Inflation, Gravity Waves, and Dark Matter

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Units

- $\hbar = c = k_B = 1$
- $M_P = 1.2 \times 10^{19}$ GeV ($m_P = 2.4 \times 10^{18}$ GeV)
- $l_P = 1.6 \times 10^{-33}$ cm, $t_P = 5.4 \times 10^{-44}$ sec
- G = Newton's constant = M_P^{-2}
- $\text{GeV}^{-1} = 10^{-14}$ cm = 10^{-24} sec
- 1 MeV = 10^{10} K

ΛCDM Model (current paradigm)

Λ stands for Dark Energy with Einstein's cosmological constant being the leading candidate

($P_\Lambda = w_\Lambda \rho_\Lambda$, with $w_\Lambda = -1$)

$$\rho_{Total} = \rho_\Lambda + \rho_{CDM} + \rho_M \approx \rho_c$$

$$\rho_\Lambda \approx 10^{-120} m_P^4$$ ← Fine tuning?

CDM denotes 'cold dark matter' (particle have tiny velocities)

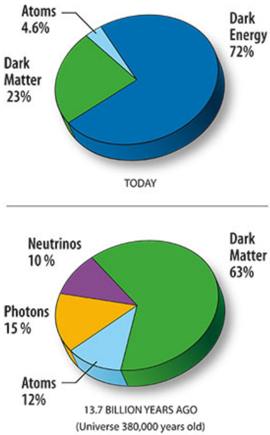


Image courtesy of NASA / WMAP Science Team

Where does ΛCDM come from?

Four Fundamental Forces

Force	Strength	
• Strong	~ 1	Standard Model of HE Physics
• Electromagnetic	$\sim 10^{-2}$	
• Weak	$\sim 10^{-5}$	
• Gravity	$\sim 10^{-38}$	General Relativity

STANDARD MODEL OF HE PHYSICS

Provides excellent description of strong, weak and electromagnetic interactions.

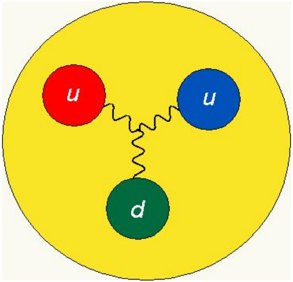
Based on local gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

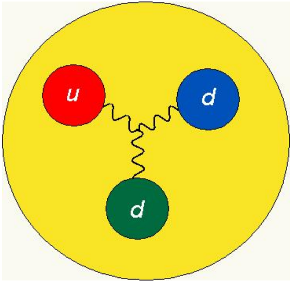
QCD - strong interactions involving 'colored' quarks & gluons

Electromagnetic and weak interactions mediated by $W^{\pm}$, Z^0 bosons and γ, which have been found

Only 'color neutral' states exist in nature proton



neutron

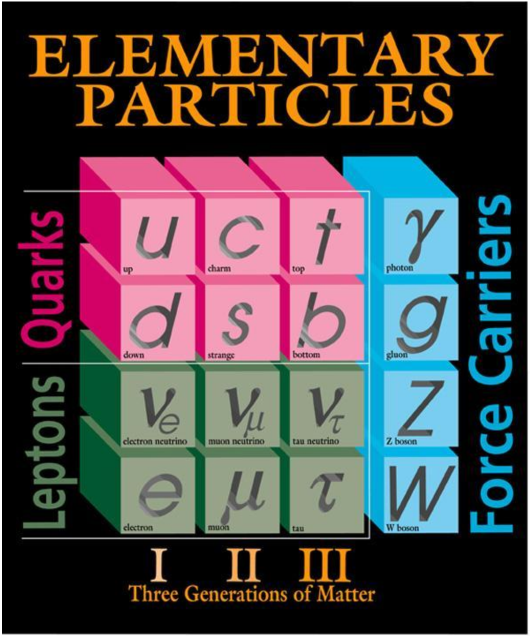


Color neutral 'atoms'

Two Key features:

Color confinement;

Asymptotic freedom;



Fermilab 95-759

Higgs Boson

- Spin zero particle from spontaneous breaking of electroweak symmetry:

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle} U(1)_{\mathrm{EM}}$$

$$\langle\phi\rangle \sim 10^2 \ \mathrm{GeV}(t \sim 10^{-10} \ \mathrm{sec})$$

$m_h \approx 125\mathrm{GeV}$ (Huge discovery announced by ATLAS and CMS on July 4, 2012)

- Compare: Superconductor (Cooper pairs $\longleftrightarrow \langle\phi\rangle$)

IS THERE “NEW” PHYSICS BEYOND THE STANDARD MODEL?

Most Likely Yes!

1) Neutrino Oscillations (solar & atmospheric):

These require non-zero (albeit ‘tiny’) neutrino masses $\sim 10^{-1} – 10^{-2}$ eV.

In the SM, neutrinos have zero mass.

2) Dark Matter (non-baryonic)

SM has no plausible DM candidate

Oscillation Data

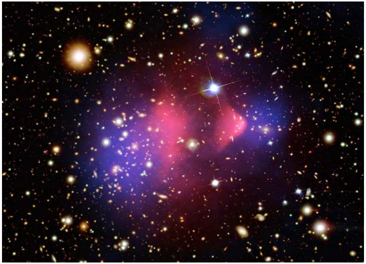
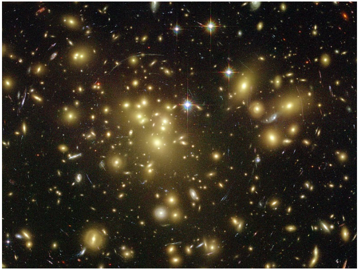
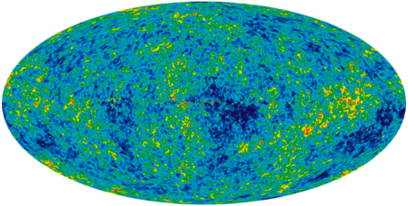
- $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$
- $\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\text{eV}^2$
- $\sin^2(2\theta_{23}) = 0.999^{+0.001}_{-0.018}$ (normal mass hierarchy)
  $\sin^2(2\theta_{23}) = 1.000^{+0.000}_{-0.017}$ (inverted mass hierarchy)
- $\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\text{eV}^2$ (normal mass hierarchy)
  $\Delta m^2_{32} = (2.52 \pm 0.07) \times 10^{-3}\text{eV}^2$ (inverted mass hierarchy)
- $\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}$
- Tiny masses (compared to quarks and charged leptons)
- Mixing angles $\rightarrow$ large (compared to quark sector)

Dark Matter in The Universe

- Zwicky ($\sim$1930)
  Galaxies in the Coma cluster seem to be moving too rapidly to be halo together by the gravitational attraction of the visible matter.

- Rotation curves of velocity versus radial distance for stars and gax provide indirect evidence for the existence of 'missing' non-luminous mass.

- $\delta\rho/\rho \sim 10^{-5} \Longrightarrow$ structure formation (galaxies, clusters) hard without non-baryonic dark matter.


Hot Big Bang Cosmology

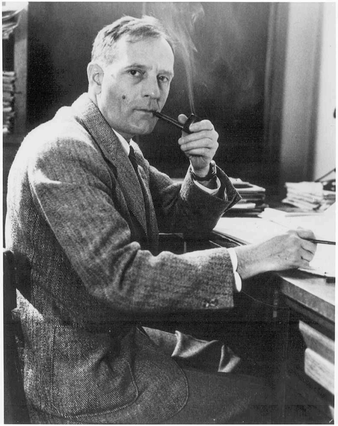
- Comes from combining Standard Model (SM) of high energy physics with Einstein' general relativity, and the assumption that on sufficiently large scales, the universe is isotropic and homogeneous.

isotropic ↑ same in all directions

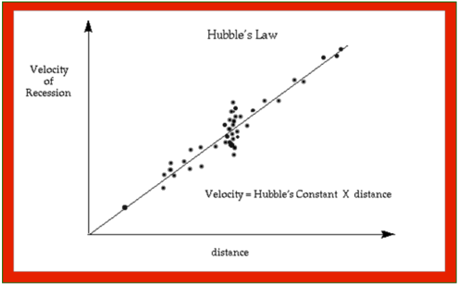
homogeneous ↑ position independent

Three remarkable predictions (Consequences):

1. Expanding Universe
2. Cosmic Microwave Background Radiation (CMB)
3. Nucleosynthesis



Edwin Hubble

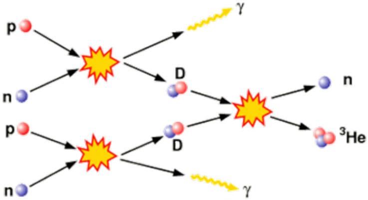


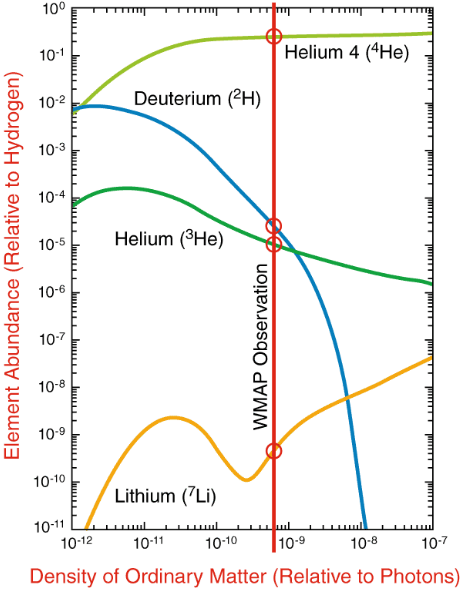
$H_0 = 67.8 \pm 0.9$ (km/s)/Mpc

$t_0 = 1/H_0 = 13.813 \pm 0.038$Gyr

(Planck, arXiv:1502.01589)

In natural units, $H_0 \approx 10^{-33}$ eV!





NASA/WMAP Science Team
WMAP101087

Element Abundance graphs: Steigman, Encyclopedia of Astronomy and Astrophysics (Institute of Physics) December, 2000

- A homogeneous and isotropic universe is described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right],$$

where r, ϕ and θ are 'comoving' polar coordinates, which remain fixed for objects that follow the general cosmological expansion.

k is the scalar curvature of 3-space, with k = 0, +1, -1 describing a flat, closed and open universe respectively.

Geometry of the Universe

- Friedmann Equation

$$\Omega \equiv \frac{\rho}{\rho_c} = 1 + \frac{k}{(aH)^2}, \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} = \text{critical density}$$

- Closed ($\Omega > 1$ or $k = 1$)
- Open ($\Omega < 1$ or $k = -1$)
- Flat ($\Omega = 1$ or $k = 0$)

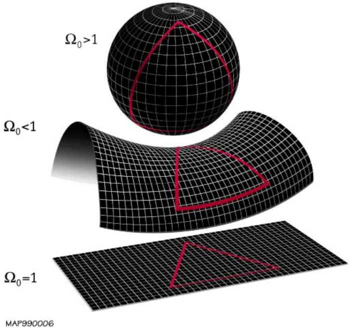


Image courtesy of NASA: http://map.gsfc.nasa.gov/universe/uni_shape.html

Solving Friedmann Equations:

- For flat universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \propto \rho$$

- Matter $\left(\rho_m = \frac{NM}{V}\right)$

$$\rho_m \propto a^{-3} \Rightarrow a(t) \propto t^{2/3}$$

- Radiation $\left(\rho_\gamma = \frac{Nhc}{V\lambda}\right)$

$$\rho_\gamma \propto a^{-4} \Rightarrow a(t) \propto t^{1/2}$$

- Vacuum $(\rho_\Lambda = \text{const.})$

$$\rho_\Lambda \propto a^0 \Rightarrow a(t) \propto e^{Ht}$$

Cosmological Problems

- Flatness Problem

Present energy density of the universe is determined to be equal to its critical value corresponding to a flat universe. This means that in the early universe

$$\Omega - 1 = \frac{k}{(aH)^2} \propto t \qquad \text{(for a radiation dominated universe)}$$

$$\Rightarrow \quad |\Omega_{BBN} - 1| \leq 10^{-16} \qquad \left(|\Omega_{GUT} - 1| \leq 10^{-55}\right)$$

How does this come about?

Horizon Problem

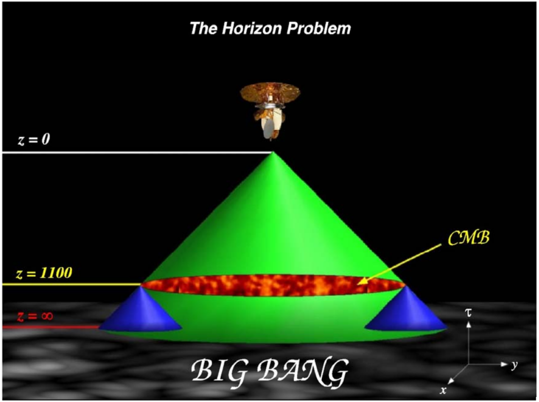


Image courtesy of W. Kinney

Why the CMB is so uniform on large scales?

- Origin of primordial density fluctuation which lead to Large Scale Structure and also explain

$$\delta T/T \sim 10^{-5}$$

observed by COBE/WMAP and other experiments?

- Origin of baryon asymmetry ($n_b/n_\gamma \sim 10^{-10}$)?

Inflationary Cosmology

[Guth, Linde, Albrecht & Steinhardt, Starobinsky, Mukhanov, Hawking, . . .]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?

Cosmic Inflation

- Inflation can be defined as:

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0,$$ a decreasing comoving horizon

$$\ddot{a} > 0,$$ an accelerated expansion

$$P < -\rho/3,$$ a negative pressure → repulsive gravity

↓

drives inflation

- Consider a scalar field ϕ

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \approx V,$$

$$a(t) \approx e^{Ht}$$ → inflation

Slow rolling scalar field acts as an inflaton

Cosmic Inflation

Tiny patch ~10^{-28} cm ⇨ > 1 cm after 60 e-foldings (time constant ~10^{-38} sec)

Inflation over ⇨ radiation dominated universe (hot big bang)

Quantum fluctuations of inflation field give rise to nearly scale invariant, adiabatic, Gaussian density perturbations

⇨ Seed for forming large scale structure

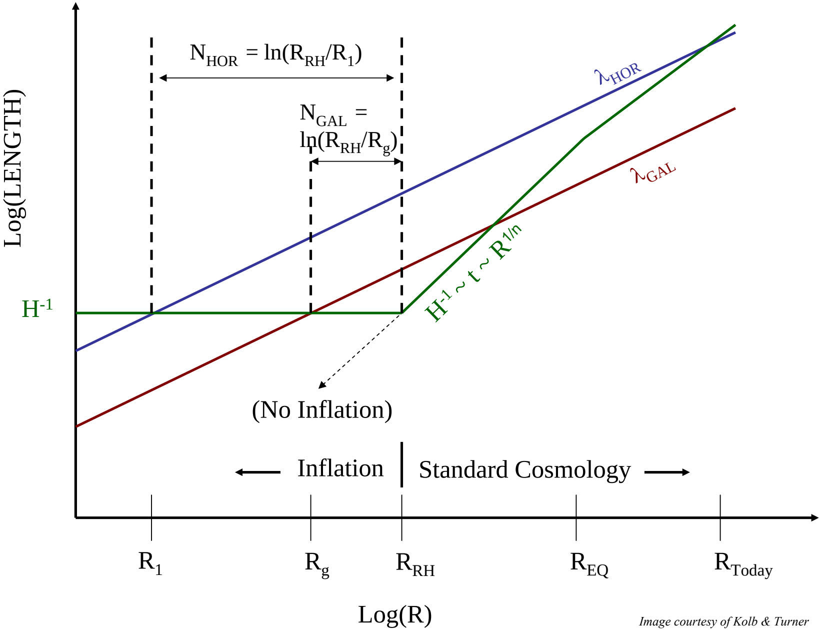


Image courtesy of Kolb & Turner

- Solution to the Flatness Problem $\left(\Omega - 1 = \frac{k}{(aH)^2} \right)$

$$|\Omega_f - 1| = |\Omega_i - 1| e^{-2N} \to 0,$$ where $N = \mathrm{H}\,\Delta\mathrm{t} \geq 50$

- Solution to the Horizon Problem

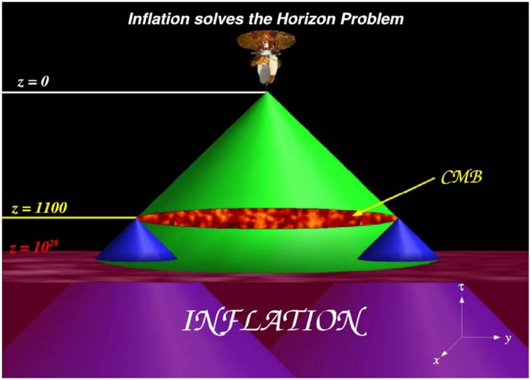


Image courtesy of W. Kinney

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.

- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

Slow-roll Inflation

- The tensor to scalar ratio r can be related to the energy scale of inflation via
$$V(\phi_0)^{1/4} = 3.3 \times 10^{16}\, r^{1/4} \text{ GeV}.$$
- The amplitude of the curvature perturbation is given by
$$\Delta_{\mathcal{R}}^2 = \frac{1}{24\pi^2}\left(\frac{V/m_p^4}{\epsilon}\right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.$$
- The spectrum of the tensor perturbation is given by
$$\Delta_h^2 = \frac{2}{3\,\pi^2}\left(\frac{V}{m_P^4}\right)_{\phi=\phi_0}.$$
- The number of e-folds after the comoving scale $l_0 = 2\,\pi/k_0$ has crossed the horizon is given by
$$N_0 = \frac{1}{m_p^2}\int_{\phi_e}^{\phi_0}\left(\frac{V}{V'}\right) d\phi.$$

  Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$.

Scalar and Tensor Perturbations

During inflation, the universe contains a uniform scalar (inflaton) field and a uniform background metric.

There are quantum mechanical fluctuations about this zero-order scheme. According to inflationary cosmology, this generates $\delta\rho/\rho$ as well as gravity waves (from tensor fluctuations in the gravitational metric).

(1) $V = m^2\phi^2$

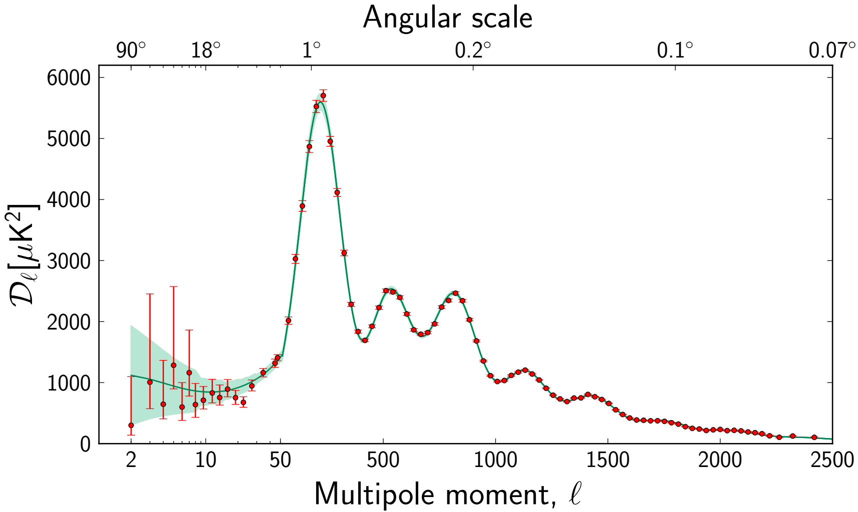
$$\implies \frac{\delta\rho}{\rho} \sim \frac{m}{M_\mathsf{P}} \implies m \sim 10^{13}\ \text{GeV}$$

(2) $V = \lambda\phi^4$

$$\implies \frac{\delta\rho}{\rho} \propto \sqrt{\lambda} \implies \lambda \sim 10^{-12}$$

(tiny quartic coupling)

Can Standard Model Higgs field drive inflation?



Planck (2013), arXiv:1303.5075

CMB to Parameters

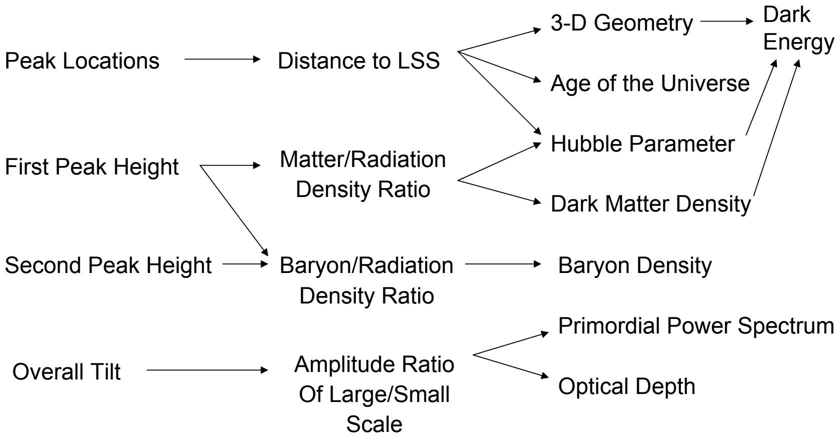


Image courtesy of E. Komatsu

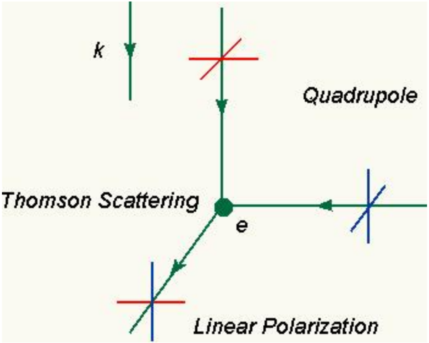
Gravity Waves from Inflation

Inflation also generates tensor fluctuations in the gravitation metric which correspond to gravity waves. They induce fluctuations in the CMB and provide a unique signature of inflation. Their discovery would have far reaching implications for inflationary cosmology. The PLANCK satellite now in orbit has an excellent chance to 'detect' gravity waves if inflation is 'driven' by a grand unified theory with a characteristic energy scale $\sim 10^{16}$ GeV.

(note LHC cm energy $\sim 10^{4}$ GeV!)

CMB Polarization

- CMB radiation is expected to be polarized from Compton scattering during (matter-radiation) decoupling.
- To produce polarized radiation the incoming radiation must have a non-zero quadrupole. One expects the polarization signal to be small.



Polarization is generated by both scalar and tensor perturbations.

E modes

(varies in strength in the same direction as its orientation)

B modes

(varies in strength in a direction different from that in which it is pointing)

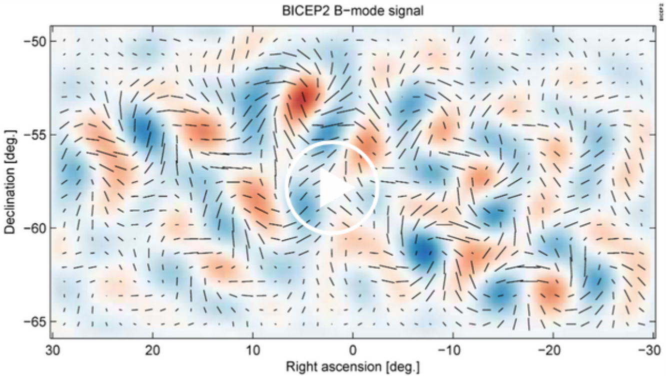
BICEP 2 Result

- BICEP 2 a few months ago surprised many people with their results that $r \sim 0.2$ (0.16).
- Some tension with the Planck upper bound $r < 0.11$.
- Somewhat earlier WMAP 9 stated that $r < 0.13$.

Big Bang breakthrough announced; gravitational waves detected

By Elizabeth Landau, CNN

Updated 10:37 AM ET, Tue March 18, 2014



SCIENCE

66 COMMENTS

Speck of Interstellar Dust Obscures Glimpse of Big Bang

By DENNIS OVERBYE JAN. 30, 2015

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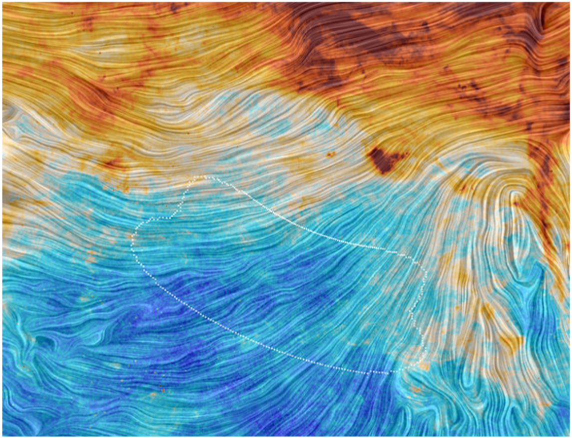
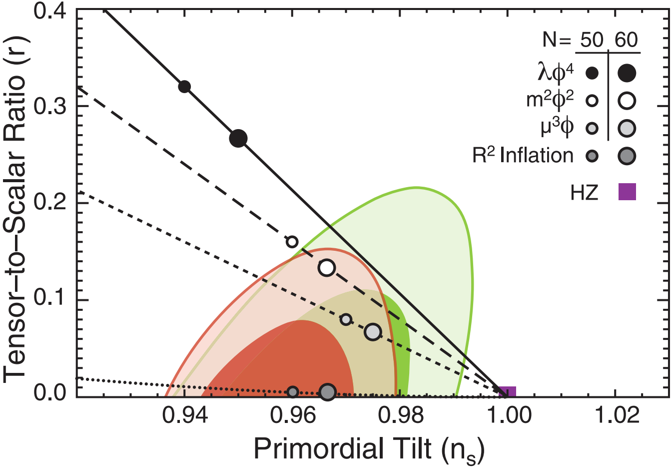
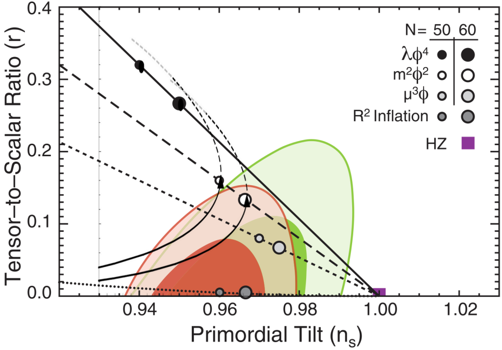




WMAP nine year data

Radiatively Corrected ϕ^2 Potential:

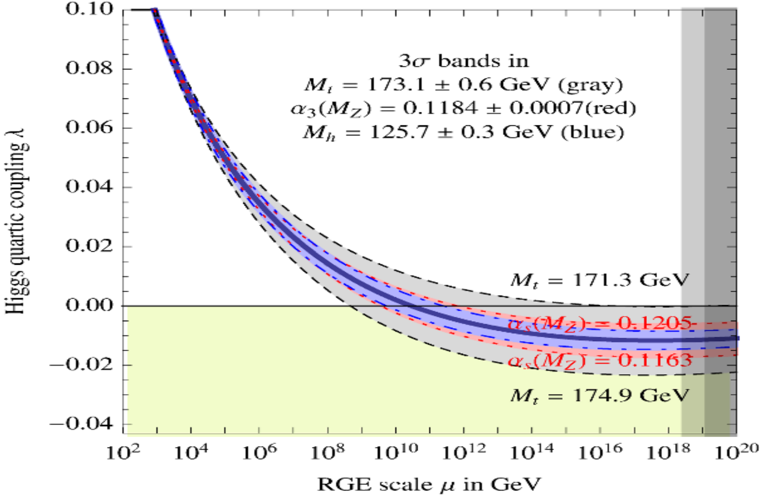


n_s vs. r for radiatively corrected ϕ^2 potential. The dashed portions are for $\kappa < 0$. The one loop radiative correction is larger than the tree level potential in the portions displayed in gray. N is taken as 50 (left curves) and 60 (right curves).

Standard Model Higgs Inflation?

Update of RGE analysis (@ 3-loop level)

Buttazzo et al., JHEP 12 (2013) 089

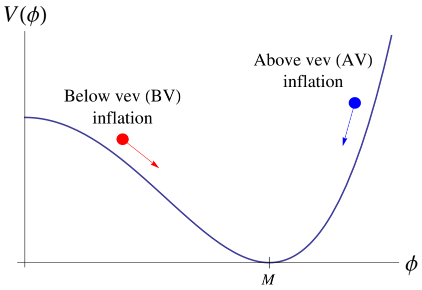


Tree Level Gauge Singlet Higgs Inflation

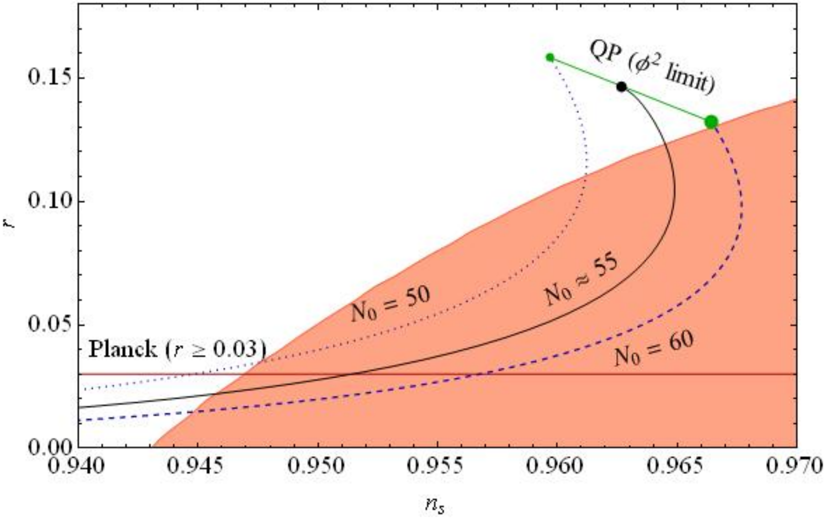
- Consider the following Higgs Potential:

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 \quad \longleftarrow \text{(tree level)}$$

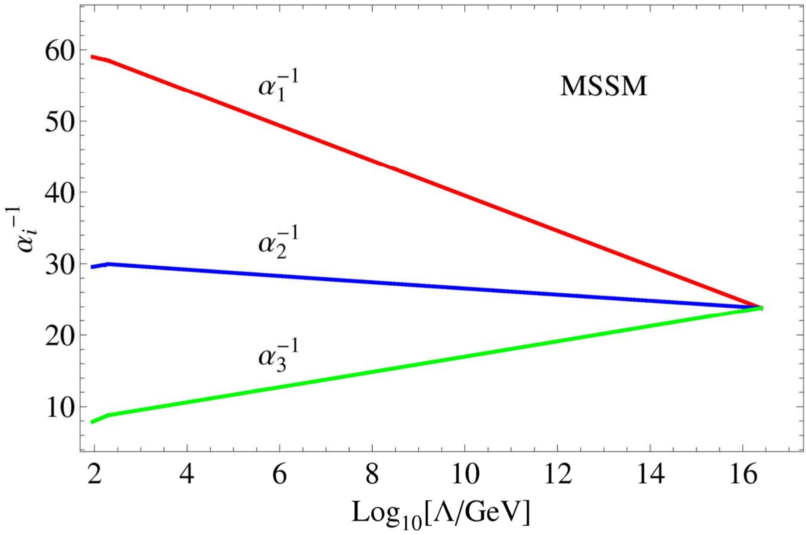
Here ϕ is a gauge singlet field.

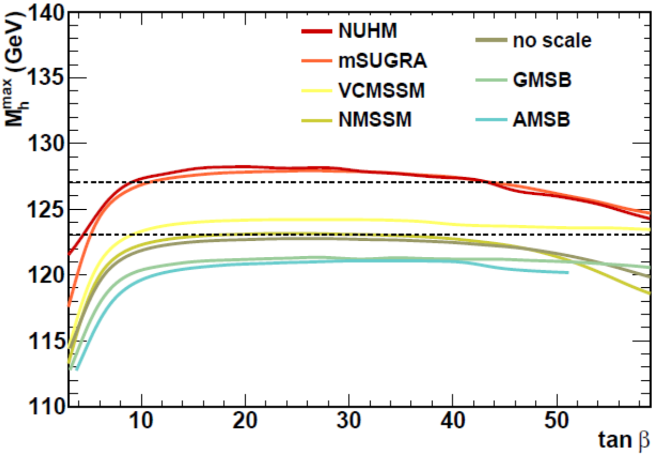


- WMAP/Planck data favors BV inflation


QP (ϕ^2 limit)
$N_0 = 50$
$N_0 \approx 55$
$N_0 = 60$
Planck ($r \geq 0.03$)
0.15
0.10
0.05
0.00
r
0.940
0.945
0.950
0.955
0.960
0.965
0.970
n_s

Supersymmetry





A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, Phys. Lett. B **708**, 162 (2012)

Supersymmetric Higgs (Hybrid) Inflation

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- Ground State

$$|\langle \Phi \rangle| = M, \;\; \langle S \rangle = 0$$

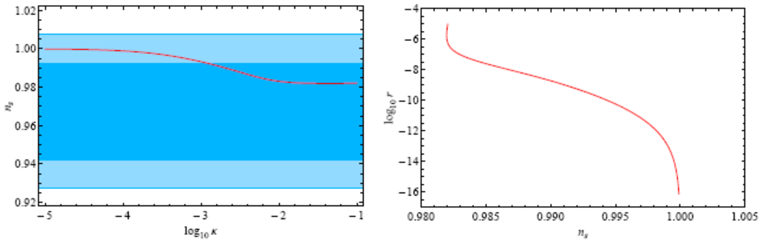
Cf: Superconductor, $\langle \Phi \rangle \to$ cooper pair, $\langle S \rangle \to$ temperature

- To realize inflation

$$S \gg M \text{ in early universe } (T \gg T_c)$$

$\Rightarrow$ At tree level, $V \approx \kappa^2 M^4 \quad \Rightarrow \quad$ exponential expansion

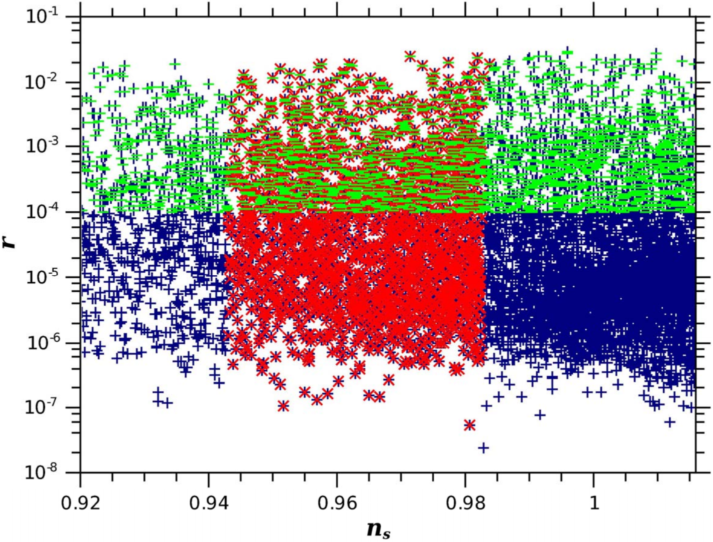
Tree Level plus radiative corrections:



$$n_s \approx 1 - \frac{1}{N_0} \approx 0.98$$

$\delta T/T \propto (M/M_P)^2 \sim 10^{-5} \longrightarrow$ attractive scenario ($M \sim M_G$)

More complete analysis:



Dark Matter Candidates

Candidates includes:

- WIMP (weakly interaction massive ($10^2 - 10^3$ GeV) particle)
- Axions – very light (~10^{-5} eV), very weakly interaction particle
- Wimpzilla – very massive (10^{12} GeV), perhaps not entirely stable, particle
- Gravitino – keV mass partner of graviton; behaves as ‘warm’ dark matter?

WIMP Candidates (10^2 - 10^3 GeV in mass)

- Neutralino (neutral, spin ½, stable, light supersymmetric particle)

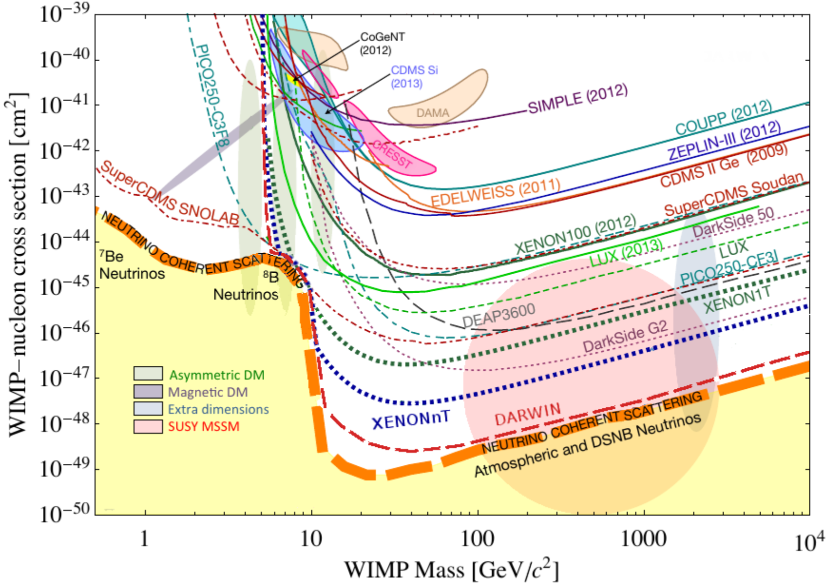
- Lightest neutral Kaluza Klein particle

(E.g. KK excitation of some suitable known particle)

- Dark (mirror/hidden universe) baryons

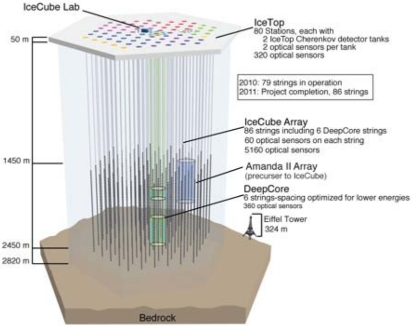
SUSY DM CANDIDATES

Spin	U(1) M_1	SU(2) M_2	Up-type μ	Down-type μ	$m_{\tilde{\nu}}$	$m_{3/2}$
2						G graviton
3/2		Neutralinos: $\{\chi \equiv \chi_1, \chi_2, \chi_3, \chi_4\}$				$\tilde{G}$ gravitino
1	B	W^0				
1/2	$\tilde{B}$ Bino	$\tilde{W}^0$ Wino	$\tilde{H}_u$ Higgsino	$\tilde{H}_d$ Higgsino	ν	
0			H_u	H_d	$\tilde{\nu}$ sneutrino	



WIMP–nucleon cross section [cm²]
WIMP Mass [GeV/c²]
CoGeNT (2012)
CDMS Si (2013)
DAMA
SIMPLE (2012)
COUPP (2012)
ZEPLIN-III (2012)
CDMS II Ge (2009)
CRESST
EDELWEISS (2011)
SuperCDMS Soudan
SuperCDMS SNOLAB
PICO250-C3F8
XENON100 (2012)
LUX (2013)
DarkSide 50
LUX
PICO250-CF3I
XENON1T
DEAP3600
DarkSide G2
XENONnT
DARWIN
NEUTRINO COHERENT SCATTERING
Atmospheric and DSNB Neutrinos
⁷Be Neutrinos
⁸B Neutrinos
Asymmetric DM
Magnetic DM
Extra dimensions
SUSY MSSM

Indirect Search



The predictions of r (primordial gravity waves) for various inflation models:

1. Gauge Singlet Higgs Inflation:

$r \geq 0.02$ for $n_s \geq 0.96$

2. SM Higgs Inflation:

$r \sim 0.003$, $n_s \sim 0.968$

3. Non-Minimal ϕ^4 Inflation:

$r \geq 0.002$ for $n_s \geq 0.96$

4. Dark Matter Inflation:

$0.003 \leq r \leq 0.007$

5. MSSM Inflation:

$r \sim 10^{-16}$ with $0.93 \leq n_s \leq 1$

6. Susy Higgs (Hybrid) Inflation:

$r \leq 10^{-4}$ (minimal), $r \leq 0.03$ (non-minimal)

Planck (2015) says that $r < 0.09$

Summary

Many Challenges:

- Dark Matter
- Supersymmetry
- Gravity Waves
- Neutrino Physics
- Proton Decay
- Dark Energy