Problems in human motion planning

Brian Skinner

26 February 2015

Argonne NATIONAL LABORATORY

Part 1: The interaction law between pedestrians

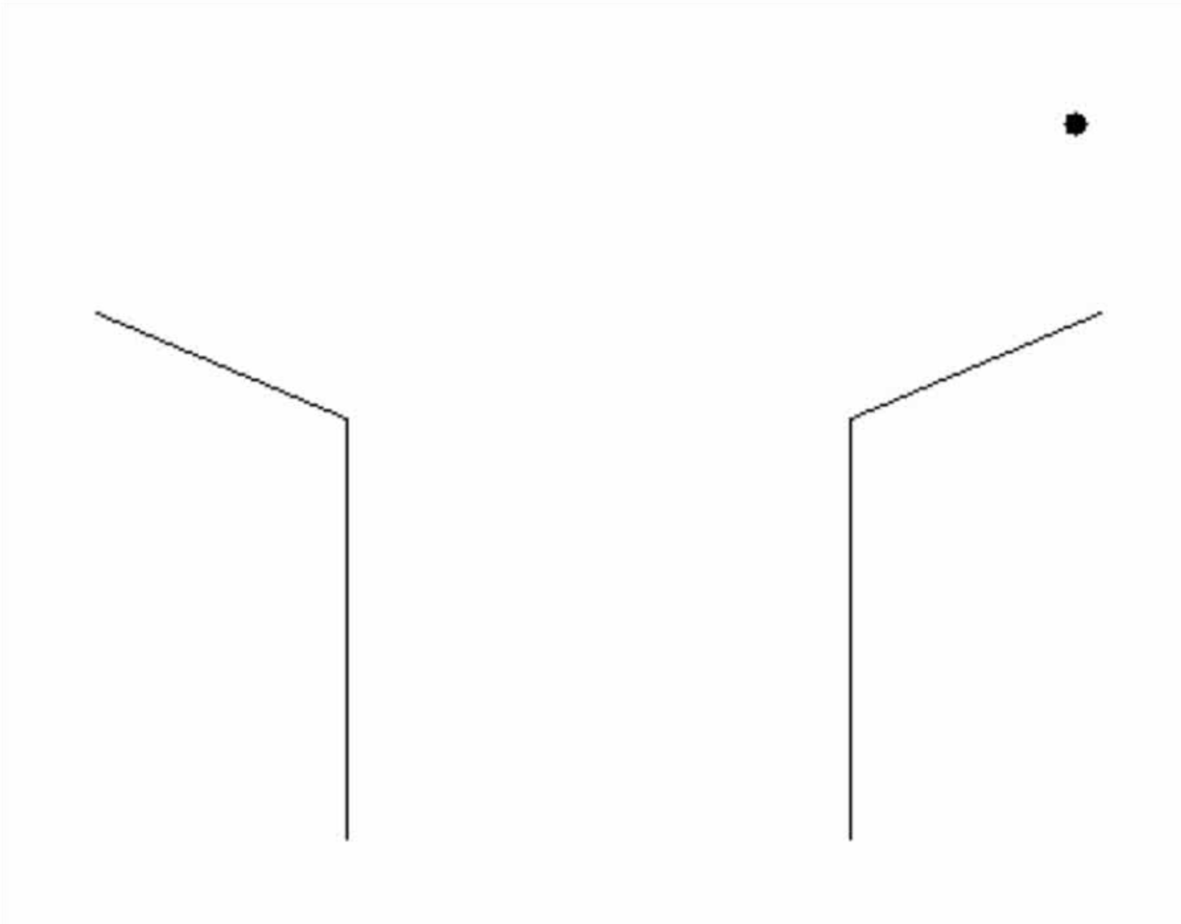
Computer Science & Engineering

Applied Motion Lab

Ioannis Karamouzas

Stephen J. Guy

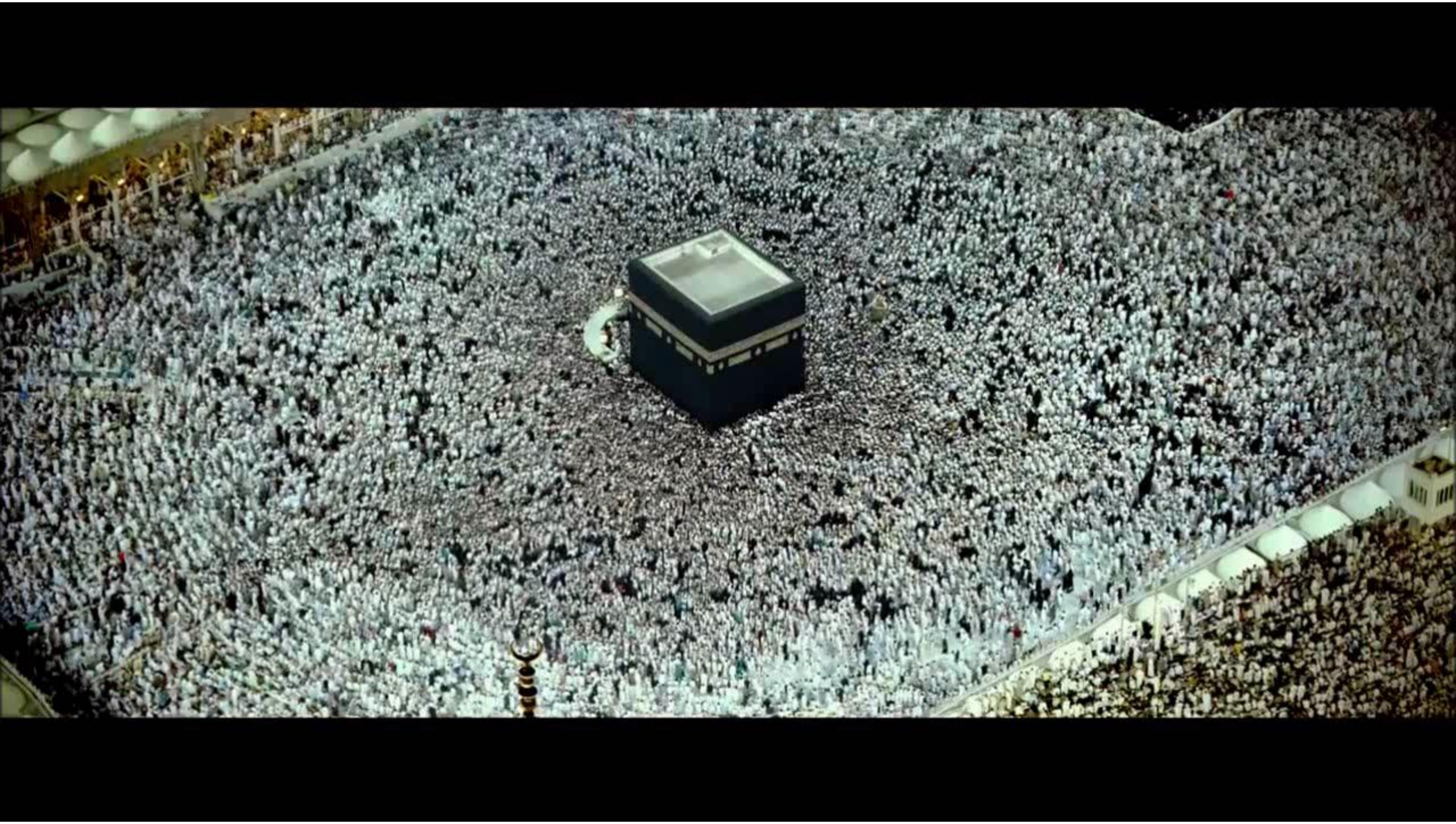
What is this?



People!

A. Seyfried, O. Passon, B. Steffen, M. Boltes, T. Rupprecht and W. Klingsch *New insights into pedestrian flow through bottlenecks* arXiv:physics/0702004

Human "particle systems" on a large scale

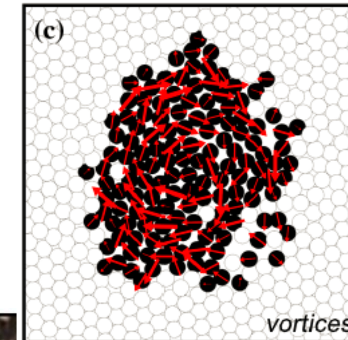


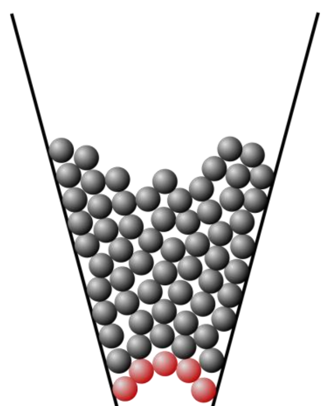
Human "particle systems" on a large scale

Emergent "particle" behaviors in crowds:

- compression waves
- vortices
- "fingering instability"
- jamming transitions

How seriously can these similarities be taken?





The "social force" model

PHYSICAL REVIEW E VOLUME 51, NUMBER 5 MAY 1995

Social force model for pedestrian dynamics

Dirk Helbing and Péter Molnár
II. Institute of Theoretical Physics, University of Stuttgart, 70550 Stuttgart, Germany
(Received 14 April 1994; revised manuscript received 5 January 1995)

An overdamped "goal force" that pulls pedestrians to their goal:

$$\overrightarrow{F_g} = \frac{1}{\tau}\left(\overrightarrow{v_g} - \overrightarrow{v_i}\right)$$

and a repulsive "social force" that keeps pedestrians from colliding:

$$\overrightarrow{F}_{ij} = -\nabla_{r_{ij}} V(r_{ij})$$

What is the interaction law V?

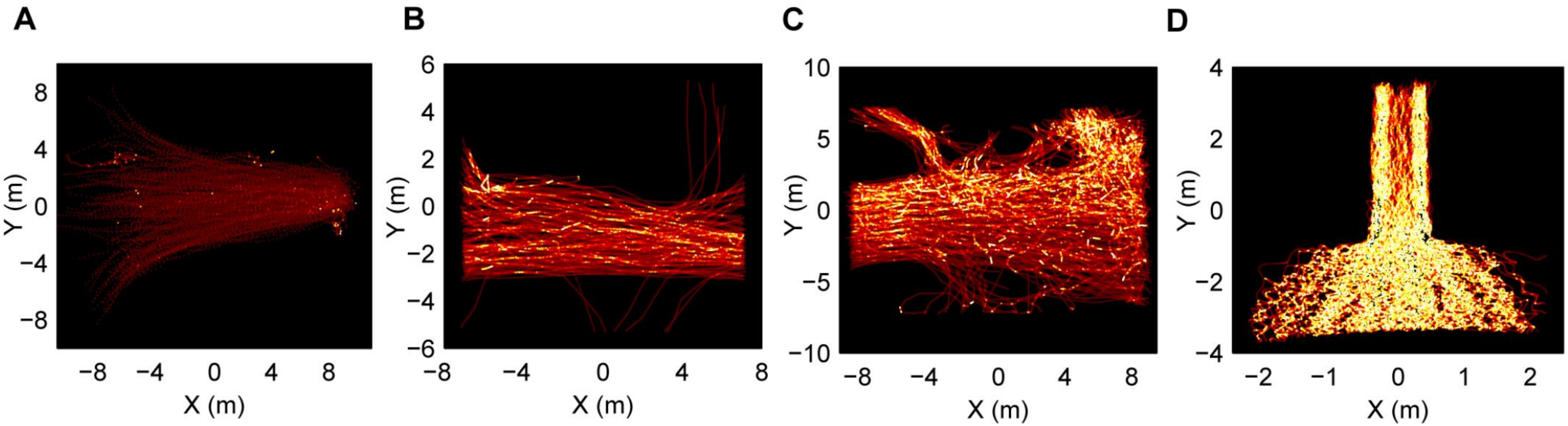
Helbing and Molnar's guess:

$$V = V_0 e^{-r/R}$$

...the literature has many more "guesses"

Can we *measure* the pedestrian interaction law?

Start with data:



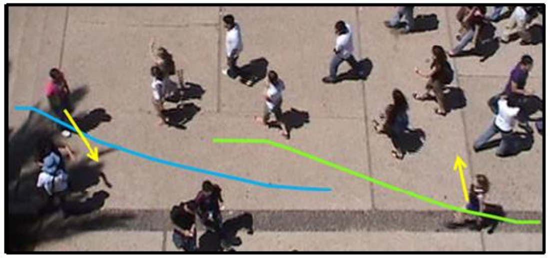
college campus, sparse unidirectional

college campus, sparse bidirectional

college campus, moderate multidirectional

controlled experiment, dense unidirectional

Correlating acceleration with relative position is too hard:

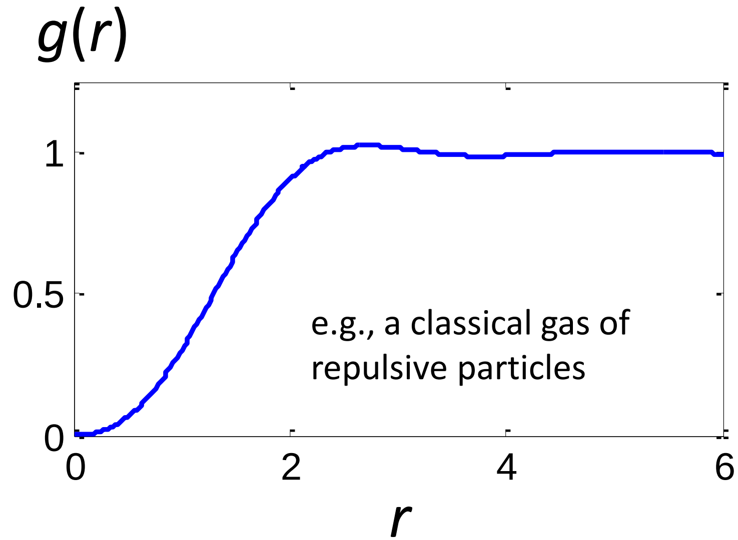


...try a probabilistic description

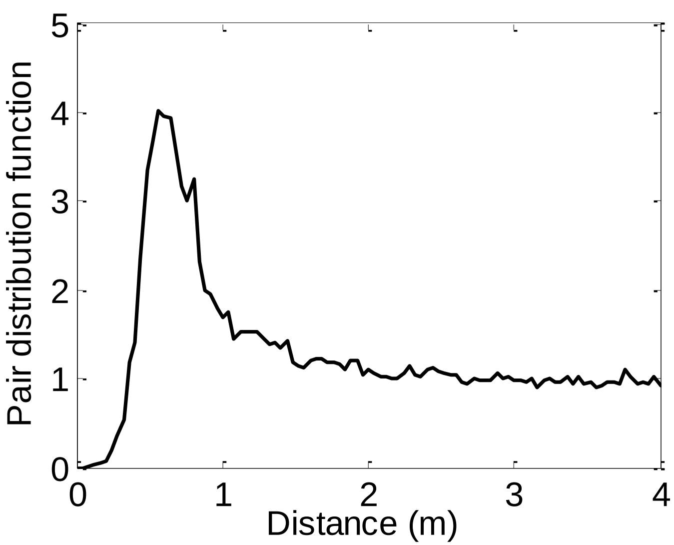
Pair distribution function

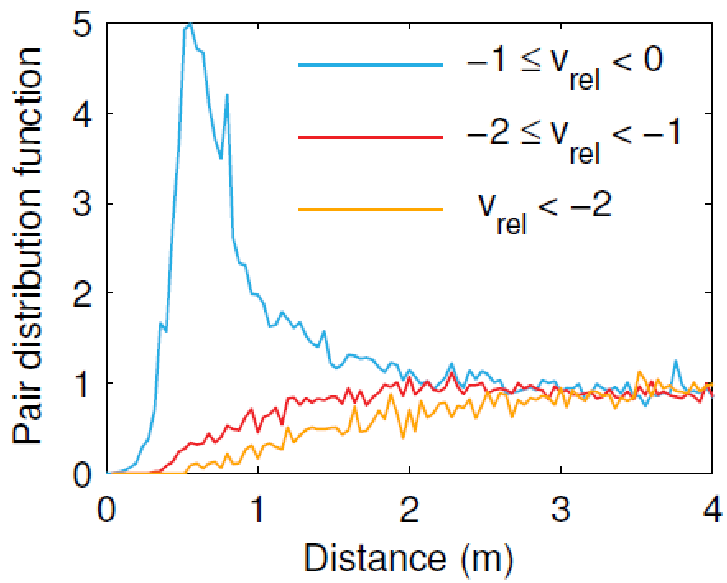
Look for statistical suppression of certain configurations:

$$g(r) = \frac{\text{Prob. density of pair separation } r}{\text{Prob. density of } r \text{ for non-interacting particles}}$$



Result (from "natural" settings):

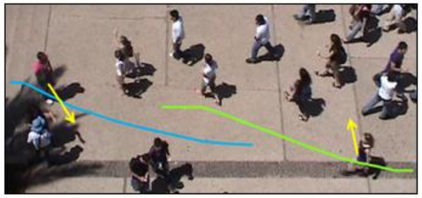




Interaction depends on relative velocity!

Anticipatory interaction

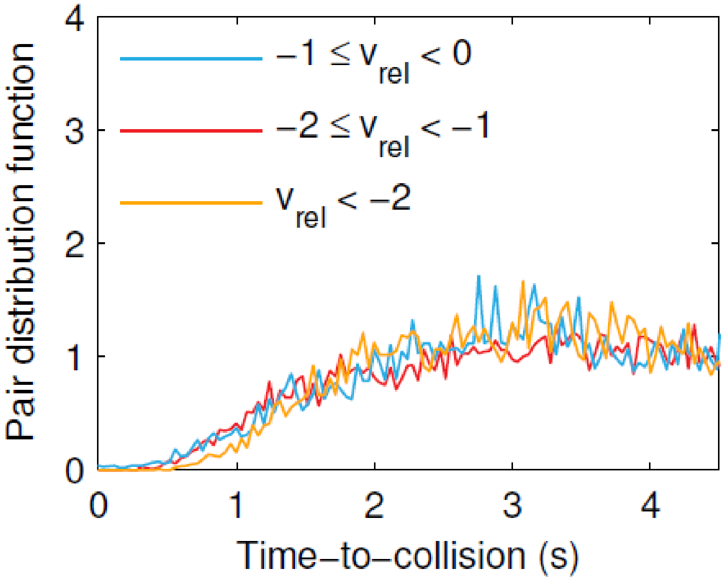
Interaction between people is influenced by *anticipation* effects:

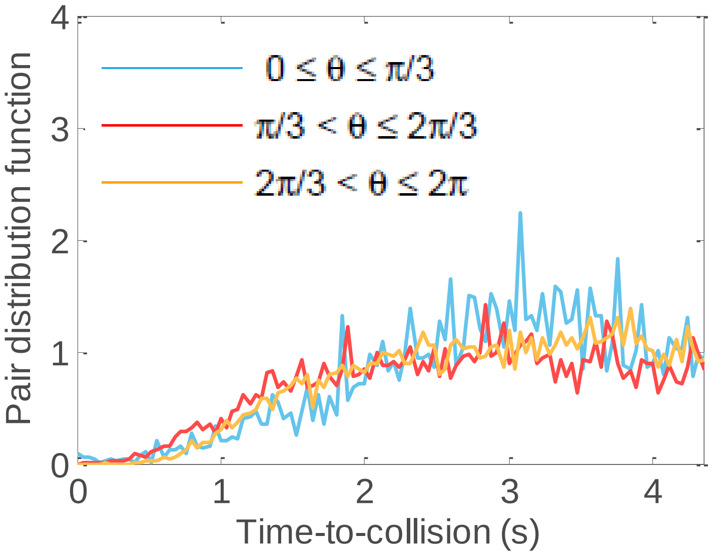


noticeable acceleration when approaching head-on, even at large separation

no acceleration when walking side-by-side, even at small separation

Define τ = projected time to collision





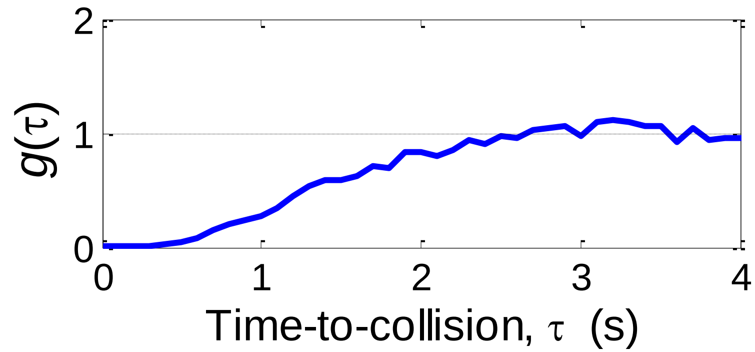
Interaction is a function of τ only!

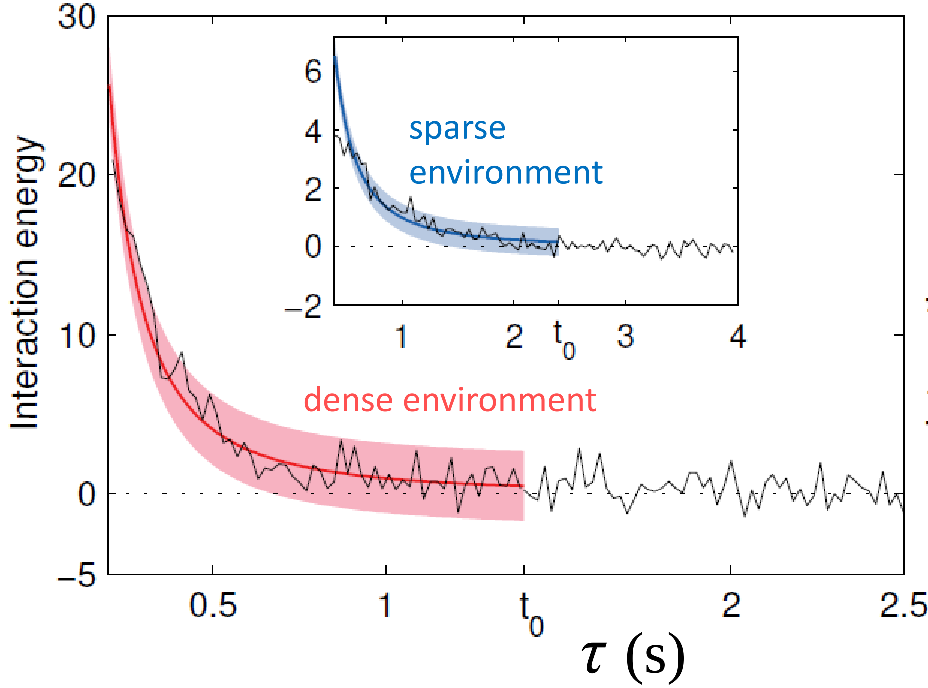
The interaction “energy”

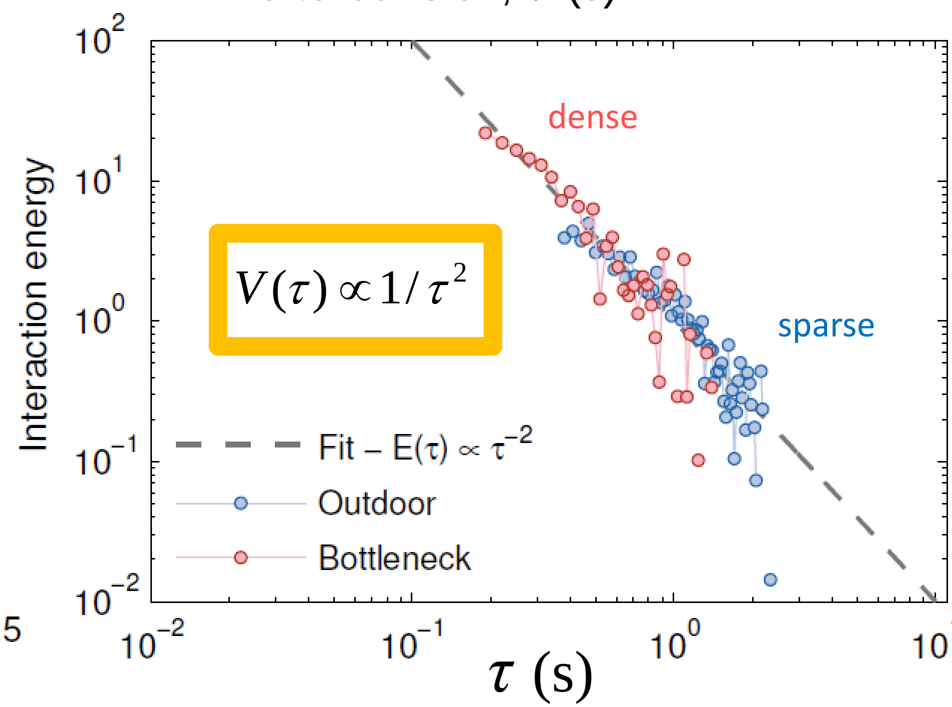
Define a Boltzmann factor: $$g(\tau) \propto \exp[-V(\tau)/\text{"}k_B T\text{"}]$$

At small τ, pair interaction produce a strong suppression of $g(\tau)$

$$V(\tau) \propto \ln[1/g(\tau)]$$





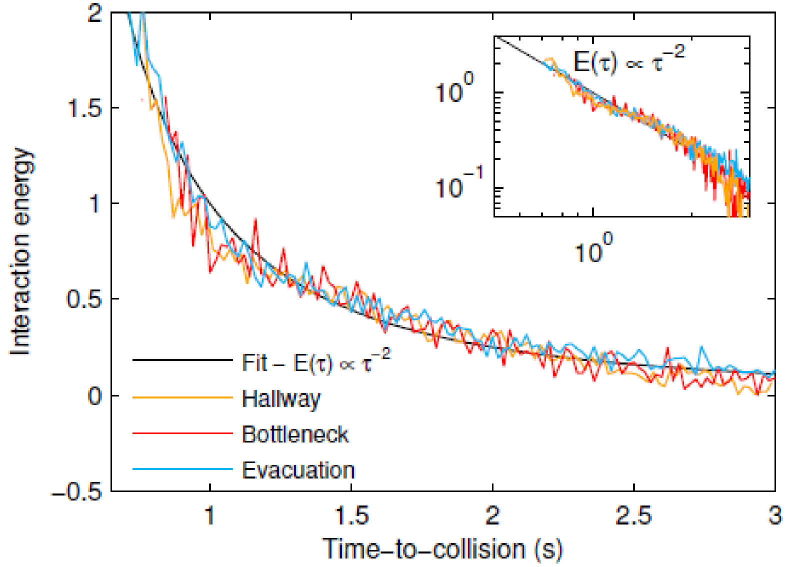


Simulating pedestrians

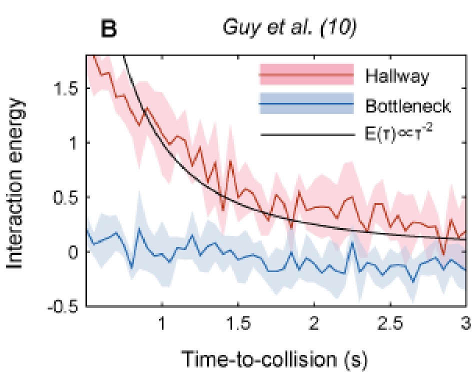
Natural choice for simulating dynamics:

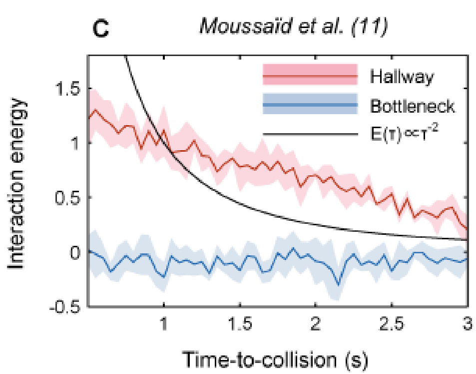
$$\vec{F} = -\vec{\nabla} U$$

Simulation reproduces statistical distributions:



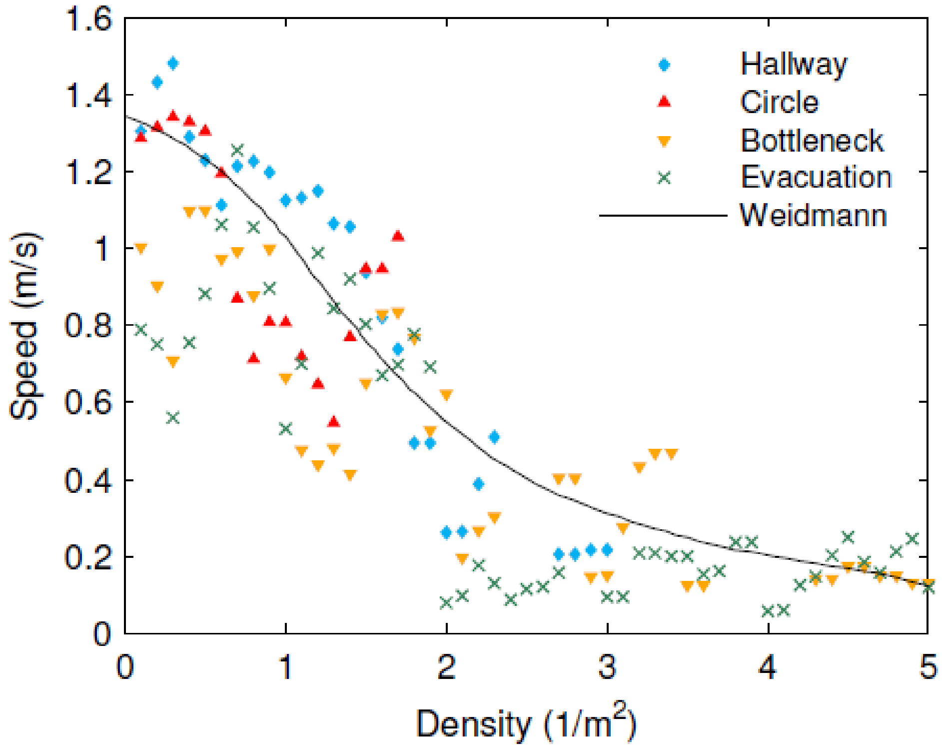
...other methods do not:





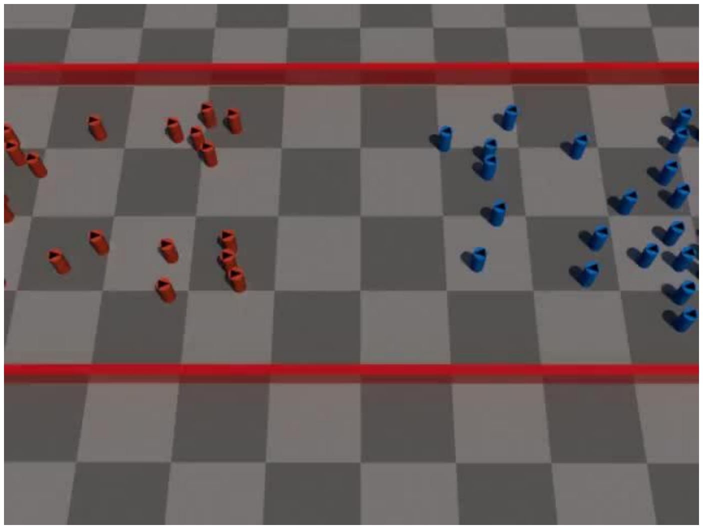
Simulating pedestrians

Reproduces known relationship between pedestrian density and speed:

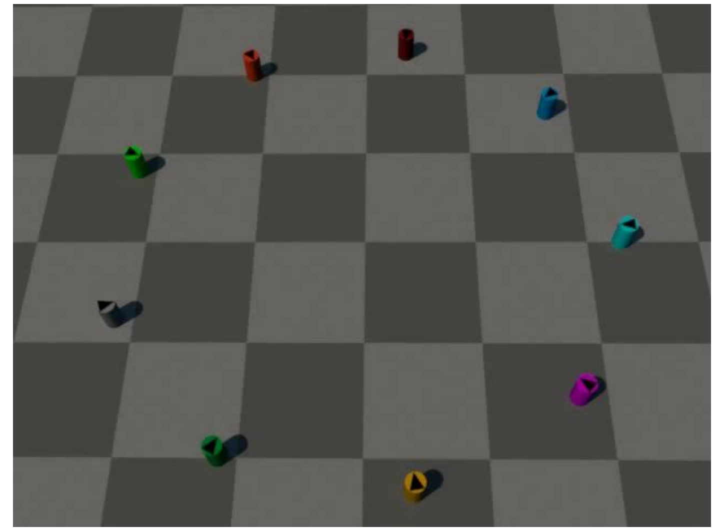


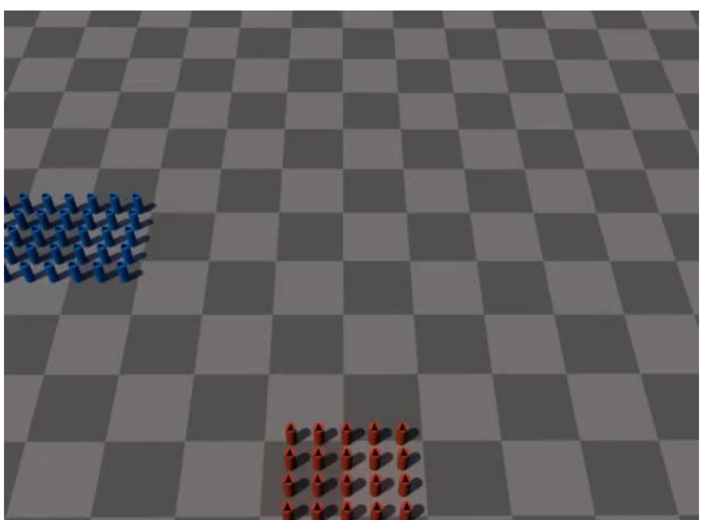
Simulations:

Lane formation:

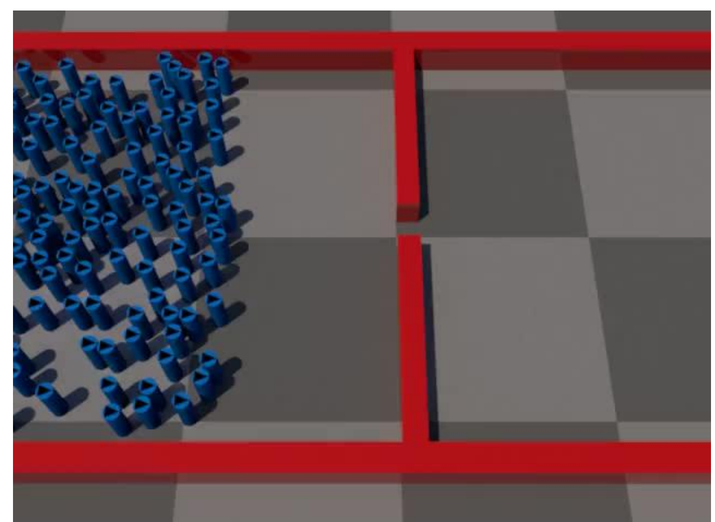


vortices:



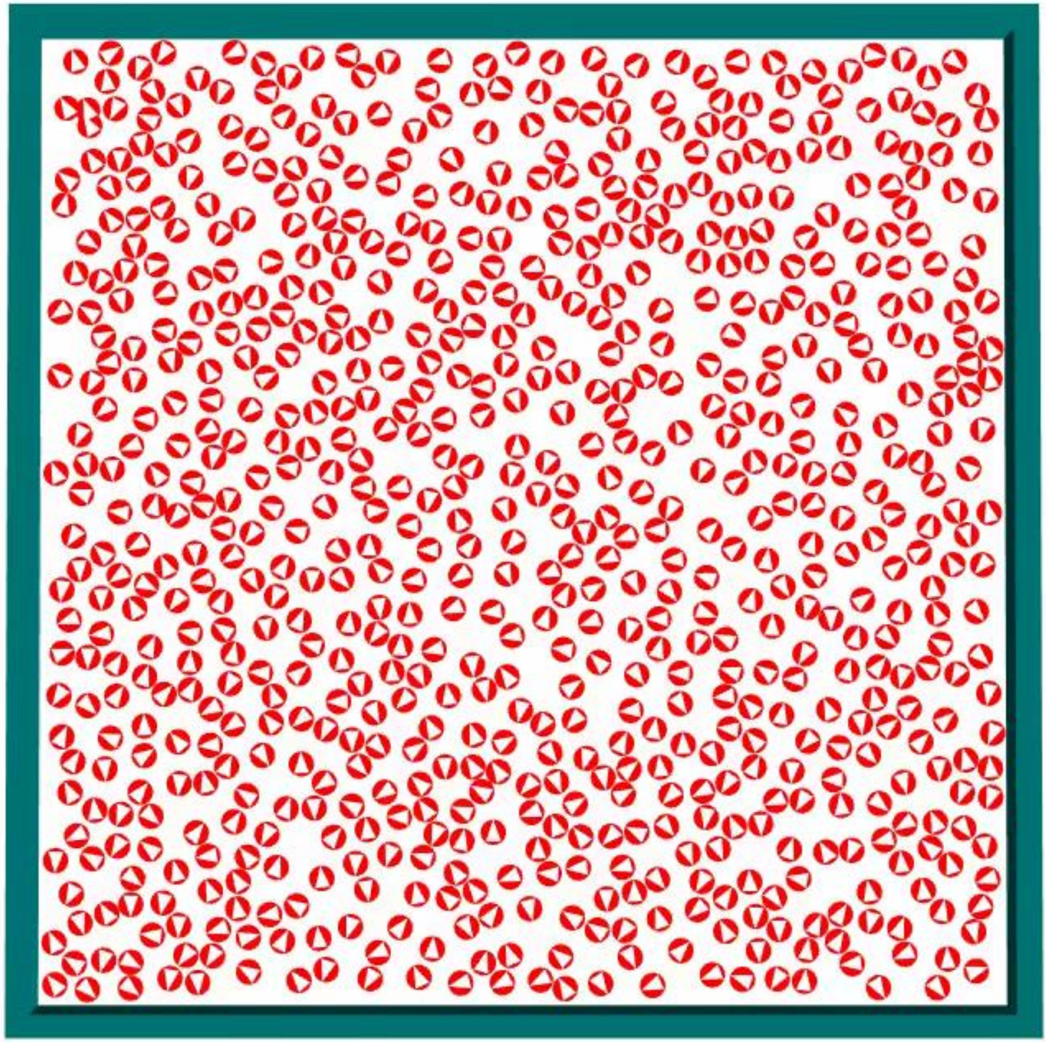


arching:



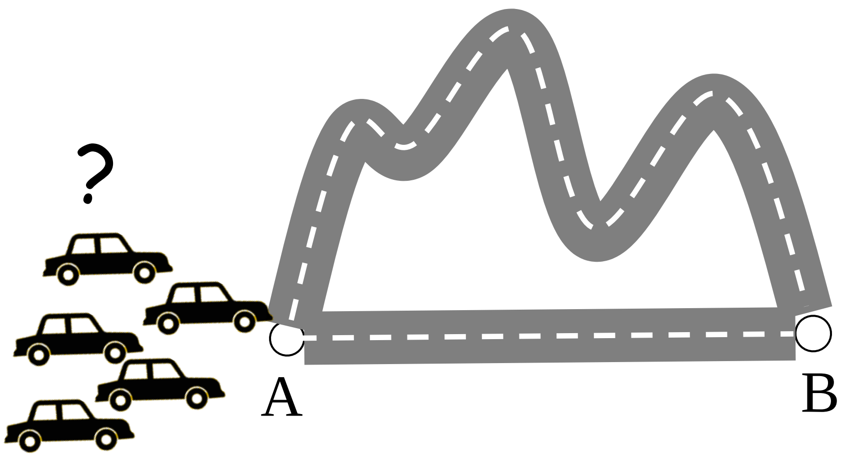
Flocking

What if pedestrians have no “goal force”, but only a preferred walking speed?



...Also represents a fast algorithm for large-scale crowd simulation

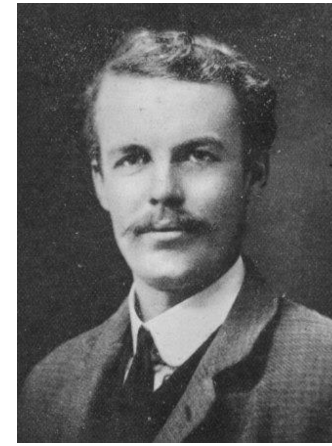
Part 2: The Price of Anarchy in congestible networks



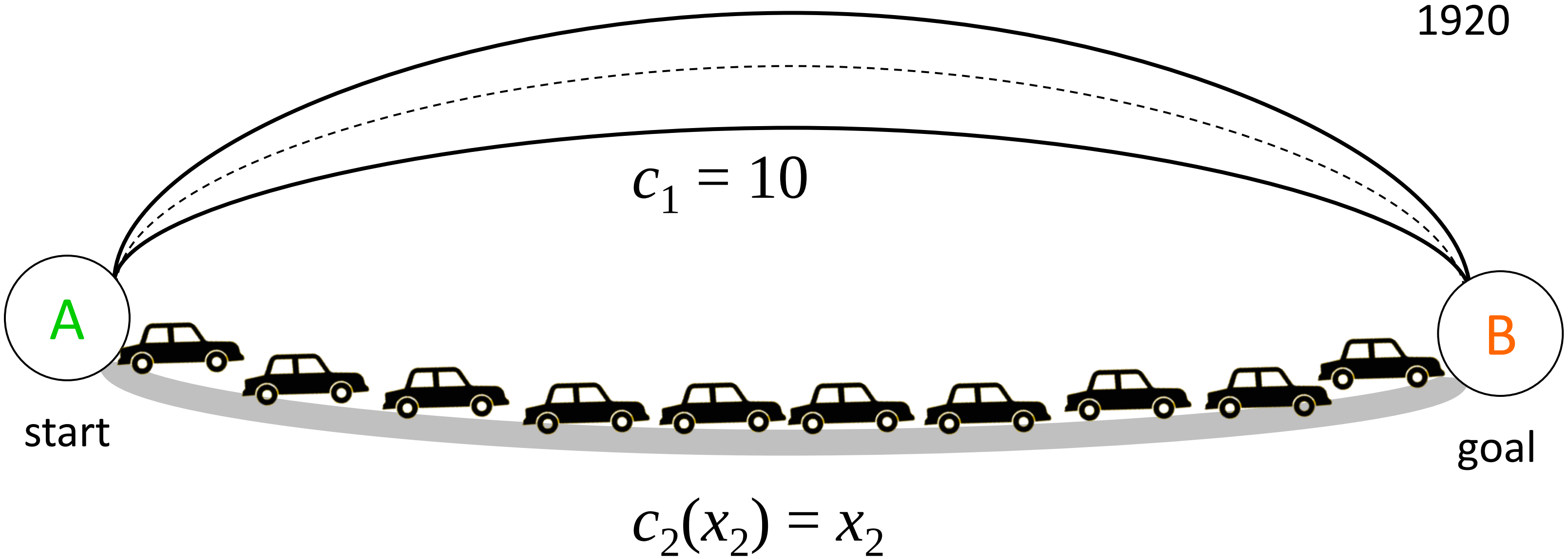
How do we choose between discrete paths when the transit time depends on what other people are choosing?

How efficient are our choices?

Pigou's example

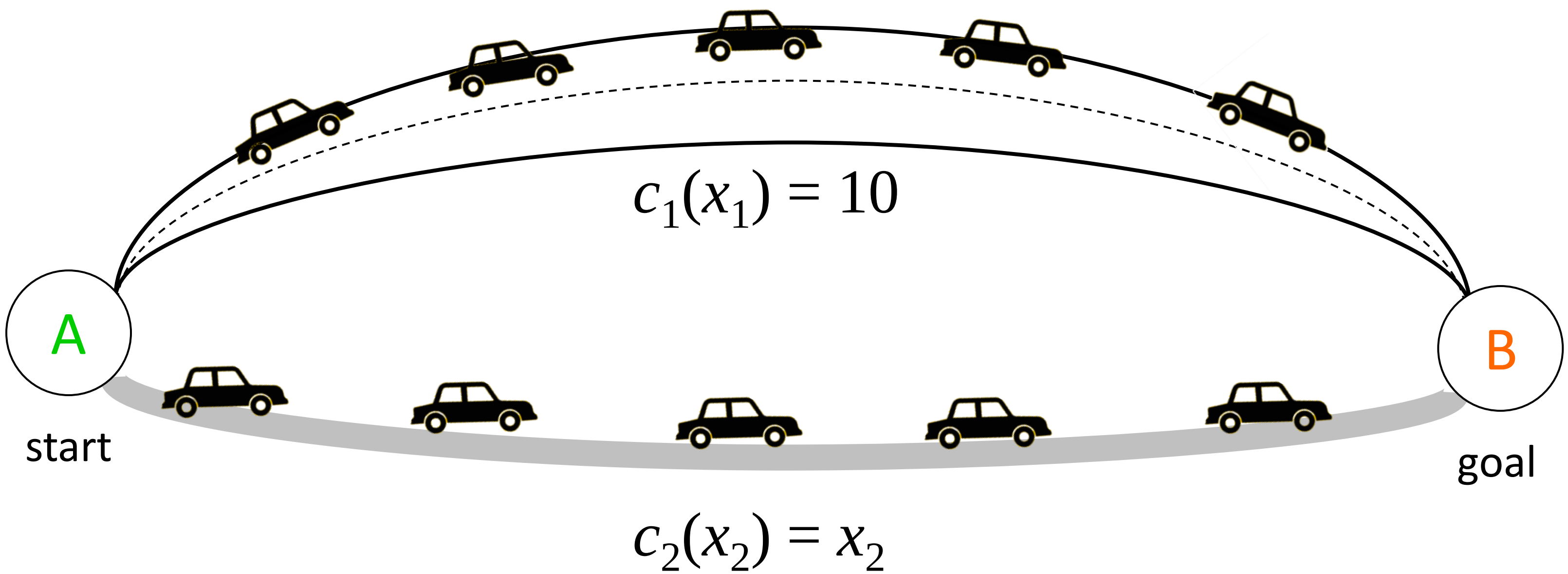


Arthur Pigou,
1920



"Nash Equilibrium" : $\langle C \rangle = 10$

The "price of anarchy"



How do you optimize the performance of the network?

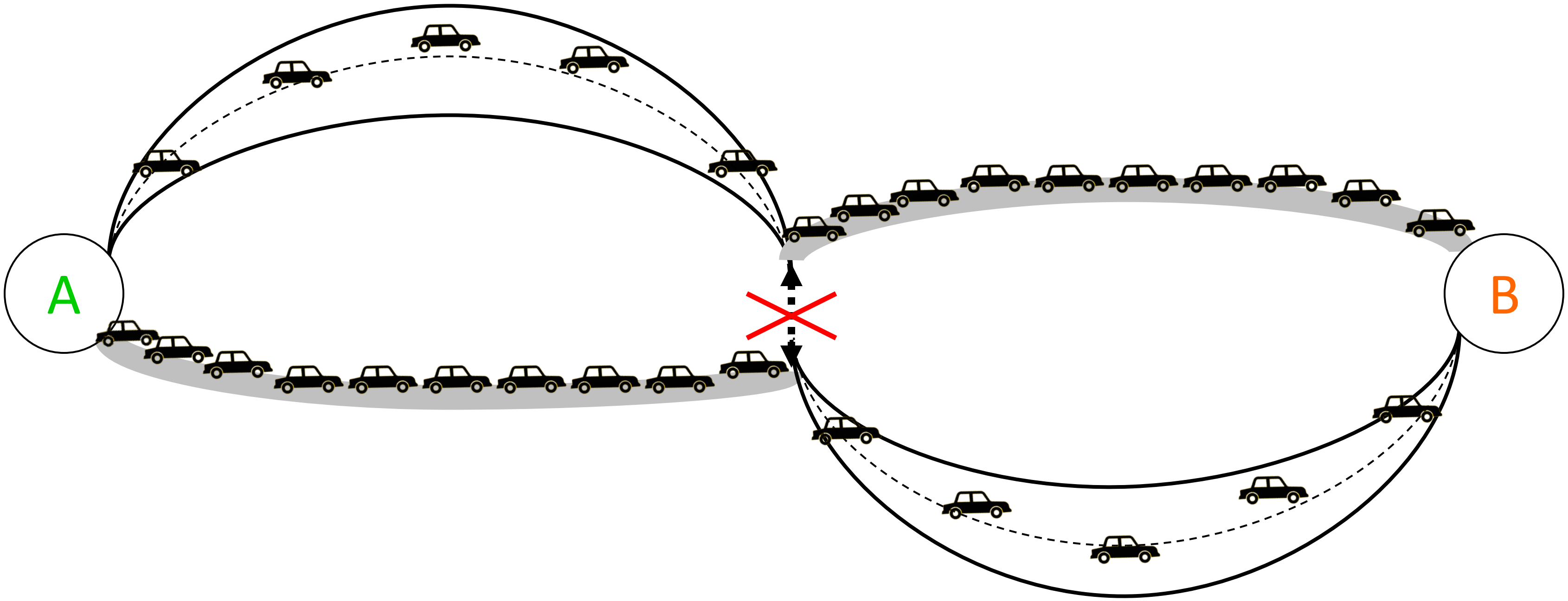
Look for the minimum of

$$\langle C \rangle = \frac{x_1 c_1(x_1) + x_2 c_2(x_2)}{10}$$

$$\langle C \rangle_{opt} = 7.5$$

"Price of Anarchy": 2.5 minutes = 33%

Braess's Paradox



Nash Equilibrium: $\langle C \rangle_{NE} = 20$

True optimium: $\langle C \rangle_{opt} = 15$

Traffic can *improve* when a road is closed

What if They Closed 42d Street and Nobody Noticed?

By GINA KOLATA
Published: December 25, 1990

ON Earth Day this year, New York City's Transportation Commissioner decided to close 42d Street, which as every New Yorker knows is always congested. "Many predicted it would be doomsday," said the Commissioner, Lucius J. Riccio. "You didn't need to be a rocket scientist or have a sophisticated computer queuing model to see that this could have been a major problem."

EMAIL
PRINT

But to everyone's surprise, Earth Day generated no historic traffic jam. Traffic flow actually improved when 42d Street was closed.

San Francisco:

Seoul:

On the road:

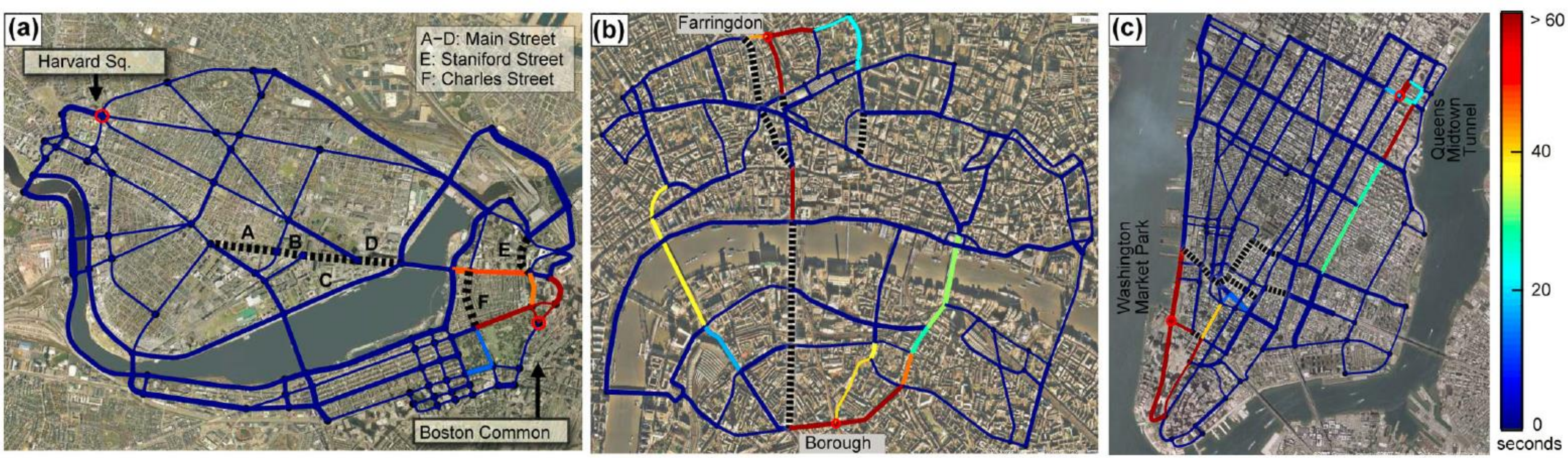
PRL **101,** 128701 (2008) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2008

Price of Anarchy in Transportation Networks: Efficiency and Optimality Control

Hyejin Youn,[1] Michael T. Gastner,[2,3] and Hawoong Jeong[1,*]

[1] *Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea*
[2] *Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
[3] *Department of Computer Science, University of New Mexico, Albuquerque, New Mexico 87131, USA*
(Received 3 January 2008; published 17 September 2008)

Uncoordinated individuals in human society pursuing their personally optimal strategies do not always achieve the social optimum, the most beneficial state to the society as a whole. Instead, strategies form Nash equilibria which are often socially suboptimal. Society, therefore, has to pay a *price of anarchy* for the lack of coordination among its members. Here we assess this price of anarchy by analyzing the travel times in road networks of several major cities. Our simulation shows that uncoordinated drivers possibly waste a considerable amount of their travel time. Counterintuitively, simply blocking certain streets can partially improve the traffic conditions. We analyze various complex networks and discuss the possibility of similar paradoxes in physics.



In computer networks:

Selfish Routing and the Price of Anarchy

Tim Roughgarden*

January 7, 2006

*Department of Computer Science, Stanford University, 462 Gates Building, 353 Serra Mall, Stanford, CA 94305. Supported in part by ONR grant N00014-04-1-0725, DARPA grant W911NF-04-9-0001, and an NSF CAREER Award. Email: tim@cs.stanford.edu.

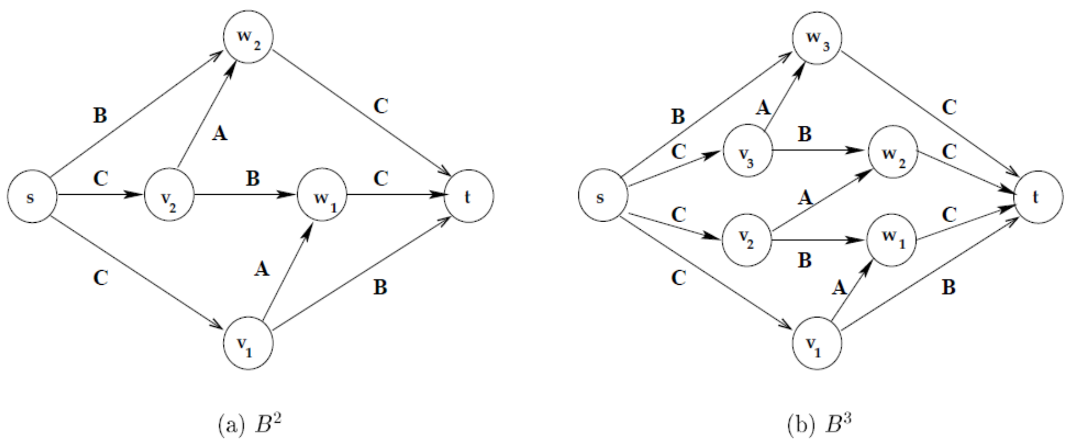


Figure 4: The second and third Braess graphs. Edges are labeled with their types.

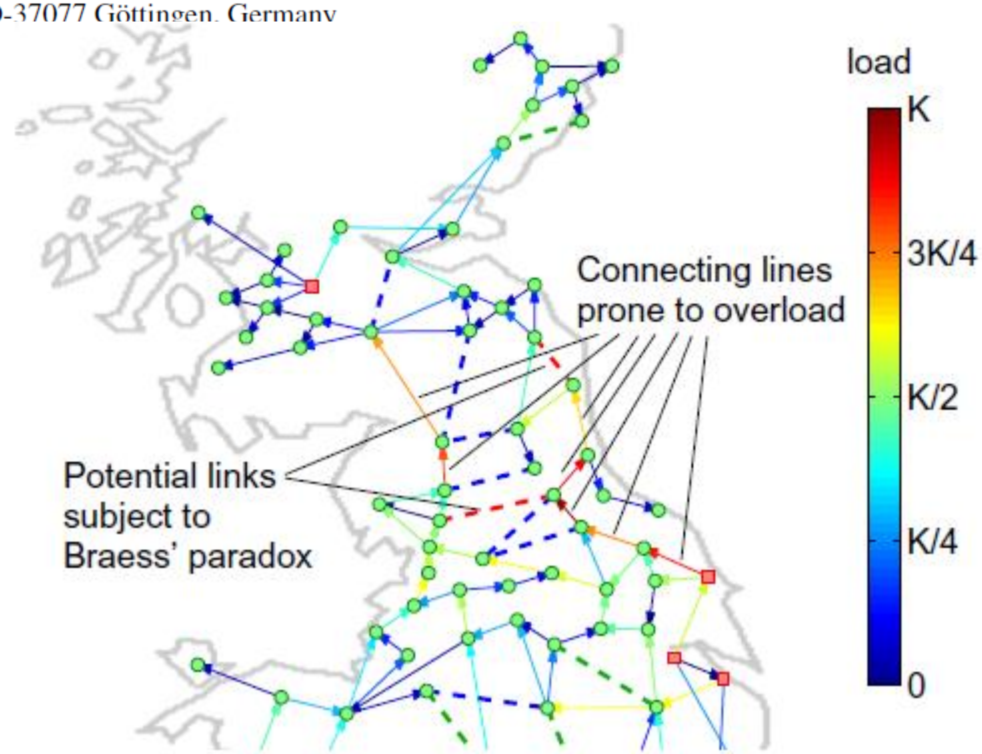
In power transmission:

New Journal of Physics
The open-access journal for physics

Braess's paradox in oscillator networks, desynchronization and power outage

Dirk Witthaut[1,3] **and Marc Timme**[1,2]
[1] Network Dynamics Group, Max Planck Institute for Dynamics and Self-Organization (MPIDS), D-37073 Göttingen, Germany
[2] Faculty of Physics, University of Göttingen, D-37077 Göttingen, Germany
E-mail: witthaut@nld.ds.mpg.de

New Journal of Physics **14** (2012) 083036 (16p
Received 3 June 2012
Published 29 August 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/8/083036



In health care:

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Selfish routing in public services

Vincent A. Knight *, Paul R. Harper

School of Mathematics, Cardiff University, Cardiff, UK

A R T I C L E I N F O

Article history:
Received 8 March 2012
Accepted 2 April 2013
Available online 10 April 2013

Keywords:
Game theory
Queueing theory
Health care
OR in health services

A B S T R A C T

It is well observed that individual behaviour can have an effe
impact of this behaviour on the economic efficiency of publi
we present results concerning the congestion related implica
choosing between facilities. The work presented has importa
level when considering the effect of allowing individuals to
general the introduction of choice in an already inefficient s
ducing choice in a system that copes with demand will hav

Fig. 8. Service nodes (crosses) and demand nodes (flags) in Wales.

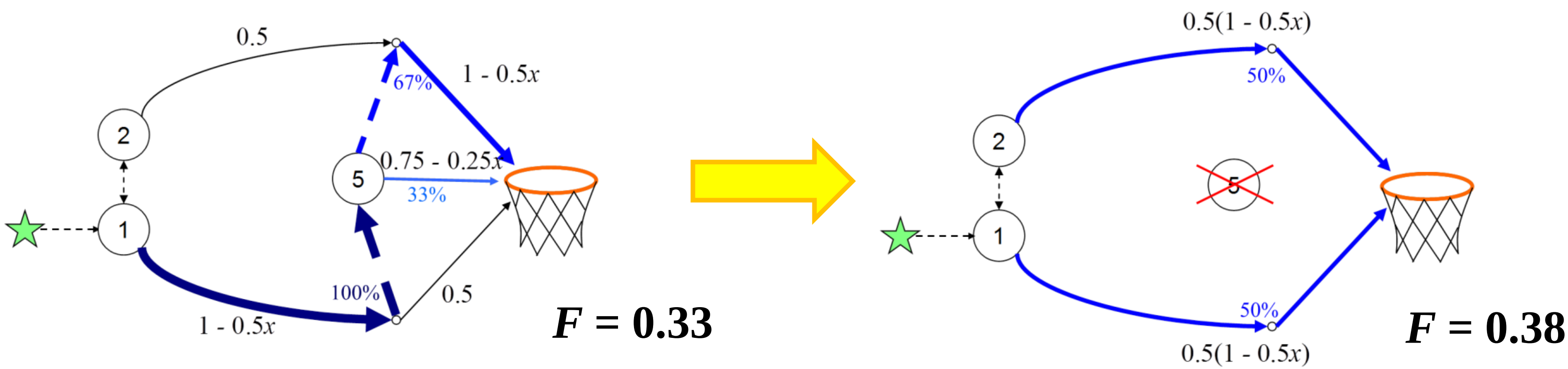
In sports:

Journal of Quantitative Analysis in Sports

Volume 6, *Issue* 1 2010 *Article* 3

The Price of Anarchy in Basketball

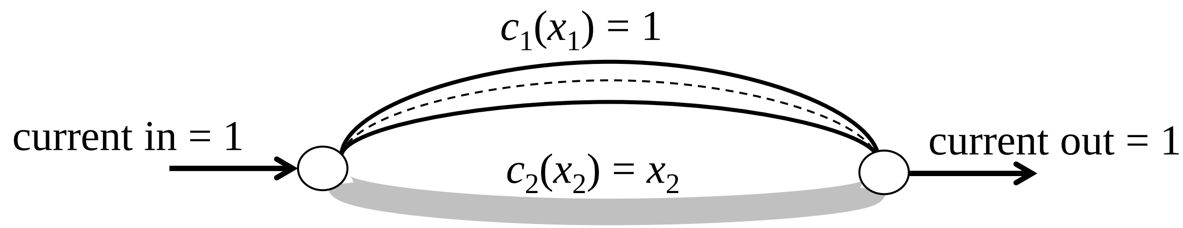
Brian Skinner*



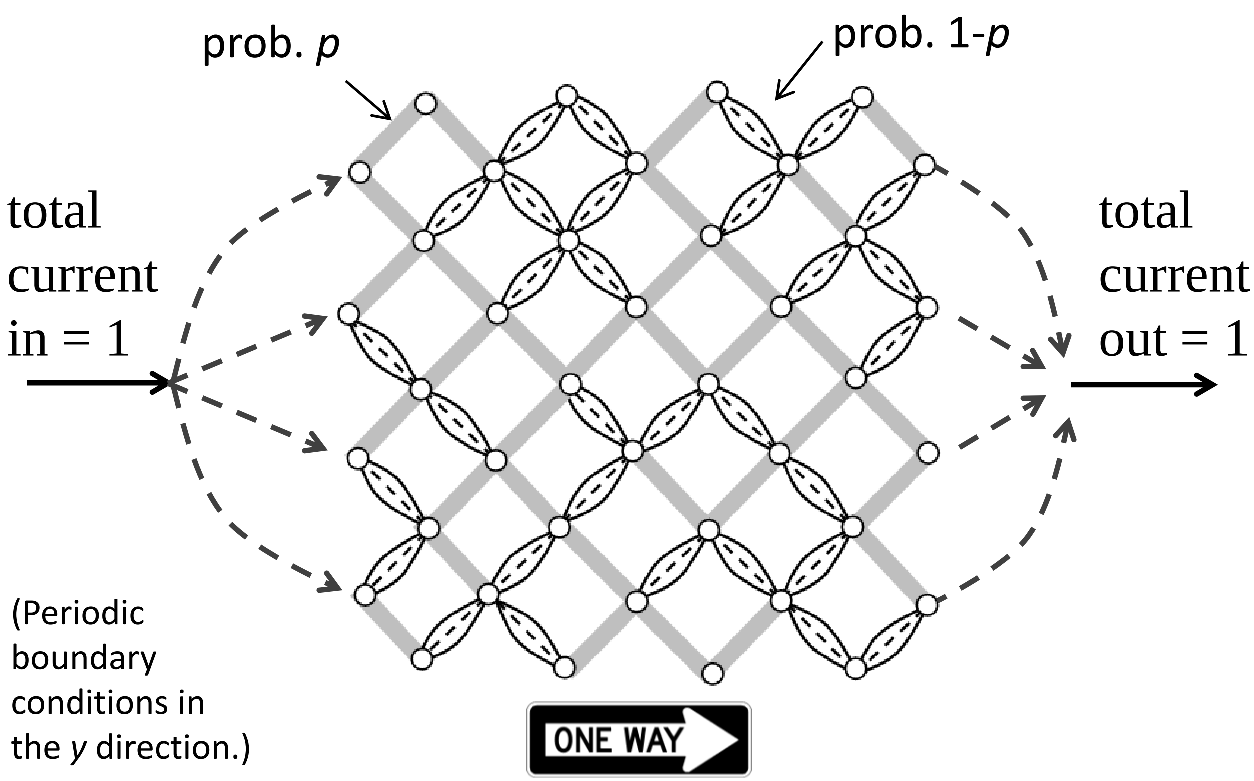
MIT Sloan Sports Analytics Conference
Presented by ESPN

What happens when "congestible" and "incongestible" roads are combined into a lattice?

Pigou's example:



Model:



(Periodic boundary conditions in the *y* direction.)

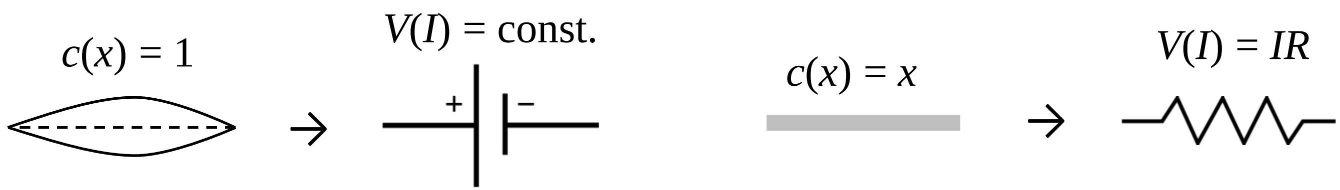
Every current path has the same number of steps.

What is the POA as a function of p?

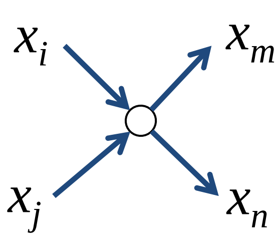
Traffic networks as electrical circuits

Finding the traffic pattern can be mapped onto a problem of electrical circuits:

traffic → current, commute time → voltage drop

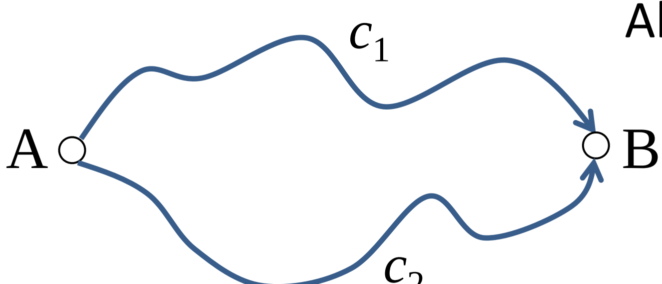


"Kirchoff's Laws":



current in = current out

$$x_i + x_j = x_m + x_n$$



All paths between A and B have the same voltage drop

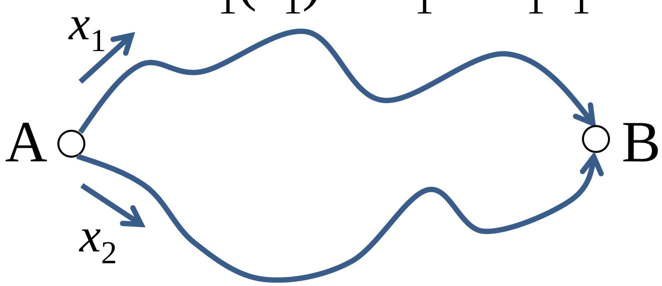
$$c_1 = c_2$$

Solving the circuit produces the *equilibrium* result

Optimum flow in the circuit model

Optimizing commute time across two paths:

$$c_1(x_1) = a_1 + b_1 x_1$$



$$c_2(x_2) = a_2 + b_2 x_2$$

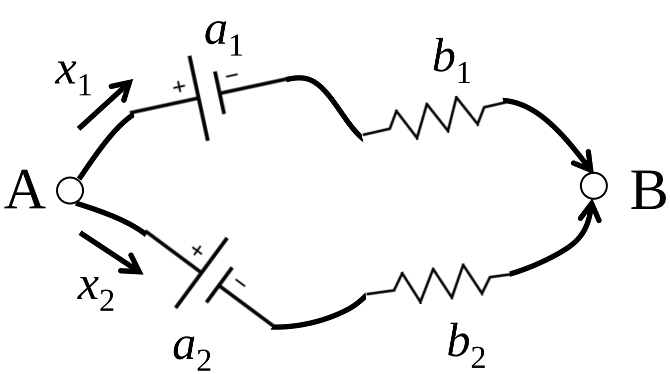
Total commute time:

$$C = x_1\, c_1(x_1) + x_2\, c_2(x_2)$$

Optimize:

$$\frac{\partial C}{\partial x_1} = \frac{\partial C}{\partial x_2} \quad \Rightarrow \quad a_1 + 2b_1 x_1 = a_2 + 2b_2 x_2$$

Circuit analog:



Optimum flow in the circuit model

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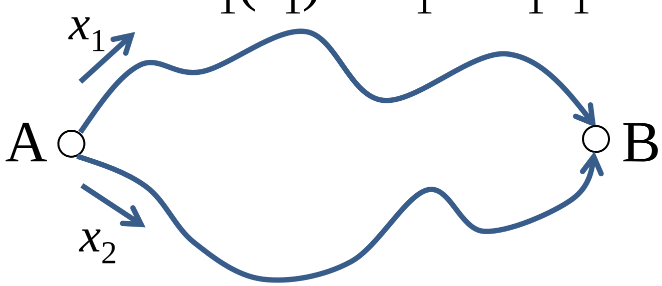

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Optimize:

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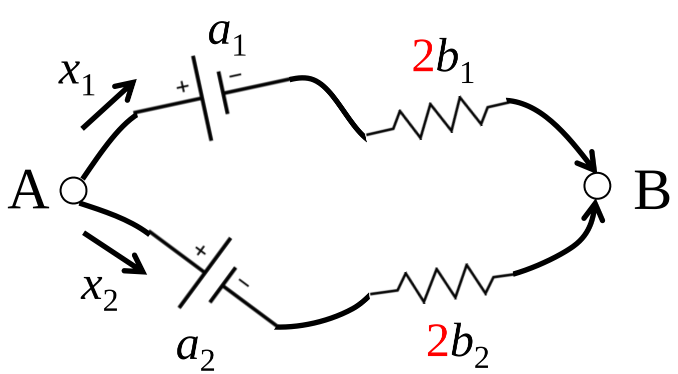
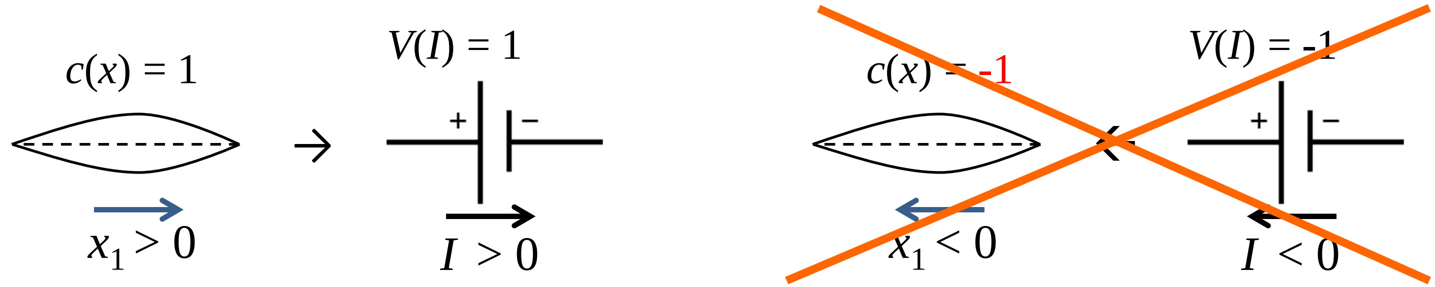
Circuit analog:



Optimal currents arise when “resistance” is doubled.

A voltage-resistor-*diode* circuit

All currents must be positive

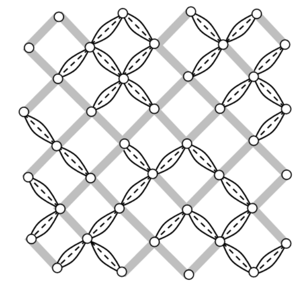


Circuit elements have diodes:

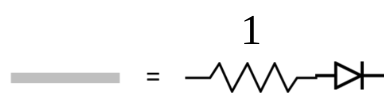
Must find the configuration of each diode that gives a valid solution of Kirchoff's equations.

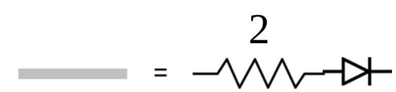
Solution is guaranteed to be unique:
There is only one equilibrium, and one optimum.

Numerical procedure



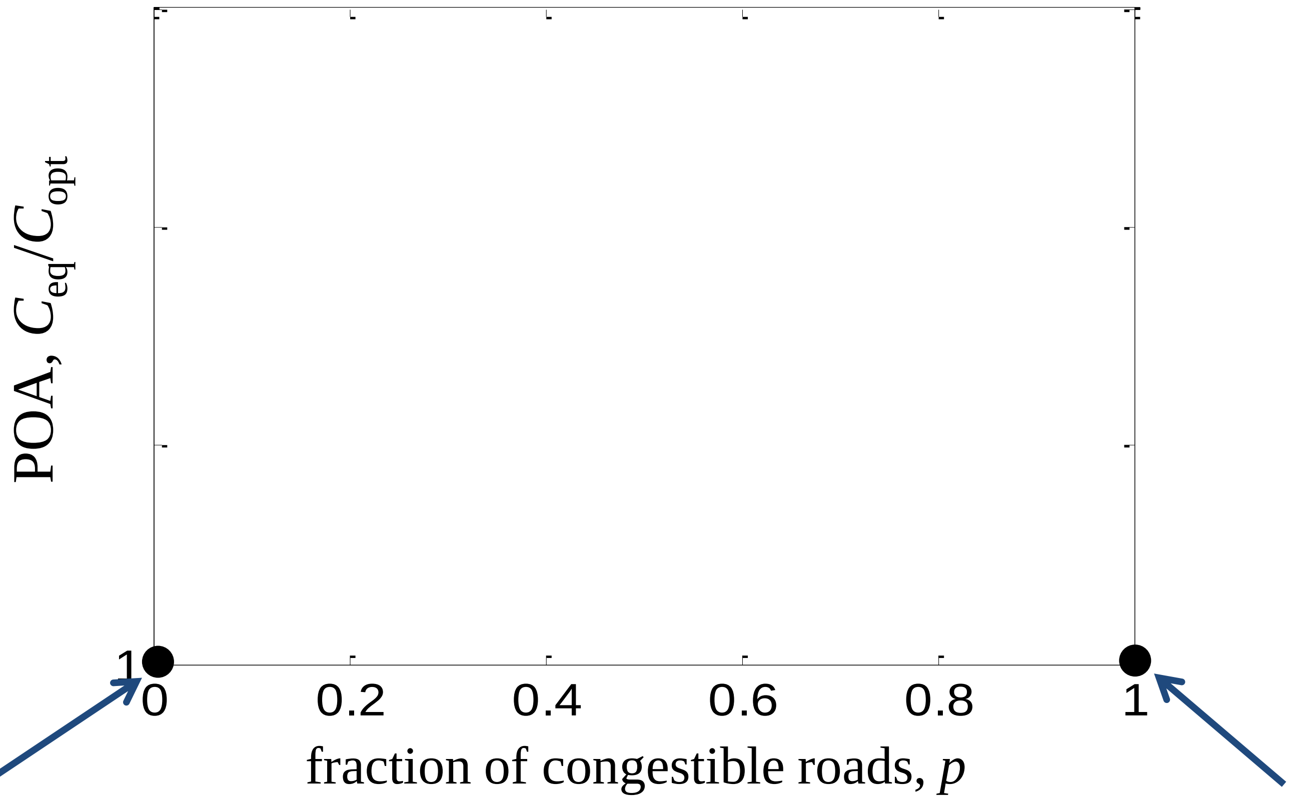
- For a given *p*, randomly assign the network links
- Map the network onto a battery-resistor-diode circuit

equilibrium: 

optimum: 

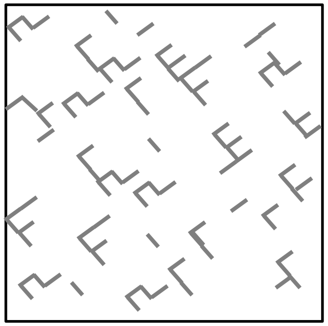
- Search numerically for the correct configuration of diodes and the currents $\{x_i\}$
- Calculate the total commute time: $$C = \sum_{\text{roads } i} x_i c(x_i)$$
- Define the "price of anarchy": $$POA = C_{eq} / C_{opt}$$

Results: the price of anarchy

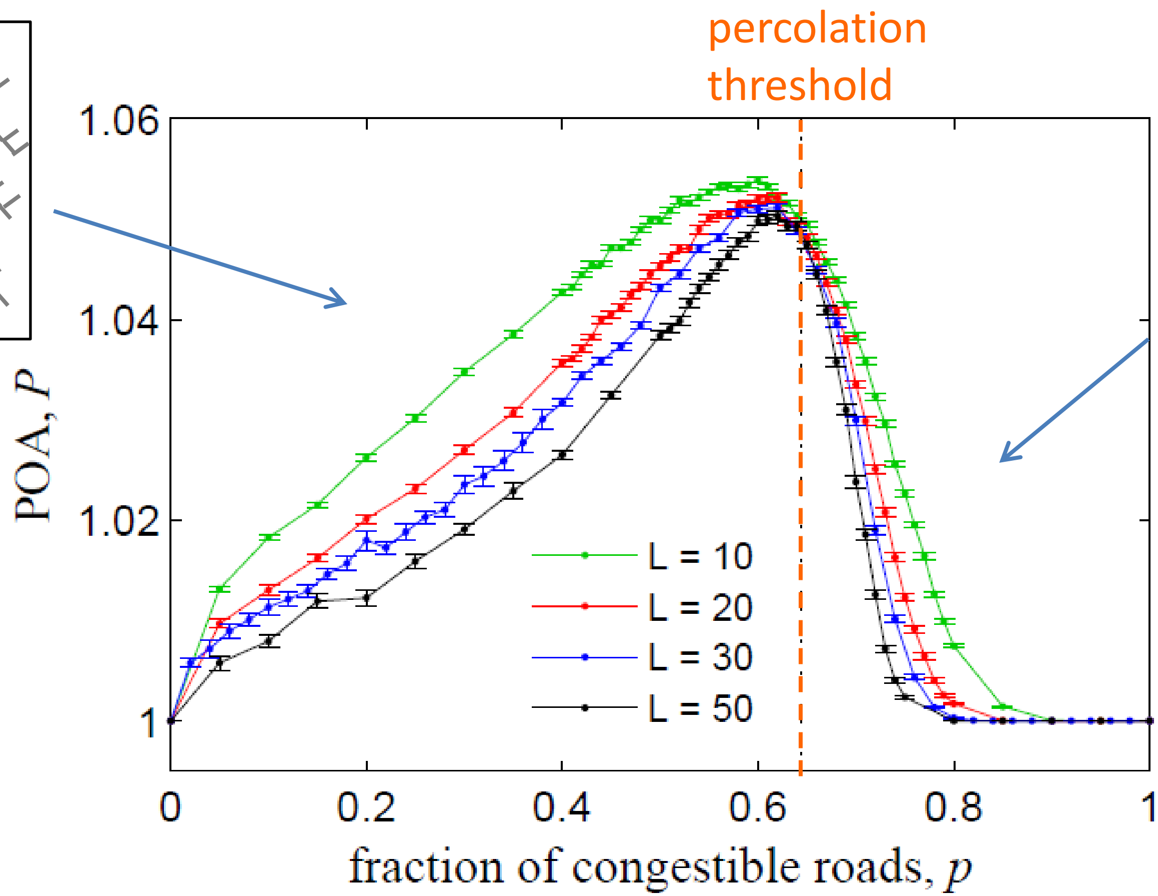


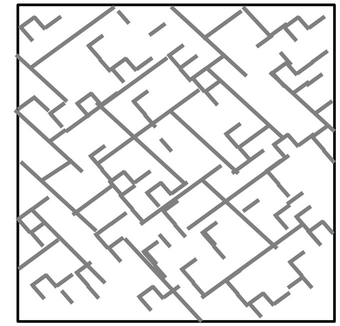
network is uniform
→ POA is 1

Results: the price of anarchy



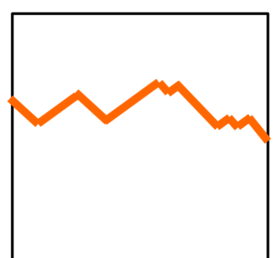
At $p < p_c$ fast congestible roads form only finite clusters





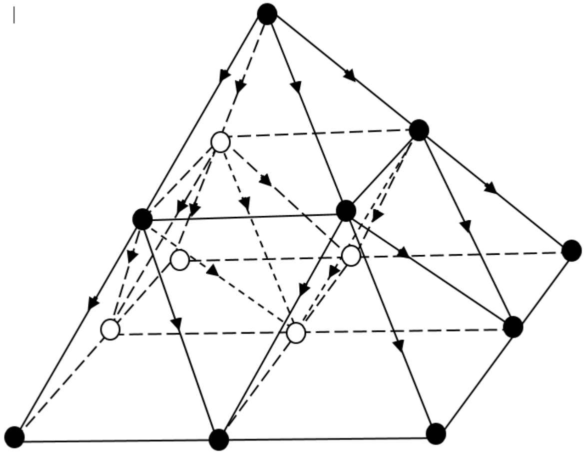
At $p > p_c$ "percolating" pathways of congestible roads connect system edges

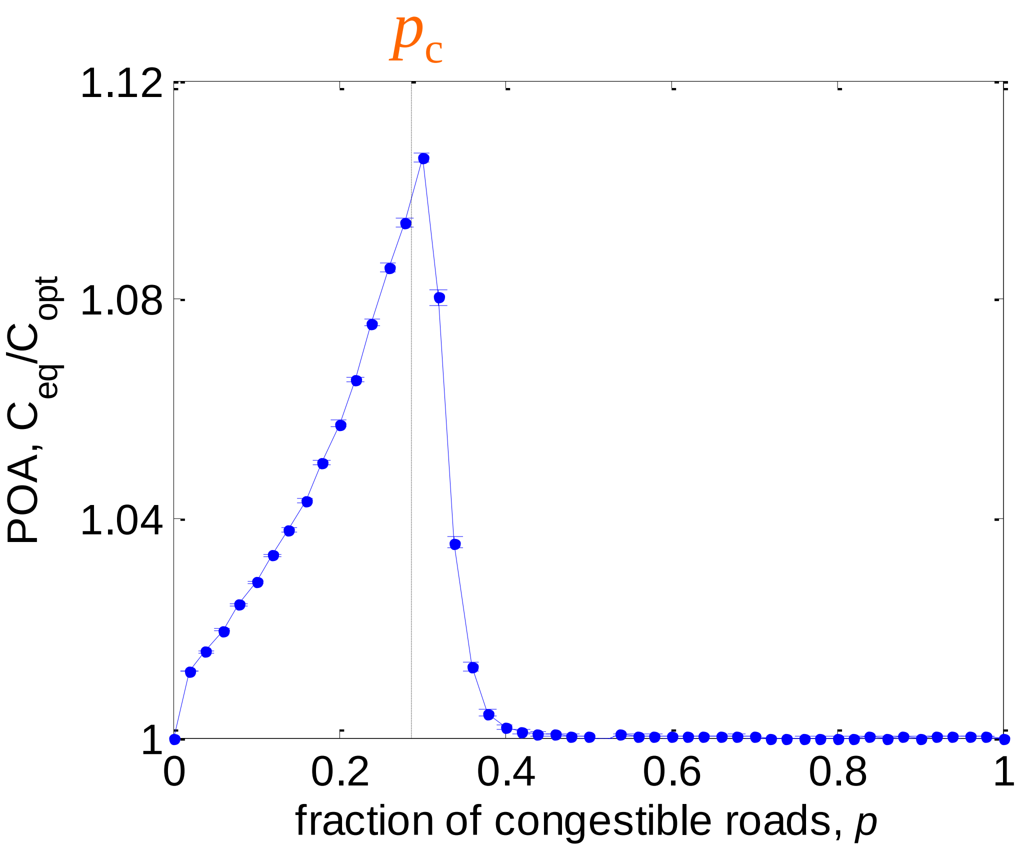
At $p = p_c$, a *single* pathway exists connecting system edges



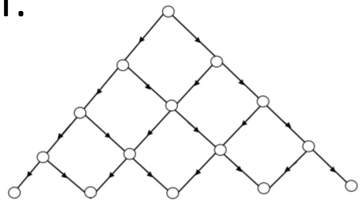
The POA is maximized at the percolation threshold

POA for a 3D lattice



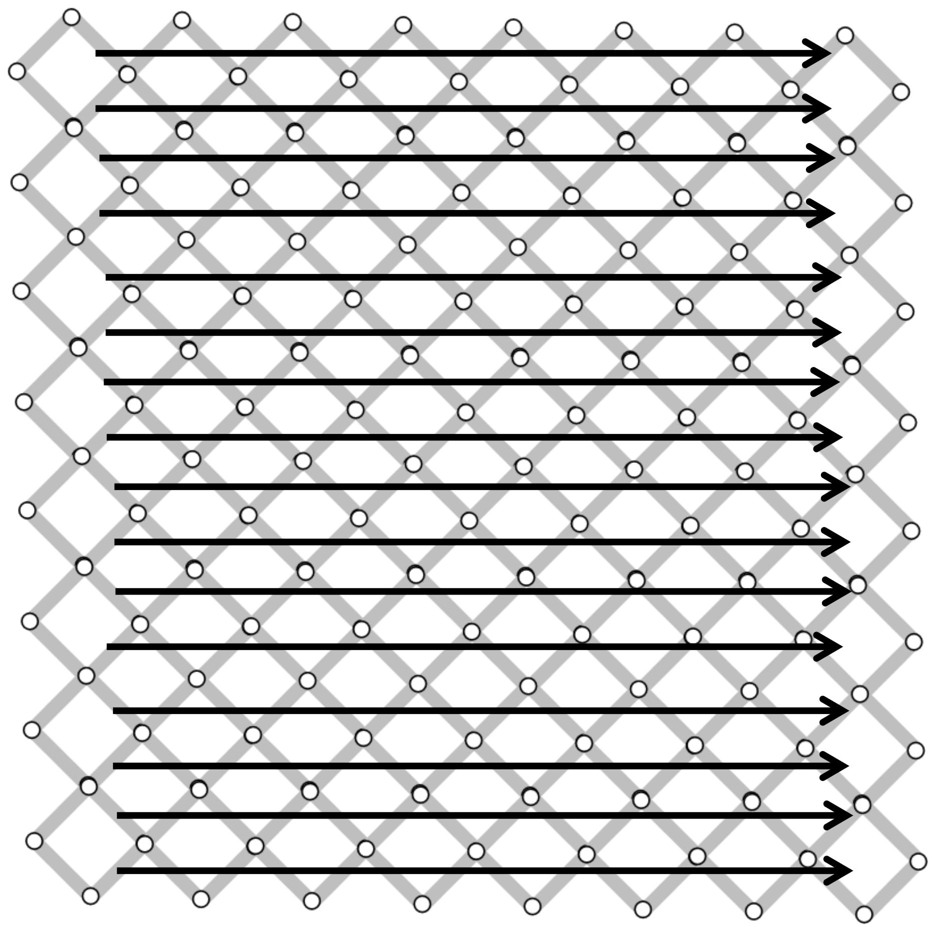


instead of:



Current paths

$p = 1$: uniform lattice



same current in every link:

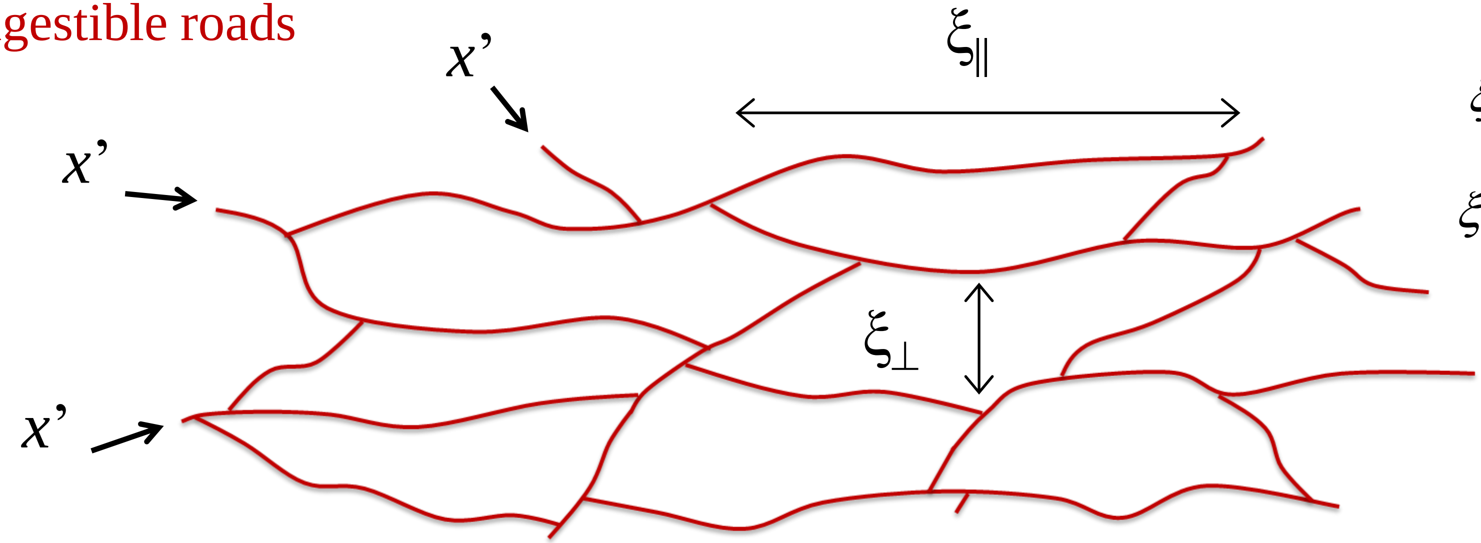
$x = 1/2L$

Commute time:

$C = x \cdot 2L = 1$

Current paths at $p > p_c$

percolating network of fast, congestible roads



$\xi_\parallel \sim |p - p_c|^{-\nu_\parallel}$

$\xi_\perp \sim |p - p_c|^{-\nu_\perp}$

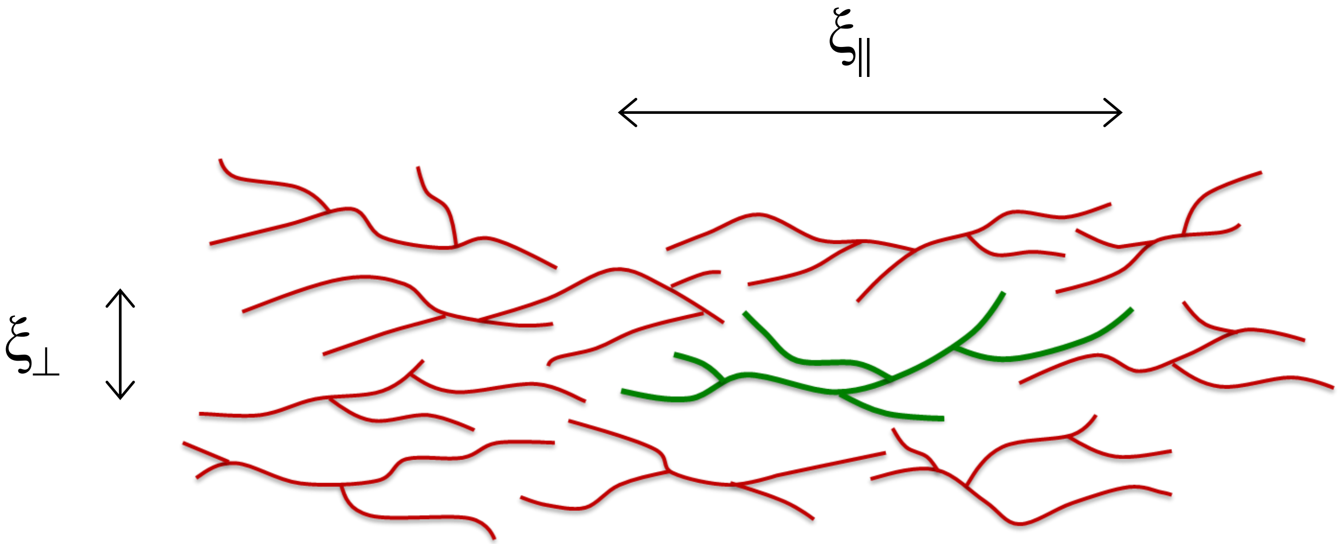
$x' \sim 1 \times (\xi_\perp / L)$

$C \sim x' L$

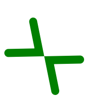
$$C \sim \xi_\perp \sim |p - p_c|^{-\nu_\perp}$$

Commute time is constant at $L \to \infty$

Current paths at $p < p_c$



$$C \sim \frac{L}{\xi_{\parallel}} \sim L|p - p_c|^{\nu_{\parallel}}$$

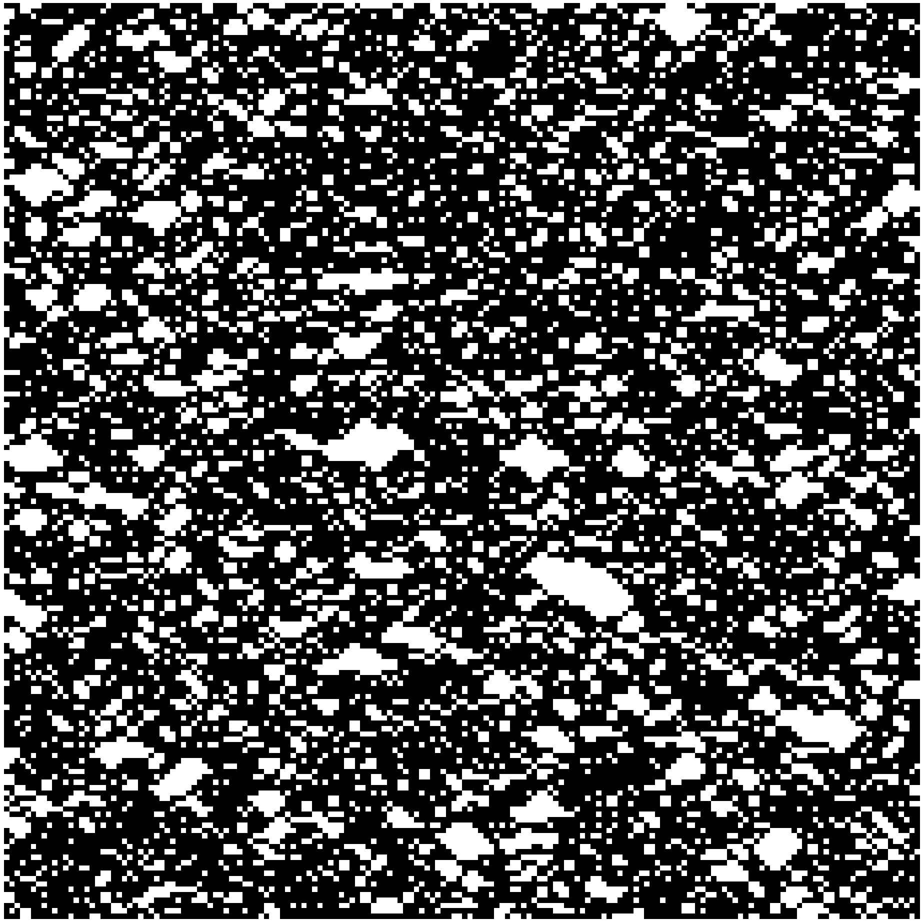


Commute time grows extensively with system size

"Holes" in the current path

$p_c < p < 1$: small concentration of slow incongestible roads

Showing all roads with $x > 0$

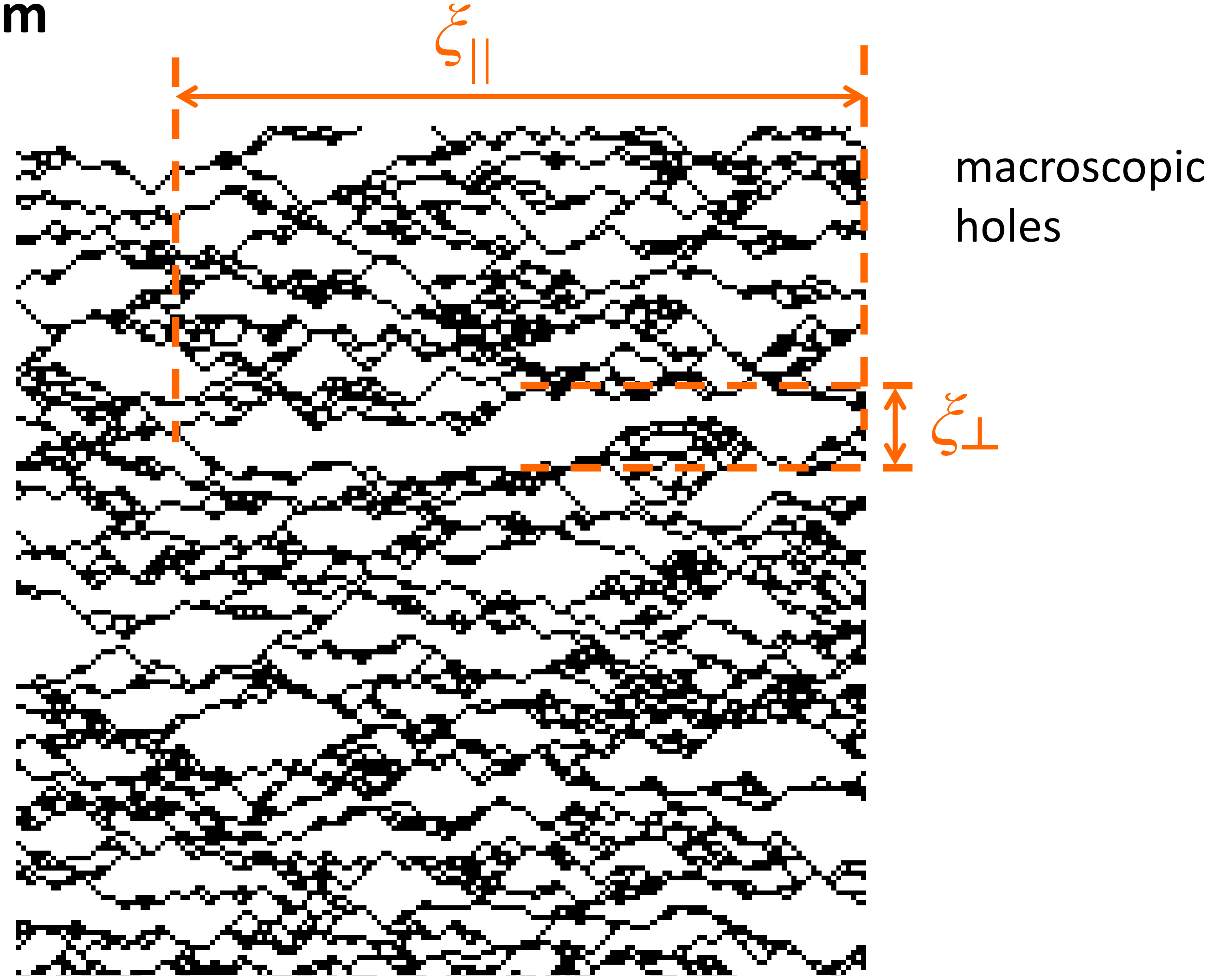


small holes start to open in the current paths

“Holes” in the current path

$p \sim p_c$, **equilibrium**

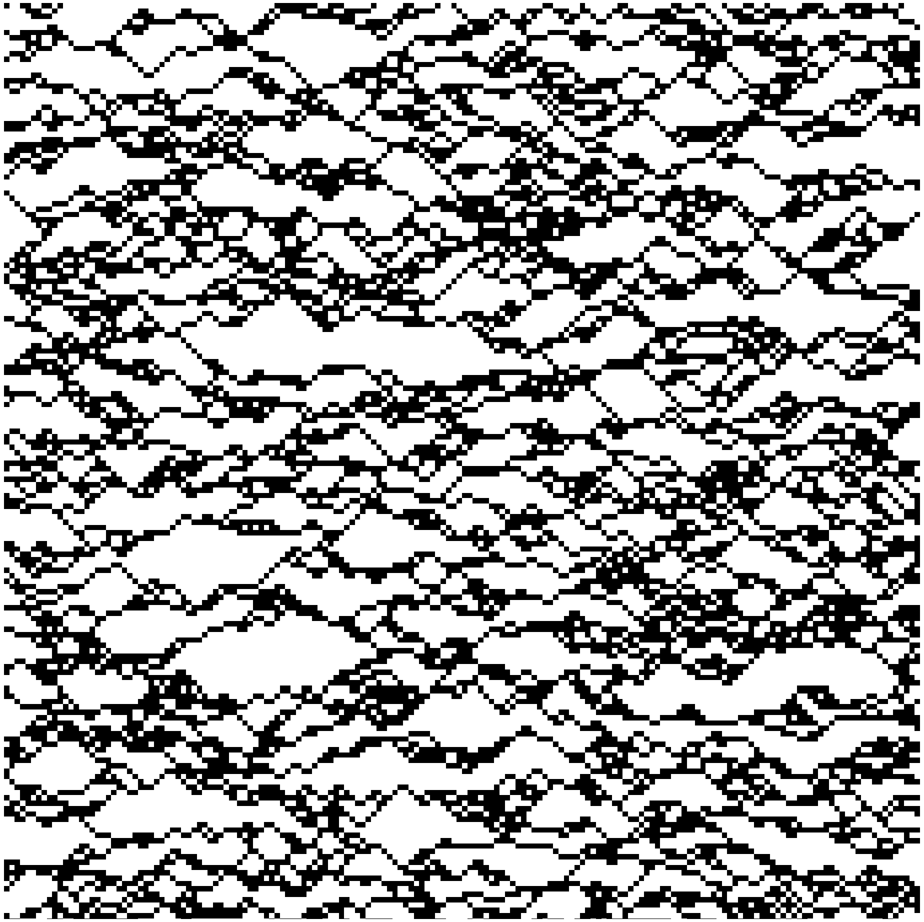
Showing all roads with $x > 0$



“Holes” in the current path

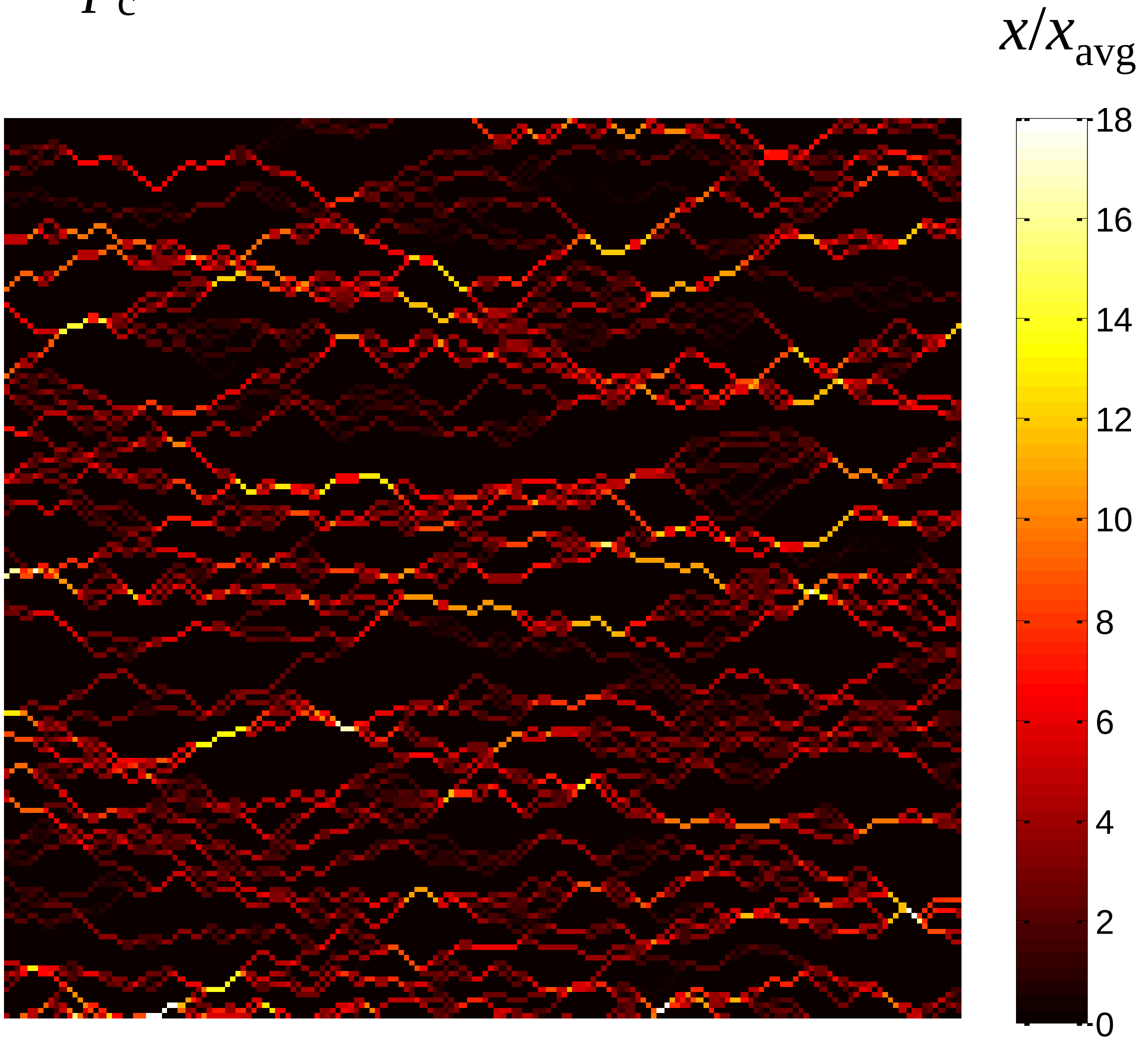
$p \sim p_c$, **optimum**

Showing all roads with $x > 0$



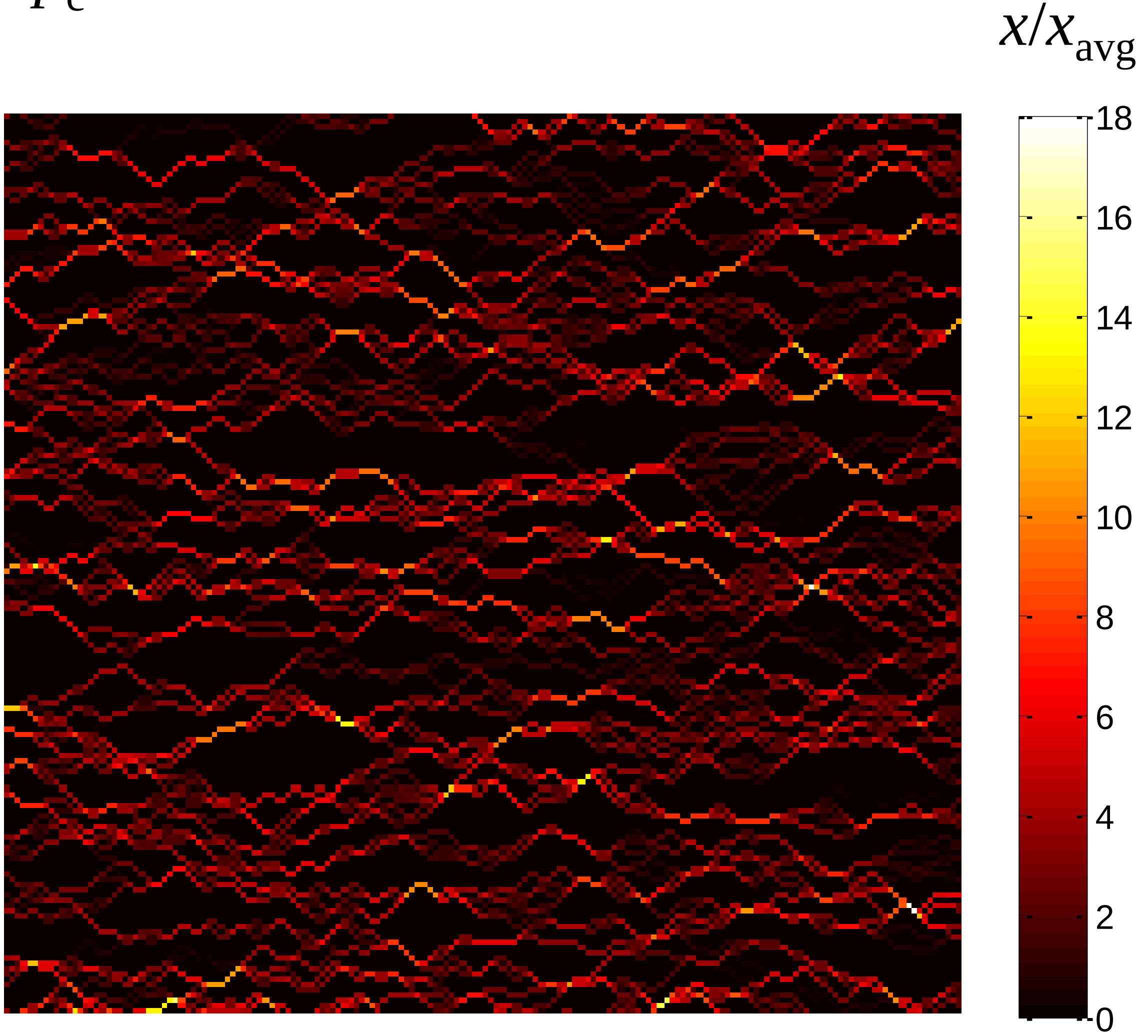
Current paths

Equilibrium, $p \sim p_c$:



Current paths

Optimum, $p \sim p_c$:



Critical Scaling

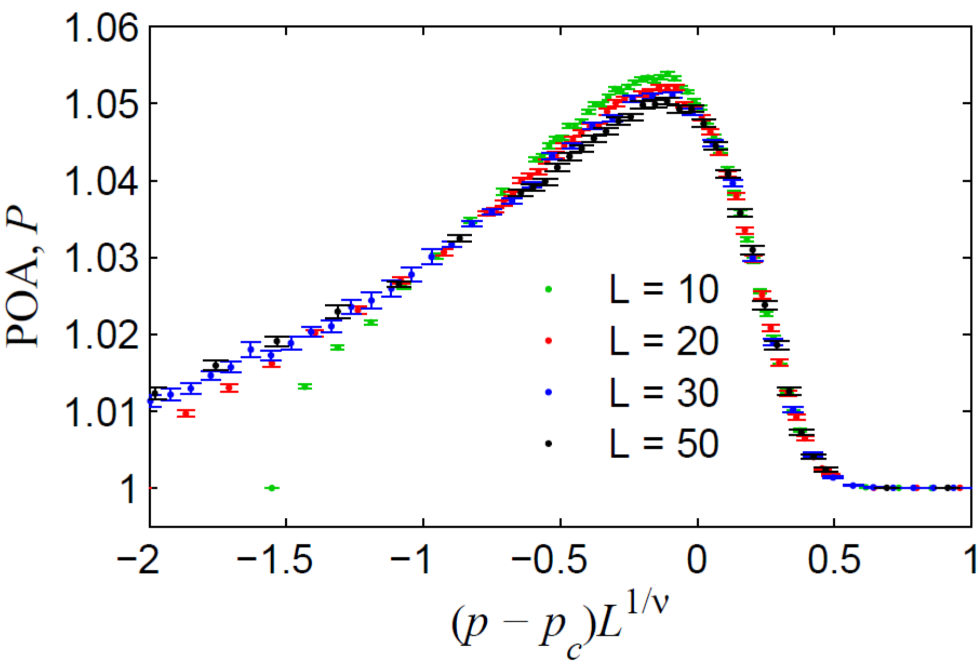
In the presence of large "percolation clusters"

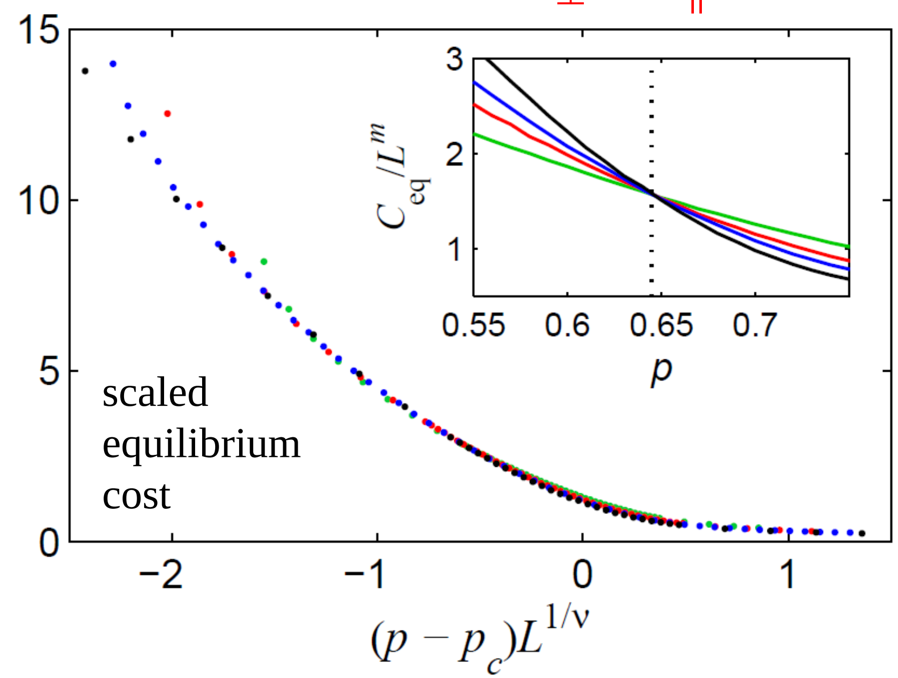
$$\xi \sim (p - p_c)^{-\nu}$$

system properties can be written as

$$P = f\left(L/\xi\right) = f\left((p - p_c)L^{1/\nu}\right)$$

$\nu = \nu_\perp + \nu_\parallel \approx 2.8$

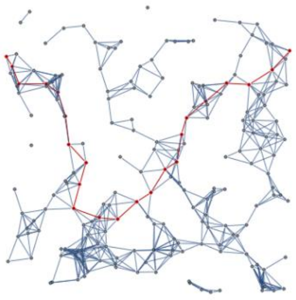




Some open questions:

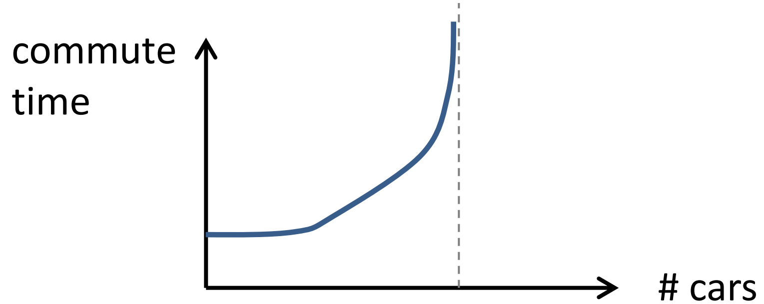
- Is there a more general connection between percolation and network inefficiency?

  Can we exploit it to improve networks?



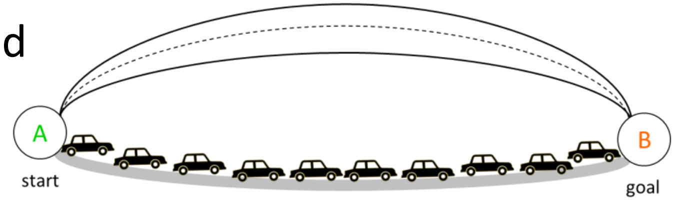
- What happens when the cost functions become nonlinear?

  e.g.

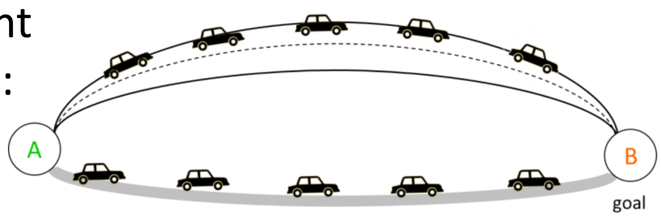


- Is user ignorance a good thing or a bad thing?

informed drivers:

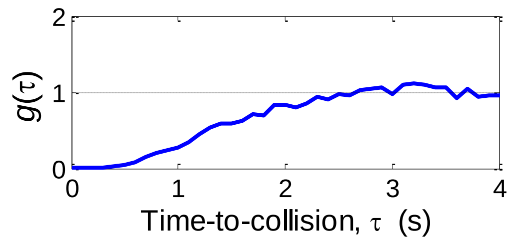


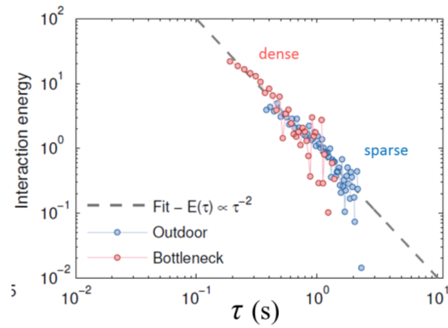
ignorant drivers:

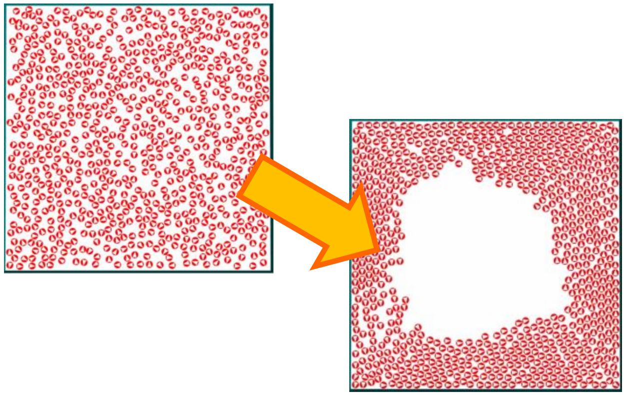


Conclusions

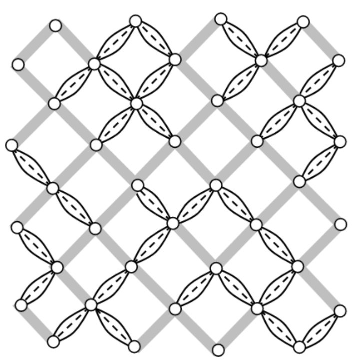
- **Part 1**: The interaction "energy" between pedestrians in a crowd is $V \sim 1/(\text{time to collision})^2$

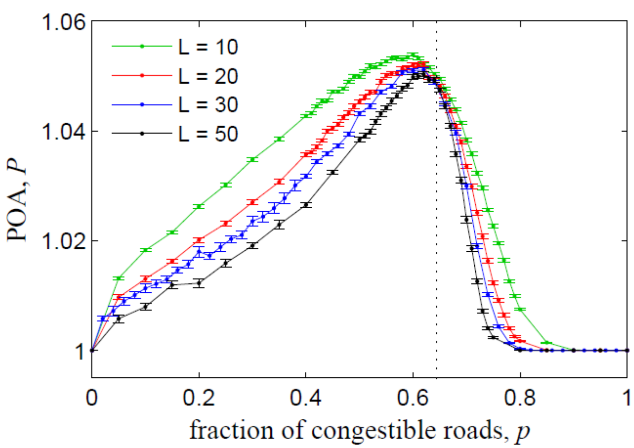






- **Part 2**: The price of anarchy in a model network is maximized at the percolation threshold for congestible links

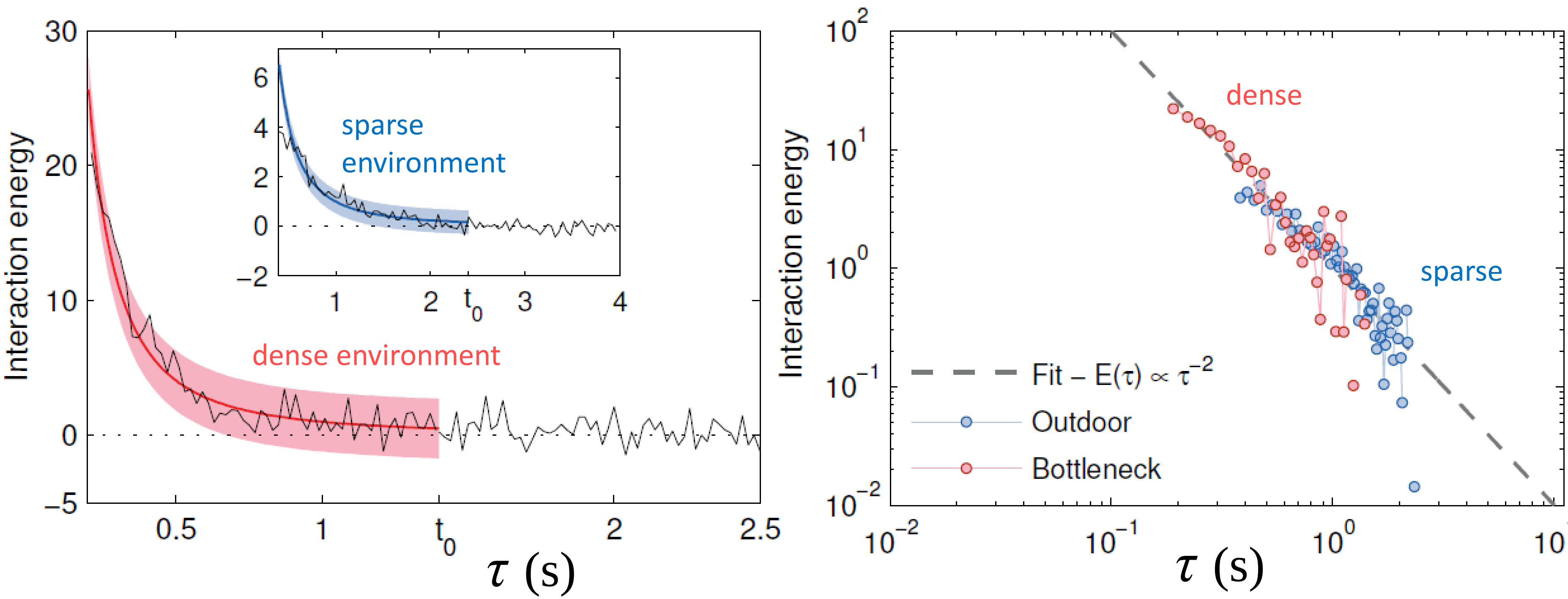




Thank you.

Reserve Slides

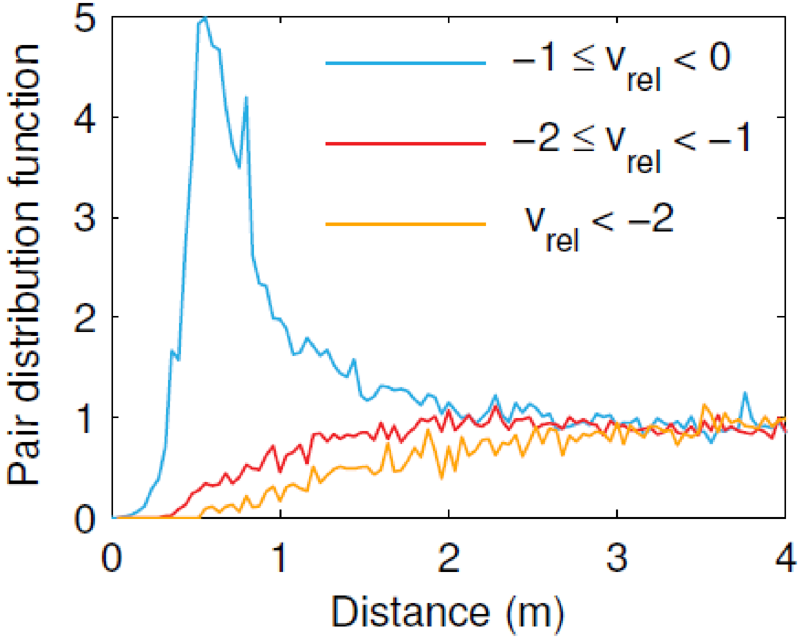
Truncation of the interaction



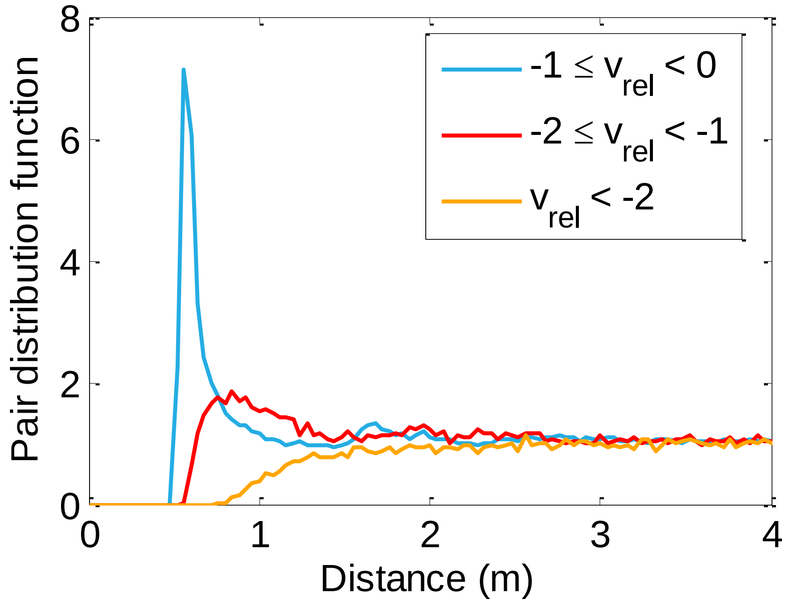
$$V(\tau) \propto (1/\tau^2)\exp[-\tau/\tau_0]$$

velocity-resolved pair distribution

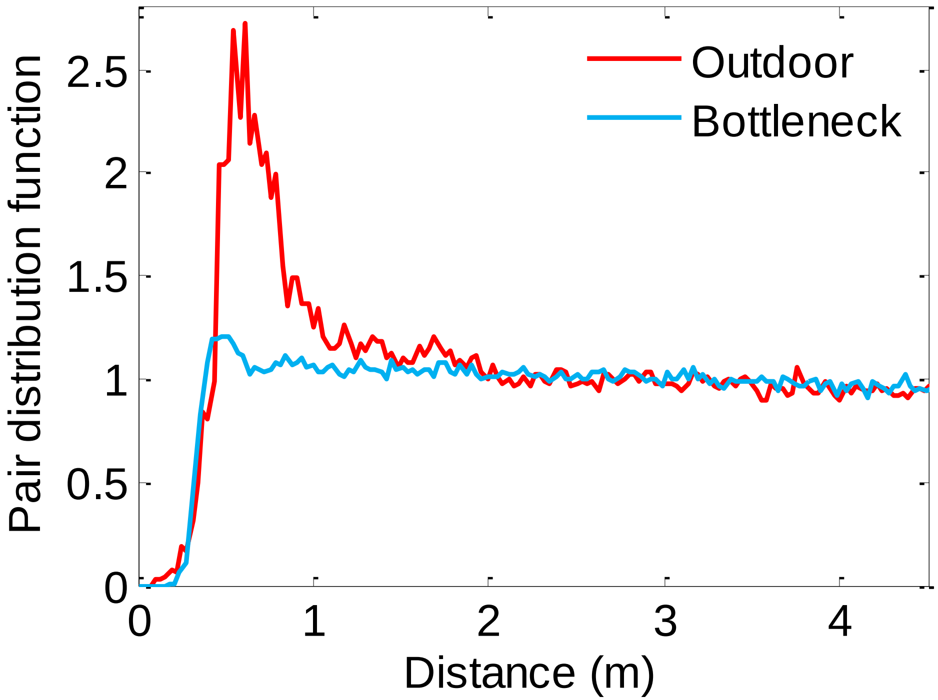
college campus – real data



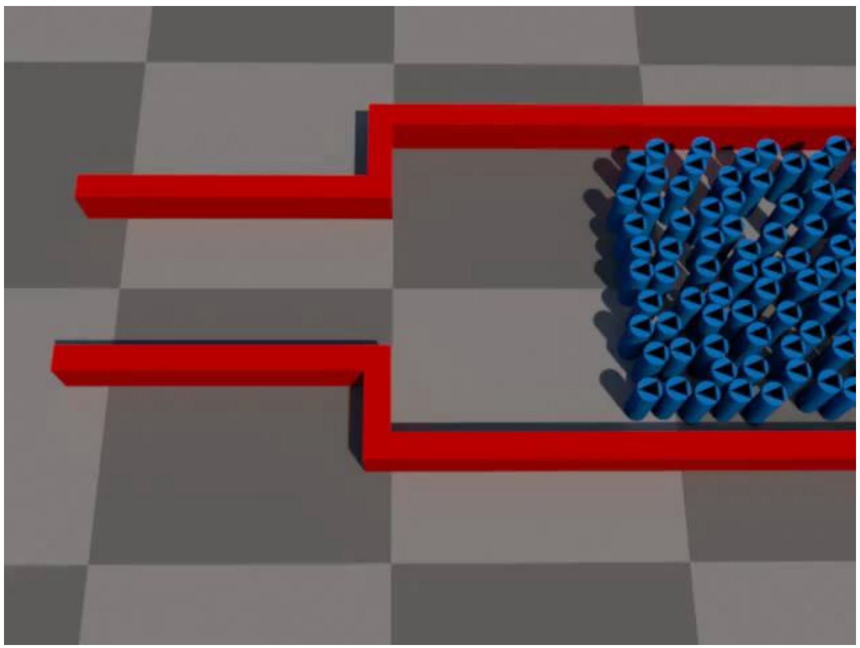
“hallway” scenario – simulation

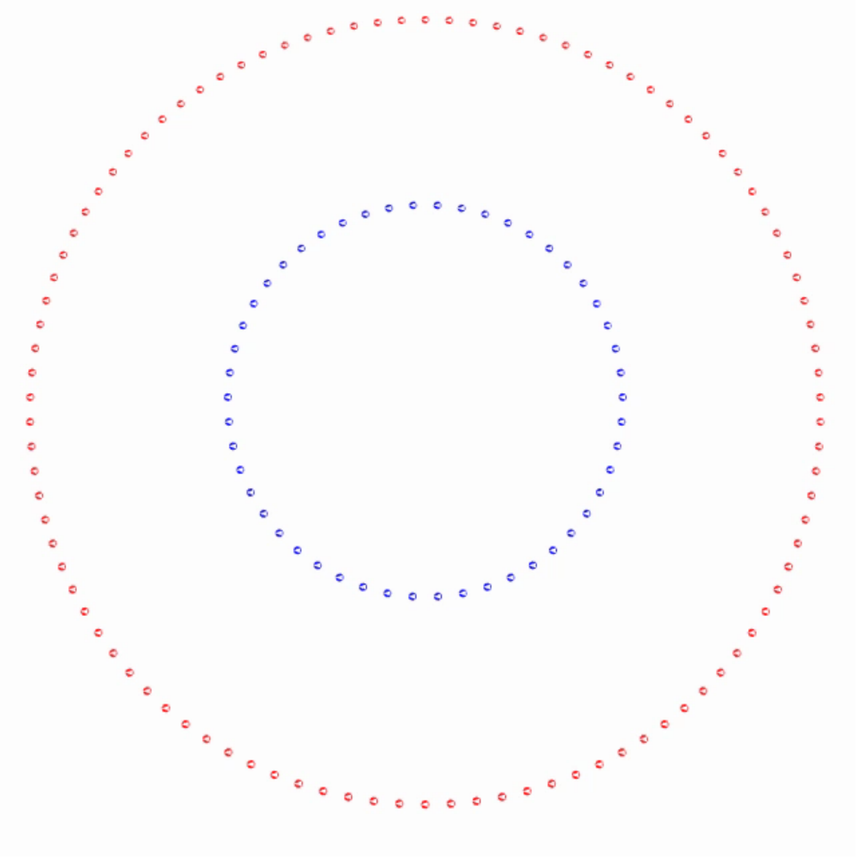


pair distribution for $\tau = \infty$

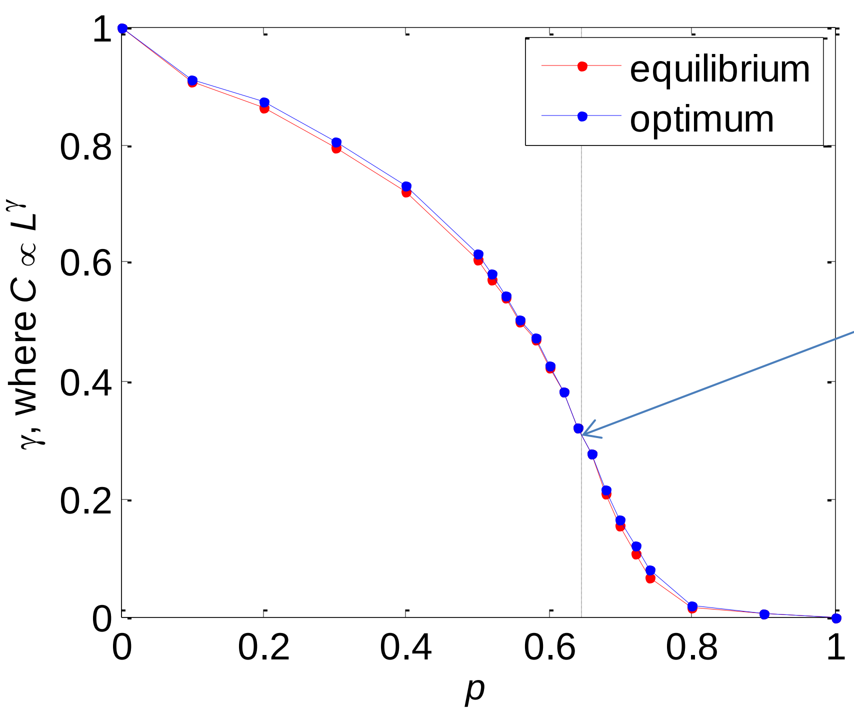


More videos





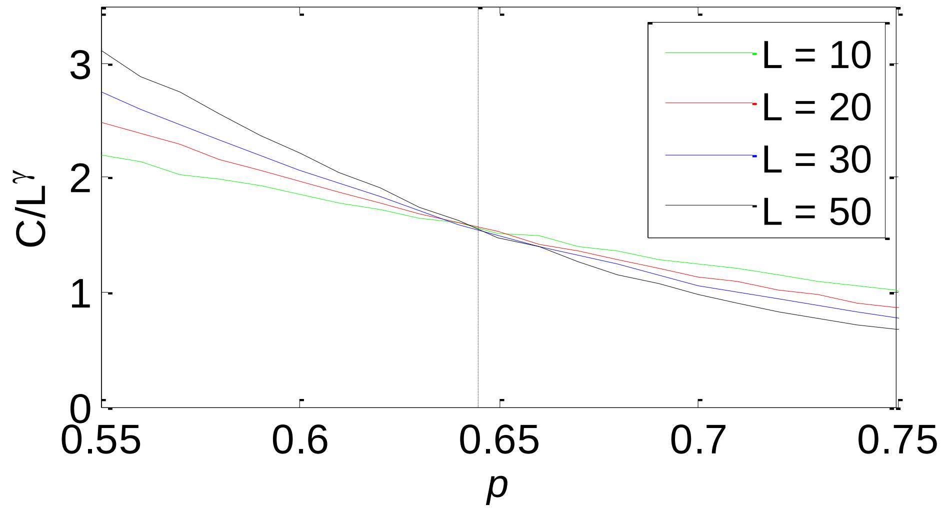
Scaling



$\gamma(p_c) \sim 0.31$

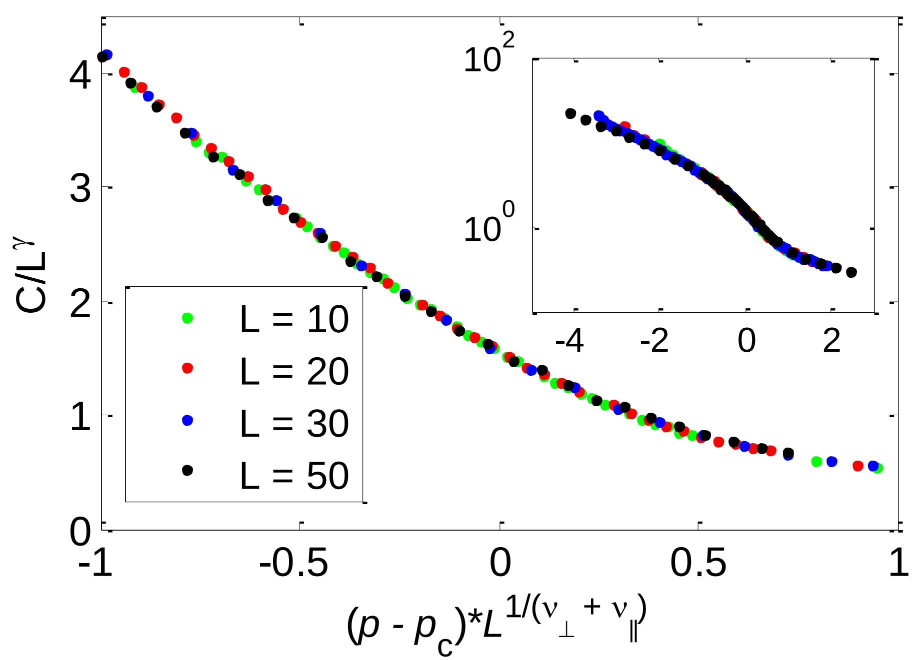
Scaling

$$\xi \sim (p - p_c)^{-\nu}$$

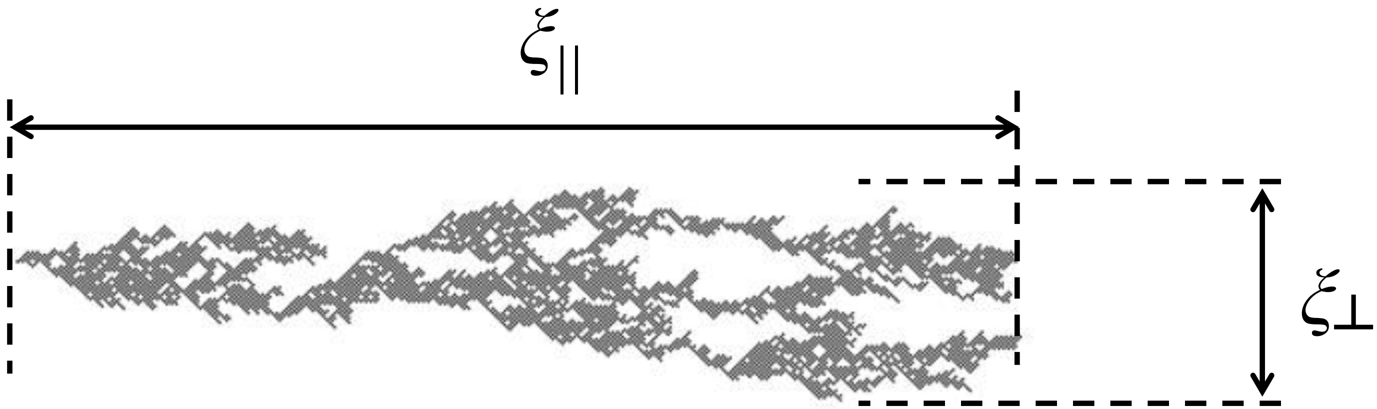


$$C / L^{\gamma} \sim f(L/\xi)$$

$$C / L^{\gamma} \sim f((p - p_c)L^{1/\nu})$$



Critical exponents in DP



$$\xi_{\|} \sim \left|p - p_c\right|^{-\nu_{\|}}$$

$\nu_{\|} \sim 1.295$

$$\xi_{\perp} \sim \left|p - p_c\right|^{-\nu_{\perp}}$$

$\nu_{\perp} \sim 0.733$