Non-Constant Models for Dark Energy and the Possible Fates of the Universe

Kevin J. Ludwick

November 6, 2013

Relativistic Cosmology

- Einstein (with help from others): general relativity
- Apply GR to our universe: relatvistic cosmology
- Cosmological principle: assumes homogeneity and isotropy on a large scale

FLRW Metric

- The Friedmann-Lemaître-Robertson-Walker metric describes a universe that follows the cosmological principle
- $ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$
- Throughout this work, we assume $c = 1$ and $k = 0$
- Comoving distance: r
- Proper distance: $a(t)r$

Friedmann Equations

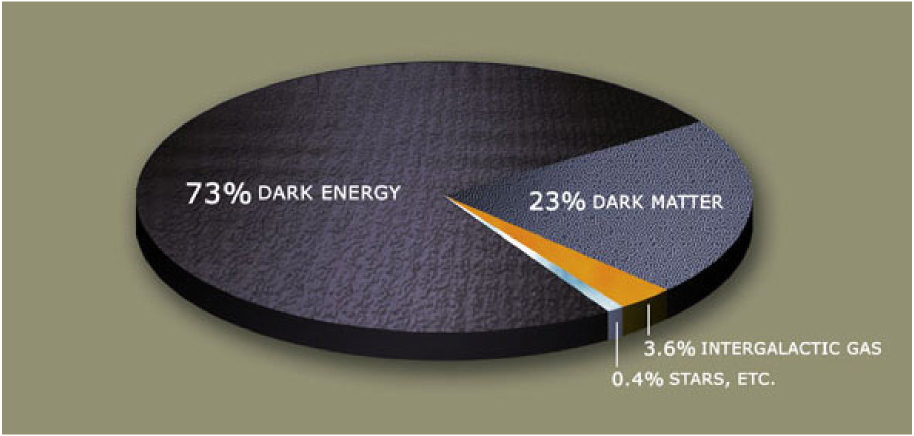
- Solve Einstein's equation using the FLRW metric and modeling the universe as a perfect fluid (no shear stresses, isotropic pressure) $\rightarrow$ Friedmann equations
- $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
- $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
- Sometimes written in terms of the Hubble parameter, $H \equiv \frac{\dot{a}}{a}$
- Also can be written in terms of redshift: $Z = \frac{a(t_0)}{a(t)} - 1$
- $Z = \frac{\lambda_{observed}}{\lambda_{emitted}} - 1$

Discovery of Dark Energy

- High-z Supernova Search Team in 1998, Supernova Cosmology Project in 1999: SNIa spectra
- Conclusion: dark energy, responsible for cosmic acceleration
- Other evidence: galaxy surveys, late-time integrated Sachs-Wolfe effect
- 2011 Nobel Prize: Schmidt, Riess, Perlmutter

Characteristics of Dark Energy

- About 68% of our universe is dark energy
- Physical intution of the nature and dynamics of DE lacking
- Strange feature: unlike normal matter, as volume increases, DE density does NOT decrease


73% DARK ENERGY
23% DARK MATTER
3.6% INTERGALACTIC GAS
0.4% STARS, ETC.

Theories of Dark Energy

- explanations/candidates: modified gravity, back-reaction, dynamical component of stress-energy tensor (quintessence), constant vacuum energy
- $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
- Cosmological constant term usually subsumed as an extra component of the density in the stress-energy tensor: $\rho = \sum_i \rho_i = \rho_{rad} + \rho_m + \rho_\Lambda$
- When only component (or dominant one) is ρ_Λ: de Sitter space

Modeling Dark Energy

- Relationship between pressure and density usually assumed to be $p_i = w_i \rho_i$
- For the cosmological constant (CC) model, $w_\Lambda = -1$, and this gives constant DE density
- We generalize w_Λ to be w_{DE}, which is not restricted to be -1

Continuity Equation

- Combine the two Friedmann equations $\Rightarrow \frac{d\rho}{da} = -\frac{3}{a}(\rho + p)$
- separable equation for ρ and p as functions of a if components independent of each other
- Continuity equation governs dynamics of individual components

Plan

Equations of State

- DE density models as functions of Z
- Focus on redshift of transition (Z^*) from matter to dark energy domination (deceleration to acceleration)
- For ΛCDM model (assuming 2 components):

$$Z^* = \left(\frac{2\Omega_\Lambda(t_0)}{\Omega_m(t_0)}\right)^{\frac{1}{3}} - 1 \quad = \quad 0.743$$

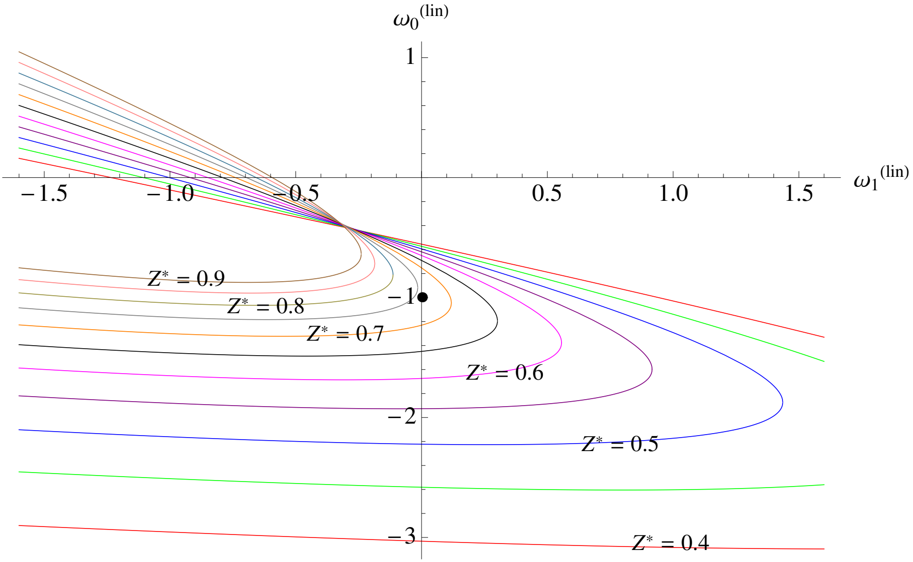
Equations of State

$$w^{(lin)}(Z) = w_0^{(lin)} + w_1^{(lin)} Z$$

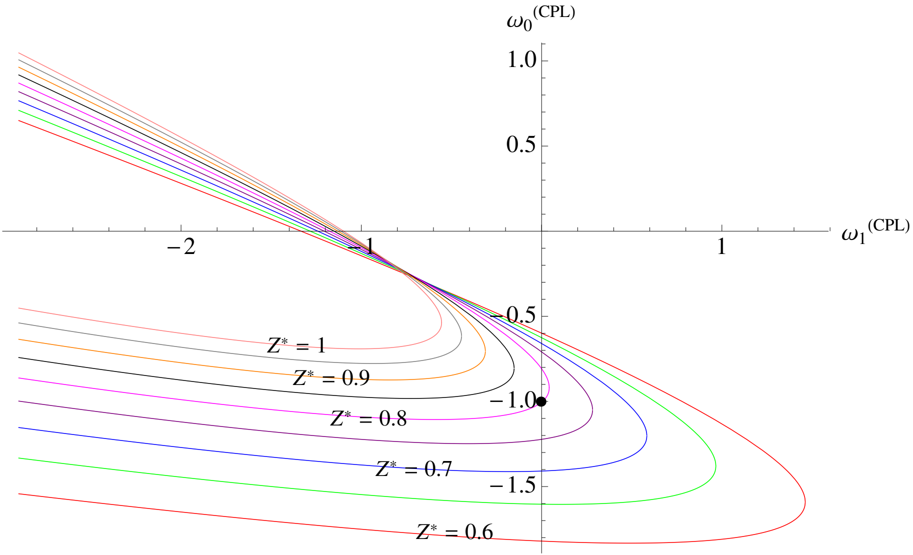
$$w^{(CPL)}(Z) = w_0^{(CPL)} + w_1^{(CPL)} \frac{Z}{1+Z}$$

$$w^{(SSS)}(Z) = -\frac{1+\tanh[(Z-w_0^{(SSS)})w_1^{(SSS)}]}{2}$$

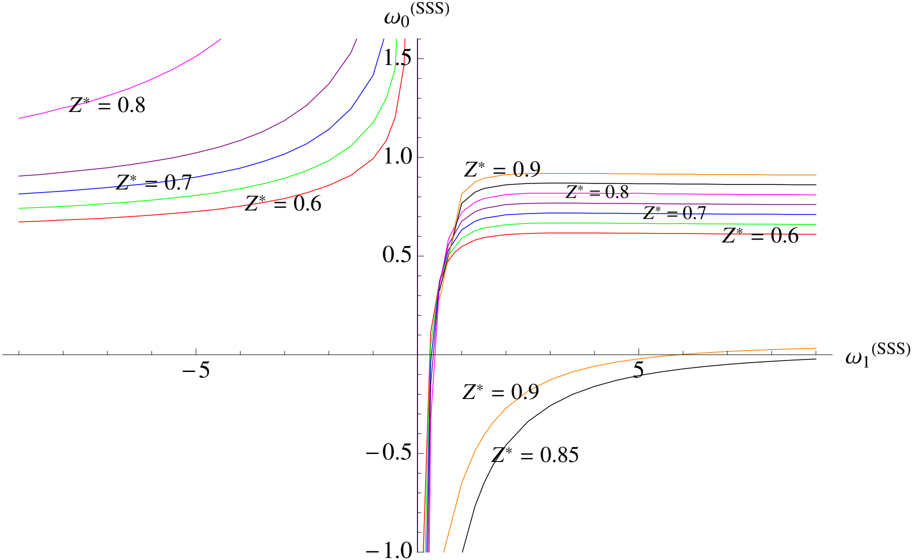
$$w^{(new)}(Z) = w_0^{(new)} + w_1^{(new)} \frac{Z}{2+Z}$$

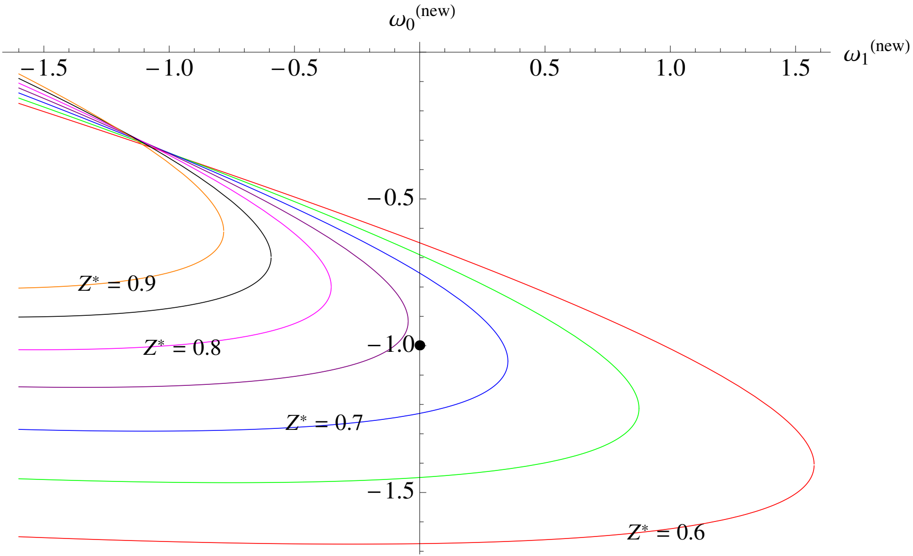


$\omega_0^{(\mathrm{lin})}$
$\omega_1^{(\mathrm{lin})}$
1
−1
−2
−3
−1.5
−1.0
−0.5
0.5
1.0
1.5
$Z^* = 0.9$
$Z^* = 0.8$
$Z^* = 0.7$
$Z^* = 0.6$
$Z^* = 0.5$
$Z^* = 0.4$

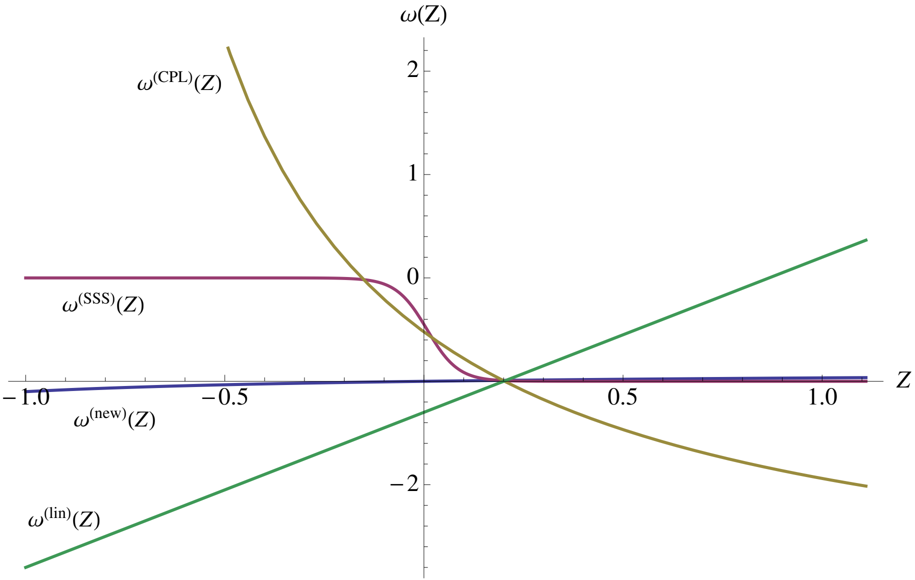


$\omega_0^{(CPL)}$
1.0
0.5
$\omega_1^{(CPL)}$
−2
−1
1
−0.5
−1.0
−1.5
$Z^* = 1$
$Z^* = 0.9$
$Z^* = 0.8$
$Z^* = 0.7$
$Z^* = 0.6$


$\omega_0^{(SSS)}$
$\omega_1^{(SSS)}$
1.5
1.0
0.5
−0.5
−1.0
−5
5
$Z^* = 0.8$
$Z^* = 0.7$
$Z^* = 0.6$
$Z^* = 0.9$
$Z^* = 0.8$
$Z^* = 0.7$
$Z^* = 0.6$
$Z^* = 0.9$
$Z^* = 0.85$



$\omega_0^{(new)}$
$\omega_1^{(new)}$
−1.5
−1.0
−0.5
0.5
1.0
1.5
−0.5
−1.0
−1.5
$Z^* = 0.9$
$Z^* = 0.8$
$Z^* = 0.7$
$Z^* = 0.6$


$\omega(Z)$
2
1
0
−2
−1.0
−0.5
0.5
1.0
Z
$\omega^{(\mathrm{CPL})}(Z)$
$\omega^{(\mathrm{SSS})}(Z)$
$\omega^{(\mathrm{new})}(Z)$
$\omega^{(\mathrm{lin})}(Z)$

Evolutionary Equation of State

- No *a priori* physical intuition that helps determine equation of state
- Recent literature seems to indicate that the equation of state is approximately constant or, if varying, slowly varying around $w = -1$
- New model: nonsingular for all allowed values of redshift, $-1 < Z < \infty$ $(0 < a < \infty)$, and with appropriate parameters in the equation of state, slowly varying
- $Z = -1$ (i.e., when a is infinite) is achieved at a finite time in the future in Big Rip models

Exploring Models with Increasing Dark-Energy Density

- Big Rip model has $w < -1$, and $w < -1$ implies increasing cosmic acceleration
- $w < -1$: phantom models
- Such models violate general relativity's energy conditions, but even CC model violates SEC
- *WMAP 9*: $w = -1.037^{+0.071}_{-0.070}$ (95% CL); *Planck*: $w = -1.13^{+0.13}_{-0.14}$ (95% CL)
- General relativity has not been tested past the scale of our solar system; all observations that led to these best fit values of w were very far beyond that scale
- Phantom field models can avoid vacuum instabilities ("ghosts") if treated as effective field theories with momentum cutoffs
- Perhaps such theories will be fleshed out better in the future as we learn more about the nature of dark energy and the quantum nature of gravity

Plan

Incomplete List of Fates of the Universe for Monotonically Increasing DE Density

- Big Rip: $H(t) \longrightarrow +\infty, \qquad t \longrightarrow t_{rip} < \infty$
- **Little Rip**: $H(t) \longrightarrow +\infty, \qquad t \to +\infty$
- "No-rip": $H(t) = H(t_0)$ (CC model during DE domination)

Models

$$\text{Model 1: } \frac{\rho_1}{\rho_0} = \left(\frac{3A}{2\sqrt{\rho_0}}\ln(a/a_0) + 1\right)^2$$

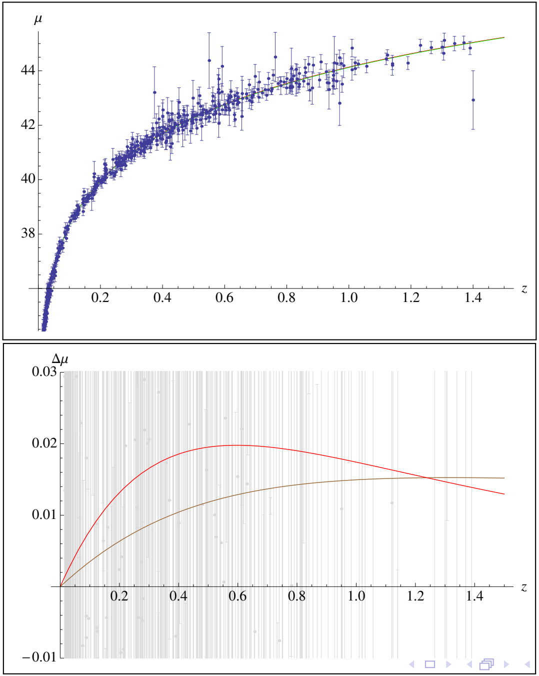
$$\text{expansion law: } \frac{a}{a_0} = e^{(2\sqrt{\rho_0}/3A)[e^{(\sqrt{3}A/2)(t-t_0)}-1]}$$

$$\text{Model 2: } \frac{\rho_2}{\rho_0} = N\left(\frac{a}{a_0}, B\right)\frac{(1+\ln(\frac{a}{a_0}+B))^2}{(1+\ln(1+B))^2}\frac{(\ln(1+\ln(\frac{a}{a_0}+B)))^2}{(\ln(1+\ln(1+B)))^2}, \text{ where}$$

$$N\left(\frac{a}{a_0}, B\right) = \frac{(\frac{a}{a_0}+B)^2}{(1+B)^2(\frac{a}{a_0})^2}$$

$$\text{expansion law: } \frac{a}{a_0} = e^{(e^{\ln(1+\ln(1+B))e^{\left[\frac{\sqrt{\rho_0/3}(t-t_0)}{(1+B)(1+\ln(1+B))\ln(1+\ln(1+B))}\right]}}-1)} - B$$

- Note: $\rho_1(a_0) = \rho_2(a_0) = \rho_0$; free parameters A and B determined from fitting to SNIa data from the Supernova Cosmology Project (SCP)


μ
44
42
40
38
0.2
0.4
0.6
0.8
1.0
1.2
1.4
z
Δμ
0.03
0.02
0.01
−0.01
0.2
0.4
0.6
0.8
1.0
1.2
1.4
z

Spectrum of Parameterizations Between CC and Big Rip

- To avoid Big Rip singularity, $t \to \infty$ as $a \to \infty$: $\int_{x_0}^{\infty} \frac{1}{\sqrt{\rho(x)}} dx \to \infty$ where $x \equiv \ln a$
- Define $\ln_j(a) \equiv \ln \ln \ln \ldots . \ln(a)$ (iterated j times)
- $\rho \sim (\ln a)^2 (\ln_2 a)^2 (\ln_3 a)^2 \ldots (\ln_m a)^2$ implies $a \sim \exp(\exp(\exp \ldots (\exp(t))\ldots))$ ($m + 1$ exponentials)
- Models 1 and 2 are examples of the $m = 1$ and $m = 2$ cases respectively
- The slowest growing power-law modification: $\rho \sim (\ln a)^2 (\ln_2 a)^2 (\ln_3 a)^2 \ldots (\ln_m a)^{2+\epsilon}$ where $\epsilon > 0 \to$ Big Rip singularity

Scalar-Field Models

If the stress-energy tensor is described by the presence of a homogeneous DE scalar field, local conservation of mass/energy and momentum ($\nabla_\mu T^\mu_\nu = 0$) gives the equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

For $V(\phi) > 0$, $w_\phi < -1 \rightarrow$ negative kinetic term (phantom field). Field "rolls up" the potential.

Little Rip Described by Scalar-Field Models

- For monotonically increasing potential $V(\phi)$, Little Rip: $\rho \to \infty$ and $V(\phi) \to \infty$ as $\phi \to \infty$

Plan

Coupling with Dark Matter

- Can coupling dark matter with dark energy with an equation of state characteristic of a Little Rip avoid the effects of a Little Rip?
- Consider these conservation laws (DE decays into DM): $\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q\rho_{DE}$, $\dot{\rho}_{DM} + 3H\rho_{DM} = Q\rho_{DE}$ (Q is a positive constant)
- We consider the DM and DE components to be dominant: $\frac{3}{\kappa^2}H^2 = \rho_{DE} + \rho_{DM}$ ($\kappa \equiv \sqrt{8\pi G}$)
- Assume de Sitter evolution ($H = H_0 > 0$). Does Q exist such that the Little Rip is avoided?
- Use a DE equation of state for a Little Rip: $w = -1 - \frac{2\lambda}{3H_0}e^{-\lambda t}$
- Yes: we find $\frac{\lambda}{Q} < \frac{1}{2}$
- Is the de Sitter solution for this coupling stable? Perturb H, ρ_{DE}, ρ_{DM}, see if the time derivative of the perturbations (according to their conservation laws) are negative. STABLE.

Scalar-Tensor Models

- Scalar-tensor models: a theory in which gravity is described by the action of a scalar field along with the usual tensor field of general relativity
- We use this formulation:
  $S = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\}$
- Without the second and third terms, minimizing the action would result in Einstein's equation for an empty universe, i.e., for a stress-energy tensor $T_{\mu\nu} = 0$. We leave out terms for matter and radiation because we consider this model only when dark energy dominates over other density components, so these terms are negligible
- $w < -1 \rightarrow$ non-canonical, negative kinetic term

Scalar-Tensor Models

- Scalar-tensor theory written as Little Rip model, agrees with data
- Stability: perturb H and ϕ and find that the time derivatives of the perturbations are negative $\rightarrow$ STABLE

Plan

Complete List of Fates of the Universe for Monotonically Increasing DE Density

- Big Rip: $H(t) \longrightarrow +\infty, \qquad t \longrightarrow t_{rip} < \infty$
- Little Rip: $H(t) \longrightarrow +\infty, \qquad t \rightarrow +\infty$
- **Pseudo-rip**: $H(t) \longrightarrow H_\infty < \infty \qquad t \rightarrow +\infty$
- "No-rip": $H(t) = H(t_0)$ (CC model during DE domination)
- Analogous conditions for ρ via 1st Friedmann equation
- No other possible fate of universe for monotonically increasing DE density

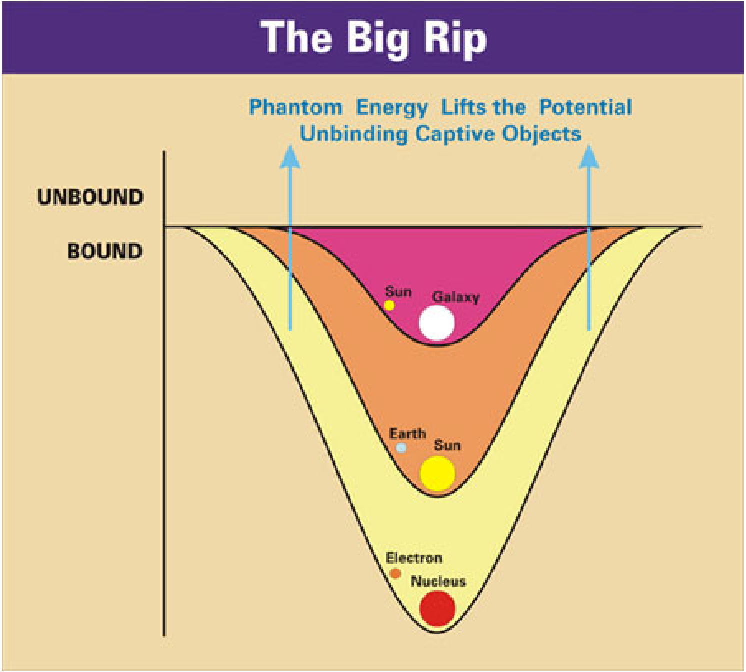
Use the Force

The inertial force F_{inert} on a mass m as seen by a gravitational source separated by a comoving distance l is given by

$$F_{inert} = ml(\dot{H}(t) + H(t)^2)$$

$$= -ml\frac{4\pi G}{3}(\rho(a) + 3p(a))$$

$$= ml\frac{4\pi G}{3}(2\rho(a) + \rho'(a)a)$$


The Big Rip
Phantom Energy Lifts the Potential
Unbinding Captive Objects
UNBOUND
BOUND
Sun
Galaxy
Earth
Sun
Electron
Nucleus

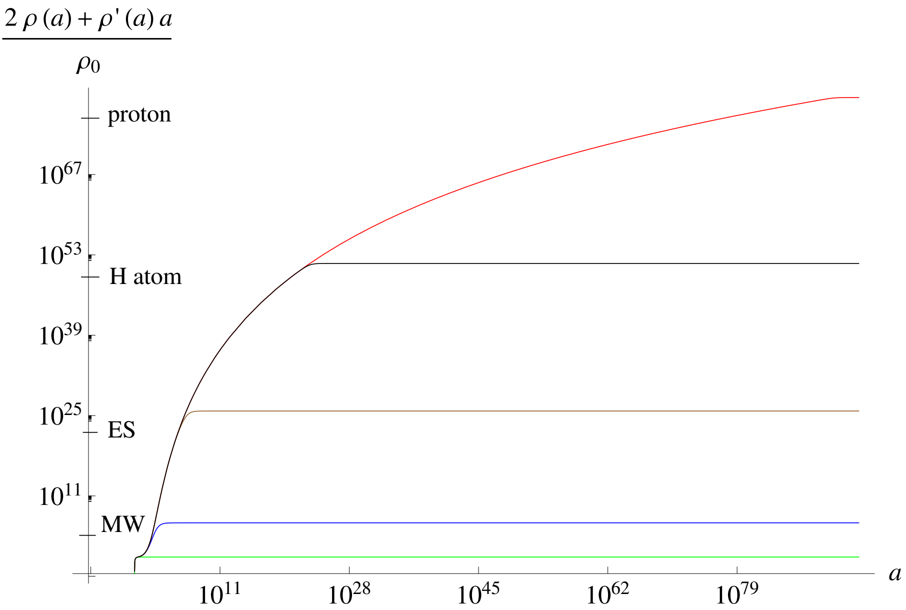
Complete List of Fates of the Universe for Monotonically Increasing DE Density

- Big Rip: dissociates all bound structures in a finite time
- Little Rip: dissociates all bound structures eventually
- **Pseudo-rip**: dissociates all bound structures at or below a certain threshold determined by the parameterization (could dissociate nothing)
- "No-rip": bound structures held together by a binding force greater than the constant force produced by the constant DE density stay together

Model 1

$$\rho_1(a, B, f, s) = \rho_0 \frac{\ln\left[\frac{1}{f+1/a} + \frac{1}{B}\right]^s}{\ln\left[\frac{1}{f+1} + \frac{1}{B}\right]^s}$$

- f, s fixed to control strength of rip
- B: free parameter for fitting data from the Supernova Cosmology Project
- $\rho_1(a = 1) = \rho_0$
- All parameterizations: $\chi^2_{reduced} \sim 0.98$



$\frac{2\,\rho(a) + \rho'(a)\,a}{\rho_0}$
proton
10^{67}
10^{53}
H atom
10^{39}
10^{25}
ES
10^{11}
MW
10^{11}
10^{28}
10^{45}
10^{62}
10^{79}
a

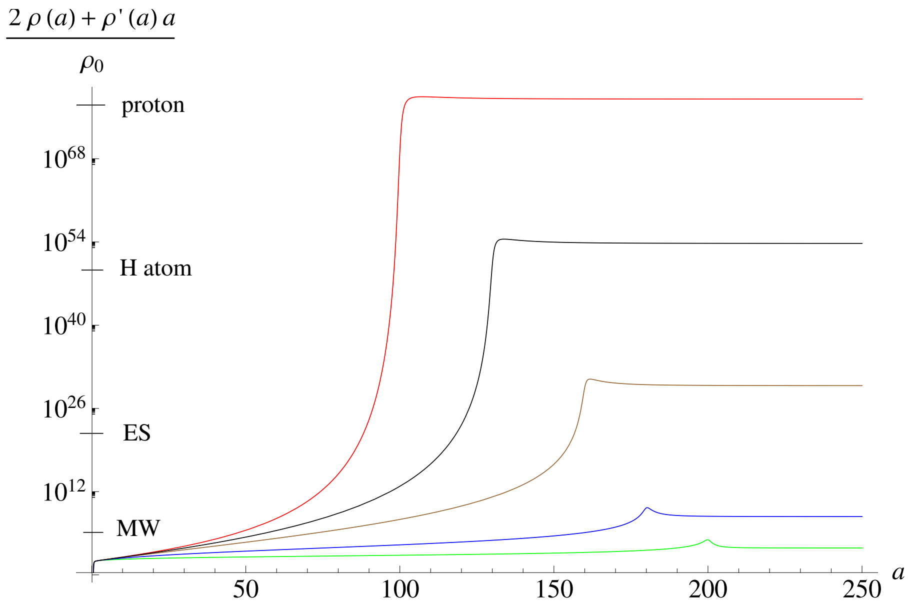
Finding When Objects Rip Apart

- I use a metric that interpolates between a static Schwarzschild metric for small distances away from the center of the object and the FLRW metric for large distances:
$dt^2 = -\left(1 - \frac{2GM}{a(t)r}\right) dt^2 + a(t)^2[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$
- Incorporating DE model in metric, I find when bound structure starts to rip apart by looking at the geodesic (Milky Way, Earth-Sun system)
- For smaller structures (H atom, proton), I use the Coloumb force and experimentally determined expression for strong force

Model 2

$$\rho_2(a, A, n, m) = \rho_0 \frac{A}{2}(\tan^{-1}(a-n) - \tan^{-1}(1-n) + 1)^m$$

- n, m fixed to control strength of rip
- A: free parameter for fitting data from the Supernova Cosmology Project
- $\rho_2(a = 1) = \rho_0$
- All parameterizations: $\chi^2_{reduced} \sim 0.98$


$\frac{2\,\rho(a) + \rho'(a)\,a}{\rho_0}$
proton
10^{68}
10^{54}
H atom
10^{40}
10^{26}
ES
10^{12}
MW
50
100
150
200
250
a

Non-Monotonicity from Monotonicity

- Local maxima in scaled inertial force curves
- Possible for structures to recombine

Inflections

- Needed for monotonically increasing functions to grow a certain amount arbitrarily quickly
- $F_{inert} \sim 2\rho(a) + \rho'(a)a$ can grow arbitrarily fast via peak in $\rho'(a)$
- The Sun may not rise tomorrow....

Scalar Field Realization of Pseudo-Rip

- For monotonically increasing potential $V(\phi)$, Pseudo-rip: $V(\phi) \to V_0$ and $\rho \to V_0$ as $\phi \to \infty$

Summary

- We examined models with non-constant dark-energy densities anticipating their evolution in the future
- We categorized the possible fates of the universe for monotonically increasing DE, introducing two new categories: Big Rip (all bound objects dissociate in finite time), Little Rip (all bound objects dissociate eventually), Pseudo-rip (all bound objects at or below a certain threshold dissociate), No-rip (constant cosmic acceleration, so bound objects with greater binding force than DE's force stay together)

Summary

- Little Rip: parameterizations for extra fluid component (models 1 and 2), scalar-tensor models (Big and Little Rip classically stable), coupled DM and DE (classically stable)
- Pseudo-rip: several parameterizations for models 1 and 2, dissociation times for chosen bound structures (and they may recombine)
- All models for the Little Rip and Pseudo-rip fit well with SNIa data. We could have also fit to BAO and CMB data, but we just wanted to introduce these new categories that involve the future of the universe (for which there are no data)

Sources for Figures in the Order of Appearance

- Rotation curve: http://en.wikipedia.org/wiki/File:GalacticRotation2.svg.
- CMB: http://en.wikipedia.org/wiki/File:WMAP_2010.png.
- DM data: D. P. Quinn *et al.*, Mon. Not. Roy. Astr. Soc. **396**, L11 (2009).
- Pie chart: http://scienceblogs.com/startswithabang/2009/09/dark_matter_part_i_how_much_ma.php.
- Next 5 figures: P. H. Frampton and K. J. Ludwick, Eur. Phys. J. C **71**, 1735 (2011).
- Little Rip fit: P. H. Frampton, Kevin J. Ludwick, R. J. Scherrer, Phys. Rev. D **84**, 063003 (2011).
- Structures rip apart: http://www.aip.org/png/2003/200.htm.
- Pseudo-rip inertial force plots: P. H. Frampton, K. J. Ludwick, and R. J. Scherrer, Phys. Rev. D **85**, 083001 (2012).

Coupling with Dark Matter

- Use a DE equation of state for a Little Rip: $w = -1 - \frac{2\lambda}{3H_0} e^{-\lambda t}$
- Rewritten as $(\rho_{DE} + p_{DE})^2 = \frac{4\lambda^2}{3\kappa^2}\rho_{DE}$
- In the end, we get $H_0 = \frac{4\lambda^2}{3Q\left(1-\frac{4\lambda^2}{Q^2}\right)}$, which requires $\frac{\lambda}{Q} < \frac{1}{2}$.
- $\rho_{DM} = \frac{16\lambda^4}{3\kappa^2 Q^2\left(1-\frac{4\lambda^2}{Q^2}\right)}$, $\quad \rho_{DE} = \frac{64\lambda^6}{3\kappa^2 Q^4\left(1-\frac{4\lambda^2}{Q^2}\right)^2}$
- The phantom DE density is increasing because of expansion, but it's decreasing because of decay into DM
- The cold DM is decreasing because of expansion, but it's decreasing because of DE's decay
- The present-day ratio of DM to DE is $1/3 \to \lambda^2/Q^2 \sim 3/16$

Is de Sitter a Stable Solution for Coupled DM and DE?

Is the de Sitter spacetime we assumed stable, an attractor solution?

$$H = \frac{4\lambda^2}{3Q\left(1-\frac{4\lambda^2}{Q^2}\right)} + \delta H, \quad \rho_{DM} = \frac{16\lambda^4}{3\kappa^2 Q^2\left(1-\frac{4\lambda^2}{Q^2}\right)} + \delta\rho_{DM},$$

$$\rho_{DE} = \frac{64\lambda^6}{3\kappa^2 Q^4\left(1-\frac{4\lambda^2}{Q^2}\right)^2} + \delta\rho_{DE}$$

Is de Sitter a Stable Solution for Coupled DM and DE?

First Friedmann equation: $\frac{8\lambda^2}{\kappa^2 Q\left(1-\frac{4\lambda^2}{Q^2}\right)}\delta H = \delta\rho_{DE} + \delta\rho_{DM}$

Conservation laws: $\delta\dot{\rho}_{DE} = \frac{16\lambda^4}{\kappa^2 Q^2\left(1-\frac{4\lambda^2}{Q^2}\right)}\delta H - \frac{Q}{2}\delta\rho_{DE},$

$$\delta\dot{\rho}_{DM} = -\frac{16\lambda^4}{\kappa^2 Q^2\left(1-\frac{4\lambda^2}{Q^2}\right)}\delta H + Q\delta\rho_{DE} - \frac{4\lambda^2}{Q\left(1-\frac{4\lambda^2}{Q^2}\right)}\delta\rho_{DM}$$

Is de Sitter a Stable Solution for Coupled DM and DE?

$$\frac{d}{dt}\begin{pmatrix} \delta\rho_{DE} \\ \delta\rho_{DM} \end{pmatrix} = \begin{pmatrix} -\frac{Q}{2}\left(1-\frac{4\lambda^2}{Q^2}\right) & \frac{2\lambda^2}{Q} \\ Q\left(1-\frac{2\lambda^2}{Q^2}\right) & -\frac{\frac{2\lambda^2}{Q}\left(3-\frac{4\lambda^2}{Q^2}\right)}{1-\frac{4\lambda^2}{Q^2}} \end{pmatrix}\begin{pmatrix} \delta\rho_{DE} \\ \delta\rho_{DM} \end{pmatrix}$$

Stability requires the derivative of the perturbations to be negative $\rightarrow$ negative eigenvalues $\rightarrow$ negative trace, positive determinant $\rightarrow$

$$-\frac{Q}{2}\left(1-\frac{4\lambda^2}{Q^2}\right)-\frac{\frac{2\lambda^2}{Q}\left(3-\frac{4\lambda^2}{Q^2}\right)}{1-\frac{4\lambda^2}{Q^2}}<0\,, \quad \lambda^2>0$$

The first condition is satisfied if $\frac{\lambda}{Q}<\frac{1}{2}$ (from before), and the second condition is satisfied trivially $\rightarrow$ STABLE

Little Rip Described by Scalar-Tensor Models

- Use $\omega(\phi) = -\frac{2}{\kappa^2} f''(\phi)\,, \quad V(\phi) = \frac{1}{\kappa^2}\left(3f'(\phi)^2 + f''(\phi)\right)$
- Solve Friedmann equations from this theory: $\phi = t\,, \quad H = f'(t)$
- Example of a Little Rip (same model used for coupled DE and DM): $w = -1 - \frac{2\lambda}{3H_0} e^{-\lambda t}$
- $\omega(\phi) = -\frac{2\lambda H_0}{\kappa^2} e^{\lambda\phi}\,, \quad V(\phi) = \frac{1}{\kappa^2}\left(3H_0^2 e^{2\lambda\phi} + \lambda H_0 e^{\lambda\phi}\right)$
- Field redefinition: $\varphi = \frac{2e^{\frac{\lambda}{2}\phi}}{\kappa}\sqrt{\frac{2H_0}{\lambda}}$

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{3\lambda^2\kappa^2}{64}\varphi^4 - \frac{\lambda^2}{8}\varphi^2\right\}$$

- The parameter A from before corresponds to $2\lambda/\sqrt{3}$ and is bounded as $2.74 \times 10^{-3}\ \text{Gyr}^{-1} \leq A \leq 9.67 \times 10^{-3}\ \text{Gyr}^{-1}$, or $2.37 \times 10^{-3}\ \text{Gyr}^{-1} \leq \lambda \leq 8.37 \times 10^{-3}\ \text{Gyr}^{-1}$

Is the Scalar-Tensor Model Stable?

Perturb: $\phi = t + \delta\phi(t)\,, \quad H = f'(t) + \delta h(t)$

Friedmann equations:
$\frac{3}{\kappa^2}H^2 = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi)\,, \quad -\frac{1}{\kappa^2}\left(2\dot{H} + 3H^2\right) = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi)$

$$\frac{d}{dt}\begin{pmatrix} \delta h \\ \delta\phi \end{pmatrix} = \begin{pmatrix} -6f'(t) & 6f'(t)f''(t) + f'''(t) \\ -3\frac{f'(t)}{f''(t)} & 3f'(t) \end{pmatrix}\begin{pmatrix} \delta h \\ \delta\phi \end{pmatrix}$$

Is the Scalar-Tensor Model Stable?

- Stability requires the derivative of the perturbations to be negative $\rightarrow$ negative eigenvalues $\rightarrow$ negative trace, positive determinant
- $-3f'(t) < 0\,, \quad 3\frac{f'(t)f'''(t)}{f''(t)} > 0$
- First condition: trivially satisfied for expanding universe since $f'(t) = H > 0$. If universe in phantom phase, then $f''(t) = \dot{H} > 0$, and second condition reduces to $f'''(t) = \ddot{H} > 0$. For Big and Little Rip: $H \rightarrow \infty$ as $t \rightarrow \infty$, which requires $\ddot{H} > 0$, so they are STABLE.
- For example, model corresponding to $w = -1 - \frac{2\lambda}{3H_0}e^{-\lambda t}$ (which gives $H = H_0 e^{\lambda t}$) is stable.
- Note: Little Rip and Big Rip *classically* stable, but we don't investigate quantum effects (vacuum fluctuations, coupling to gravitons, etc.)