Exotic Zero Energy Modes at Topological Defects in Crystalline Superconductors

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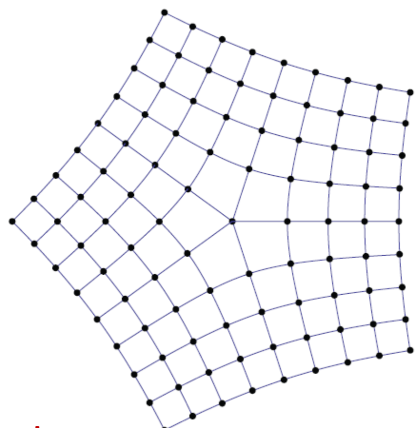

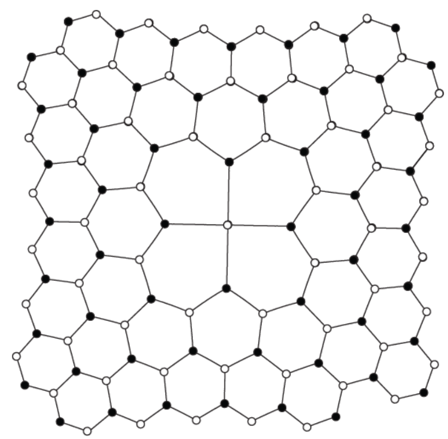

Collaborators:

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Wladimir A. Benalcazar

Kam Tuen Law

Noah Fan Qi Yuan

Phys. Rev. Lett. 111, 047006 (2013)

arXiv:1311.0496 (2013)

To appear soon

Outline

- Topology and exotic particles
  - Quantum Hall effect
  - Topological insulators

  Bulk topology => exotic boundary excitations

- Topological defects
  - Vortices in unconventional 2D superconductors
  - Ising anyons (Majorana zero modes)

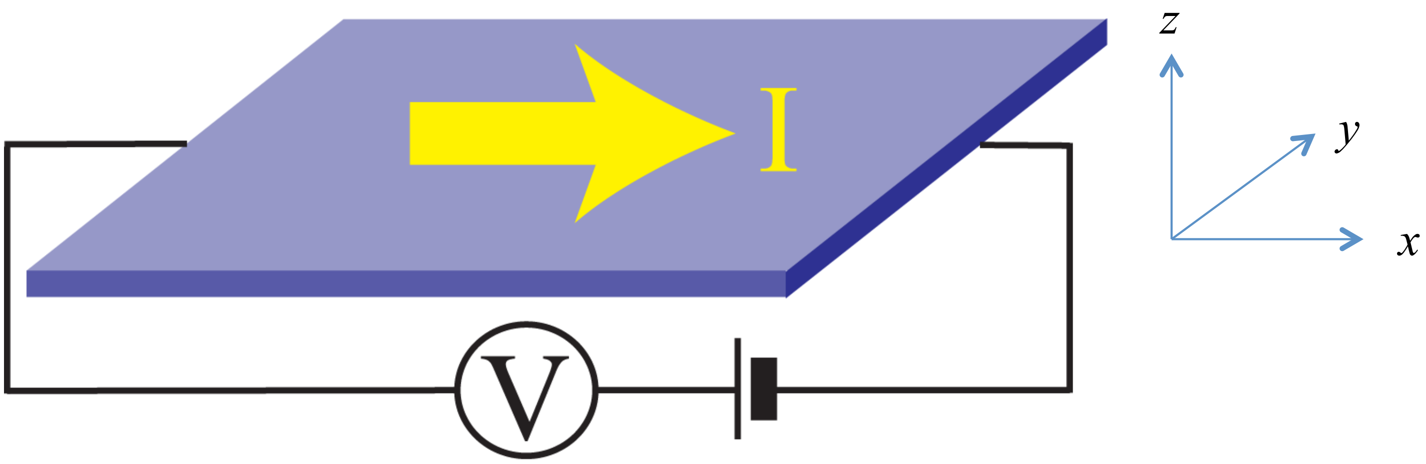
- Majorana zero modes in crystalline superconductors

  [**JT** *and Taylor L Hughes*; Phys. Rev. Lett. 111, 047006 (2013)]
  [*Wladimir A. Benalcazar*, **JT** *and Taylor L Hughes*; arXiv:1311.0496 (2013)]
  [*Noah F.Q. Yuan*, **JT** *and K.T. Law*; to appear soon (2014)]

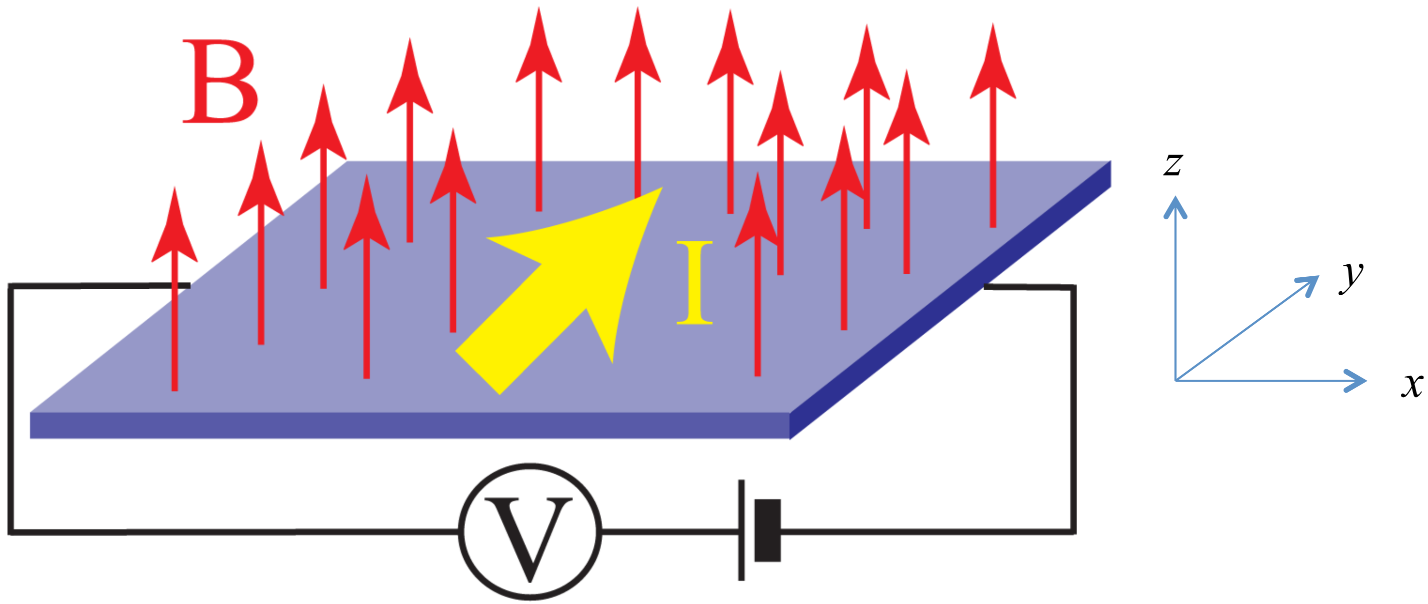
  - Broken translation and rotation lattice symmetries
  - Crystalline defects: dislocation, disclinations
  - Majorana zero modes at
    - Strontium Ruthenate
    - Graphene and silicene
    - MoS_2

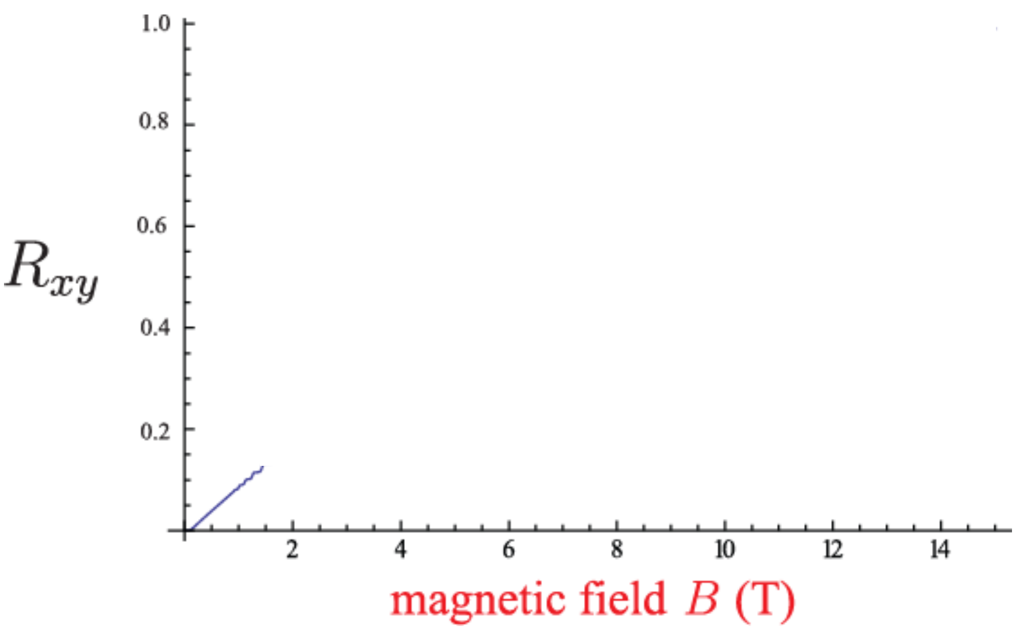
TOPOLOGY AND EXOTIC EXCITATIONS

Quantum Hall Effect



Quantum Hall Effect





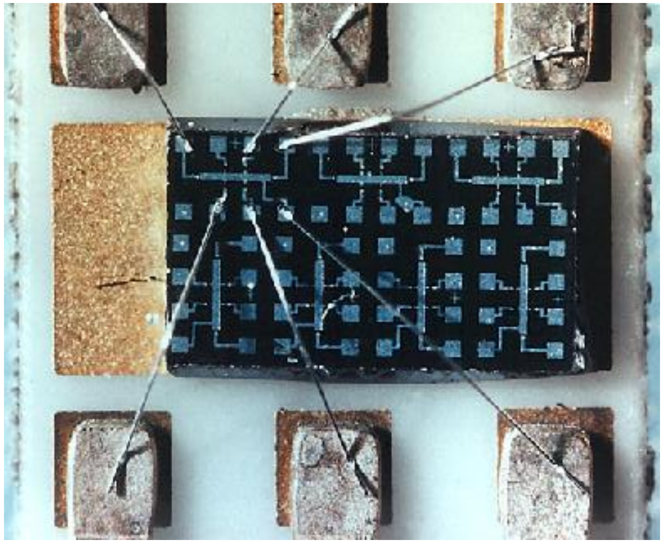
$$V_x = R_{xy} I_y$$

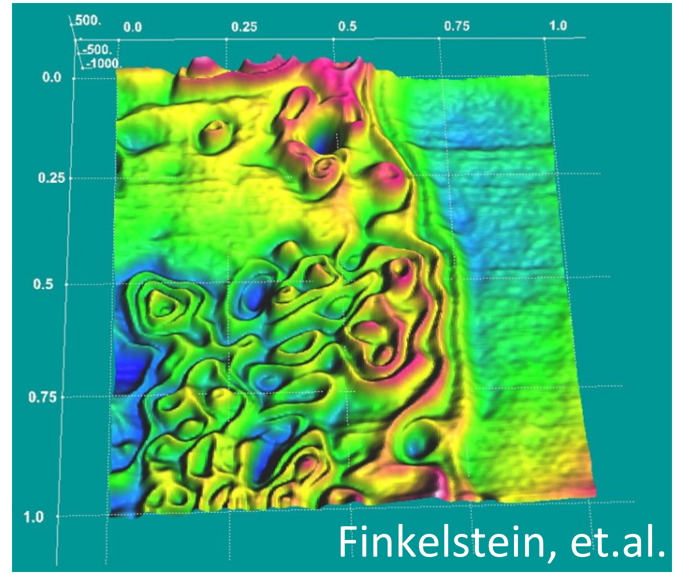
$$R_{xy} = \frac{B}{\rho}$$

$$\boxed{h/e^2 = 25812.807557(18)\Omega}$$

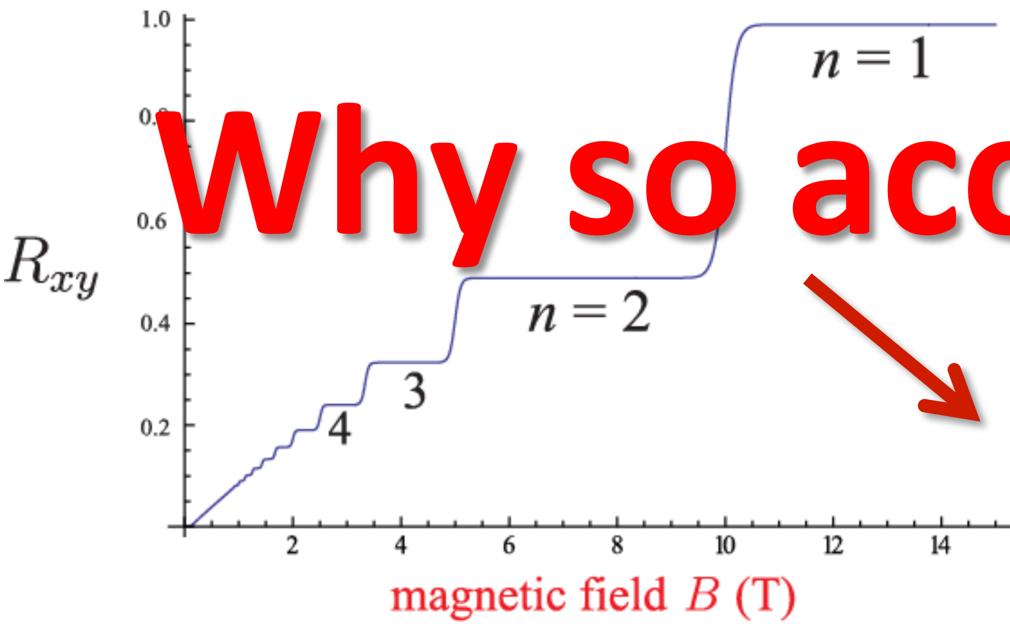
von Klitzing, et.al. 80’

Quantum Hall Effect





Finkelstein, et.al.



Why so accurate?

$$V_x = R_{xy} I_y$$

$$R_{xy} = \frac{1}{n}\frac{h}{e^2}$$

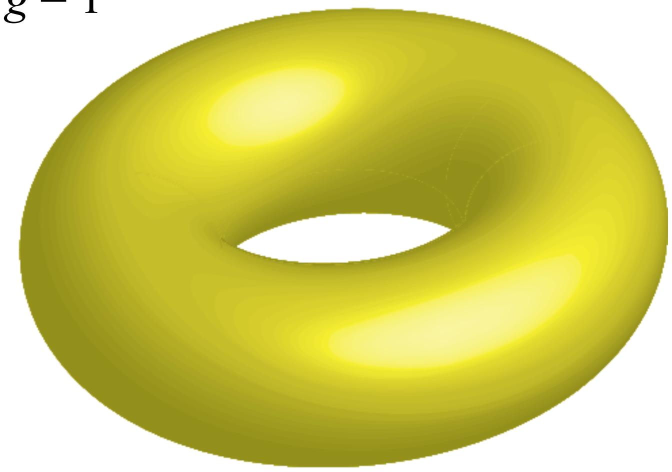
$$\boxed{h/e^2 = 25812.807557(18)\Omega}$$

von Klitzing, et.al. 80’

Topology of electronic structure

$g = 0$

$g = 1$



Gauss-Bonnet theorem

$$2 - 2g = \frac{1}{2\pi}\int_{\Sigma} R\,dA$$

Gaussian curvature

Chern number

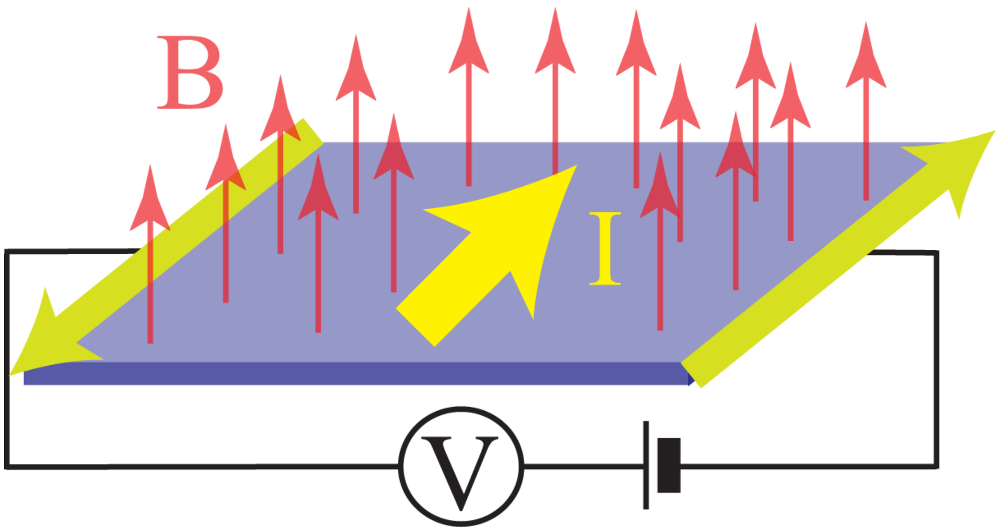
Kubo formula

$$\frac{1}{R_{xy}} = \frac{e^2}{h}\boxed{\int \frac{dk_x dk_y}{2\pi}\underbrace{2\mathrm{Im}\langle \partial_{k_x} u(\mathbf{k}) | \partial_{k_y} u(\mathbf{k}) \rangle}_{\text{Berry curvature}}}$$

Thouless, *et.al.* 82

Topologically protected excitations

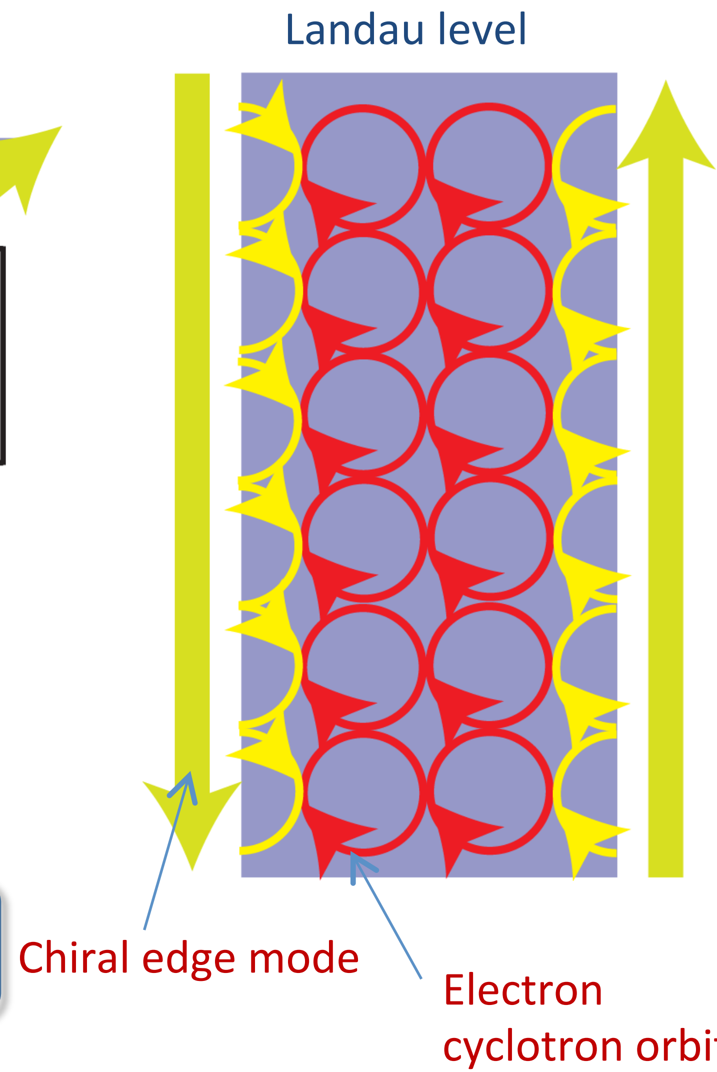
Chiral edge mode of a Landau level



Bulk-boundary correspondence

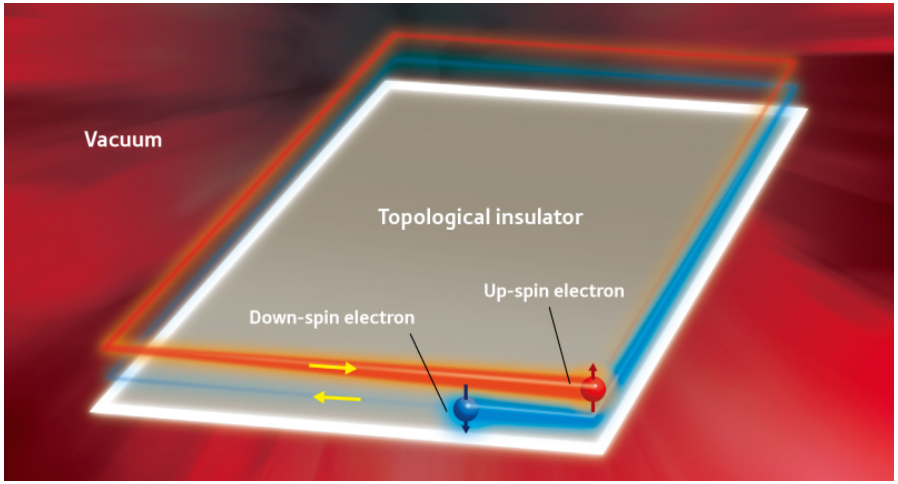
Single direction electronic channel
No backscattering
Dissipationless transport

NOT realizable in any 1D systems



Topologically protected excitations

Helical edge mode of a quantum spin Hall insulator

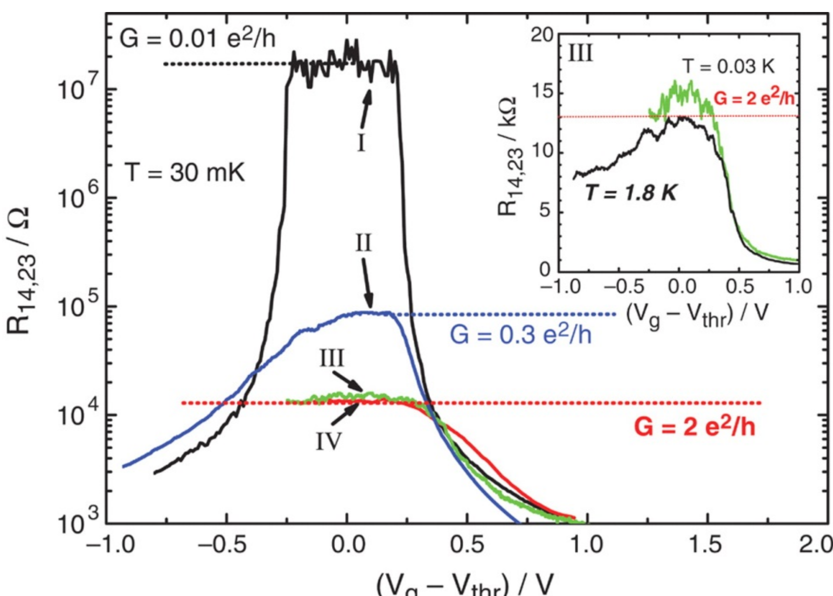


Kane, Mele 05 $\mathbb{Z}_2$ bulk topology

Bulk-boundary correspondence

Bulk – spin-orbit coupled time reversal symmetric 2D insulator
Boundary – gapless helical edge mode
Backscattering prohibited by time reversal symmetry

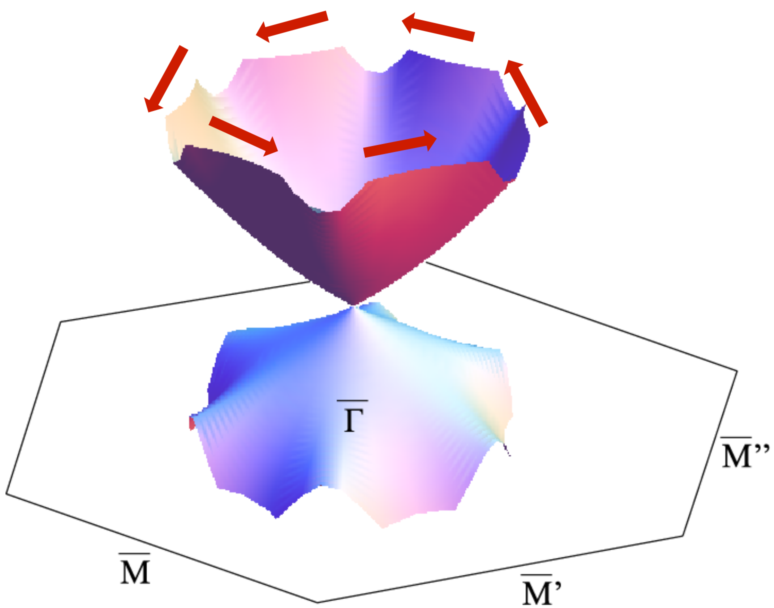
NOT realizable in any time reversal symmetric 1D systems



Konig, *et.al.* 07

Topologically protected excitations

Surface Dirac cone of a 3D topological insulator



$\mathbb{Z}_2$ bulk topology

Bulk-boundary correspondence

Bulk – spin-orbit coupled time reversal symmetric 3D insulator
Boundary – gapless surface Dirac cone
Weak anti-localization

Fu, Kane, Mele, 07
Moore, Balents, 07
Roy, 07
Hsieh, *et.al.* 08, 09

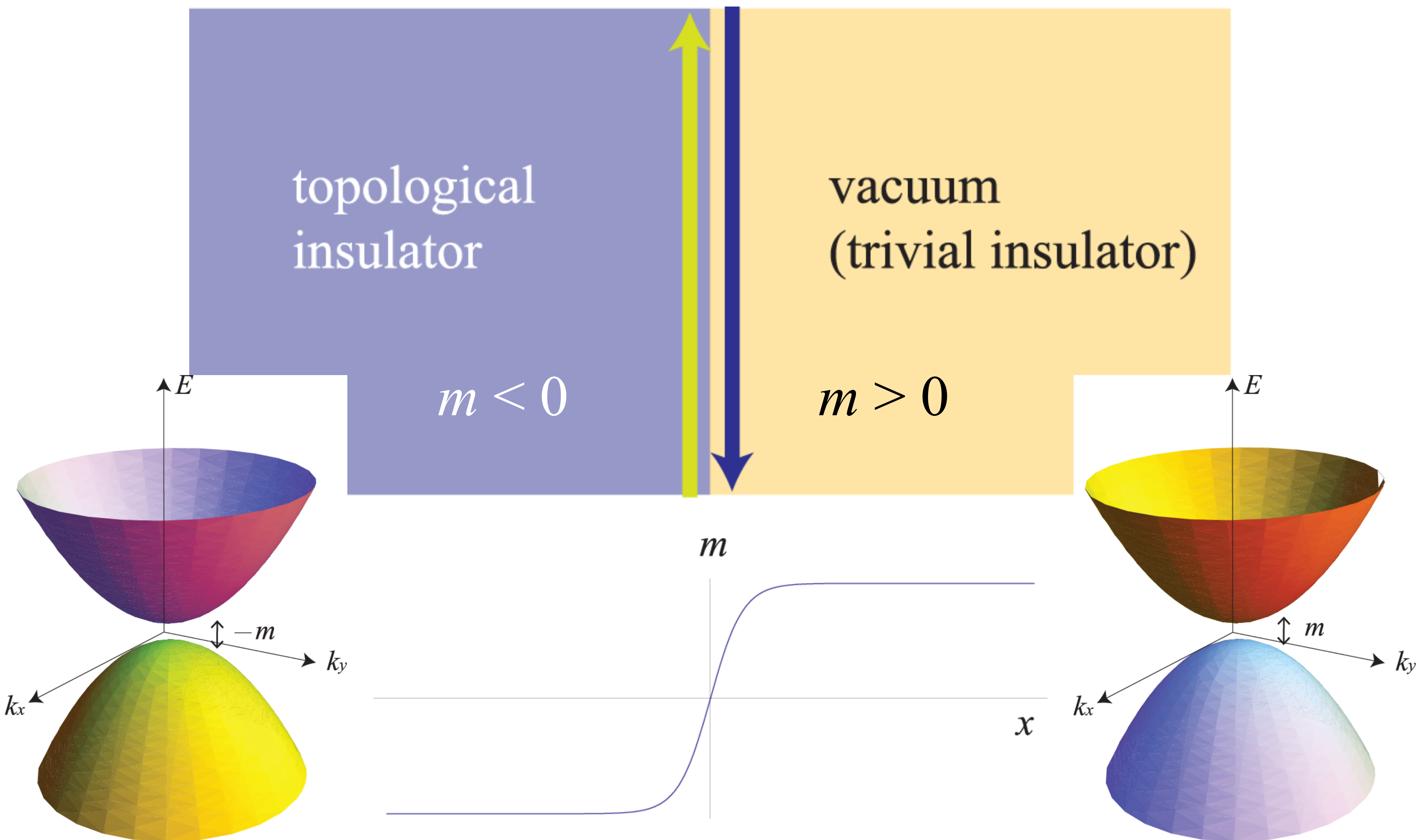
NOT realizable in any time reversal symmetric 2D systems

Topology of bulk electronic band structure

Exotic boundary excitations

TOPOLOGICAL DEFECTS

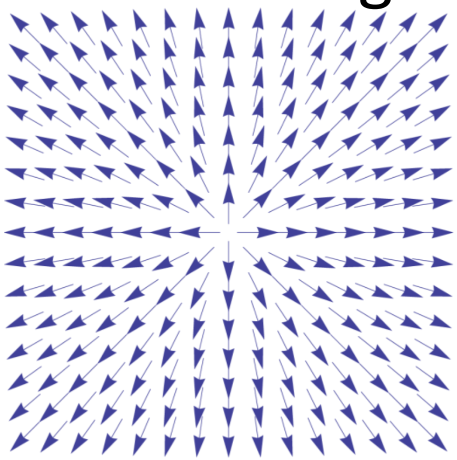
Boundary = Domain wall

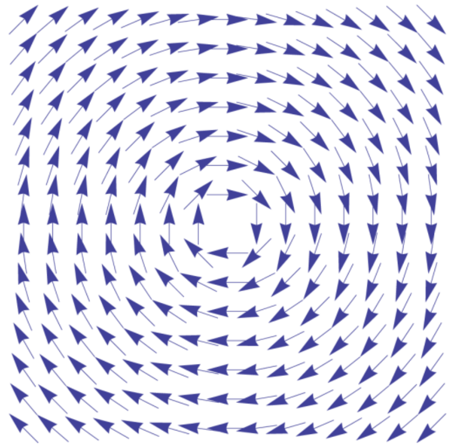


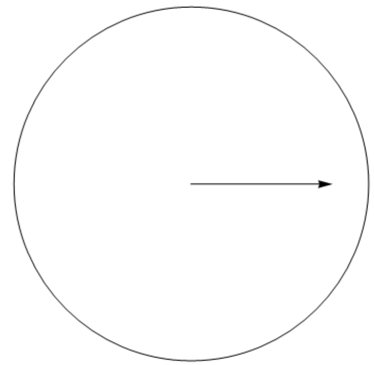
$$H = v(k_x\sigma_x + k_y\sigma_y)\tau_z + m\tau_x$$

Topological point defects

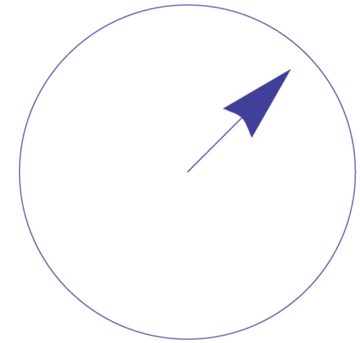
Winding number = +1



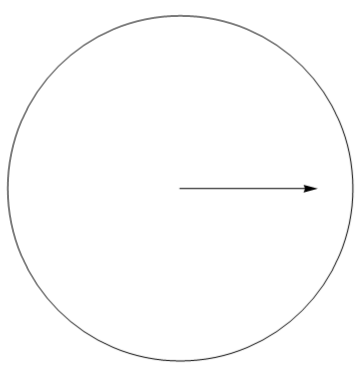


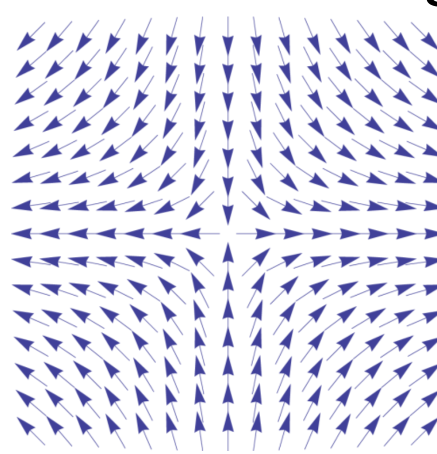


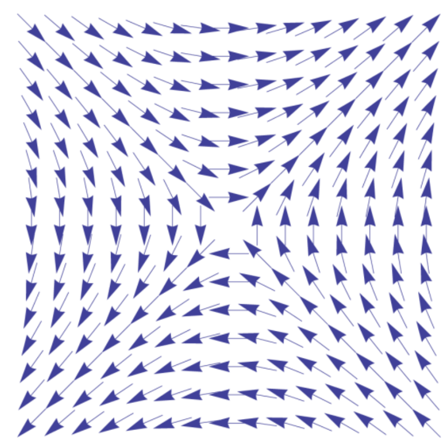
Order parameter



Winding number = -1







Topological point defects

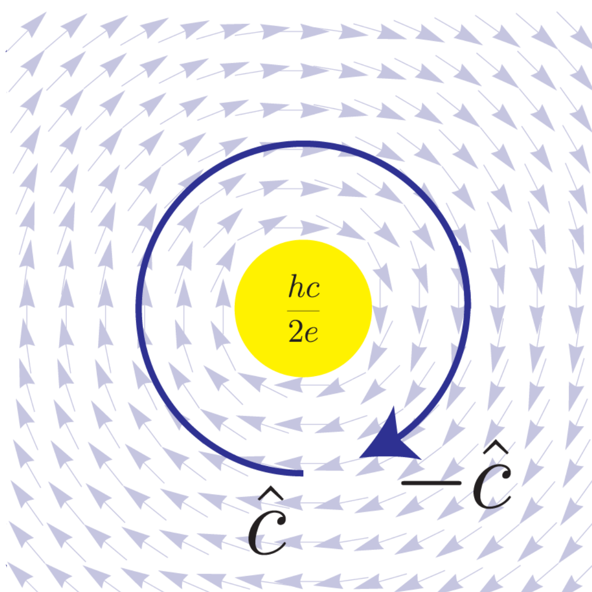
Vortices in conventional superconductors

BCS ground state $|GS\rangle \sim a_0|0\rangle + a_2\hat{c}_a^\dagger\hat{c}_b^\dagger|0\rangle + a_4\hat{c}_a^\dagger\hat{c}_b^\dagger\hat{c}_c^\dagger\hat{c}_d^\dagger|0\rangle + \ldots$

Broken charge conservation $U(1)$-symmetry

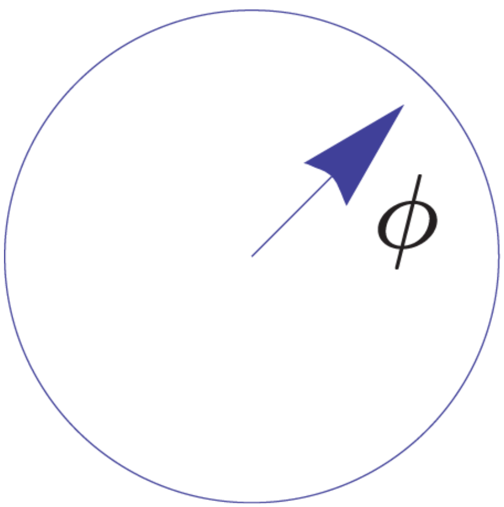
$$\left\langle c_\uparrow^\dagger(\mathbf{k})c_\downarrow^\dagger(-\mathbf{k})\right\rangle \sim \Delta = |\Delta|e^{i\phi}$$

Flux vortex



Feynman, 55
Abrikosov, 57

Superconductor pairing phase



Topological point defects

Vortices in **unconventional** superconductors

Topological defects

- Winding of classical order parameter in real space

+

Topological superconductors

- Winding of quantum states in momentum space

Something really amazing !!!

Topological point defects

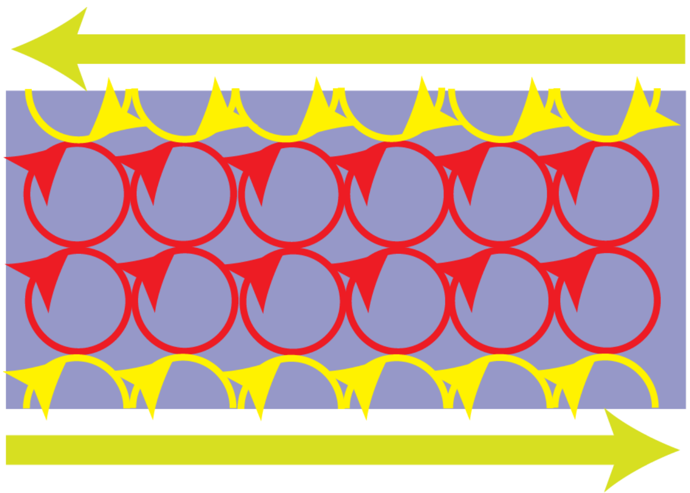
Vortices in chiral $p+ip$ superconductors

Pairing order: time reversal breaking
odd parity, spin-triplet

$$\langle c^\dagger(\mathbf{k})c^\dagger(-\mathbf{k})\rangle \sim \Delta(k_x+ik_y) = |\Delta|e^{i\phi}(k_x+ik_y)$$

Bulk-boundary correspondence

Quantum Hall effect



Chiral charged complex (Dirac) fermion

Chiral p-superconductor

Chiral neutral real (Majorana) fermion

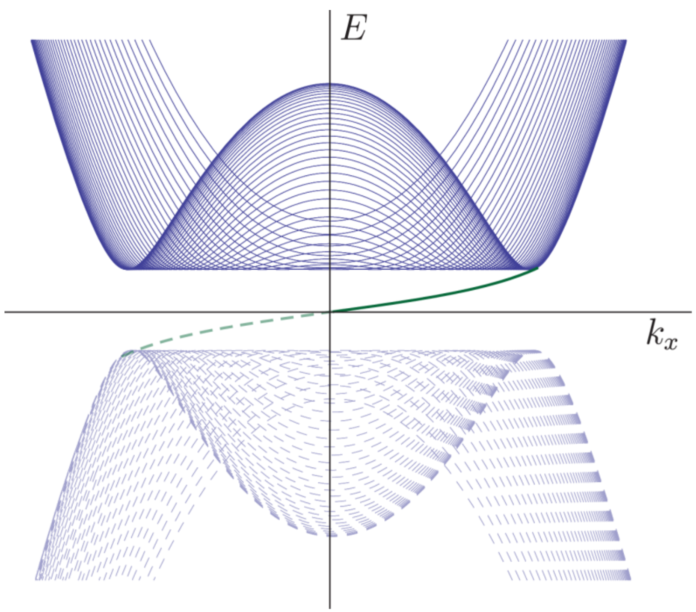
Topological point defects

Vortices in chiral $p+ip$ superconductors

Pairing order: time reversal breaking
odd parity, spin-triplet

$$\langle c^\dagger(\mathbf{k})c^\dagger(-\mathbf{k})\rangle \sim \Delta(k_x + ik_y) = |\Delta|e^{i\phi}(k_x + ik_y)$$

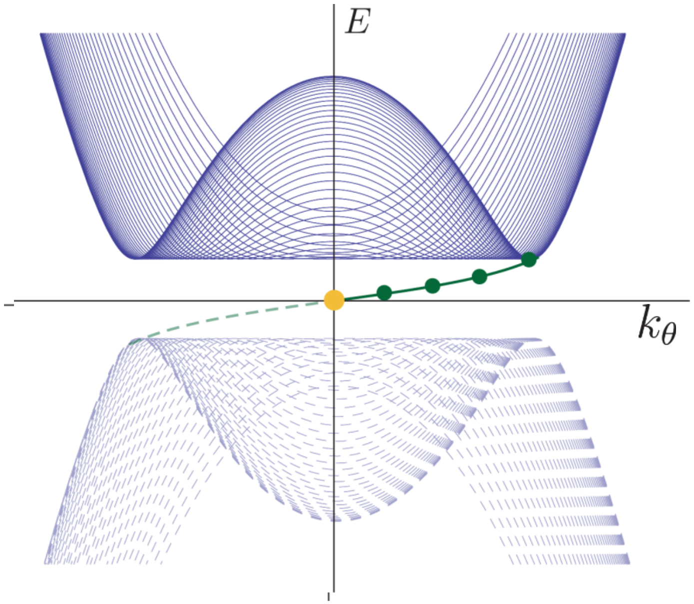
Bulk-boundary correspondence



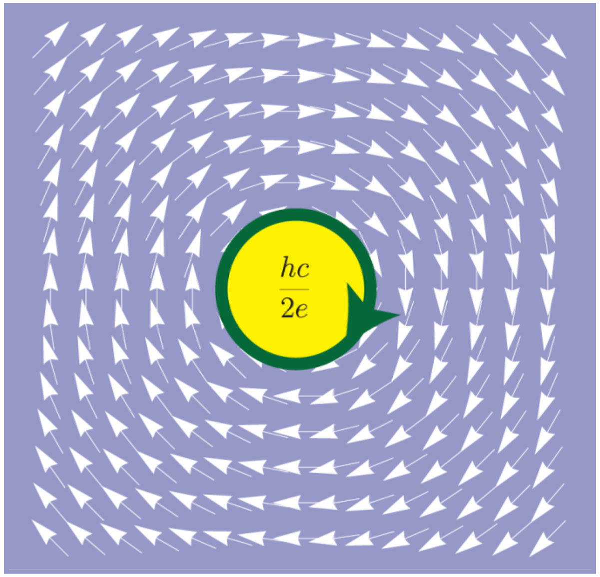
Topological point defects

Vortices in chiral $p+ip$ superconductors

Discrete chiral Majorana modes



Chiral p-superconductor with a flux vortex



Periodic boundary condition:

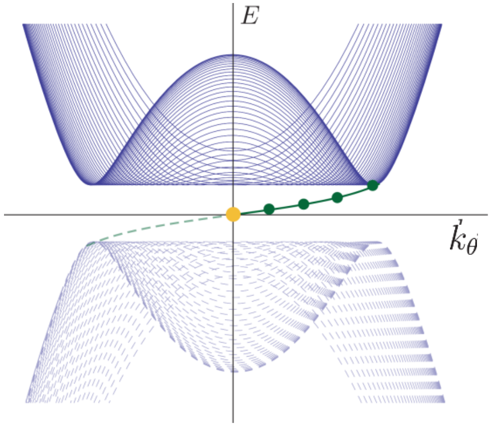
360-deg rotation of fermion = -1

π Berry phase from flux vortex

$$e^{2\pi i k_\theta R} = +1$$

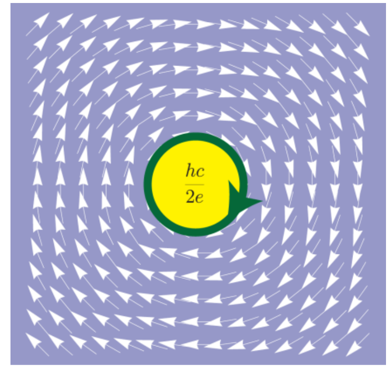
N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000)

Ising anyons



Zero energy Majorana mode

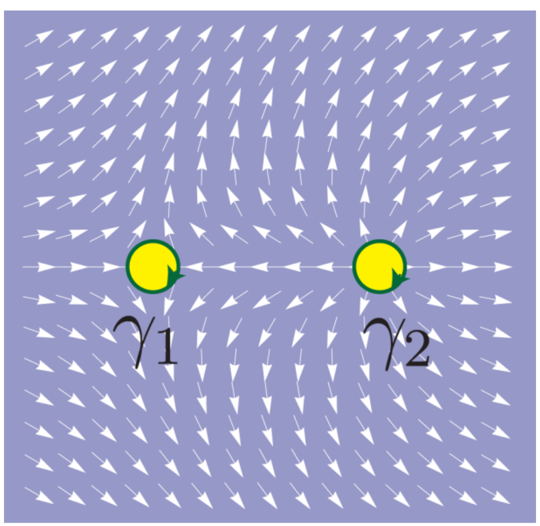
@



Flux vortex

= **Ising anyon**

Zero energy Majorana (real) fermion operators



$$\gamma_i^\dagger = \gamma_i$$

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta_{ij}$$

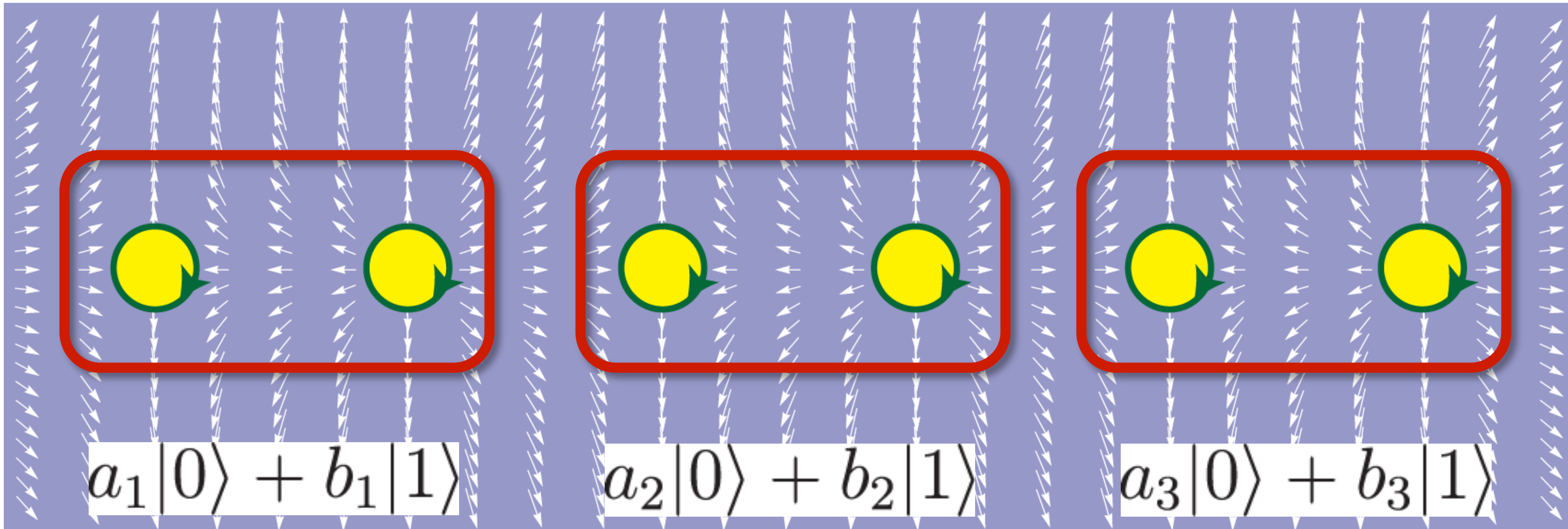
$$\hat{c} = \frac{\gamma_1 + i\gamma_2}{2}$$

$$|0\rangle$$

$$|1\rangle = \hat{c}^\dagger|0\rangle$$

N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000)

Non-local Quantum Information Storage



$$\dim = 2^{n/2-1}$$

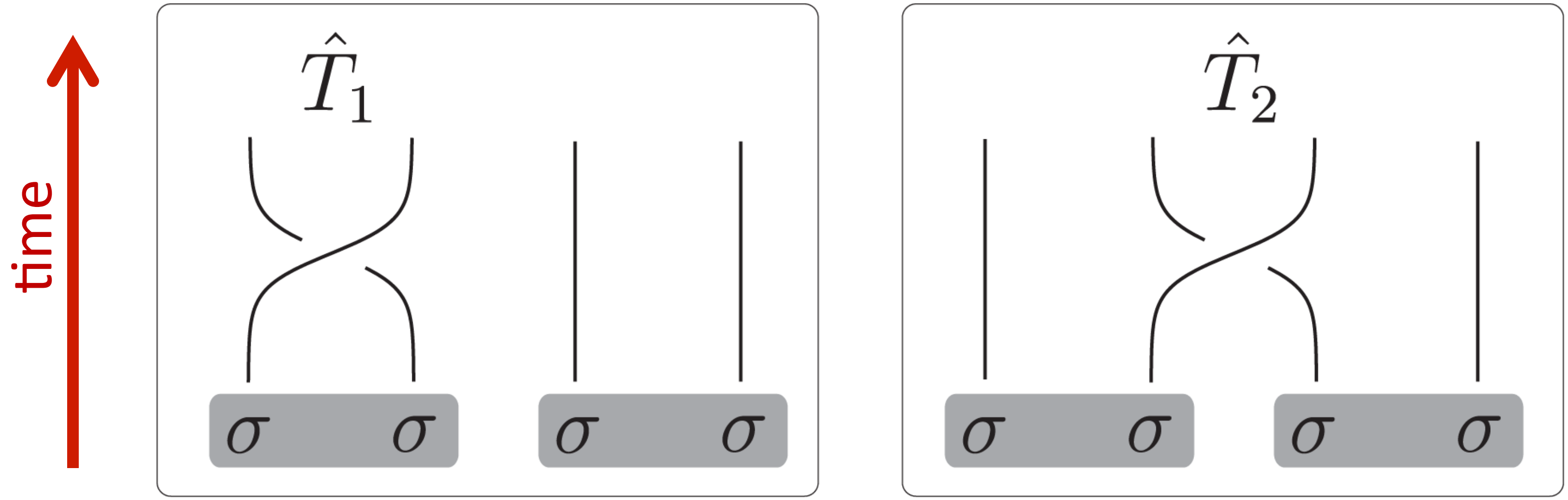
Qubits

$$d_\sigma = \sqrt{2}$$

Robust against

local perturbation
accidental measurement
quantum decoherence

Non-Abelian braiding operations



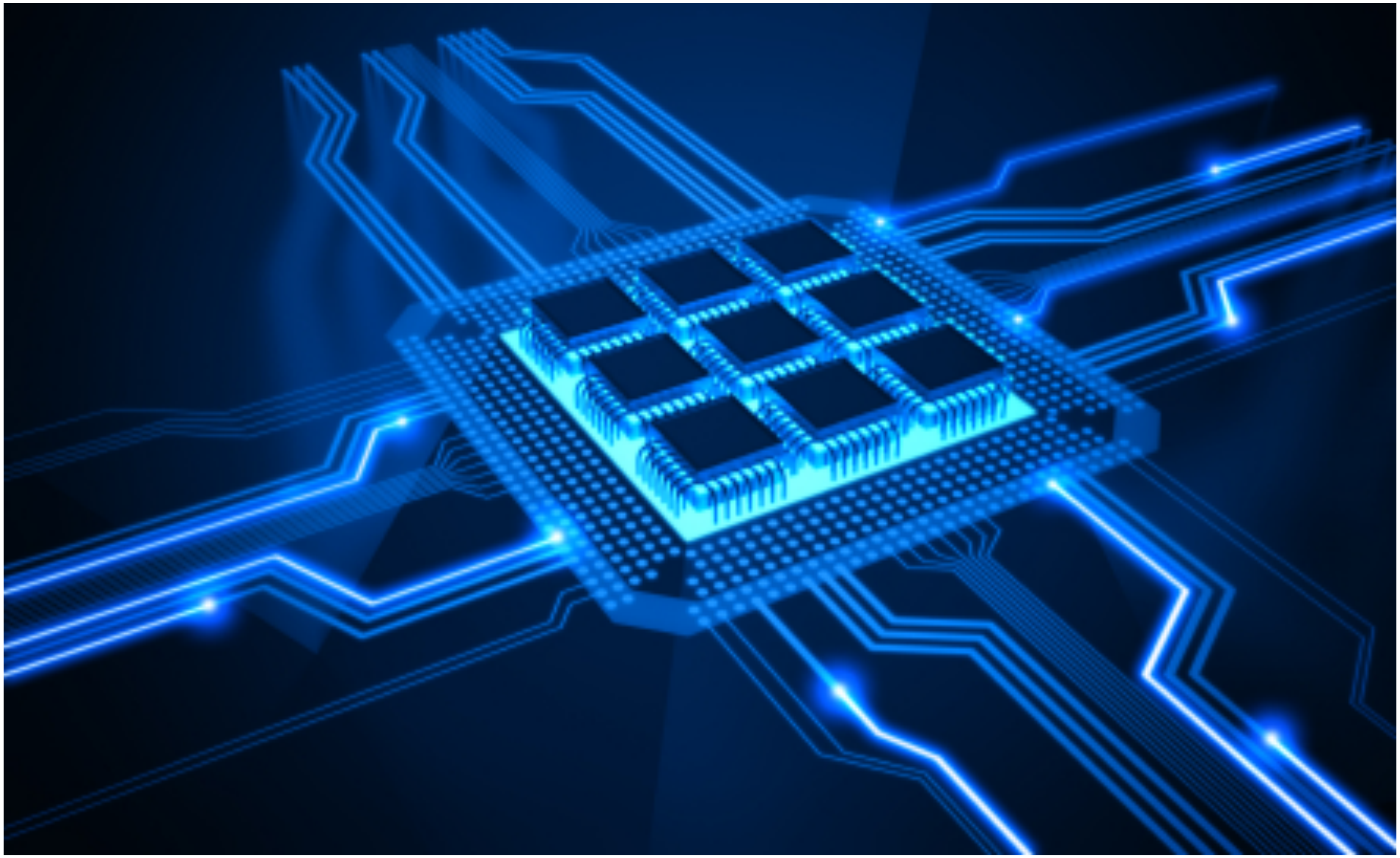
Ground states: $|00\rangle, \quad |11\rangle$

$$\hat{T}_1 \propto \exp\left(i\frac{\pi}{4}\sigma_z\right) \qquad \hat{T}_2 \propto \exp\left(i\frac{\pi}{4}\sigma_x\right)$$

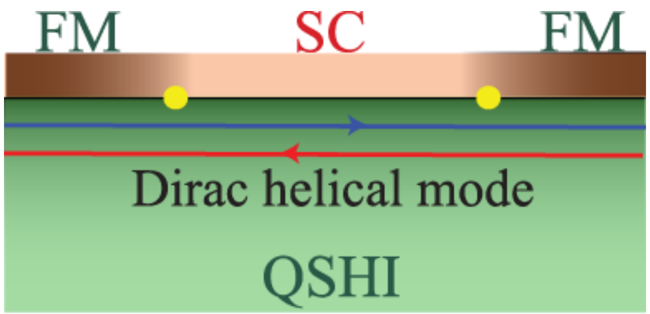
Entangled state: $\hat{T}_2|00\rangle = \dfrac{|00\rangle + i|11\rangle}{\sqrt{2}}$

C. Nayak and F. Wilczek, Nucl. Phys. B **479**, 529 (1996); D.A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001)

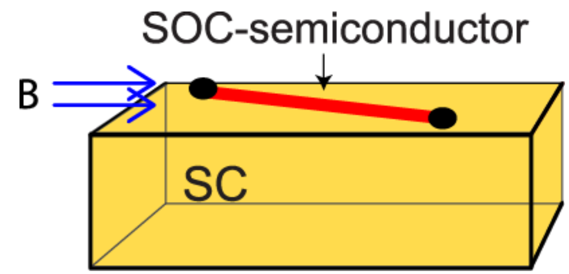
Topological Quantum Computing



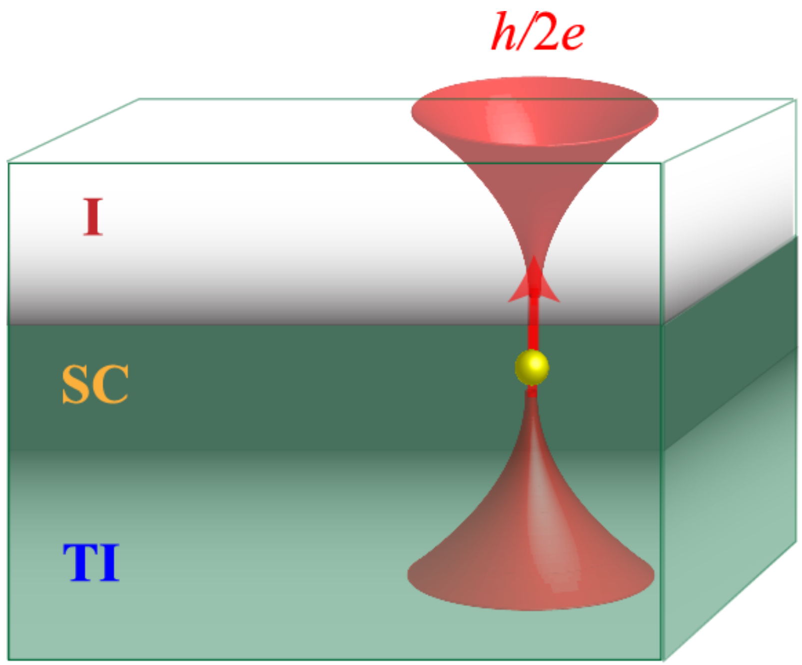
Majoranas at proximity interfaces



Fu and Kane, 09



Sau, Lutchyn, Tewari, Das Sarma, 10

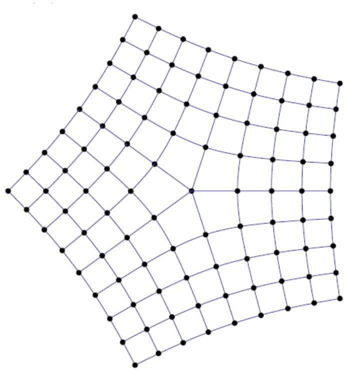


J.C.Y. Teo and C.L. Kane,
Phys. Rev. Lett. **104**, 046401 (2010)
Phys. Rev. B **82**, 115120 (2010)

- # What is the problem?
  - Moore-Read $\nu=5/2$ FQH state requires very low temperature, high mobility and high magnetic field
  - Is Sr_2RuO_4 chiral? (Raghu, Kapitulnik, Kivelson, 2010)
  - TI-SC-FM heterostructures require smooth interface
  - Majoranas in **non-chiral homogeneous** materials in reasonable temperature **without external magnetic field**?
  - Non-abelian objects from abelian systems?

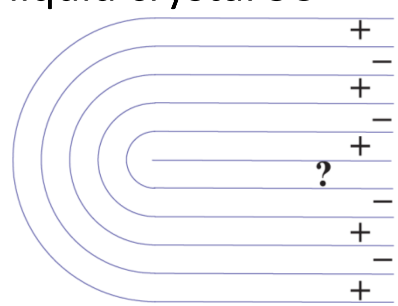
- # Defect related topics

Disclinations in topological crystalline superconductors



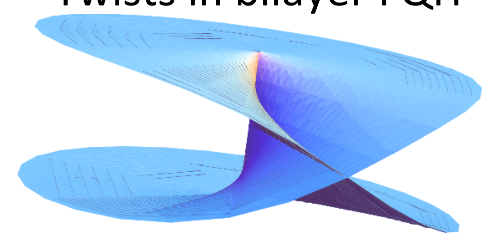
JT , Hughes
PRL **111**, 047006 (2013)

Fractional vortices in liquid crystal SC



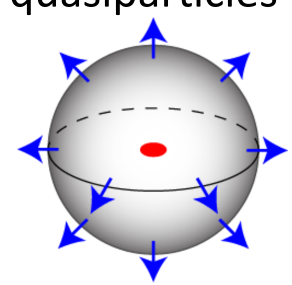
Gopalakrishnan, **JT**, Hughes;
PRL **111**, 025304 (2013)

Twists in bilayer FQH



JT, Roy, Chen; arXiv:1308.5984 (2013)

Ising quasiparticles in 3D



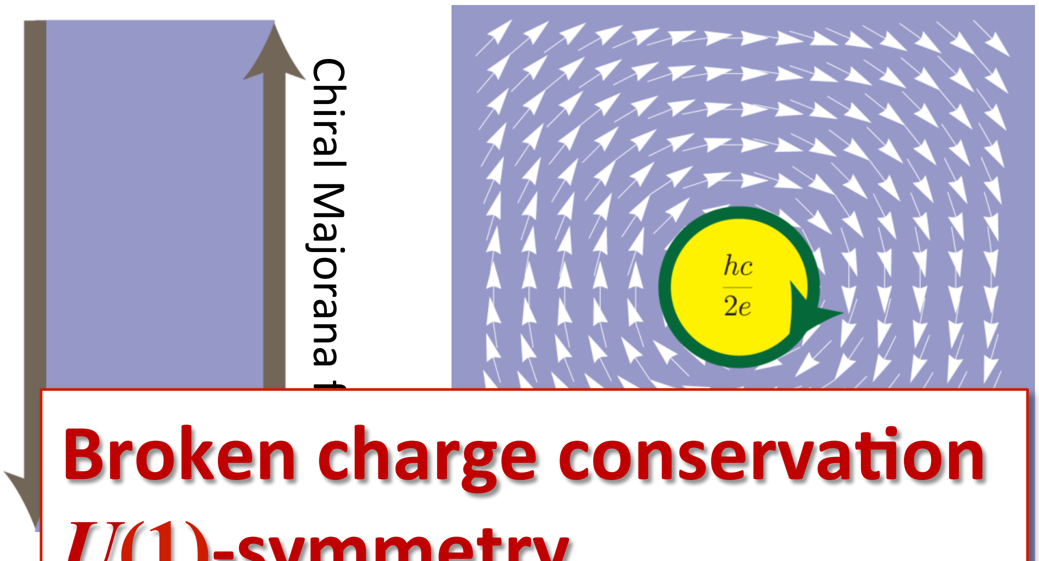
JT, Kane; PRL **104**, 046401 (2010)

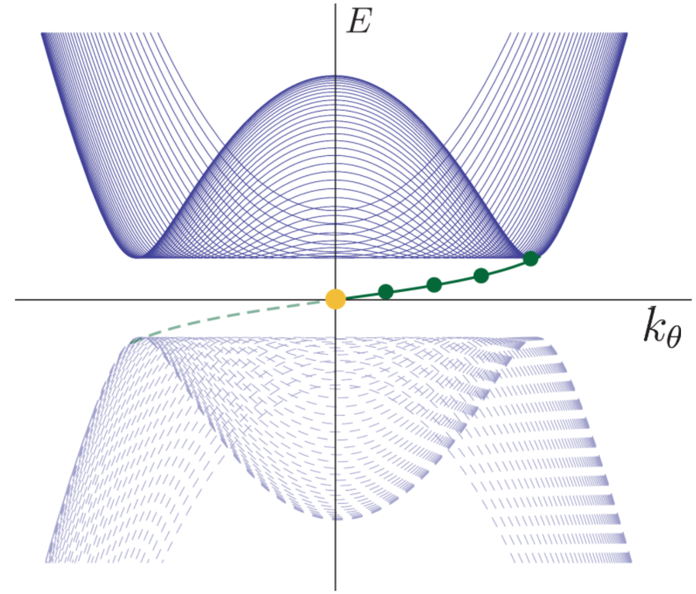
MAJORANA ZERO MODES IN TOPOLOGICAL CRYSTALLINE SUPERCONDUCTORS

JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)
Benalcazar, **JT**, Hughes; arXiv:1311.0496 (2013)

Topological point defects

Vortices in chiral $p+ip$ superconductors





Broken charge conservation
$U(1)$-symmetry

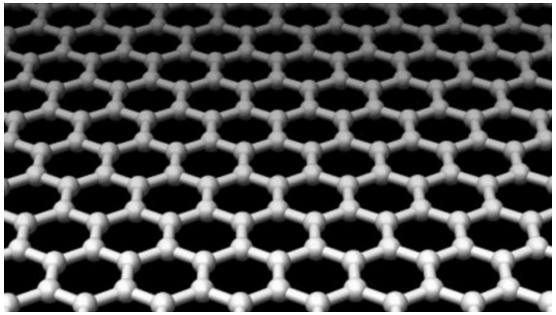
Bulk topology + Winding of order parameter = Majorana zero mode

Are there any broken symmetries?

Topological crystalline defects

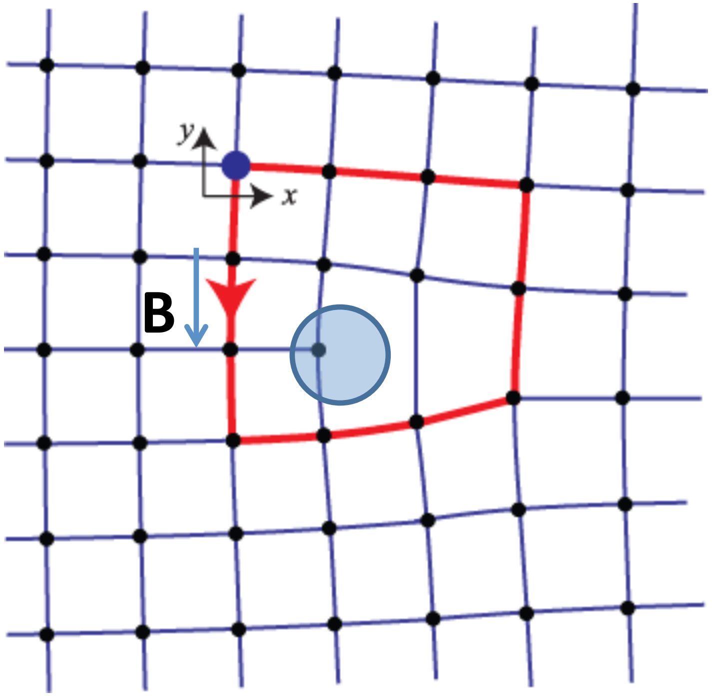
Crystalline order

- Broken translation and rotation symmetry



Dislocation (torsion singularity)

Burgers' vector

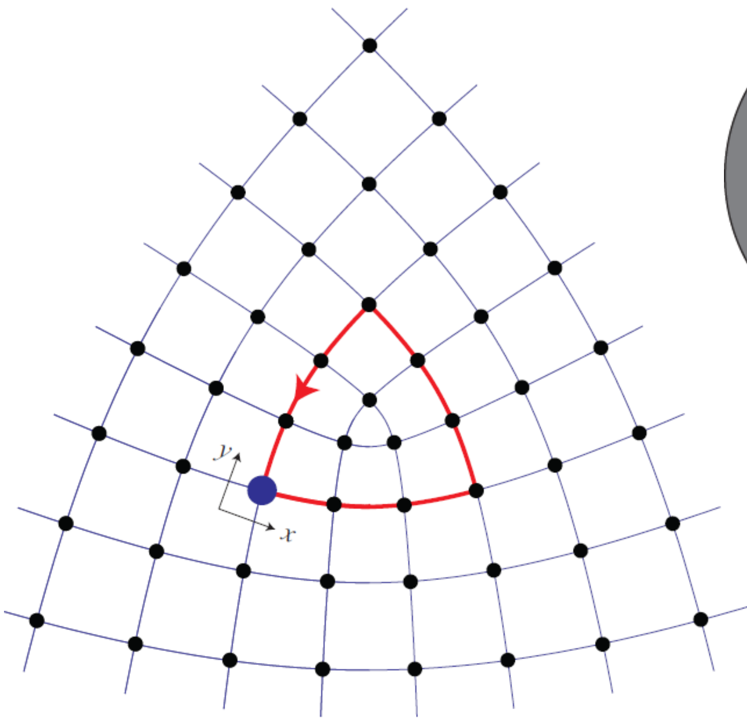


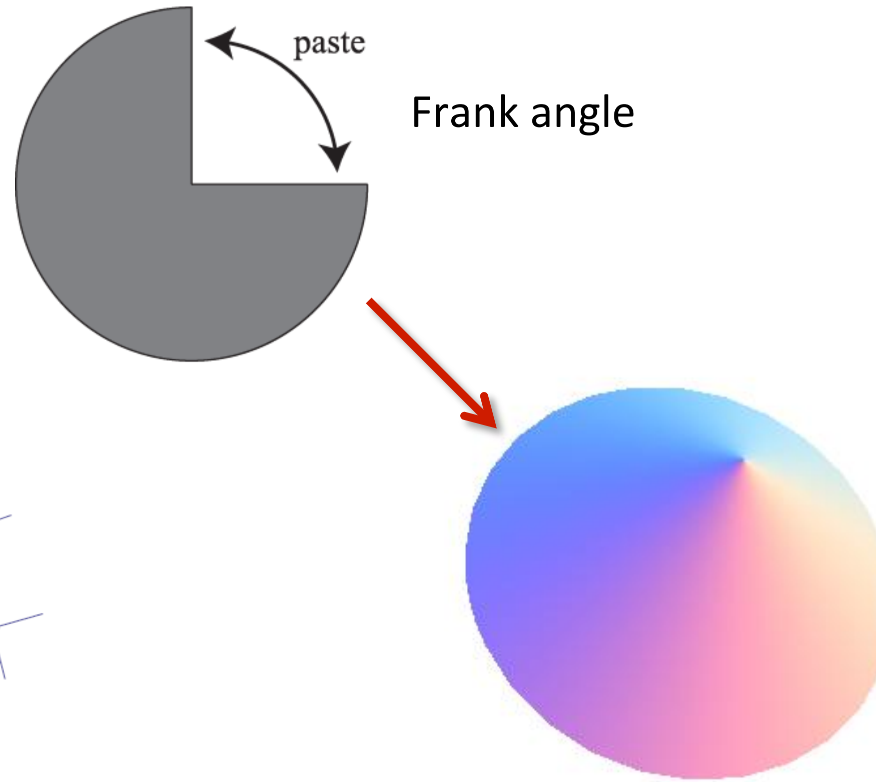
Topological crystalline defects

Crystalline order

- Broken translation and rotation symmetry

Disclination (curvature singularity)



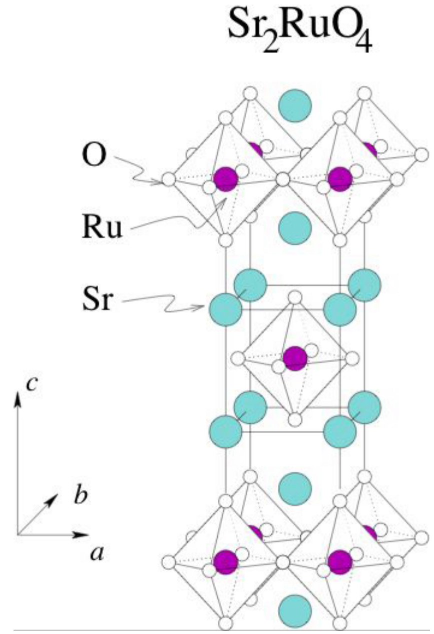


Topological crystalline defects

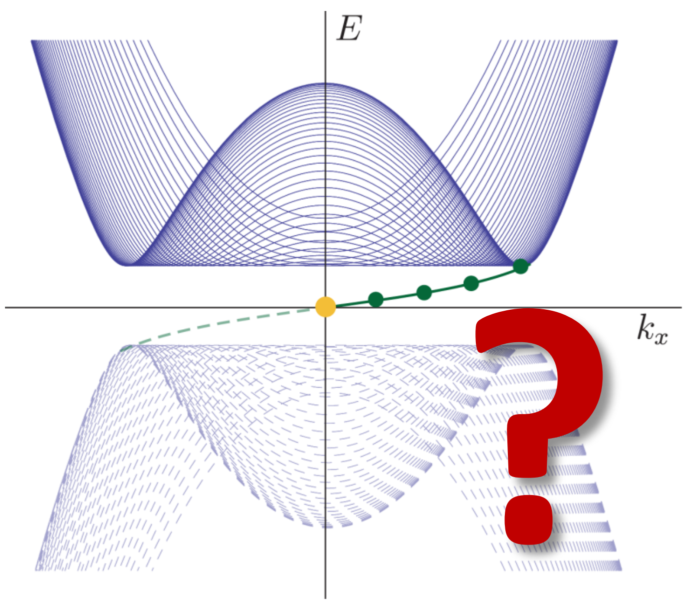
Lattice symmetry protect bulk topology + **Topological crystalline defects**

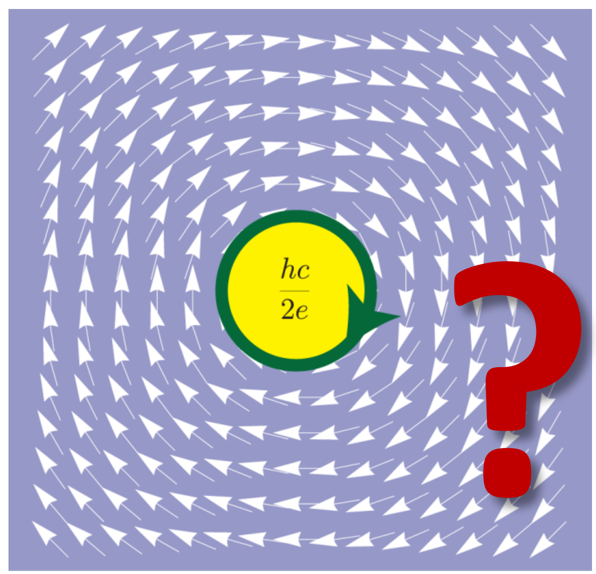
Majorana zero mode

Strontium Ruthenate

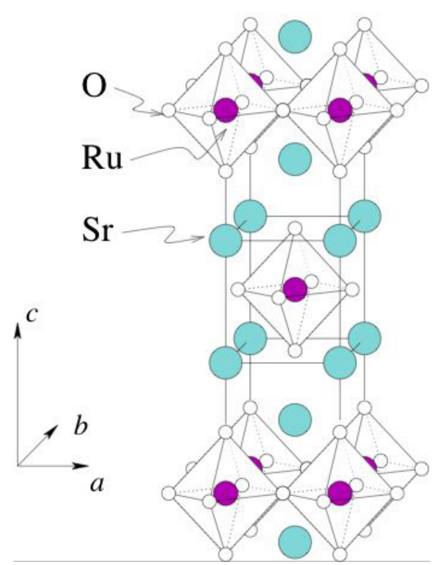


- Layered perovskite structure, quasi-2D, fourfold rotation symmetry
- Unconventional SC (Kidwingira, *et.al.*, 2004)
  - Spin-triplet *p*-wave, breaks time reversal, odd parity
- Chiral $p_x + ip_y$?
  - Weak edge current (Stone, Roy, 2004; Kirtley, *et.al.*, 2007)





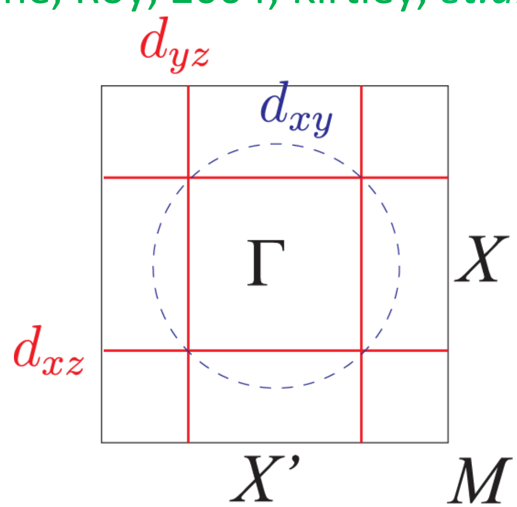
Strontium Ruthenate



- Layered perovskite structure, quasi-2D, fourfold rotation symmetry
- Unconventional SC (Kidwingira, *et.al.*, 2004)
  - Spin-triplet *p*-wave, breaks time reversal and parity
- Chiral p_x+ip_y?
  - Weak edge current (Stone, Roy, 2004; Kirtley, *et.al.*, 2007)
  - ~~d_{xy} ➔ chiral p+ip pairing~~
  - Quasi-1D nature

    from d_{xz}, d_{yz} bands?

    (Raghu, *et.al.*, 2010)

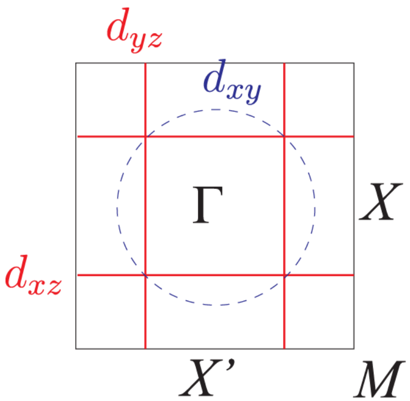


Fermi lines of
Ru d-bands on Brillouin zone

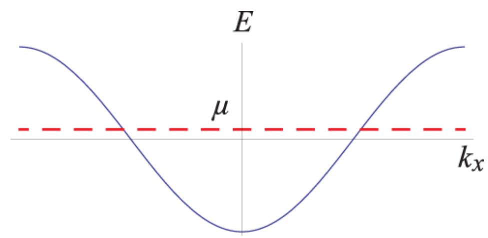
  - STM on superconducting DOS (Firmo, *et.al.*, 2013)
  - Incommensurate antiferromagnetic fluctuations (K. Iida, *et.al.*, 2011)

Strontium Ruthenate

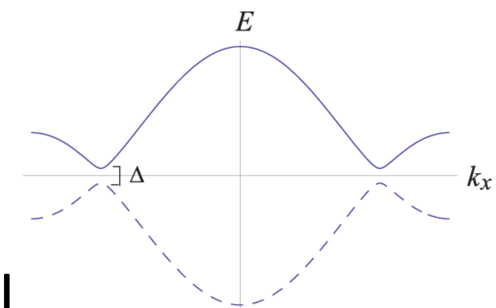
- Quasi-1D model on d_{xz}, d_{yz} bands (Raghu, *et.al.*, 2010)



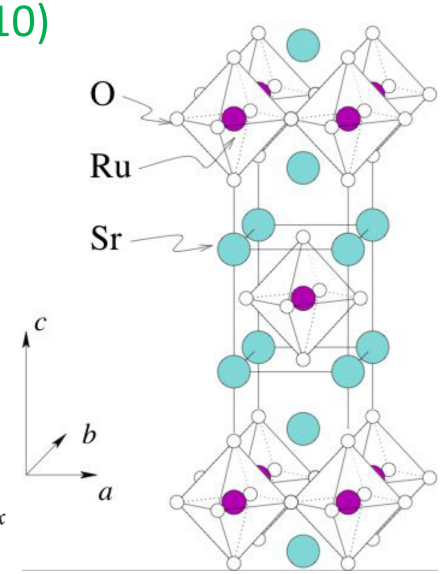
d_{yz} band
➔ 1D metal



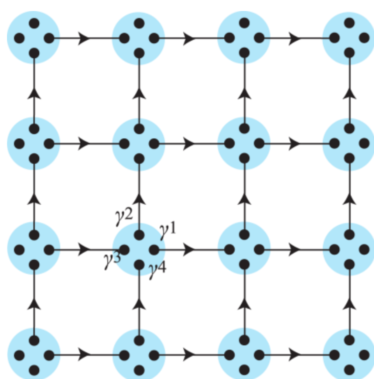
1D spin-triplet
superconductor



Sr_2RuO_4



- Majorana tight binding model



Real space Majorana lattice
representing SC d_{xz}, d_{yz} bands

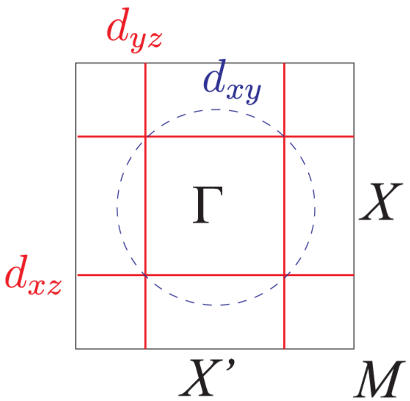
BdG Hamiltonian of SC d_{xz}, d_{yz} bands

$$H = [(t\cos k_x - \mu)\tau_z + \Delta \sin k_x \tau_y] \\ \oplus [(t\cos k_y - \mu)\tau_z + \Delta \sin k_y \tau_y]$$

JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

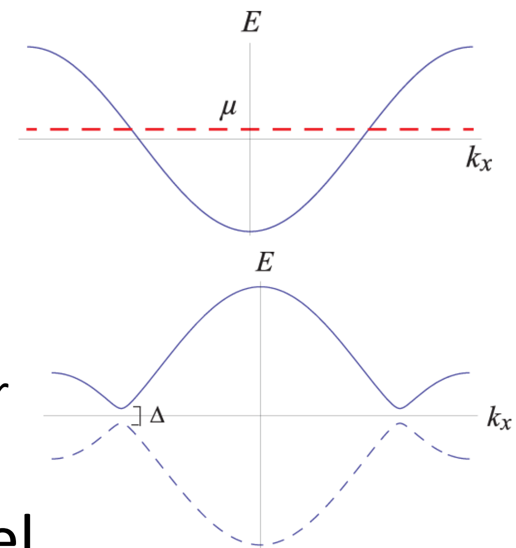
Strontium Ruthenate

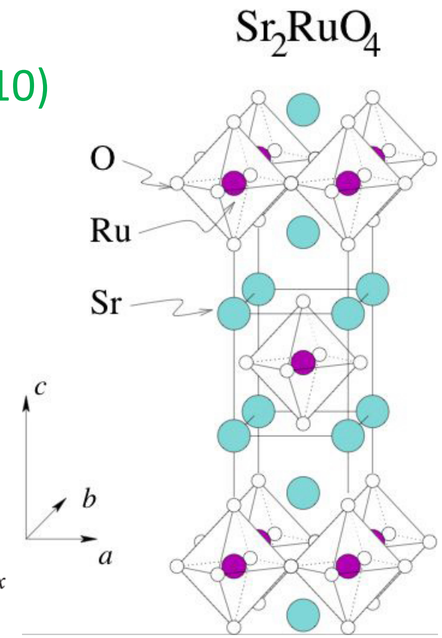
- Quasi-1D model on d_{xz}, d_{yz} bands (Raghu, *et.al.*, 2010)



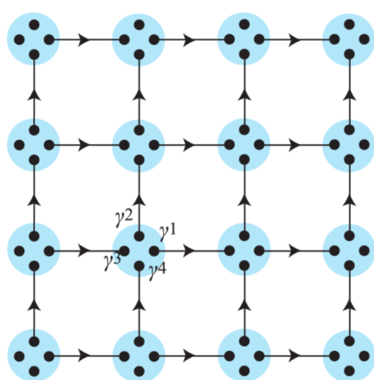
d_{yz} band
➔ 1D metal

1D spin-triplet superconductor

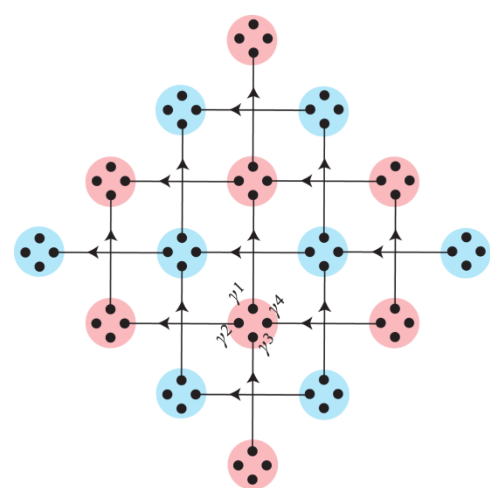




- Majorana tight binding model



Real space Majorana lattice representing SC d_{xz}, d_{yz} bands



Majorana lattice of two layers

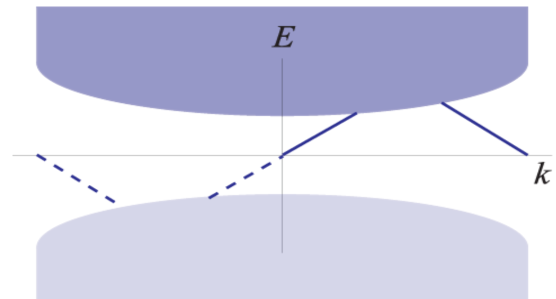
JT, Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Strontium Ruthenate

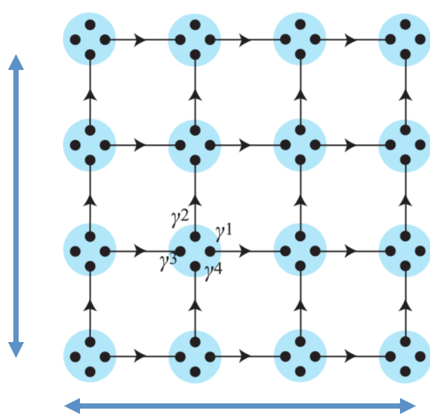
- Is Sr_2RuO_4 topologically trivial?

Hidden translation symmetry protected topology

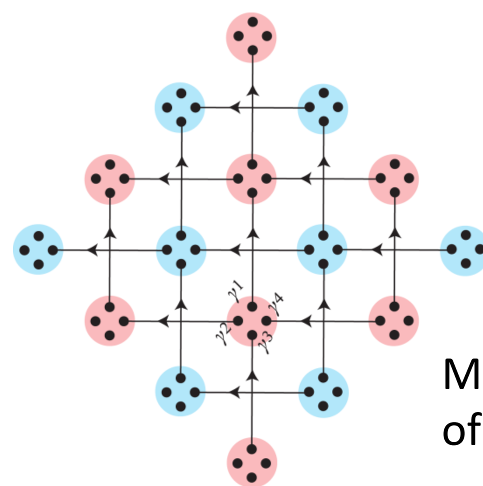
Weak $\mathbf{Z}_2$ topology $\mathbf{G}_\nu = (1, 1)$



- Majorana tight binding model



Real space Majorana lattice representing SC d_{xz}, d_{yz} bands



Majorana lattice of two layers

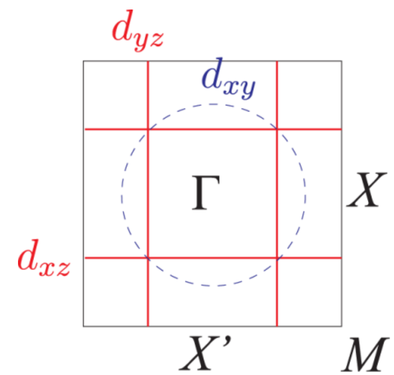
JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Strontium Ruthenate

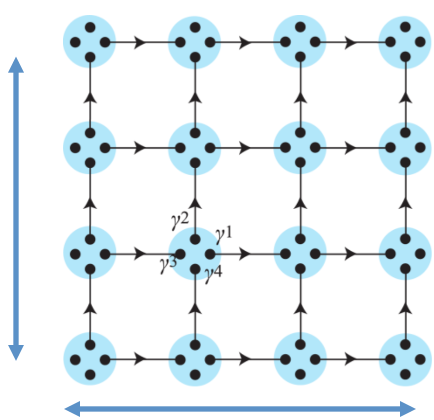
- Is Sr_2RuO_4 topologically trivial?

Hidden rotation symmetry protected topology

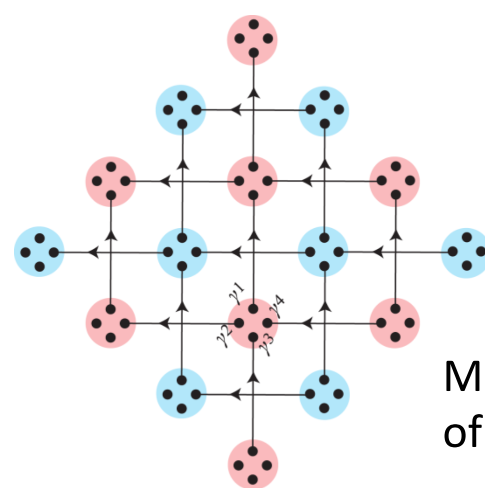
	C_4	C_2
Γ	$\pm e^{-i\pi/4}$	$-i$
M	$\pm e^{+i\pi/4}$	$+i$
X		$\pm i$



- Majorana tight binding model



Real space Majorana lattice representing SC d_{xz}, d_{yz} bands



Majorana lattice of two layers

How to get Majorana bound states?

JT, Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Majorana Bound States at Disclinations

- Kitaev's 1D superconducting chain

$$H = (t\cos k_x - \mu)\tau_z + \Delta \sin k_x \tau_y$$

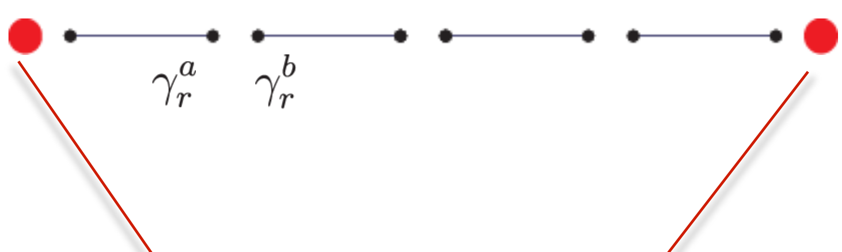
  - Trivial limit $\boxed{t < \mu}$

$$t = \Delta = 0$$

$$c_r = (\gamma_r^a + i\gamma_r^b)/2$$

$$H = i\mu \sum \gamma_r^a \gamma_r^b$$

  - Topological limit $\boxed{t > \mu}$



Majorana zero energy bound state

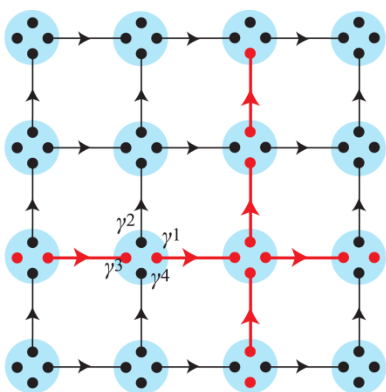
$$\mu = 0, \quad t = \Delta$$

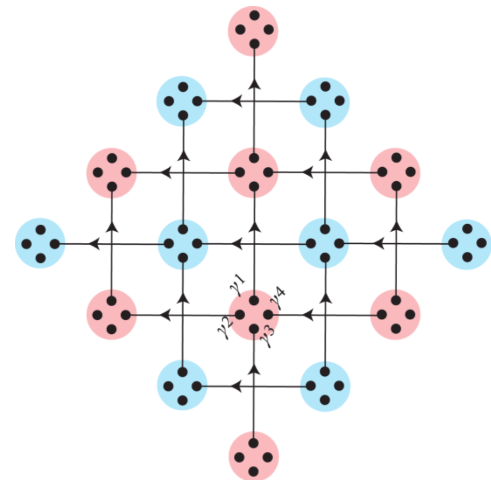
$$H = i\Delta \sum_r \gamma_r^a \gamma_{r+1}^b$$

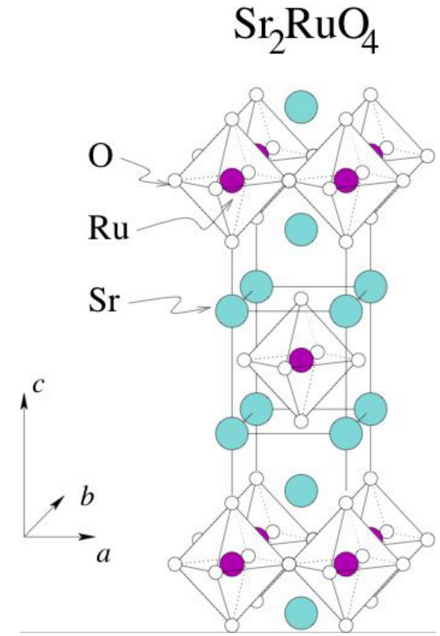
Majorana Bound States at Disclinations

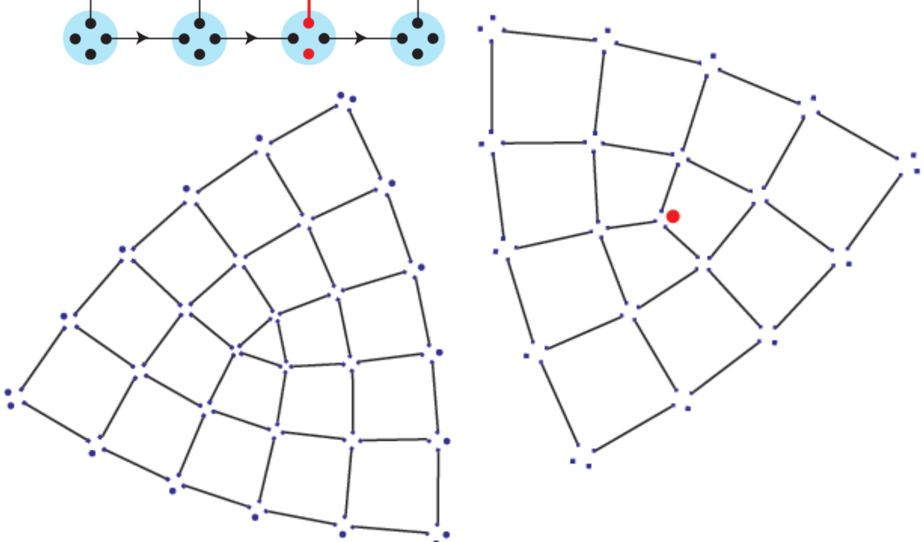
- The tight binding models of Sr_2RuO_4

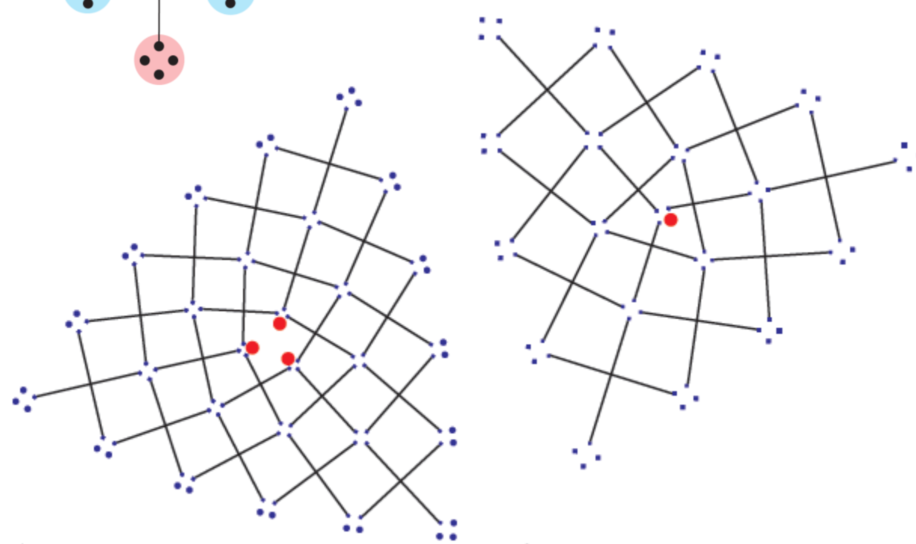
  2D version of Kitaev's chain







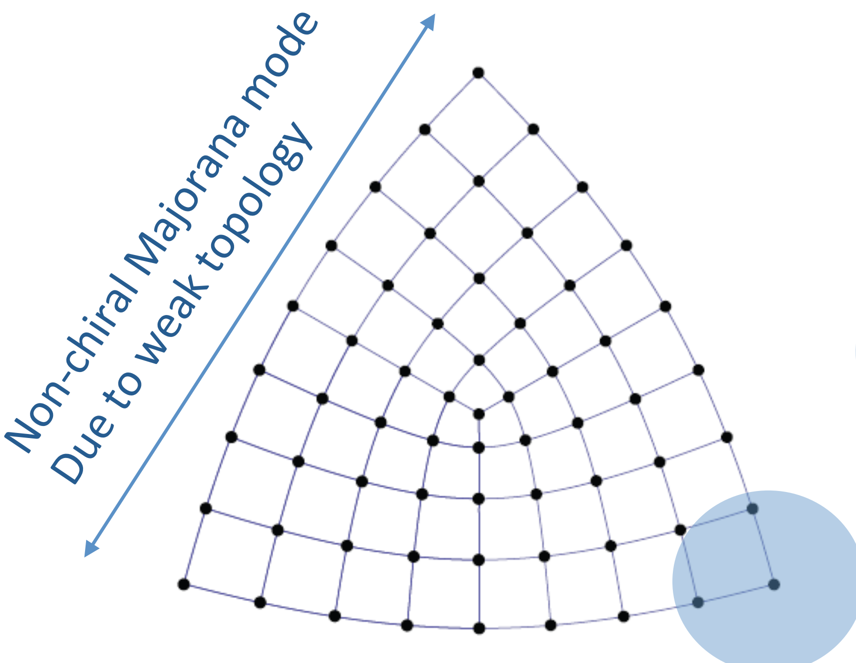


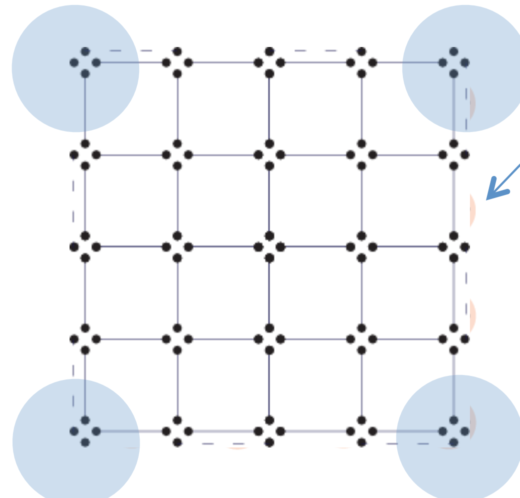


JT, Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Corner MBS in Strontium Ruthenate

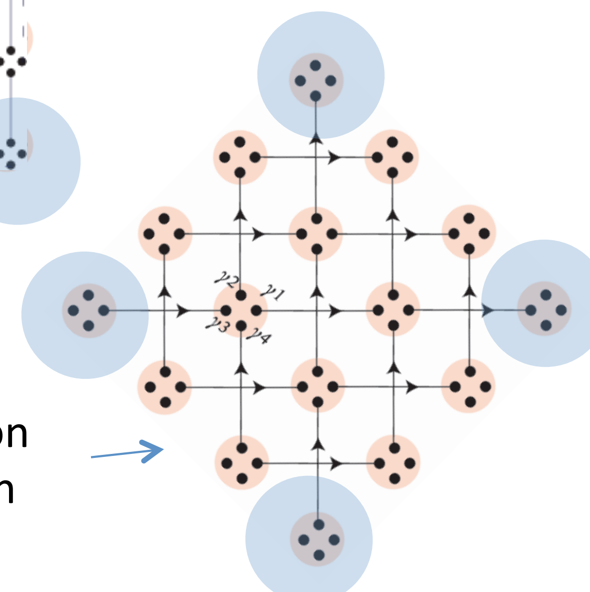
#MBS at disclination = #MBS at corners + #MBS along edges (mod 2)





Gapped out by perturbation

Sr_2RuO_4



Gapped out by on site hybridization

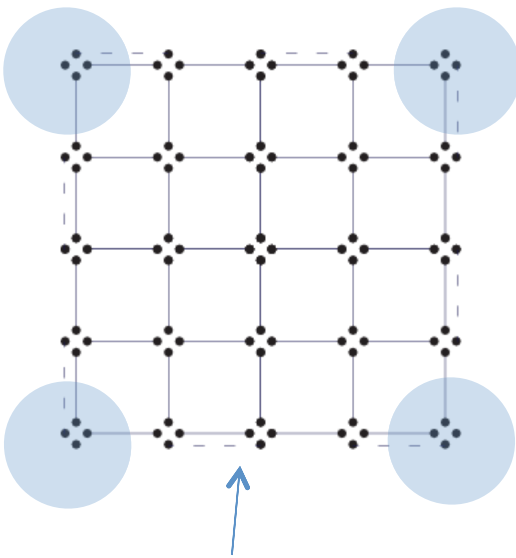
$2 \times Sr_2RuO_4$

JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Non-chirality is actually good!!

Quasi-1D
Sr_2RuO_4

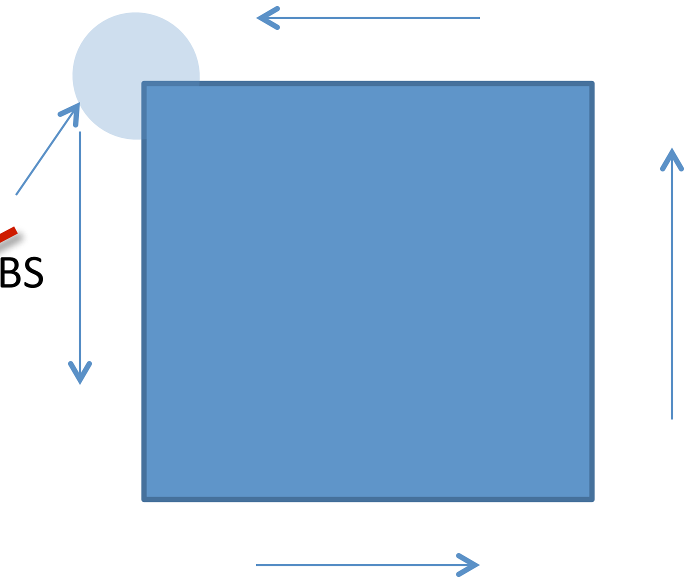
Corner MBS



Gapped out by
perturbation

Chiral p+ip
Sr_2RuO_4

~~Corner MBS~~



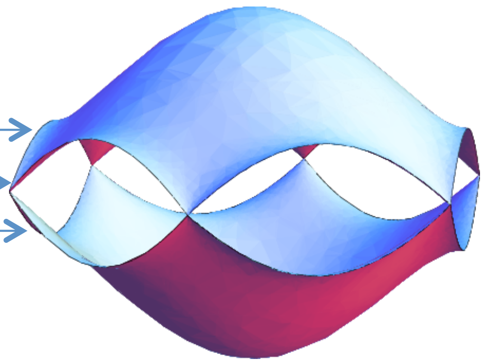
Robust against
perturbations

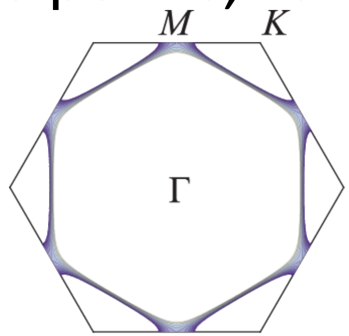
JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Doped Graphene / Silicene

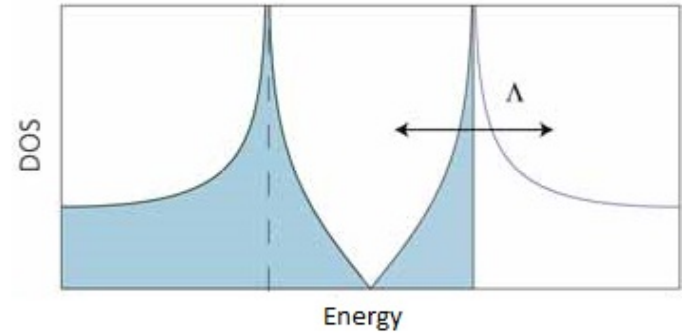
- Filling ν~5/8 (or 3/8)
- Fermi energy at M saddle points, van Hove singularity in DOS

$\nu = 5/8$
$\nu = 1/2$
$\nu = 3/8$



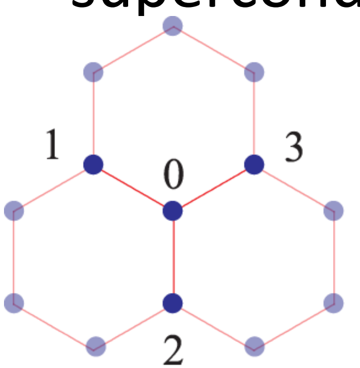


Fermi line



Nandkishore, *et.al.*, 2012
Black-Schaffer, *et.al.*, 2007

- Chiral $d_{xy} + i d_{x^2-y^2}$ spin-singlet superconductivity



$$\Delta = (\Delta_1, \Delta_2, \Delta_3)$$

$$\Delta_1(c^\dagger_{0\uparrow}c^\dagger_{1\downarrow} - c^\dagger_{0\downarrow}c^\dagger_{1\uparrow}) + h.c.$$

$$\Delta_2(c^\dagger_{0\uparrow}c^\dagger_{2\downarrow} - c^\dagger_{0\downarrow}c^\dagger_{2\uparrow}) + h.c.$$

$$\Delta_3(c^\dagger_{0\uparrow}c^\dagger_{3\downarrow} - c^\dagger_{0\downarrow}c^\dagger_{3\uparrow}) + h.c.$$

$$\Delta_s \propto (1, 1, 1)$$

$$\boxed{\begin{aligned}\Delta_{d_{xy}} &\propto (0, 1, -1)\\ \Delta_{d_{x^2-y^2}} &\propto (2, -1, -1)\end{aligned}}$$

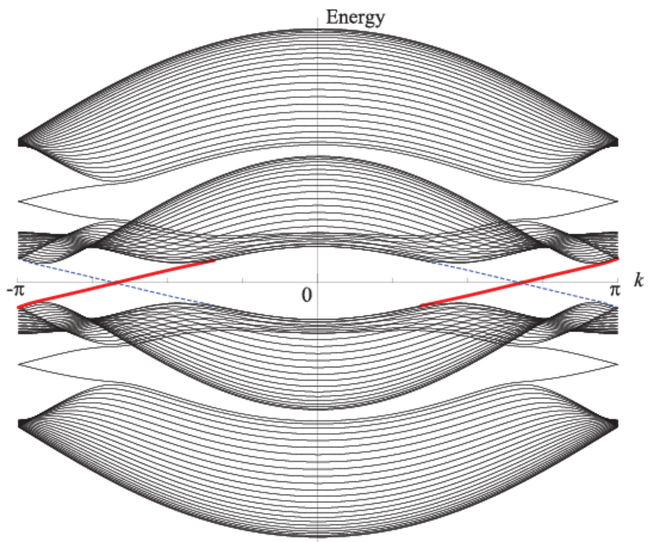
$$\rightarrow \Delta_{d_{xy}} + i\Delta_{d_{x^2-y^2}}$$

Doped Graphene / Silicene

- Broken threefold rotation and time reversal symmetry

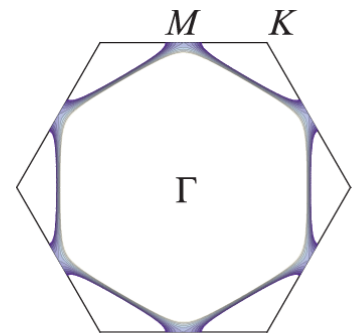
$$\Delta_{d_{xy}} + i\Delta_{d_{x^2-y^2}}$$

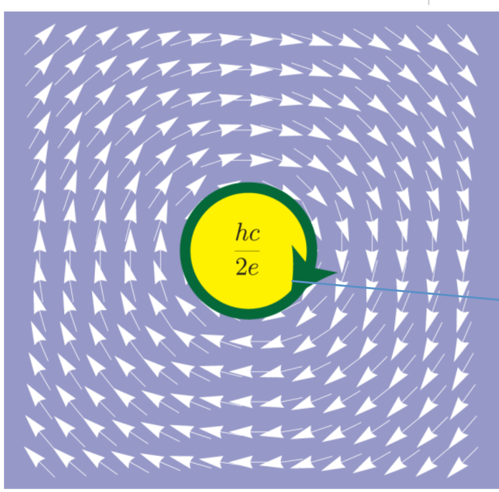
Zigzag edge



$ch = 2$

	C_2
Γ	$\pm i$
M	$\pm i$





Even number of
Majorana zero modes
➔ Not protected

How to get protected Majorana bound states?

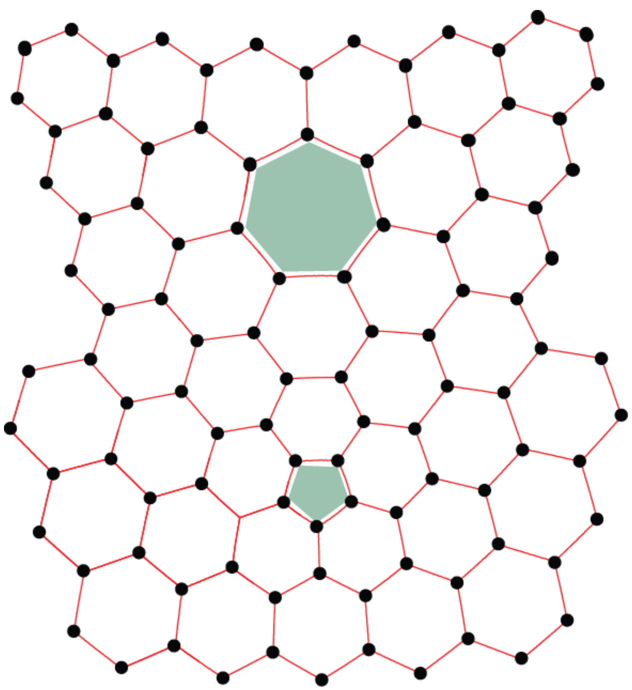
Benelcazar, **JT**, Hughes; arXiv:1311.0496 (2013)

Majoranas at Conical Defects in Graphene

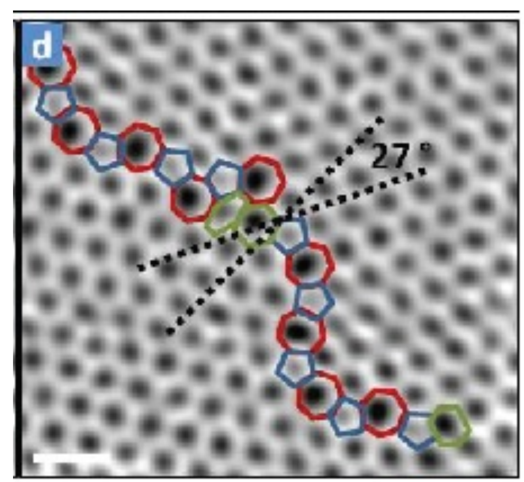
- MBS at 180-deg disclination

$$\Theta = \frac{1}{2\pi}\mathbf{T}\cdot \cancel{\mathbf{G}_\nu} + \frac{\Omega}{2\pi}(ch + \cancel{[M]})$$

- 180 = 60 + 60 + 60 ➔ MBS at 60-deg disclination



Majorana 1D chain along grain boundary?

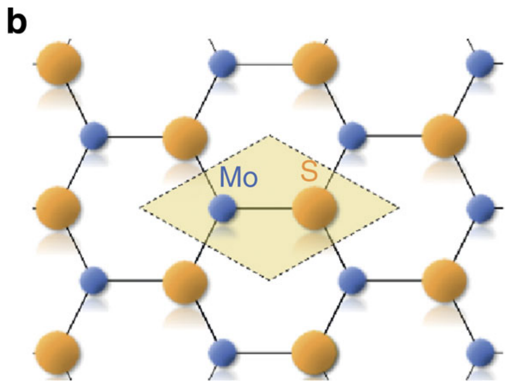


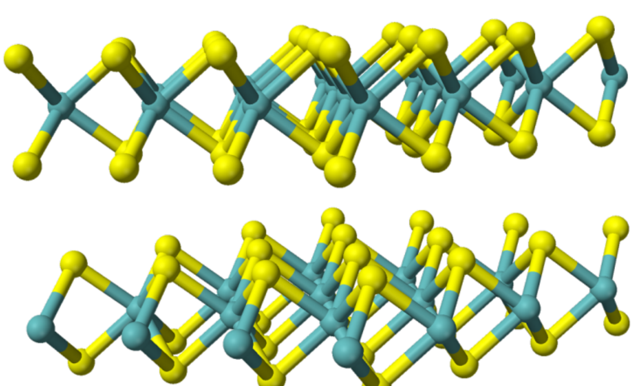
ADF-STEM image
Huang, *et.al.*, 2010

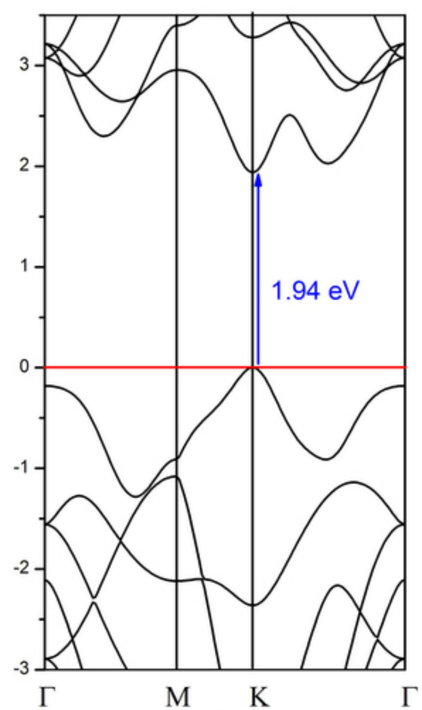
Benelcazar, **JT**, Hughes; arXiv:1311.0496 (2013)

Majoranas at Dislocations in MoS_2

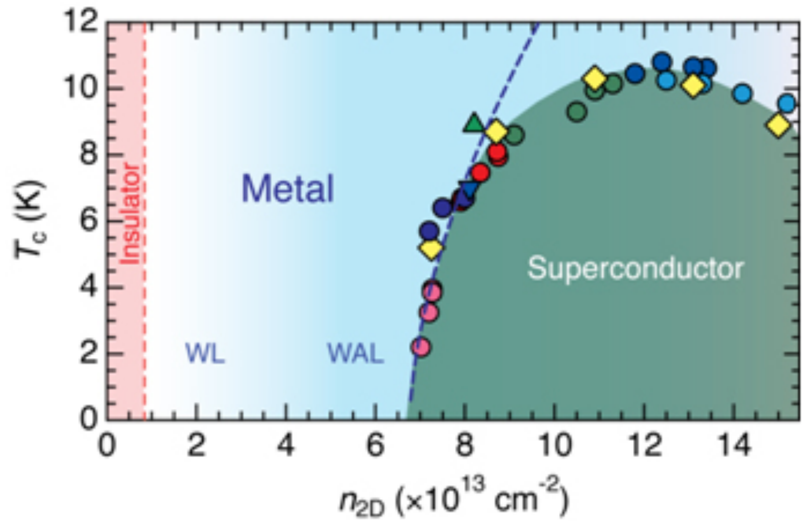
- Drawback of doped graphene / silicene
  - No observed superconductivity
  - Need huge dopant amount or gate voltage → alter band structure
- Superconducting MoS_2







Terrones, *et.al.* 2013

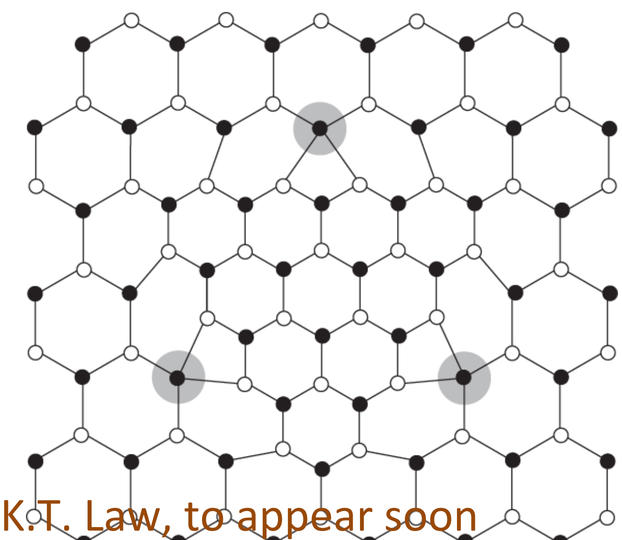


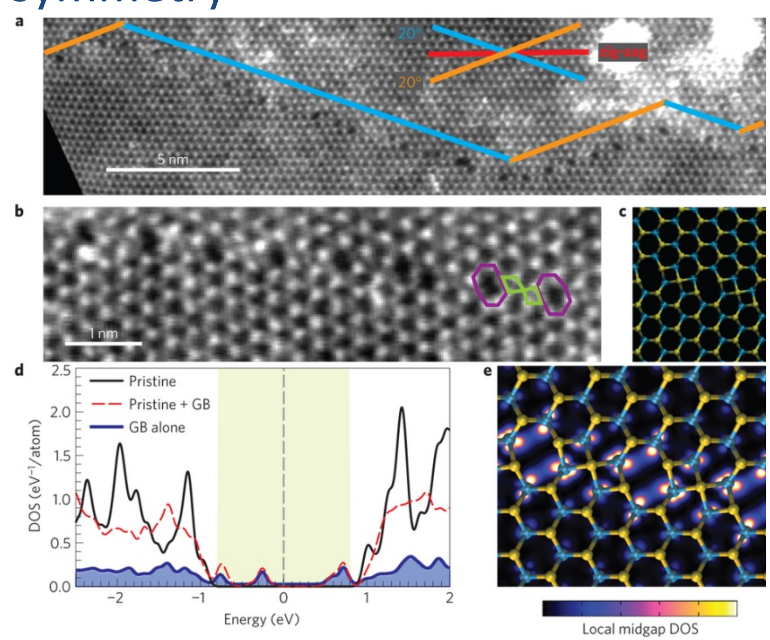
Ye, *et.al.*, Science 338, 1193 (2012)
Taniguchi, *et.al.*, Appl. Phys. Lett. 101, 042603 (2012).

Majoranas at Dislocations in MoS_2

- Drawback of doped graphene / Silicene
  - No observed superconductivity
  - Need huge dopant amount or gate voltage → alter band structure
- Superconducting MoS_2
  - Possible $d+id$ superconducting phase
  - Breaks 3-fold symmetry
  - Weak topology protected by translation symmetry
  - Majorana zero mode at dislocation

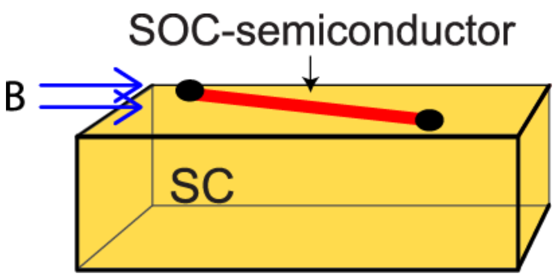
van der Zande, et.al., Nature materials, (2013)





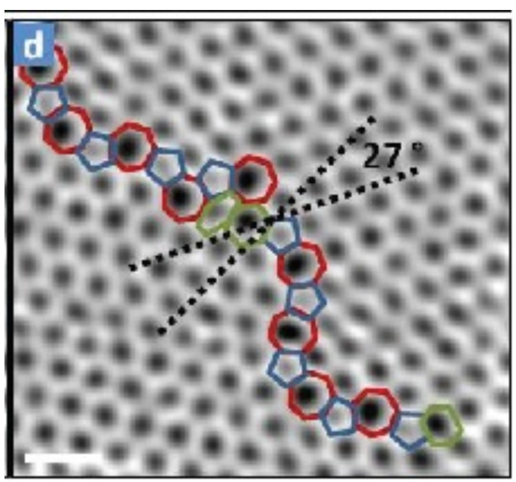
Noah F.Q. Yuan, **JT**, K.T. Law, to appear soon

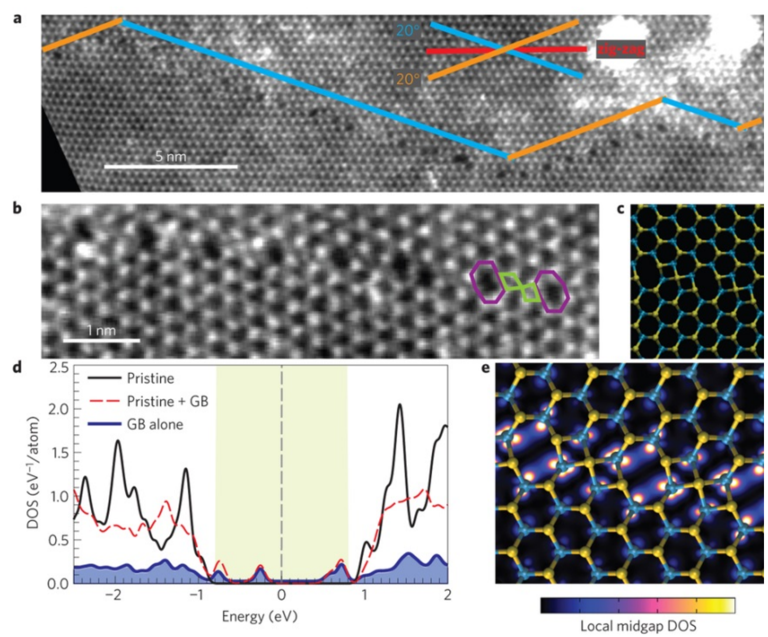
Majorana chain along grain boundaries



Sau, Lutchyn, Tewari, Das Sarma, 10

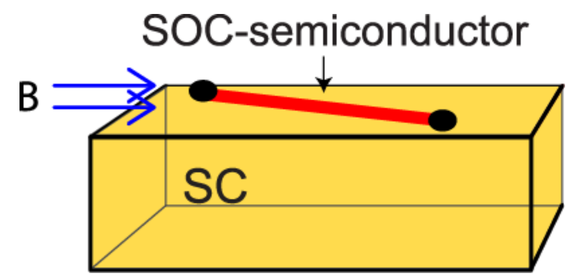
Kitaev, 01

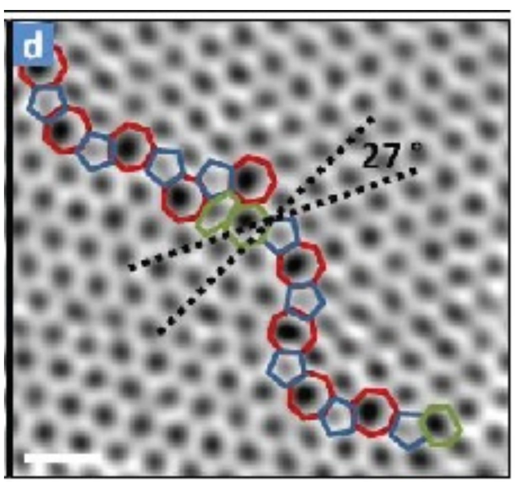


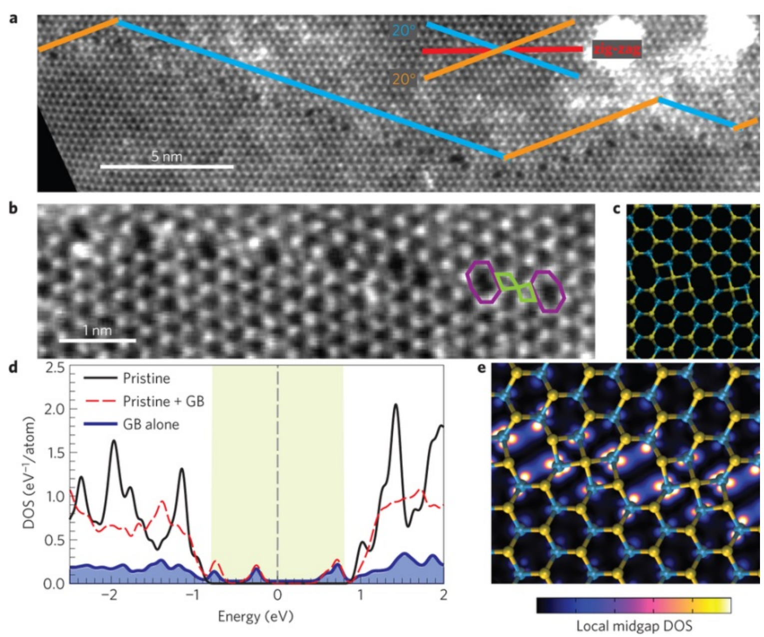


Majorana chain along grain boundaries

- Advantages
  - No heterostructures or proximity induced superconductivity
  - Clean system
  - Possible STM experiments







Conclusion

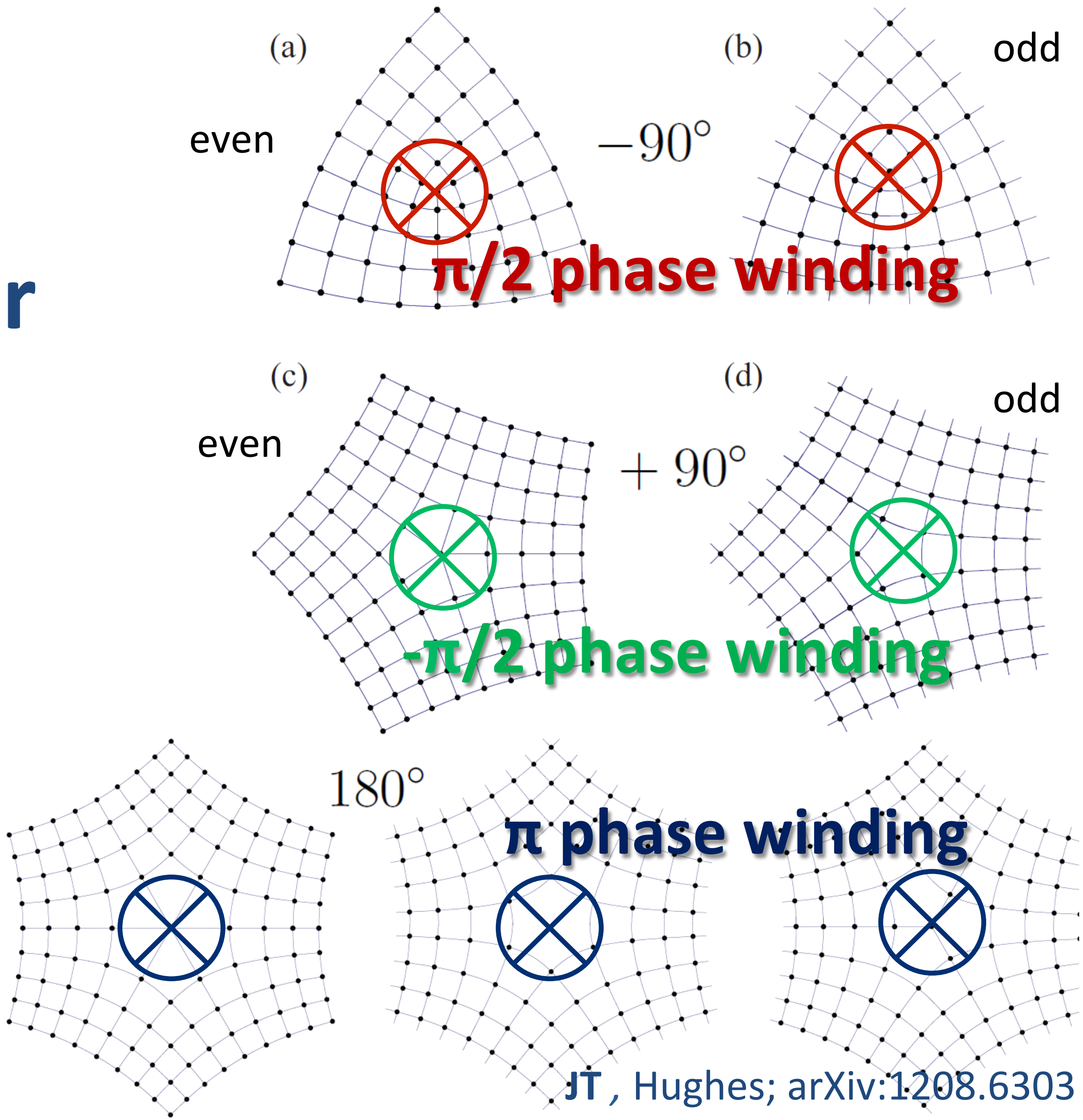
Topological defects + Topological superconductors

Something really amazing !!!

- Majorana bound states in Topological crystalline SC
  - Dislocation and disclination
  - Crystalline symmetry protected topology
  - Strontinum Ruthenate
  - Doped graphene and silicene
  - MoS_2
- What's next?
  - Revisit materials with overlooked topology

Fractional Vortices

Say *p*-wave pairing order



JT , Hughes; arXiv:1208.6303

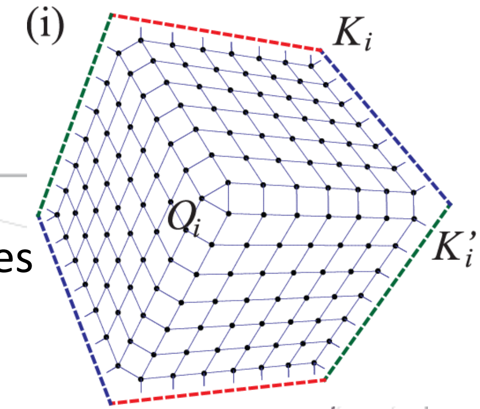
How about a non-tight binding limit?

- Chiral p+ip model

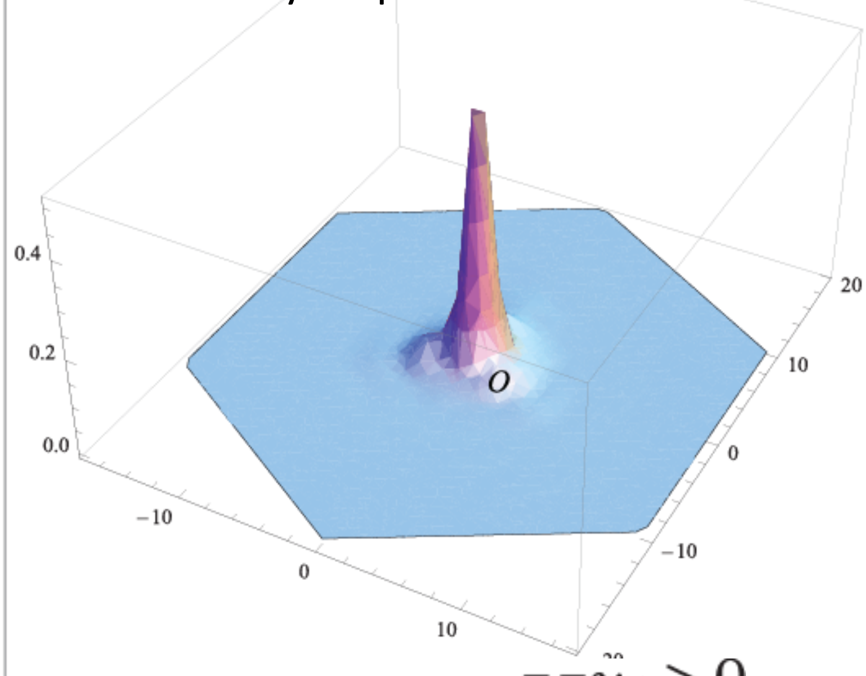
$$H_a = \Delta(\sin k_x \tau_x + \sin k_y \tau_y) + u_1(\cos k_x + \cos k_y)\tau_z + 2u_2 \cos k_x \cos k_y \tau_z$$

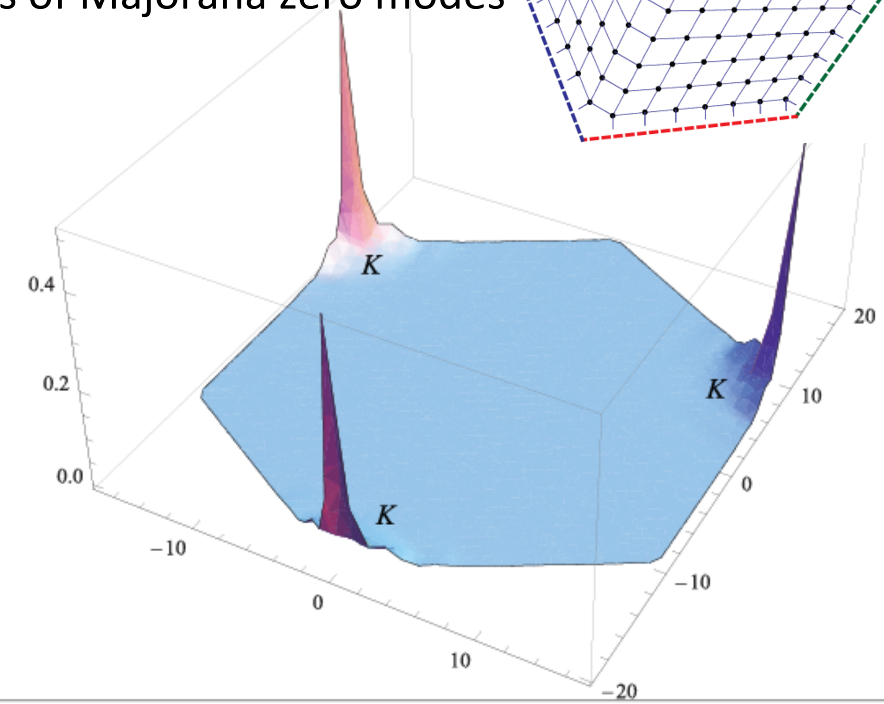
$$|u_1| > |u_2| > 0$$

Exponentially localized zero modes $\boxed{H_a^{u_1<0}}$



Probability amplitude for wave functions of Majorana zero modes





No zero mode for $H_a^{u_1>0}$

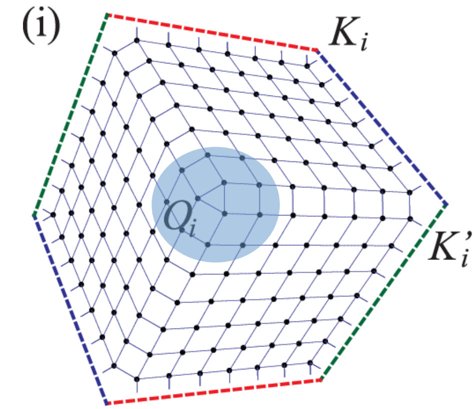
JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

How about a non-tight binding limit?

- Chiral p+ip model

$$H_a = \Delta(\sin k_x \tau_x + \sin k_y \tau_y) + u_1(\cos k_x + \cos k_y)\tau_z + 2u_2 \cos k_x \cos k_y \tau_z$$

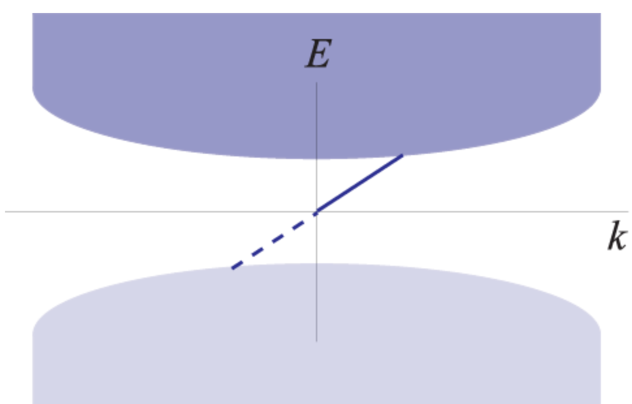
$$|u_1| > |u_2| > 0$$



No MBS at disclination

$$\boxed{H_a^{u_1>0}}$$

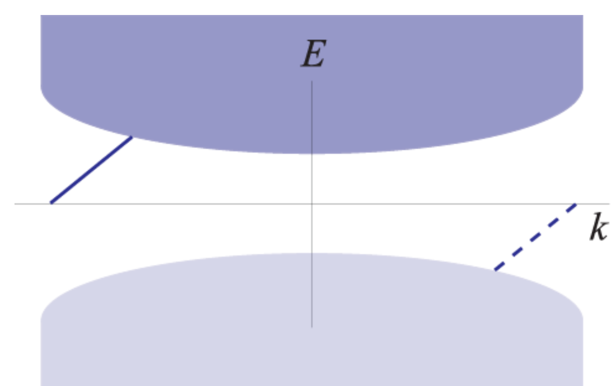
$$ch = 1 \qquad \mathbf{G}_\nu = (0,0)$$



Single MBS at disclination

$$\boxed{H_a^{u_1<0}}$$

$$ch = 1 \qquad \mathbf{G}_\nu = (1,1)$$



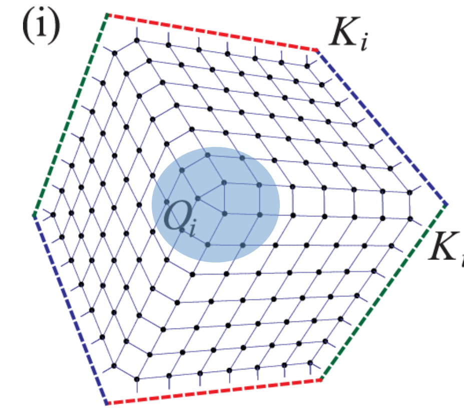
JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

How about a non-tight binding limit?

- Chiral p+ip model

$$H_a = \Delta(\sin k_x \tau_x + \sin k_y \tau_y) + u_1(\cos k_x + \cos k_y)\tau_z + 2u_2 \cos k_x \cos k_y \tau_z$$

$$|u_1| > |u_2| > 0$$



No MBS at disclination

$$\boxed{H_a^{u_1>0}}$$

$$ch = 1 \qquad \mathbf{G}_\nu = (0,0)$$

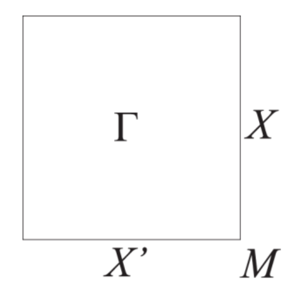
	C_4	C_2
Γ	$e^{-i\pi/4}$	$-i$
M	$e^{+i\pi/4}$	$+i$
X		$+i$

Single MBS at disclination

$$\boxed{H_a^{u_1<0}}$$

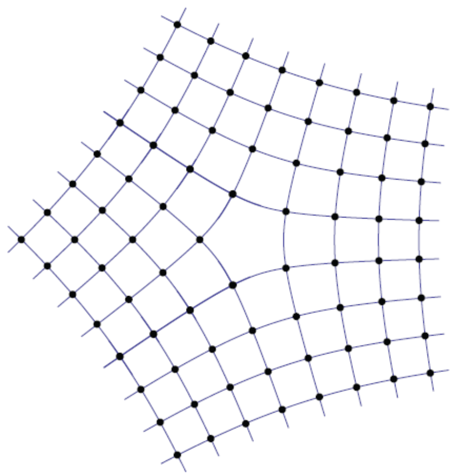
$$ch = 1 \qquad \mathbf{G}_\nu = (1,1)$$

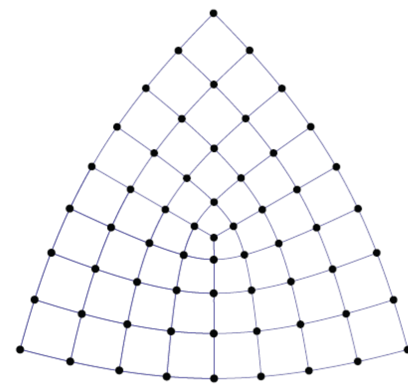
	C_4	C_2
Γ	$e^{+i\pi/4}$	$+i$
M	$e^{-i\pi/4}$	$-i$
X		$+i$



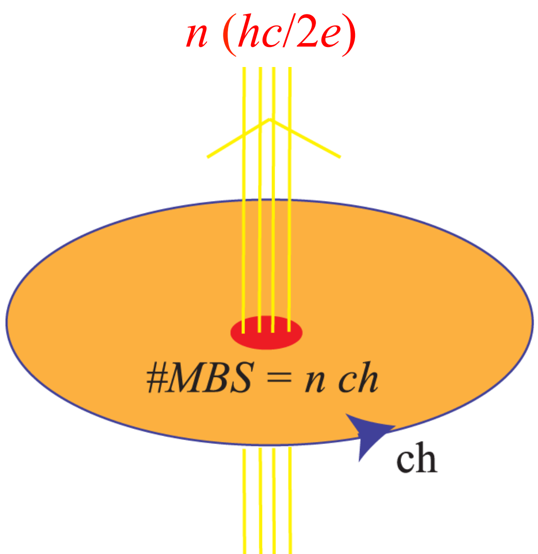
JT, Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

How do I know there is a MBS in general?

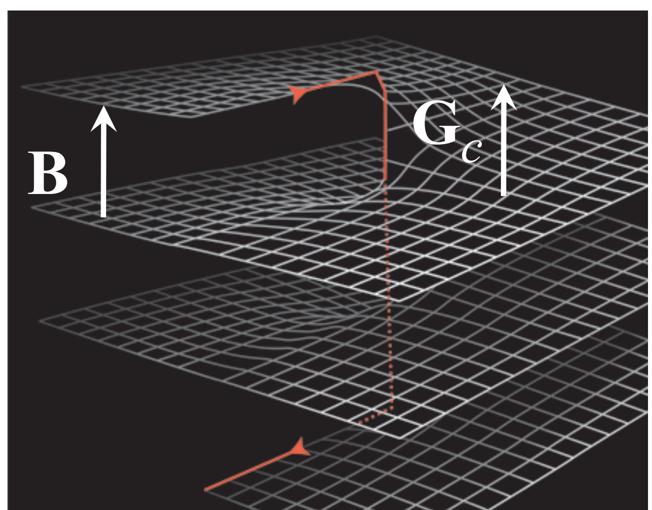




Product between some
- **defect quantities**
- **bulk topological invariants**



Chiral superconductor
Read, Green, 00



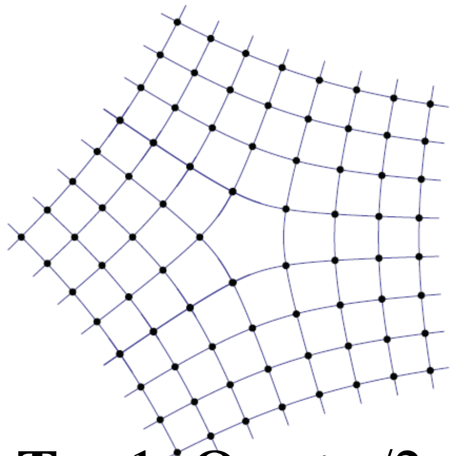
Weak topological insulator
Ran, et.al., 09

Existence of helical mode

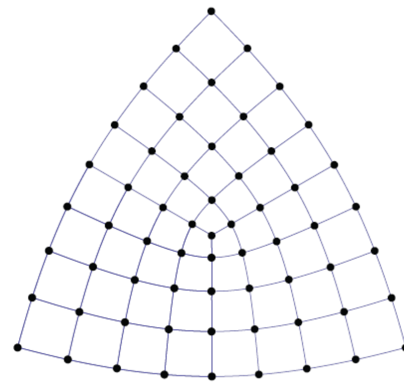
$$\nu = \frac{1}{2\pi}\mathbf{B}\cdot\mathbf{G}_c$$

Weak $\mathbf{Z}_2$ indices

How do I know there is a MBS in general?



$\mathbf{T} = 1, \Omega = +\pi/2$



$\mathbf{T} = 0, \Omega = -\pi/2$

Product between some
- **defect quantities**
- **bulk topological invariants**

Number of Majoranas at disclination

$$\Theta^{(4)} = \frac{1}{2\pi}\mathbf{T}\cdot\mathbf{G}_\nu + \frac{\Omega}{2\pi}(ch + \underbrace{2[X] + [M_1] + 3[M_2]}_{\text{Rotation eigenvalues}})$$

Even/odd translations around defect

Weak invariant

$$\mathbf{G}_\nu = [X] + [M_1] + [M_2]$$

Frank angle

Chern invariant

$$ch = \frac{i}{2\pi}\int_{BZ}\mathrm{Tr}(d\mathcal{A})$$

JT, Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Topological Classification of TCS

- Stable topological invariants (all T-breaking)
  - Chern invariant
  - Rotation invariants

4-fold momenta rotation eigenvalues
at $\prod = \Gamma, M$

$$\Pi_5 = e^{-i\pi/4},\ \Pi_6 = e^{i\pi/4}$$

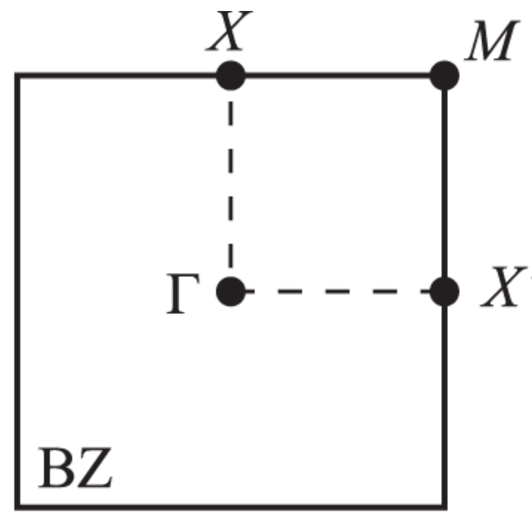
$$\Pi_7 = e^{i3\pi/4},\ \Pi_8 = e^{-i3\pi/4}$$

2-fold momenta rotation eigenvalues

$$X_3 = i, \quad X_4 = -i$$

Rotation spectra discrepancies in *valence* bands

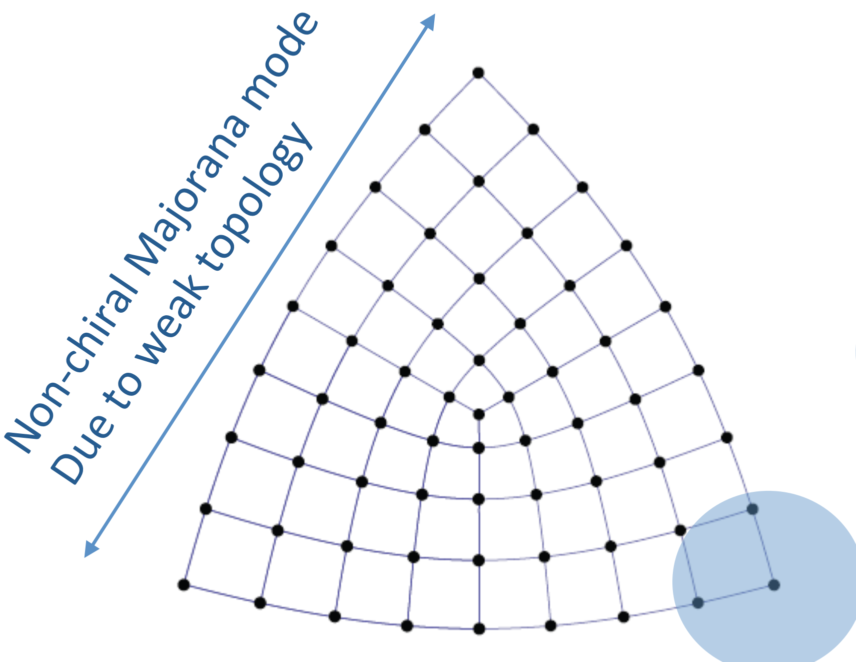
$$\boxed{\begin{array}{l} [X] = \#X_4 - \#\Gamma_5 - \#\Gamma_7 \\ [M_1] = \#M_6 - \#\Gamma_6 \quad [M_2] = \#M_7 - \#\Gamma_7 \end{array}}$$

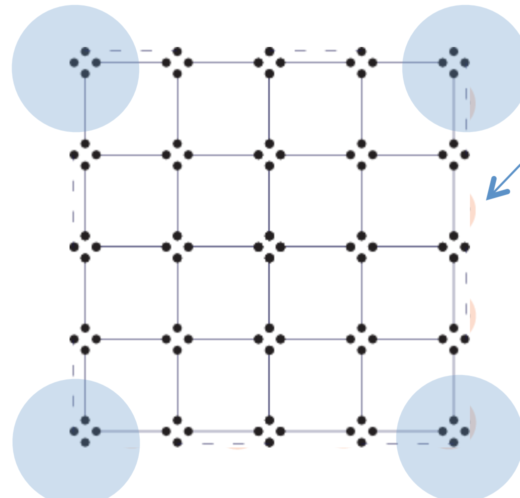


JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Corner MBS in Strontium Ruthenate

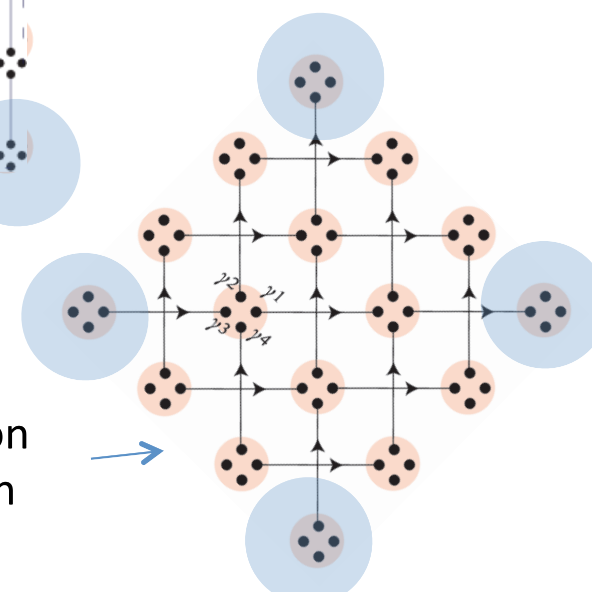
#MBS at disclination = #MBS at corners + #MBS along edges (mod 2)





Sr_2RuO_4

Gapped out by perturbation



Gapped out by on site hybridization

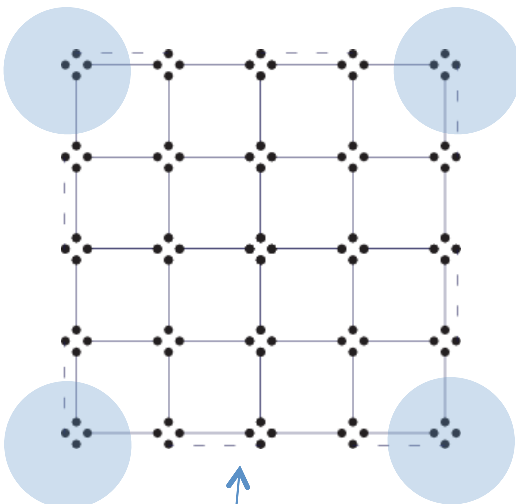
$2 \times Sr_2RuO_4$

JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

Non-chirality is actually good!!

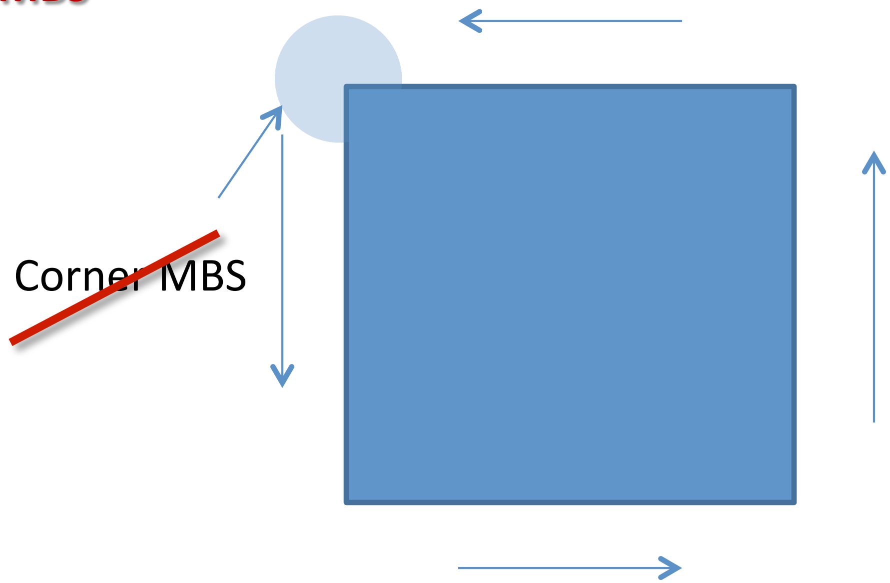
Quasi-1D
Sr_2RuO_4

Corner MBS



Gapped out by
perturbation

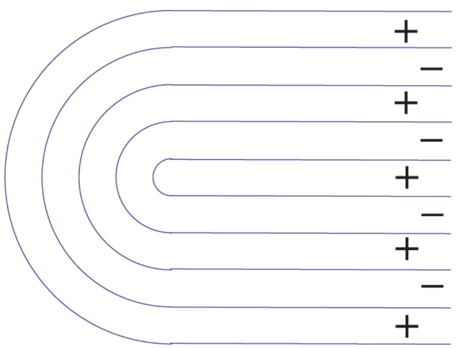
Chiral p+ip
Sr_2RuO_4

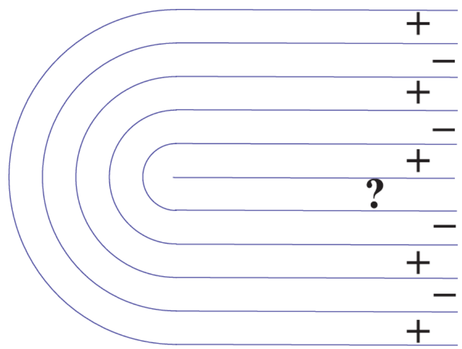


Robust against
perturbations

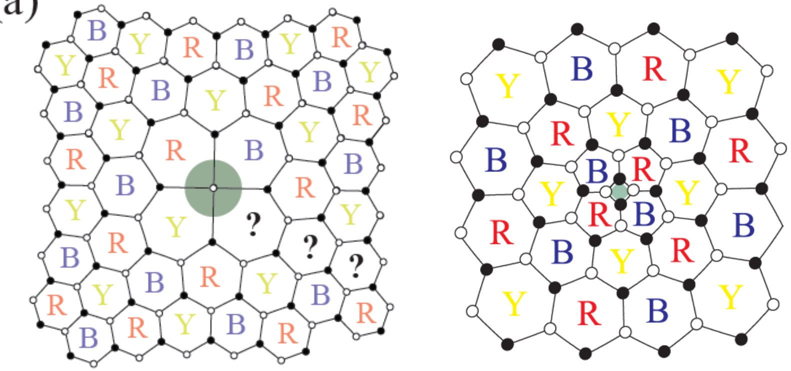
JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

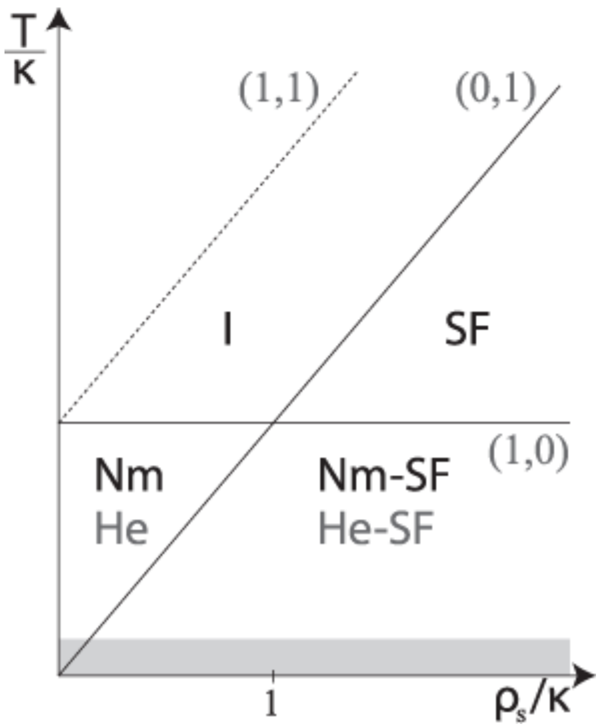
Melting Phases of Liquid Crystal Superfluid

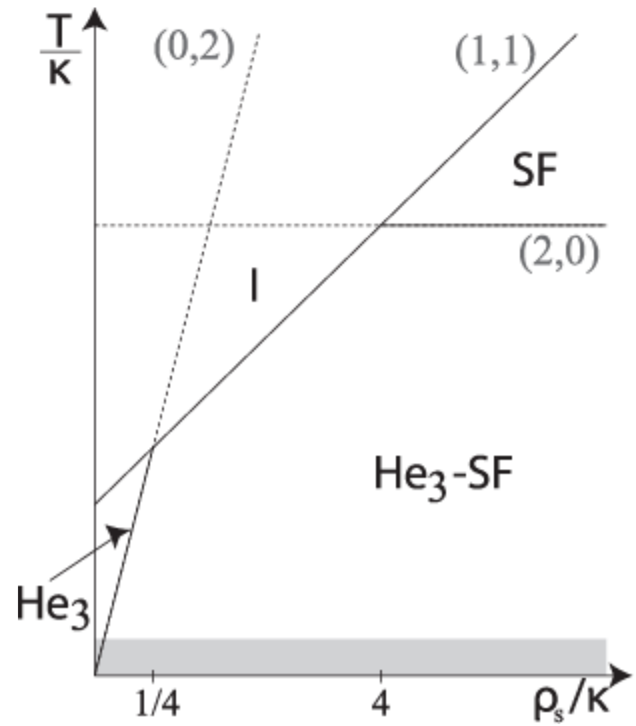




(a)



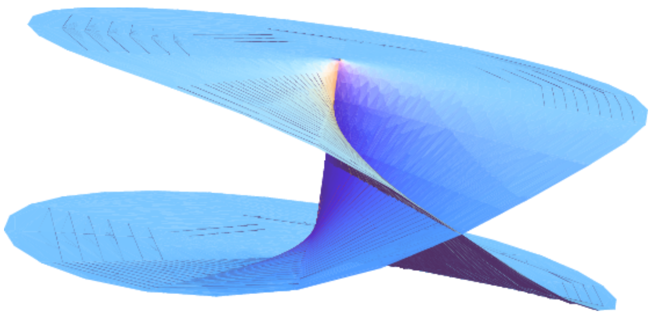


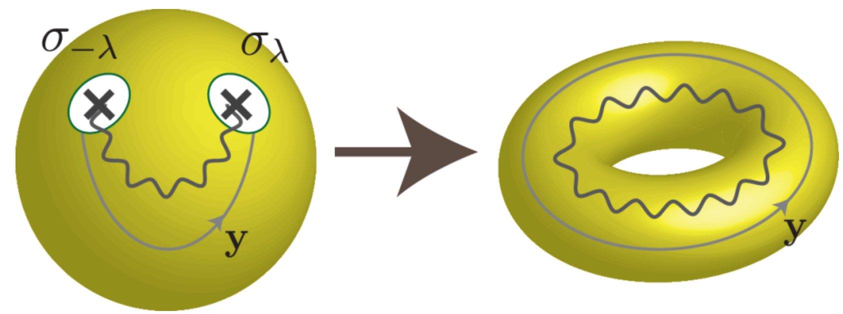


Gopalakrishnan, **JT**, Hughes; Phys. Rev. Lett. 111, 025304 (2013)

Non-abelian Twist Defects

- Topological order intertwined with classical order





$$\xcancel{SL(2;\mathbb{Z}) = \left\langle S, T \;\middle|\; \begin{array}{c} (ST)^3 = C, S^2 = C \\ C^2 = 1, SCS^{-1} = TCT^{-1} = C \end{array} \right\rangle}$$

braiding exchanging

$$\Gamma_0(2) = \left\langle \begin{array}{c} S = t_y \\ T = t_x^2 \end{array} \;\middle|\; \begin{array}{c} (ST^{-1})^2 = C, C^2 = 1 \\ SCS^{-1} = TCT^{-1} = C \end{array} \right\rangle$$

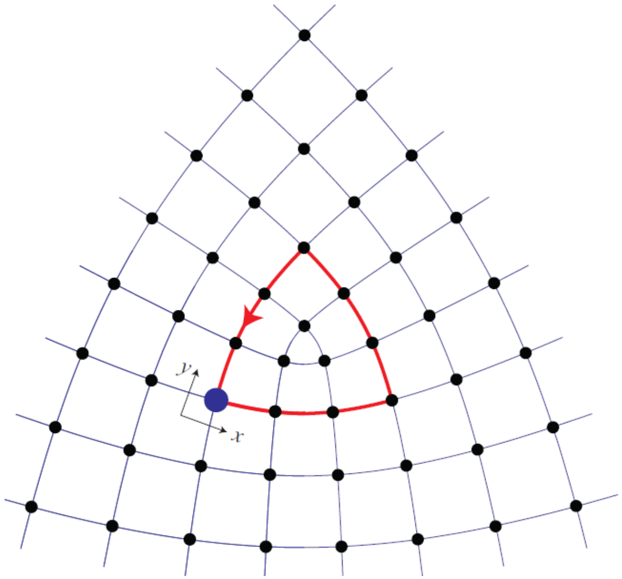
braiding exchanging

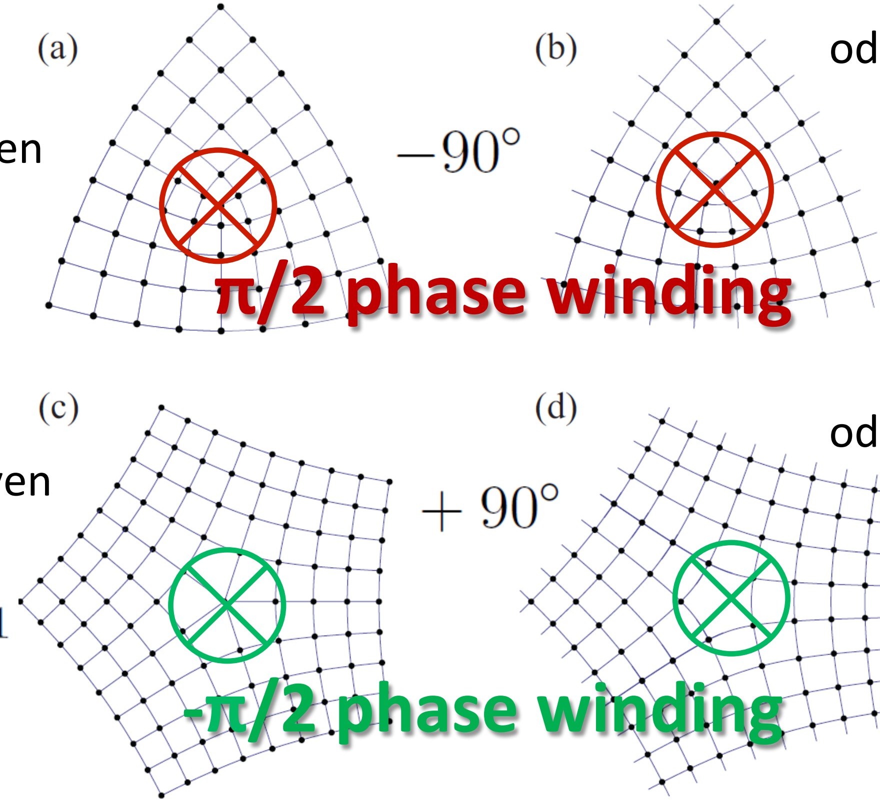
JT, Roy, Chen; arXiv:1308.5984 (2013)
JT, Roy, Chen; arXiv:1306. 1538 (2013)

Conclusion

- Majorana bound states in Topological crystalline SC
  [**JT** *and Taylor L Hughes*; Phys. Rev. Lett. 111, 047006 (2013); arXiv: 1208.6303]
  - Lattice symmetry protected topological superconductor
  - Topological Classification of TCS
  - Revisit classification of dislocation-disclination composite
  - Lattice defect MBS - Homogeneous environment
    - No external field
    - No proximity interfaces
  - Sr_2RuO_4 corners
  - 5-7 defects in doped graphene
- What's next?
  - Revisit materials with overlooked topology
  - Melting phases with intertwined topological – classical order

Classification of Disclinations



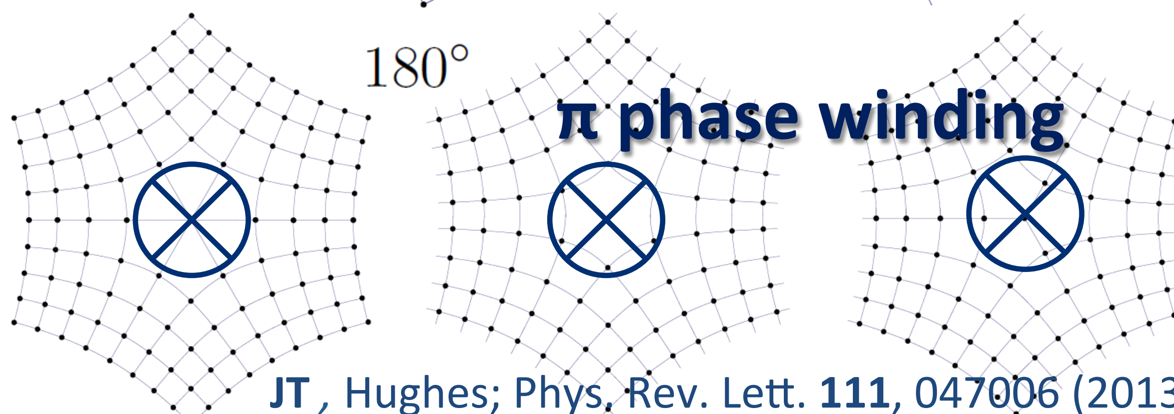


$$r(\mathbf{e_x})^3 r(\mathbf{e_x})^3 r(\mathbf{e_x})^3 = (-3\mathbf{e}_x) r^{-1}$$

Classification:

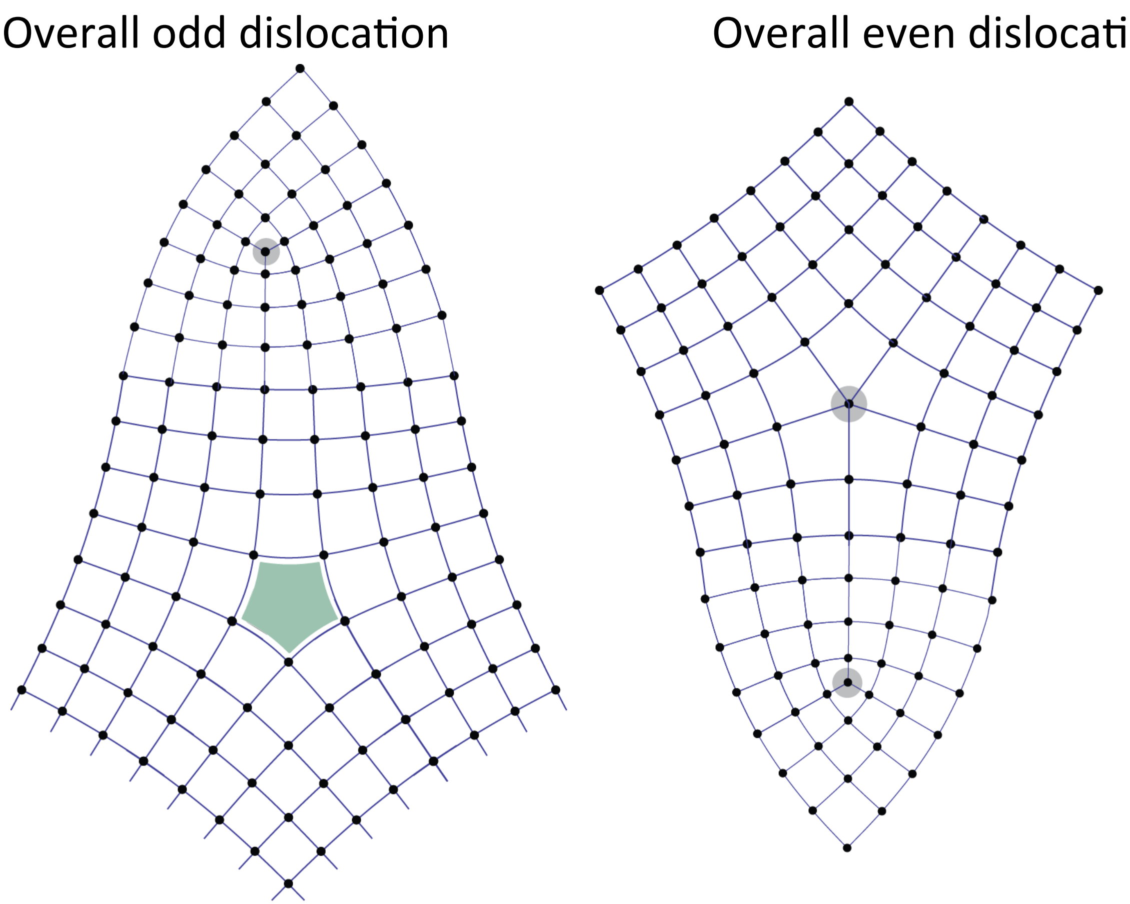
$$C_4 \times \mathbb{Z}_2$$

Eveness / oddness
of number of translations



JT , Hughes; Phys. Rev. Lett. **111**, 047006 (2013)

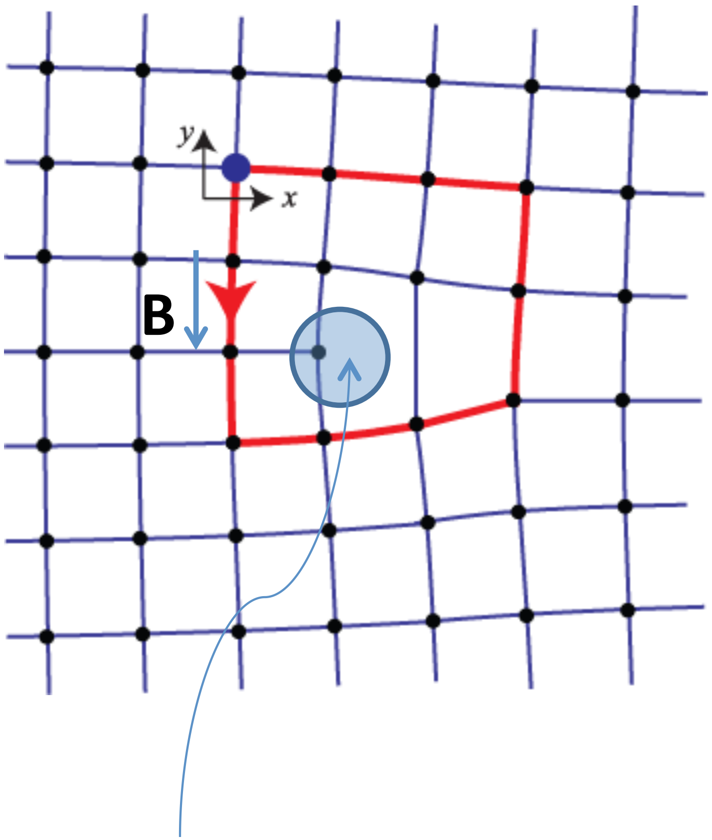
Dislocation = Disclination dipole



JT , Hughes; Phys. Rev. Lett. 111, 047006 (2013)

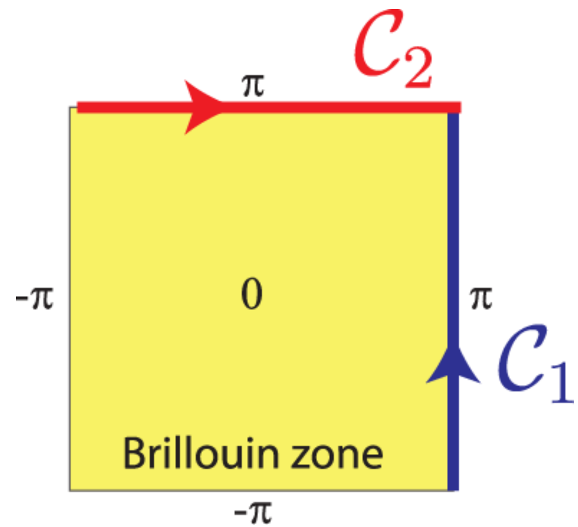
Gopalakrishnan, **JT**, Hughes; Phys. Rev. Lett. 111, 025304 (2013)

Majorana Bound states at Dislocations



$$\mathbf{G}_\nu = \nu_1 \mathbf{b}_1 + \nu_2 \mathbf{b}_2$$

$$\nu_i \equiv \frac{i}{\pi} \oint_{\mathcal{C}_i} \mathrm{Tr}(\mathcal{A}) \mod 2$$



number of Majorana zero mode

$$\equiv \frac{1}{2\pi} \mathbf{B} \cdot \mathbf{G}_\nu \mod 2$$

JT, Kane, 2010

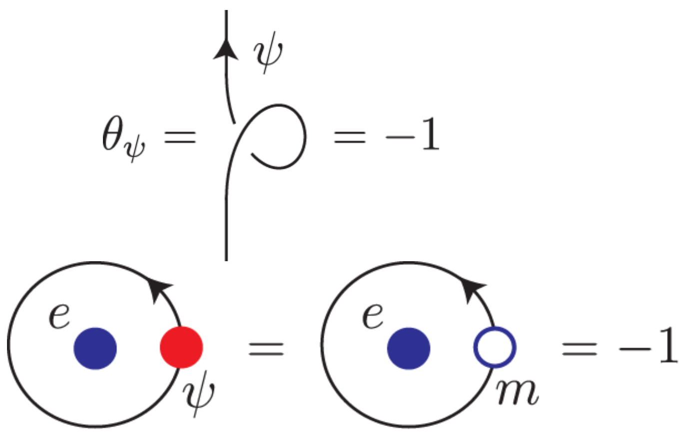
Abelian bosonic bilayer FQH states

- Kitaev toric code

$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$$1 = (0,0) \quad e = (1,0)$$

$$m = (0,1) \quad \psi = e \times m = (1,1)$$



$$\overset{\sigma_x}{e \longleftrightarrow m}$$

A. Kitaev, Ann. Phys. 303, 2 (2003)

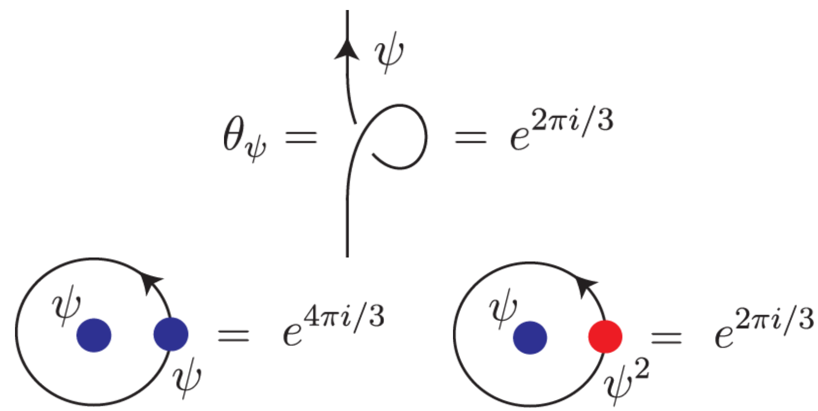
- Second hierarchy state

$$K = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

SU(3)$_1$

$$1 = \psi^3 = (0,0) \quad \psi = (1,0)$$

$$\psi^2 = (2,0) \equiv (0,-1)$$



$$\overset{\sigma_x}{\psi \longleftrightarrow \psi^2}$$

F.D.M. Haldane, PRL 51, 605 (1983)