

# QCD in Five Dimensions

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# Introduction

- ▷ Not really 5d-QCD, but a way of performing first principles 4-dimensional QCD calculations, that uses a fifth dimension as a trick:
  - ▷ Domain Wall Fermions(DWF) (a type of lattice QCD)
- ▷ I will pick one particular quantity to display the advantage of this approach  $B_K$  - the kaon B-parameter.
  - ▷ Phenomenologically Interesting for constrain the standard model
  - ▷ “Easy” with Domain Wall Fermions

# The Standard Model

The standard model of particle physics involves 12 fermionic particles:

$$\begin{array}{c}
 \text{leptons} \\
 \left( \begin{array}{c} \nu_e \\ e^- \end{array} \right) \quad \left( \begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right) \quad \left( \begin{array}{c} \nu_\tau \\ \tau^- \end{array} \right) \quad \Bigg| \quad \begin{array}{c} \text{quarks} \\ \left( \begin{array}{c} u \\ d \end{array} \right) \quad \left( \begin{array}{c} c \\ s \end{array} \right) \quad \left( \begin{array}{c} t \\ b \end{array} \right)
 \end{array}$$

$$\begin{array}{c}
 Q \\
 +\frac{2}{3}e \\
 -\frac{1}{3}e
 \end{array}$$

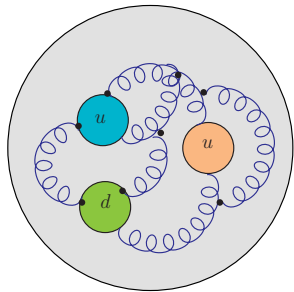
and 3 types of – force carrying – bosons:

$$\begin{array}{c}
 \text{EM} \\
 \text{Weak} \\
 \text{Strong}
 \end{array}
 \begin{array}{c}
 \gamma \\
 W^\pm, Z^0 \\
 g
 \end{array}
 \begin{array}{c}
 \text{photon} \\
 \text{vector bosons} \\
 (8) \text{ gluons}
 \end{array}$$

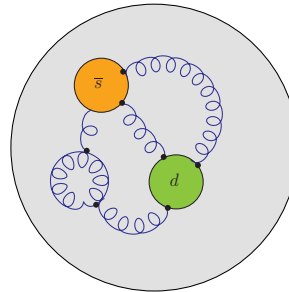
and one – unobserved – scalar boson: the Higgs.

# Asymptotic freedom + Confinement

- ▷ 2004's Nobel Prize was awarded to Gross, Politzer and Wilczek for the discovery of Asymptotic Freedom.
  - ▷ For high energy (short distance) processes the quarks look weakly coupled : perturbation theory makes sense
  - ▷ For low energy (long distance) processes the Strong force is justly named : higher order processes not suppressed.
- ▷ We live in a low energy world. In fact, the Strong force is so strong that free quarks are never observed, just bound states:



- ▷ Baryons: 3 quarks
  - ▷ Proton
  - ▷ Neutron...



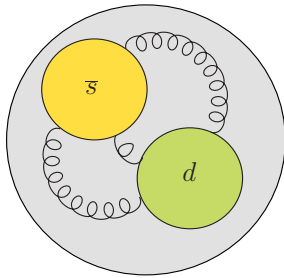
- ▷ meson: quark + anti-quark
  - ▷ pions
  - ▷ kaons...

- ▷ Lattice QCD provides both a definition of the Strong(QCD) force beyond perturbation theory, and a practical method of calculating it's predictions.

# This talk

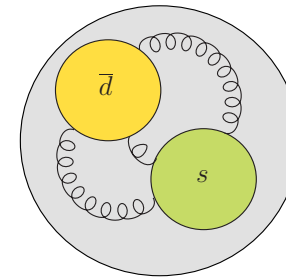
In this talk I describe some of the advances made in lattice QCD in recent years, taking the particular example of a calculation of CP-violating effects in the neutral Kaon system.

- ▷ Kaons are mesons.



- ▷ The  $K^0$  is the mesonic state containing a d-quark and a s-antiquark.

- ▷ The  $\overline{K}^0$  is the mesonic state containing a s-quark and a d-antiquark.



- ▷ C and P are symmetries broken by the Weak force.

Plan of talk: C and P, the Weak force, CP-violation in the kaon system, lattice QCD.

# C

## What is CP?

C, P (and T) are potential symmetries of particle physics

- ▷ C stands for Charge Conjugation
  - ▷ Relativistic quantum field theory requires every particle to have an anti-particle : opposite quantum numbers ( charge, etc... )
  - ▷ charge conjugation interchanges each particle with it's anti-particle

$$C|e^{-}\rangle = |e^{+}\rangle; C|\gamma\rangle = -|\gamma\rangle$$

- ▷ Note: some particles are there own anti-particles.

# P

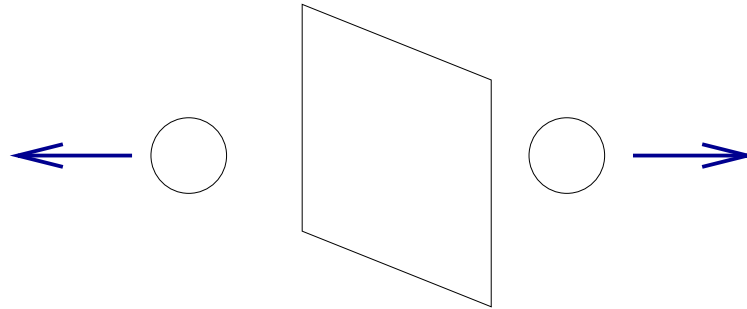
- ▷ P stands for Parity
- ▷ Maps to the “mirror image” world
  - ▷ flips both **spatial** co-ordinates and **momenta**,

$$\vec{p} = \frac{d\vec{x}}{dt}$$

but leave the **spin** (intrinsic angular momentum;  $\pm 1/2$ )

$$\vec{J} \sim \vec{L} = \vec{x} \times \vec{p}$$

the same



**helicity** – component of spin in direction of motion – **flips**

$$+\frac{1}{2} \equiv \text{right-handed}; -\frac{1}{2} \equiv \text{left-handed}$$

# CPT

- ▷ T stands for Time Reversal
- ▷ Reverses momenta, and flips spins; leaves positions the same.
- ▷ cf. non-relativistic QM: Time reversal is anti-unitary operator

$$e^{iHt} \rightarrow e^{-iHt}$$

complex phases are not T invariant.

CPT:

The **Standard Model** is invariant under the combined symmetry of **CPT**, where **T** is time reversal (true for any relativistic quantum field theory).

$$\text{Break CP} \Leftrightarrow \text{Break T}$$



## C, P breaking

- ▷ Electromagnetic and Strong forces are individually symmetric under C, P and T.
- ▷ The Weak force breaks P [Lee and Yang (Columbia), 1957]
  - ▷ first measured in  $\beta$  decay of polarized Cobalt [Wu *et al*, 1957]

Still a chance that CP was not violated:

- ▷ 1964 Fitch and Cronin (working at BNL) detected a tiny CP violating effect in neutral K decays.
  - ▷ Better be the case: CP-violation needed to produce matter-antimatter asymmetry (although levels of CP-violation observed seem too small to explain this).

## Weak Force

- ▷ The only force mediated by **massive** force carriers.

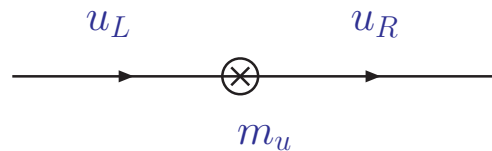
$W^+$	$80.4 \text{ GeV}/c^2$
$W^-$	$80.4 \text{ GeV}/c^2$
$Z^0$	$91.187 \text{ GeV}/c^2$

Here masses are given in terms of energies ( $E = mc^2$ ):

- ▷ To give some point of reference : Mass of a proton  $\approx 1\text{GeV}/c^2$
- ▷ Later in the talk we'll be discussing physics involving the lightest mesons:  
**Pions** and **Kaons**
  - ▷ Mass of **pions** ( $\pi^\pm, \pi^0$ )  $\approx 100 \text{ MeV}/c^2$  and **kaons** ( $K^\pm, K^0, \overline{K}^0$ )  $\approx 500 \text{ MeV}/c^2$ .
- ▷ Much lower energy scales than the mass of the **Weak** bosons; the **Weak** force is very short range.

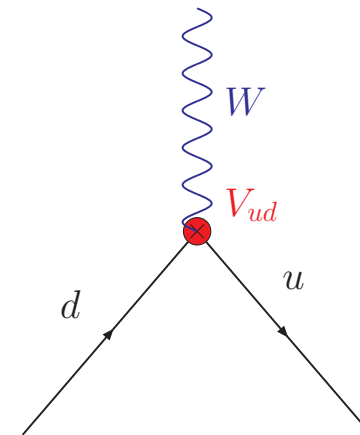
# Weak Force

- ▷ Each quark can occur in two chiralities: **left-** and **right-** handed
  - ▷ If the left- and right-handed particles are completely independent then the theory is said to be **chirally symmetric**.
  - ▷ Chiral symmetry is broken by the masses of the particles:



- ▷ As for **helicities**, **Parity** invariance implies physics same for **left-** and **right-** handed chiralities.
- ▷ As mentioned before the quarks come in different flavours:

Quarks			$Q$
$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{2}{3}e$
			$-\frac{1}{3}e$



the **Weak** force allows quark flavours to mix

# CKM

- ▷ The **Weak** interaction effects only **left-handed** particles and **right-handed** anti-particles

$$\mathcal{L}_{\text{int}} \propto (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + \text{HC}$$

and so maximally breaks **P** and **C**.

- ▷ **Cabibbo-Kobayashi-Maskawa** mixing matrix encodes the possible mixings:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- ▷ This is a  $3 \times 3$  **unitary matrix** and so has **9** free parameters (3 angles, 6 phases). Can remove **5** phases by global redefinitions of the quark fields:
  - ▷ 1 phase left over: **complex phase** means **T violation**, which means **CP violation**
  - ▷ These remaining 4 parameters are **fundamental** parameters in the **Standard Model**

## CKM matrix...

- ▷ A common approximation to the CKM matrix is the Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

- ▷ Not unique, but encodes the experimental reality that elements get smaller as you move off the diagonal
  - ▷  $\lambda \sim 0.23$
- ▷ All standard model CP violation enters through the parameter  $\bar{\eta}$  (suppressed by three factors of  $\lambda$ )
- ▷ The calculation I will discuss later constrains the relationship between  $\bar{\rho}$  and  $\bar{\eta}$ .

# What I'm trying to do

CP-violation discovered in 1964.  
Why do we still care?

- ▷ See CP-violation in nature.
- ▷ Have CP-violation in SM.



- ▷ Difficult to move between the CKM matrix and a prediction of experimental results. Here I give one example of such a calculation.
- ▷ CP-violation controlled by a single parameter:
  - ▷ Over-constraining the CKM matrix could give evidence of new physics.

## The Neutral Kaon system

- ▷ If there was no **Weak** force, all forces would preserve **flavour** and the  $K^0$  and  $\overline{K}^0$  mesons would be stable (**negative parity**) particles. CP takes me between the two:

$$CP|K^0\rangle = -|\overline{K}^0\rangle$$

can arrange in **CP eigenstates**

$$|K_{\text{even}}\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle - |\overline{K}^0\rangle \right); CP+$$
$$|K_{\text{odd}}\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle + |\overline{K}^0\rangle \right); CP-$$

- ▷ CP is broken; Actually observe

$$|K_S\rangle = \frac{|K_{\text{even}}\rangle + \bar{\epsilon}|K_{\text{odd}}\rangle}{\sqrt{1 + |\bar{\epsilon}|^2}}$$
$$|K_L\rangle = \frac{|K_{\text{odd}}\rangle + \bar{\epsilon}|K_{\text{even}}\rangle}{\sqrt{1 + |\bar{\epsilon}|^2}}$$

- ▷  $|K_S\rangle$  almost CP-even;  $|K_L\rangle$  almost CP-odd

## The Neutral Kaon system...

- ▷ look at decay into the (CP-even)  $|\pi\pi\rangle$  state.
- ▷ If you measure  $K_L \rightarrow \pi\pi$  there are two contributions
  - ▷ indirect CP-violation: due to  $K_{\text{even}}$  component ( $\epsilon$ )
  - ▷ direct CP-violation: due to weak decay ( $\epsilon'$ )
- ▷  $\epsilon' \ll \epsilon$  (whole other talk),

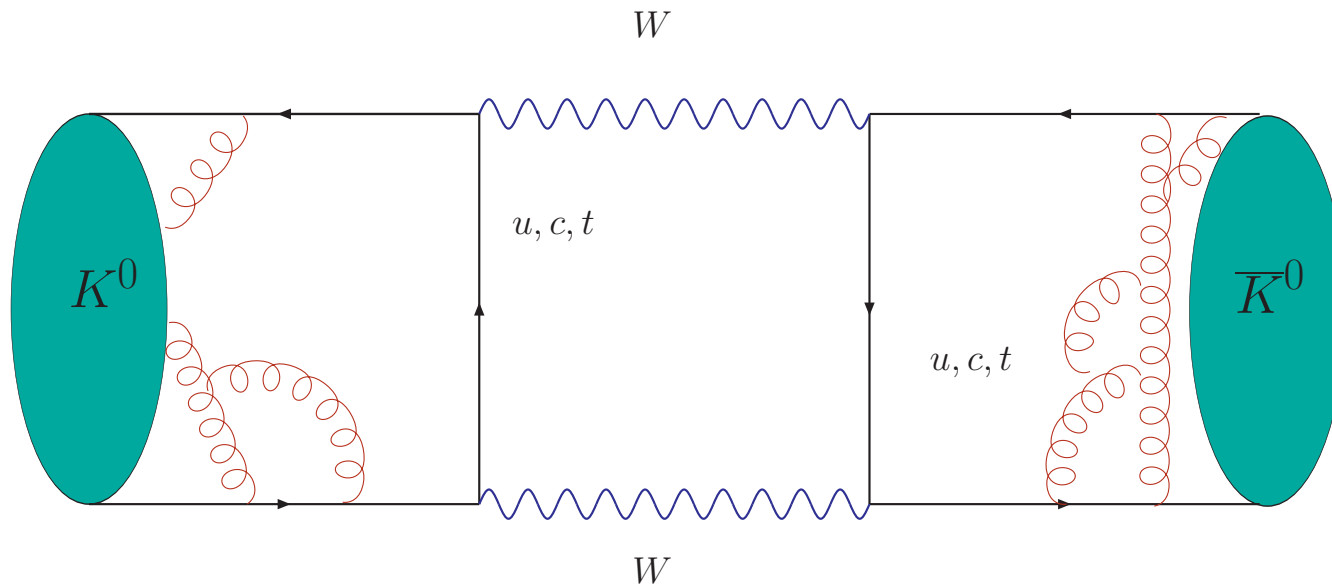
$$\epsilon_K = \frac{A(K_L \rightarrow \pi\pi)}{A(K_S \rightarrow \pi\pi)}$$

- ▷ Experimentally
  - ▷  $\epsilon = 2.271(17) \times 10^{-3}$
  - ▷  $\epsilon'/\epsilon$ 
    1. KTEV (FNAL 2001) :  $20.7(2.8) \times 10^{-4}$
    2. NA48 (CERN 2001) :  $15.3(2.6) \times 10^{-4}$



# Matter-antimatter Oscillations

Diagrammatically the CP-violating mixture arises from the oscillation between  $K^0$  and  $\bar{K}^0$  mediated by:



- ▷ Weak interactions: 80GeV ; Strong interactions: 1GeV
- ▷ Approach: use the operator product expansion and renormalisation group.

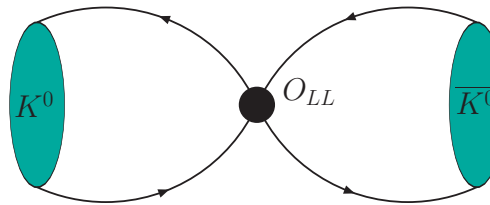
$$B_K \dots$$

- ▷ **Weak** particle much heavier than the scales at which we work:
  - ▷ Replace by **effective** point interaction (**Higher order terms**  $\sim p^2/M_W^2$ )
  - ▷ Continue this for all particle which are high energy enough that perturbation theory sensible (top, bottom, charm quarks; not up, down or strange).
  - ▷ Leave low energy excitations ( $\sim 1\text{GeV}$ ) to the lattice calculation

$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \bar{\eta} \left[ -\eta_1 S(x_c) + \eta_2 S(x_t) (A^2 \lambda^4 (1 - \bar{\rho}) + \eta_3 S(x_c, x_t)) \right] \hat{B}_K$$

- ▷ need to calculate this on the lattice:

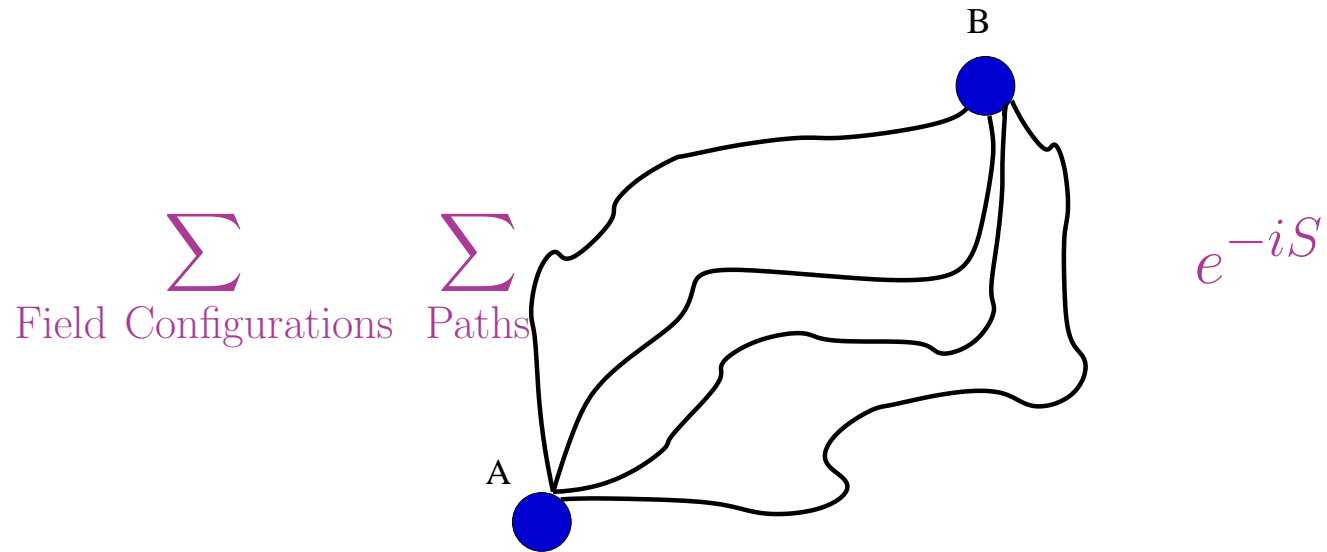
$$B_K = \frac{\langle \bar{K}^0 | O_{LL} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$



defined in some **renormalisation scheme** at some **scale**.

# Path Integrals

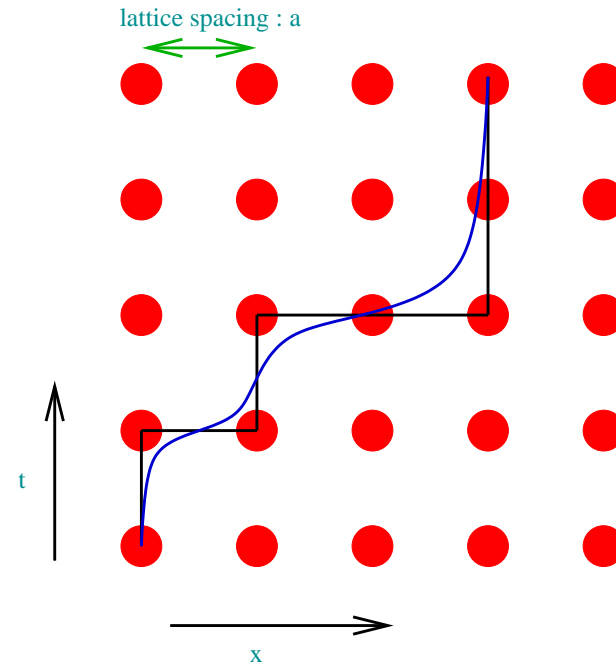
Physical observables in QFT can be calculated in the path integral formulation:



$S$  is classical action.

# Lattice QCD

- ▷ Lattice QCD : Discretise QCD on a **four-dimensional**, Euclidean, space-time lattice
  - ▷ quarks live on sites
  - ▷ the links between the sites encode the gluons
- ▷ Introduce a discretised Dirac equation, coupled to gluons.
- ▷ This provides:
  1. A **regularisation** of QCD.
  2. A way to calculate it's **predictions** using a computer.
    - ▷ path integral  $\rightarrow$  sum of integrals over links



# Numerical Simulation: Summing over Field Configurations

$$Z = \int dU e^{-\beta S}$$

$10^4$  lattice :

- ▷  $10^4 \times 4 \times 8 = 320,000$  dimensional integral
- ▷ 2 points/dimension (two possible values of force per point)

$$2^{320,000} \sim 10^{96,000} \text{ terms}$$

- ▷ age of the universe  $\sim 10^{27}$  nanoseconds (currently using lattice sizes of  $32^3 \times 64$ ).

Need a better way to calculate the answer:

- ▷ Importance sampling:
  - ▷ Generate an ensemble of “typical” configurations

$$P(C) \propto e^{-\beta S}$$

- ▷ Use a Markov process:  $O(50)$  “independent” configurations needed to start to get results.

## Numerical Simulation: Summing over Paths

- ▷ For a given configuration of gluons, we need to sum over all paths the quarks may take:
- ▷ The quark's physics is described by a modified Dirac Operator.
- ▷ Summing over all paths is equivalent to inverting the discretised form of this Dirac Operator
  - ▷ a matrix of size 10's of millions  $\times$  10's of millions
  - ▷ can only approximate the solution to this.
  - ▷ the smaller the quark masses, the longer it takes.
  - ▷ this is what the supercomputers used in lattice QCD spend most of their time doing.

# Systematic Errors in Lattice Calculations

Still a hard problem. Just some of the **systematic errors** that have to be deal with are:

- ▷ Finite Volume
  - ▷ Want – at least – two volumes
- ▷ Unphysical quark masses [**The lighter the mass, the more expensive**]
  - ▷ May not be able to get down to the up and down quark masses.
  - ▷ But use many different values.
  - ▷ Extrapolate to physical point using **effective theory** for mass dependence of low energy QCD: **Chiral Perturbation Theory**.
- ▷ Finite Lattice Spacing
  - ▷ Choose different ways of discretising the Dirac equation (and gluon action).
  - ▷ Extrapolate from multiple lattice spacings.

# Discretising QCD

- ▷ Various ways to discretise QCD
  - ▷ all (**should**) be the same in **continuum limit**.
  - ▷ different trade-offs at finite lattice spacing

Two traditional Fermionic Actions:

## 1. “Staggered Fermions”:

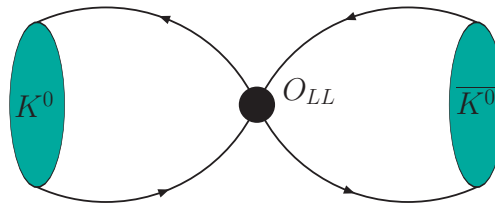
- ▷ Cheap, but have extra flavours of quark (**tastes**); **taste** mixing problem.
- ▷ “**Improved**” staggered fermions recently been very successful.

## 2. “Wilson Fermions”:

- ▷ Exact flavour symmetry, (**badly**) broken chiral symmetry
  - ▷ characteristic energy scale of chiral symmetry breaking a few **GeV**, not a few **MeV**



## Calculating $B_K$



- ▷ If you have exact **chiral** and **flavour** symmetry at finite lattice spacing, then there is only one operator that contributes to  $B_K$ . It is of the form:

$$O_{LL} = \bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma_\mu d_L$$

- ▷ If **chiral symmetry** is **broken** four other operators may mix.

$$\langle \bar{K}^0 | O_{LL} | K^0 \rangle = Z_{11} \langle \bar{K}^0 | O_{LL} | K^0 \rangle_{\text{latt}} + \sum_{i \geq 2} z_{1i} \langle \bar{K}^0 | O_{\text{MIX},i} | K^0 \rangle_{\text{latt}}$$

In principle I have to calculate these **five** separate quantities and then take just the right combination so that all chiral symmetry breaking effects cancel.

$B_K \dots$

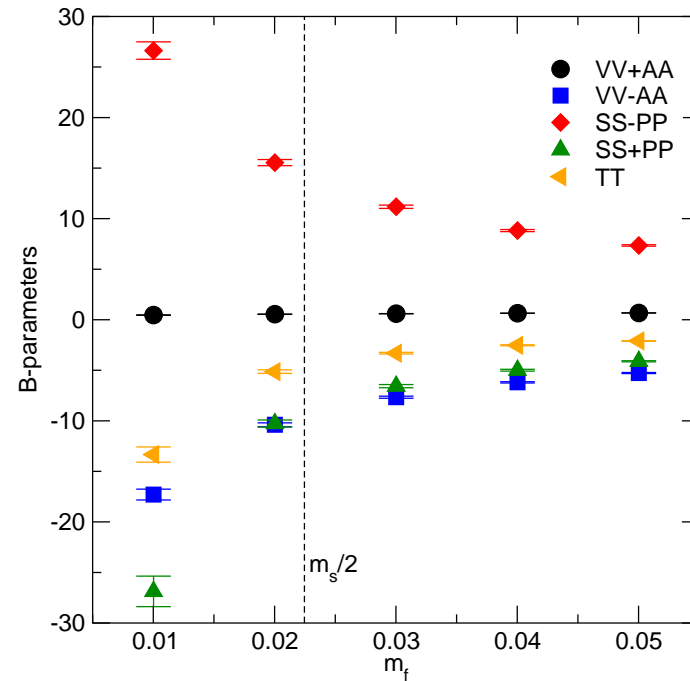
- ▷ First order chiral perturbation theory predicts that

$$\langle \bar{K}^0 | O_{LL} | K^0 \rangle \propto M_k^2$$

and, **unfortunately**, that

$$\langle \bar{K}^0 | O_{\text{THE REST}} | K^0 \rangle \propto 1$$

so... as the chiral limit is approached the wrong chirality operators will **dominate**.

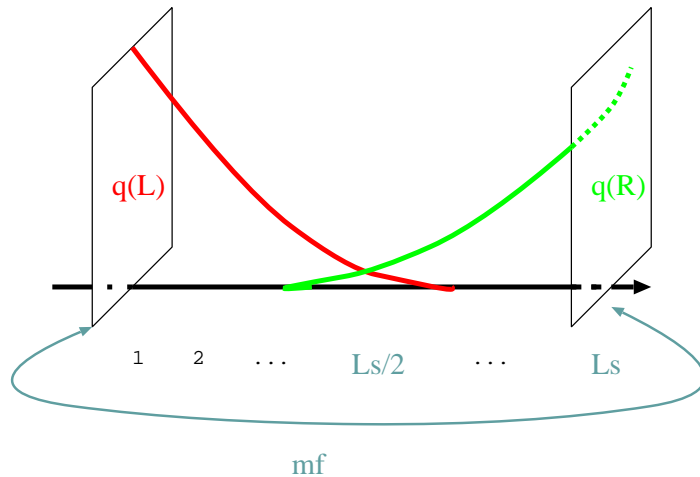


- ▷ Conclusion: **need good chiral symmetry**.

$$B_K \dots$$

- ▷ If **flavour symmetry** is broken (**Staggered fermions**) many other operators may mix
  - ▷ Rough estimate is  $16^4$  possible operators
  - ▷ Working to first order in perturbation theory only four are **commonly** included.
    - ▷ The **dominant** source of error is from neglecting higher orders
- ▷ Conclusion: **would really like good flavour symmetry.**
- ▷ Domain Wall fermions provide both.

# Domain Wall Fermions



▷ Lattice fermions traditionally break either **flavour** or **chiral** symmetry.

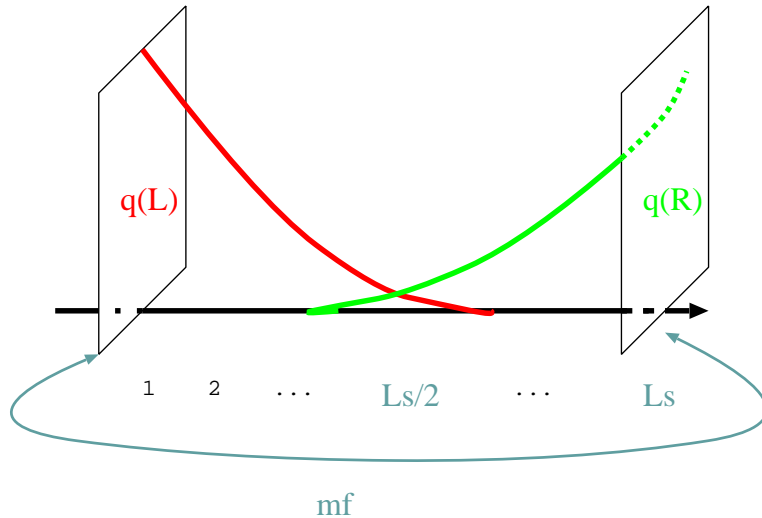
▷ **Domain Wall Fermions** preserve **flavour** symmetry and have **greatly reduced** chiral symmetry breaking.

▷ at the **expense** adding a extra, **fifth**, dimension.

▷ The nearest neighbour derivative in the 5th dimension distinguishes left- and right- handed fermions

$$\begin{aligned}
 & -\gamma_\mu \frac{1}{2} (\nabla_\mu^+ + \nabla_\mu^-) + \frac{1}{2} \nabla_\mu^- \nabla_\mu^+ + M_5 && \text{4d piece} \\
 & + P_L \partial_5^+ - P_R \partial_5^- && \text{5d piece}
 \end{aligned}$$

# Domain Wall Fermions



- ▷ Define 4d quark fields on the wall

$$q_x = P_L \Psi_{x,0} + P_R \Psi_{x,L_s-1}$$

- ▷ Couple the two walls with a mass term

$$m_f \bar{q} q$$

- ▷ For finite  $L_s$  **chiral symmetry is broken**, leading to an additive shift of the mass

$$m_f \rightarrow m_f + m_{\text{res}}$$

- ▷  $m_{\text{res}} \rightarrow 0$  as  $L_s \rightarrow \infty$  ; The **cost** in computer time  $\propto L_s$ 
  - ▷ Need small  $m_{\text{res}}$  (**few MeV**) for reasonable  $L_s$  ( $O(10)$ )

## Setting the scale

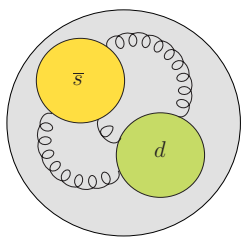
- ▷ The lattice action is (**necessarily**) dimensionless.
  - ▷ This is also the case for anything calculated on the lattice.
  - ▷ Can connect to dimensionful **continuum** fields by putting in factors of the lattice spacing ( $a$ ).
    - ▷ When a pion mass (**say**) is calculated on the lattice get  $aM_\pi$  not  $M_\pi$ .
- ▷ Need to compare to one dimensionful parameter from experiment to extract the scale.
  - ▷ A **Corollary** to this is that we don't know the lattice spacing till after we have collected many configurations (**just set the gauge-coupling and input masses**).
  - ▷ Should be able to choose any quantity.

## Strange quark point

If I want to calculate  $B_K$ , then I have to know what masses to put in for the **strange** (and **up/down**) quark masses.

- ▷ These aren't well known quantities: masses smaller than scale at which perturbation theory works. (need to work them out using LQCD).

**Simple answer:**  $K^0$  is made up of **d-quark** and a **s-antiquark**.



- ▷ The **correct** s and d quark masses are those that give the physical value for the  $K^0$  mass.
  - ▷ The **correct** u and d quark masses are those that give the physical values for the pion masses.
  - ▷ Use the pion, kaon and omega-baryon as inputs to set the scale, and up, down and strange quark masses.
  - ▷ All other quantities calculated are predictions
- Problem:** Not sure what masses of d-quark is exactly, but sure it's **much less** than any mass I can simulate on this lattice.
- ▷ → Chiral Perturbation Theory

# Chiral Perturbation Theory

- ▷ Effective theory describing low energy QCD
  - ▷ expansion about the **zero quark mass** limit (stops working for quark masses which are **“too heavy”**).
  - ▷ degree's of freedom are pions and kaons, not quarks and gluons...
- ▷ For the lightest pseudo-scalar mesons (for example the  $K^0$ ), to first order

$$M_{PS}^2 = (2B_0) (m_{Q1} + m_{Q2})$$

- ▷ Realistically we work to next-to-leading order (NLO).
  - ▷ NLO: 4 coefficients
  - ▷ NNLO:  $\sim 20$
- ▷ How Heavy is **“too heavy”**?



# Calculating a mass on the Lattice

- ▷ Put an operator with right quantum numbers to be a **Kaon** at timeslice 0 (say), and study it's propagation to timeslice  $T$



- ▷ Standard Time evolution operator

$$e^{iHT}$$

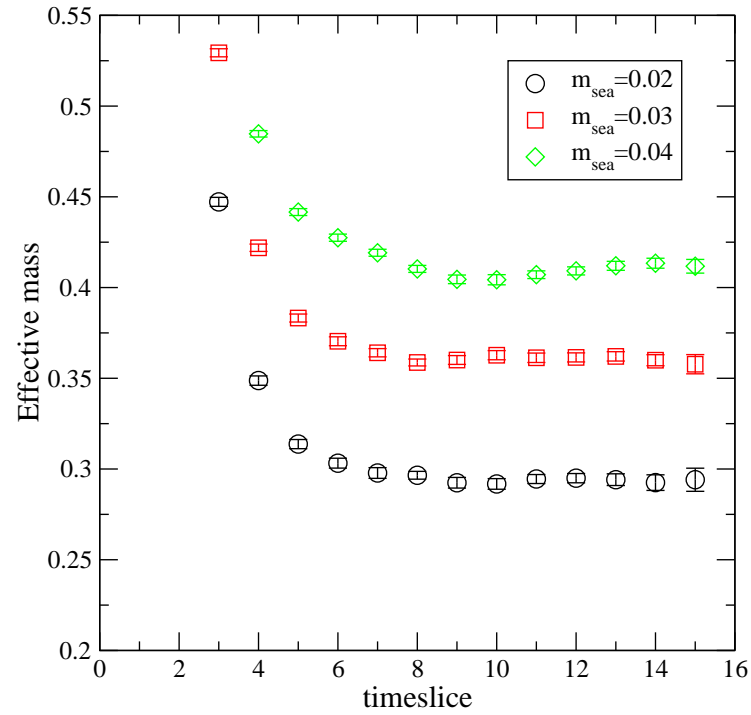
- ▷ Working in Euclidean Space via "**Wick Rotation**":  $T \rightarrow iT$

$$e^{-HT}$$

- ▷ **Large times**: lowest energy state dominates. For particles at rest, this is just the mass:

$$e^{-HT} \equiv e^{-M_K T}$$

# Effective Mass



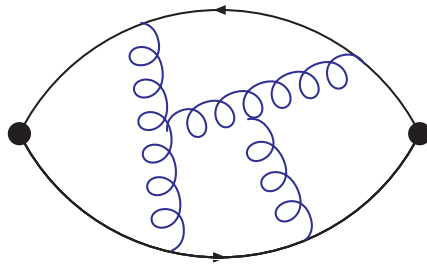
$$C(T) = \langle K(0) \overline{K}(T) \rangle \sim A e^{-M_K T} ; M_K = -\ln \frac{C(T+1)}{C(T)}$$

# Quenched Approximation

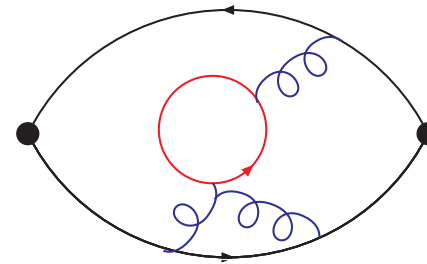
Until a few years ago most lattice calculation were performed in the **Valence** or **Quenched** approximation:

- ▷ Quarks propagate in background gauge field.
- ▷ In perturbative language:

include:



neglect:



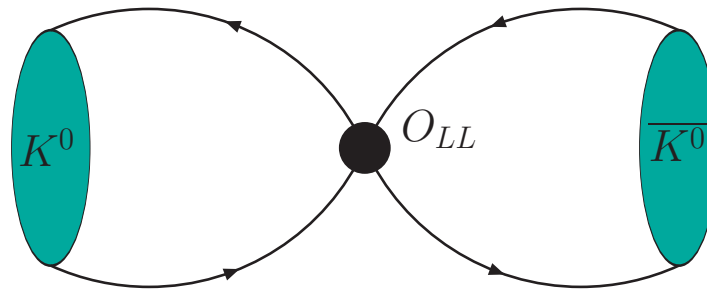
$O(100)$  times less computationally demanding than full QCD.

- ▷ This is an uncontrolled approximation: **the only way we know how to do better, is not to do it at all.**

## Approach

I'm going to show some RBC results in the quenched approximation from two different lattice spacings..

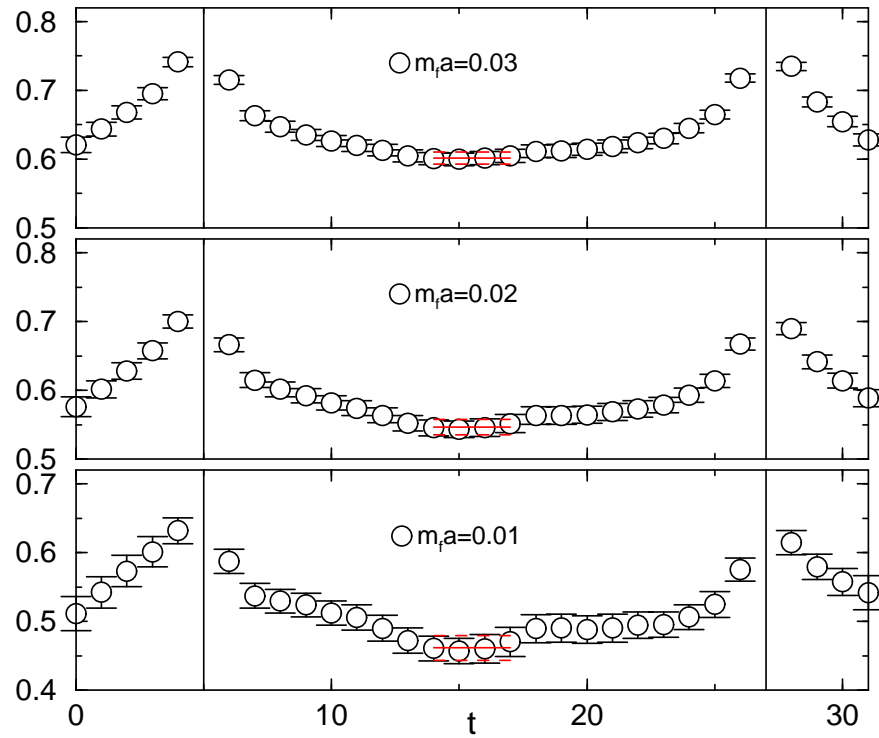
- ▷  $a^{-1} = 2 \text{ GeV}$  ( $L_s = 16$ ) and  $3 \text{ GeV}$  ( $L_s = 10$ )
- ▷  $\sim 1.5\text{fm}^3 \times 3\text{fm}$  box
- ▷ degenerate masses ( strange quark mass equal to down quark mass )



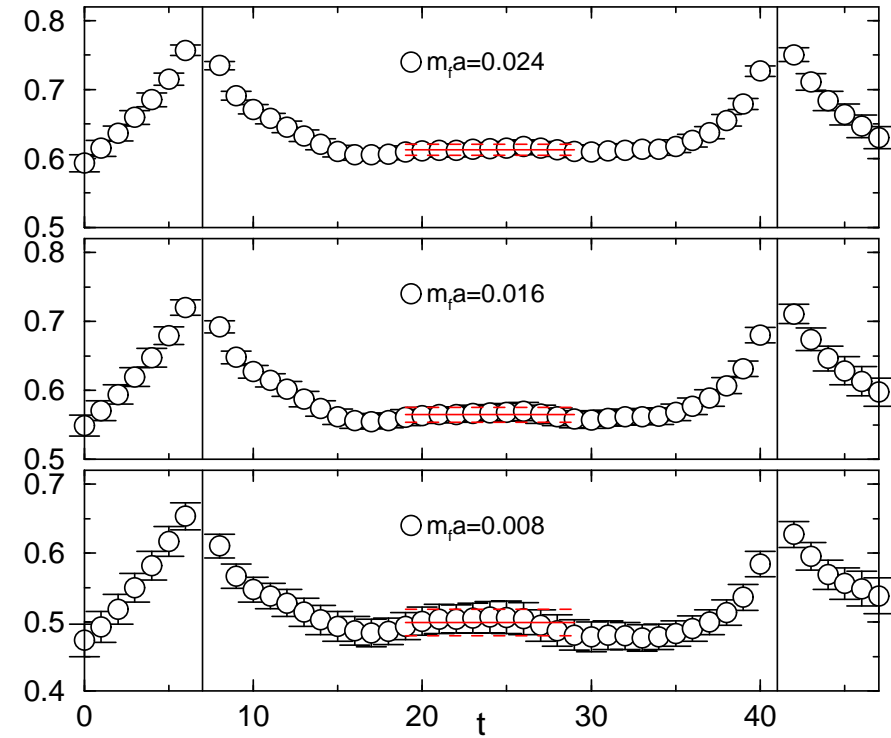
- ▷ Put operator with quantum numbers for Kaon at timeslices 4 and 28 (2 GeV)
- ▷ Move effective Weak vertex over all timeslices
  - ▷ Plateau should appear for large separation.

## Bare $B_K$ Plateaus

$$a^{-1} = 2\text{GeV}$$



$$a^{-1} = 3\text{GeV}$$



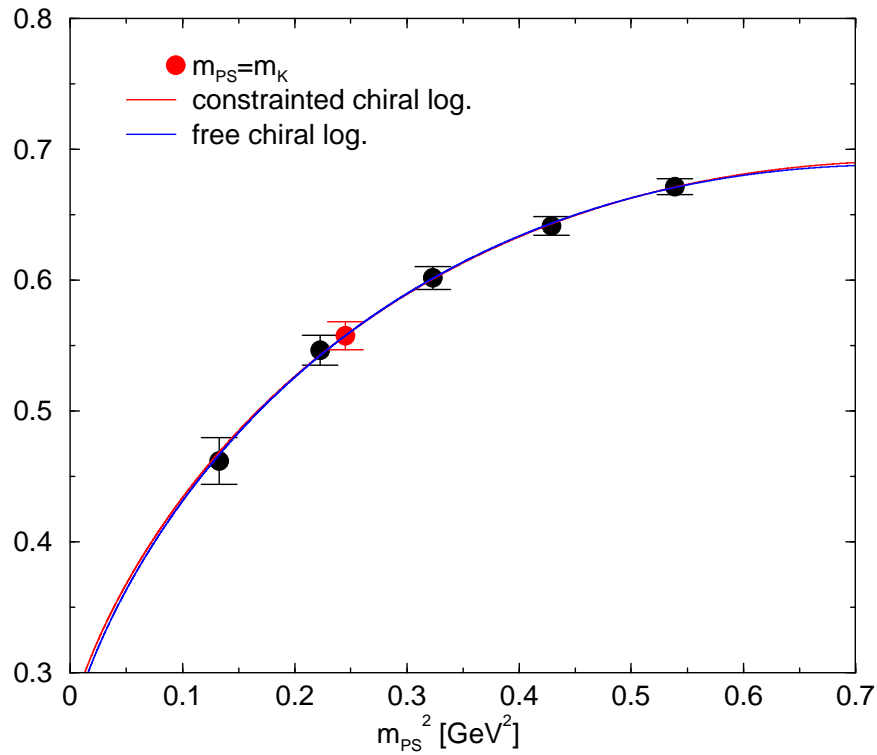
► Quenched Domain wall fermions results for the ratio used to extract  $B_K$ .

# Chiral Fits and Extraction of $B_K$

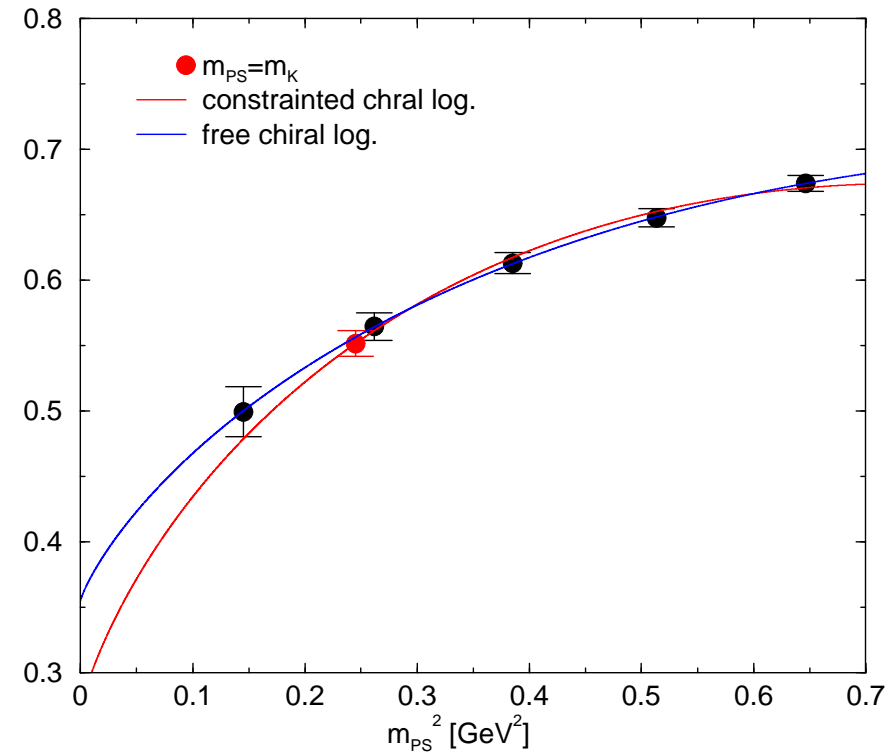
Predicted NLO ChiPT:

$$B_K = b_0 \left( 1 - \frac{6}{(4\pi f)^2} M_K^2 \ln \left[ \frac{M_K^2}{(4\pi f)^2} \right] \right) + b_1 M_K^2$$

DWF(RBC)  $a^{-1} = 2\text{GeV}$



DWF(RBC)  $a^{-1} = 3\text{GeV}$

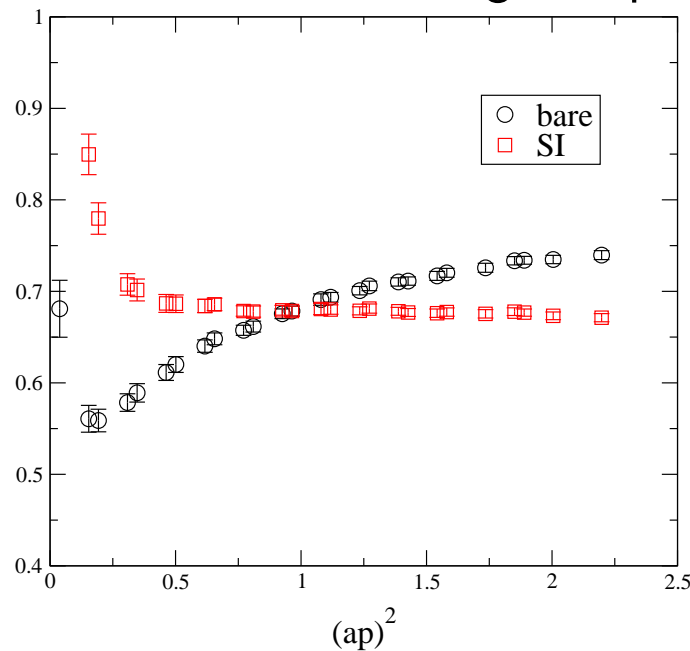


## Fixing the normalisation

To get an answer “renormalised” at the same scale and in the same scheme and the perturbative calculation. Apply same condition on the lattice as in the perturbative calculation.

- ▷ lattice perturbation theory (tedious, slow convergene)
- ▷ NPR (directly on the lattice)

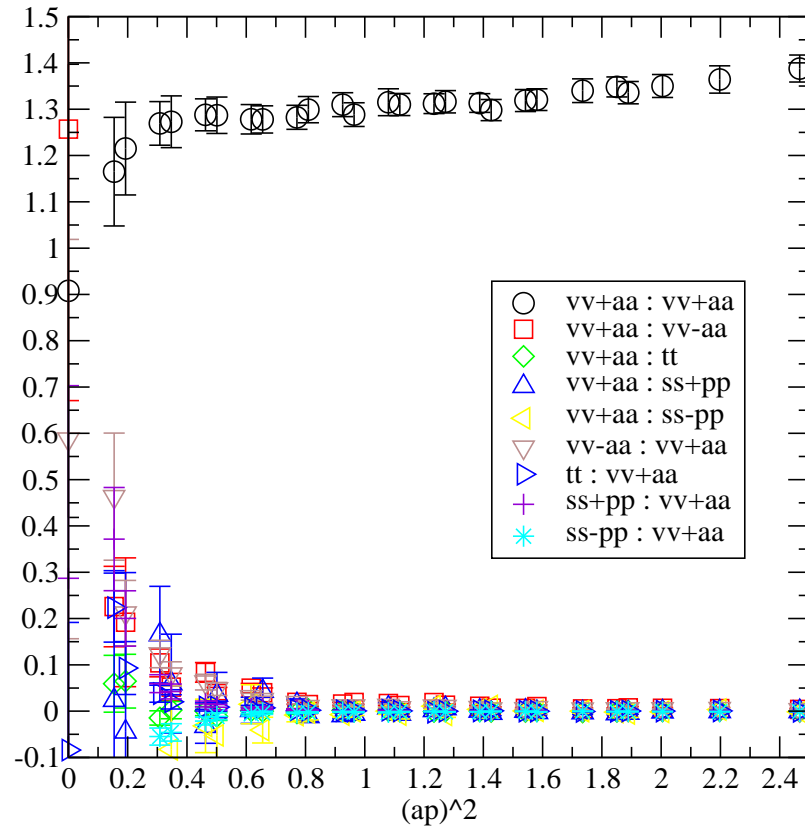
Same issue for defining the quark mass:



- ▷ Mass renormalisation
- ▷ Divide by 3-loop perturbation theory prediction for the scale dependence
- ▷ residual scale dependence is small.

# $B_K$ renormalisation

Elements of  $(Z/Z_q)^{-1}$  (chiral limit)



- ▶ When performing this calculation for  $B_K$ , can allow for possibility of mixing with wrong chirality operators.
- ▶ no evidence for mixing.

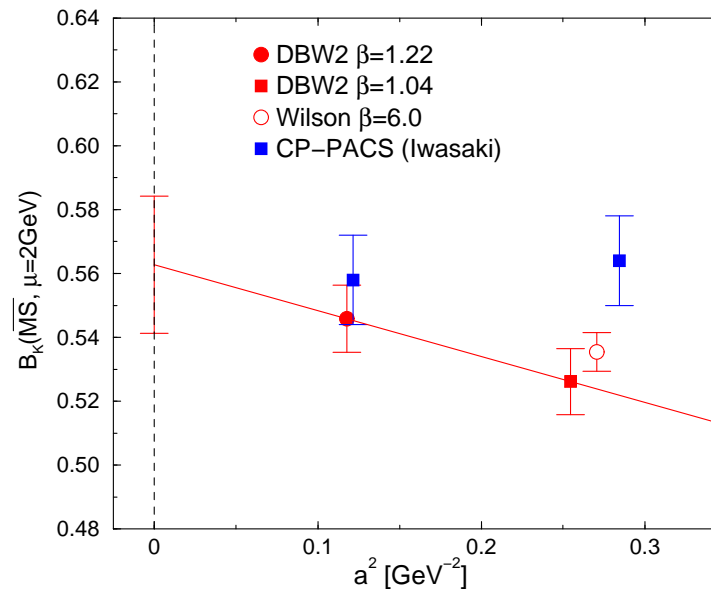


## Quenched continuum limit of $B_K$

- ▷ Extrapolate to the continuum as

$$A + Ba^2$$

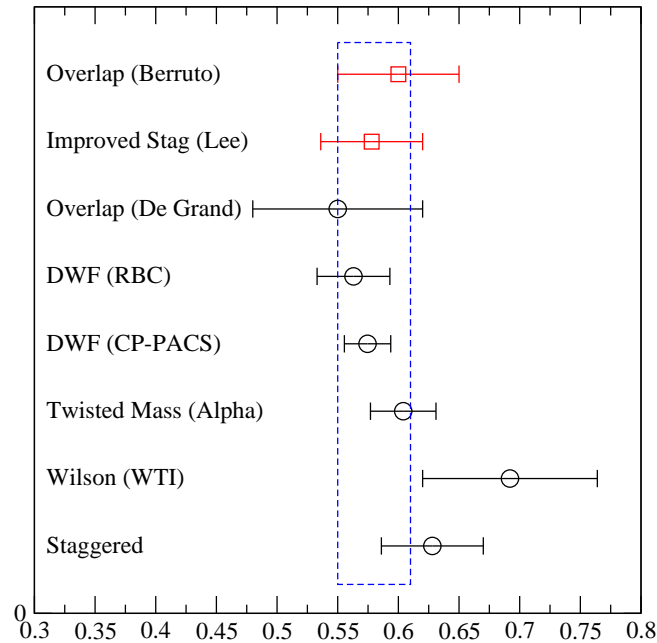
(would be **linear** in **a** without chiral symmetry).



- ▷ Continuum limit consistent with a similar calculation by the **CP-PACS** collaboration.

# Quenched Summary

- ▷ So how do the results stack up?



World Average (Lattice 2005):

$$B_K^{NDR}(2\text{GeV}) = 0.58(3)$$

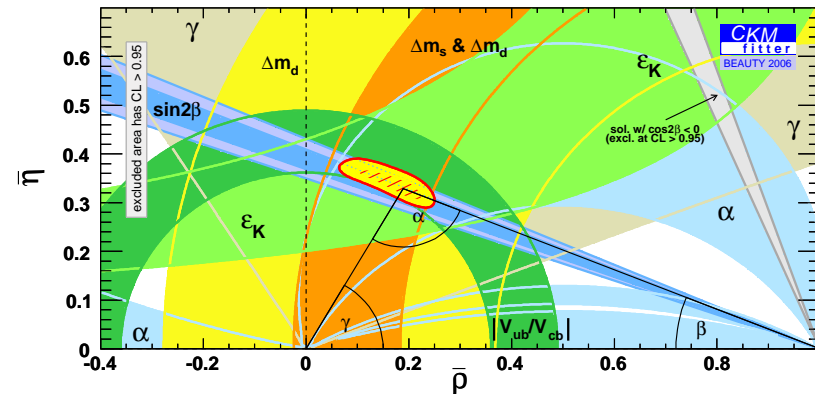
- ▷ Good agreement between the various **formulations** for the quenched value.
- ▷ The common thread for the results with the smallest error-bars are that they are using discretisations for which only a **single operator** had to be used.

# $B_K$ and the CKM

Recall:

$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \bar{\eta} \left[ -\eta_1 S(x_c) + \eta_2 S(x_t) A^2 \lambda^4 (1 - \bar{\rho}) + \eta_3 S(x_c, x_t) \right] \hat{B}_K$$

- ▷ If the standard model is correct, multiple different measurements should agree on  $\bar{\rho}$  and  $\bar{\eta}$ .
- ▷ This is the CKMfitter group's plot from Beauty 2006.



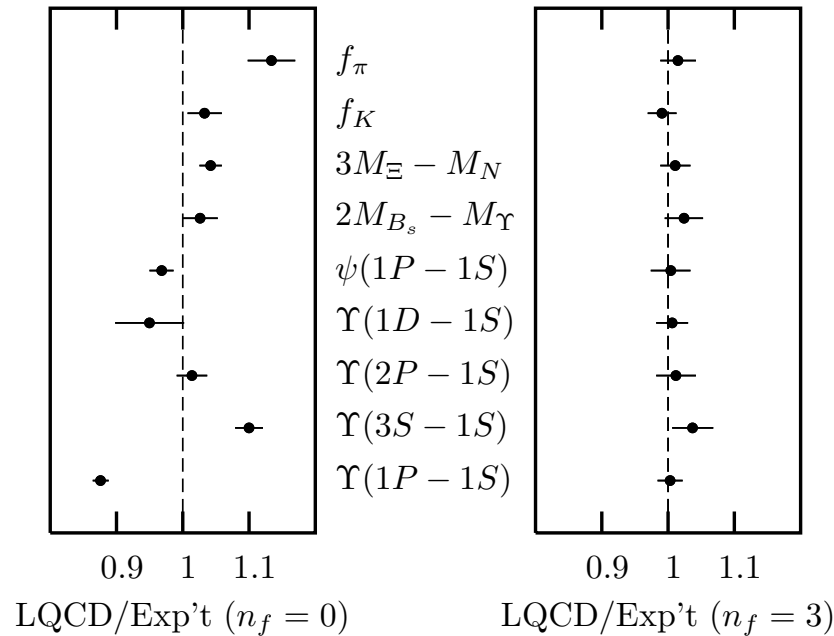
- ▷ For these results, the value quoted is the result of lattice calculations

$$B_K^{NDR}(2\text{GeV}) = 0.58(3)(6)$$

The second (**dominant**) error is a (slightly educated) guess of the **quenching error**.

# Dynamical Fermions

Recently, full QCD (three flavours) has become practical:



► Results from the MILC collaboration, showing the difference between results using improved staggered fermions in the quenched and 3-flavour theories.

► “Improved” staggered fermions still break flavour symmetry (smaller effect)

► Big problem for  $B_K$

► Require “quartic-root trick”.

Obvious approach: dynamical domain wall fermions.

# Dynamical Domain Wall Fermions

- ▷  $\sim 8$  years ago a small group of us based at **Brookhaven Lab** and **Columbia** started to explore dynamical **DWF**.
- ▷ 3/2 years running on a 400GF partition of the 1TF **QCDSP** (**the cell-phone supercomputer**)
- ▷ two degenerate dynamical flavours
- ▷  $16^3 \times 32 ; ((2\text{fm})^3 \times 4\text{fm})$

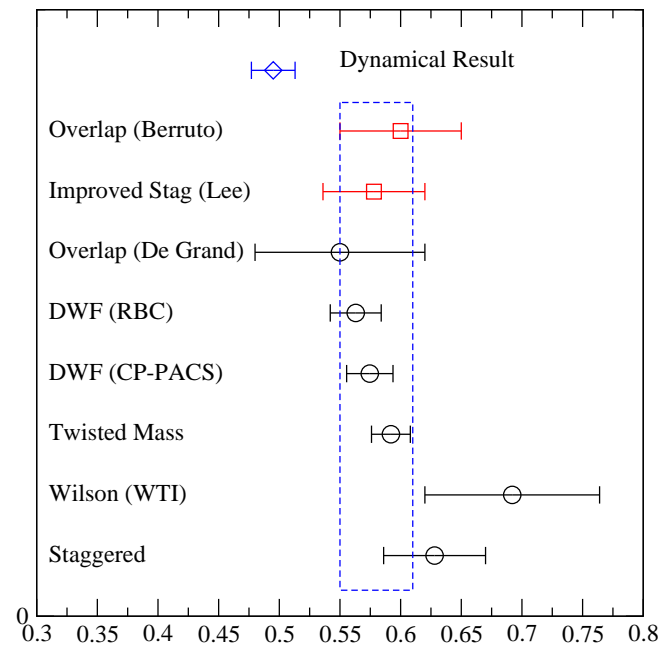


- ▷ **Note:** this is the “**less quenched**” approximation.
  - ▷ **dynamical** u and d; **quenched** s quark.

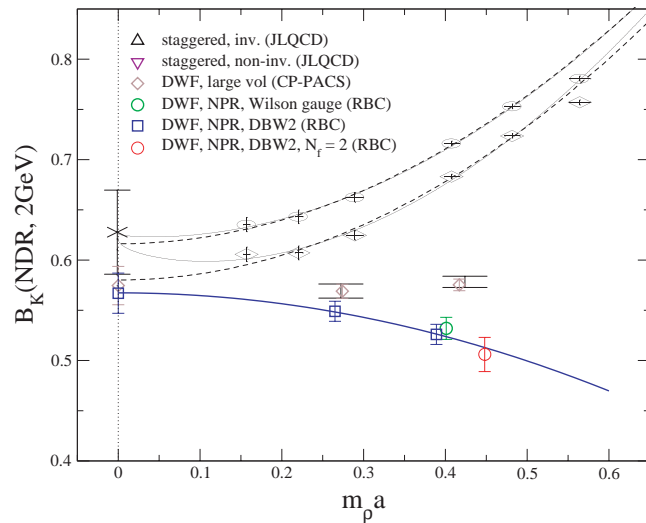
Made possible by a lot of work on improving **fermion algorithms** and learning how the Domain Wall Fermion mechanism success depends on the **Gauge Action** used.

- ▷ **Stepping stone** to 3 flavour dynamical DWF on **QCDOC**.

# $N_f = 2$ Dynamical Domain Wall Fermions



# $N_f = 2$ Dynamical Domain Wall Fermions



- ▷ “Suggestive” graph with the quenched and dynamical DWF results on, with the  $a^2$  extrapolation on it.
- ▷ Not a very sensible thing to plot
- ▷ Our dynamical result is only 3% lower than the quenched results closest in lattice spacing.

This was the information used to estimate the error due to quenching on the world average.

- ▷ need :
  - ▷ two lattice spacings
  - ▷ larger volumes
  - ▷ smaller masses
  - ▷ correct number of quarks

# The QCDOC supercomputers



The QCDOC computers: RBRC (BNL), UKQCD (Edinburgh), US Machine (BNL).

▷ each  $\sim 10$  TFlops (peak).

There is a joint project of (parts of) the UKQCD, RBC and LHPC collaborations using (parts of) all three machines: 2+1 flavour Dynamical DWF



# USQCD/Bluegene

▷ USQCD: a loose confederation of all US lattice theorists

▷ ~ 150 members; 22M funding over 5 years.

▷ Executive committee: writes applications for computer time.

▷ Program committee: decides who gets computer time.

▷ Software committee: oversees software infrastructure.



▷ USQCD has its own resources, but most of the computer time for the past year has come from DOE “leadership class machines” such as Argonne’s BG/P pictures above.

▷ Starting from very little a few years ago, the DWF project now has the single largest allocation of time, with over 50% of the time devoted to lattice generation.

## Historical Document: The “Master Plan” (January 2005)

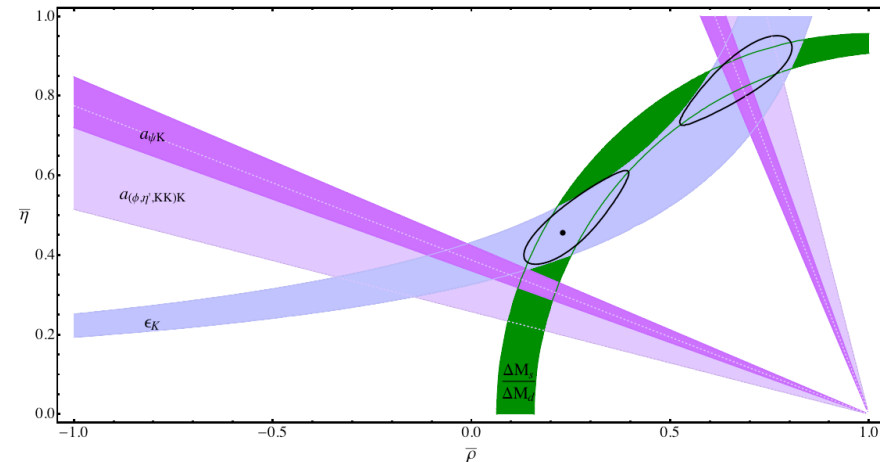
$m_f/m_s$	$L_s$	$L * a$ (Fm)	$L * a * m_\pi$	Nodes	Trajs.	Time (days)	Proc. Hrs.
$16^3 \times 32, 1/a = 1.8 \text{ GeV}, a = 0.11 \text{ Fm}:$							
0.6	12	1.78	3.44	2,048	4,833	8	3.84E+05
0.5	12	1.78	3.14	2,048	5,294	12	6.13E+05
0.4	12	1.78	2.81	2,048	5,919	23	1.11E+06
$24^3 \times 64, 1/a = 1.8 \text{ GeV}, a = 0.11 \text{ Fm}:$							
0.4	12	2.67	4.22	4,096	5,919	123	1.21E+07
0.3	12	2.67	3.65	4,096	6,835	270	2.65E+07
0.2	16	2.67	2.98	6,144	8,371	758	1.12E+08
$32^3 \times 64, 1/a = 1.8 \text{ GeV}, a = 0.11 \text{ Fm}:$							
0.3	16	3.56	4.87	8,192	6,835	529	1.04E+08
$32^3 \times 64, 1/a = 2.4 \text{ GeV}, a = 0.083 \text{ Fm}:$							
0.5	12	2.67	4.71	8,192	7,059	273	5.37E+07

▷ 3 volumes, 2 lattice spacings, various quark masses

# Full QCD: Initial Results

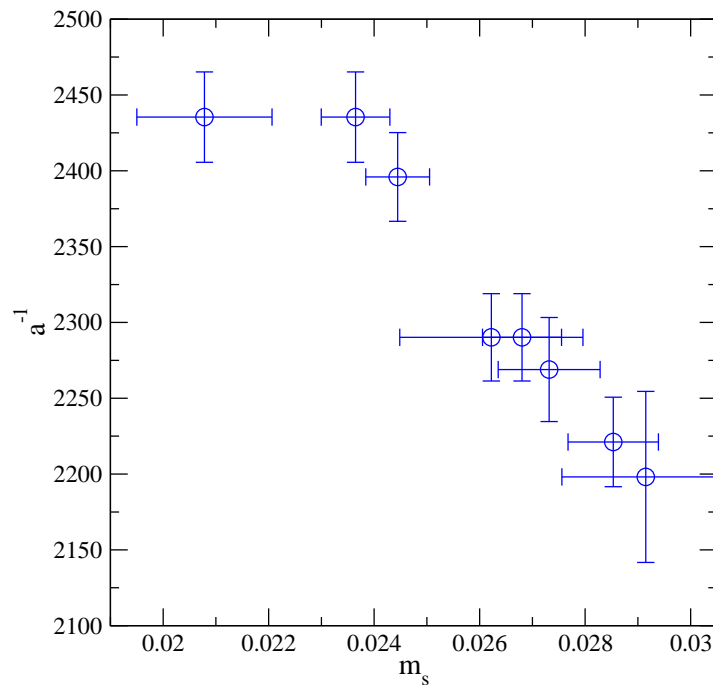
- ▷ [hep-lat/0612005](#): Have completed extensive action/scale finding study
  - ▷ Switched to another gauge action.
  - ▷ Found a **factor of 6** in simulation speed.
- ▷ [hep-lat/0701013](#): Basic spectrum and chiral fits for the chosen parameter set and the small lattices.
- ▷ [0804.0473 \[hep-lat\]](#): Larger lattices, full analysis, but only a single lattice spacing.
  - ▷ Found **Chiral Perturbation Theory** doesn't work with Kaons: moved to a version using only the Pions.

- ▷  $B_K(2\text{GeV}, \overline{\text{MS}}) = 0.524(30)$
- ▷ Tightness in the  $\bar{\rho} - \bar{\eta}$  plane **Lungi and Soni (2008)**
- ▷ Still no Scaling error.



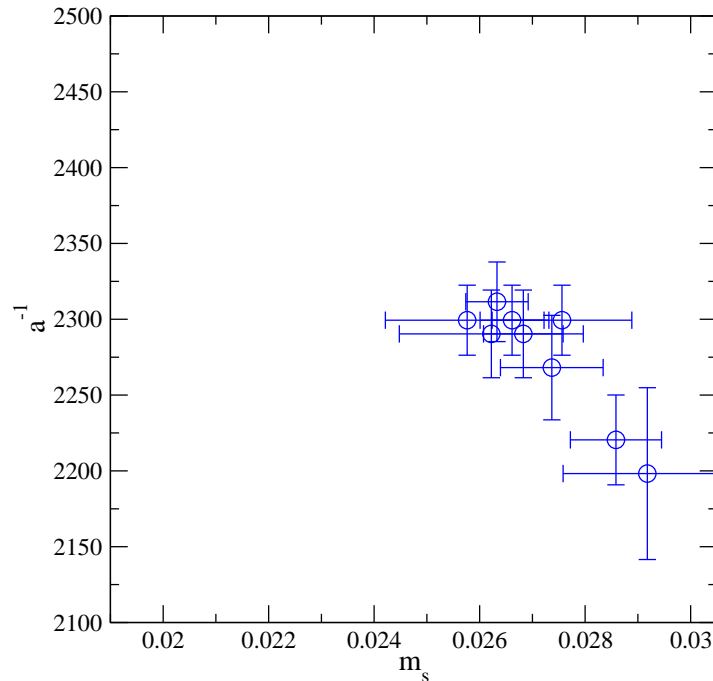
## Full QCD: Second Lattice Spacing

- ▷ For the last year or so, we have been working on adding the second, finer, lattice spacing
- ▷ Why so long? Data didn't make sense:



- ▷ Different quantities get a different lattice spacing.

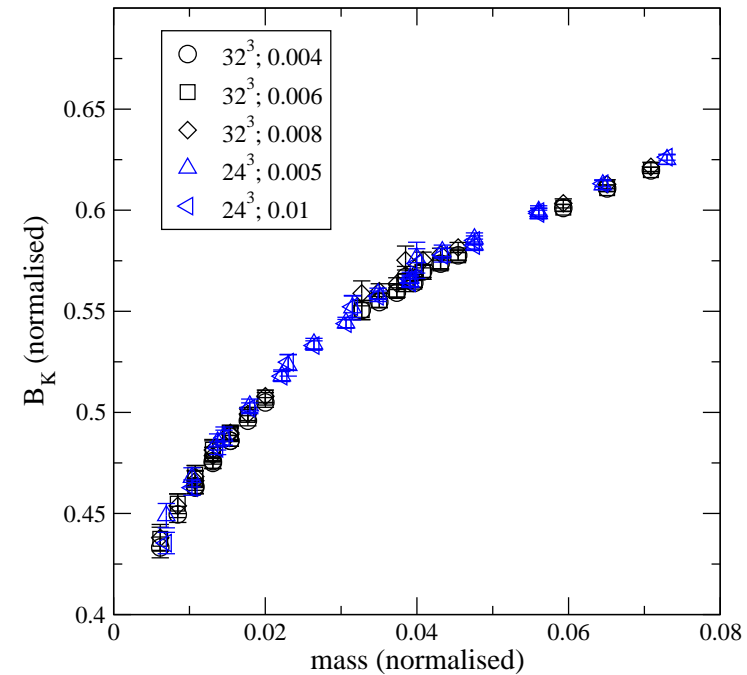
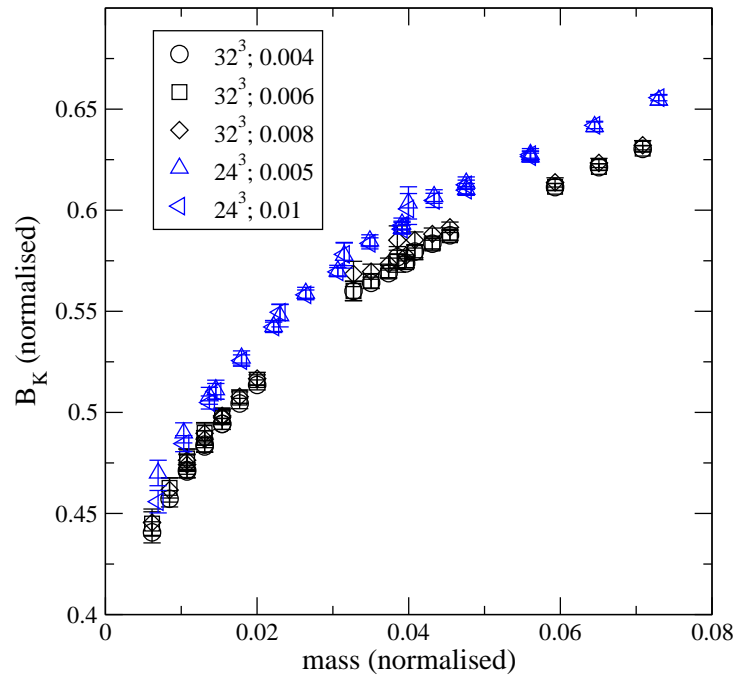
# Full QCD: Matching Lattices



- ▶ If we switch to “naive” fits (Taylor expanding in the quark masses), things look fine.
- ▶ Even our improved Chiral Perturbation Theory isn’t working.

- ▶ Developed a matching technique that doesn’t use Chiral Perturbation Theory to compare different lattice spacings.

## Scaling for $B_K$



- ▷ Seeing great scaling for  $B_K$ .
- ▷ Are using both Chiral Perturbation Theory and Naive Fits.
- ▷ Should reduce the error by a factor of  $\sim 2$
- ▷ Details being finalised now....

## Summary

- ▷ Lattice QCD is a vital tool for both defining the theory of Strong interactions and calculating it's predictions.
- ▷ For many years Lattice calculations have been performed solely in the quenched approximation; with theoretical and computational advances it has finally become possible to perform calculations in full QCD.
- ▷ The Domain wall fermion approach to lattice QCD preserves both flavour and chiral symmetries (almost) at finite lattice spacing, reducing many source of systematic error.
  - ▷ Full scale calculations with control of all/most systematics are underway
- ▷ For the past year we have been producing lattice with yet another discretisation of the gauge field, which always us to use a larger lattices spacing
  - ▷ smaller quark masses

# This isn't all...

## Physics Goals + Configuration Policy

A sample of the physics we wish to study on these lattices:

Hadronic spectrum  
Light quark masses  
Topological charge  
 $K \rightarrow \pi\pi$  decay

epsilon-prime

Vus

$K \rightarrow \pi l \nu$

Exotic hadrons, pentaquarks

Neutron EDM

Electromagnetic structure of hadrons

$\eta'$  meson

Structure functions

Decay constants  
Static quark potential  
Kaon B-parameter  
Nucleon matrix elements

Excited nucleon states

Nucleon decay

Muon  $g - 2$

$U(1)_A$  problem

Charm and bottom physics

Hyperon beta-decay

Vus

Proton Decay  
(beyond SM)

Heavy-light

Vus

Of course, any configs generated using **USQCD** resources are immediately available to the US community; no physics topics will be reserved for our initial exclusive study.

As **UKQCD** also benefit from these configurations, we will release a further set of lattices equal in number to those generated with **USQCD** resources, with the same (lack of) restrictions.

- full release within 6 months of the paper describing the configurations.

Political Stuff