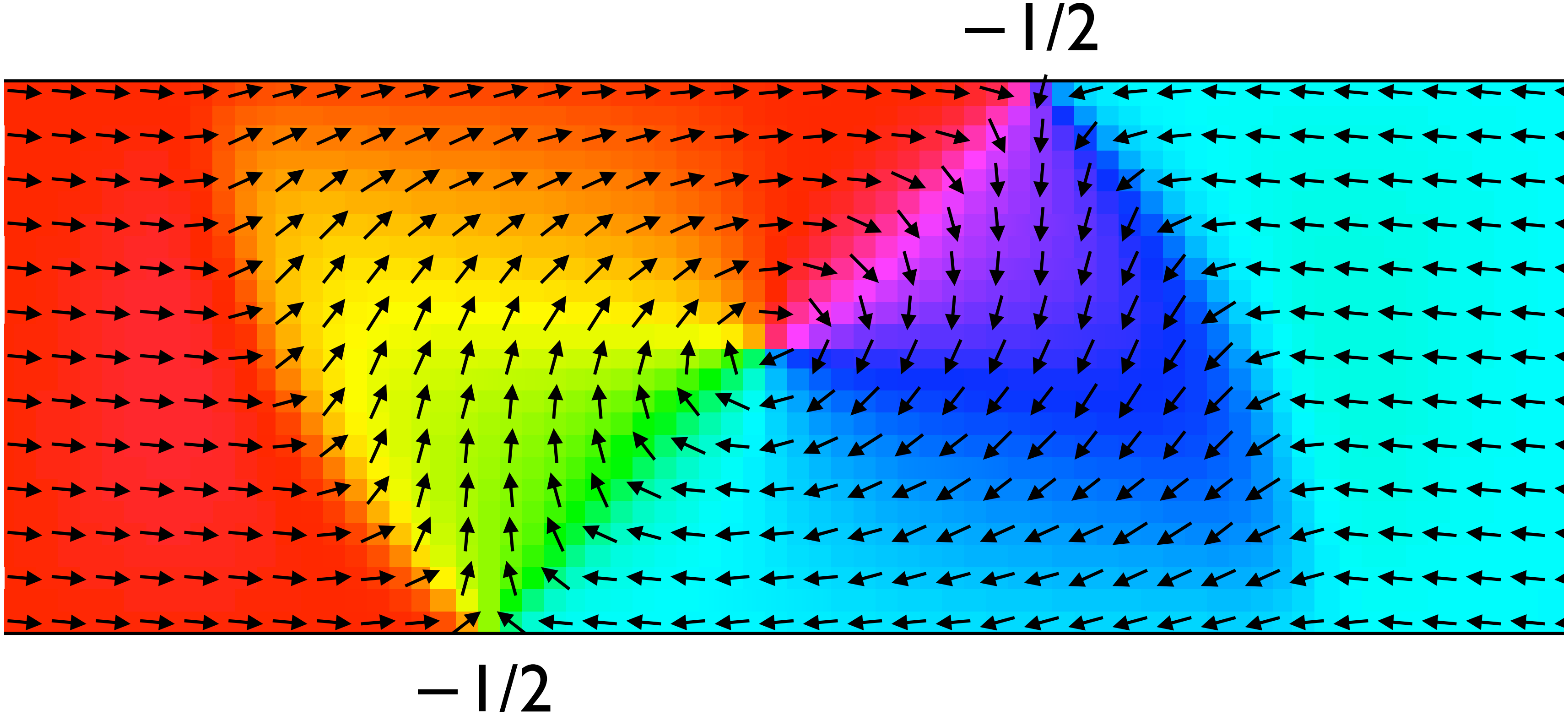
- Seen in thicker and wider strips.

“Vortex” wall



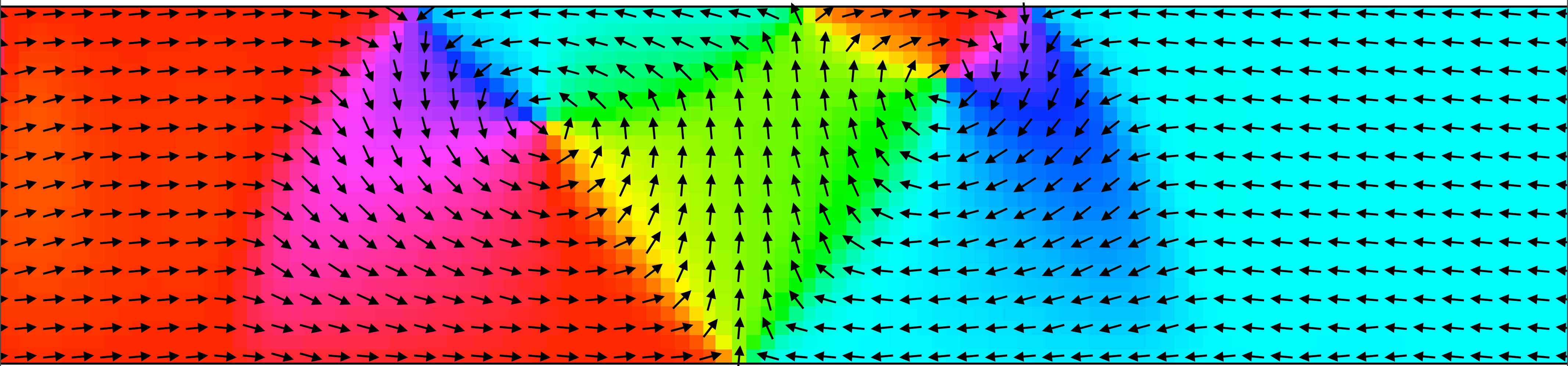
- Seen in thicker and wider strips.
- Most economical if +1/2 defects “forbidden.”

“Vortex” wall



- Seen in thicker and wider strips.
- Most economical if $+1/2$ defects “forbidden.”
- $+1/2$ has magnetic charge $\Rightarrow$ high dipolar energy.

An exotic wall



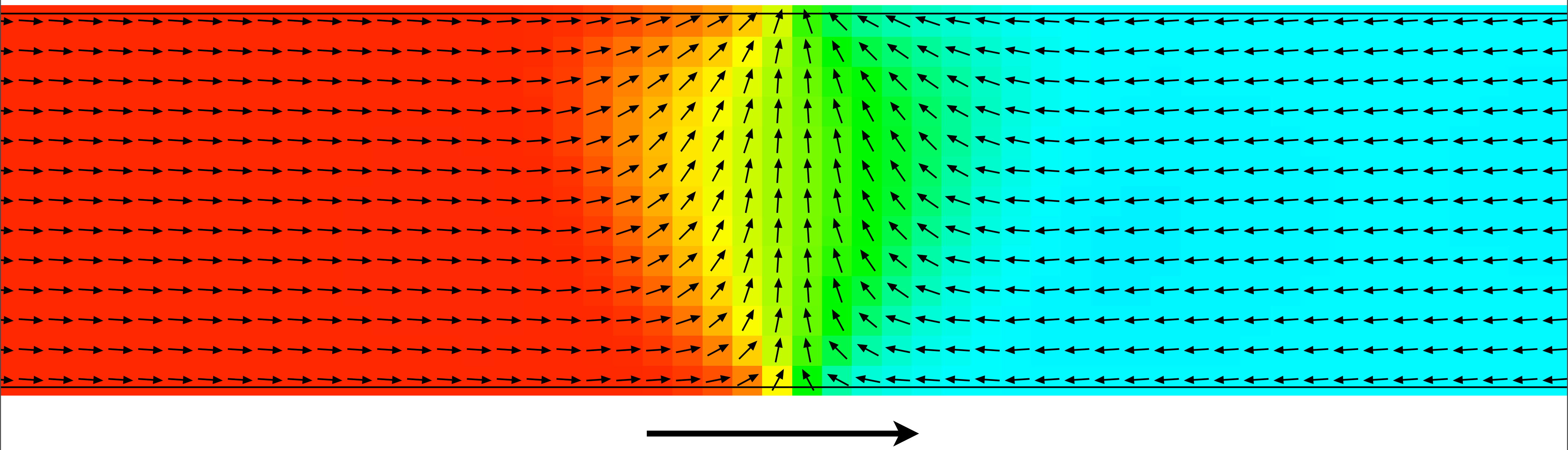
$$4 \times (-1/2) + 2 \times (+1) = 0.$$

Transient state far out of equilibrium.

Summary: DW statics

- DWs in nanostrips are composite objects...
- ...made from integer and fractional vortices...
- ...following simple topological rules.
- Typical makeup of a domain wall:
  - $+1/2$ and $-1/2$ edge defects,
  - $2\times(-1/2)$ edge defects, $+1$ bulk vortex.
  - other compositions are metastable states.

DW dynamics



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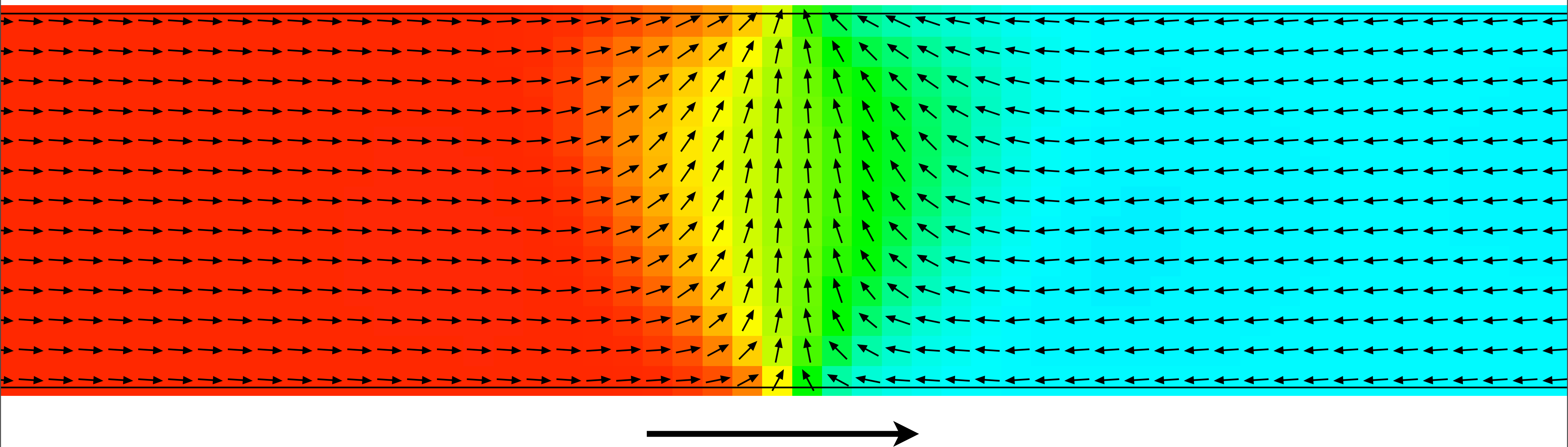
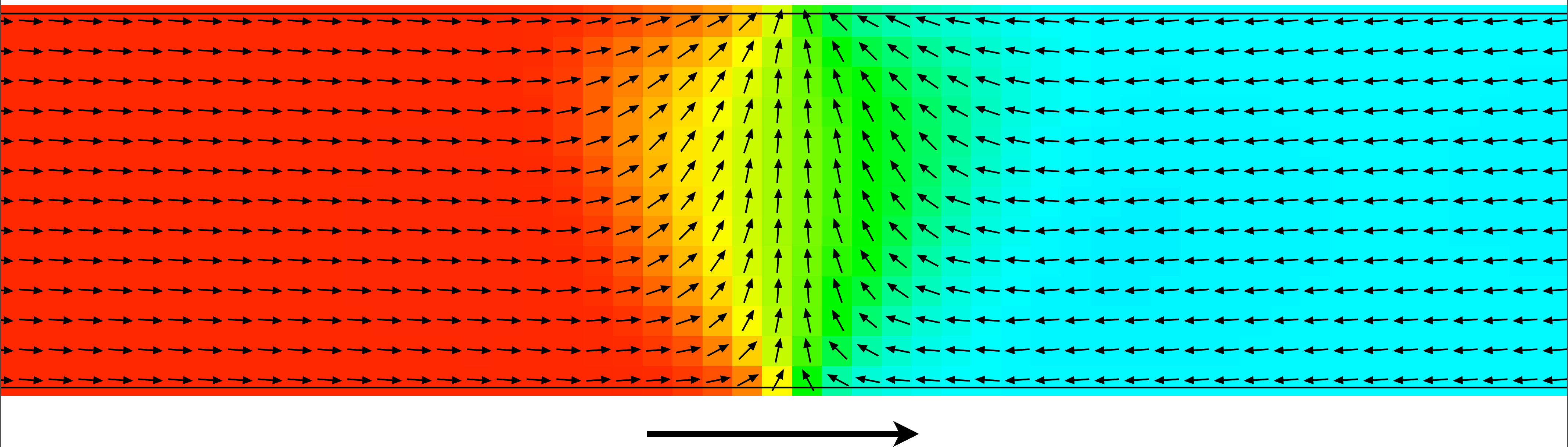


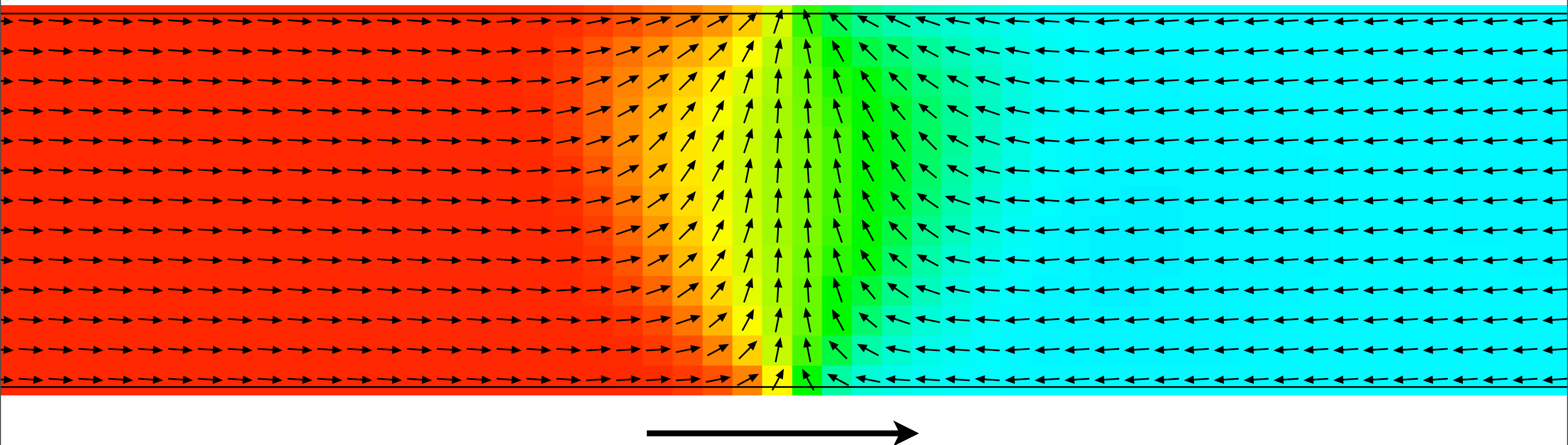
- DWs in nanostrips carry magnetic charge,

DW dynamics



- DWs in nanostrips carry magnetic charge,
- hence can be moved by applying magn. field.

DW dynamics



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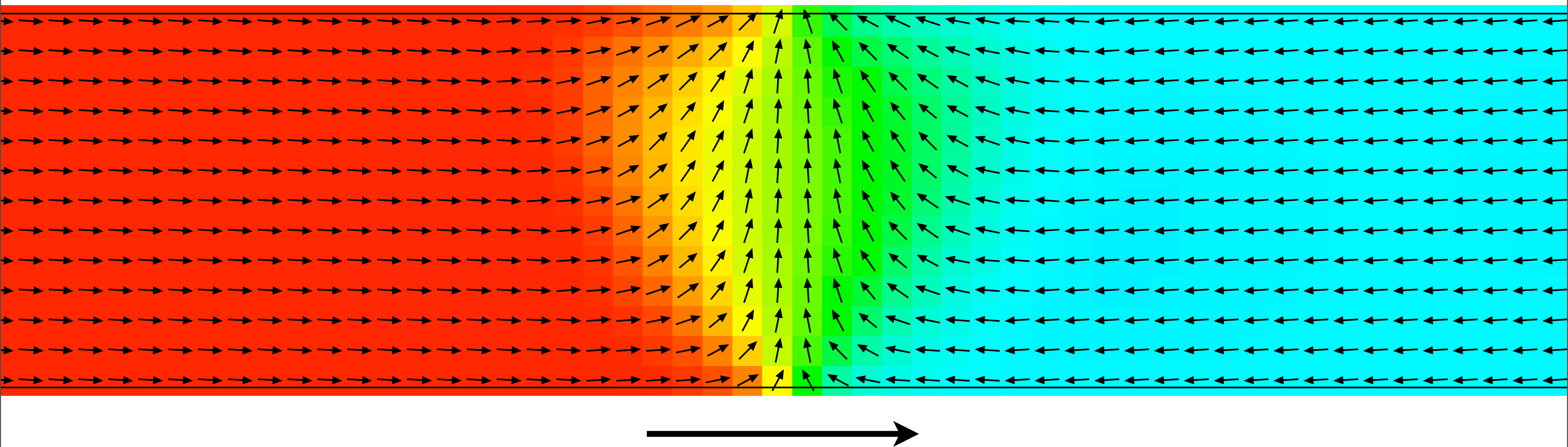
- Even a weak field (20 Oe) can do that!

DW dynamics



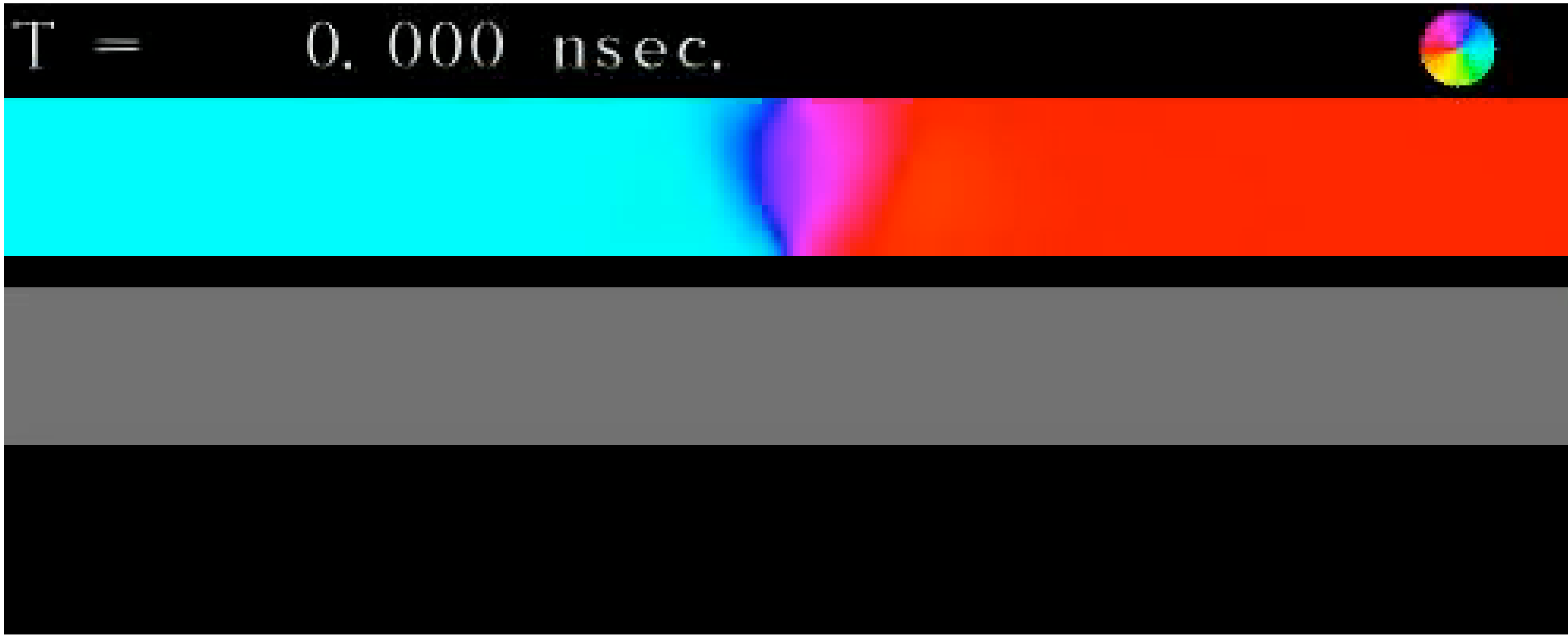
- DWs in nanostrips carry magnetic charge,
- hence can be moved by applying magn. field.
- Even a weak field (20 Oe) can do that!
- (Spin) current can, too. Less efficient.

Weak field: viscous motion

XY

Z

∇

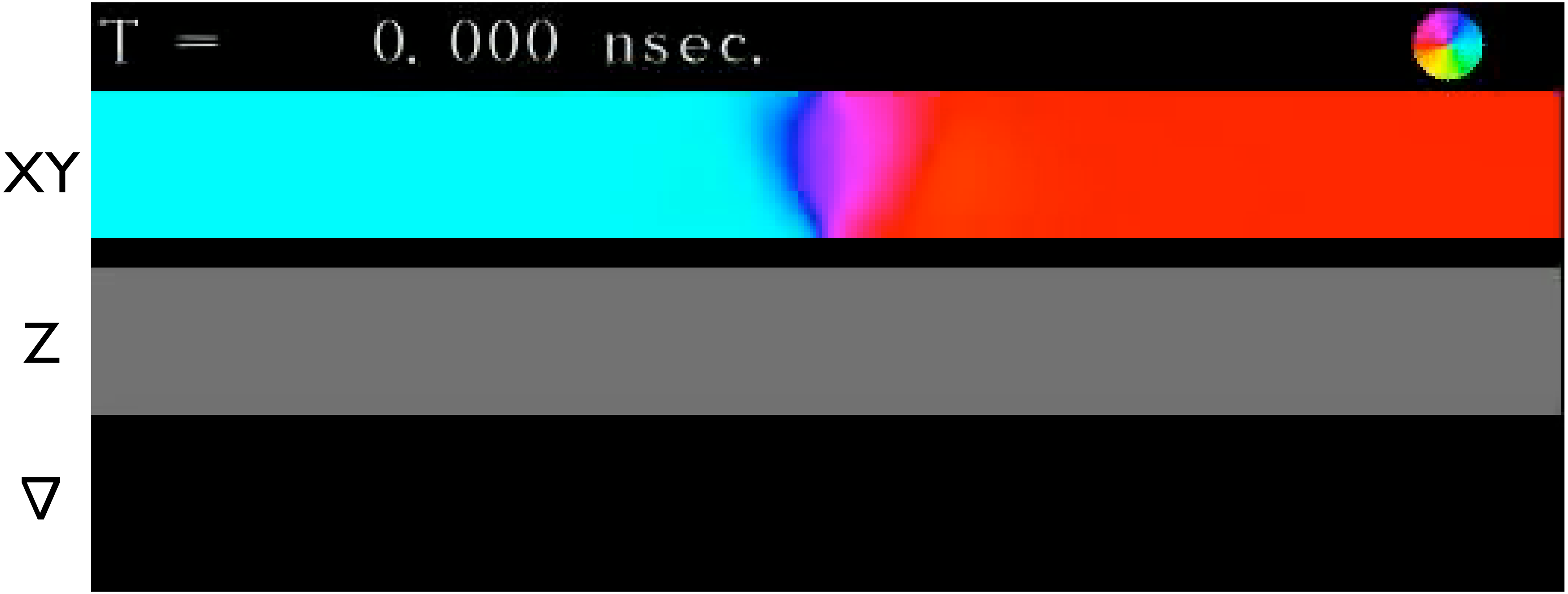


- DW moves steadily, slightly deformed.
- Speed is set by rate of energy dissipation:

$$\sum_i F_i^x - \Gamma v^x = 0.$$

DW viscosity

Stronger field: complex motion



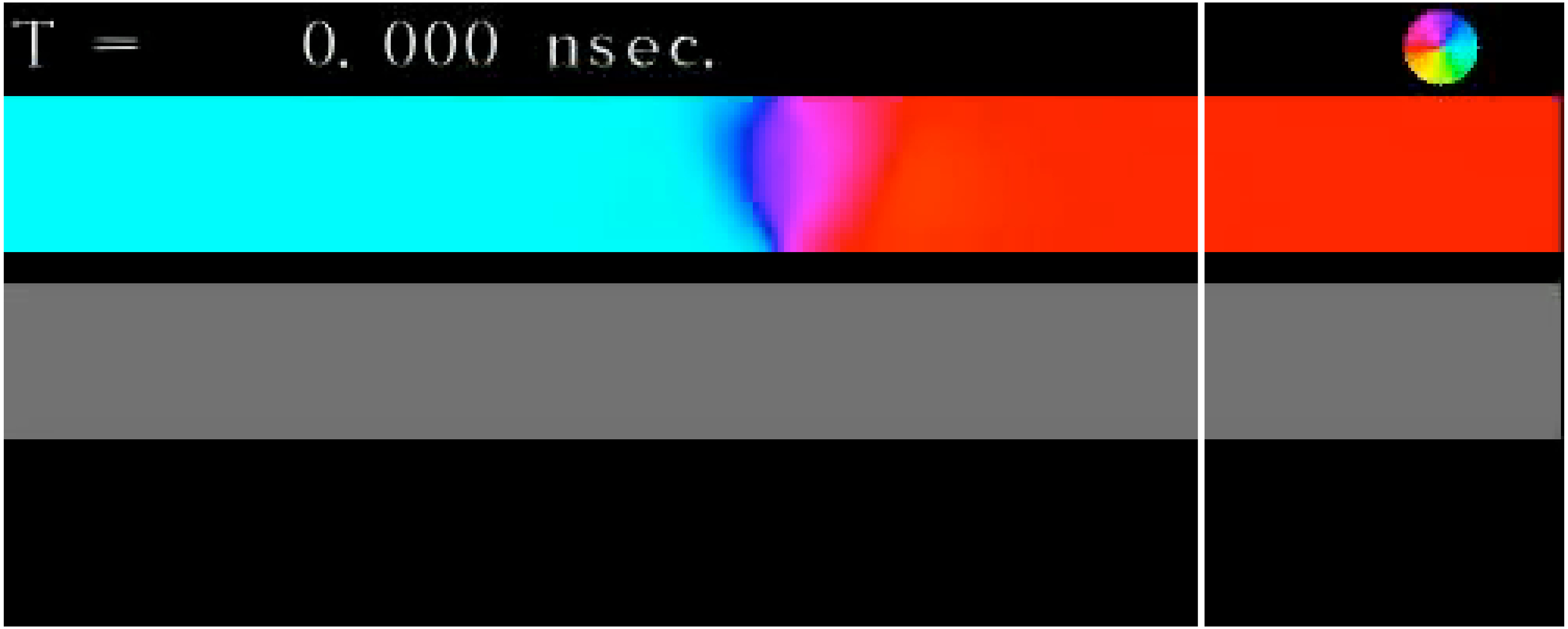
- Periodic creation, annihilation of ±1 vortices.
- Viscous motion if no ±1 vortex is present.
- Oscillations in the presence of a ±1 vortex. ⇐

Stronger field: complex motion

XY

Z

∇



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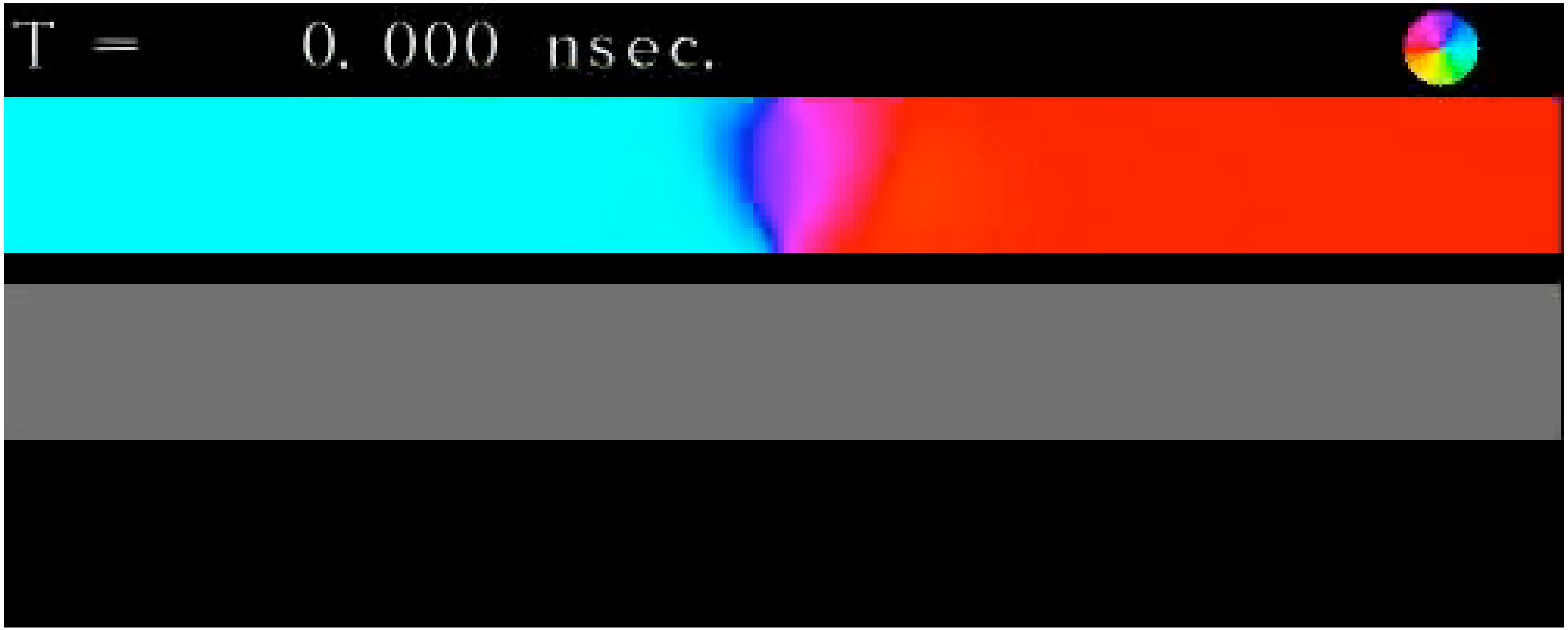
XY

Z

∇



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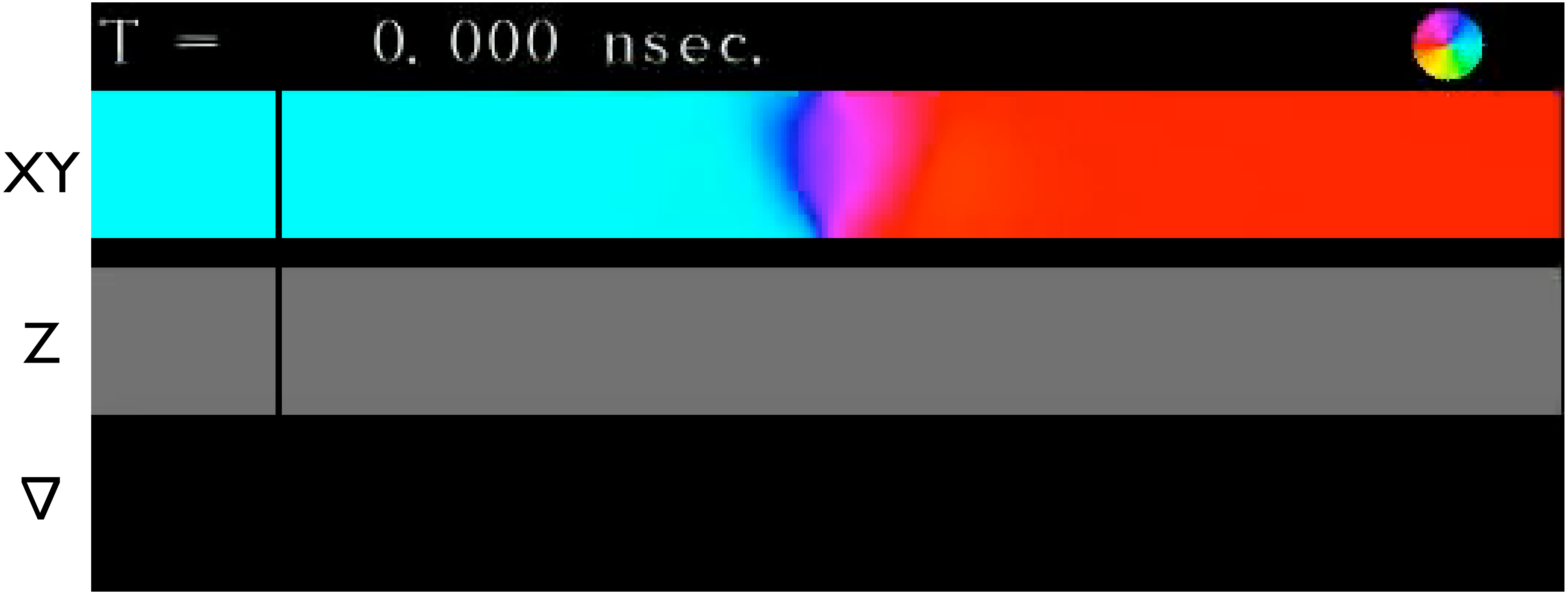

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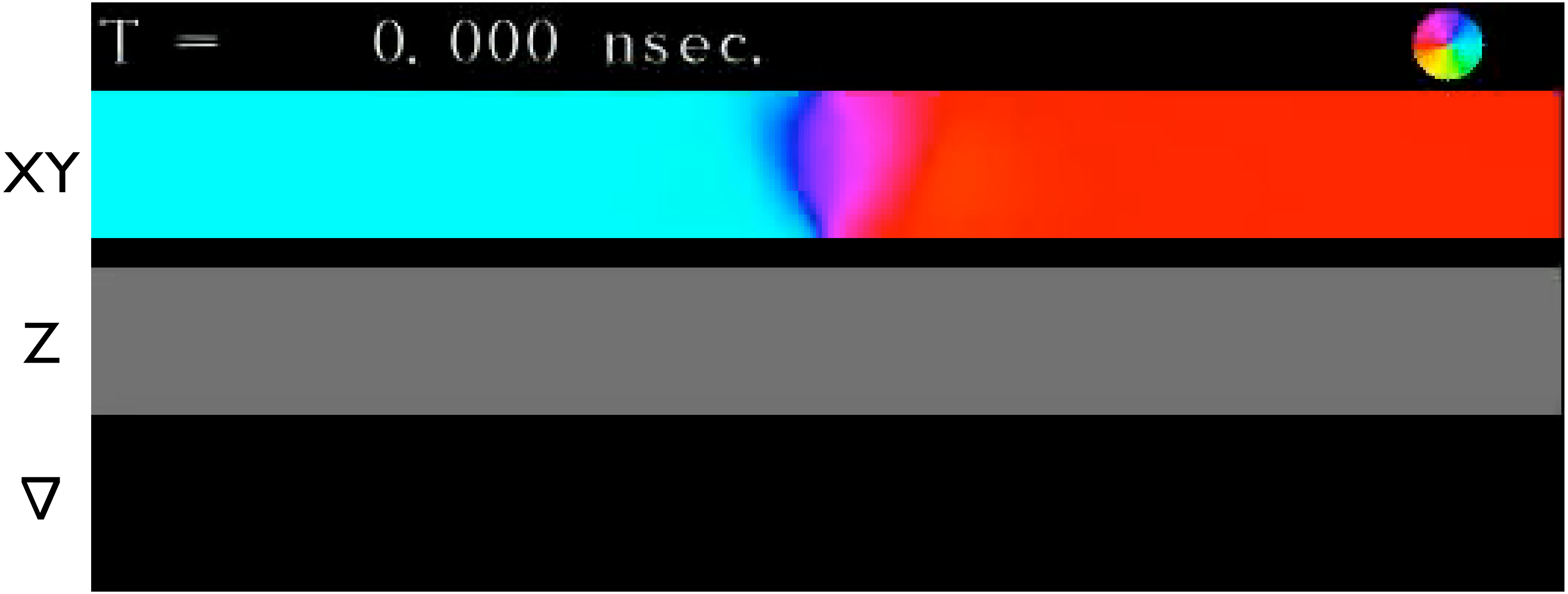

- Periodic creation, annihilation of ±1 vortices.
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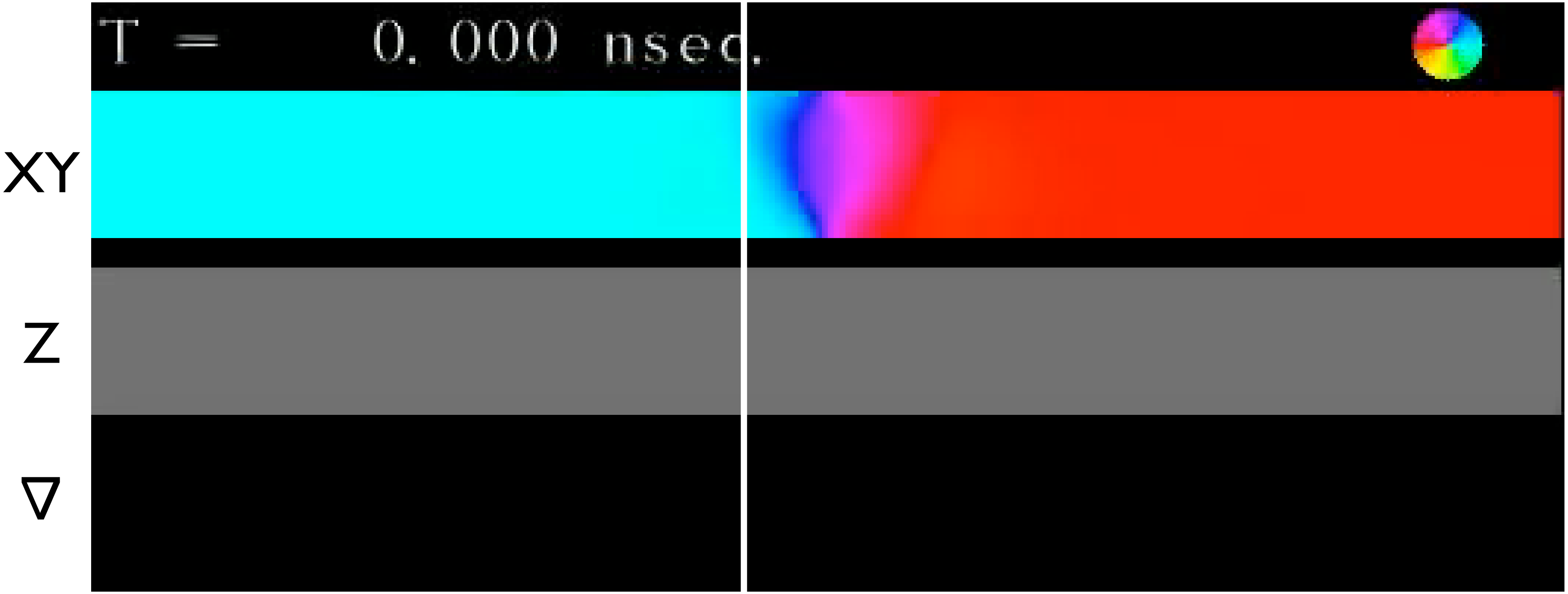
- Oscillations in the presence of a ±1 vortex. $\Leftarrow$

Stronger field: complex motion



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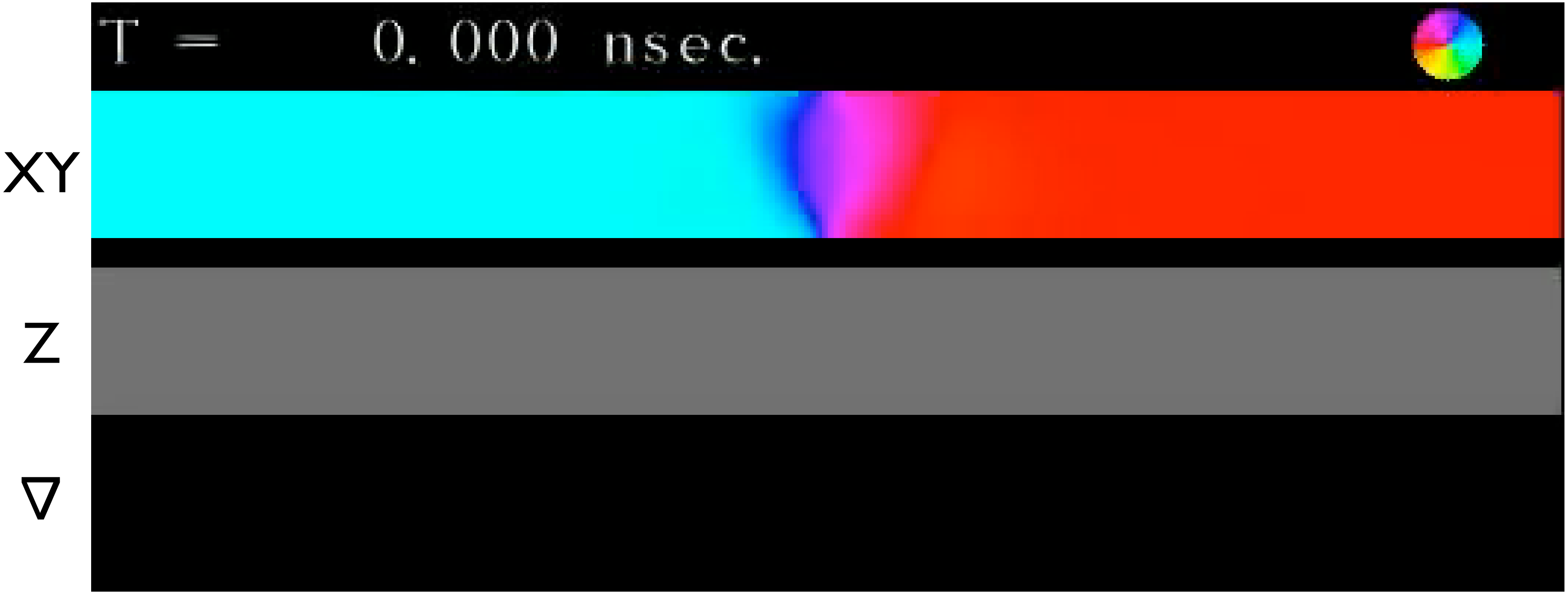
⇚

Stronger field: complex motion



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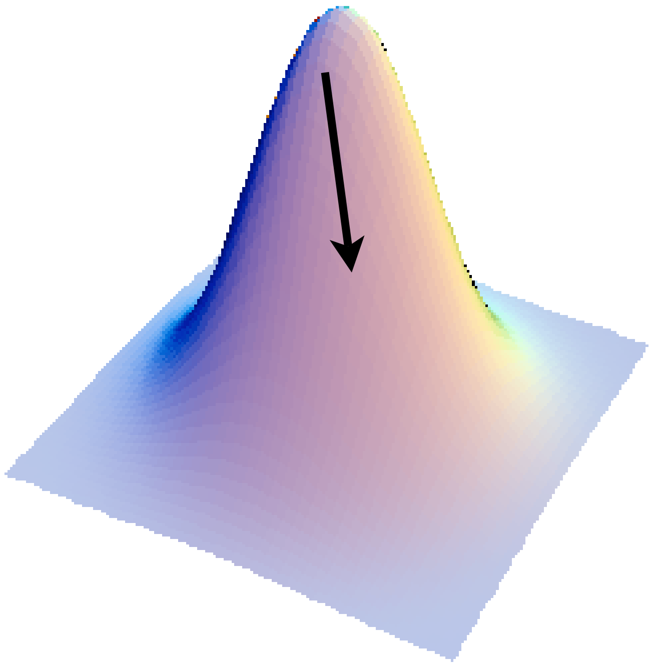
Underdamped regime

$$\sum_i \mathbf{F}_i - \Gamma \mathbf{v} + \mathbf{G} \times \mathbf{v} = 0, \quad G \gg \Gamma.$$



- Gyrotropic force:
  - acts on bulk vortices (±1),
  - has a topological origin,
  - overwhelms the viscous force.
- Vortex absent: viscous motion downhill.
- Vortex present: motion along equipotential lines.

Underdamped regime

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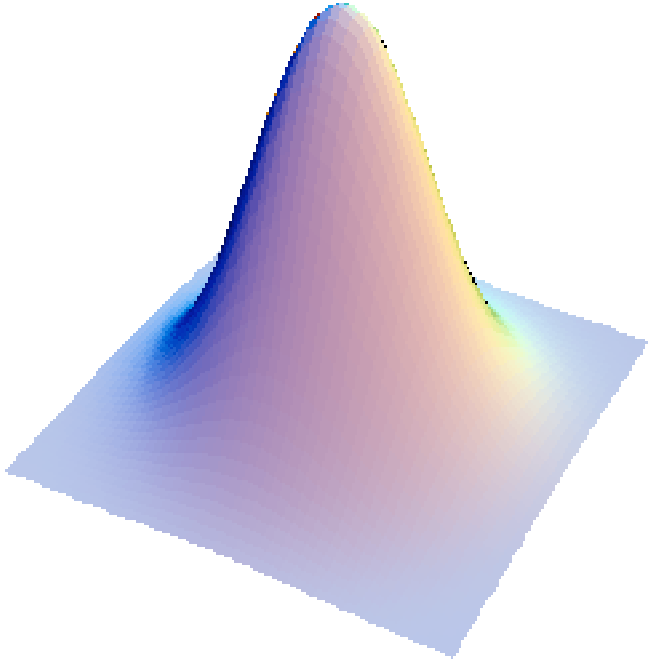

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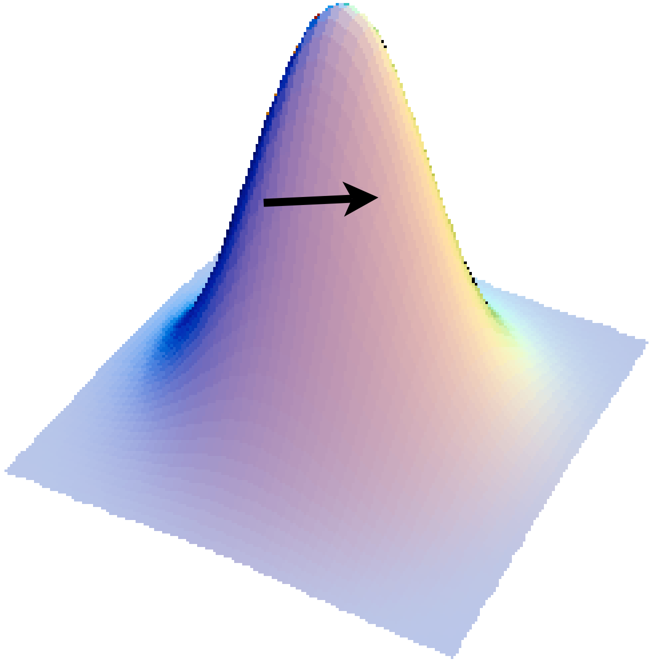

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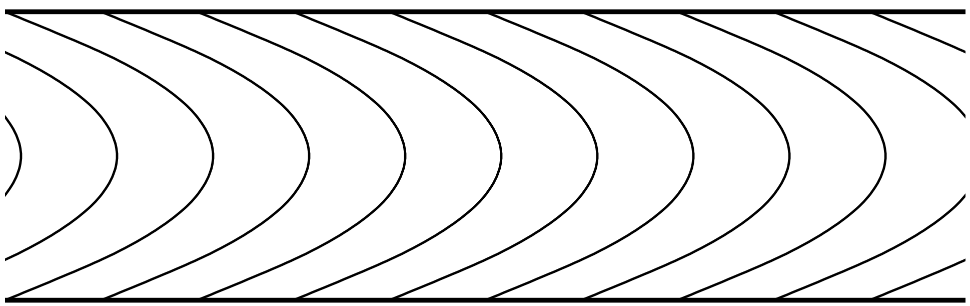


$$\mathbf{G} \times \mathbf{v} = \nabla U, \quad \Rightarrow \quad U(x, y) = \text{const.}$$

$$U(x, y) = -2HMwx + V(y).$$

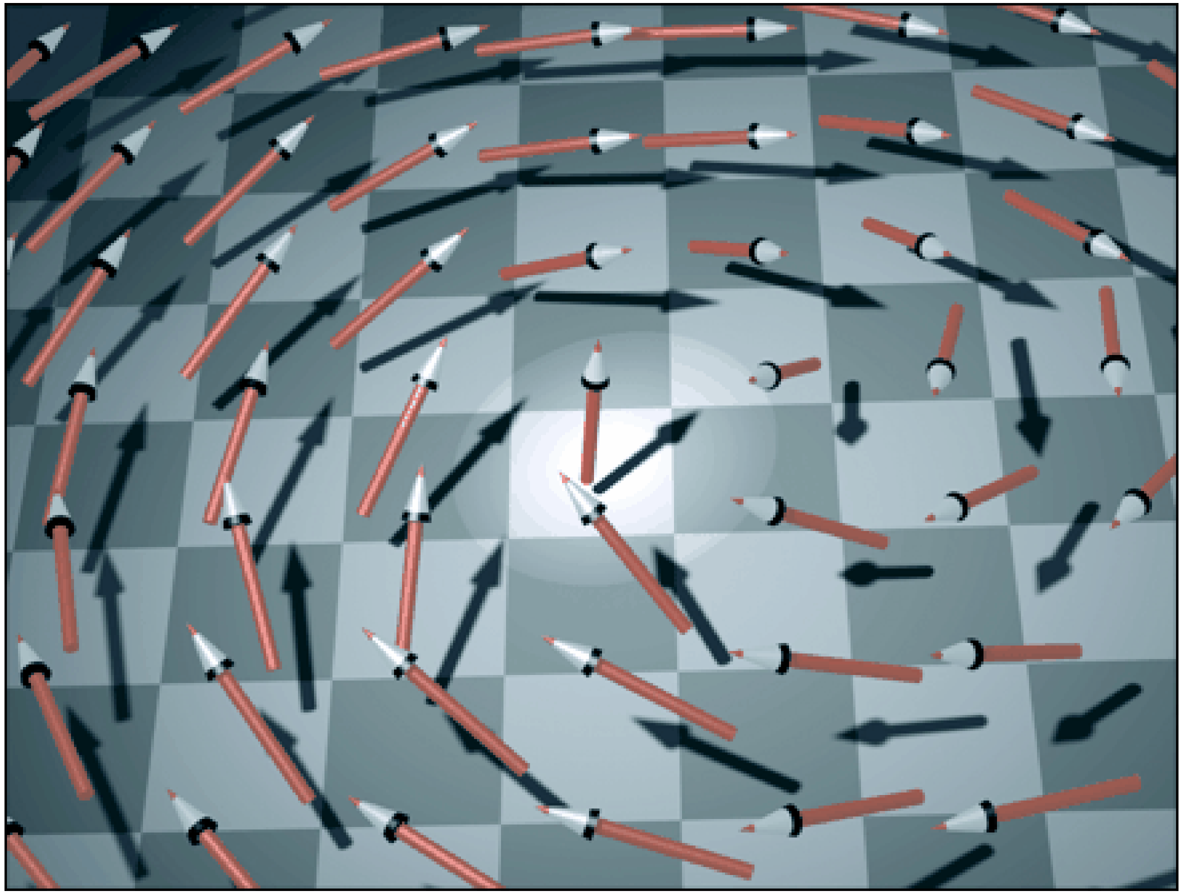
$$\partial_x U(x, y) = -F^{\text{Zeeman}} = -2HMw,$$

$$v_y = 2HMw/G, \quad T = w/|v_y| = \pi/\gamma H.$$

Crossing time = half a period of Larmor precession.

K. Yu. Guslienko (2007); O. T. (2007). $\Rightarrow$

Vortex core: a close-up



M points out of the plane (↑ or ↓) at the core (≈10 nm).

A. Wachowiack *et al.*, Science (2002).

Topological nature of gyroforce

$$\mathbf{F}^{\text{gyro}} = \mathbf{G} \times \mathbf{v} = 4\pi J q \,\hat{\mathbf{z}} \times \mathbf{v},$$

$$q = \frac{1}{8\pi} \int d^2 r \, \epsilon^{ij} \, \hat{\mathbf{m}} \cdot (\partial_i \hat{\mathbf{m}} \times \partial_j \hat{\mathbf{m}}) = np/2,$$

vorticity

z polarization

A. A. Belavin and A. M. Polyakov (1975).

E. Feldtkeller (1965); A. A. Thiele (1973).

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- *q* is the skyrmion number, a conserved charge.

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vorticity

z polarization

- *q* is the skyrmion number, a conserved charge.
- Counts how many times **M** wraps on the sphere.

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- Vortex with core ↑ covers Northern hemisphere (+1/2).

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vorticity

z polarization

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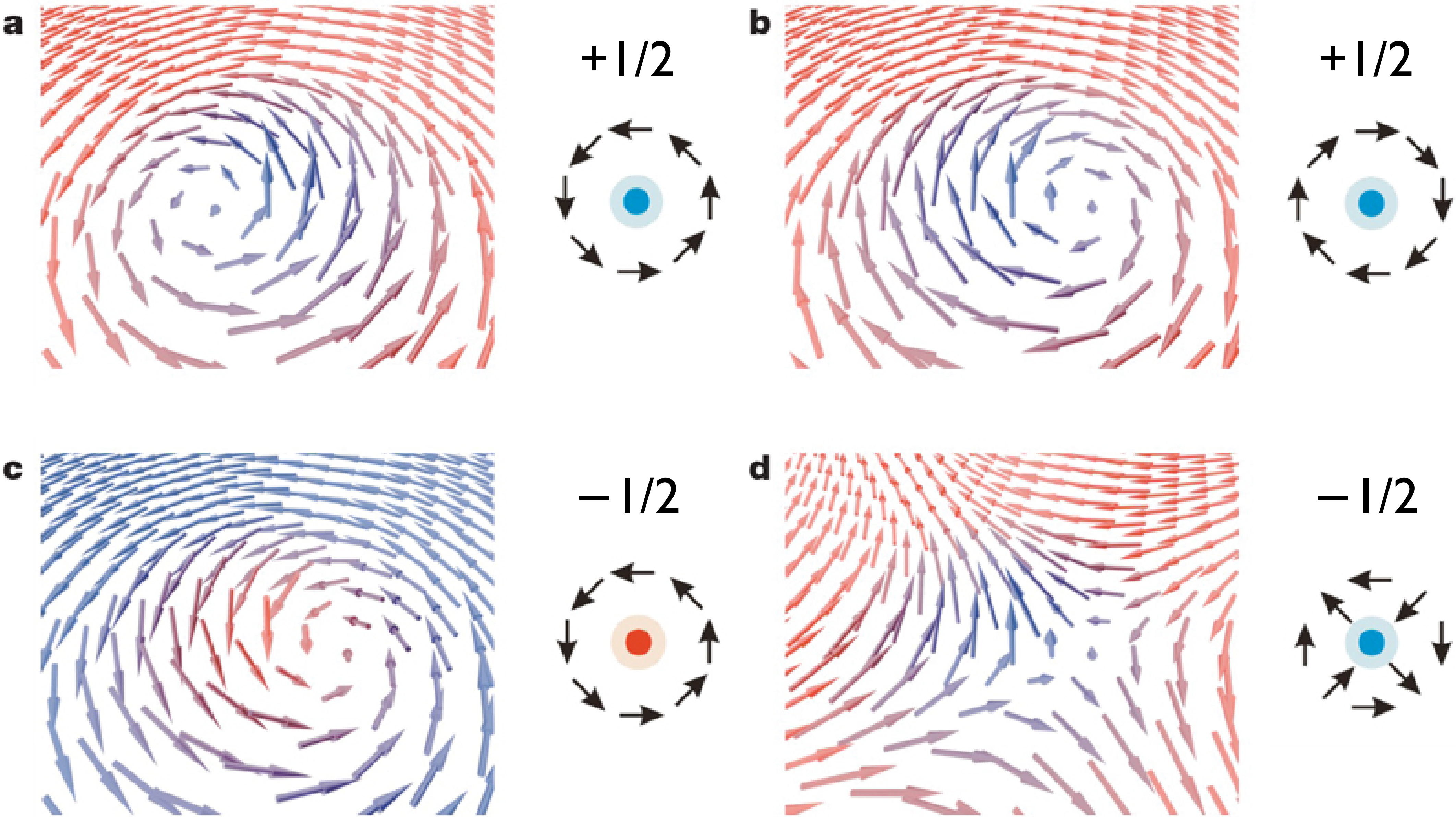
A. A. Belavin and A. M. Polyakov (1975).

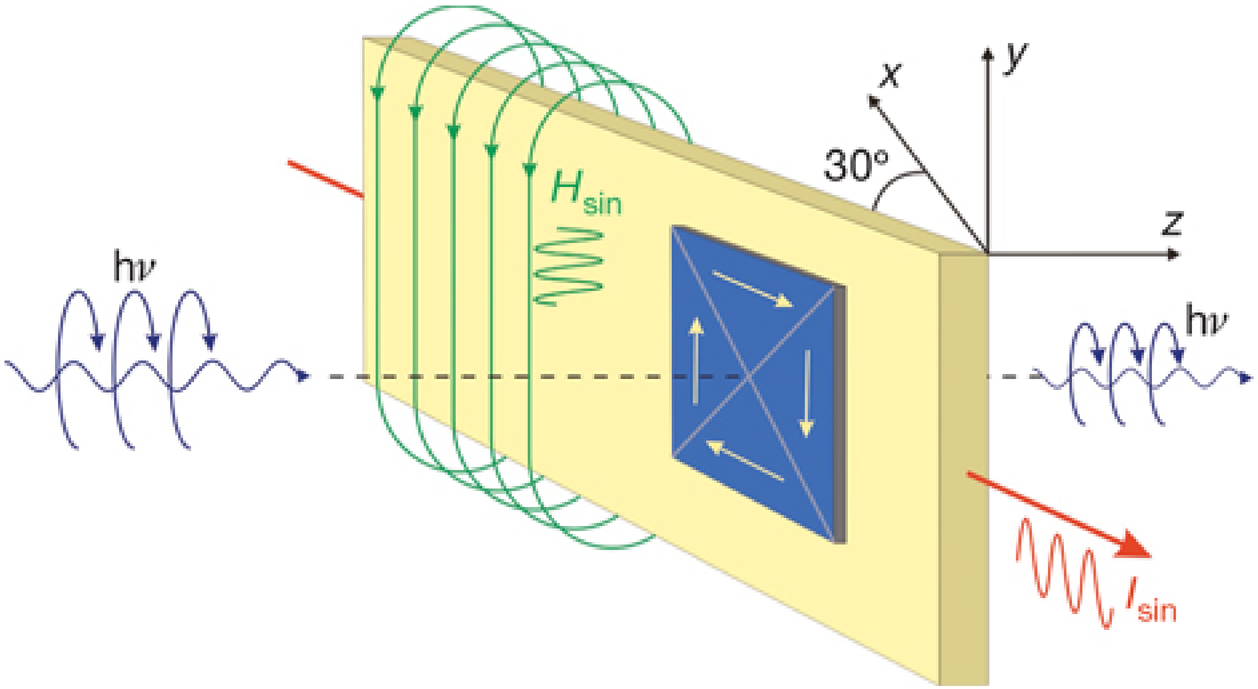
E. Feldtkeller (1965); A. A. Thiele (1973).

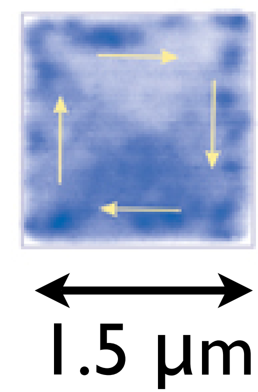
Skyrmion charge of a vortex



Skyrmion charge = vorticity × polarization / 2

Observing gyrotropic effects

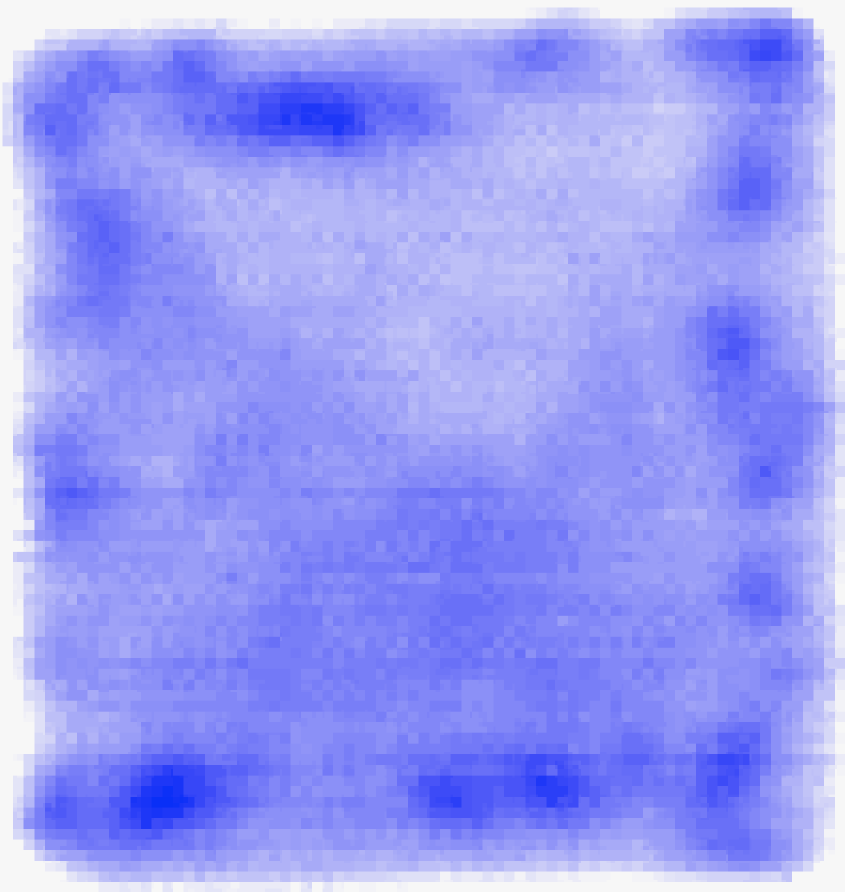




X-ray dichroism provides real-time info about **M**.

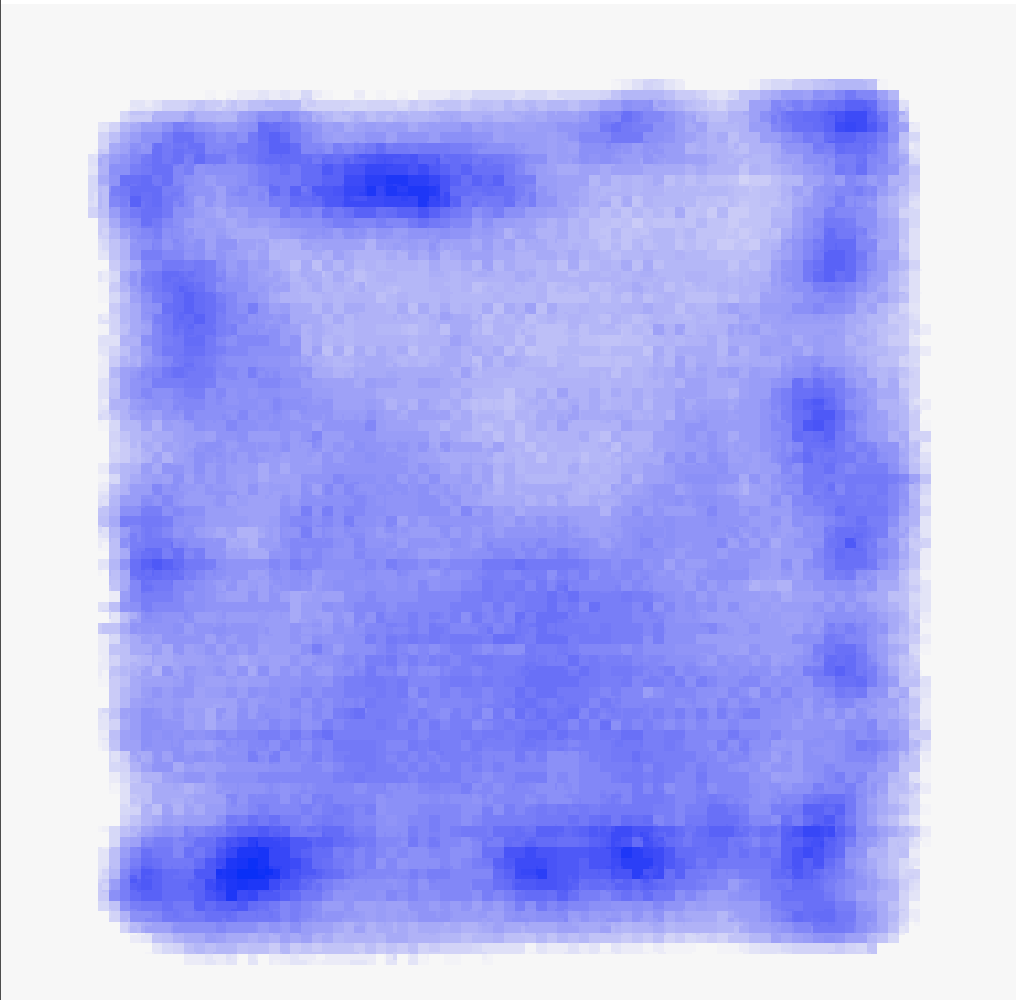
B. Van Waeyenberge *et al.*, Nature (2006).

Gyrating vortex



$q = +1/2$

Gyrating vortex



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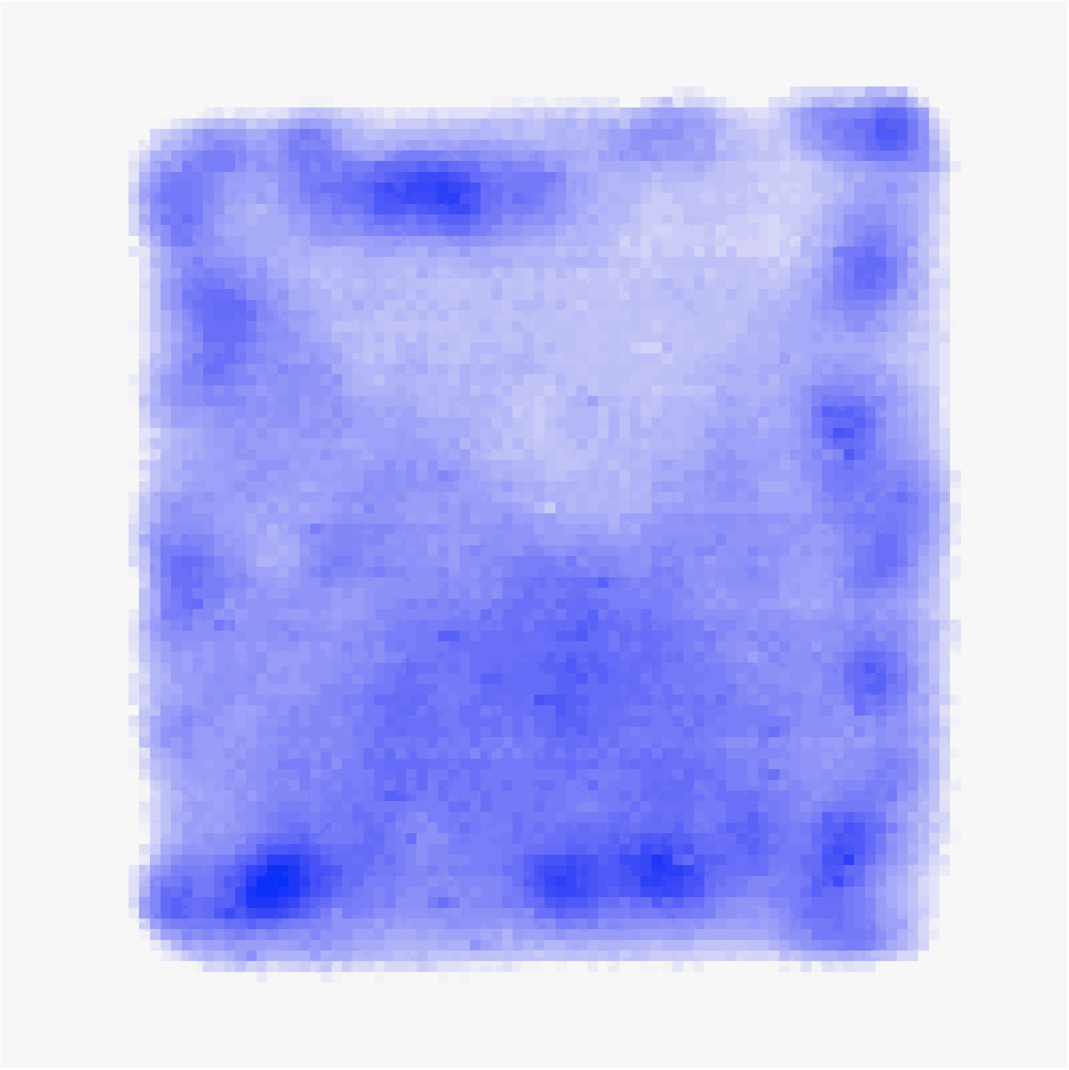

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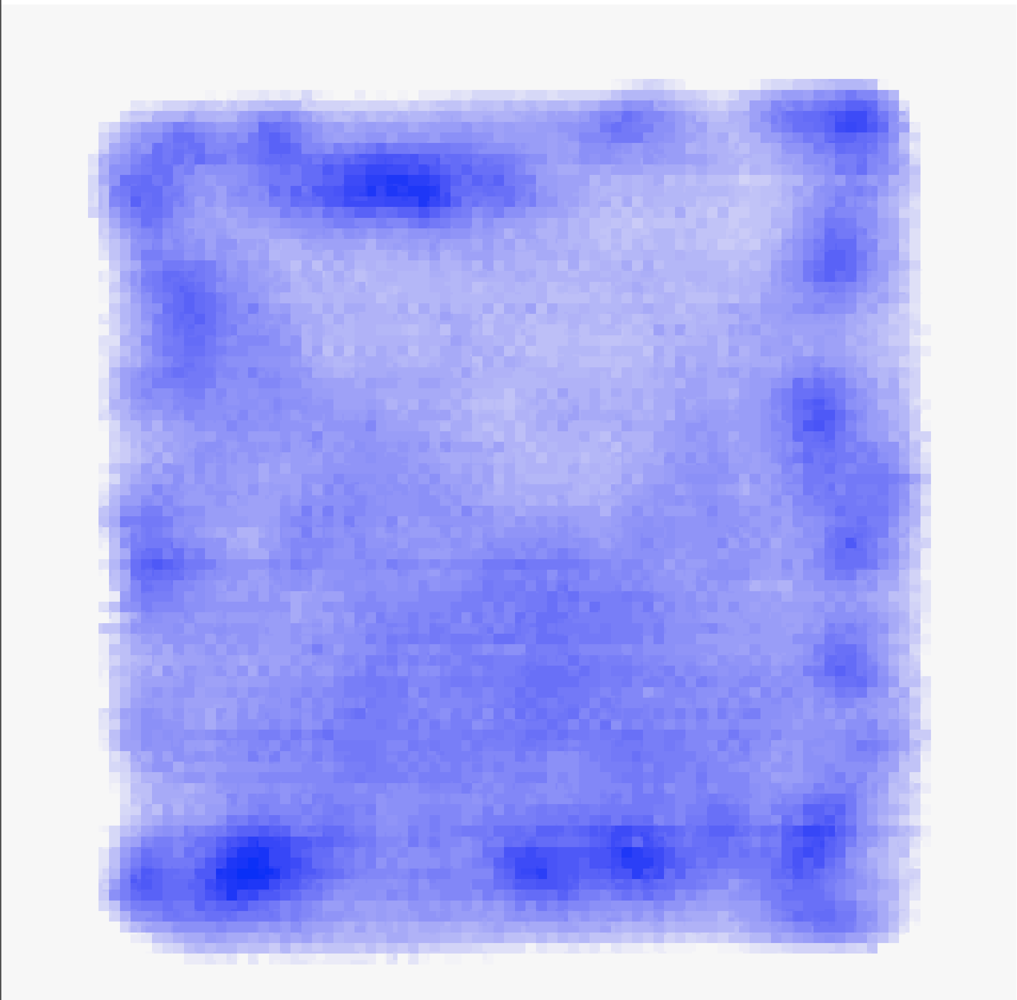

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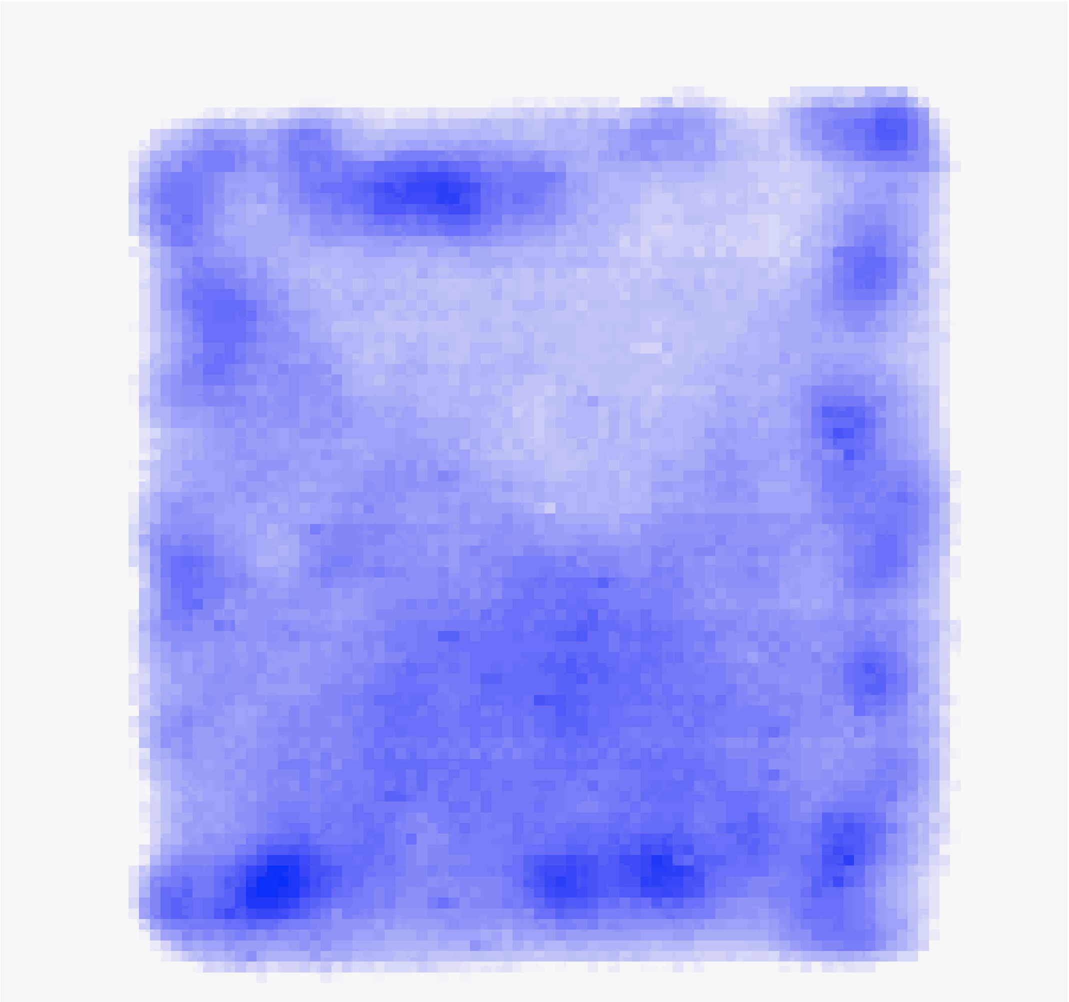
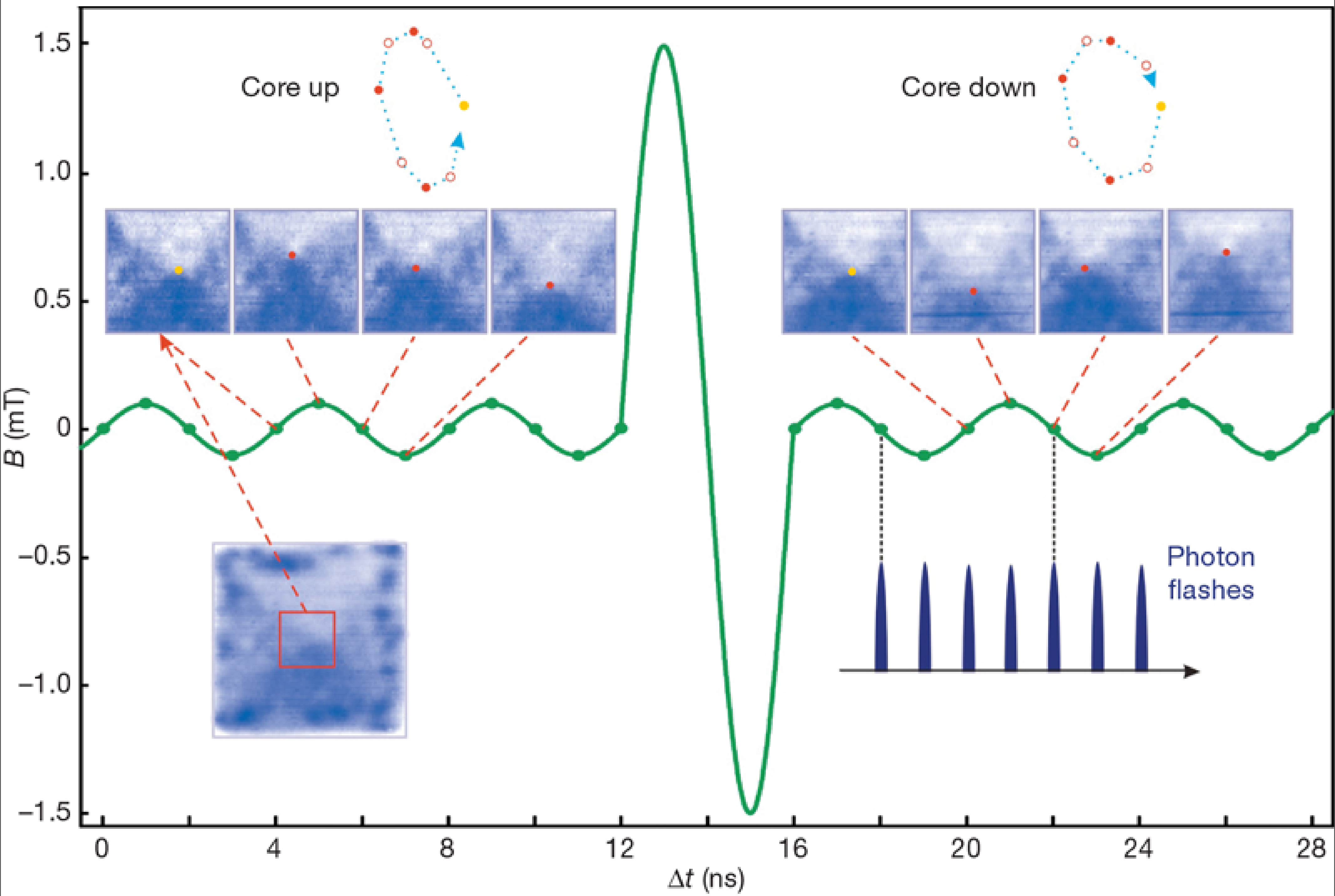


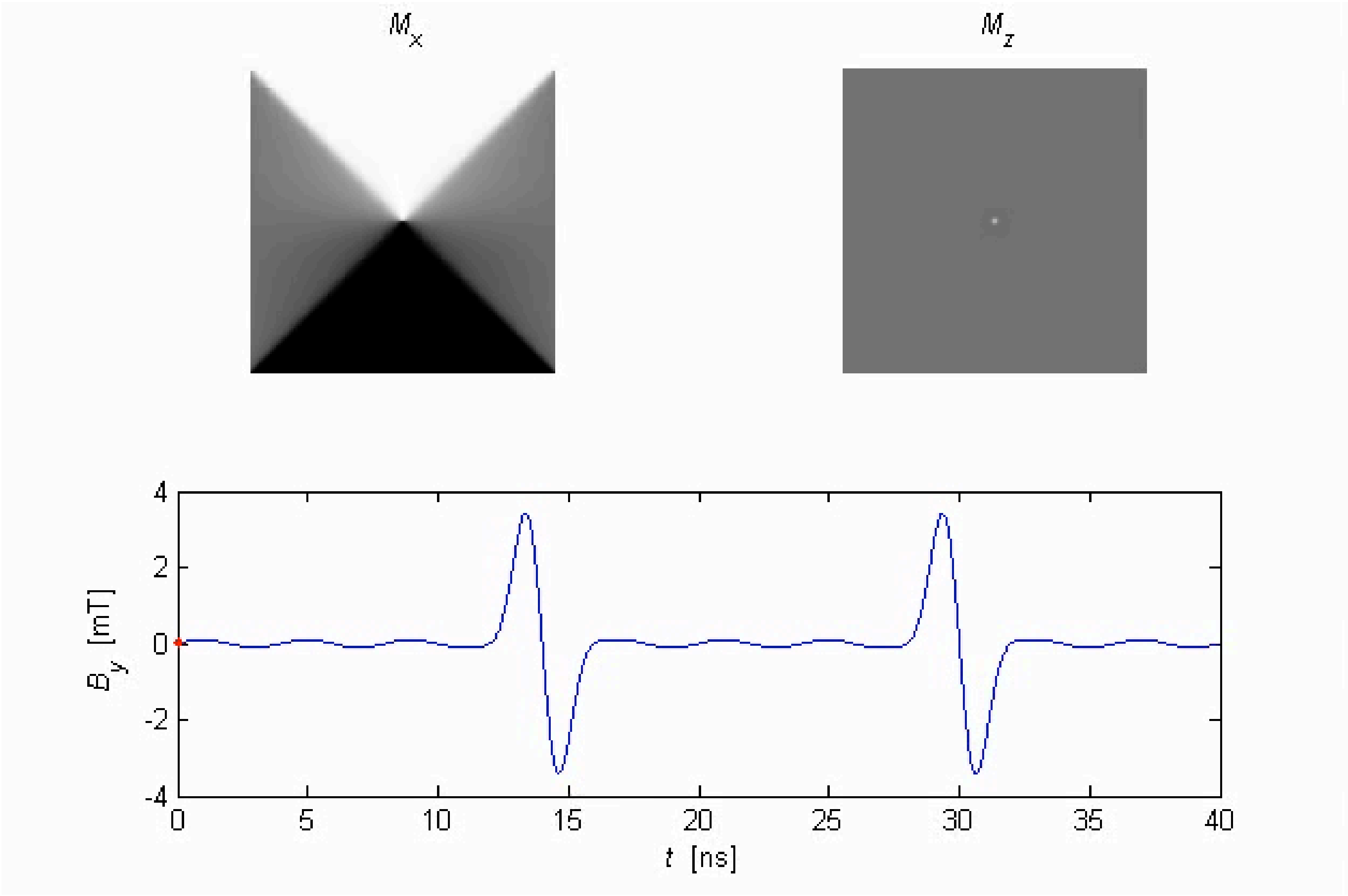
$q = -1/2$

The skyrmion number has changed!



Core up
Core down
Photon
flashes
1.5
1.0
0.5
0
−0.5
−1.0
−1.5
B (mT)
0
4
8
12
16
20
24
28
Δt (ns)

Flipping the core

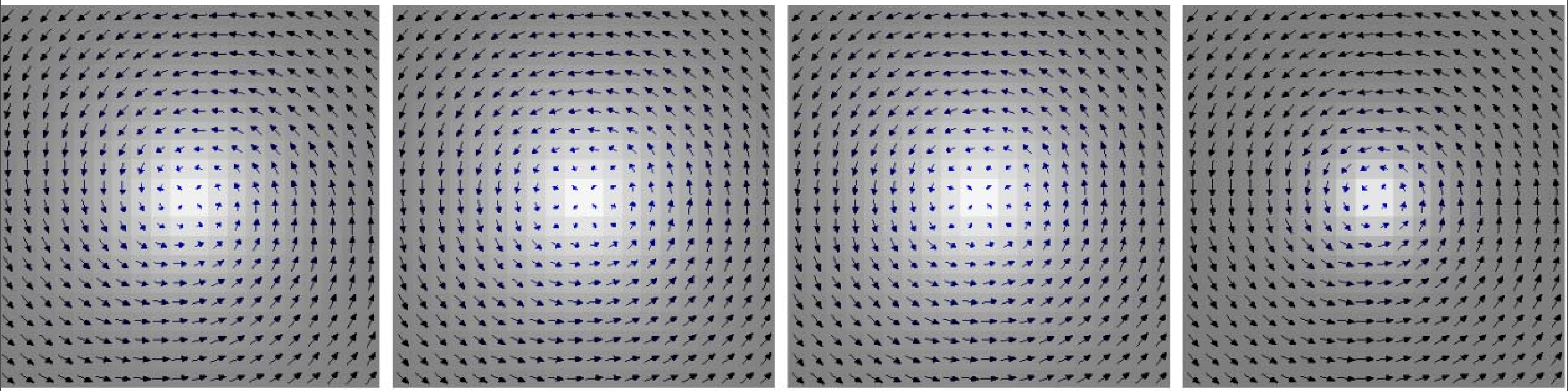


Numerical simulation

Direct core flipping

depth ⟶

Black = ↓, white = ↑.



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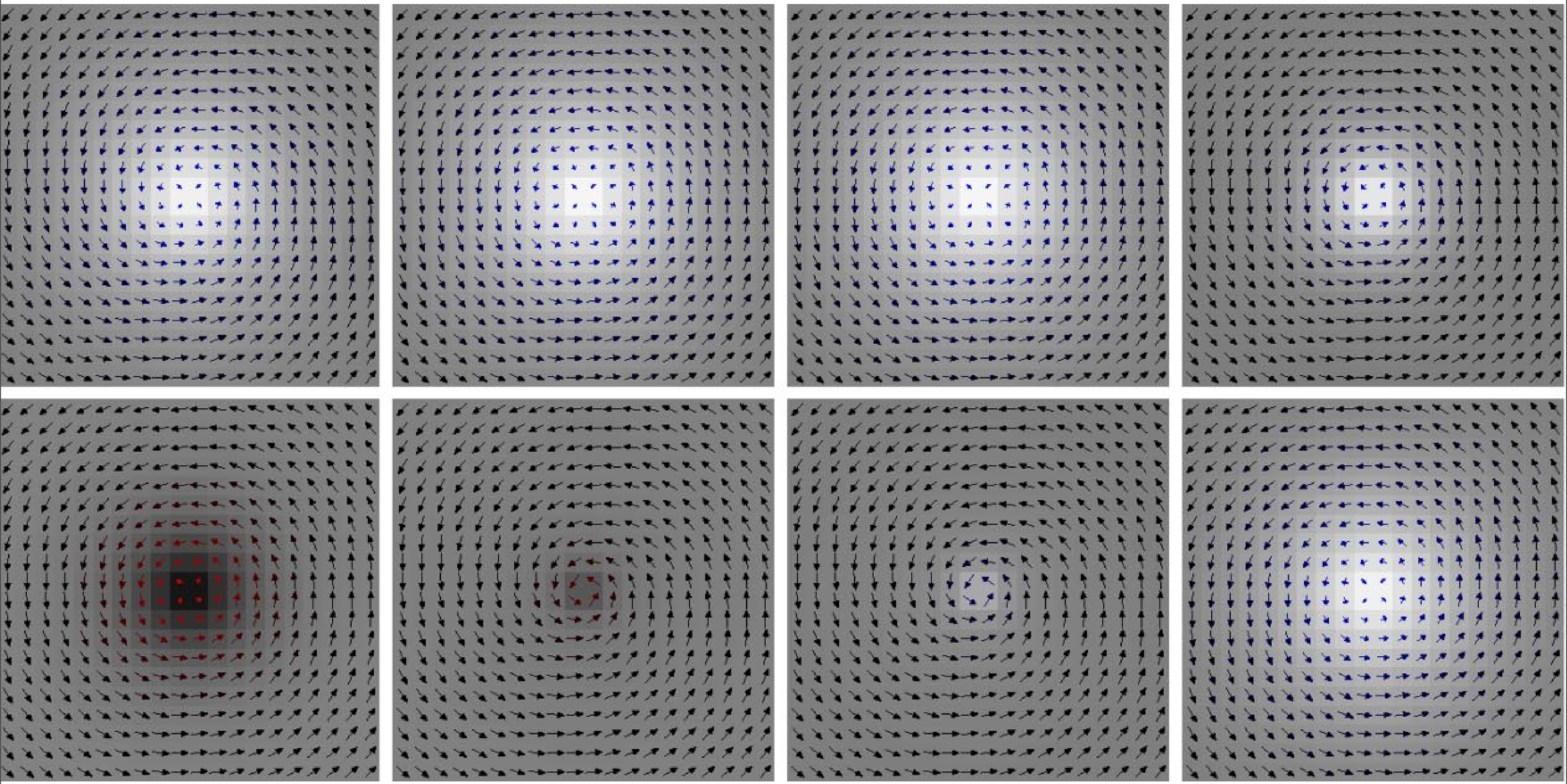
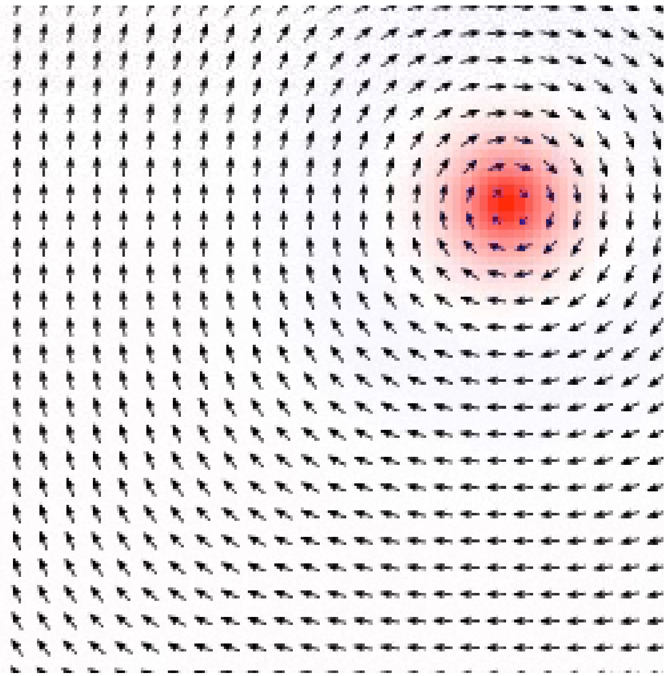


Mediated by a monopole (hedgehog). A. Thiaville *et al.*, PRB (2003).

Two-stage core flipping

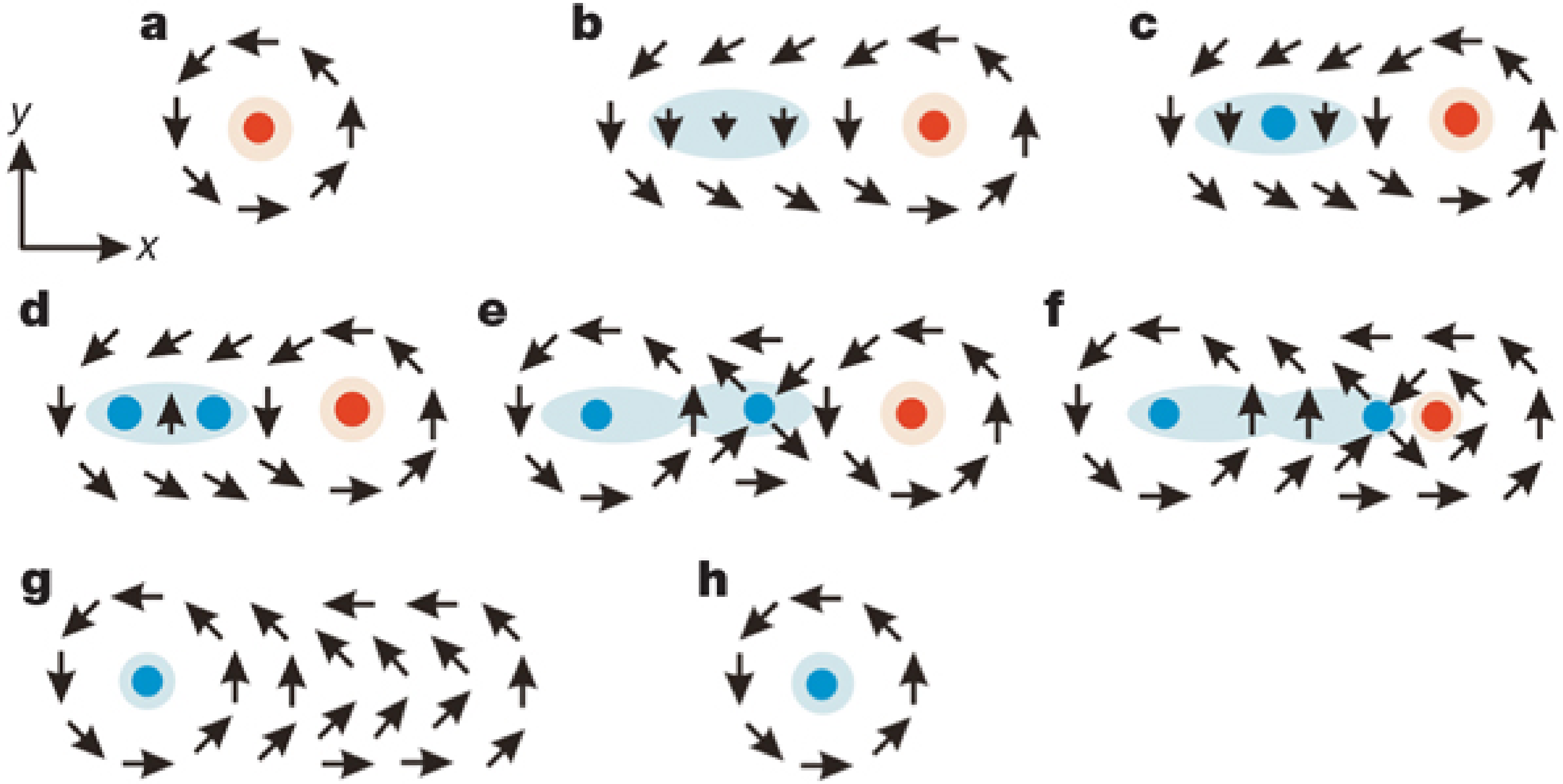


$M_z > 0$

$M_z < 0$

- Vortex-antivortex pair is created (Δq = 0).
  - requires a modest field.
- Antivortex + old vertex annihilate (Δq = 1);
  - spin-wave explosion is the death of a skyrmion.

Two-stage core flipping



B. Van Waeyenberge *et al.*, Nature (2006).

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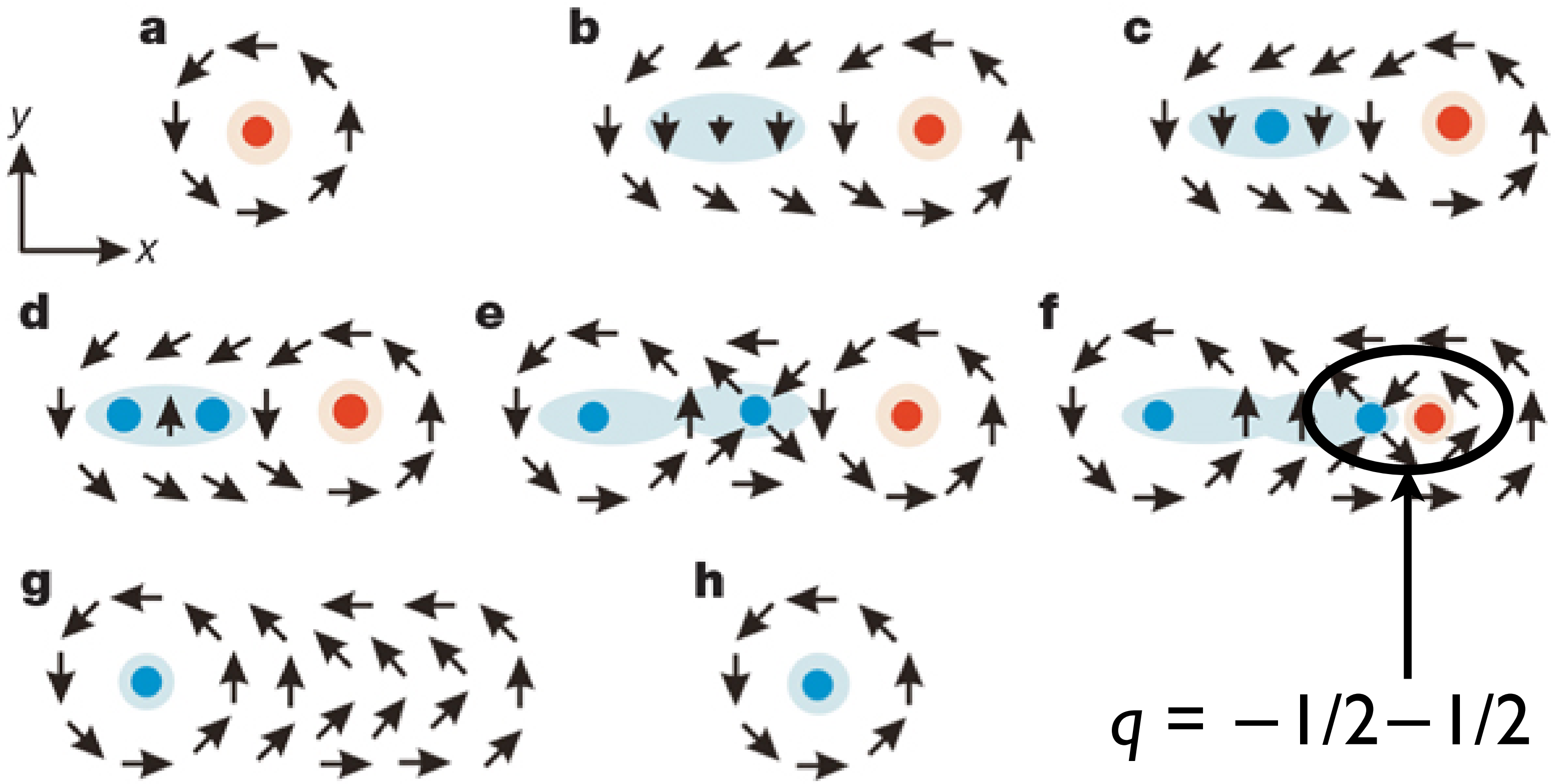
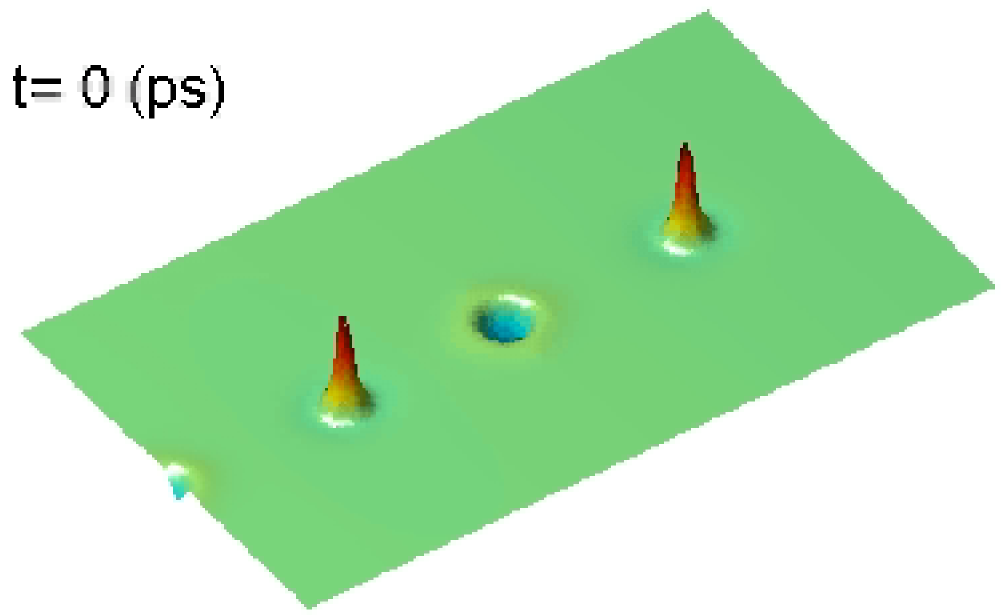


$q = -1/2 - 1/2 = -1$

B. Van Waeyenberge *et al.*, Nature (2006).

Vortex-antivortex annihilation



Vortex + antivortex → skyrmion → spin waves

Energy of spin waves = $8\pi A$ (skyrmion energy).

K. S. Lee *et al.*, APL (2005); R. Hertel and C. M. Schneider, PRL (2006).

O. Tretiakov and O. T., PRB (2007).

Summary

- Submicron magnets have just the right scale:
  - dipolar and exchange energies are comparable.
- Thin films with isotropic exchange have
  - bulk vortices with skyrmion charge $\pm 1/2$,
  - edge defects with fractional vorticity $\pm 1/2$.
- Domain walls in nanostrips and nanorings are made from integer and fractional vortices.
- Dynamics of DWs reduces to the creation, propagation, and annihilation of these defects.
- Skyrmion charge of vortices affects dynamics.
- Skyrmion number violation directly observed.