

Jet Energy Loss and Stopping Distances in Weakly-Couple Quark-Gluon Plasmas

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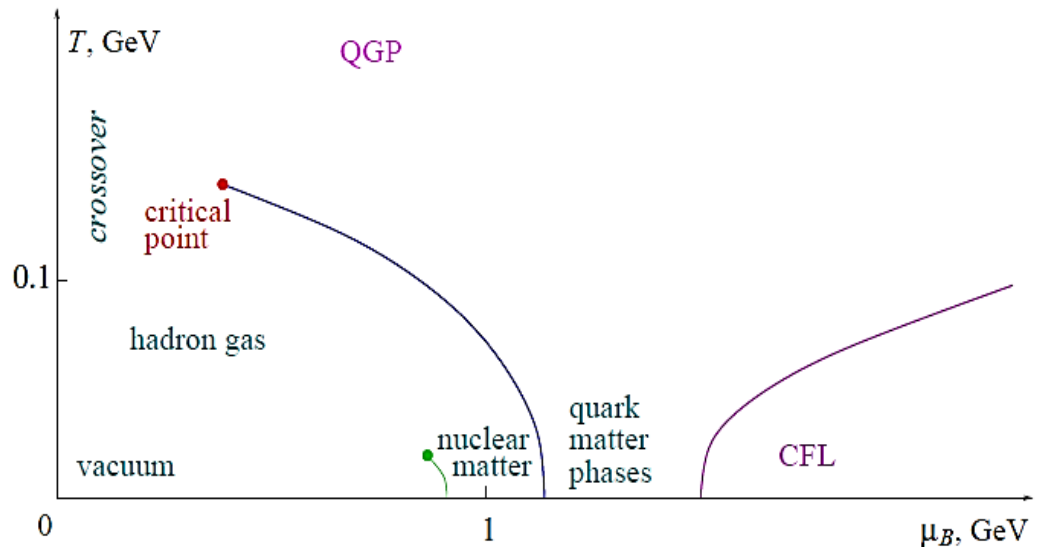
1. Peter Arnold, Wei Xiao: Phys. Rev. D78, 125008 (2008) [arXiv: 0810.1026]
2. Peter Arnold, Sean Cantrell, and Wei Xiao: Phys. Rev. D81, 045017 (2010) [arXiv: 0912.3862 [hep-ph]].

Outlines

- Introductions
- Calculation of transport parameter
- Calculation of energy loss rate
- Calculation of stopping distances
- Numerical results and discussions

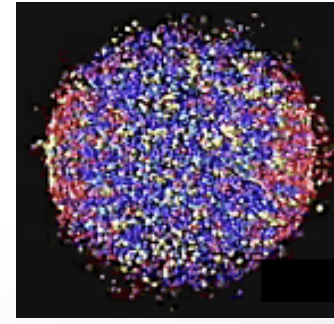
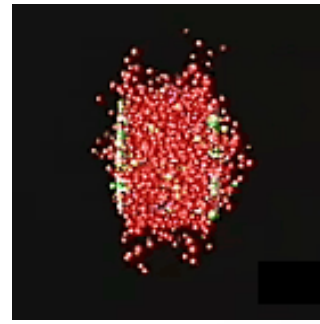
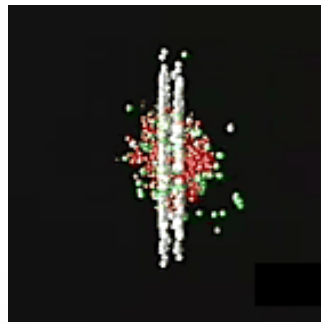
Introduction: Quark-Gluon Plasma

- QGP is a phase of QCD which exists in extreme high temperature and/or high density.
- In RHIC of BNL, gold nuclei are collided at 100GeV per nucleon. It is believed QGP is created at $T \sim 350\text{MeV}$ ($4 \times 10^{12}\text{K}$, 250,000 time hotter than the center of Sun)



QCD phase diagram – a semiquantitative sketch

M. Stephanov hep – lat/0701002

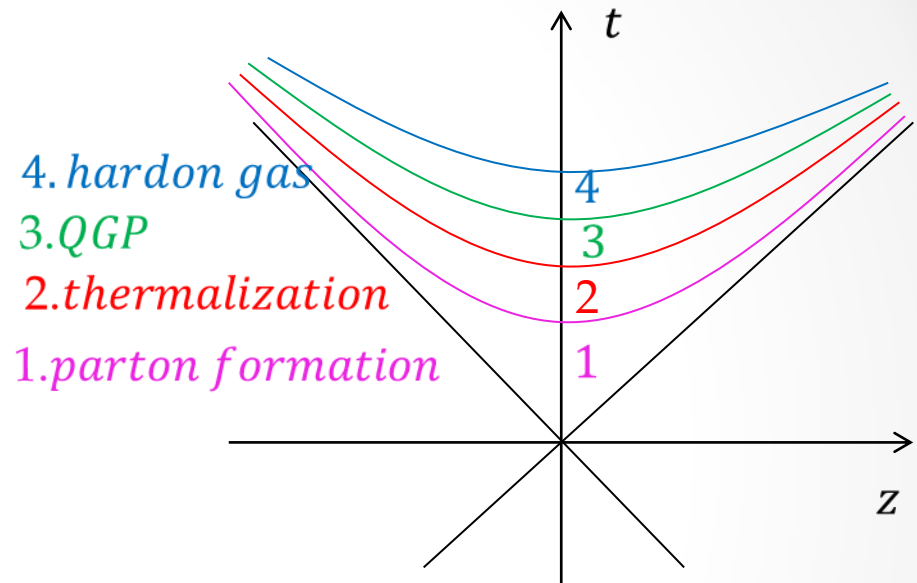


1. ions about to collide 2. ion collision 3. quarks, gluons freed 4. plasma created

Heavy Ion Collision

- Suppose two gold nuclei move along z and $-z$ direction respectively, near the speed of light.
- Highly Lorentz contracted in z direction, define proper time:

$$\tau = \sqrt{t^2 - z^2}$$



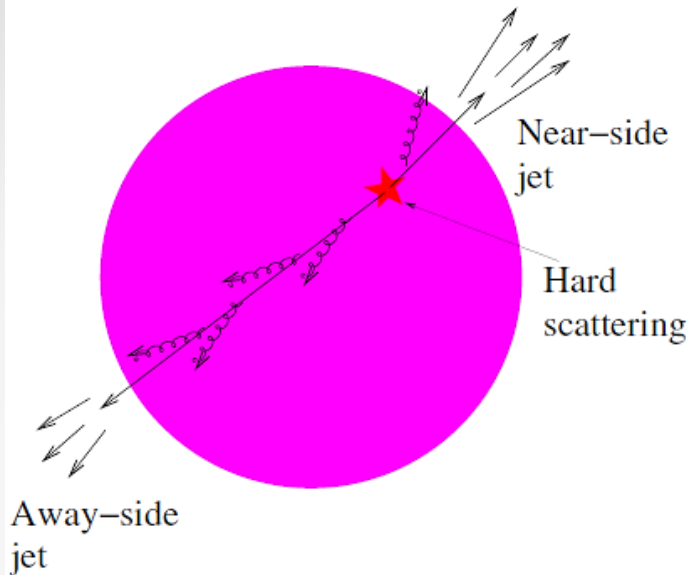
- Gold nuclei Collide at $\tau = 0$
- Stage 1 & 2:
Parton formation and thermalization, little is known.
- Stage 3: $\tau \sim 0.1 - 1 fm$, studied by transport theory and hydrodynamics.
- Stage 4: free streaming gas of hadrons, eventually arrive at detector.

Lightcone Cartoon of Heavy Ion Collision

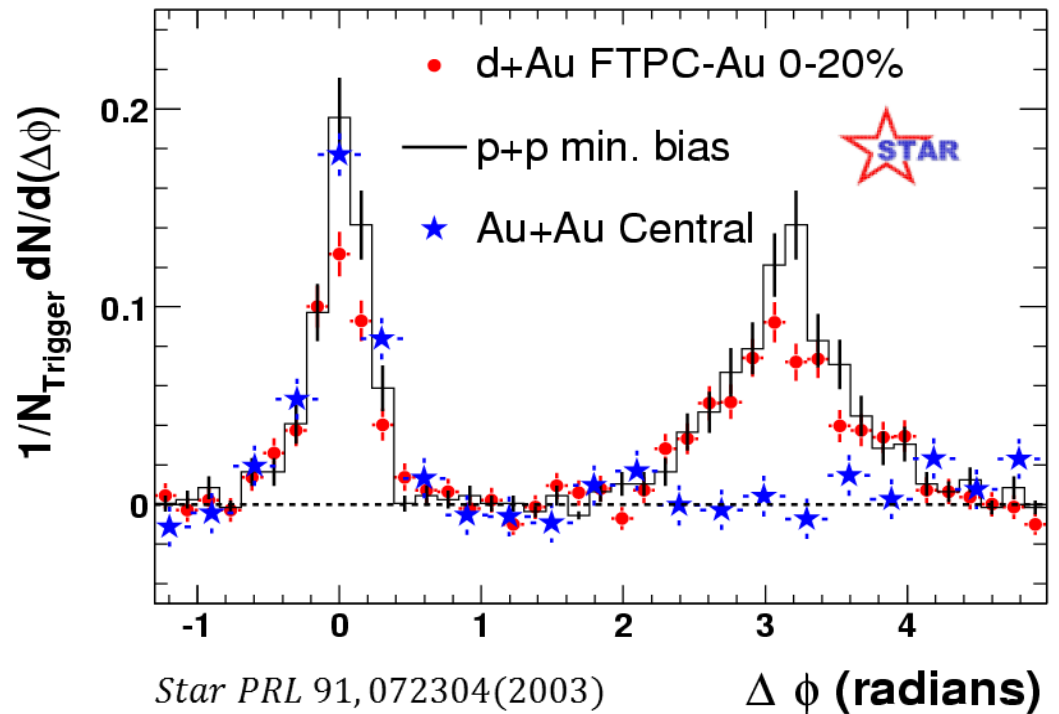
L. McLerran hep-ph/0104285

Jet quenching

- In RHIC Au-Au collision, define azimuthal angle ϕ on the transverse plane. correlation function shows “absence of away-side jet”.

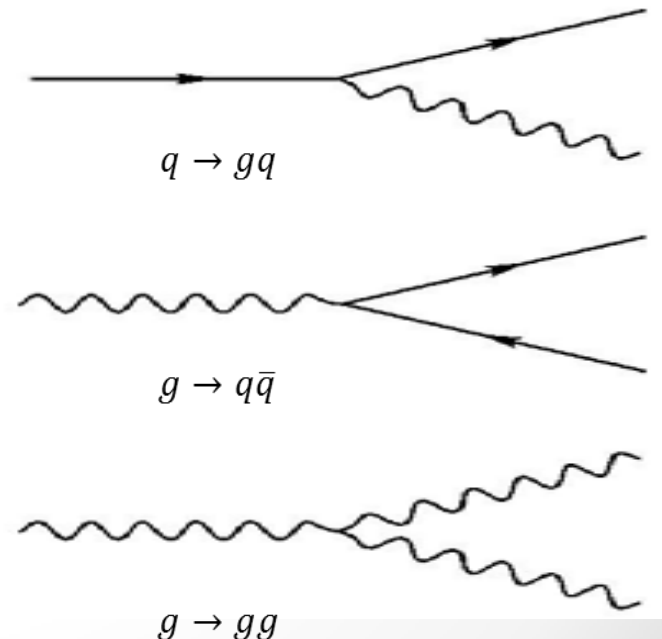


- Partons interact with medium and deposit energies before hadronization.
- Studying medium properties and how parton lose energy is interesting.



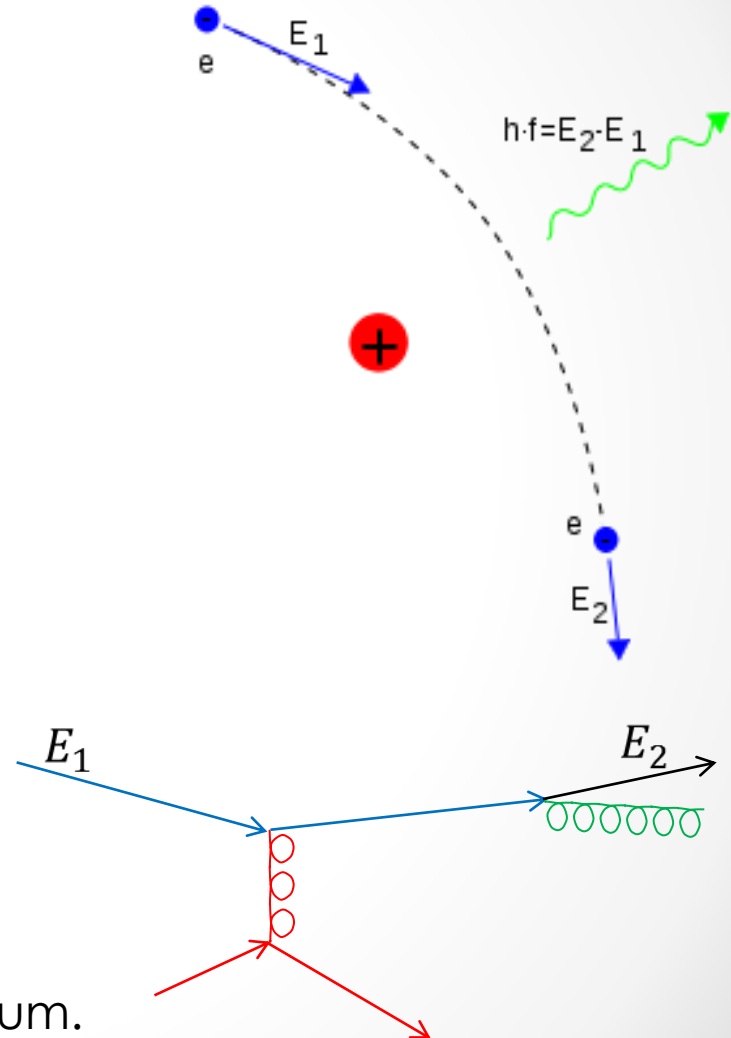
Parton Energy Loss in Weak Coupling

- We don't know whether quark-gluon plasma created in RHIC is weakly-coupled or strongly-coupled, but we can only carry out perturbative calculations in **weak coupling**.
- When high energy quarks or gluons travel through a weakly-coupled quark-gluon plasma, they lose energy, and finally equilibrate with the plasma.
- For weakly-coupled QCD plasma, energy loss is dominated by **hard bremsstrahlung** and **pair production** in the limit $E \gg T$.
- Right figure shows the only three lowest order ($\sim g$) interaction vertices in QCD.



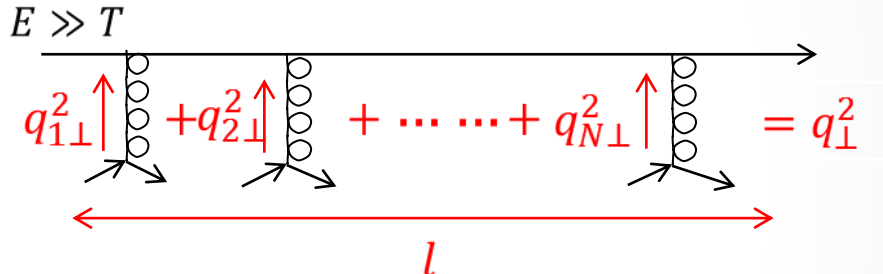
From “Collision Rate” to Bremsstrahlung

- **bremsstrahlung** refers to any radiation due to the acceleration of a charged particle.
- If quarks or gluons in the plasma kick a moving parton (through exchange a virtual gluon), then the parton will deflect from its original direction and may bremsstrahlung a final gluon.
- In order to evaluate the gluon bremsstrahlung rate, we must study the property of the medium, and find out the rate of “energy transfer” via elastic collisions with medium.



Characterization of the Medium

- We can define a **transport coefficient \hat{q}** : **average transverse momentum square transferred from medium per unit path length** for a particle travelling through the plasma, parametrically:

$$\hat{q} \sim \frac{\langle q_{\perp}^2 \rangle}{l}$$


- One of the most important parameters used to characterize the property of medium.
- \hat{q} can be defined precisely by the elastic collision rate Γ_{el} :

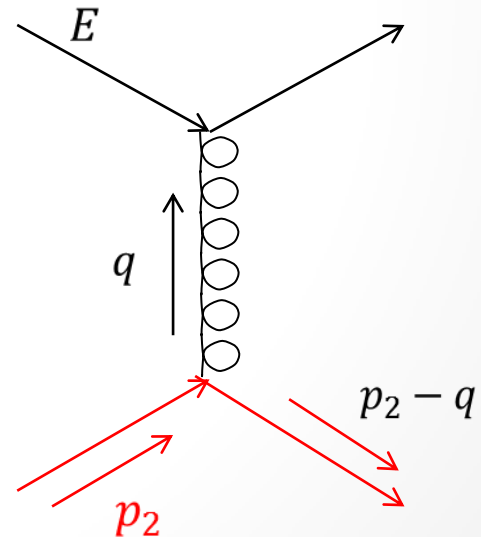
$$\hat{q} \equiv \int d^2 q_{\perp} \frac{d\Gamma_{el}}{d^2 q_{\perp}} q_{\perp}^2$$

Differential Elastic Scattering Rate

- In general, the differential rate for a high-energy particle elastically scatter from the plasma is given by:

$$\frac{d\Gamma_{el,s}}{d^2q_{\perp}} \sim \int dq_z \int d^3p_2 \frac{d\sigma_{el}}{d^3q} f(\vec{p}_2)[1 \pm f(\vec{p}_2 - \vec{q})]$$

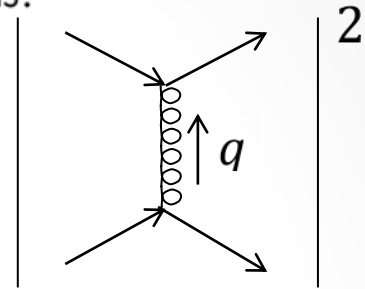
- $f(\vec{p}_2)$ is a Bose or Fermi distribution that accounts for the probability of encountering the plasma particle.
- For high-energy particles ($E \gg gT$), elastic scattering from the plasma is dominated by t -channel gluon exchange.



Integral Expression of \hat{q}

- Differential elastic scattering rate has the limit forms:

$$\frac{d\Gamma_{el,s}}{d^2q_{\perp}} \propto \begin{cases} \frac{1}{q_{\perp}^2(q_{\perp}^2 + m_D^2)}, & (q_{\perp} \ll T) \\ \frac{1}{q_{\perp}^4}, & (q_{\perp} \gg T) \end{cases}$$



- $m_D \sim gT$ is Debye Screening mass, which characterizes the modified Coulomb law: $V(r) \sim e^{-m_D r}/r$, it also acts as an infrared cutoff of q_{\perp} .
- Recall the definition of transport coefficient $\hat{q} \equiv \int d^2q_{\perp} \frac{d\Gamma_{el}}{d^2q_{\perp}} q_{\perp}^2$, which is UV divergent, we introduce a cutoff Λ for the transverse momentum transfer q_{\perp} :

$$\hat{q}(\Lambda) \equiv \int_{q_{\perp} < \Lambda} d^2q_{\perp} \frac{d\Gamma_{el}}{d^2q_{\perp}} q_{\perp}^2$$

- For the application to bremsstrahlung, there will turn out to be a natural physical cutoff scale $\Lambda \sim Q_{\perp}$, which means the total transverse momentum transfer during formation time. (roughly $Q_{\perp} \sim (\hat{q}E)^{1/4}$)

Analytical Results for \hat{q}

- When $\Lambda \ll T$, simply use the $q_\perp \ll T$ limit of $\frac{d\Gamma_{els}}{d^2q_\perp}$, integrate to get:

$$\hat{q}(\Lambda) \simeq 2C_R \alpha T m_D^2 \ln\left(\frac{\Lambda}{m_D}\right) \quad \text{when } (m_D \ll \Lambda \ll T)$$

where $\alpha = g^2/(4\pi)$ is strong coupling constant, and quadratic Casimir $C_R = 4/3$ for quarks and 3 for gluons.

- Bremsstrahlung of sufficiently high energy particles depends on $\hat{q}(\Lambda)$ for $\Lambda \gg T$. In this case, we need to find $\frac{d\Gamma_{els}}{d^2q_\perp}$ where $q_\perp \sim T$ by carrying out field theory calculation.
- we can obtain a general expression for transport coefficient $\hat{q}(\Lambda)$ when $\Lambda \gg T$. For example: in weakly-coupled 3-flavor QCD:

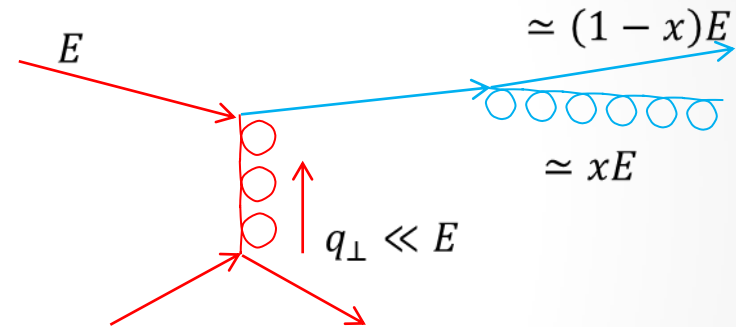
$$\hat{q}(\Lambda) \simeq \frac{C_R}{\pi^3} g^4 T^3 \left(6.31 \ln \frac{\Lambda}{m_D} + 1.09 \ln \frac{T}{m_D} - 0.52 \right)$$

Differential Rate for Parton Splitting

- Now we can find the differential rate for gluon bremsstrahlung when $E \gg T$ in weakly-coupled quark gluon plasma:

$$\frac{d\Gamma_{s \rightarrow gs}(E, T, x)}{dx} = f(E, T, x, \hat{q}(E, T), P_{s \rightarrow gs}(x))$$

- $P_{s \rightarrow gs}$ is QCD splitting function.
and we have a similar result for the rate of pair production.



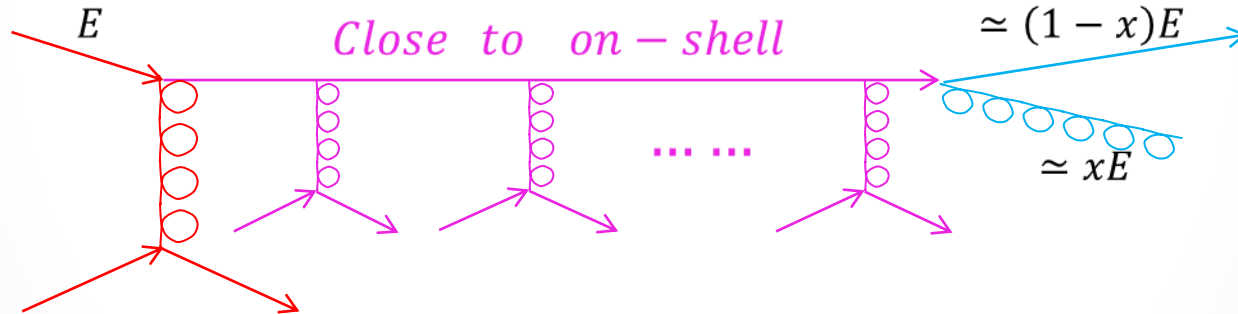
- expansion in powers of inverse log:

$$\frac{d\Gamma_{s \rightarrow gs}(E, x)}{dx} = \alpha^2 T \sqrt{\frac{\ln(E/T)}{E/T}} P_{s \rightarrow gs}(x) \mathcal{A}(x) \left[1 + \frac{\ln(\ln(E/T)) + 2\mathcal{B}(x)}{2\ln(E/T)} + \dots \right]$$

- $\mathcal{A}(x)$ and $\mathcal{B}(x)$ are from iterative definition of $\hat{q}(E, T)$, since the cutoff Λ in the integral expression of \hat{q} itself is a function of \hat{q} . ($\Lambda \sim (\hat{q}E)^{1/4}$)
I will use this expansion to calculate stopping distances.

A Closer Look of $\Gamma_{s \rightarrow gs}$ LPM Effect

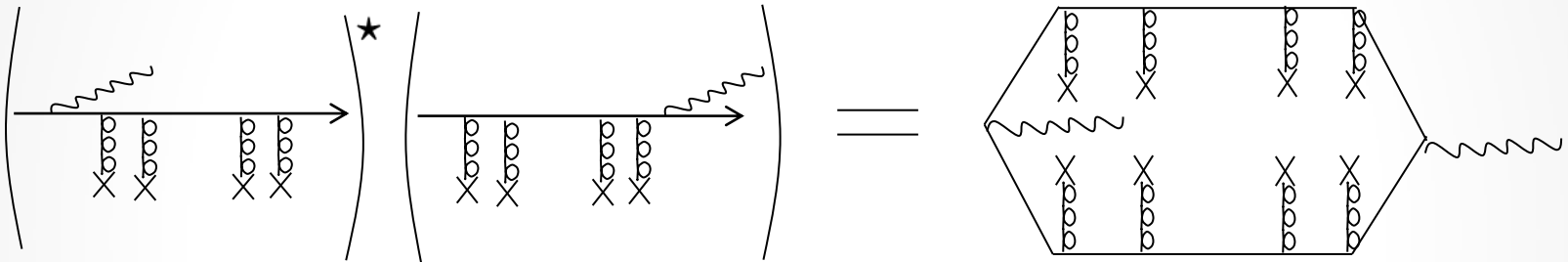
- In real world, the collision with medium particles is more complicated.
- **Landau-Pomeranchuk-Migdal(LPM) effect:** For very high energy particle, the quantum mechanical duration of the splitting process exceeds the mean free time between collisions, so successive collisions cannot be treated independently.



- When initial E is quite large, the momentum transfer q may bring the parton just a little bit off mass-shell. Then $\Delta t \sim 1/\Delta E$ may exceed the mean free path, and the parton can undergo multiple collision during bremsstrahlung.

LPM Effect

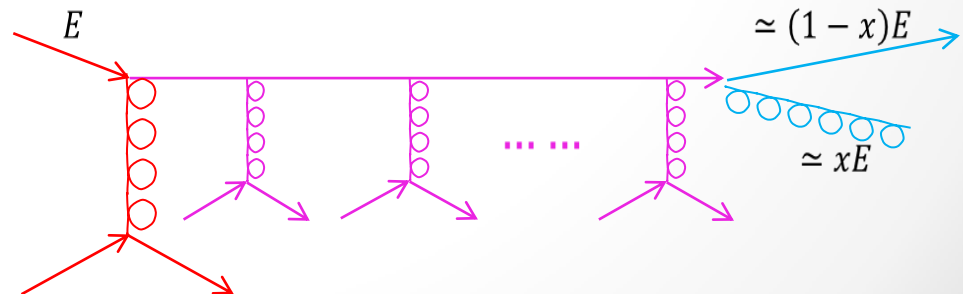
- Because of multiple collisions during the long formation time, the bremsstrahlung gluons from before and after scatterings can interfere:



- The interference will **suppress** bremsstrahlung rate when initial energy of parton is very large.

- Parametrically:

$$\Gamma_{s \rightarrow gs} \sim \alpha \cdot \alpha T \sqrt{\frac{\ln(E/T)}{E/T}}$$



Stopping Distances

- As we have obtained parton splitting rate (energy loss rate) Γ :

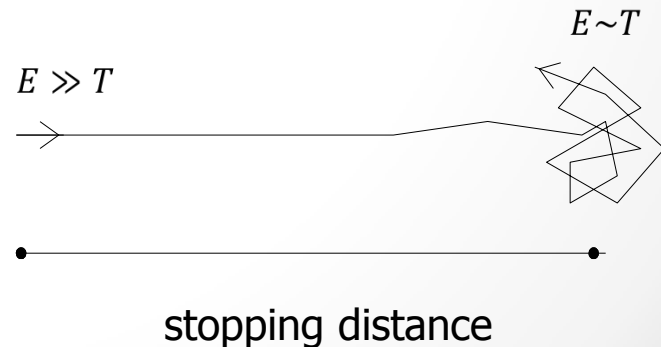
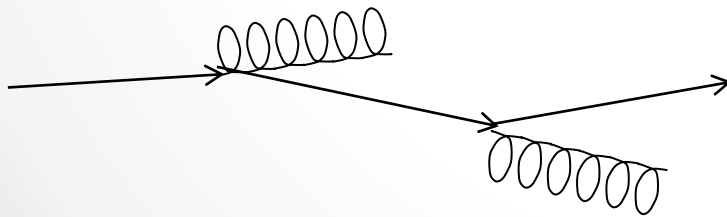
$$\Gamma \sim \alpha^2 T \sqrt{\frac{\ln(E/T)}{E/T}}$$

- We can use it to calculate parton stopping distances $l_{s,g}$ in weakly-coupled quark gluon plasmas.
- Discussion of stopping distances $l_{s,g}$ has a theoretical advantage over discussion of energy loss rates Γ . Because the stopping distances can be generalized to the case of strong coupling, where one may not speak of individual partons.

Quark Number Stopping Distances. 1

- From the QCD interaction vertices, after an original quark splits, it can only convert into a quark and a gluon, in this case, we can still follow the path of an individual quark.
- suppose we prepare an initial quark with energy $E \gg T$, throw it into the plasma then follow it's path, and repeat this trial for N time, and define the position of quark at time t as:

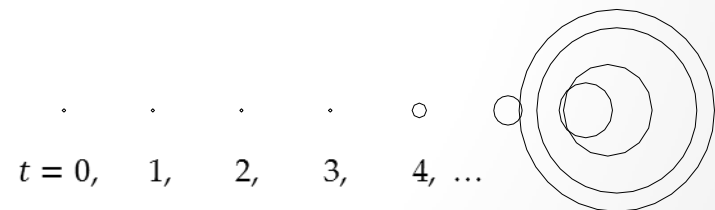
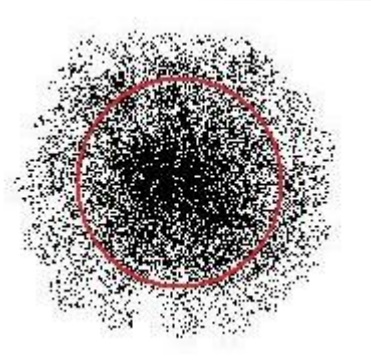
$$\bar{x}(t) = \frac{1}{N} \sum_{i=0}^N x_i(t)$$



Quark Number Stopping Distances. 2

We can also define quark number stopping in another way:

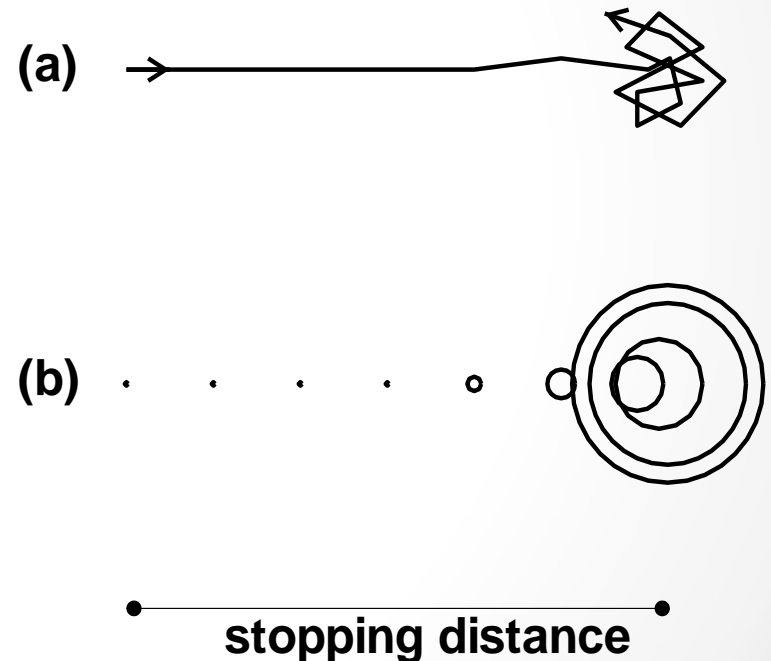
- Right figure, we plot the quark number probability distribution at time t . and make a contour to include a certain percentage of the total quarks, for example: 60% .
- As time progresses , the center of the contour moves forward, slowing down with time, and the diameter of the contour increases.



stopping distance

Quark Number Stopping Distances. 3

- To summarize the two different definitions we presented in the last two slides:
- Fig.(a) only applies to weak coupling where we can talk about individual particles.
- Fig.(b) can be generalized to strongly coupled case, since quark number density $\bar{q}\gamma^0 q$ is defined non-perturbatively.



Calculation of Stopping Distances

General Analysis for Quark Number Stopping:

- For weakly-coupled plasmas, when $E \gg T$ (parton energy \gg plasma temperature), the energy loss rate is parametrically:

$$\Gamma \sim \alpha^2 T \sqrt{\frac{\ln(E/T)}{E/T}}$$

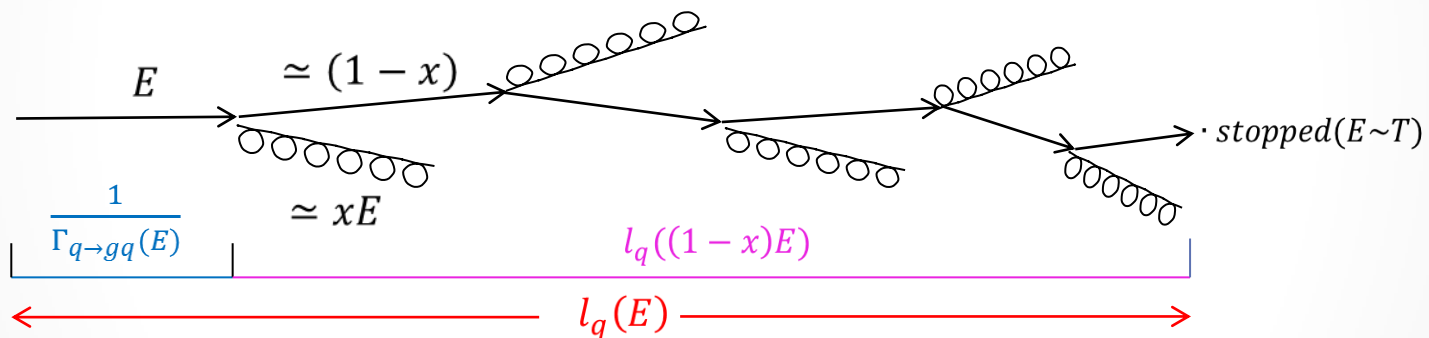
- $\alpha = g^2/(4\pi)$ is strong coupling constant.
- If we suppose in each splitting, the quark losses half of its energy, then the stopping distance is parametrically:

$$l_{stop} \sim \frac{1}{\Gamma(E)} + \frac{1}{\Gamma(E/2)} + \frac{1}{\Gamma(E/4)} + \dots \sim \frac{1}{\Gamma(E)} \sim \frac{1}{\alpha^2 T} \sqrt{\frac{E/T}{\ln(E/T)}}$$

Calculation of Quark Number Stopping

- To make a precise calculation, suppose initial quark's energy is E , while the gluon emitted from this quark carries energy xE . We have the self-consistent equation for l_q :

$$l_q(E) = \frac{1}{\Gamma_{q \rightarrow gq}(E)} + \int_0^1 dx \frac{d\Gamma_{q \rightarrow gq}(E, x)/dx}{\Gamma_{q \rightarrow gq}(E)} l_q((1-x)E)$$



- the 1st term on the right hand side represents the distance travelled before first splitting, the 2nd term is the remainder of the stopping distance after that, and $(d\Gamma/dx)/\Gamma$ is the probability that the emitted gluon has energy fraction x .

A Simple Leading Order Analysis

- For quark number stopping, the self consistent equation is:

$$l_q(E) = \frac{1}{\Gamma_{q \rightarrow gq}(E)} + \int_0^1 dx \frac{d\Gamma_{q \rightarrow gq}(E, x)/dx}{\Gamma_{q \rightarrow gq}(E)} l_q((1-x)E)$$

- Use $\Gamma_{q \rightarrow gq}(E) = \int_0^1 dx \frac{d\Gamma_{q \rightarrow gq}(E, x)}{dx}$, we can convert it into a integral equation:

$$\int_0^1 dx \frac{d\Gamma_{q \rightarrow gq}(E, x)}{dx} [l_q(E) - l_q((1-x)E)] = 1$$

- And we know from the previous slide, parametrically:

$$l_{q,stop} \sim \frac{1}{\alpha^2 T} \sqrt{\frac{E/T}{\ln(E/T)}}$$

- At leading order, we can write $l_q(E) \simeq \sqrt{E}/A_q$, plug it into integral equation for l_q . We find A_q can be expressed as an integration of x .

$$A_q = \int_0^1 dx \frac{\sqrt{E} d\Gamma_{q \rightarrow gq}(E, x)}{dx} [1 - (1-x)^{\frac{1}{2}}]$$

Next-to-Leading Logarithmic Order Analysis

- As presented in the previous slides, analytical expression for $d\Gamma/dx$ exists when $\ln(E/T)$ is large:

$$\frac{d\Gamma(E, x)}{dx} = \alpha^2 T \left(\frac{T}{E}\right)^{\frac{1}{2}} \ln^{\frac{1}{2}}(\kappa E) P_{s \rightarrow gs}(x) \mathcal{A}(x) \left[1 + \frac{\ln(\ln(\kappa E)) + 2\mathcal{B}(x)}{2\ln(\ln(\kappa E))} + \dots \right]$$

- plug it into the self consistent equation for l_q

$$\int_0^1 dx \frac{d\Gamma_{q \rightarrow gq}(E, x)}{dx} [l_q(E) - l_q((1-x)E)] = 1$$

- To NLL0, quark number stopping distance has the form:

$$l_q(E) = \frac{1}{A} \sqrt{\frac{E}{\ln(\kappa E)}} \left[1 - \frac{\ln(\ln(\kappa E)) + 2B}{2 \ln(\kappa E)} + \dots \right]$$

- A and B can be expressed as integral of $\mathcal{A}(x)$ and $\mathcal{B}(x)$, they are only functions of T , and independent of initial quark energy E .

Numerical Results for Quark Number Stopping

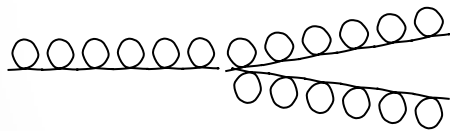
- the NLO expression for quark number stopping distance:

$$l_{stop,q} \simeq \frac{1}{a\alpha^2 T} \sqrt{E/(TL)} \quad \text{where: } L = \ln\left(\frac{bEL}{\alpha^c T}\right)$$

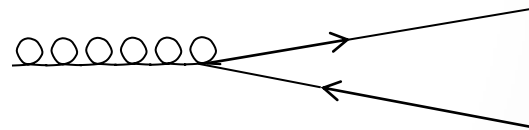
- a, b, c are numerical constants, they depend on number of flavors we take into account, and on our assumption of E and T .
- In 3-flavor QCD, the numerical constants are:
 - (1) when $E \gg T/(\alpha^2 \ln(\alpha^{-1}))$: $(a, b, c) = (2.230, 0.105, 0.346)$
 - (2) when $T \ll E \ll T/(\alpha^2 \ln(\alpha^{-1}))$: $(a, b, c) = (2.415, 0.474, 0)$
- if we take $\alpha = \frac{1}{3}$, $T = 350 \text{ MeV}$, $E = 10 \text{ GeV}$, then: $l_{stop,q} \simeq 9.72 \text{ fm}$
compare with the diameter of gold nucleon $d_{Au} \simeq 15 \text{ fm}$.

Gluon Energy Stopping. 1

- Even in the weak coupling, gluon splitting is quite different from quark splitting.
- After gluon bremsstrahlung a final gluon, or create a quark-antiquark pair through pair production, we don't know how to follow an individual gluon's path.



which gluon to follow?



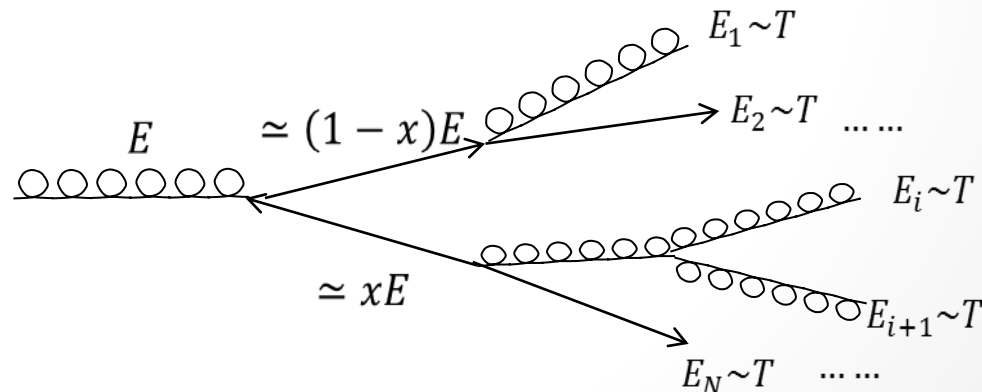
no gluon to follow!

- Try to take all the particles cascaded from an initial gluon into account.

Gluon Energy Stopping. 2

- Consider the probability distribution for energy in excess of equilibrium (rather than quark number), we can measure how far it travels before stops. This is so called “**gluon energy stopping distance**”.
- For a weakly-coupled plasma, consider the splitting of an initial gluon moving in the z direction, which cascades through splitting into N particle, which are “stopped” at positions z_1, z_2, \dots, z_N relative to the initial position of the gluon, then:

$$l_{stop,g}^{(energy)} \simeq \frac{\sum_i E_i z_i}{E}$$



Calculation of Gluon Energy Stopping

- A gluon may split into a quark-antiquark pair, so when calculate gluon energy stopping, we have to take quark energy stopping into account.
- we can write the self-consistent equations of gluon energy stopping and quark energy stopping.

$$l_g^{(e)}(E) = \frac{1}{\Gamma_g(E)} + \int_0^1 dx \left\{ \frac{1}{2} \frac{d\Gamma_{g \rightarrow gg}(E, x)/dx}{\Gamma_g(E)} \left[x l_g^{(e)}(xE) + (1-x) l_g^{(e)}((1-x)E) \right] \right. \\ \left. + \frac{d\Gamma_{g \rightarrow q\bar{q}}(E, x)/dx}{\Gamma_g(E)} \left[x l_q^{(e)}(xE) + (1-x) l_q^{(e)}((1-x)E) \right] \right\}$$

$$l_q^{(e)}(E) = \frac{1}{\Gamma_{q \rightarrow gq}(E)} + \int_0^1 dx \frac{d\Gamma_{q \rightarrow gq}(E, x)/dx}{\Gamma_{q \rightarrow gq}(E)} \left[x l_g^{(e)}(xE) + (1-x) l_q^{(e)}((1-x)E) \right]$$

- The gluon splitting rate is $\Gamma_g \equiv \Gamma_{g \rightarrow gg} + \Gamma_{g \rightarrow q\bar{q}}$, considering both gluon bremsstrahlung and pair production processes.

Results for Energy Stopping Distances

- The principle for calculation is the same as quark number stopping, but more complicated, we found to **leading logarithmic order**, their analytical results have the similar form as quark number stopping:

$$l_s^{(e)} \simeq \frac{1}{a_s^{(e)} \alpha^2 T} \sqrt{\frac{E}{TL}}, \quad L = \ln \left(\frac{E}{\alpha^c T} \right)$$

- In 3-flavor QCD, the numerical constants are:

(1) when $E \gg T/(\alpha^2 \ln(\alpha^{-1}))$:

$$(a_g^{(e)}, a_q^{(e)}, c) = (6.272, 3.875, 0.346)$$

(2) when $T \ll E \ll T/(\alpha^2 \ln(\alpha^{-1}))$:

$$(a_g^{(e)}, a_q^{(e)}, c) = (6.793, 4.197, 0)$$

- if we take $\alpha = \frac{1}{3}$, $T = 350 \text{ MeV}$, $E = 10 \text{ GeV}$, then:

$$l_g^{(e)} \simeq 8.36 \text{ fm}, \quad l_q^{(e)} \simeq 13.53 \text{ fm}, \quad \text{compare with } l_{\text{stop},q} \simeq 9.72 \text{ fm}$$

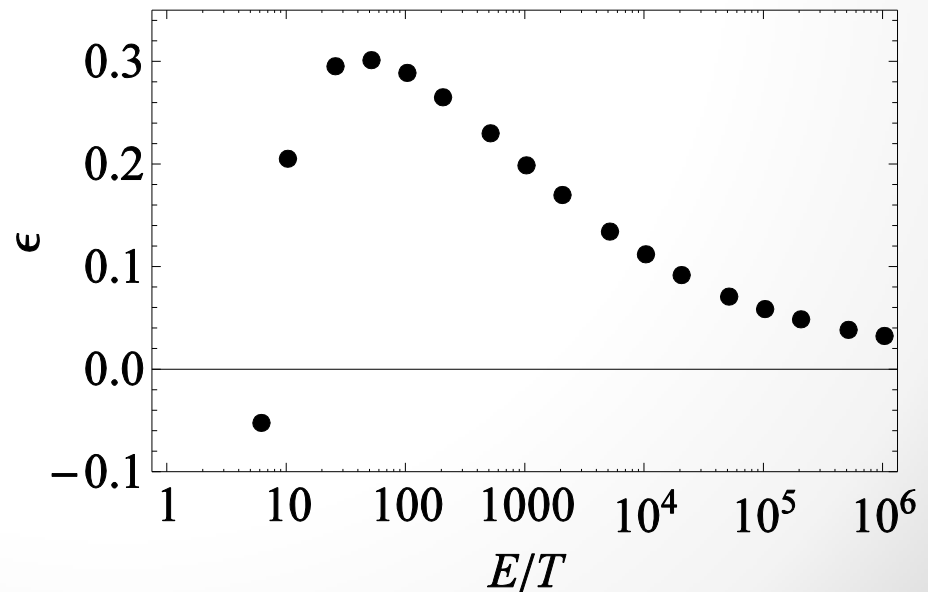
Numerical Test of the Result

- How precise is our analytical result in high E ? Can the analytical expressions of l_{stop} give reasonable numerical results when the parton energy E is not so large compared to plasma temperature T ?
- We carry out Monte Carlo evolution of a large sample of quarks with energy E , using the full weak-coupling bremsstrahlung rate $d\Gamma/dx$ to randomly determine whether each quark lose energy xE in each small time step Δt :

- The right figure shows the relative error

$$\epsilon = 1 - \frac{l_{nlo}}{l_{numeric}}$$

as a function of E/T .



Discussions and Conclusions

- Quantitatively: we found UV-regulated value of \hat{q} in weak coupling, and used this value to generalized gluon bremsstrahlung and pair production rates for massless high-energy particles in a weakly-coupled quark-gluon plasma.
- Quantitatively: we derived simple formulas for parton stopping distances in weakly-coupled QCD plasma, they behave like $\sqrt{E/\ln(E)}$ in the high energy limit.
- Contrast: stopping distances in strong coupling is calculated by Chesler and his collaborators (arXiv: 0810.1985 [hep-ph])

$$l_{stop,q} \approx \frac{0.5}{T} \left(\frac{E}{T\sqrt{\lambda}} \right)^{\frac{1}{3}} \quad (\lambda \equiv N_c g_s^2)$$

stopping distance for a colored excitation(represented by a classical string) in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in large N_c limit.

Thank You