

(Flat) Extra Dimensions: Where Do We Stand?

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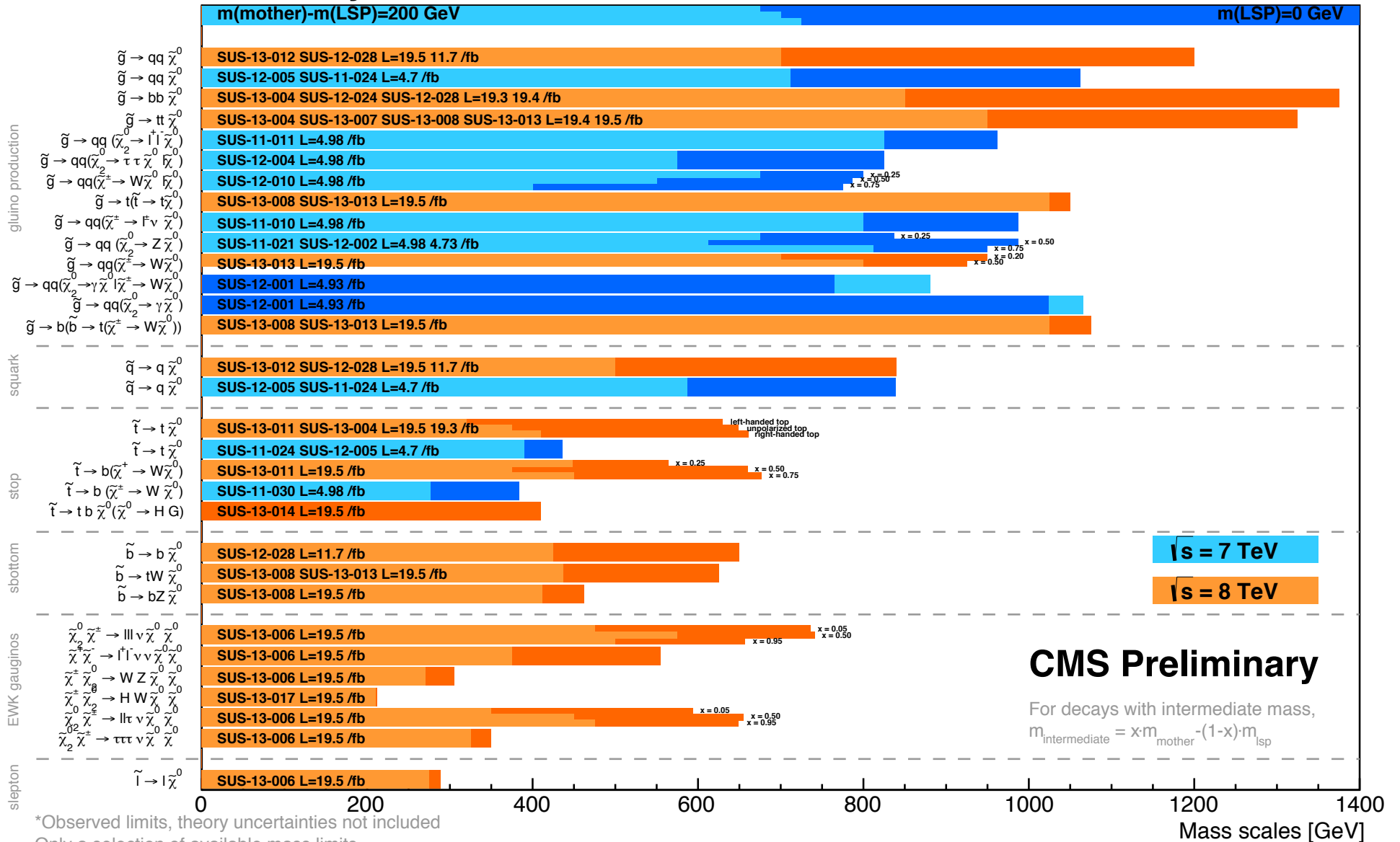
High Energy Physics Seminar
Department of Physics
University of Virginia
April 16, 2014



SUSY Searches

Summary of CMS SUSY Results* in SMS framework

SUSY 2013



*Observed limits, theory uncertainties not included

Only a selection of available mass limits

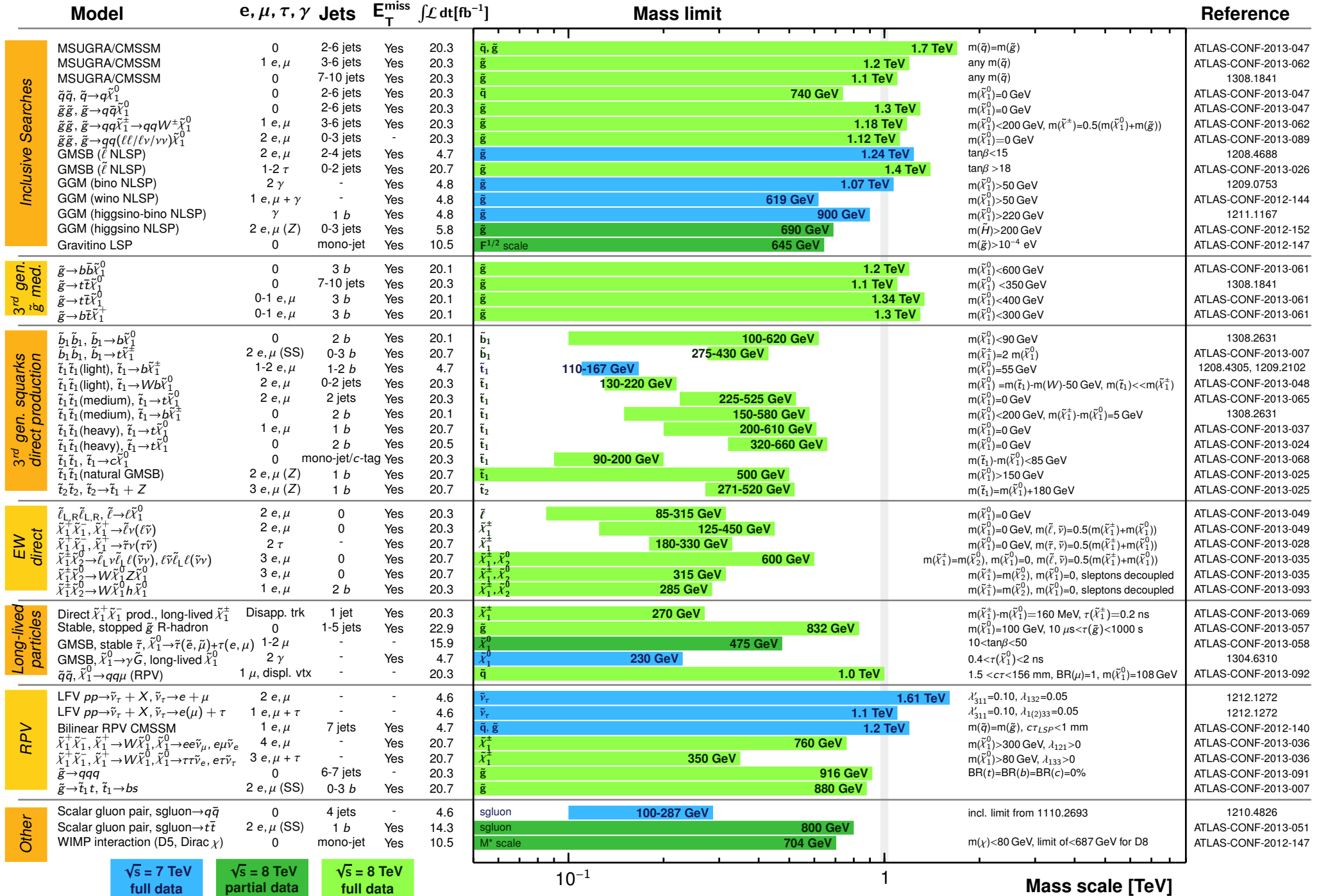
Probe *up to* the quoted mass limit

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: SUSY 2013

ATLAS Preliminary

$$\int \mathcal{L} dt = (4.6 - 22.9) \text{ fb}^{-1} \quad \sqrt{s} = 7, 8 \text{ TeV}$$

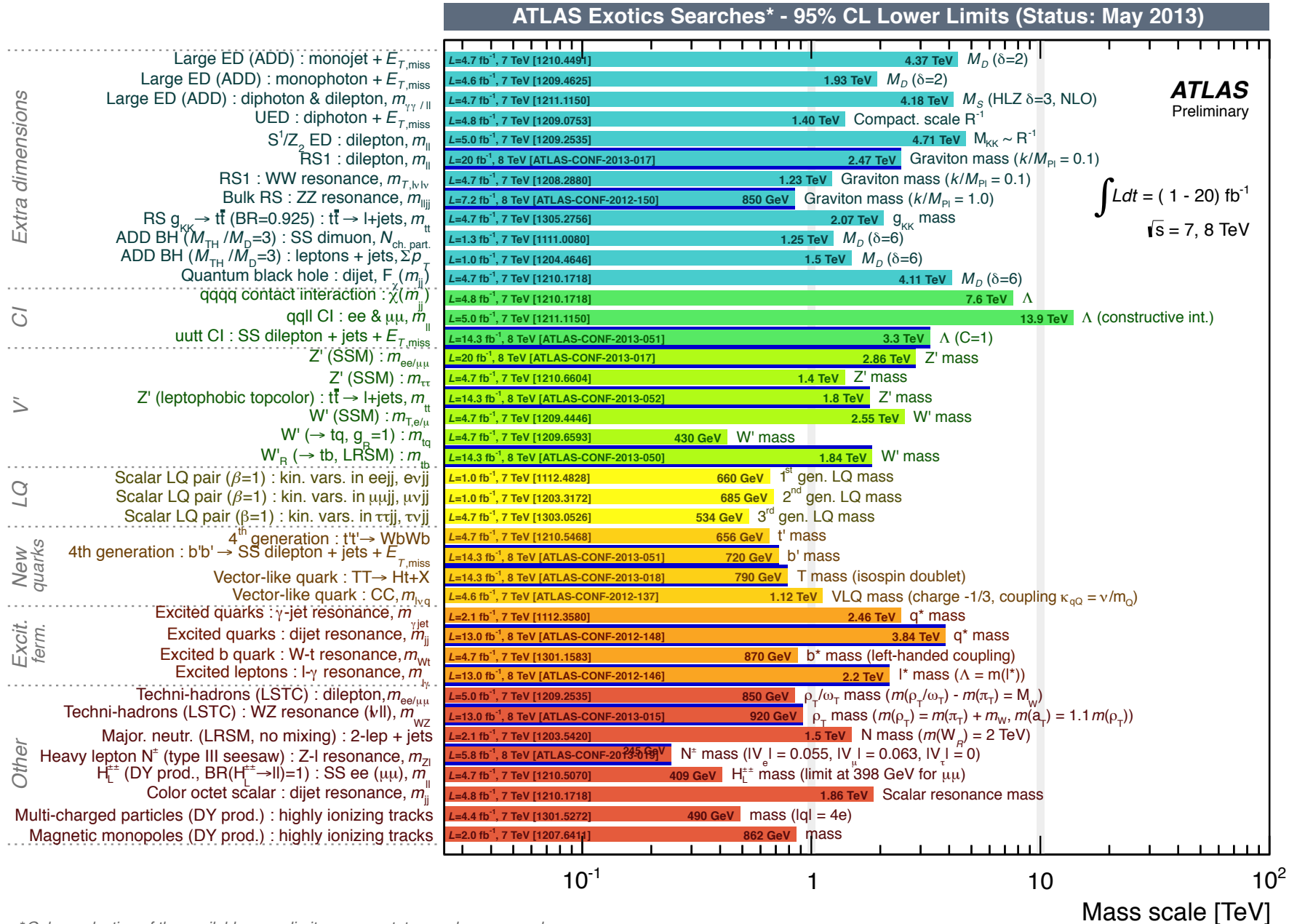


*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.

Why SUSY is not there?

- Perhaps SUSY scale is higher than we thought
 - but there are no 2-3 sigma indications at all
- How to hide (?):
 - make spectrum more degenerate
 - make SUSY scale higher and higher (and higher ...)
 - we haven't really looked at every single channel;
 - find exotic productions / decays
 - introduce more parameters;
 - weaken current bounds

Non-SUSY Searches



- What are current LHC bounds on (flat) ED?
- Is KK-photon dark matter ruled out?
- Is Universal Extra Dimensions still alive?

- My response was:
 - Why do you ask me?
 - I am not in CMS or ATLAS collaboration
 - I did not make these models anyway
 - I am working on something different these days...

Short Answers

- $M_H=126$ GeV and relic abundance disfavors 2UED (6D) with minimal mass spectrum
- MUED (5D) is very constrained as any other models
 - $R_{\text{inv}} > 1.2\text{-}1.3$ TeV from tri-lepton search (8 TeV)
 - $R_{\text{inv}} < 1.5$ TeV from relic abundance
- There are ways to introduces more parameters
 - brane-localized terms and fermion-bulk masses
 - MUED exists in various event generators: CalcHEP, PYTHIA, MG/ME, Herwig, Sherpa, etc
 - For NMUED, coupling and mass spectrum can be modified

Outline

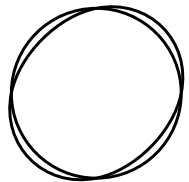
- Universal Extra Dimensions (TeV^{-1} ED)
 - basic review
 - collider and dark matter
 - 5D and 6D
 - current LHC bounds (2012)
- Beyond Minimal Model (2013-2014)
 - more parameters
 - AMS-02 data (positrons)
 - exotic signatures?
- (no gravity in this talk)

If there are extra dimensions...

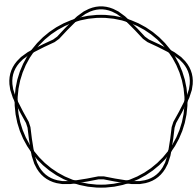
- Energy-momentum relation in 5D.

$$E^2 = p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + m^2 \qquad E^2 = p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + p_y^2 + m^2$$

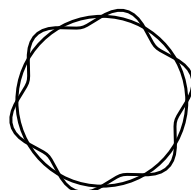
- If an extra dimension is a circle,



$$2\pi R = 2\lambda$$



$$2\pi R = 5\lambda$$



$$2\pi R = 6\lambda$$

$$p_y = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi R}{n} \Rightarrow p_y = \frac{2\pi n}{2\pi R} = \frac{n}{R}$$

$$E^2 = p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + \frac{n^2}{R^2} + m^2 \equiv \vec{p}^2 + M_n^2$$

$$M_n = \sqrt{\frac{n^2}{R^2} + m^2}$$

A Scalar Field in 5 Dimensions

- Action for a scalar of mass m

$$\mathcal{S} = \int d^4x dy \left[\partial_M \Phi^*(x, y) \partial^M \Phi(x, y) - m^2 \Phi^*(x, y) \Phi(x, y) \right]$$

$$M, N = 0, 1, 2, 3, 5 \equiv \mu, 5,$$

$$g_{MN} = (+ - - - -)$$

$$\partial_M = (\partial_\mu, \partial_5)$$

- ASSUME an extra dimension is a circle (S_1),

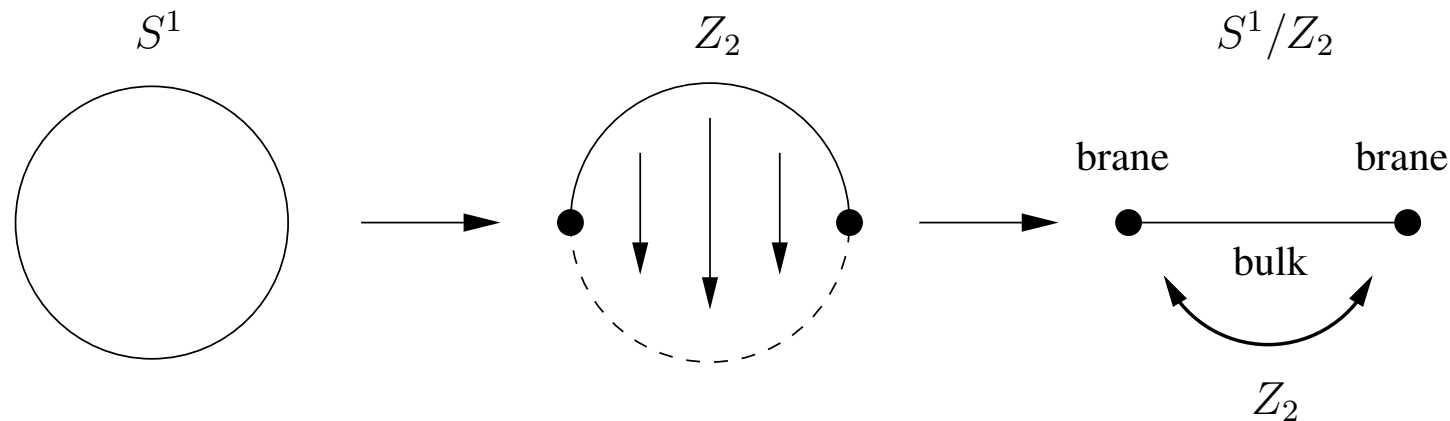
$$\Phi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi_n(x) \exp\left(\frac{iny}{R}\right) \quad 2\pi R \delta_{n,m} = \int_0^{2\pi R} dy \exp\left(\frac{i(n-m)y}{R}\right)$$

- Compactify ED (integrate out unknown coordinate)

$$\mathcal{S} = \sum_{n=-\infty}^{\infty} \int d^4x \left[\partial_\mu \phi_n^*(x) \partial^\mu \phi_n(x) - \left(\frac{n^2}{R^2} + m^2 \right) \phi_n^*(x) \phi_n(x) \right] \quad m_n = \sqrt{\frac{n^2}{R^2} + m^2}.$$

Problems with circular ED

- No (two components) chiral fermions in $D > 4$.
- $SO(1,3) \sim SU(2) \times SU(2) \sim SO(3) \times SO(3)$
- Each gauge field has 5 components, $(G_\mu(x, y), G_5(x, y))$
- What particle corresponds to this 5-th component?
- Introduce “Orbifold” (manifold with fixed points)



A Scalar Field on an Interval

- Action for a scalar with potential V

$$S_{bulk} = \int d^4x \int_0^{\pi R} \left(\frac{1}{2} \partial^M \phi \partial_M \phi - V(\phi) \right) dy$$

- Variational principle leads to $\delta S = 0$

$$\delta S = \int d^4x \int_0^{\pi R} \left(\partial^M \phi \partial_M \delta \phi - \frac{\partial V}{\partial \phi} \delta \phi \right) dy$$

$$\delta S = \int d^4x \int_0^{\pi R} dy \left[-\partial_\mu \partial^\mu \phi \delta \phi - \frac{\partial V}{\partial \phi} \delta \phi - \partial_y \phi \partial_y \delta \phi \right]$$

- Keep the boundary terms when integrating by parts

$$\delta S = \int d^4x \int_0^{\pi R} \left[-\partial_M \partial^M \phi - \frac{\partial V}{\partial \phi} \right] \delta \phi - \left[\int d^4x \partial_y \phi \delta \phi \right]_0^{\pi R}$$

- Bulk EOM and boundary terms

$$\partial_M \partial^M \phi = -\frac{\partial V}{\partial \phi} \quad \text{Neumann BC } \partial_y \phi| = 0 \quad H(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos\left(\frac{ny}{R}\right) \right\}$$

Standard Model on S_1/Z_2

$$\mathcal{L}_{Gauge} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W_{MN}^a W^{aMN} - \frac{1}{4} G_{MN}^A G^{AMN} \right\} ,$$

$$\begin{aligned} \mathcal{L}_{GF} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{2\xi} (\partial^\mu B_\mu - \xi \partial_5 B_5)^2 - \frac{1}{2\xi} (\partial^\mu W_\mu^a - \xi \partial_5 W_5^a)^2 \right. \\ \left. - \frac{1}{2\xi} (\partial^\mu G_\mu^A - \xi \partial_5 G_5^A)^2 \right\} , \end{aligned}$$

$$\mathcal{L}_{Leptons} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{L}(x, y) \Gamma^M D_M L(x, y) + i \bar{E}(x, y) \Gamma^M D_M E(x, y) \right\} ,$$

$$\begin{aligned} \mathcal{L}_{Quarks} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{Q}(x, y) \Gamma^M D_M Q(x, y) + i \bar{U}(x, y) \Gamma^M D_M U(x, y) \right. \\ \left. + i \bar{D}(x, y) \Gamma^M D_M D(x, y) \right\} , \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{Yukawa} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_u \bar{Q}(x, y) U(x, y) i \tau^2 H^*(x, y) + \lambda_d \bar{Q}(x, y) D(x, y) H(x, y) \right. \\ \left. + \lambda_e \bar{L}(x, y) E(x, y) H(x, y) \right\} , \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{Higgs} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left[(D_M H(x, y))^\dagger (D^M H(x, y)) + \mu^2 H^\dagger(x, y) H(x, y) \right. \\ \left. - \lambda (H^\dagger(x, y) H(x, y))^2 \right] , \end{aligned}$$

$$\begin{aligned}
H(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos\left(\frac{ny}{R}\right) \right\} , & D_M L(x, y) &= \left(\partial_M + i g_2^{(5)} W_M + i \frac{y_1}{2} g_1^{(5)} B_M \right) L(x, y) , \\
B_\mu(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ B_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} B_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\} , & D_M E(x, y) &= \left(\partial_M + i \frac{y_2}{2} g_1^{(5)} B_M \right) E(x, y) , \\
B_5(x, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} B_5^n(x) \sin\left(\frac{ny}{R}\right) , & D_M Q(x, y) &= \left(\partial_M + i g_3^{(5)} G_M + i g_2^{(5)} W_M + i \frac{y_3}{2} g_1^{(5)} B_M \right) Q(x, y) , \\
W_\mu(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ W_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} W_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\} , & D_M U(x, y) &= \left(\partial_M + i g_3^{(5)} G_M + i \frac{y_4}{2} g_1^{(5)} B_M \right) U(x, y) , \\
W_5(x, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} W_5^n(x) \sin\left(\frac{ny}{R}\right) , & D_M D(x, y) &= \left(\partial_M + i g_3^{(5)} G_M + i \frac{y_5}{2} g_1^{(5)} B_M \right) D(x, y) . \\
G_\mu(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ G_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} G_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\} , \\
G_5(x, y) &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G_5^n(x) \sin\left(\frac{ny}{R}\right) , \\
Q(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_L Q_L^n(x) \cos\left(\frac{ny}{R}\right) + P_R Q_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} , \\
U(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_R u_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L u_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} , \\
D(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ d_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_R d_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L d_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} , \\
L(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ L_0(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_L L_L^n(x) \cos\left(\frac{ny}{R}\right) + P_R L_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} , \\
E(x, y) &= \frac{1}{\sqrt{\pi R}} \left\{ e_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_R e_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L e_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) &= \frac{\pi R}{2} \delta_{m,n} , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) &= \frac{\pi R}{2} \delta_{m,n} , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) &= \frac{\pi R}{4} \Delta_{mnl}^1 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= \frac{\pi R}{8} \Delta_{mnlk}^2 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \sin\left(\frac{ly}{R}\right) \sin\left(\frac{ky}{R}\right) &= \frac{\pi R}{8} \Delta_{mnlk}^3 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) &= \frac{\pi R}{4} \Delta_{mnl}^4 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= \frac{\pi R}{8} \Delta_{mnlk}^5 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) &= 0 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \sin\left(\frac{ly}{R}\right) &= 0 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) &= 0 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= 0 , \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \sin\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= 0 ,
\end{aligned}$$

$$\Delta_{mnl}^1 = \delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n} ,$$

$$\Delta_{mnlk}^2 = \delta_{k,l+m+n} + \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} + \delta_{k+m,l+n} + \delta_{k+l,m+n} + \delta_{k+n,l+m} ,$$

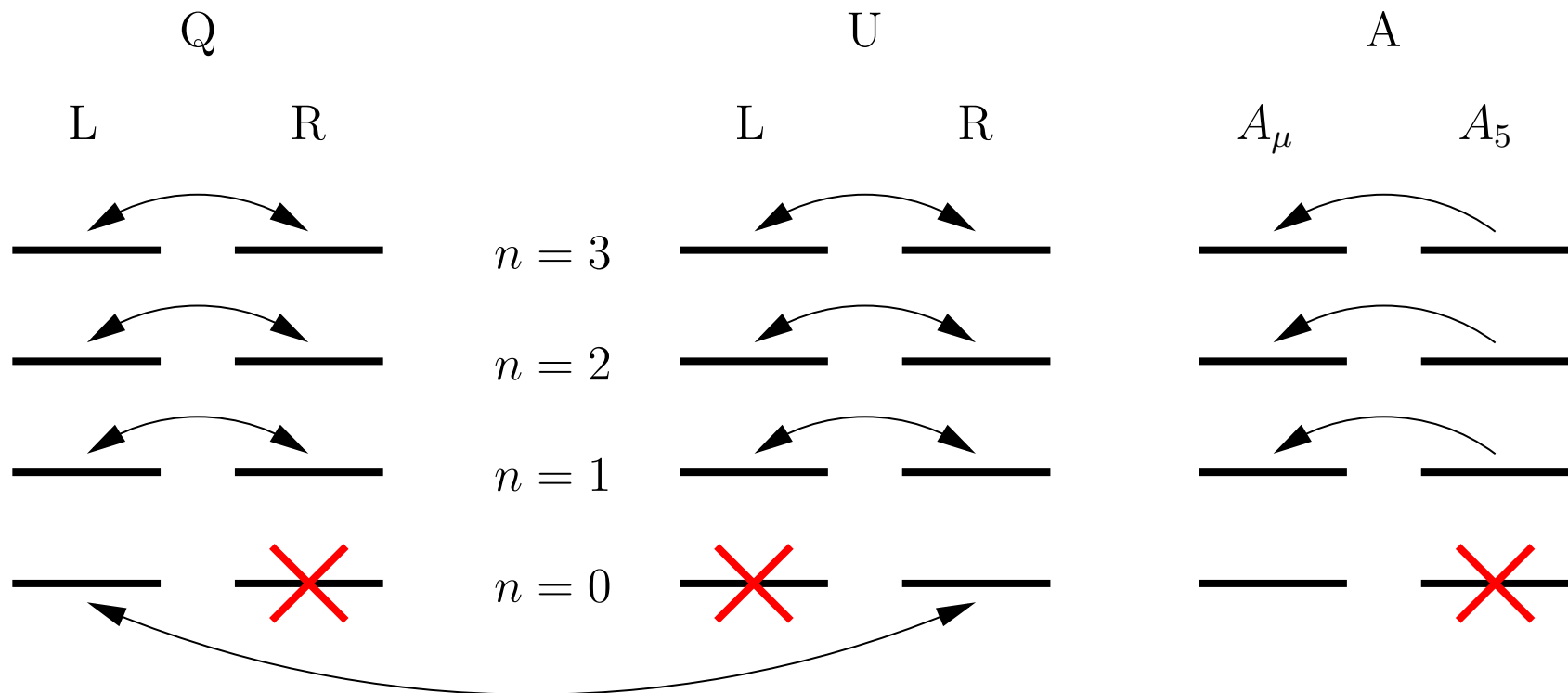
$$\Delta_{mnlk}^3 = -\delta_{k,l+m+n} - \delta_{l,m+n+k} - \delta_{m,n+k+l} - \delta_{n,k+l+m} + \delta_{k+l,m+n} + \delta_{k+m,l+n} + \delta_{k+n,l+m} ,$$

$$\Delta_{mnl}^4 = -\delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n} ,$$

$$\Delta_{mnlk}^5 = -\delta_{k,l+m+n} - \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} - \delta_{k+l,m+n} + \delta_{k+m,l+n} + \delta_{k+n,l+m} .$$

KK states after Compactification

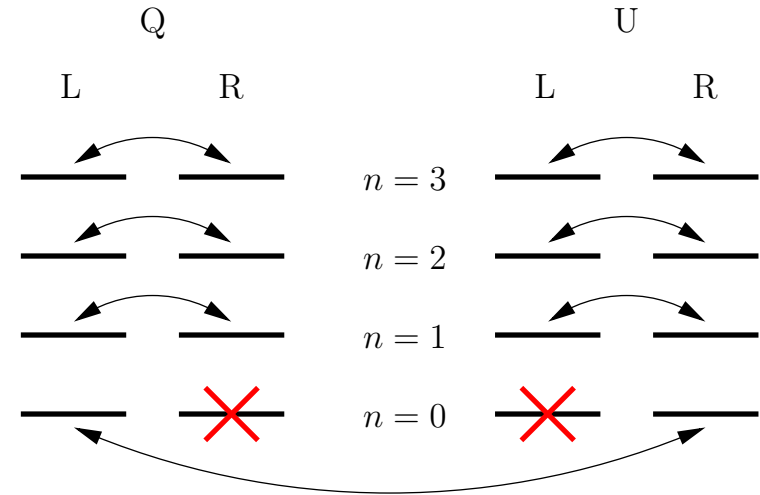
- Each SM particle has an infinite number of KK partners, with mass $\sqrt{m^2 + \frac{n^2}{R^2}}$
- KK particles have the same spin as SM particles.
- There are TWO Dirac KK fermions for each SM fermion.



KK states after Compactification

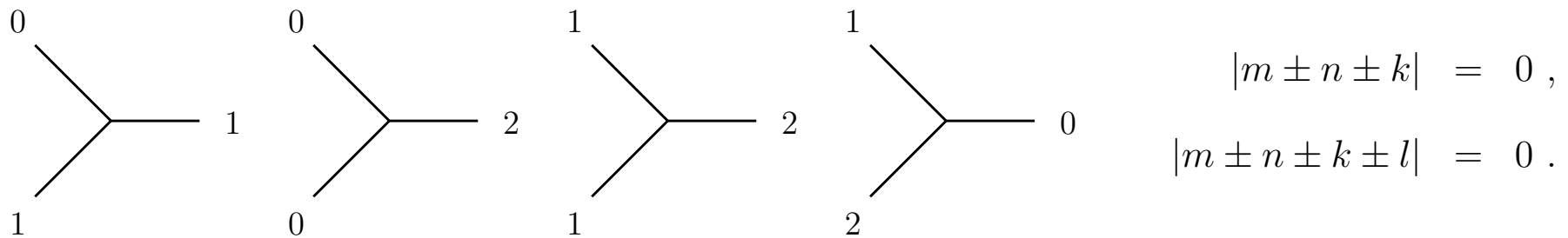
SU(2) Symmetry	SM mode	KK mode
Quark doublet	$q_L(x) = \begin{pmatrix} U_L(x) \\ D_L(x) \end{pmatrix}$	$Q_L^n(x) = \begin{pmatrix} U_L^n(x) \\ D_L^n(x) \end{pmatrix}, Q_R^n(x) = \begin{pmatrix} U_R^n(x) \\ D_R^n(x) \end{pmatrix}$
Lepton doublet	$L_0(x) = \begin{pmatrix} \nu_L(x) \\ E_L(x) \end{pmatrix}$	$L_L^n(x) = \begin{pmatrix} \nu_L^n(x) \\ E_L^n(x) \end{pmatrix}, L_R^n(x) = \begin{pmatrix} \nu_R^n(x) \\ E_R^n(x) \end{pmatrix}$
Quark Singlet	$u_R(x)$	$u_R^n(x), u_L^n(x)$
Quark Singlet	$d_R(x)$	$d_R^n(x), d_L^n(x)$
Lepton Singlet	$e_R(x)$	$e_R^n(x), e_L^n(x)$

	KK Fermions	I_3	Y	$Q = I_3 + \frac{Y}{2}$
Quark Doublet	$U_n = U_L^n(x) + U_R^n(x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
	$D_n = D_L^n(x) + D_R^n(x)$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
Quark Singlet	$u_n = u_L^n(x) + u_R^n(x)$	0	$\frac{2}{3}$	$\frac{2}{3}$
	$d_n = d_L^n(x) + d_R^n(x)$	0	$-\frac{2}{3}$	$-\frac{1}{3}$
Lepton Doublet	$\nu_n = \nu_L^n(x) + \nu_R^n(x)$	$\frac{1}{2}$	-1	0
	$E_n = E_L^n(x) + E_R^n(x)$	$-\frac{1}{2}$	-1	-1
Lepton Singlet	$e_n = e_L^n(x) + e_R^n(x)$	0	-2	-1
	no KK singlet ν^n	-	-	-



KK states after Compactification

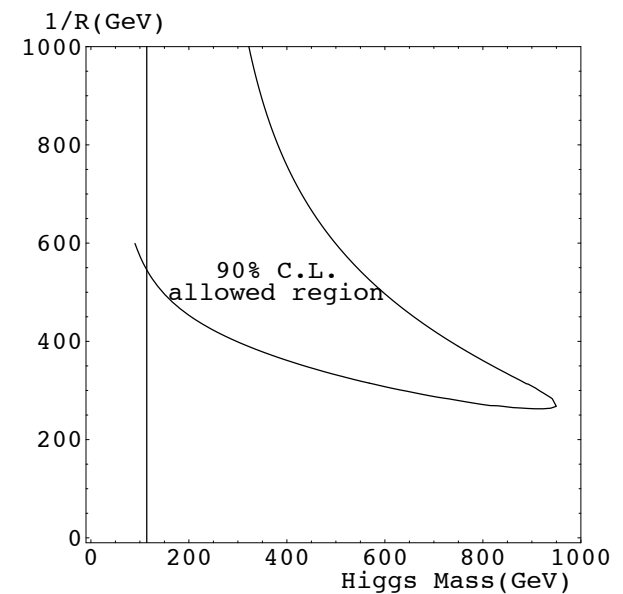
- All vertices at tree level satisfy KK number conservation.



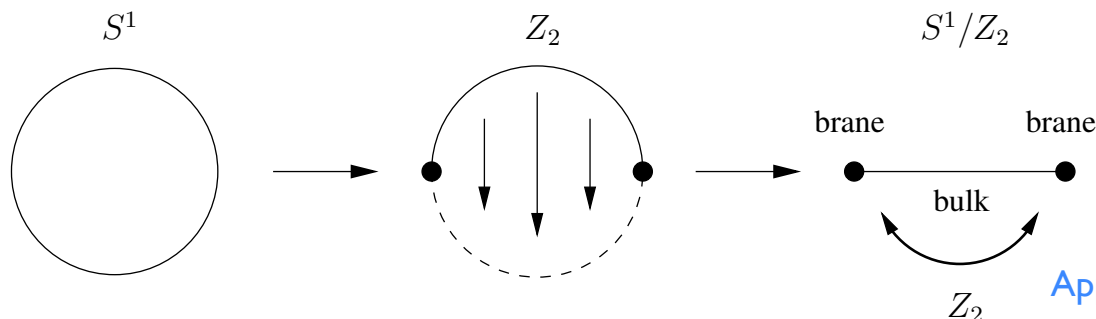
- KK parity, $(-1)^n$ is always conserved even at higher order.
- New vertices are basically the same as the SM couplings (up to normalization factor).

Overview on UED

- Universal: all SM particles in flat ED (no gravity)
- The simplest model: S^1/Z_2 (5D)
- KK tower after compactification with n/R
- KK-parity: $(-1)^n$
 - all SM particles (zero mode) are even
 - level 1 KK particles ($n=1$) are odd
 - level 2 KK particles ($n=2$) are even
 - electroweak precision constraints are avoided
 - new contributions are loop-suppressed
 - the LKP is stable and a DM candidate



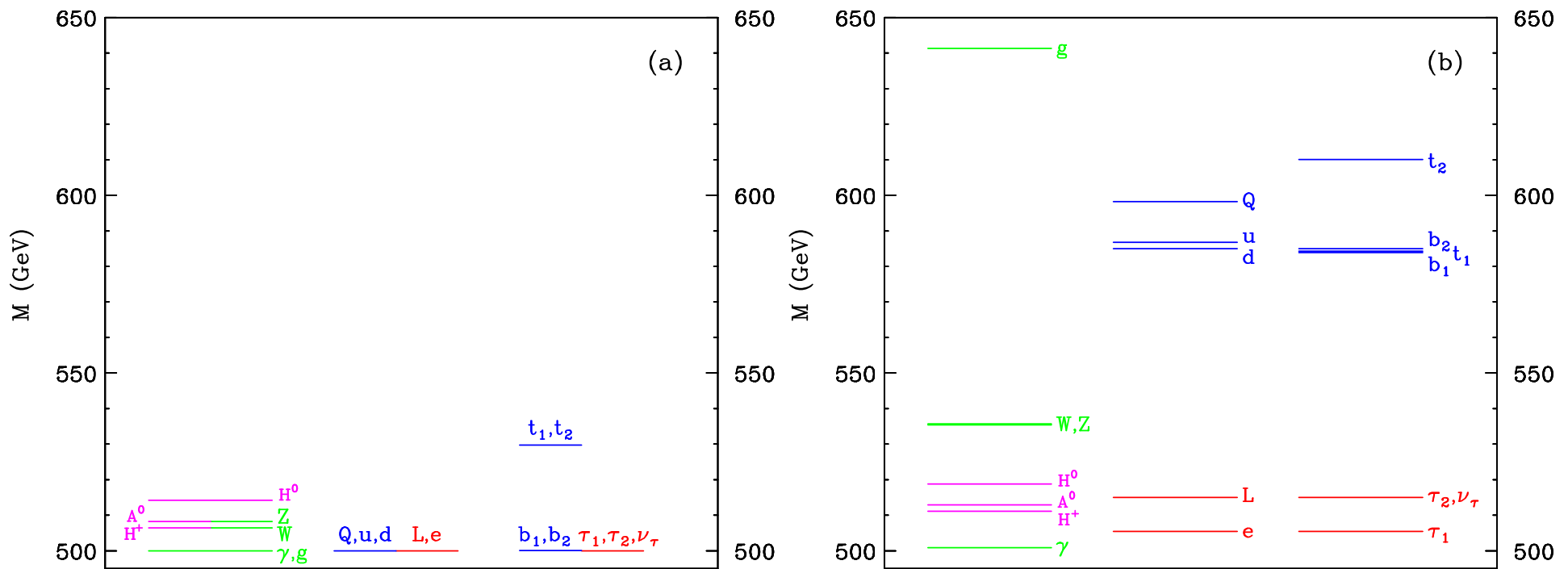
Appelquist, Yee 2002



Appelquist, Cheng, Dobrescu 2001

More on UED

- Minimal UED: mass splitting be generated by radiative corrections (assuming no boundary terms and no bulk masses)
- Short RG running leads to **compressed** mass spectrum
- Two parameters: R , Λ (cutoff)

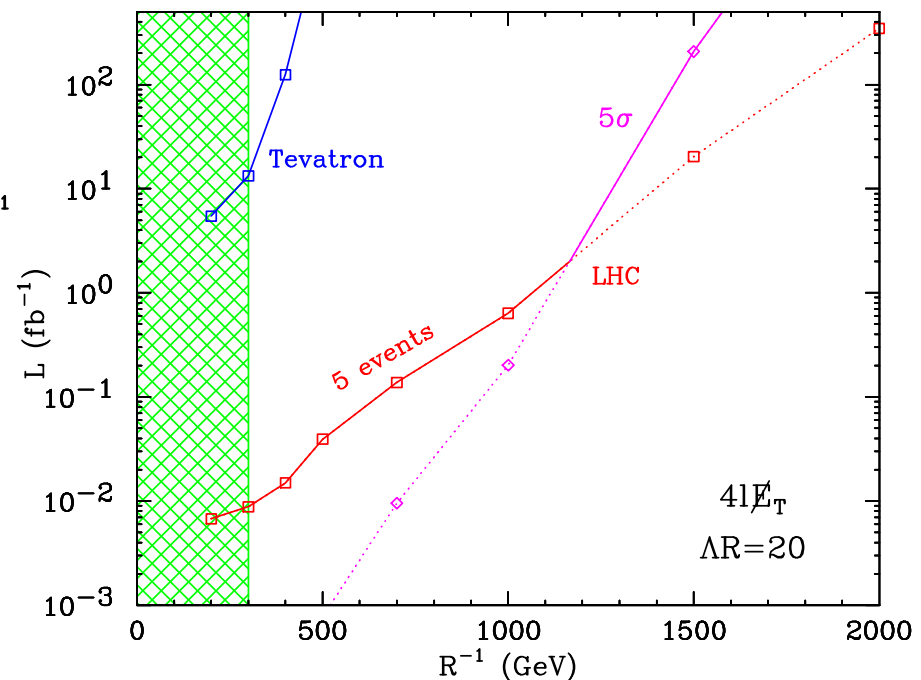
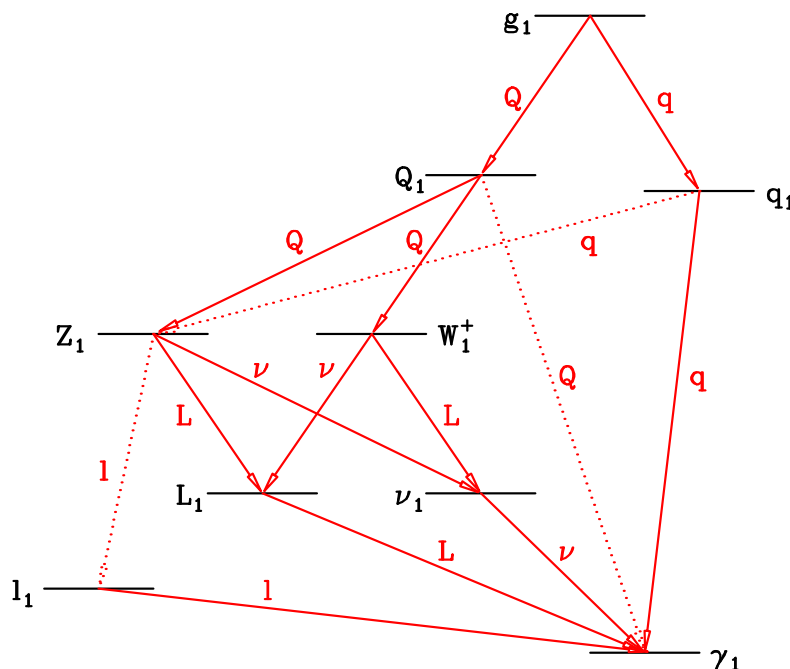


$1/R = 500$ GeV

More on UED

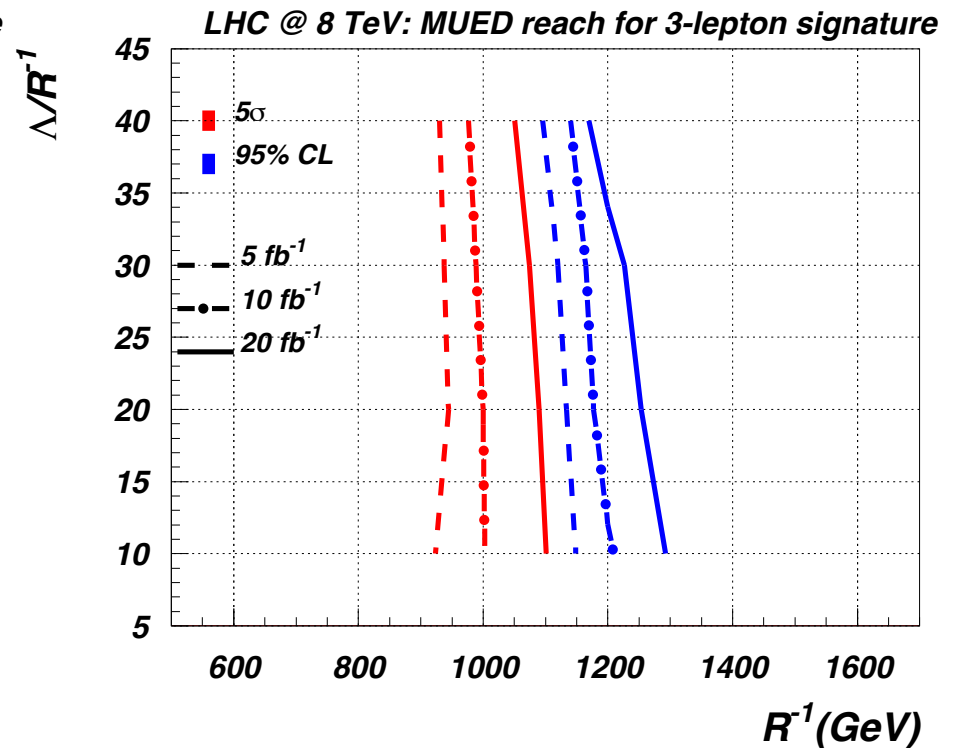
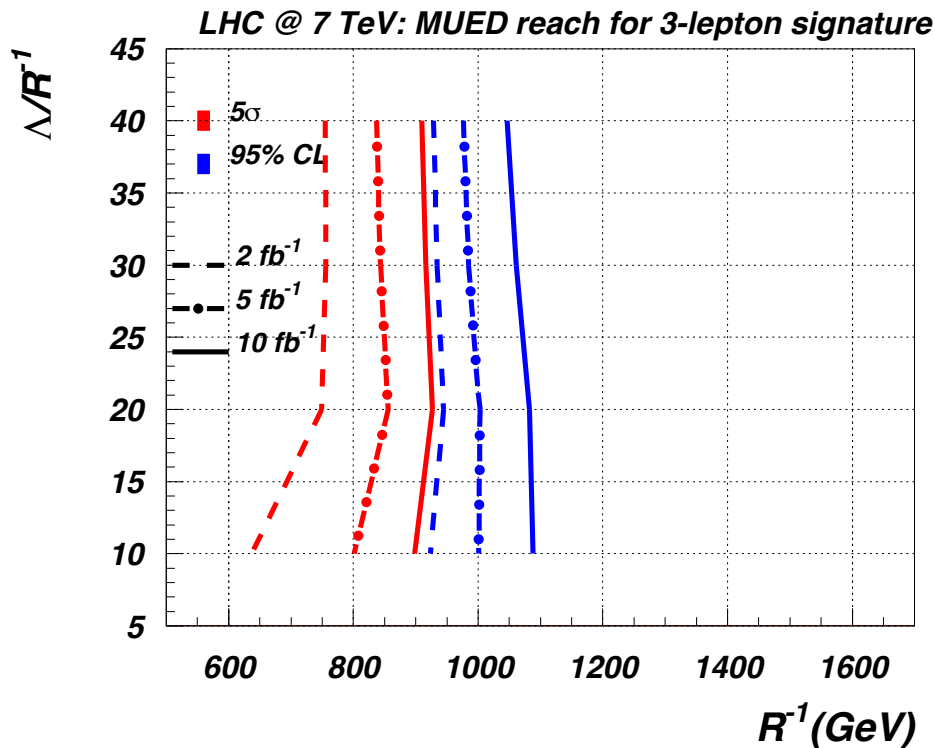
- Two parameters: R , Λ (cutoff)
- The same spin: SM and KK partners
- Larger production cross sections (compared to SUSY productions), i.e., KK gluon, KK quark productions
- Decay products are softer
- 4 leptons with large branching fractions

0205314, Cheng, Matchev, Schmaltz



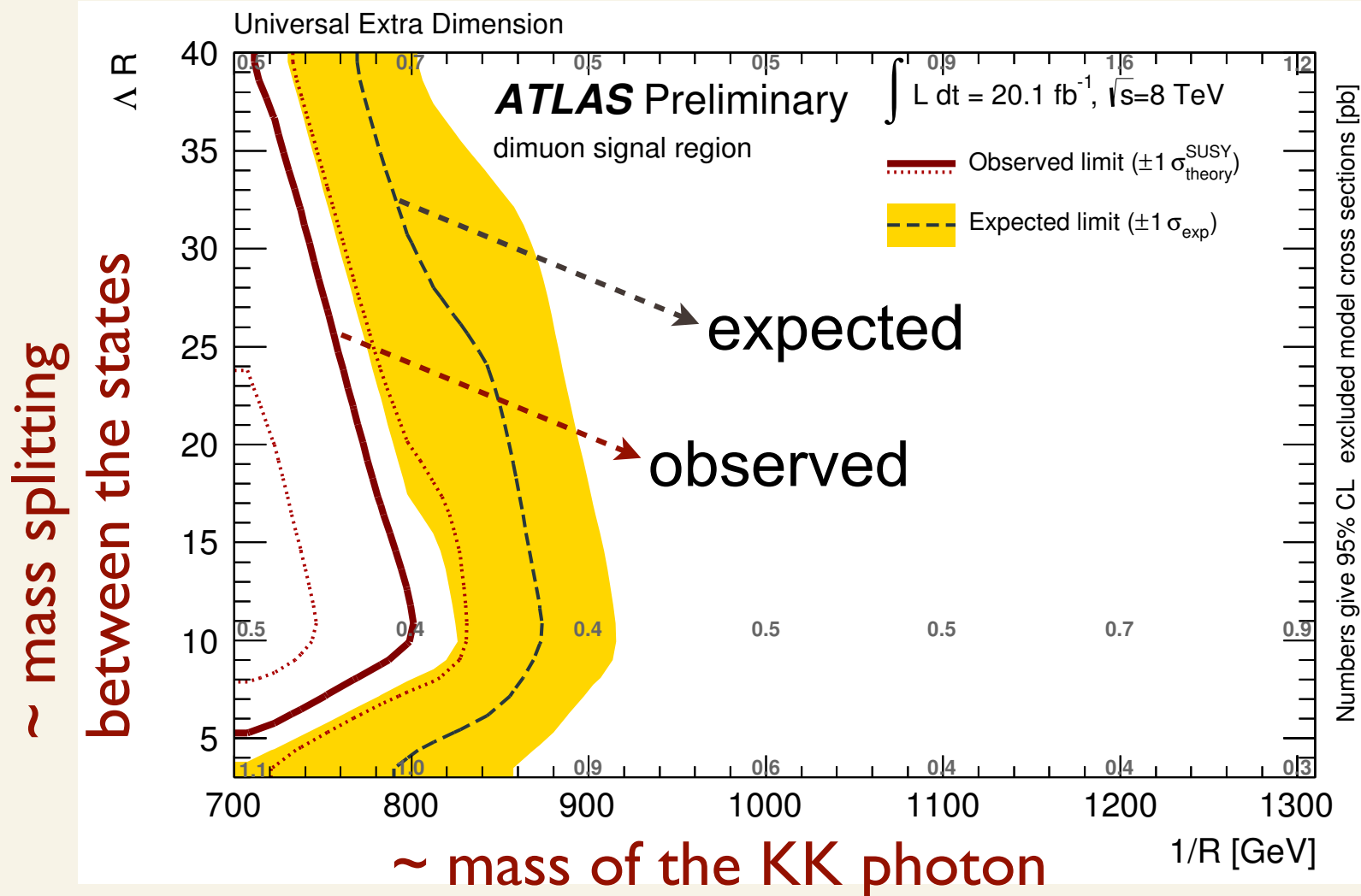
Current bounds from LHC

Belyaev, Brown, Moreno, Papineau 2012



- Collider and low energy experiments provide lower bound on KK scale ($1/R$)
- Minimal UED (2 parameters) is very constrained
- Cutoff dependence is logarithmic

ATLAS limit (dimuon)



SUSY is an ED theory

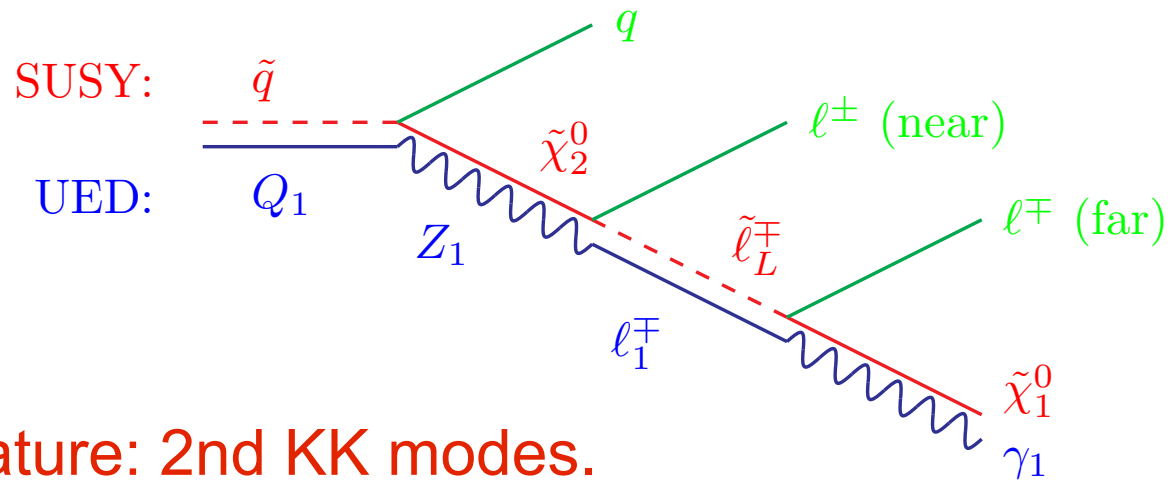
- SUSY is an extra dimension theory with anti commuting coordinate

$$\Phi(x^\mu, \theta) = \phi(x^\mu) + \psi^\alpha(x^\mu)\theta_\alpha + F(x^\mu)\theta^\alpha\theta_\alpha$$

	SUSY	UED
Field	$\Phi(x^\mu, \theta, \theta^*)$	$\Phi(x^\mu, y^i)$
Symmetry	Supersymmetry	5D Lorentz and gauge symmetry (broken by compactification and boundary interactions)
Component fields	(SM, Superpartner) Φ in terms of θ and θ^*	(SM, KK partners) Φ in terms of bases (exp(y), cos(y), sin(y))
Spins	differ by $\frac{1}{2}$	same spins
\mathcal{L}_{eff}	$\int d\theta d\theta^* S[\Phi(x, \theta, \theta^*)]$	$\int dy S[\Phi(x, y)]$
Discrete symmetry \rightarrow DM	R-parity	KK-parity = $(-1)^n$
# of parameters	many (soft terms) 5 in MSUGRA	many (boundary terms) 2 in MUED (R, Λ)
Gauge bosons	maybe	KK partners of SM gauge bosons
Renormalizability	Yes	No

SUSY vs UED

- SUSY-like cascade decays at the LHC from the first KK modes.



- Distinct feature: 2nd KK modes.

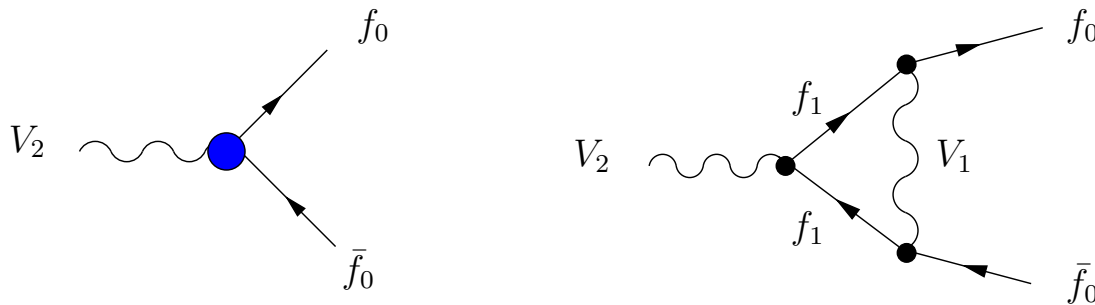
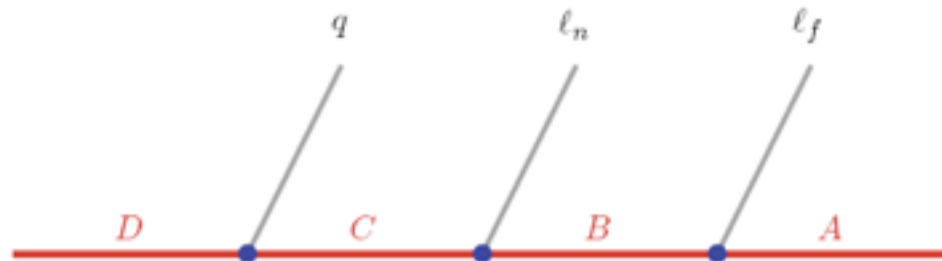


Figure 3. The effective $\bar{f}_0 V_2^\mu f_0$ KK-number violating coupling on the left is generated at one loop order from the one loop diagram on the right.

Spin and Couplings of Dark Matter: Why is it difficult to measure them?

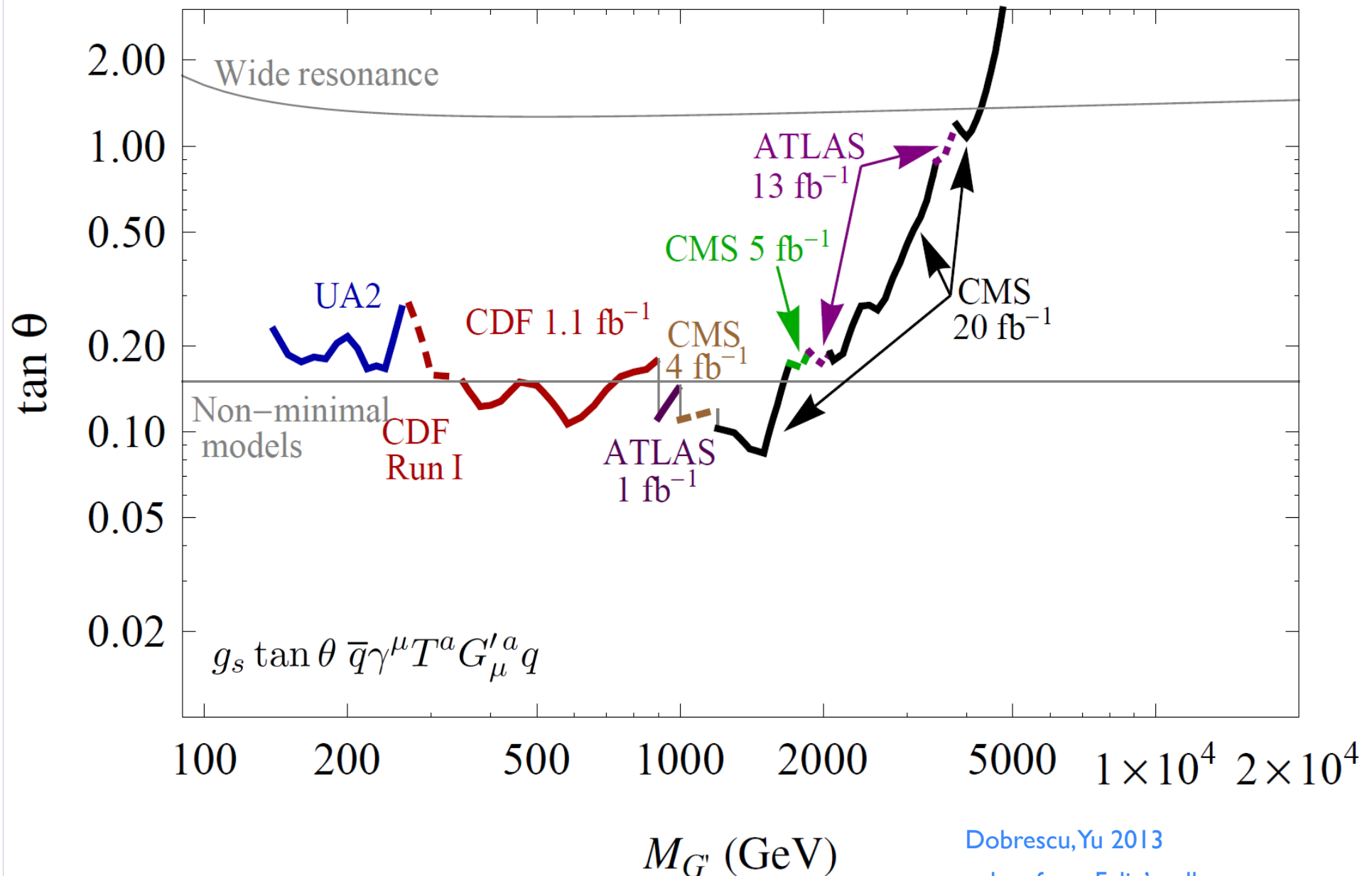
- Missing energy signatures arise from something like:



- Several alternative explanations:

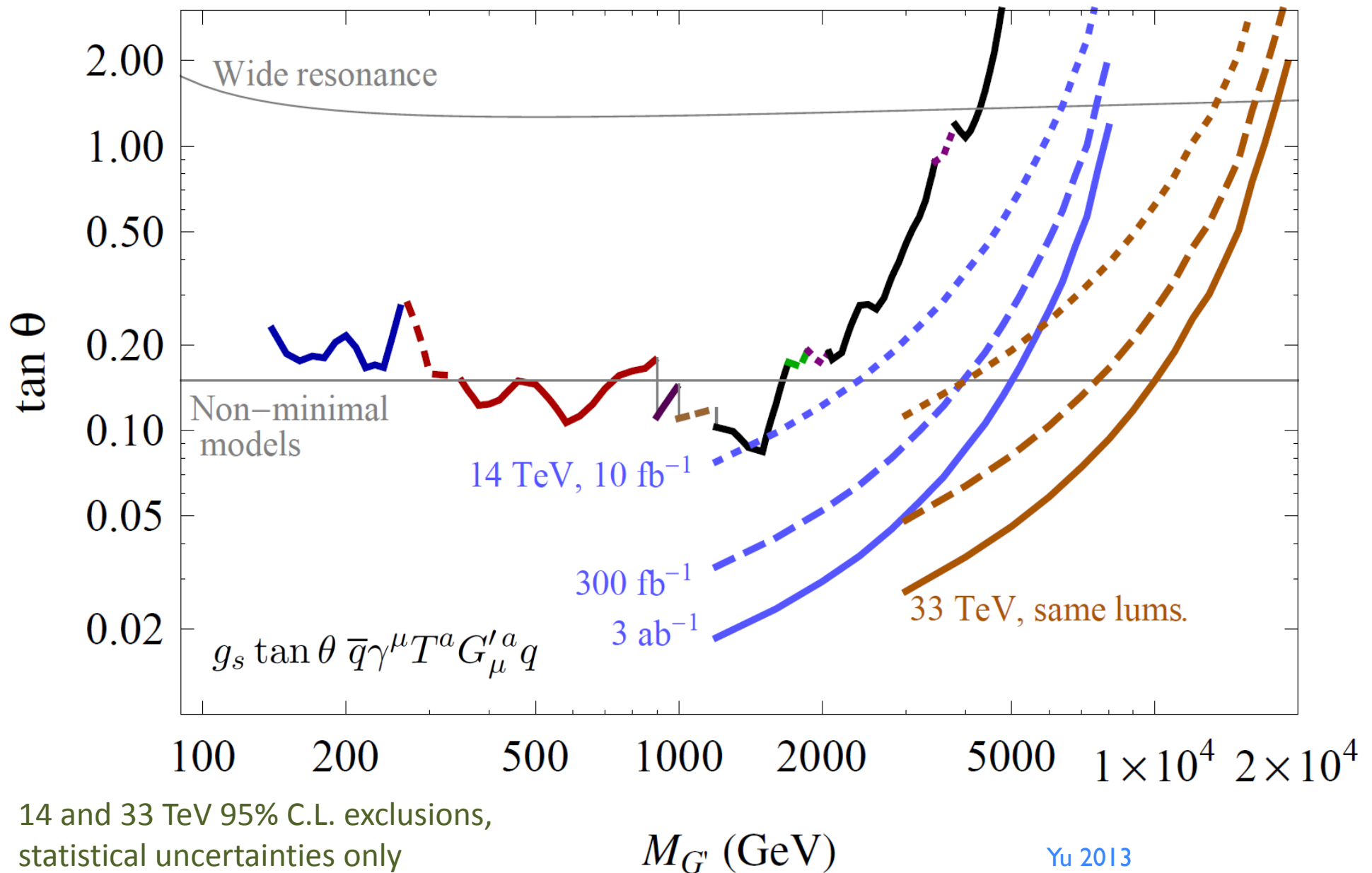
S	Spins	D	C	B	A	Example
1	SFSF	Scalar	Fermion	Scalar	Fermion	$\tilde{q} \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\ell} \rightarrow \tilde{\chi}_1^0$
2	FSFS	Fermion	Scalar	Fermion	Scalar	$q_1 \rightarrow Z_H \rightarrow \ell_1 \rightarrow \gamma_H$
3	FSFV	Fermion	Scalar	Fermion	Vector	$q_1 \rightarrow Z_H \rightarrow \ell_1 \rightarrow \gamma_1$
4	FVFS	Fermion	Vector	Fermion	Scalar	$q_1 \rightarrow Z_1 \rightarrow \ell_1 \rightarrow \gamma_H$
5	FVFV	Fermion	Vector	Fermion	Vector	$q_1 \rightarrow Z_1 \rightarrow \ell_1 \rightarrow \gamma_1$
6	SFVF	Scalar	Fermion	Vector	Fermion	—

Coupling vs. mass **current** limits: G'



Dobrescu, Yu 2013
taken from Felix's talk

Coupling vs. mass projections: G'

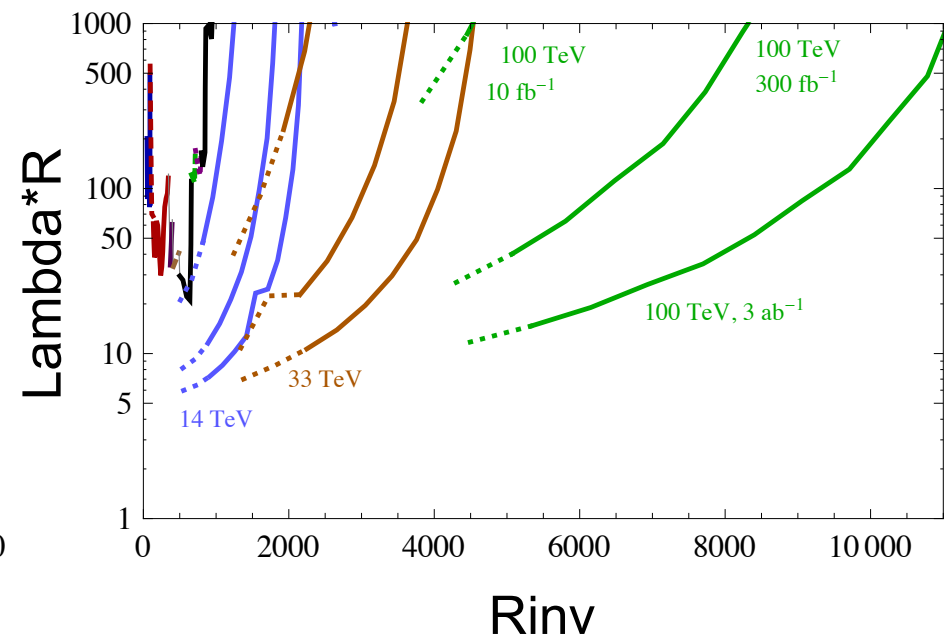
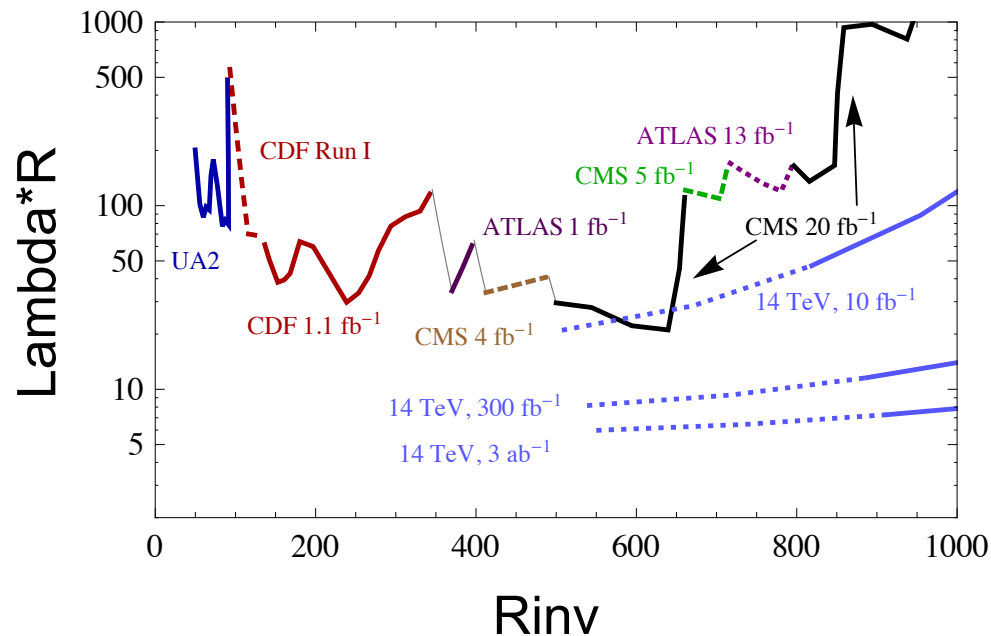


14 and 33 TeV 95% C.L. exclusions,
statistical uncertainties only

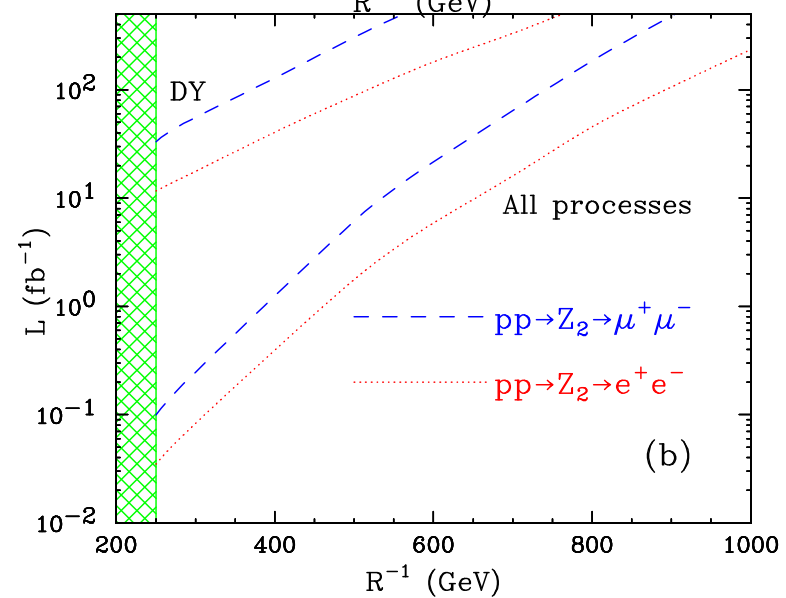
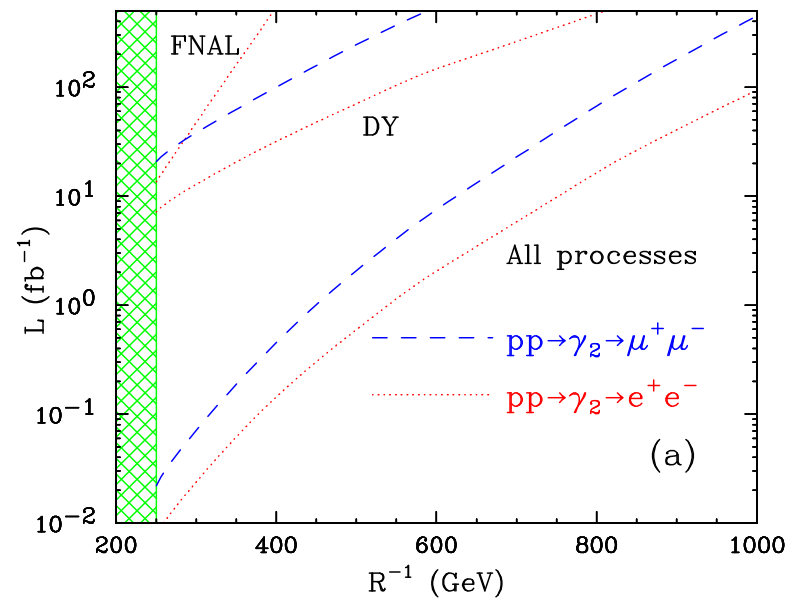
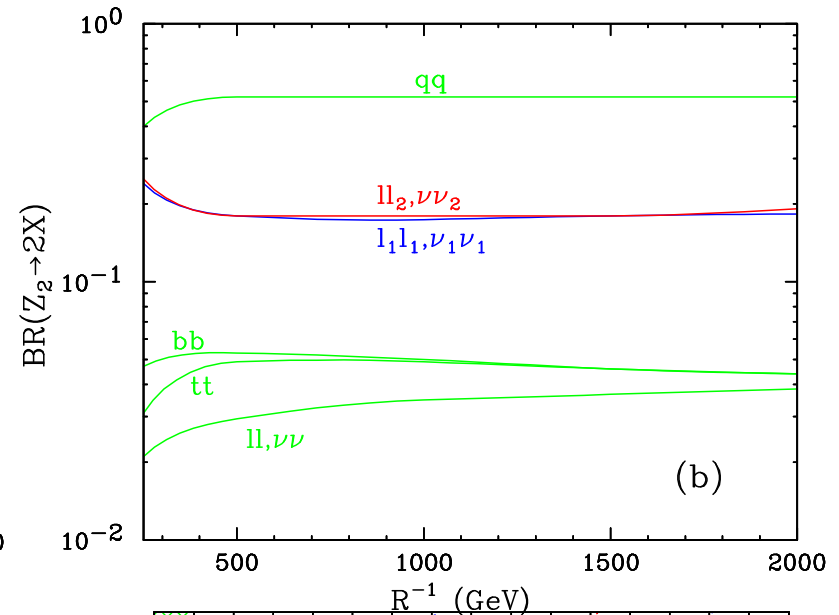
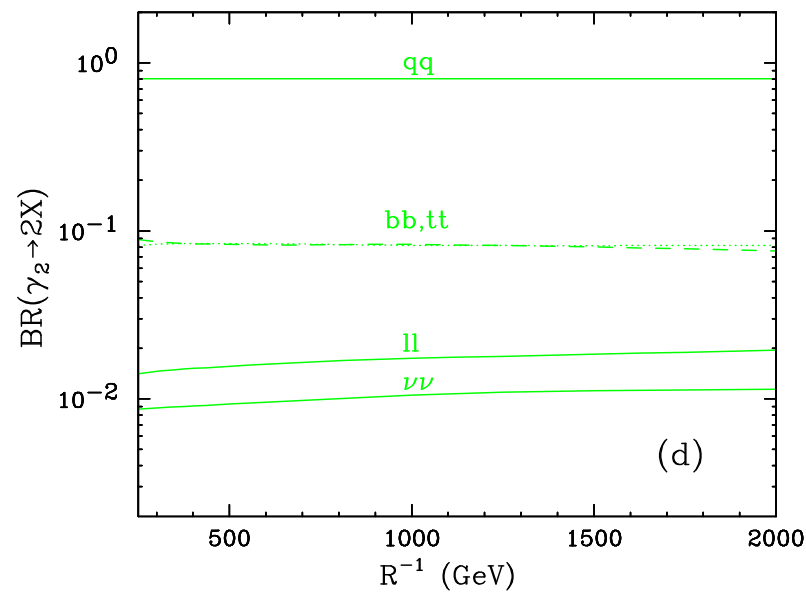
Current/projected bounds on level-2 KK gluon

Kong, Yu 2013

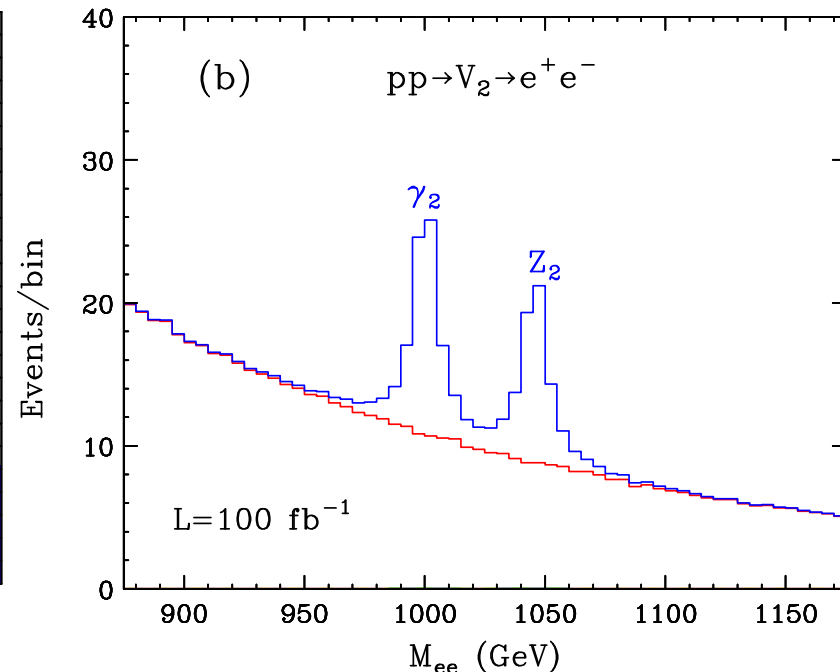
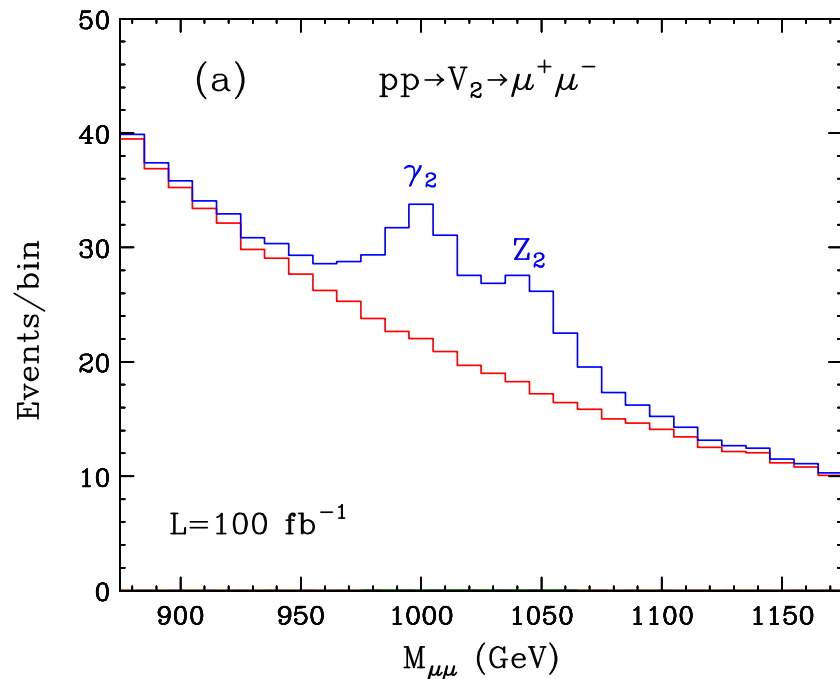
$SU(3)_c$ gauge boson	Quark (up)	$ig_3 \frac{\lambda^A}{2} \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16\pi^2} \ln \left(\frac{\Lambda}{\mu} \right)^2 \left[P_L \left(\frac{1}{8}g_1^2 + \frac{27}{8}g_2^2 - \frac{11}{2}g_3^2 \right) + P_R \left(2g_1^2 - \frac{11}{2}g_3^2 \right) \right]$
G_2	Quark (down)	$ig_3 \frac{\lambda^A}{2} \gamma^\mu \frac{1}{\sqrt{2}} \frac{1}{16\pi^2} \ln \left(\frac{\Lambda}{\mu} \right)^2 \left[P_L \left(\frac{1}{8}g_1^2 + \frac{27}{8}g_2^2 - \frac{11}{2}g_3^2 \right) + P_R \left(\frac{1}{2}g_1^2 - \frac{11}{2}g_3^2 \right) \right]$



Level 2: KK (dilepton) resonances



Level 2: KK (dilepton) resonances

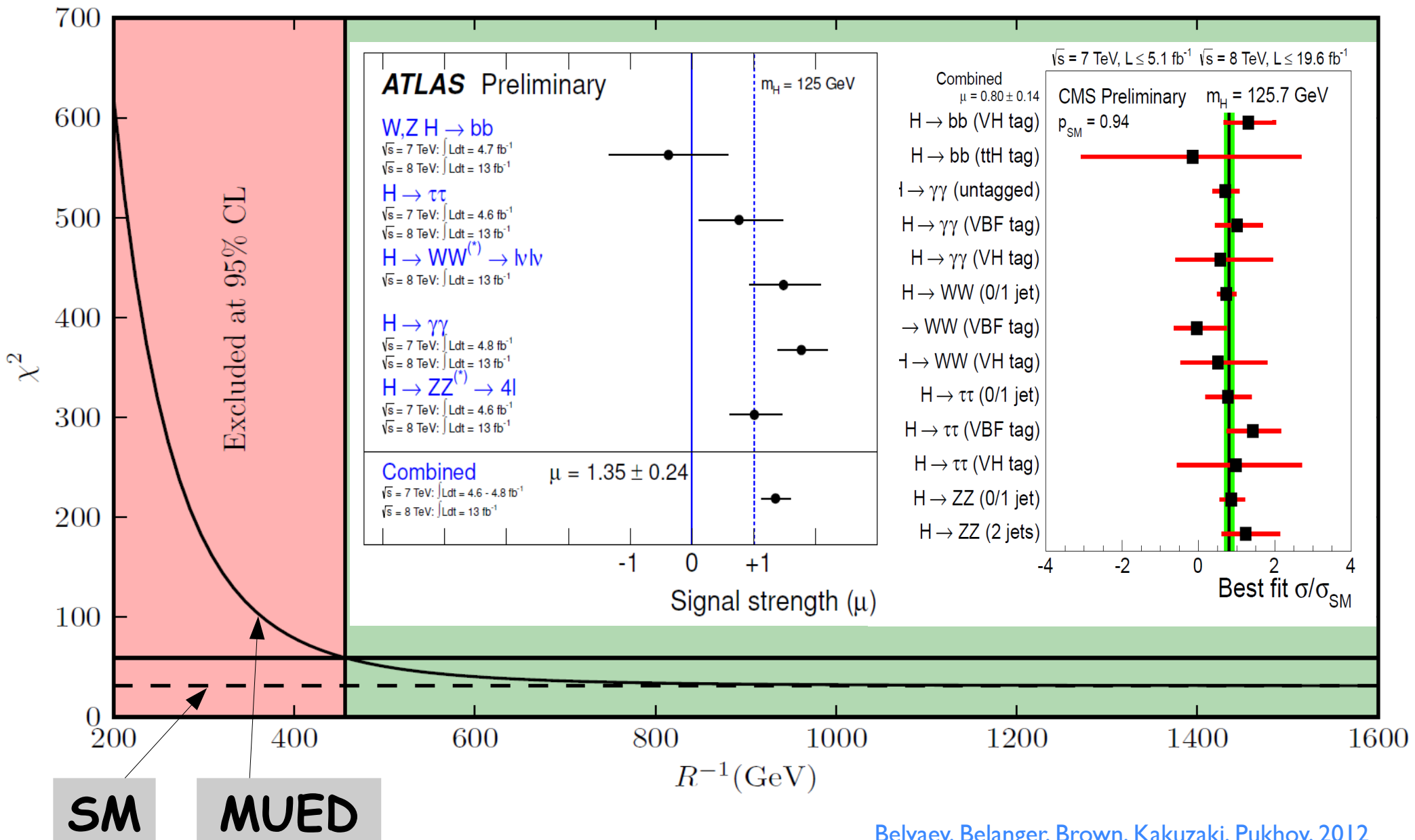


$$\Delta m_{ee}/m_{ee} \approx 1\%$$

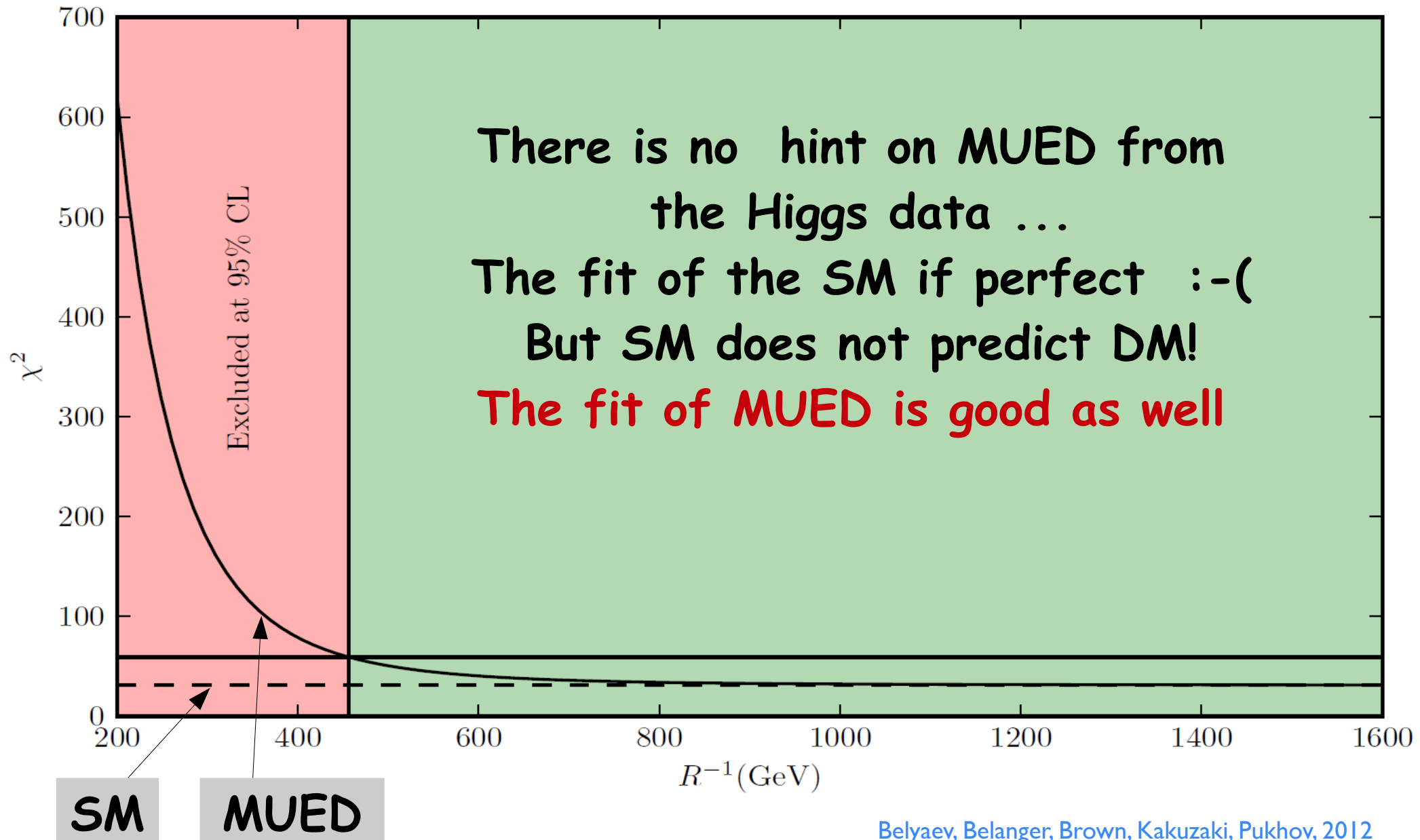
$$\frac{\Delta m_{\mu\mu}}{m_{\mu\mu}} = 0.0215 + 0.0128 \left(\frac{m_{\mu\mu}}{1 \text{ TeV}} \right)$$

Datta, Kong, Matchev 2005

Data Fit with MUED vs SM



Data Fit with MUED vs SM



KK Photon Dark Matter

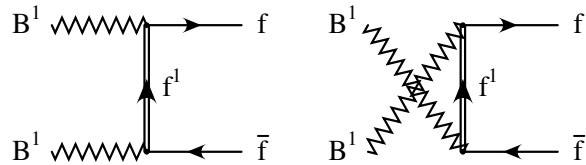


Figure 4: Feynman diagrams for $B^{(1)}B^{(1)}$ annihilation into fermions.

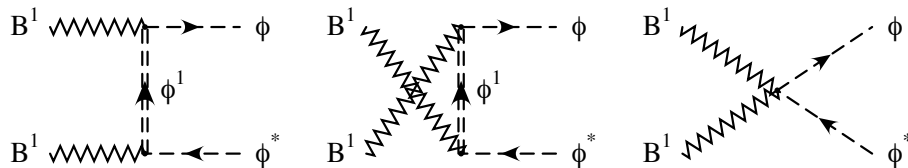
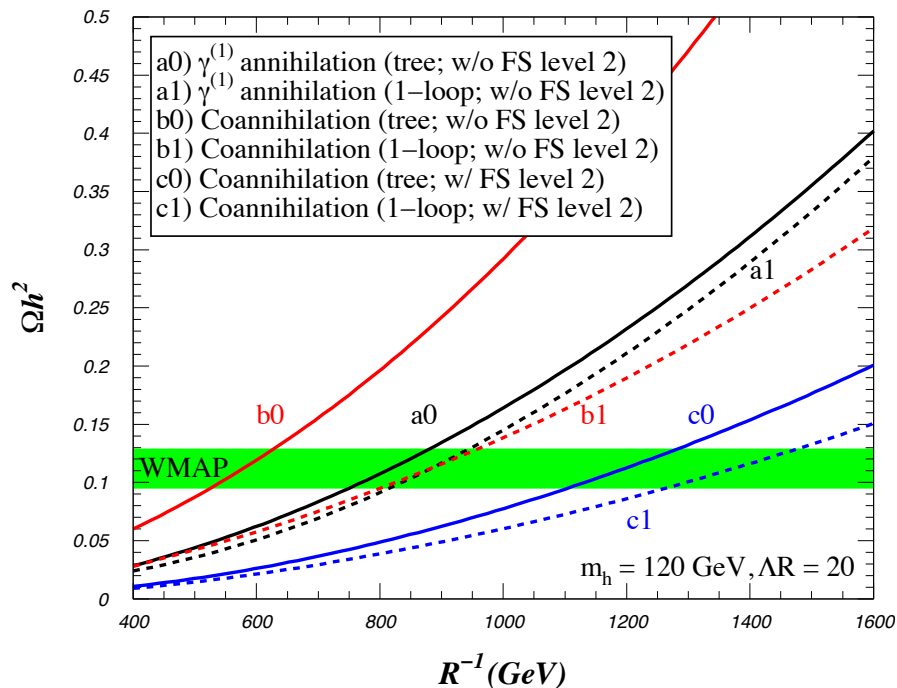
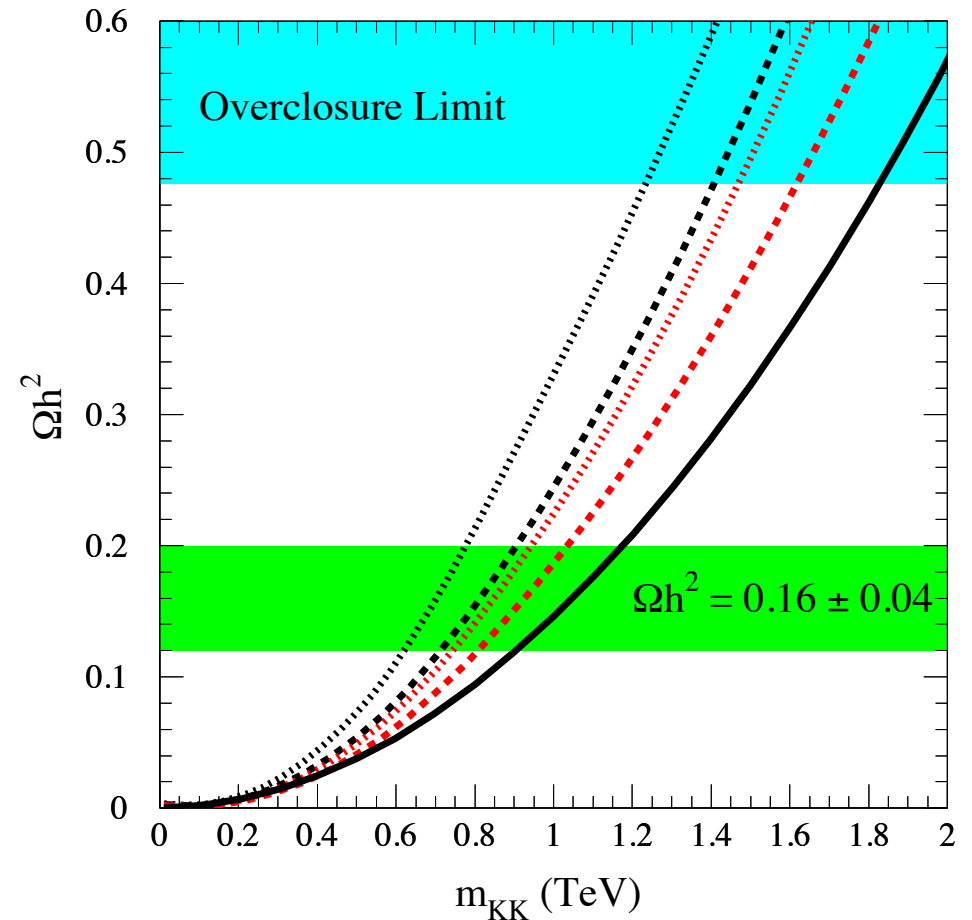


Figure 5: Feynman diagrams for $B^{(1)}B^{(1)}$ annihilation into Higgs scalar bosons.



Servant, Tait 2002



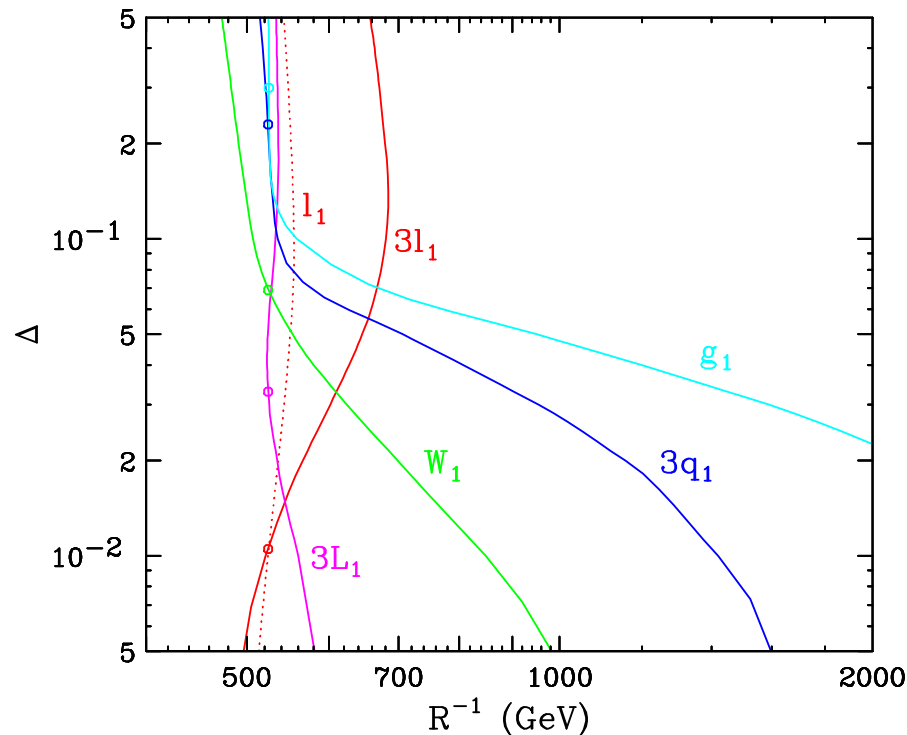
Belanger, Kakizaki, Pukhov, 2010

Kong, Matchev, 2005

KKDM in non-minimal model

- The change in the cosmologically preferred value for R^{-1} as a result of varying the different KK masses away from their nominal MUED values (along each line, $\Omega h^2 = 0.1$)

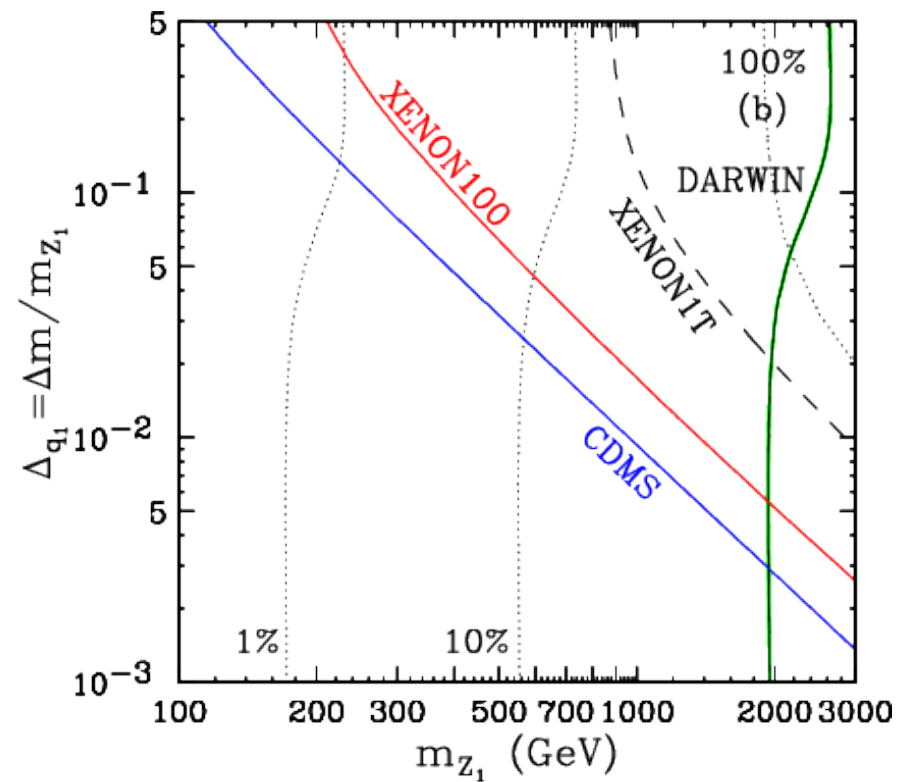
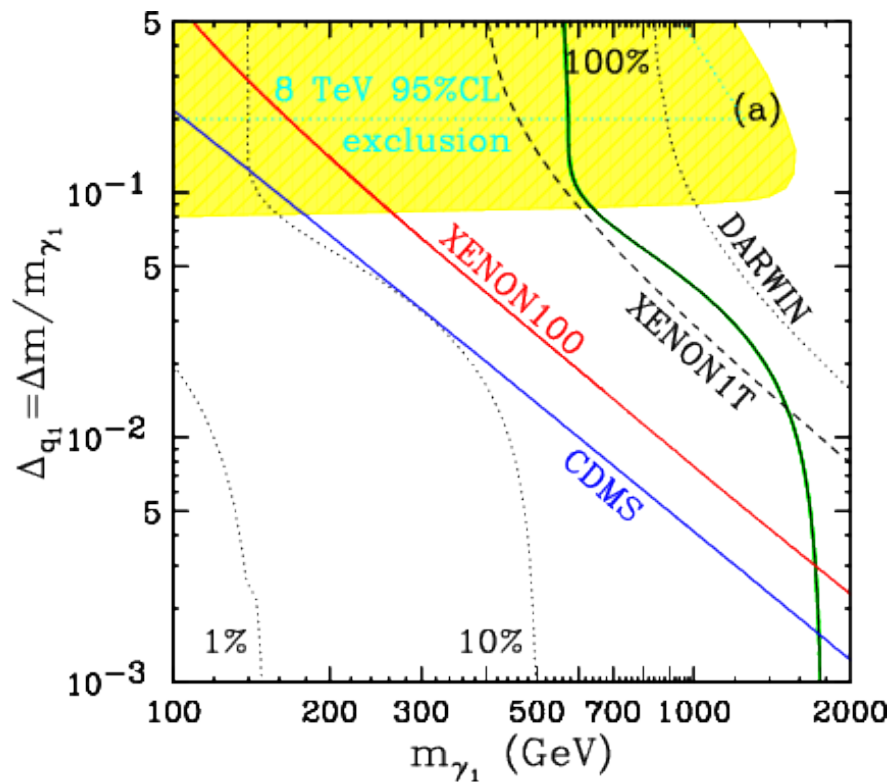
(Kong, Matchev, hep-ph/0509119)



- In nonminimal UED, Cosmologically allowed LKP mass range can be larger
 - If $\Delta = \frac{m_1 - m_{\gamma_1}}{m_{\gamma_1}}$ is small, m_{LKP} is large, UED escapes collider searches
 - But, good news for dark matter searches

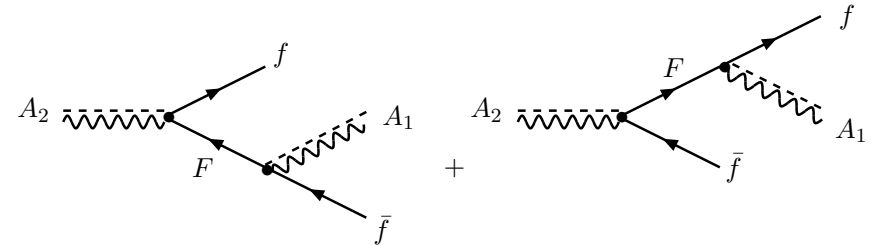
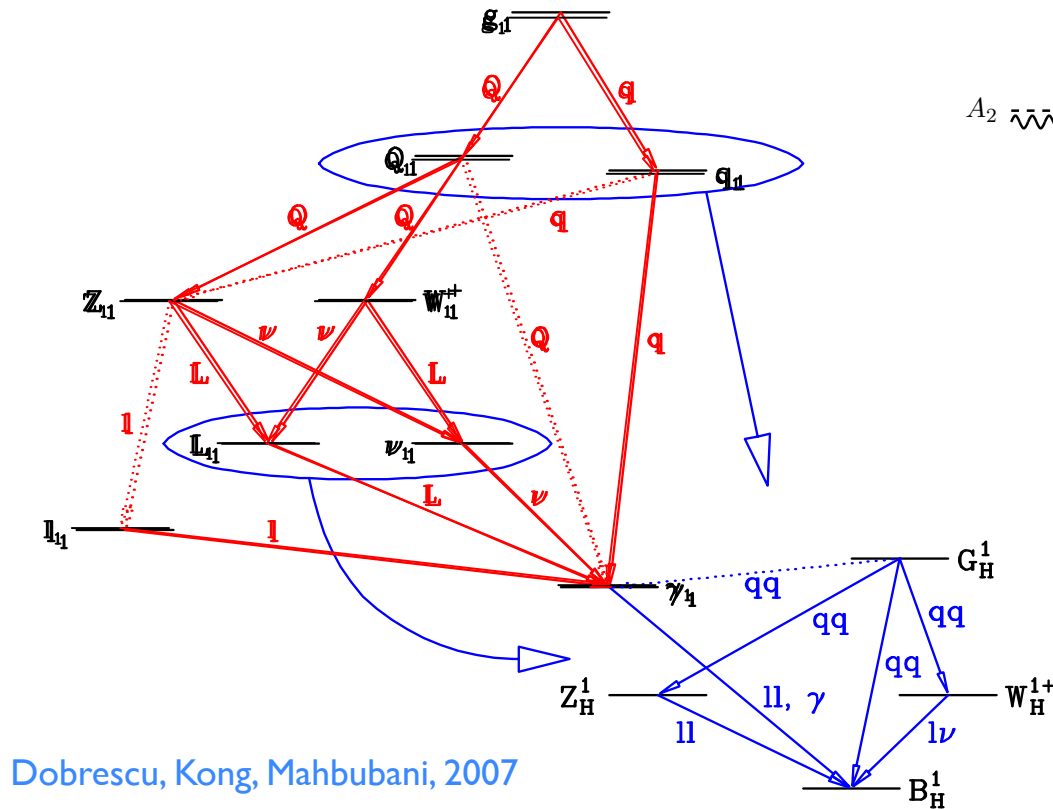
KK Dark Matter: complementarity

- Treat the LKP mass and mass splitting as free parameters.
- Gives a better chance for the LHC, and direct detection.



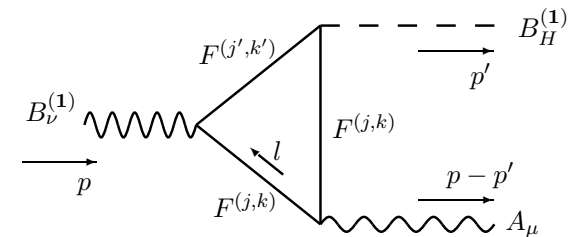
- Yellow: 4 leptons plus MET at 14 TeV LHC with 100 fb⁻¹
- Green: relic abundance

How Many Extra Dimensions ?



$$\text{Br} \left(B_\mu^{(1)} \rightarrow B_H^{(1)} \gamma \right) \equiv b_{B\gamma} \approx 34.0\%$$

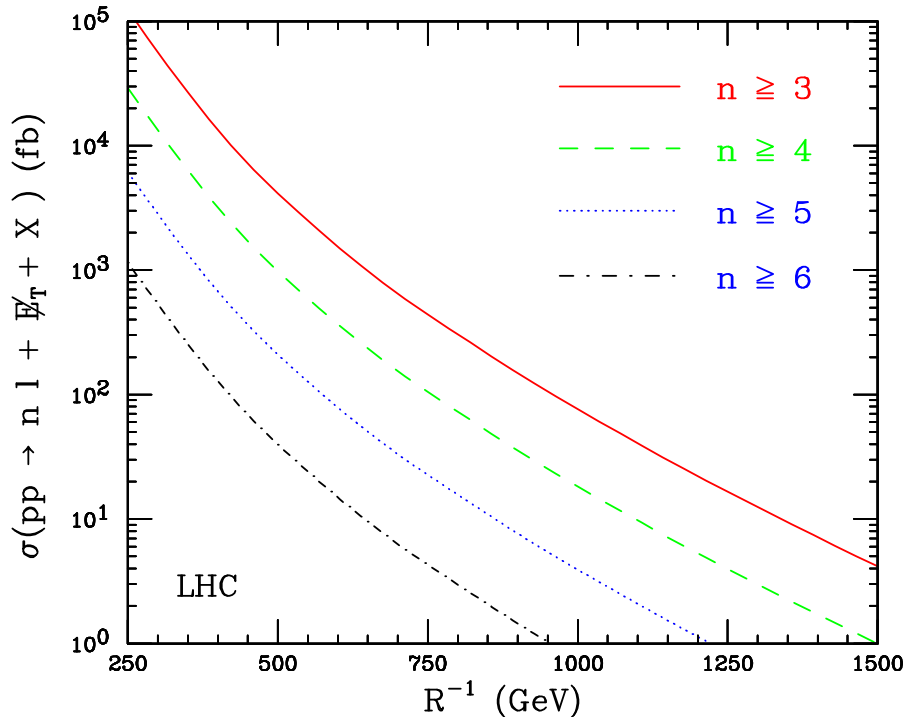
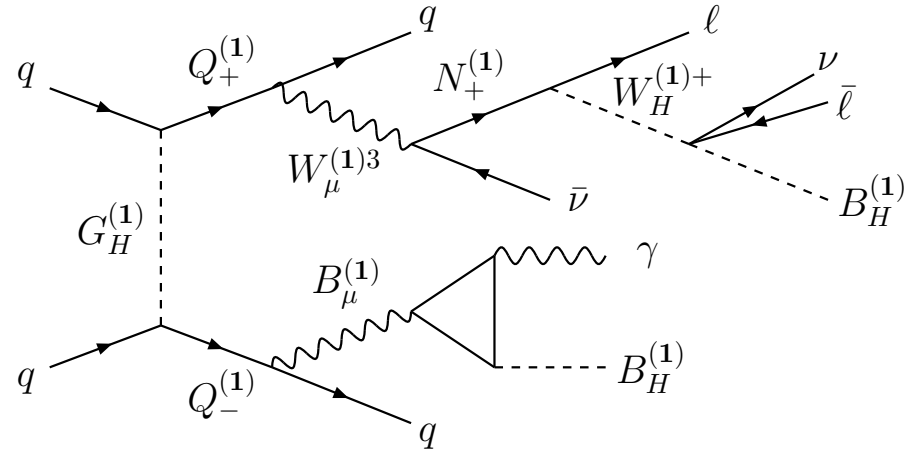
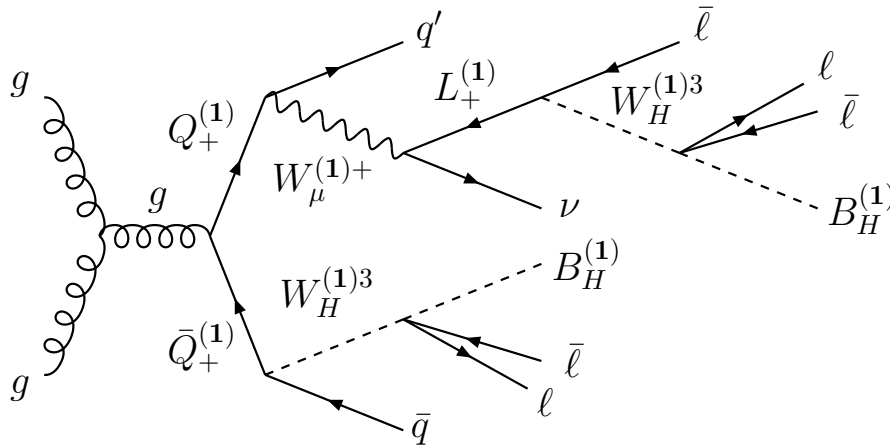
$$\text{Br} \left(B_\mu^{(1)} \rightarrow B_H^{(1)} e^+ e^- \right) \equiv b_{Be} \approx 21.3\%$$



$$-\frac{R}{4} \left(\mathcal{C}_B \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} B_{\alpha\beta}^{(1)} B_H^{(1)} + \mathcal{C}_G \epsilon^{\mu\nu\alpha\beta} G_{\mu\nu} B_{\alpha\beta}^{(1)} G_H^{(1)} \right)$$

- Extra “spinless” states: GH, ZH, WH, BH
- KK photon is not DM and decays to spinless photon via 1-loop 2 body or tree-level 3 body decay (with vanishing BC at cutoff scale)

Multi-leptons from 2UED



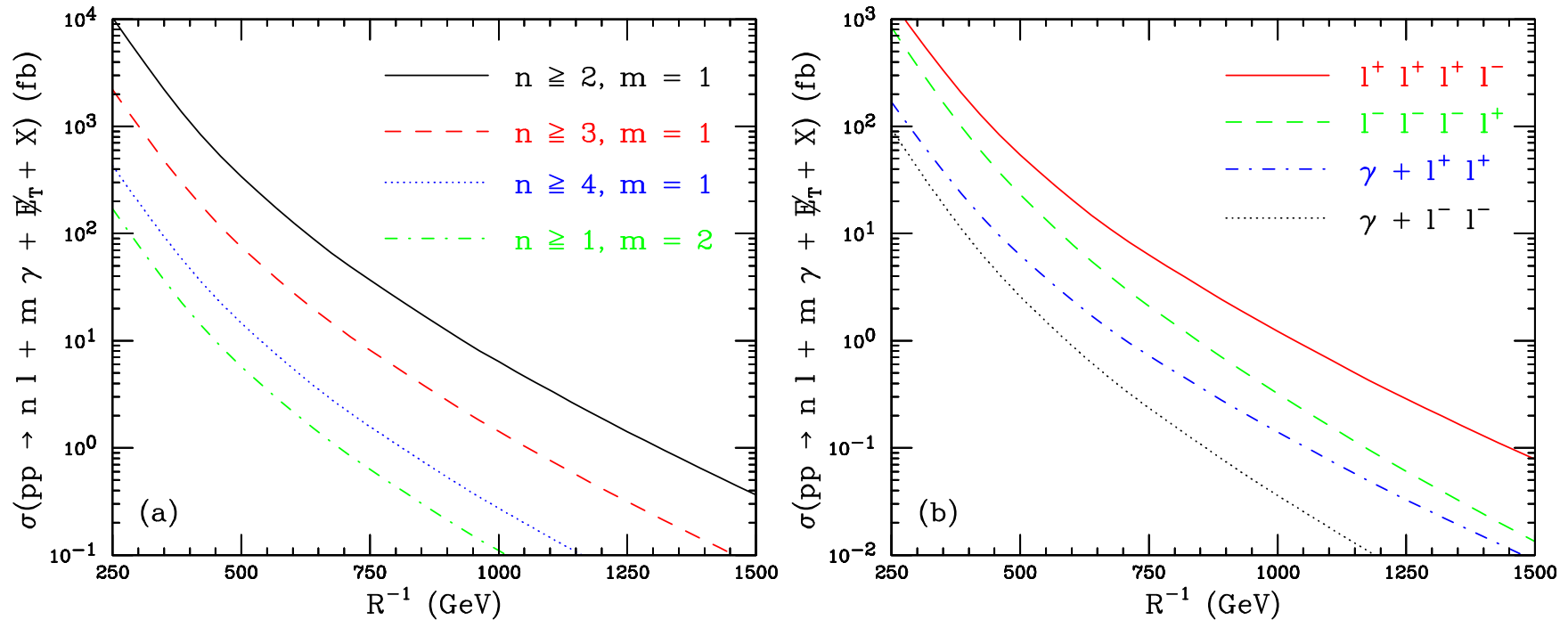
$$\sigma(pp \rightarrow n\ell + m\gamma + \cancel{E}_T, n \geq n_{min}) = \sum_{i=1}^{11} \sum_{j \geq i}^{11} \sigma(pp \rightarrow A_i^{(1)} A_j^{(1)}) B_{ij}$$

$$B_{ij} = \sum_{\substack{a,b=0 \\ a+b \geq n_{min}}}^4 \sum_{\substack{a',b'=0 \\ a'+b'=m}}^1 \text{Br}(i, a, a') \text{Br}(j, b, b')$$

$$0 \leq n + 2m \leq 8$$

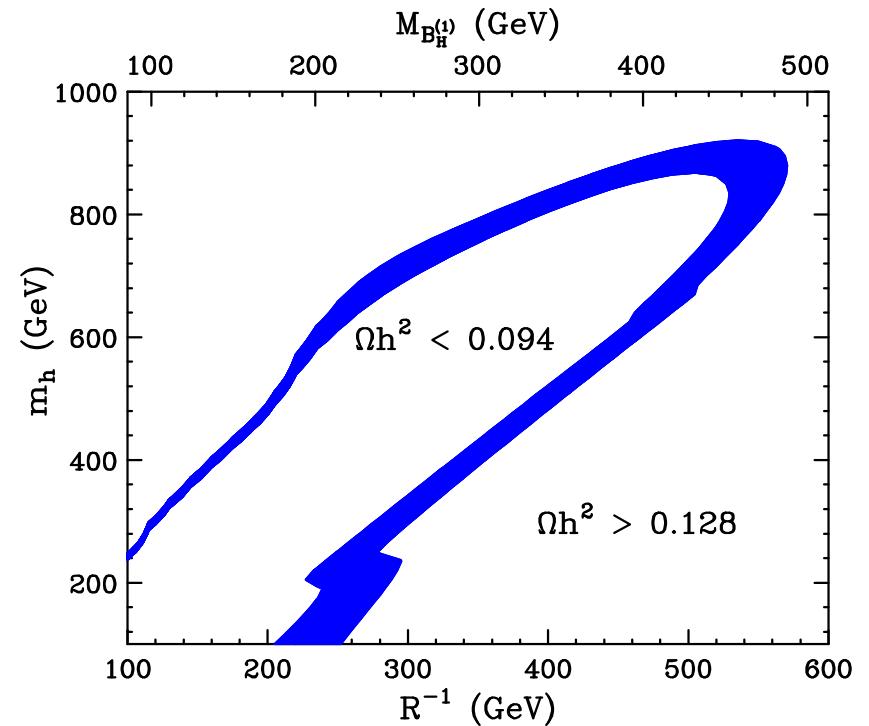
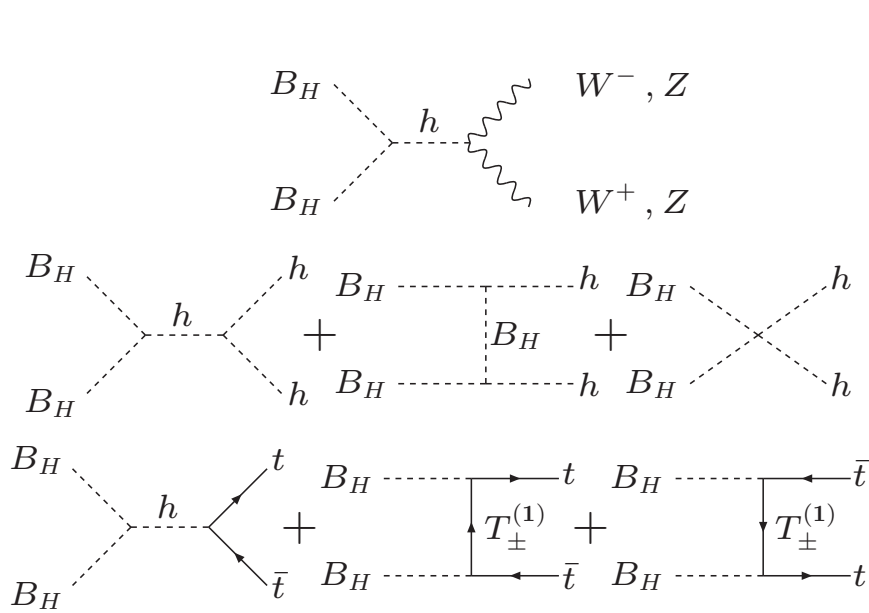
- The number of multi-lepton events at 14 TeV LHC
- No acceptance cuts

Leptons and Photons from 2UED



- The number of lepton + photon events (14 TeV)

Spinless Photon Dark Matter



Dobrescu, Hooper, Kong, Mahbubani, 2007

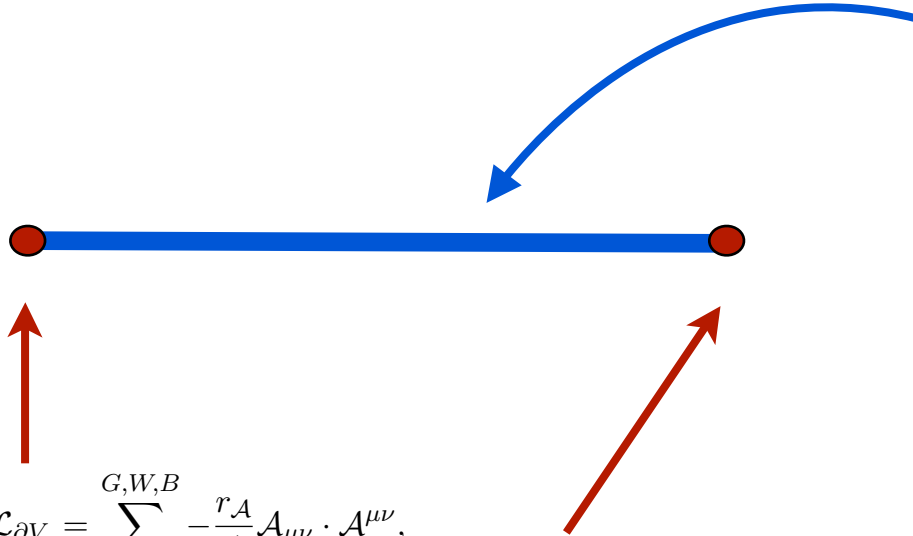
- $R^{-1} < 600$ GeV
- Light higgs requires light KK particles
→ large production cross-sections at the LHC/Tevatron
- Oblique corrections: $R_{\text{inv}} > 900$ GeV
- Relic abundance: $R_{\text{inv}} < 600$ GeV
- $M_H = 125-126$ GeV

Many Variations

- MUED: Minimal Universal Extra Dimensions (cf. mSUGRA)
- 2UED: Two Universal Extra Dimensions (cf. GMSB)
- nUED: non-minimal Universal Extra Dimensions
 - boundary terms
- SUED: Split Universal Extra Dimensions (cf. Split SUSY)
 - bulk terms
- sUED: UED with singlet extension
- NMUED: Next-to-Minimal UED (cf. pMSSM)
 - (with boundary and bulk terms)
- Many others with larger gauge groups (cf. $SU(2)_L \times SU(2)_R$)

Next-to-Minimal UED

Flacke, Kong, Park 2013



$$S_5 = \int d^4x \int_{-L}^L dy [\mathcal{L}_V + \mathcal{L}_\Psi + \mathcal{L}_H + \mathcal{L}_{Yuk}]$$

$$\mathcal{L}_V = \sum_A^{G,W,B} -\frac{1}{4} \mathcal{A}^{MN} \cdot \mathcal{A}_{MN}$$

$$\mathcal{L}_\Psi = \sum_\Psi^{Q,U,D,L,E} i\bar{\Psi} \overleftrightarrow{D}_M \Gamma^M \Psi - M_\Psi \bar{\Psi} \Psi$$

$$\mu\theta(y) = M_{Q,L} = -M_{U,D,E}$$

$$M_\Psi(y) = -M_\Psi(-y).$$

$$\mathcal{L}_H = (D_M H)^\dagger D^M H - V(H),$$

$$V(H) = -\mu_5^2 |H|^2 + \lambda_5 |H|^4,$$

$$\mathcal{L}_{Yuk} = \lambda_5^E \bar{L} H E + \lambda_5^D \bar{Q} H D + \lambda_5^U \bar{Q} \tilde{H} D + \text{h.c.}$$

$$\mathcal{L}_{\partial V} = \sum_A^{G,W,B} -\frac{r_A}{4} \mathcal{A}_{\mu\nu} \cdot \mathcal{A}^{\mu\nu},$$

$$\mathcal{L}_{\partial \Psi} = \sum_{\Psi=Q,L} i r_\Psi \bar{\Psi}_L D_\mu \gamma^\mu \Psi_L + \sum_{\Psi=U,D,E} i r_\Psi \bar{\Psi}_R D_\mu \gamma^\mu \Psi_R,$$

$$\mathcal{L}_{\partial H} = r_H (D_\mu H)^\dagger D^\mu H + r_\mu \mu_5^2 |H|^2 - r_\lambda \lambda_5 |H|^4,$$

$$\mathcal{L}_{\partial Y_{uk}} = r_{\lambda^E} \lambda_5^E \bar{L} H E + r_{\lambda^D} \lambda_5^D \bar{Q} H D + r_{\lambda^U} \lambda_5^U \bar{Q} \tilde{H} D + \text{h.c.}$$

$$S_{bdy} = \int d^4x \int_{-L}^L dy (\mathcal{L}_{\partial V} + \mathcal{L}_{\partial \Psi} + \mathcal{L}_{\partial H} + \mathcal{L}_{\partial Y_{uk}}) [\delta(y-L) + \delta(y+L)]$$

$$L = \pi R/2$$

NMUED

Flacke, Kong, Park 2013

fermion bulk masses $M_{Q,U,D,L,E}$

boundary gauge parameters r_G, r_W, r_B

boundary Higgs parameters r_H, r_μ, r_λ

boundary fermion parameters $r_{Q,U,D,L,E}$

boundary Yukawa couplings $r_{\lambda^{U,D,E}}$

- To avoid tree-level FCNC, set all M and r flavor blind --> 19.
- For $r_\mu \neq r_\lambda$, bulk VEV and boundary VEV different.
- To avoid KK mode mixing, set all r's the same.
- Assume universal bulk masses --> two extra parameters in addition to R and Lambda (cutoff)

Fermions

$$\Psi(x, y) = \sum_{n=0}^{\infty} \left(\psi_L^{(n)}(x) f_n^{\Psi_L}(y) + \psi_R^{(n)}(x) f_n^{\Psi_R}(y) \right)$$

$$n = 0 : f_0^{\Psi_L} = \mathcal{N}_0^{\Psi} e^{\mu|y|},$$

$$\text{odd } n : \begin{cases} f_n^{\Psi_L} = \mathcal{N}_n^{\Psi} \sin(k_n y), \\ f_n^{\Psi_R} = \mathcal{N}_n^{\Psi} \left(-\frac{k_n}{m_{f_n}} \cos(k_n y) + \frac{\mu}{m_{f_n}} \theta(y) \sin(k_n y) \right), \end{cases}$$

$$\text{even } n : \begin{cases} f_n^{\Psi_L} = \mathcal{N}_n^{\Psi} \left(\frac{k_n}{m_{f_n}} \cos(k_n y) + \frac{\mu}{m_{f_n}} \theta(y) \sin(k_n y) \right), \\ f_n^{\Psi_R} = \mathcal{N}_n^{\Psi} \sin(k_n y). \end{cases}$$

$$\begin{aligned} k_n \cos(k_n L) &= (r (m_{f_n})^2 + \mu) \sin(k_n L) && \text{for odd } n, \\ r k_n \cos(k_n L) &= -(1 + r\mu) \sin(k_n L) && \text{for even } n, \end{aligned}$$

$$m_{f_n} = \sqrt{k_n^2 + \mu^2},$$

$$\mathcal{N}_n^{\Psi} = \begin{cases} \sqrt{\frac{\mu}{(1+2r\mu) \exp(2\mu L) - 1}} & \text{for } n = 0, \\ \frac{1}{\sqrt{L - \frac{\cos(k_n L) \sin(k_n L)}{k_n} + 2r \sin^2(k_n L)}} & \text{for odd } n, \\ \frac{1}{\sqrt{L - \frac{\cos(k_n L) \sin(k_n L)}{k_n}}} & \text{for even } n, \end{cases}$$

$$\int_{-L}^L dy f_m^{\Psi_L} f_n^{\Psi_L} [1 + r (\delta(y + L) + \delta(y - L))] = \delta_{mn},$$

$$\int_{-L}^L dy f_m^{\Psi_R} f_n^{\Psi_R} = \delta_{mn}.$$

Bosons

$$\mathcal{A}_\mu(x, y) = \sum_{n=0}^{\infty} \mathcal{A}_\mu^{(n)}(x) f_n^{\mathcal{A}}(y),$$

$$H(x, y) = \sum_{n=0}^{\infty} H^{(n)}(x) f_n^{\mathcal{A}}(y).$$

$$\cot(k_n L) = r k_n \quad \text{for odd } n,$$

$$\tan(k_n L) = -r k_n \quad \text{for even } n,$$

$$m_{\gamma_n} = k_n.$$

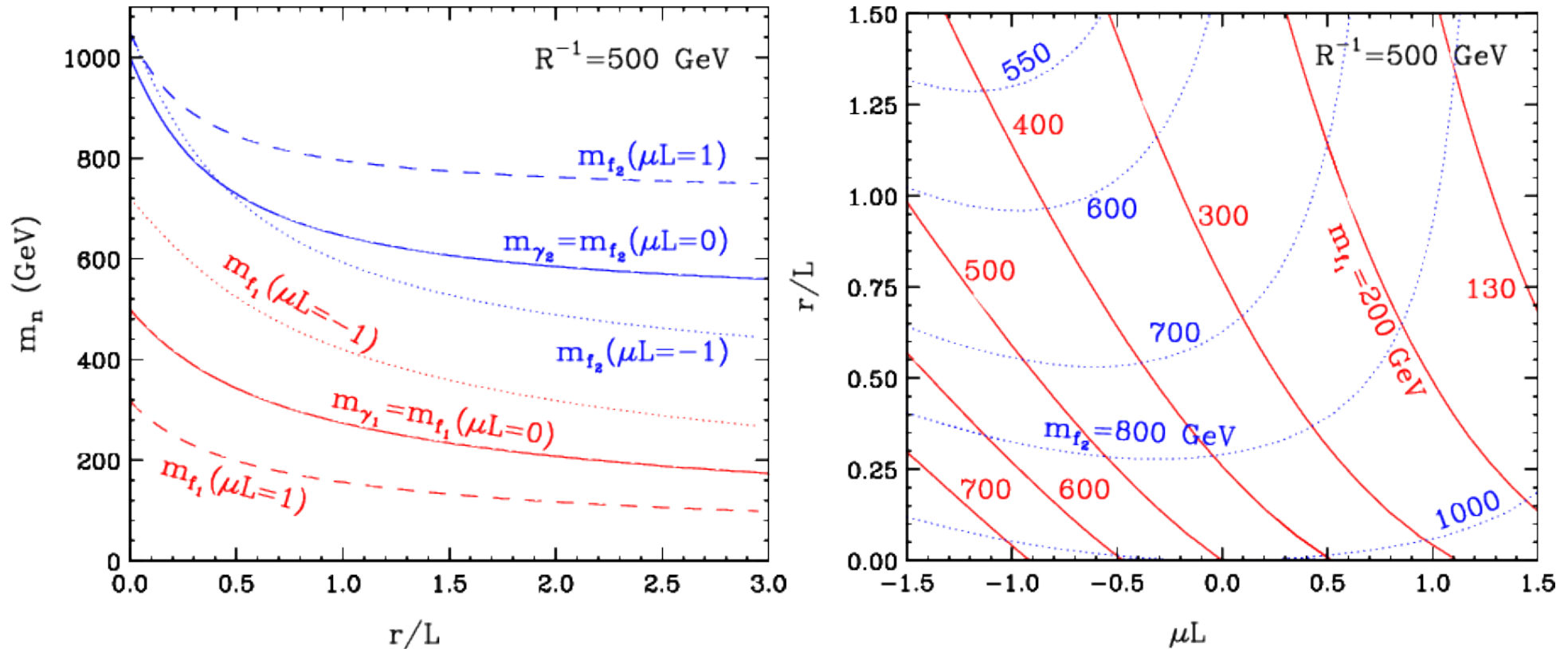
$$n = 0 : \quad f_0^{\mathcal{A}}(y) = \frac{1}{\sqrt{2L(1 + \frac{r}{L})}}$$

$$\text{odd } n : \quad f_n^{\mathcal{A}}(y) = \sqrt{\frac{1}{L + r \sin^2(k_n L)}} \sin(k_n y),$$

$$\text{even } n : \quad f_n^{\mathcal{A}}(y) = \sqrt{\frac{1}{L + r \cos^2(k_n L)}} \cos(k_n y),$$

$$\int_{-L}^L dy f_m^{\mathcal{A}} f_n^{\mathcal{A}} [1 + r (\delta(y + L) + \delta(y - L))] = \delta_{mn}.$$

NMUED: tree-level spectrum



$$L = \pi R/2$$

- “r” decreases masses of KK bosons and KK fermions
- “mu” increases masses of KK fermions (demand: $\mu < 0$)
- No loop corrections --> no dependence on cutoff

Couplings

$$S_{eff} \supset \int d^4x i g_A^5 \bar{\psi}_{L/R}^{(0)} \mathcal{A}^{(0)} \psi_{L/R}^{(0)} \int_{-L}^L dy f_0^{\mathcal{A}} f_0^{\Psi_{L/R}} f_0^{\Psi_{L/R}} [1 + r (\delta(y+L) + \delta(y-L))]]$$

$$= \int d^4x i \frac{g_A^5}{\sqrt{2L(1+r/L)}} \bar{\psi}_{L/R}^{(0)} \mathcal{A}^{(0)} \psi_{L/R}^{(0)},$$

$$g_{110}^{\mathcal{A}} = g_A^5 \int dy [1 + r (\delta(y+L) + \delta(y-L))] f_1^{\mathcal{A}} f_1^{\Psi_L} f_0^{\Psi_L} \equiv g^{\mathcal{A}} \mathcal{F}_{110}$$

$$g_{220}^{\mathcal{A}} = g_A^5 \int dy [1 + r (\delta(y+L) + \delta(y-L))] f_2^{\mathcal{A}} f_2^{\Psi_L} f_0^{\Psi_L} \equiv g^{\mathcal{A}} \mathcal{F}_{220}$$

$$g_{211}^{\mathcal{A}} = g_A^5 \int dy [1 + r (\delta(y+L) + \delta(y-L))] f_2^{\mathcal{A}} f_1^{\Psi_L} f_1^{\Psi_L} \equiv g^{\mathcal{A}} \mathcal{F}_{211}$$

$$g_{200}^{\mathcal{A}} = g_A^5 \int dy [1 + r (\delta(y+L) + \delta(y-L))] f_2^{\mathcal{A}} f_0^{\Psi_L} f_0^{\Psi_L} \equiv g^{\mathcal{A}} \mathcal{F}_{200}$$

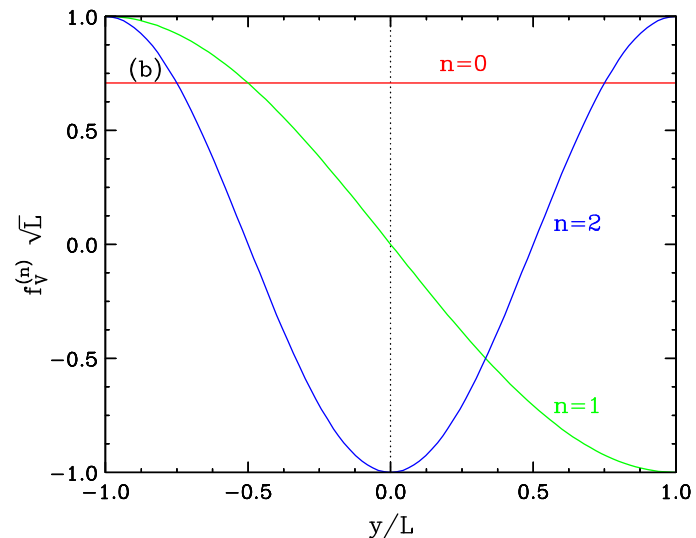
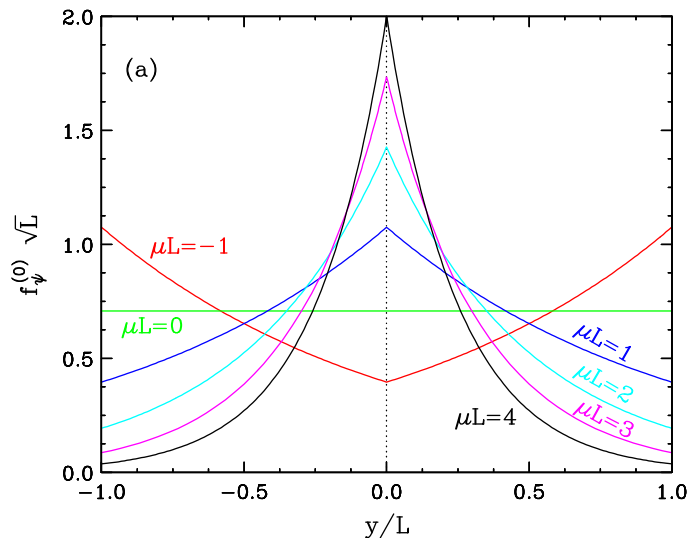
$$g_A^5 = g_A \sqrt{2L \left(1 + \frac{r}{L}\right)}$$

$$\mu_5 = \mu_H,$$

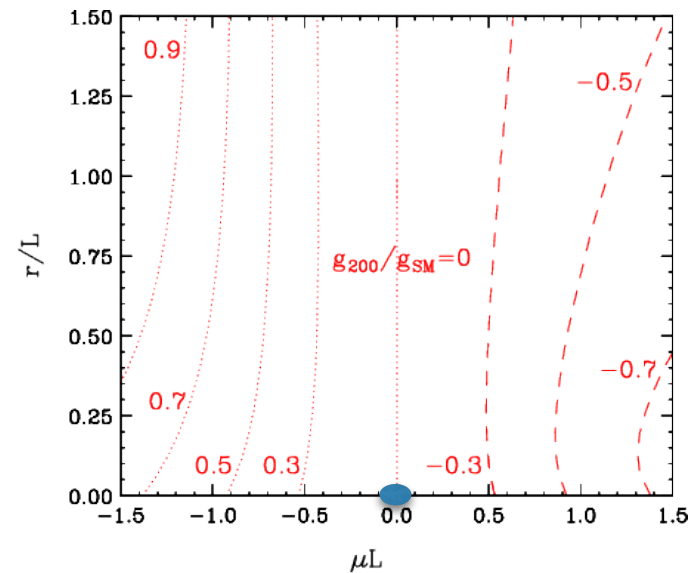
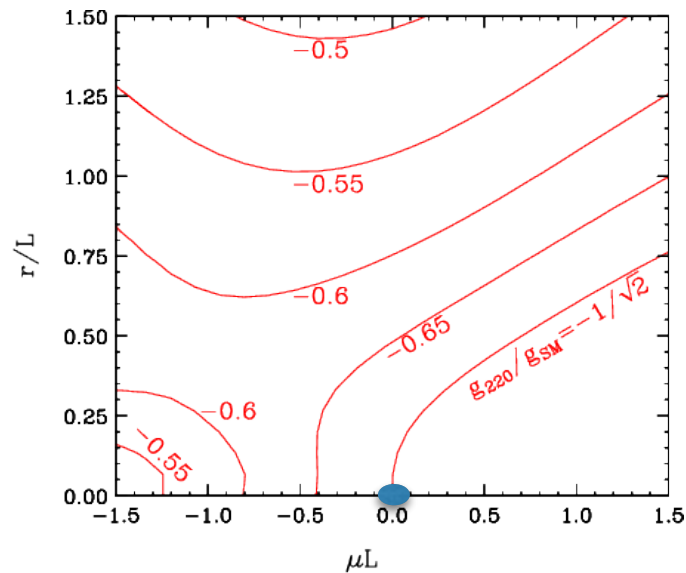
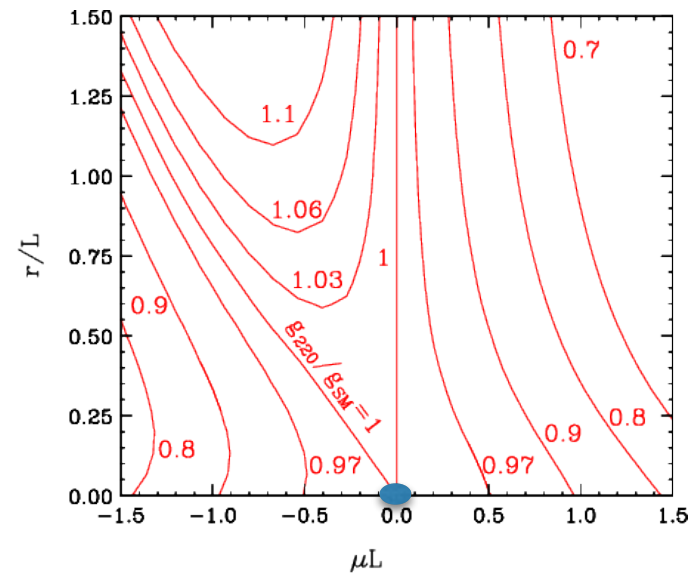
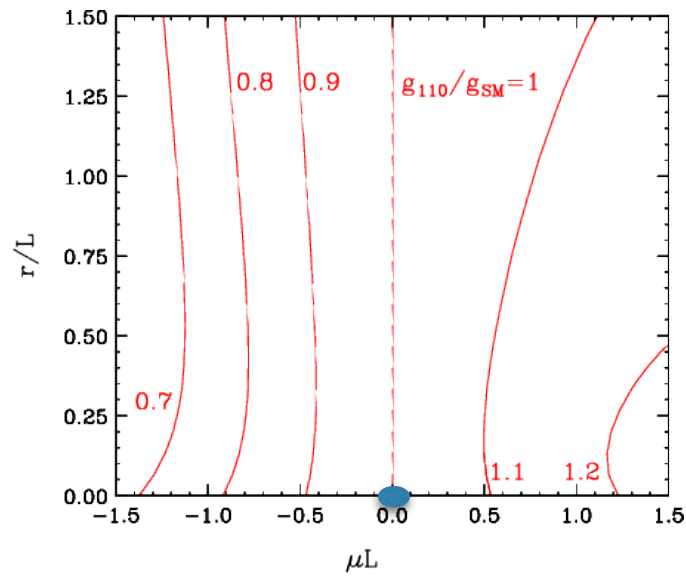
$$\lambda_5 = \lambda_H \left(2L \left(1 + \frac{r}{L}\right)\right),$$

$$\lambda_5^{U,D,E} = \lambda^{U,D,E} \sqrt{2L \left(1 + \frac{r}{L}\right)}.$$

Kong, Park, Rizzo 2010



NMUED: couplings ($\mu < 0$)



W' in UED

CMS PAPER EXO-11-024

$\mathcal{L} = 4.7 \text{ fb}^{-1}$

Models and interpretations

- $W - W'$ interferences considered (left-handed W')
- UED: $W'_{KK}(n = 2, 4, \dots)$ (coupling to SM fermions)

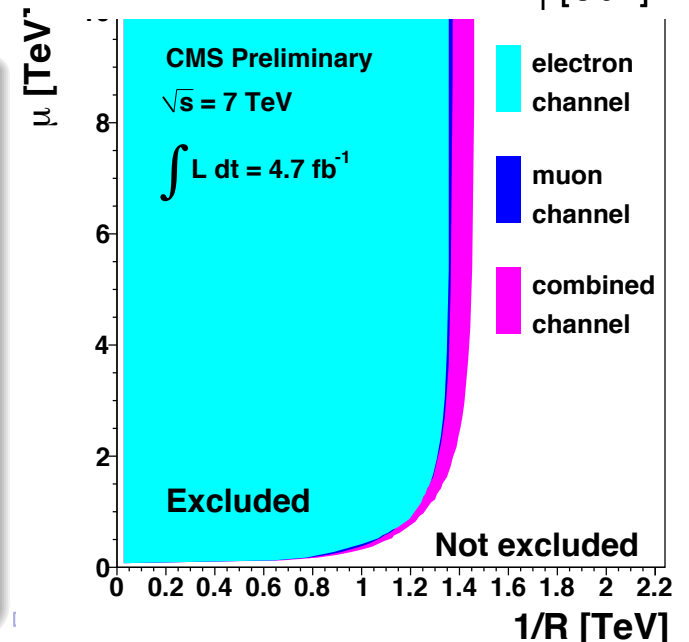
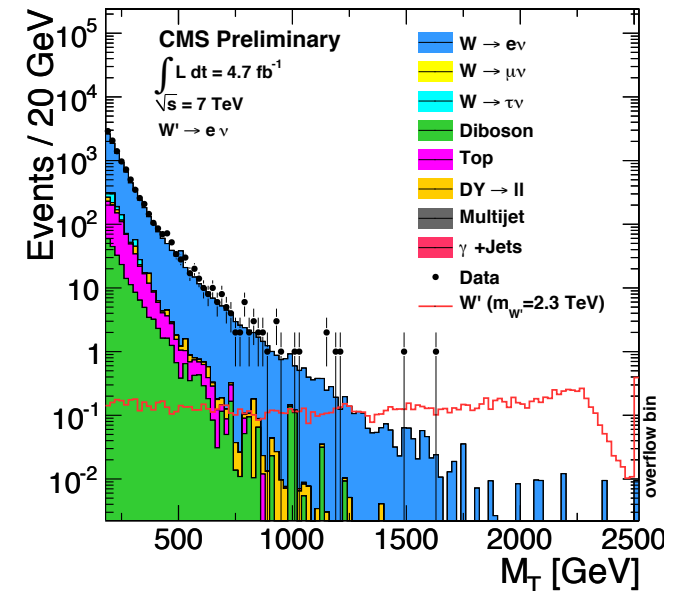
Lepton channels $\ell = e, \mu$ (+ E_T^{miss})

- W boson transverse mass reconstruction

$$M_T = \sqrt{2 \cdot \ell \cdot E_T^{\text{miss}} \cdot (1 - \cos \Delta\phi_{\ell, \nu})}$$

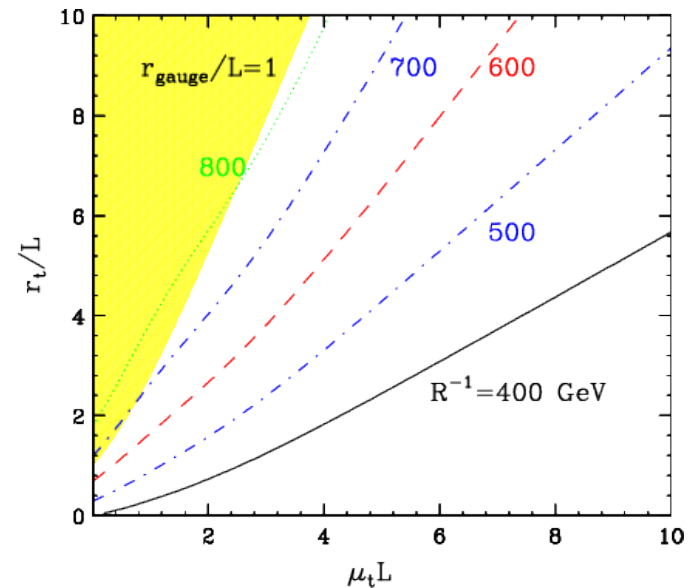
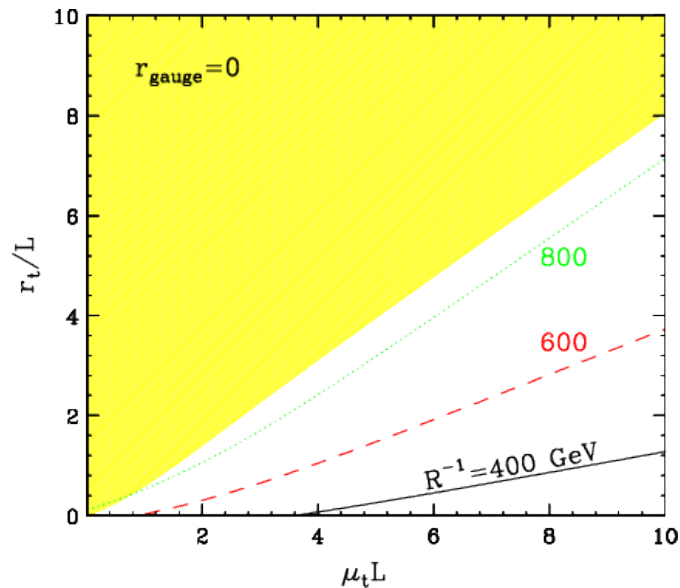
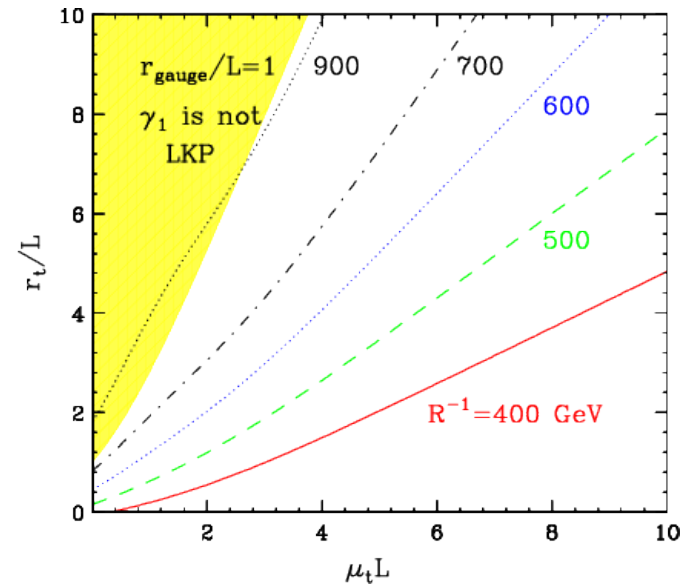
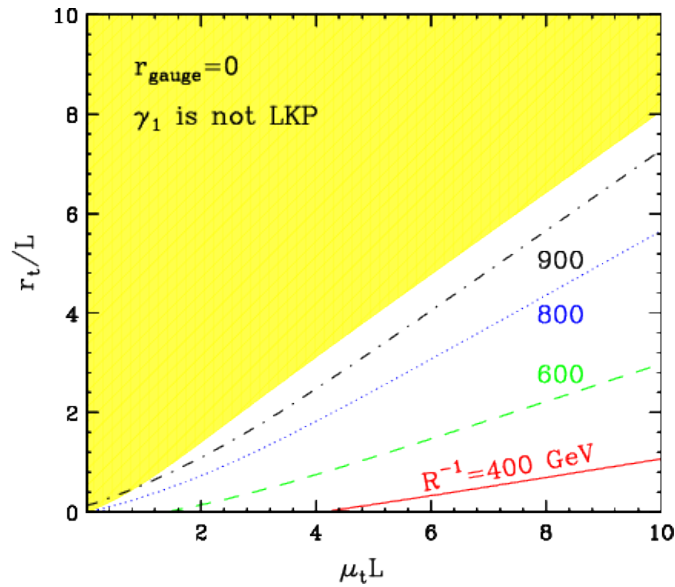
Bayesian exclusion limits at 95% C.L.

- Higher order EW corrections (not plotted) at high masses reduce interference effects
- Limit on $m_{W'}$ (right-handed): 2.5 TeV, on $m_{W'}$ (left-handed): 2.63 TeV [2.43 TeV] for constructive [destructive] $W - W'$ interference
- Universal Extra Dimension re-interpretation:
Limits in terms of ED Radius R and Dirac mass term μ
No sensitivity to $n \geq 4$ modes (yet)

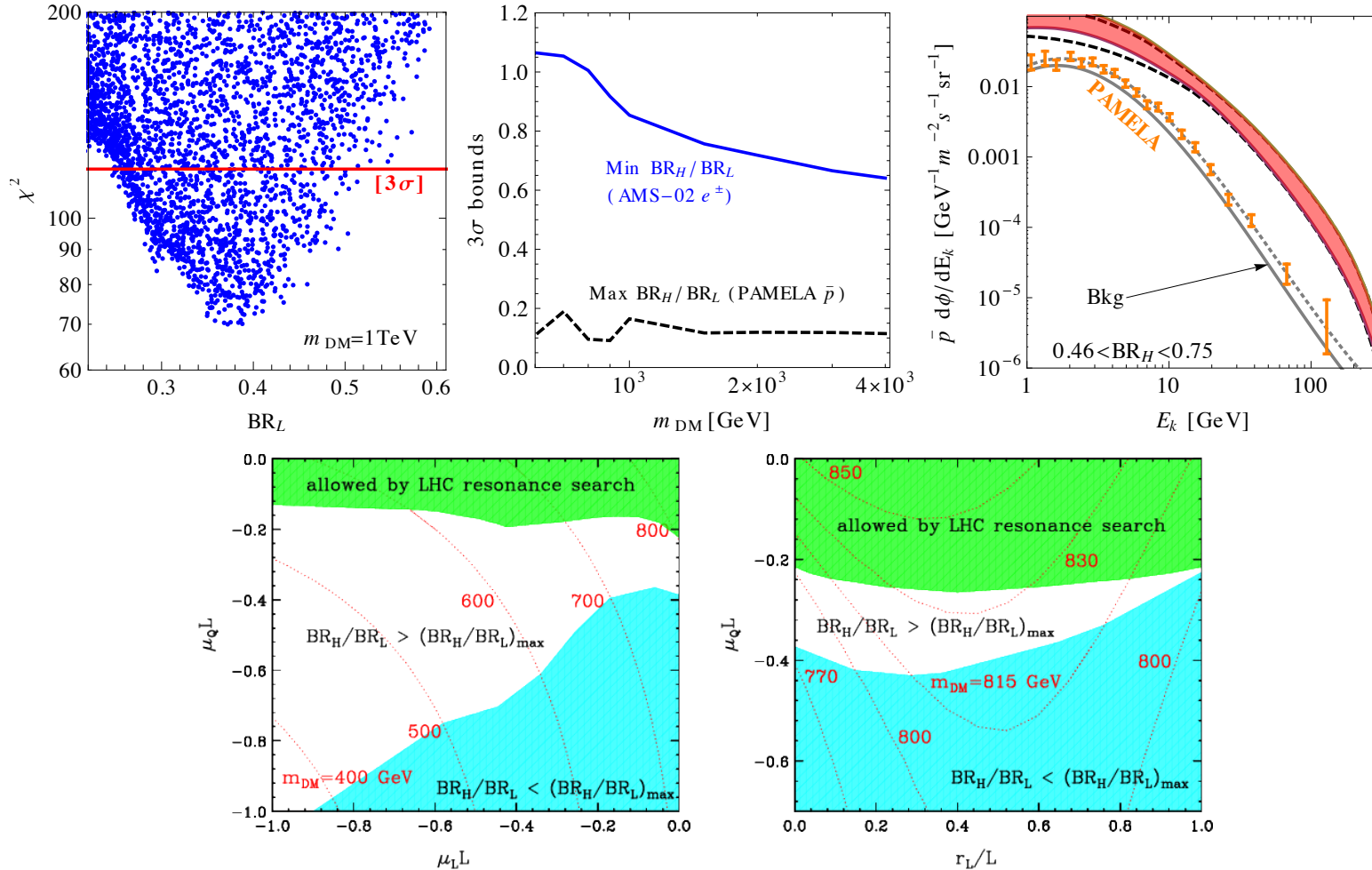


MH=126 GeV in UED

Flacke, Kong, Park, 2013 PLB

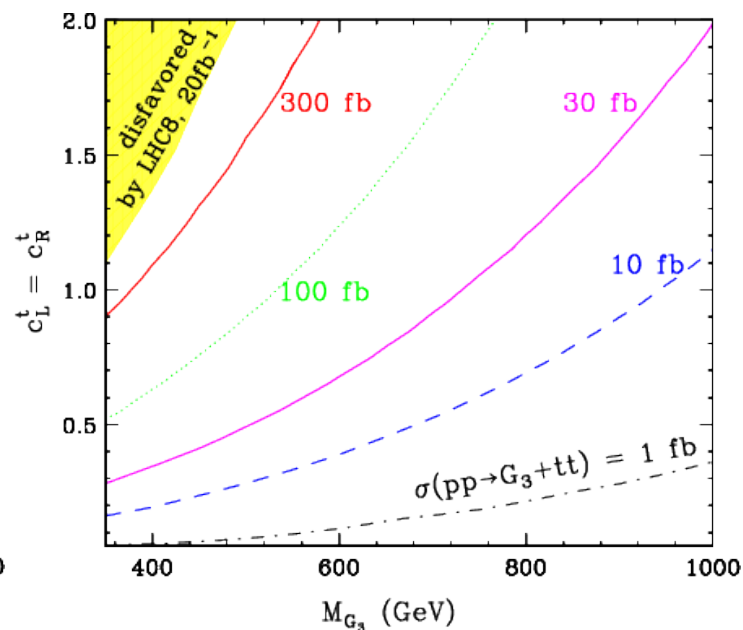
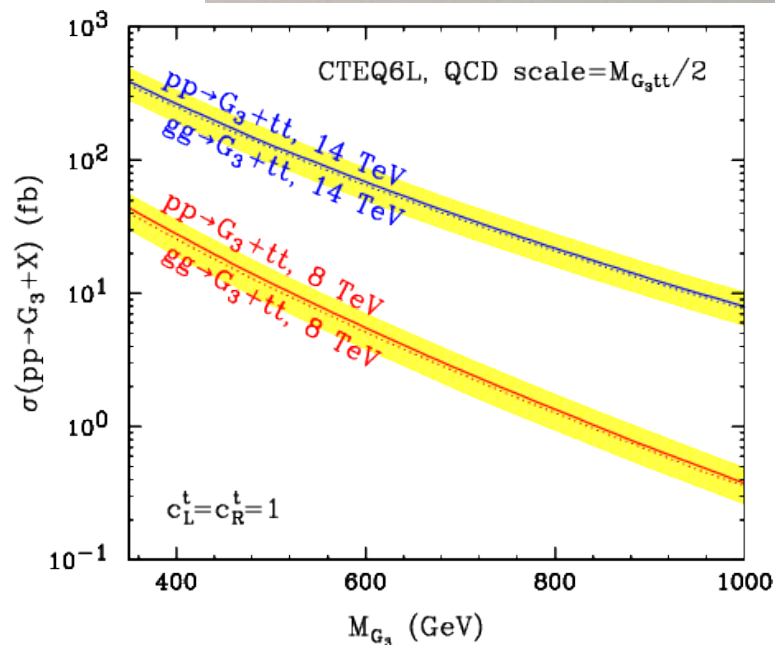
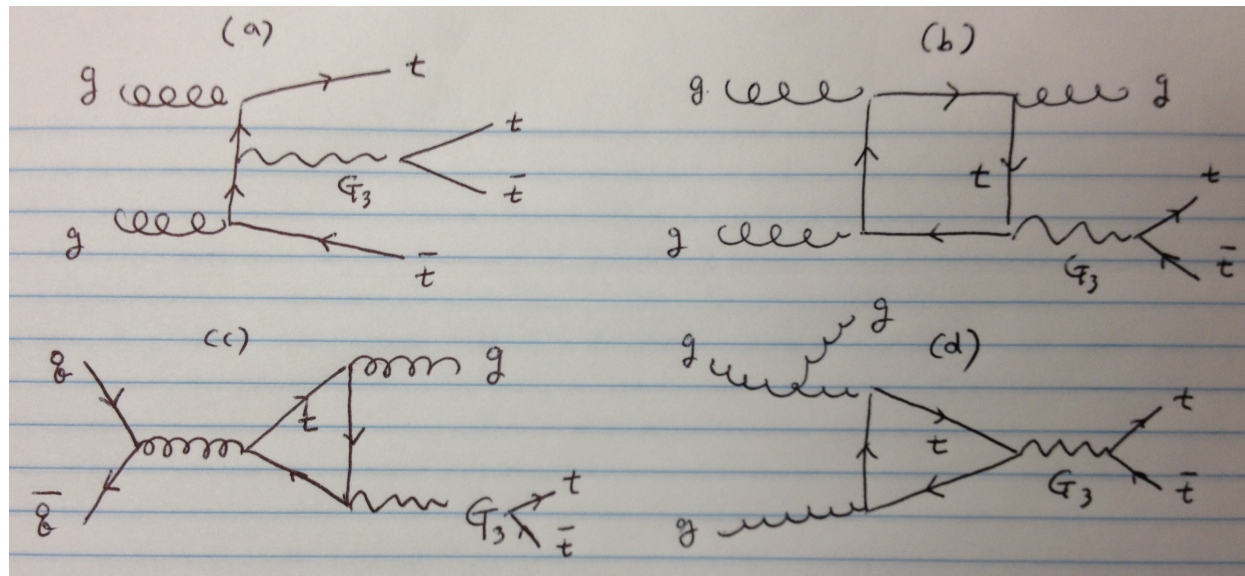


AMS-02 and NMUED



- positron data disfavors universal boundary term and universal bulk mass
 - Dijet constraints and indirect detection indicates more complicated structure
- $\mu_{u,d} < \mu_{c,s} < \mu_{t,b}$ $\mu_{t_L} \neq \mu_{t_R}$ and $\mu_{q_L} \neq \mu_{q_R}$ Kong, Marfatia, Gao, 1402.1723

Exotic $T\bar{T}$ resonance



Summary

- $M_H=126$ GeV and relic abundance disfavors 2UED with minimal mass spectrum
- MUED is very constrained
 - $R_{\text{inv}} > 1.2\text{-}1.3$ TeV from tri-lepton search (8 TeV)
 - $R_{\text{inv}} < 1.5$ TeV from relic abundance
- NMUED introduces brane terms and bulk masses
 - More parameter space, and hence tension reduced
 - Can accommodate Higgs in broad parameter space
 - Positron/antiproton data disfavors universal parametrization
 - dijet+positron data indicates more complicated structure
 - MUED exists in various event generators: CalcHEP, PYTHIA, MG/ME, Herwig, Sherpa, etc
 - For NMUED, coupling and mass spectrum can be modified

Why Consider Exotica?

- Some exotica aren't really all that exotic
- Urgent – real possibilities for 2015-????
- You have the potential to advance science

Would experimentalists have thought of this if you didn't do this work?

– *Witten*

- ...and you might actually advance science

Never start a project unless you have an unfair advantage.

– *Seiberg*

- It's fun

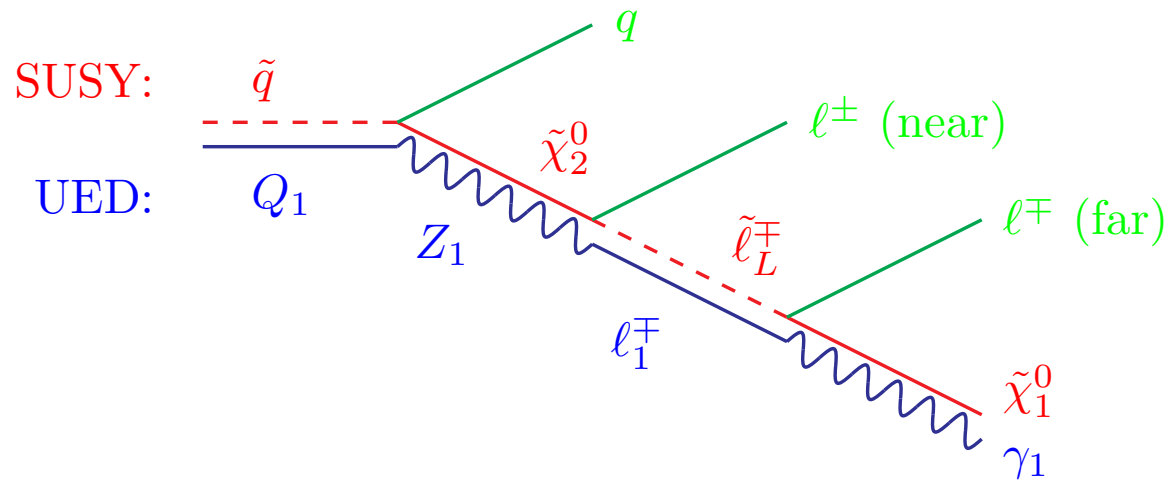
If every individual student follows the same current fashion ..., then the variety of hypotheses being generated...is limited. Perhaps rightly so, for possibly the chance is high that the truth lies in the fashionable direction. But, on the off-chance that it is in another direction - a direction obvious from an unfashionable view ... -- who will find it? Only someone who has sacrificed himself...I say sacrificed himself because he most likely will get nothing from it...But, if my own experience is any guide, the sacrifice is really not great because...you always have the psychological excitement of feeling that possibly nobody has yet thought of the crazy possibility you are looking at right now.

– *Richard Feynman, Nobel Lecture*

Back up

SUSY vs UED

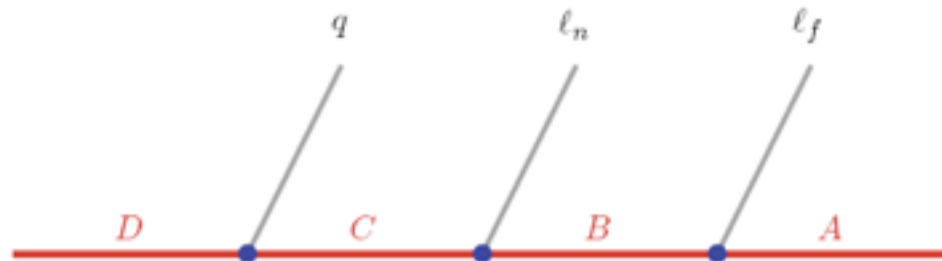
- SUSY-like cascade decays at the LHC from the first KK modes.



- Distinct feature: 2nd KK modes...

Spin and Couplings of Dark Matter: Why is it difficult to measure them?

- Missing energy signatures arise from something like:



- Several alternative explanations:

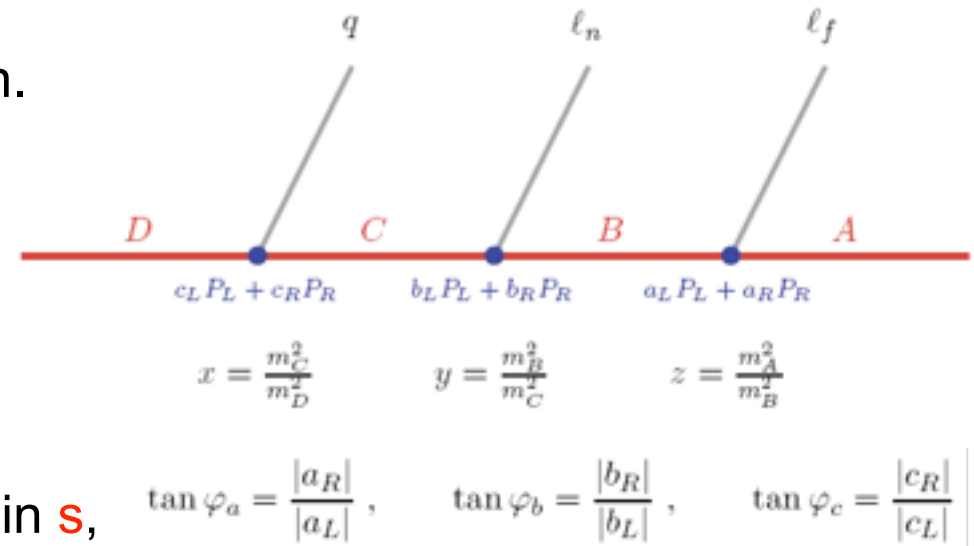
S	Spins	D	C	B	A	Example
1	SFSF	Scalar	Fermion	Scalar	Fermion	$\tilde{q} \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\ell} \rightarrow \tilde{\chi}_1^0$
2	FSFS	Fermion	Scalar	Fermion	Scalar	$q_1 \rightarrow Z_H \rightarrow \ell_1 \rightarrow \gamma_H$
3	FSFV	Fermion	Scalar	Fermion	Vector	$q_1 \rightarrow Z_H \rightarrow \ell_1 \rightarrow \gamma_1$
4	FVFS	Fermion	Vector	Fermion	Scalar	$q_1 \rightarrow Z_1 \rightarrow \ell_1 \rightarrow \gamma_H$
5	FVFV	Fermion	Vector	Fermion	Vector	$q_1 \rightarrow Z_1 \rightarrow \ell_1 \rightarrow \gamma_1$
6	SFVF	Scalar	Fermion	Vector	Fermion	—

What is a good distribution to look at?

- Invariant mass distributions!
- Advantages: well studied, know about spin.
For adjacent SM particles

$$\frac{dN}{dm^2} = a_0 + a_2 m^2 + a_4 m^4 + \dots$$

- Plot versus m^2 !
- For an intermediate BSM particle of spin \mathbf{s} , the highest order term is $m^{4\mathbf{s}}$
- For non-adjacent BSM particles, there are log terms as well.
- Disadvantage: know about many other things (hidden in the coefficients a), not all of which are measured!
 - Masses M_A, M_B, M_C, M_D (x, y, z)
 - Couplings and mixing angles (g_L and g_R)
 - Particle-antiparticle (D/D^*) fraction (f/f^*) ($f + f^* = 1$)



S	Spins	D	C	B	A
1	SFSF	Scalar	Fermion	Scalar	Fermion
2	FSFS	Fermion	Scalar	Fermion	Scalar
3	FSFV	Fermion	Scalar	Fermion	Vector
4	FVFS	Fermion	Vector	Fermion	Scalar
5	FVfV	Fermion	Vector	Fermion	Vector
6	SFVF	Scalar	Fermion	Vector	Fermion

What is the relevant question?

- Given the data, which spin configuration gives a good fit for arbitrary values of the yet unknown parameters?
 - fix mass spectrum
 - let spins, couplings, mixing angles, particle/antiparticle fraction f , etc. to float
- Previously people had asked: Given the data, which spin configuration gives a good fit for fixed values (the true ones) of the yet unknown parameters?
 - They fix: everything but the spins
 - Then let spins to float
- What is wrong with the latter approach?
 - It's the wrong chronological order
 - To measure the chirality of the couplings, we will probably need to measure the spins first
 - It's not a pure spin measurement, i.e. it is a spin measurement under certain model assumptions which still need to be verified experimentally

How do we do it?

- Separate the spin dependence from all the rest
 - Parameterize conveniently the effect from “all the rest”

$$\left(\frac{dN}{dm^2} \right)_S = F_{S;\delta}(m^2) + \alpha F_{S;\alpha}(m^2) + \beta F_{S;\beta}(m^2) + \gamma F_{S;\gamma}(m^2)$$

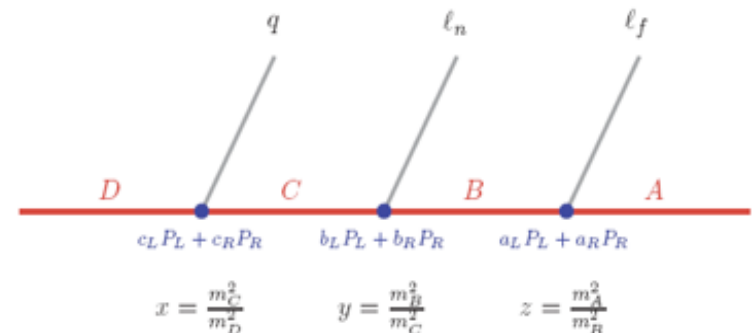
- Measure both the spin (S) as well as all the rest: α, β, γ

$$\alpha(\varphi_b, \varphi_a) = \cos 2\varphi_b \cos 2\varphi_a ,$$

$$\beta(\tilde{\varphi}_c, \varphi_b) = \cos 2\tilde{\varphi}_c \cos 2\varphi_b = (f - \bar{f}) \cos 2\varphi_c \cos 2\varphi_b$$

$$\gamma(\varphi_a, \tilde{\varphi}_c) = \cos 2\varphi_a \cos 2\tilde{\varphi}_c = (f - \bar{f}) \cos 2\varphi_a \cos 2\varphi_c$$

$$\tan \varphi_a = \frac{|a_R|}{|a_L|} , \quad \tan \varphi_b = \frac{|b_R|}{|b_L|} , \quad \tan \varphi_c = \frac{|c_R|}{|c_L|}$$



What is the method?

- Construct and then fit the three invariant mass distributions to

$$L_S^{+-}(\hat{m}_{\ell\ell}^2; x, y, z, \alpha) \equiv \left(\frac{dN}{d\hat{m}_{\ell\ell}^2} \right)_S = \mathcal{F}_{S;\delta}^{(\ell\ell)}(\hat{m}_{\ell\ell}^2; x, y, z) + \alpha \mathcal{F}_{S;\alpha}^{(\ell\ell)}(\hat{m}_{\ell\ell}^2; x, y, z) ,$$

$$S_S^{+-}(\hat{m}_{j\ell}^2; x, y, z, \alpha) \equiv \left(\frac{dN}{d\hat{m}_{j\ell}^2} \right)_S + \left(\frac{dN}{d\hat{m}_{j\ell}^2} \right)_S$$

$$= r_n^2 \mathcal{F}_{S;\delta}^{(j\ell_n)}(r_n^2 \hat{m}_{j\ell}^2; x, y, z) + r_f^2 \mathcal{F}_{S;\delta}^{(j\ell_f)}(r_f^2 \hat{m}_{j\ell}^2; x, y, z) + \alpha r_f^2 \mathcal{F}_{S;\alpha}^{(j\ell_f)}(r_f^2 \hat{m}_{j\ell}^2; x, y, z) ,$$

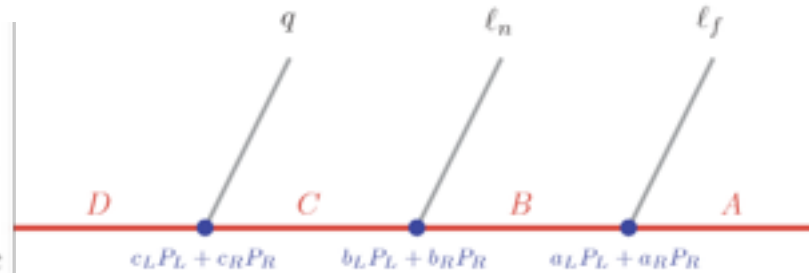
$$D_S^{+-}(\hat{m}_{j\ell}^2; x, y, z, \beta, \gamma) \equiv \left(\frac{dN}{d\hat{m}_{j\ell}^2} \right)_S - \left(\frac{dN}{d\hat{m}_{j\ell}^2} \right)_S$$

$$= \gamma r_f^2 \mathcal{F}_{S;\gamma}^{(j\ell_f)}(r_f^2 \hat{m}_{j\ell}^2; x, y, z) + \beta r_f^2 \mathcal{F}_{S;\beta}^{(j\ell_f)}(r_f^2 \hat{m}_{j\ell}^2; x, y, z) - \beta r_n^2 \mathcal{F}_{S;\beta}^{(j\ell_n)}(r_n^2 \hat{m}_{j\ell}^2; x, y, z)$$

$$\alpha(\varphi_b, \varphi_a) = \cos 2\varphi_b \cos 2\varphi_a ,$$

$$\beta(\tilde{\varphi}_c, \varphi_b) = \cos 2\tilde{\varphi}_c \cos 2\varphi_b = (f - \bar{f}) \cos 2\varphi_c \cos 2\varphi_b$$

$$\gamma(\varphi_a, \tilde{\varphi}_c) = \cos 2\varphi_a \cos 2\tilde{\varphi}_c = (f - \bar{f}) \cos 2\varphi_a \cos 2\varphi_c$$



Coupling measurements

- The fitted values of alpha, beta, gamma represent measurements of certain combinations of couplings and mixing angles
- The sign ambiguity corresponds to the chirality exchange

$$|a_L| \leftrightarrow |a_R| ,$$

$$|b_L| \leftrightarrow |b_R| ,$$

$$|c_L| \leftrightarrow |c_R| .$$

$$|a_L| = \frac{1}{\sqrt{2}} \left(1 \pm \frac{1}{\beta} \sqrt{\alpha\beta\gamma} \right)^{\frac{1}{2}} ,$$

$$|a_R| = \frac{1}{\sqrt{2}} \left(1 \mp \frac{1}{\beta} \sqrt{\alpha\beta\gamma} \right)^{\frac{1}{2}} ,$$

$$|b_L| = \frac{1}{\sqrt{2}} \left(1 \pm \frac{1}{\gamma} \sqrt{\alpha\beta\gamma} \right)^{\frac{1}{2}} ,$$

$$|b_R| = \frac{1}{\sqrt{2}} \left(1 \mp \frac{1}{\gamma} \sqrt{\alpha\beta\gamma} \right)^{\frac{1}{2}} ,$$

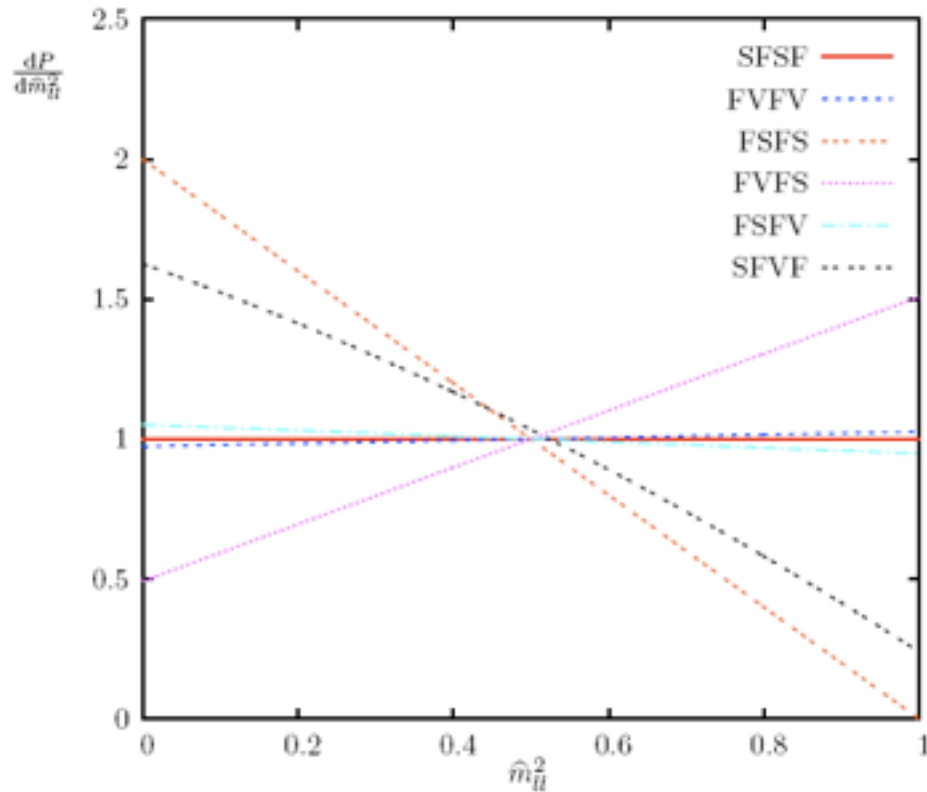
$$|c_L| = \frac{1}{\sqrt{2}} \left(1 \pm \frac{1}{f - \bar{f}} \frac{1}{\alpha} \sqrt{\alpha\beta\gamma} \right)^{\frac{1}{2}} ,$$

$$|c_R| = \frac{1}{\sqrt{2}} \left(1 \mp \frac{1}{f - \bar{f}} \frac{1}{\alpha} \sqrt{\alpha\beta\gamma} \right)^{\frac{1}{2}} ,$$

Does this really make any difference?

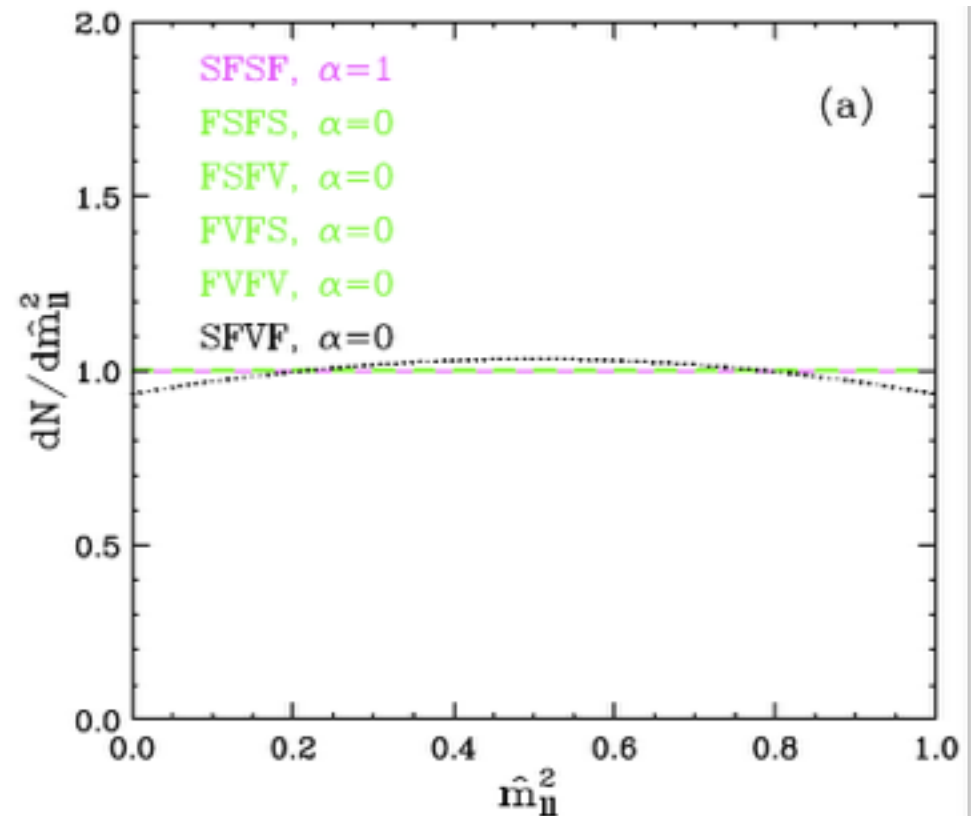
- Yes! Dilepton invariant mass distribution. Data from SPS1a.

Athanasίου, Lester, Smillie, Webber 06



- Spins vary
- Everything else fixed to SPS1a values
- Easy to distinguish!

Burns, Kong, Matchev, Park 08

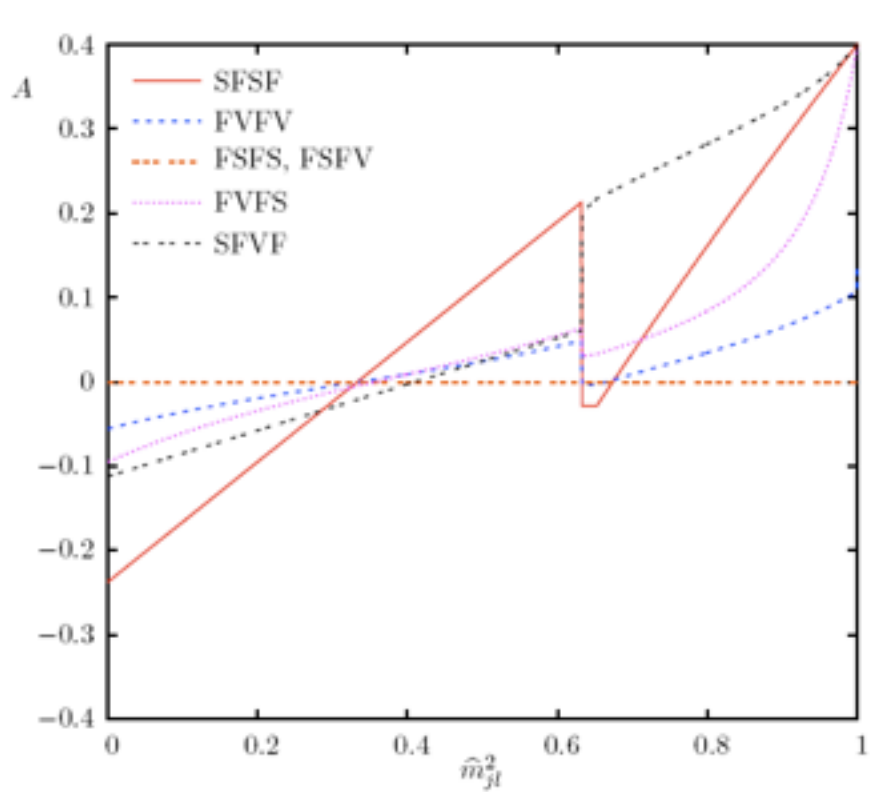


- Mass spectrum fixed to SPS1a values
- Everything else varies
- Difficult to distinguish!

Does this really make any difference?

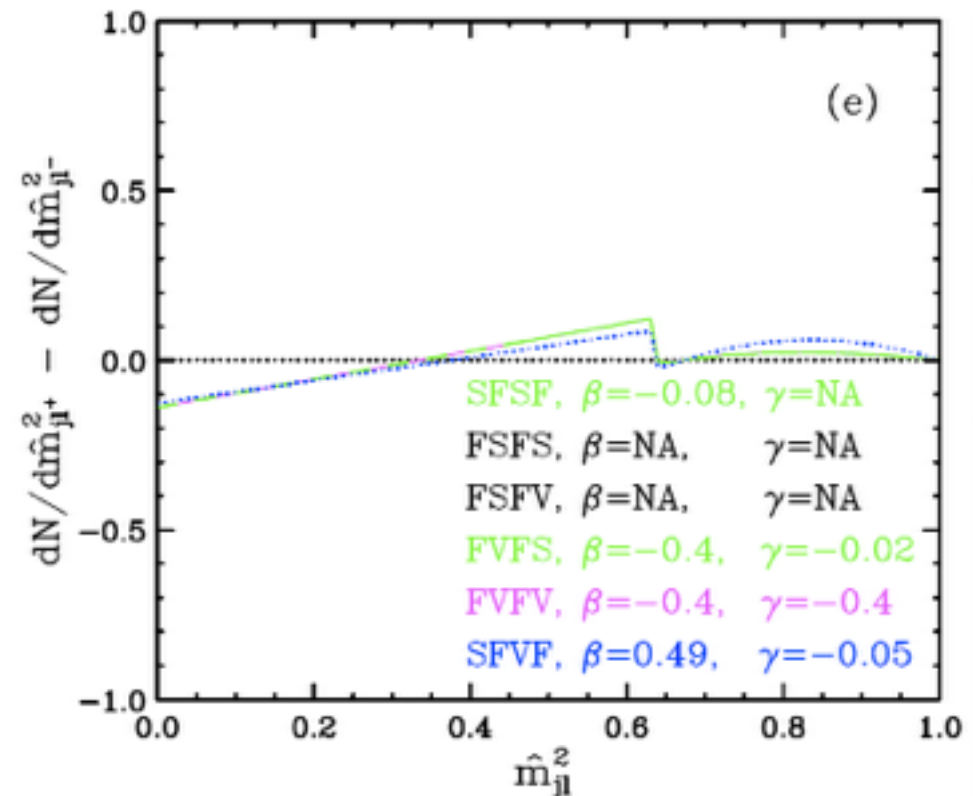
- Yes! Lepton charge (Barr) asymmetry. Data: “UED” with SPS1a mass spectrum.

Athanasίου, Lester, Smillie, Webber 06



- Spins vary
- Everything else fixed to SPS1a values
- Easy to distinguish!

Burns, Kong, Matchev, Park 08



- Mass spectrum fixed to SPS1a values
- Everything else varies
- Difficult to distinguish!

With Infinite Statistics

Burns, Kong, Matchev, Park 08

- Separate the spin dependence from all the rest
 - Parameterize conveniently the effect from “all the rest”

$$\left(\frac{dN}{dm^2} \right)_S = F_{S;\delta}(m^2) + \alpha F_{S;\alpha}(m^2) + \beta F_{S;\beta}(m^2) + \gamma F_{S;\gamma}(m^2)$$

- Measure both the spin (S) as well as all the rest: α, β, γ

Data from	Can this data be fitted by model					
	SFSF	FSFS	FSFV	FVFS	FVFF	SFVF
SFSF	yes	no	no	no	no	no
FSFS	no	yes	maybe	no	no	no
FSFV	no	yes	yes	no	no	no
FVFS	no	no	no	yes	maybe	no
FVFF	no	no	no	yes	yes	no
SFVF	no	no	no	no	no	yes