Neutrino Advances and Developments in Finite Group Models

David A. Eby Vanderbilt University, Nashville, TN

in collaboration with Paul Frampton, Shinya Matsuzaki, and Thomas Kephar

Outline

- Motivation and Neutrino Review
- Building the Minimal Renormalizable T' Model
- Connections to SM and other BSM physics
- Dark Matter from T' Symmetry
- Limitations and Future Outlook

Flavor Symmetry Motivations

- Reduction of SM Parameters
- Explain Neutrino behavior: Mixing, Mass Scale, Helicity, etc.
- Connecting Lepton & Quark Properties
- Other Particle Physics questions: Dark Matter, New Symmetries
- Clues to New Physics at the Energy Frontier

"I have done a terrible thing today, something which no theoretical physicist should ever do. I have suggested something that can never be verified experimentally."

-Wolfgang Pauli (1930)

"I have done a terrible thing today, something which no theoretical physicist should ever do. I have suggested something that can never be verified experimentally."

-Wolfgang Pauli (1930)



Strange, Surprising

The ghostly particles could pave the way to unexplored realms

SIGS

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-		
Neutrino Asymmetry	XXXXXXXX	
FTL Neutrinos		
Non-zero Reactor Angle		
MiniBooNE low-E v _e excess	XXXXXXXXXX	
Missing Reactor/Gallium v		
IceCube High Energy Events		

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-		
Neutrino Asymmetry	NEXXXXXX	
FTL Neutrinos		
Non-zero Reactor Angle		
MiniBooNE low-E v _e excess	XXXXXXXXX	
Missing Reactor/Gallium v	******	
IceCube High Energy Events		

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-		
Neutrino Asymmetry	XXXXXXXX	
FTL Neutrinos		
Non-zero Reactor Angle		
MiniBooNE low-E v_{e} excess	XXXXXXXXXX	
Missing Reactor/Gallium v		
IceCube High Energy Events		

5

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-		
Neutrino Asymmetry	XXXXXXXX	
FTL Neutrinos		
Non-zero Reactor Angle		
MiniBooNE low-E v_{e} excess	XXXXXXXXXX	
Missing Reactor/Gallium v		
IceCube High Energy Events		

5

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-		
Neutrino Asymmetry		
FTL Neutrinos		*
Non-zero Reactor Angle		
MiniBooNE low-E v _e excess	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	
Missing Reactor/Gallium v	*****	
IceCube High Energy Events		

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-	********	
Neutrino Asymmetry		
FTL Neutrinos		*
Non-zero Reactor Angle		
MiniBooNE low-E v_{e} excess		
Missing Reactor/Gallium v		
IceCube High Energy Events		

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly	X T F H X K K X X X	
MINOS Neutrino Anti-	XXXXXXXXX XXXXXXXXXX	
Neutrino Asymmetry	NEXXXXXX	
FTL Neutrinos		
Non-zero Reactor Angle		
MiniBooNE low-E v_{e} excess	??	??
Missing Reactor/Gallium v	??	??
IceCube High Energy Events		

Anomalies	New Physics	Been and Gone
Neutrino Oscillation/Mass		
LSND Anomaly		
MINOS Neutrino Anti-	XXXXXXXXX XXXXXXXXXXXXXXXXXXXXXXXXXXXX	
Neutrino Asymmetry	NAXXXXXX	
FTL Neutrinos		
Non-zero Reactor Angle		
MiniBooNE low-E v_{e} excess	??	??
Missing Reactor/Gallium v	??	??
IceCube High Energy Events	??	???

5

Neutrino Mixing

 ν_1

 ν_2

 \mathcal{V}_3

- Neutrino Oscillation
- PMNS Matrix and Parametrization
 - For simplicity we set $\delta_{\rm CP}$ to O

TBM Symmetry: Tribimaximal Mixing ν_e

Neutrino Mixing

- Neutrino Oscillation
- PMNS Matrix and Parametrization

- For simplicity we U = | set $\delta_{\rm CP}$ to O
- TBM Symmetry: Tribimaximal Mixing



 $\begin{array}{cccc} +c_{12}c_{13} & +s_{12}c_{13} & +s_{13} \\ -s_{12}c_{23} + c_{12}s_{23}s_{13} & +c_{12}c_{23} + s_{12}s_{23}s_{13} & +s_{23}c_{13} \\ -s_{12}s_{23} - c_{12}c_{23}s_{13} & +c_{12}s_{23} - s_{12}c_{23}s_{13} & +c_{23}c_{13} \\ \hline \text{with } c_{12} \equiv \cos\theta_{12}, \ s_{13} \equiv \sin\theta_{13}, \ \text{etc.} \end{array}$

Neutrino Mixing

- Neutrino Oscillation
- PMNS Matrix and Parametrization

•

- For simplicity we $^{U=|}$ set δ_{CP} to O
- TBM Symmetry: Tribimaximal Mixing

 $\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$

 $+c_{12}c_{13} +s_{12}c_{13} +s_{13}$ $-s_{12}c_{23} + c_{12}s_{23}s_{13} +c_{12}c_{23} +s_{12}s_{23}s_{13} +s_{23}c_{13}$ $-s_{12}s_{23} - c_{12}c_{23}s_{13} +c_{12}s_{23} -s_{12}c_{23}s_{13} +c_{23}c_{13}$ $with <math>c_{12} \equiv \cos\theta_{12}, \ s_{13} \equiv \sin\theta_{13}, \text{ etc.}$ $\theta_{12} \simeq 35.3^{\circ}, \ \theta_{13} = 0^{\circ}, \ \theta_{23} = 45^{\circ}$ $U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$ 6

Mass & Hierarchy

- Limited Data leads to complex picture
- Best measurements are Δ²
 between mass eigenstates

 (sign of atmospheric mass-splitting remains unknown)
- 2 possible orderings:
 Normal and Inverted
- Quasi-degenerate case (now disfavored)
- Difficult to test, but hopefully next-gen (long baseline) detectors will settle the issue



Helicity and Seesaws

- Nonzero Mass: means Sterile RH
 Neutrinos are needed
- Dirac—Higgs coupling (x10⁻¹²)
- Majorana—Possible Seesaw Mechanism
- Loss of Lepton Number Conservation
- Light LH neutrinos explained by making RH neutrinos very heavy
- Equation combines two mass matrices
- Tough to test directly as "heavy" neutrinos could be at GUT scale



Helicity and Seesaws

- Nonzero Mass: means Sterile RH
 Neutrinos are needed
- Dirac—Higgs coupling (x10⁻¹²)
- Majorana—Possible Seesaw Mechanism
- Loss of Lepton Number Conservation
- Light LH neutrinos explained by making RH neutrinos very heavy
- Equation combines two mass matrices
- Tough to test directly as "heavy" neutrinos could be at GUT scale



 $M_{\nu} = M_D M_N^{-1} M_D^T$

Helicity and Seesaws

- Nonzero Mass: means Sterile RH
 Neutrinos are needed
- Dirac—Higgs coupling (x10⁻¹²)
- Majorana—Possible Seesaw Mechanism
- Loss of Lepton Number Conservation
- Light LH neutrinos explained by making RH neutrinos very heavy
- Equation combines two mass matrices
- Tough to test directly as "heavy" neutrinos could be at GUT scale



 $M_{\nu} = M_D M_N^{-1} M_D^T$

Outline

- Motivation and Neutrino Review
- Building the Minimal Renormalizable T' Model
- Connections to SM and other BSM physics
- Dark Matter from T' Symmetry
- Limitations and Future Outlook

TíxZ

Z₂: Cyclic Group

٠

٠

- T⁻: Binary Tetrahedral Symmetry
- Double-Cover of Tetrahedral Group, A₄,
 both of which can produce TBM values
- Order 24 Non-Abelian
 (non-commuting) Finite Group
- Benefits of T´over A₄:
 Compatibility with Quark Sector
 - First modern use in 1994 as a family symmetry for Quarks

Group Irrep. Multiplication Tables

- Identical singlet and triplet structure allow significant similarities
- T´ is notable for also including doublets

 1_3

 1_{3}

 1_{1}

 1_2

 2_3

 2_1

 2_2

3

 2_1

 2_1

 2_2

 2_3

 $1_1 + 3$

 $1_2 + 3$

 $1_3 + 3$

 $2_1 + 2_2 + 2_3$

 2_{1}

 2_{2}

 $\mathbf{Z}_{\mathbf{3}}$

 $2_1 + 2_2$

 1_{2}

 1_{2}

 1_{3}

 1_{1}

 2_{2}

 2_3

 2_1

3

 1_{1}

 1_{1}

 1_{2}

 1_{3}

 2_1

 2_2

 2_3

3

 1_{1}

 1_{2}

 1_3

 2_1

 2_{2}

 2_{3}

3

A_4								
		1_1	1_{2}	1_{3}	3			
1	1	11	1_{2}	1_3	3			
1	2	1_2	1_3	1_1				
1	3	1_3	1_1	1_2	3			
3		3	3	3	$1_1 + 1_2 + 1_3 + 3 + 3$			
T ²			9-		2			
22		1	<u>- 23</u>		<u> </u>			
$\frac{Z_2}{2}$			$\frac{Z_3}{2}$	10 30	<u>j</u>			
2_{3}			2_{1}		3			
2_{1}		1	2_2	St Pr				
$1_2 + 3$		7 7	$1_3 +$	3	$2_1 + 2_2 + 2_3$			
$1_3 + 3$		16. 19	$1_1 +$	3	$2_1 + 2_2 + 2_3$			
$1_1 + 3$			$1_2 +$	3	$2_1 + 2_2 + 2_3$			

 2_3

 $\mathbf{1}_1$

12

11

 $1_3 + 3 + 3$

Particle Assignments

LH leptons in Triplet RH leptons in Singlet

 $(3,+1) \qquad \begin{array}{c} \tau_R^- (1_1,-1) & \text{and} & N_R^{(1)} \\ \mu_R^- (1_2,-1) & \text{and} & N_R^{(2)} & (1_1,+1) \\ \mu_R^- (1_2,-1) & \text{and} & N_R^{(2)} & (1_2,+1) \\ e_R^- (1_3,-1) & \text{and} & N_R^{(3)} & (1_3,+1) \end{array}$

Uses 2+1 for quarks allowing for top quark singlet

$$\begin{pmatrix} t \\ b \end{pmatrix}_{\mathrm{L}} \mathcal{Q}_{\mathrm{L}} \quad (1_{1}, +1) \qquad b \\
\begin{pmatrix} c \\ s \end{pmatrix}_{\mathrm{L}} \\
\begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{L}} \\
\end{pmatrix} \mathcal{Q}_{\mathrm{L}} \quad (2_{1}, +1), \qquad u \\
\begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{L}} \\
\end{pmatrix} \mathcal{Q}_{\mathrm{L}} \quad (2_{1}, +1), \qquad d \\
\end{pmatrix}$$

$$\left. egin{array}{ccc} {
m R} & (1_1,+1) \ {
m R} & (1_2,-1) \ {
m R} \ {
m S} \ {
m R} \ (2_2,+1) \end{array}
ight.$$

Leptons	$L_{ m L}$	$ au_{ m R}$	$\mu_{ m R}$	$e_{ m R}$	$N_{ m R}^{(1)}$	$N_{ m R}^{(2)}$	$N_{ m R}^{(3)}$
$\mathbf{T}^{'}$	3	1_1	1_{2}	1_3	1_1	1_{2}	1_3
\mathbf{Z}_{2}	+	No.	24 <u>2</u> 4		+	+	+

	1_1	1_{2}	1_{3}	2_1	2_{2}	2_{3}	3
1_1	11	1_{2}	1_3	2_1	2_2	2_{3}	3
1_{2}	1_{2}	1_{3}	1_1	2_2	2_3	2_1	3
1_3	1_{3}	1_1	1_{2}	2_{3}	2_1	2_{2}	3
2_1	2_1	2_2	2_{3}	$1_1 + 3$	$1_2 + 3$	$1_3 + 3$	$2_1 + 2_2 + 2_3$
2_2	2_{2}	2_3	2_1	$1_2 + 3$	$1_3 + 3$	$1_1 + 3$	$2_1 + 2_2 + 2_3$
2_3	2_{3}	2_1	2_{2}	$1_3 + 3$	$1_1 + 3$	$1_2 + 3$	$2_1 + 2_2 + 2_3$
3	3	3	3	$2_1 + 2_2 + 2_3$	$2_1 + 2_2 + 2_3$	$2_1 + 2_2 + 2_3$	$1_1 + 1_2 + 1_3 + 3 + 3$

Leptons	$L_{ m L}$	$ au_{ m R}$	$\mu_{ m R}$	$e_{ m R}$	$N_{ m R}^{(1)}$	$N_{ m R}^{(2)}$	$\left \begin{array}{c} N_{\mathrm{R}}^{(3)} \end{array} \right $
$\mathbf{T}^{'}$	3	1_1	1_{2}	1_{3}	1_1	1_{2}	13
\mathbf{Z}_{2}	+	-			+	÷.	+

$L_L e_R H$

	$ 1_1 $	1_2	$ 1_3 $	2_1	2_2	2_3	3
1_1	1_1	1_2	1_3	2_1	2_2	2_{3}	3
1_2	1_{2}	1_3	1_1	2_2	2_3	2_1	3
1_3	1_{3}	11	1_{2}	2_{3}	2_1	2_{2}	3
2_1	2_1	2_{2}	2_3	$1_1 + 3$	$1_2 + 3$	$1_3 + 3$	$2_1 + 2_2 + 2_3$
2_2	2_2	2_3	2_1	$1_2 + 3$	$1_3 + 3$	$1_1 + 3$	$2_1 + 2_2 + 2_3$
2_3	2_3	2_1	2_{2}	$1_3 + 3$	$1_1 + 3$	$1_2 + 3$	$2_1 + 2_2 + 2_3$
3	3	3	3	$2_1 + 2_2 + 2_3$	$2_1 + 2_2 + 2_3$	$2_1 + 2_2 + 2_3$	$1_1 + 1_2 + 1_3 + 3 + 3$

Leptons	$L_{\rm L}$	$\mid au_{ m R}$	$\mu_{ m R}$	$e_{ m R}$	$\mid N_{ m R}^{(1)}$	$N_{\mathrm{R}}^{(2)}$	$\mid N_{\mathrm{R}}^{(3)}$		
$\mathbf{T}^{'}$	3	11	1_2	13	1_{1}	1_{2}	1_3		$L_L \epsilon$
\mathbf{Z}_{2}	+				****** ****	+	+	XXX	
a at the set	e 10 e	1 10	1	34 70°				10	

	()	1)
\square_3	$\mathbf{O},$	 1)
0	````	

	$ 1_1 $	1_2	$ 1_{3} $	2_1	2_2	2_{3}	3
1_1	1_1	1_2	1_{3}	2_1	2_{2}	2_{3}	
1_2	1_{2}	1_{3}	$ 1_1 $	2_2	2_3	\cdot \cdot 2_1	3
1_3	1_{3}	1_1	1_{2}	2_{3}	2_1	2_{2}	3
2_1	2_1	2_2	2_{3}	$1_1 + 3$	$1_2 + 3$	$1_3 + 3$	$2_1 + 2_2 + 2_3$
2_2	2_2	2_3	2_1	$1_2 + 3$	$1_3 + 3$	$1_1 + 3$	$2_1 + 2_2 + 2_3$
2_3	2_{3}	2_1	2_{2}	$1_3 + 3$	$1_1 + 3$	$1_2 + 3$	$2_1 + 2_2 + 2_3$
3	3	3	3	$2_1 + 2_2 + 2_3$	$2_1 + 2_2 + 2_3$	$2_1 + 2_2 + 2_3$	$1_1 + 1_2 + 1_3 + 3 + 3$

Le	epto	ons	$\left\ L \right\ $	L	$ au_{ m R}$	$\mu_{ m R}$	$e_{ m R}$	$N_{ m R}^{(1)}$	$\mid N_{\mathrm{R}}^{(2)}$	$^{ m)} \mid N_{ m R}^{(}$	3) t		
$\mathbf{T}^{'}$			3	1_{1}	1_2	1_3	11	1_{2}	1:	3	Ç	$Q_L t_R H$	
	\mathbf{Z}_2			F	-			+	+	+			
Quarks		Ç	$2_{ m L}$	$Q_{ m L}$	$ t_{\rm R}$	b_{R}	$\Big \mathcal{C}_{\mathrm{R}} \Big $	$\mathcal{S}_{ ext{R}}$	H	/ 3	(3, -	1)	
	$\mathbf{T}^{'}$		1	-1	2_{1}	11	1_{2}	2_3	2_{2}				
	\mathbf{Z}_{2}			+	+	+			+				
4. M.	$ 1_1 $	1_2	1_{3}		2_1		2_2		3		3		
1_1	1_{1}	1_2	1_3		2_1		22	2	3	a ja	3		
1_2	12	1_{3}	1_1	C. P.	2_2	THE AN	23	\cdot	1		3	16 14 16 19 19 19 19 19 19 19 19 19 19 19 19 19	
1_{3}	1_3 1_3 1_1 1_2 2_3			2_1	2_2	2		3	<u></u>				
$\frac{2_1}{2}$	2_1	$\frac{2_2}{2}$	$\frac{2_3}{2}$		$\Gamma_1 + 3$		$\frac{1}{2} + 3$	$1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 - 1_3 $		$-2_1 + 2_1 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2$	$\frac{2_2}{2_2}$ +	$\frac{2_3}{2}$	
$\frac{Z_2}{2}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\frac{2_1}{2_2}$	$2_1 1_2 + 3$			$1_3 + 3$		$1_1 + 3$		$2_1 + 2_2 + 2_3$		
$\frac{23}{3}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$\frac{22}{3}$	21 -	+22+2	$2_3 2_1 -$	-2_2+2_3	$\begin{array}{c c} & 1_2 \\ \hline & 1_2 \\ \hline & 2_1 + 2_2 \end{array}$	$2 + 2_3$	$1_1 + 1_2 -$	$+1_3$ -	-3+3	

Lepton	$\mathbf{s} \parallel \mathbf{I}$	L	$\left \tau_{ m R} \right $	$\mu_{ m R}$	$e_{ m R}$	$N_{ m R}^{(1)}$	$ N_{\rm R}^{(2)}$	$^{2)} \mid N_{ m R}^{(3)}$	
$\mathbf{T}^{'}$		3	1_1	1_{2}	1_{3}	11	12	13	$\int \mathcal{Q}_L t_R H_{1_1}$
\mathbf{Z}_2		÷	-	_		+	+	+	
Quark	s 🏻 🤇	$2_{\rm L}$	$Q_{\rm L}$	$ t_{\mathrm{R}}$	b_{R}	$\Big \mathcal{C}_{\mathrm{R}} \Big $	$\mathcal{S}_{ ext{R}}$	$ig H'_3$	$(3, -1) H_{1_1} (1_1, +1)$
$\mathbf{T}^{'}$		1_{1}	2_1	1_1	12	2_3	2_2		
Z_2		+		+			÷		
$ 1_1 1$	$ _2 1_3$		2_1	A A	2_{2}	2	3	3	
$1_1 1_1 1_1$	$1_2 1_3$		2_1		2_{2}	- 2;	3	- 3	H X M X X X X X X X X X X X X X X X X X
1_2 1_2 1_2	$1_3 1_1$		2_2	THE AND	2_{3}		1	3	the second se
1_3 1_3 1_3	$1 1_2$	-	$\frac{2_3}{1}$		2_1	2	2	3	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\frac{2_2}{2_3}$		$\frac{1}{1} + 3$		$\frac{12+3}{12}$	1 ₃ -	- 3	$2_1 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 + 2_2 $	$+2_3$
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c} \hline z_3 & \overline{z_1} \\ \hline z_4 & \overline{z_2} \end{array}$		$\frac{1_2 + 3}{1_2 + 3}$		$\frac{3+3}{3+3}$	I ₁ -	+3 ∟?	$2_1 + 2_2 - 2_1 + 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 $	$+2_3$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c c} z_1 & z_2 \\ \hline 3 & 3 \end{array}$	2_1	$+2_2+2$	$\begin{array}{c c} & 1 \\ 3 & 2_1 + \end{array}$	-2_2+2_3	$\begin{array}{c c} & 1_2 \\ \hline & 1_2 \\ \hline & 3 \\ \end{array}$	$\frac{1}{2} + 2_3$	21 + 22 + 1 $1_1 + 1_2 + 1_3$	$\frac{+23}{3+3+3}$ 13

Lept	ons	$\parallel L$	'L	$\tau_{ m R}$	$\mu_{ m R}$	e_{R}	$N_{ m R}^{(1)}$	$\mid N_{\mathrm{R}}^{(2)}$	$N_{ m R}^{(3)}$					
T	/		3	1_1	1_2	1_3	11	1_2	13					
\mathbf{Z}_2	2		÷				+	+	+					
Qua	rks		$2_{\rm L}$	$Q_{\rm L}$	$t_{\rm R}$	b_{R}	$\Big \mathcal{C}_{\mathrm{R}} \Big $	$\mathcal{S}_{ ext{R}}$	H_3'	(3,	-1)	$H_{1_{1}}$	$(1_1, +$	1)
T	/	1	1	2_1	1_1	12	2_3	2_2	$ H_3 $	(3,	+1)	$H_{1_{3}}$	$(1_3, -$	1)
\mathbf{Z}_2	2		+		+			+						
	$ 1_2$	$ 1_3 $		2_1	1	2_2	2:	3	3					
$1_1 \mid 1_1$	1_2	1_3		2_1		-2_2	2	3	3					
1_2 1_2	13	1_1	t de	2_2		23	2	1	- 3	at the tak				
1_3 1_3	1_1	1_{2}	2	23		$\overline{2}_1$	2	2	3	1 10 10				
$\frac{2_1}{2_1}$	2_2	2_{3}		$\frac{1_1+3}{1_1+3_{$	1	$\frac{2+3}{2}$	13 -	- 3	$2_1 + 2_2 + 2_2 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 + 2_3 $	$+2_3$				
$\frac{2_2}{2}$ $\frac{2_2}{2}$	2_3	$\frac{2}{2}$		$\frac{1_2 + 3}{1_1 + 2}$		$\frac{3+3}{3-2}$			$2_1 + 2_2 - 2_1 + 2_2 - 2_2 - 2_2 + 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 - 2_2 $	$+2_3$				
$\begin{array}{c c} 2_3 & 2_3 \\ \hline 3 & 3 \\ \hline \end{array}$	$\frac{2_1}{3}$	$\overline{\begin{array}{c} 2_2 \\ 3 \end{array}}$	21 -	$\frac{1_3+3}{2_2+2}$	$\begin{array}{c c} & 1 \\ \hline 3 & 2_1 + \end{array}$	$\frac{1+3}{22+23}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	+3 2+23	$\begin{array}{r} 2_1 + 2_2 + \\ 1_1 + 1_2 + 1_3 \end{array}$	+23 +3+3				13

Leptons	$\left \begin{array}{c} L_{\mathrm{L}} \end{array} \right $	$ au_{ m R}$	$\mu_{ m R}$	$e_{\rm R}$	$N_{\mathrm{R}}^{(1)}$	$) \mid N_{\mathrm{R}}^{(2)}$	$N_{\mathrm{R}}^{(3)}$
$\mathbf{T}^{'}$	3	1_1	1_{2}	13	11	12	1_{3}
${ m Z}_2$	+				+	+	+
Quarks	\mathcal{Q}_{L}	$Q_{\rm L}$	$ t_{\rm R}$	b_{F}	$_{\rm R} \mid \mathcal{C}_{\rm F}$	\mathcal{S}_{R}	
$\mathbf{T}^{'}$	1_1	2_1	1_1	1	$_{2}$ 2 ₃	2_2	
Z_2	+	+	+			+	
Higgs	H_{1_1}	$ $ H_1	13 -	H_3	H_{3}^{\prime}		
$\mathbf{T}^{'}$	1_{1}	1;	3	3	3		
Z_2	+			÷	_		

Leptons	$L_{ m L}$	$ au_{ m R}$	$\mu_{ m R}$	$e_{ m R}$	$N_{ m R}^{(1)}$	$\mid N_{\mathrm{R}}^{(2)}$	$\left \begin{array}{c} N_{ m R}^{(3)} \end{array} \right $						
$\mathbf{T}^{'}$	3	1_1	1_2	1_{3}	1_1	1_2	13						
${ m Z}_2$	+				+	+	+						
Quarks	\mathcal{Q}_{L}	$Q_{\rm L}$	$ t_{\rm R}$	b_{R}	$\Big \mathcal{C}_{\mathrm{R}} \Big $	\mathcal{S}_{R}							
$\mathbf{T}^{'}$	1_{1}	2_1	1_1	1_2	23	2_2							
Z_2	+	+	+			+	$\mathcal{L}_Y =$	$= M_1 N_R^{(1)} N_R^{(1)} + M_{23} N_R^{(2)} N_R^{(3)} +$					
Higgs	$ S \ H_{1_1} H_{1_3} H_3 H_3 H_3 M_3 M_3 M_3 M_3 M_4 H_3 + Y_{\mu} L_L \mu_R H_3 + Y_{\tau} L_L \tau_R H_3 + Y_{\tau} L$												
$\mathbf{T}^{'}$	1_1	1	3	3	$\overline{3}$ Y_1	$Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + V_{3}L_{L}N_{R}^{(3)}H_{3} + V_{3}L_{L}N_{R$							
\mathbf{Z}_{2}	+			H		$Y_{t}(Q_{L}t_{R}H_{1_{1}}) + Y_{b}(Q_{L}b_{R}H_{1_{3}}) + Y_{c}(Q_{L}C_{R}H_{3}') + Y_{s}(Q_{L}S_{R}H_{3}) + h.c.$ ¹³							

 $\mathcal{L}_{Y} = M_{1}N_{R}^{(1)}N_{R}^{(1)} + M_{23}N_{R}^{(2)}N_{R}^{(3)} + Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{t}(\mathcal{Q}_{L}t_{R}H_{1_{1}}) + Y_{b}(\mathcal{Q}_{L}b_{R}H_{1_{3}}) + Y_{c}(\mathcal{Q}_{L}\mathcal{C}_{R}H_{3}^{'}) + Y_{s}(\mathcal{Q}_{L}\mathcal{S}_{R}H_{3}) + h.c.$

 $\mathcal{L}_{Y} = M_{1}N_{R}^{(1)}N_{R}^{(1)} + M_{23}N_{R}^{(2)}N_{R}^{(3)} + Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(3)}H_{3} + Y_{2}L$

 $\mathcal{L}_{Y} = M_{1}N_{R}^{(1)}N_{R}^{(1)} + M_{23}N_{R}^{(2)}N_{R}^{(3)} + Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(3)}H_{3} + Y_{2}L_{L}N_$

 $< H_3 >= (V, V, V) < H_3 >= (V, -2V, V)$

 $\mathcal{L}_Y = M_1 N_R^{(1)} N_R^{(1)} + M_{23} N_R^{(2)} N_R^{(3)} +$ $Y_e L_L e_R H'_3 + Y_\mu L_L \mu_R H'_3 + Y_\tau L_L \tau_R H'_3 +$ $Y_1 L_L N_R^{(1)} H_3 + Y_2 L_L N_R^{(2)} H_3 + Y_3 L_L N_R^{(3)} H_3 +$ $| < H'_{3} > = (\frac{m_{\tau}}{Y_{\tau}}, \frac{m_{\mu}}{Y_{\mu}}, \frac{m_{e}}{Y_{e}})$ $Y_t(\mathcal{Q}_L t_R H_{1_1}) + Y_b(\mathcal{Q}_L b_R H_{1_3}) +$ $< H_{1_1} >= (\frac{m_t}{Y_t}) < H_{1_3} >= (\frac{m_b}{Y_b})$ $Y_{\mathcal{C}}(Q_{L}\mathcal{C}_{R}H_{3}') + Y_{\mathcal{S}}(Q_{L}\mathcal{S}_{R}H_{3}) + h.c.$ $\langle H_3 \rangle = (V, V, V)$ $\langle H_3 \rangle = (V, -2V, V)$ $\overline{m_3} \gg \overline{m_1} = \overline{m_2}$ $m_2 \gg m_1 = m_3 = 0$

 $\mathcal{L}_{Y} = M_{1}N_{R}^{(1)}N_{R}^{(1)} + M_{23}N_{R}^{(2)}N_{R}^{(3)} + Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(3)}H_{3} + Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{2}L_{L}N_{R}^{(3)}H_{3} + Y_{2}L_{L}N_$

 $< H_3 \rightarrow (V, V, V)$

 $m_2 \gg m_1 = m_3 = 0$

 $< H_3 >= (V, -2V, V)$ $m_3 \gg m_1 = m_2$

Cabibbo Angle Prediction

- Minimal Renormalizable T´ Model (No Higgs T´ doublets)
- Shared Higgs allow connection between neutrino mixing and Cabibbo Matrix (<u>not</u> full CKM)
 - Method for Quark Mixing Prediction:
 - Assume up-type quarks and charged leptons are mass eigenstates
 - Complex Clebsch-Gordan
 coefficients from symmetry
 - Diagonalize nontrivial 2x2 quark matrices

 $\tan 2\Theta_{12} = \frac{\sqrt{2}}{3}$

 $\Theta_{12,\text{predicted}} = 12.6^{\circ}$ $\Theta_{12,\text{experimental}} = 13.0^{\circ}$

Perturbations

- Historical Development
 vs. Optimal Input
- What neutrino mixing does Cabibbo Angle demand? $\delta(M_{\nu})$
 - Perturbations in Higgs VEVs and Mixing Angles

 $\langle H_3 \rangle = V'(1, -2 + b, 1 + a)$ $\overline{\theta_{ij}} = (\theta_{ij})_{TBM} + \epsilon_k$ $\delta(M_{\nu})_{
m diag} = egin{pmatrix} \delta m_1 & 0 & 0 \ 0 & \delta m_2 & 0 \ 0 & 0 & \delta m_3 \end{pmatrix}$ $= \delta U(M_{\nu})_{\text{TBM}} U_{\text{TBM}}^T$ + $U_{\text{TBM}}\delta M_{\nu}U_{\text{TBM}}^{T}$ + $U_{\text{TBM}}(M_{\nu})_{\text{TBM}}\delta U^{T}$

Predictions

Yields correlation between Atmospheric and Reactor Mixing Angles

-

٠

While noting the trivial case restores TBM values, use of Cabibbo Angle input forces symmetry breaking

Predictions

 $\overline{\theta}_{13} = \sqrt{2} \left(\frac{\pi}{4} - \theta_{23} \right)$

-

٠

Yields correlation between Atmospheric and Reactor Mixing Angles

While noting the trivial case restores TBM values, use of Cabibbo Angle input forces symmetry breaking





Particle Data Group 2010 Review



 $2\sin^2\theta_{23}\sin^22\theta_{13} = 0.080 \text{ (DC, T2K, MINOS)}$

 $\theta_{23} = -\frac{1}{\eta}\theta_{13} + \frac{\pi}{4}$



Daya Bay Early Results





Fogli Global Results





Fogli Global Results (w/ Best Fit)

T´is blind to Octant Degeneracy



Different Global Analysis by Forero et al.

Due to redefinition of angles, model prediction is only for correlation magnitude

Though this analysis indicates lower angle precision, best fit value agrees within 2.5%

Outline

- Motivation and Neutrino Review
- Building the Minimal Renormalizable T' Model
- Connections to SM and other BSM physics
- Dark Matter from T' Symmetry
- Limitations and Future Outlook

Need a Higgs doublet to add third family quark mixing

٠

٠

٠

٠

- Several options for the NMRT M
- Many new variables limit new predictions
- Added complexity aside, this remains an intriguing avenue of investigation

Need a Higgs doublet to add third family quark mixing

٠

٠

٠

٠

- Several options for the NMRT´M
- Many new variables limit new predictions
- Added complexity aside, this remains an intriguing avenue of investigation

 $\begin{array}{c|cccc} \mathbf{A} & \overline{H_{2_{1}}(2_{1},+1)} & \overline{Y_{Qt}Q_{L}t_{R}H_{2_{1}}} \\ \mathbf{B} & \overline{H_{2_{3}}'(2_{3},-1)} & \overline{Y_{Qb}Q_{L}b_{R}H_{2_{3}}'} \\ \mathbf{C} & \overline{H_{2_{2}}'(2_{2},-1)} & \overline{Y_{Qc}Q_{L}C_{R}H_{2_{2}}'} \\ \mathbf{D} & \overline{H_{2_{3}}(2_{3},+1)} & \overline{Y_{Qs}Q_{L}S_{R}H_{2_{3}}} \end{array}$

$$\mathcal{D} = egin{pmatrix} 3 & -rac{\sqrt{2}}{3} \ -rac{\sqrt{2}}{3} & 1 \end{pmatrix}$$

Need a Higgs doublet to add third family quark mixing

٠

٠

٠

٠

- Several options for the NMRT M
- Many new variables limit new predictions
- Added complexity aside, this remains an intriguing avenue of

A $H_{2_1}(2_1, +1)$ $Y_{Qt}Q_L t_R H_{2_1}$ **B** $H_{2_3}^{'}(2_3, -1)$ $Y_{Qb}Q_{L}b_{R}H_{2_3}^{'}$ **C** $H'_{2_{2}}(2_{2},-1)$ $Y_{QC}Q_{L}C_{R}H'_{2_{2}}$ **D** $H_{2_3}(2_3,+1)$ $Y_{\mathcal{QS}}\mathcal{Q}_{\mathrm{L}}\mathcal{S}_{\mathrm{R}}H_{2_3}$

$$\mathcal{D} = \begin{pmatrix} 3 & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & 1 \end{pmatrix}$$

$$\begin{split} \textbf{investigation} \\ \mathcal{D} = \begin{pmatrix} (Y_b^{'}V_{1_3}^{'})^2 + (Y_{\mathcal{QS}}^{'}V_{2_3}^{'})^2 & \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(1 - 2\sqrt{2}\omega) & \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(\omega + \sqrt{2}) \\ \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(1 - 2\sqrt{2}\omega^2) & 3 & -\frac{\sqrt{2}}{3} \\ \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(\omega^2 + \sqrt{2}) & -\frac{\sqrt{2}}{3} & 1 \end{pmatrix} \end{split}$$

Need a Higgs doublet to add third family quark mixing

٠

٠

٠

٠

- Several options for the NMRT M
- Many new variables limit new predictions
- Added complexity aside, this remains an intriguing avenue of

A $H_{2_1}(2_1, +1)$ $Y_{Qt}Q_L t_R H_{2_1}$ **B** $H_{2_3}^{'}(2_3, -1)$ $Y_{Qb}Q_{L}b_{R}H_{2_3}^{'}$ **C** $H'_{2_{2}}(2_{2},-1)$ $Y_{QC}Q_{L}C_{R}H'_{2_{2}}$ **D** $H_{2_3}(2_3,+1)$ $Y_{\mathcal{QS}}\mathcal{Q}_{\mathrm{L}}\mathcal{S}_{\mathrm{R}}H_{2_3}$

$$\mathcal{D} = \begin{pmatrix} 3 & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & 1 \end{pmatrix}$$

$$\begin{split} \textbf{investigation} \\ \mathcal{D} = \begin{pmatrix} (Y_b^{'}V_{1_3}^{'})^2 + (Y_{\mathcal{QS}}^{'}V_{2_3}^{'})^2 & \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(1 - 2\sqrt{2}\omega) & \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(\omega + \sqrt{2}) \\ \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(1 - 2\sqrt{2}\omega^2) & 3 & -\frac{\sqrt{2}}{3} \\ \frac{1}{\sqrt{6}}Y_{\mathcal{QS}}^{'}V_{2_3}^{'}(\omega^2 + \sqrt{2}) & -\frac{\sqrt{2}}{3} & 1 \end{pmatrix} \end{split}$$

CKM Fits I



CKIM Fits II



Quartification

Toy Quiver Model

*

*

Combines T´ and SM groups

A non-specific test of compatibility

Demands an additional sub-quiver representation

 $\left\{\begin{array}{ccc} (3\bar{3}11)_{3} \supset \begin{pmatrix} t \\ b \end{pmatrix}_{\mathrm{L}} \mathcal{Q}_{\mathrm{L}} & (\mathbf{1}_{1},+1) \\ (3\bar{3}11)_{2} \supset \begin{pmatrix} c \\ s \end{pmatrix}_{\mathrm{L}} \\ (3\bar{3}11)_{1} \supset \begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{L}} \end{array}\right\} \mathcal{Q}_{\mathrm{L}} & (\mathbf{2}_{1},+1)$

 ${f SU(3)_{
m C} imes SU(3)_{
m L} imes SU(3)_{\ell} imes SU(3)_{
m R}}$

Outline

- Motivation and Neutrino Review
- Building the Minimal Renormalizable T' Model
- Connections to SM and other BSM physics
- Dark Matter from T' Symmetry
- Limitations and Future Outlook

DM & Leptogenesis

- Meanwhile in Valencia...
- New Symmetry, New RH Neutrinos, New Higgs, New Model
- Dark Matter generated from the neutral real scalar parts of Higgs triplet
- New Lagrangian Terms
- If Y₄ and Y₅ are complex, they can yield Leptogenesis
- Relic Density leads to approximate
 WIMP mass of ≈1.62 TeV

 $\begin{aligned} \mathcal{L}_{Y} &= M_{0}N_{T}N_{T} + M_{1}N_{R}^{(1)}N_{R}^{(1)} + M_{23}N_{R}^{(2)}N_{R}^{(3)} + \\ &Y_{e}L_{L}e_{R}H_{3}^{'} + Y_{\mu}L_{L}\mu_{R}H_{3}^{'} + Y_{\tau}L_{L}\tau_{R}H_{3}^{'} + \\ &Y_{1}L_{L}N_{R}^{(1)}H_{3} + Y_{2}L_{L}N_{R}^{(2)}H_{3} + Y_{3}L_{L}N_{R}^{(3)}H_{3} + \\ &Y_{4}L_{L}(N_{T}H_{3}^{''})_{3} + Y_{5}L_{L}(N_{T}H_{3}^{''})_{3'} + \\ &Y_{t}(\mathcal{Q}_{L}t_{R}H_{1_{1}}) + Y_{b}(\mathcal{Q}_{L}b_{R}H_{1_{3}}) + \\ &Y_{\mathcal{C}}(Q_{L}\mathcal{C}_{R}H_{3}^{'}) + Y_{\mathcal{S}}(Q_{L}\mathcal{S}_{R}H_{3}) + h.c. \end{aligned}$



Outline

- Motivation and Neutrino Review
- Building the Minimal Renormalizable T' Model
- Connections to SM and other BSM physics
- Dark Matter from T' Symmetry
- Limitations and Future Outlook

Limitations and Unanswered Questions

- Hopes for Future Work:
 - Higher-Order Calc. w/o Fine-Tuning
 - Combining with SUSY or other GUTs
- Leptogenesis:
 - CP-Violating Phase Practical Limitation
 - Evaluating potential in T´WIMP Model
- Structural Limitations:
 - Solar Mass Split Symmetry Limitation
 - NMRT´M with all CKM Angles



Conclusions

- We unify TBM breaking with precise CKM angles
- Successful Predictions:
 - TBM Symmetry is broken
 - Global fits indicate η lies within 5% of our expectation
- Intriguing Dark Matter Candidate with interesting behavior
- We need more data to understand neutrinos (don't we always?)
- Between new detectors, new results, and new theories this remains an exciting time in neutrino physics

Thank you for coming! Questions?