Effect of order parameter fluctuations on spectral density in d-wave superconductors

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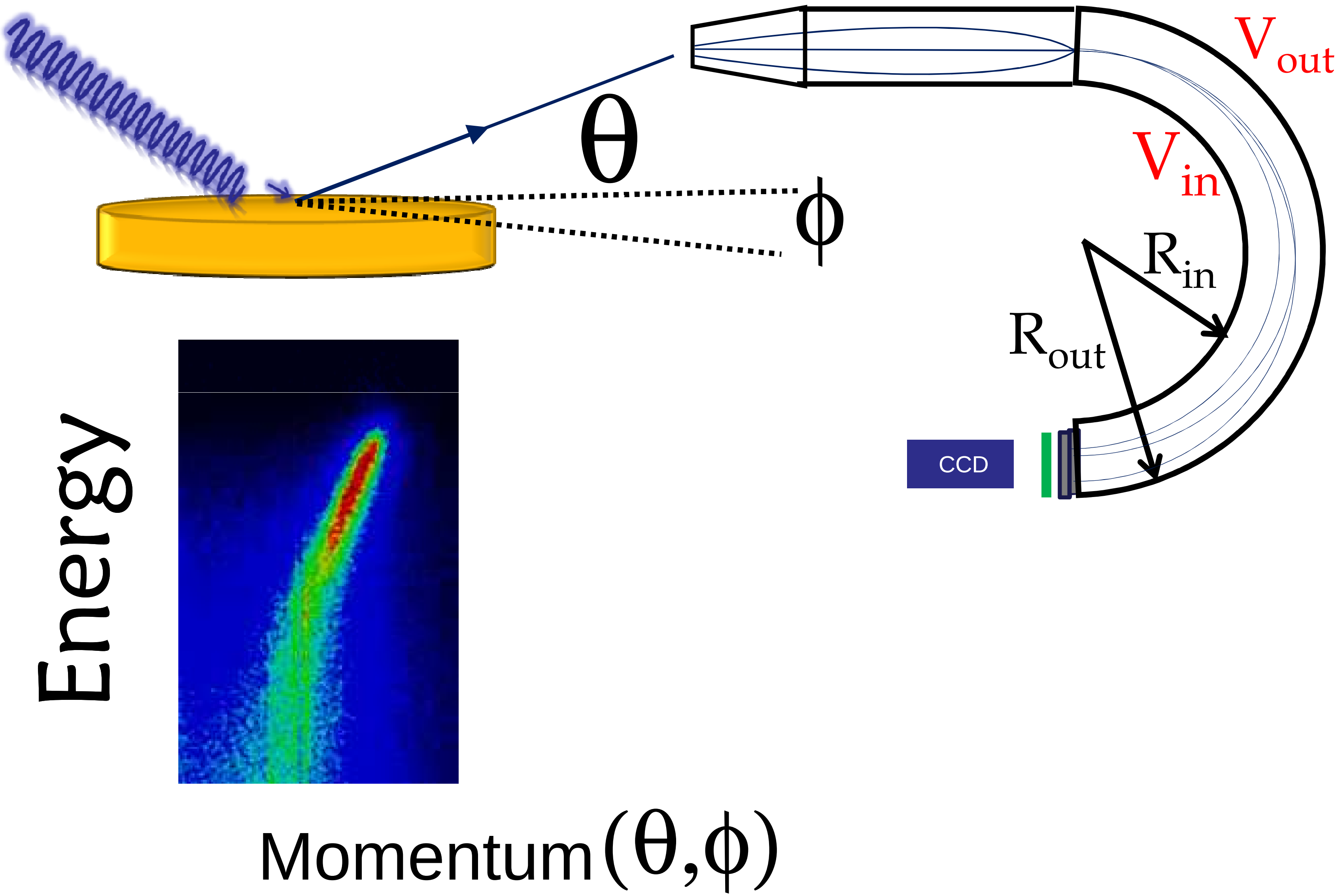
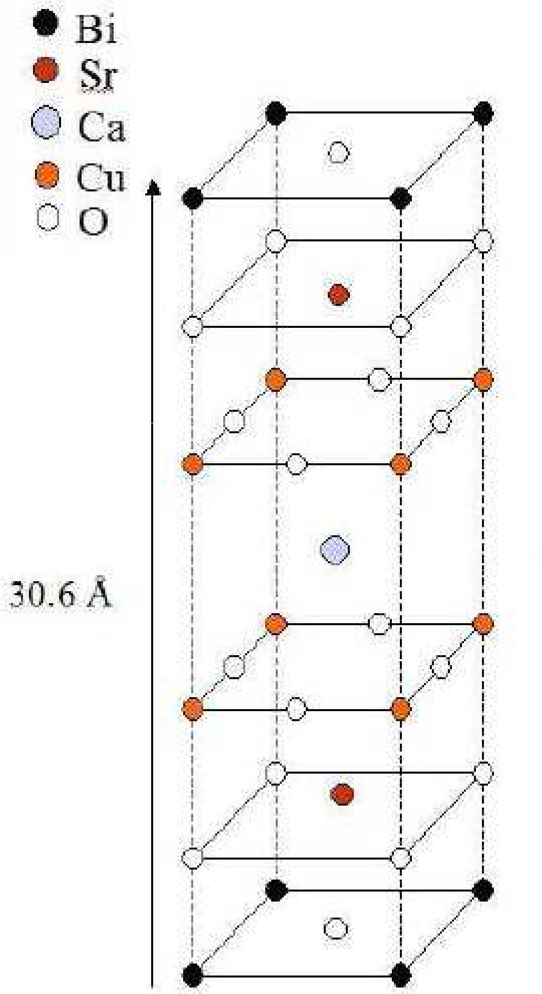
Outline

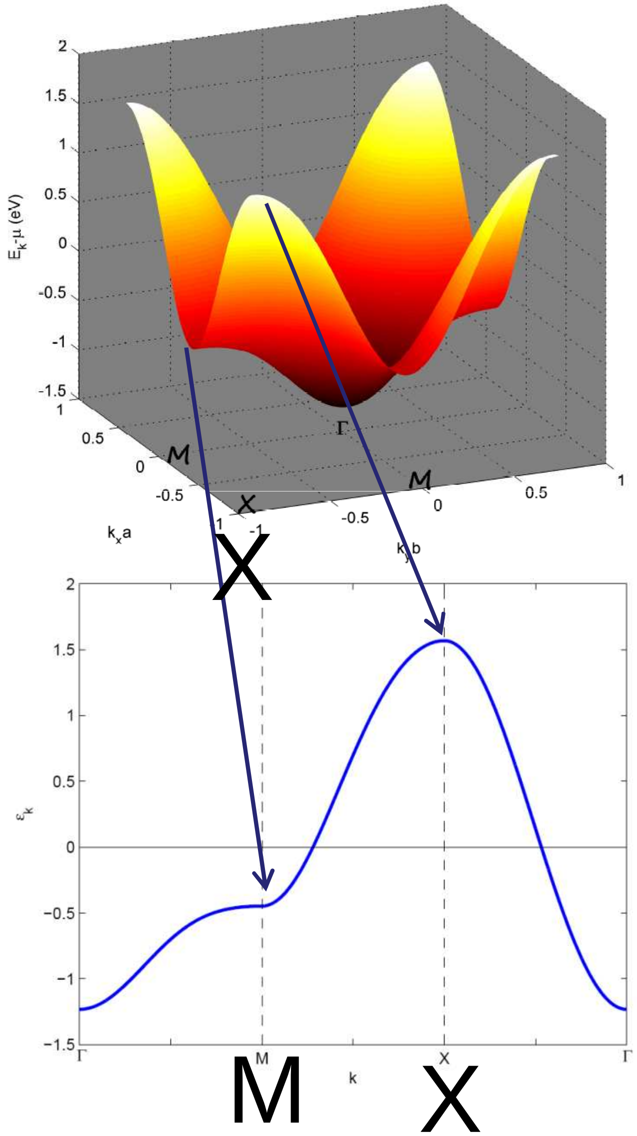
1. Motivation
2. Phase fluctuations and Spectral Density
3. Fermi Surface reconstruction
4. Conclusions

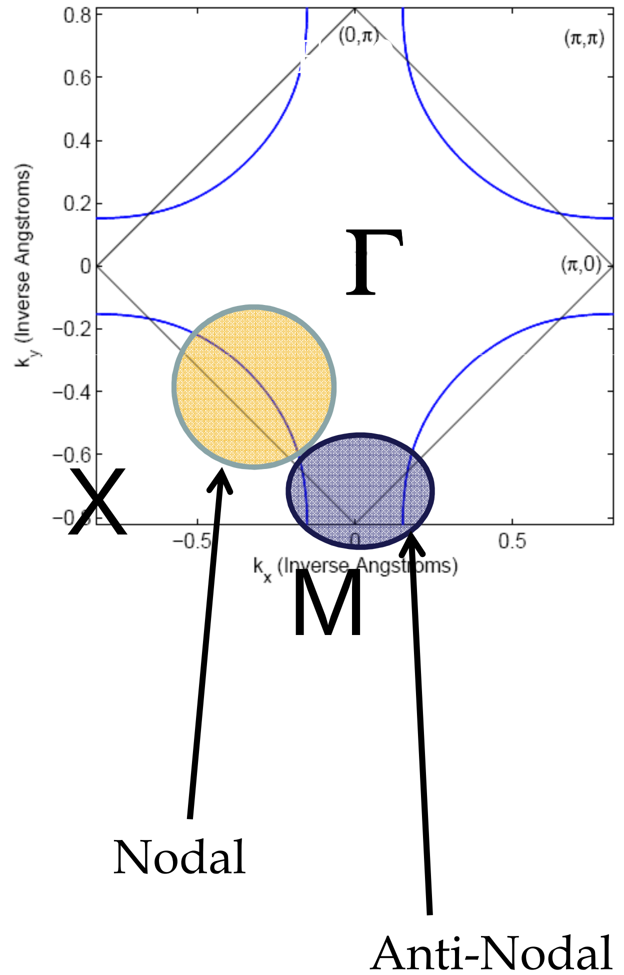
Motivation: ARPES in cuprates



Motivation: ARPES (cont)

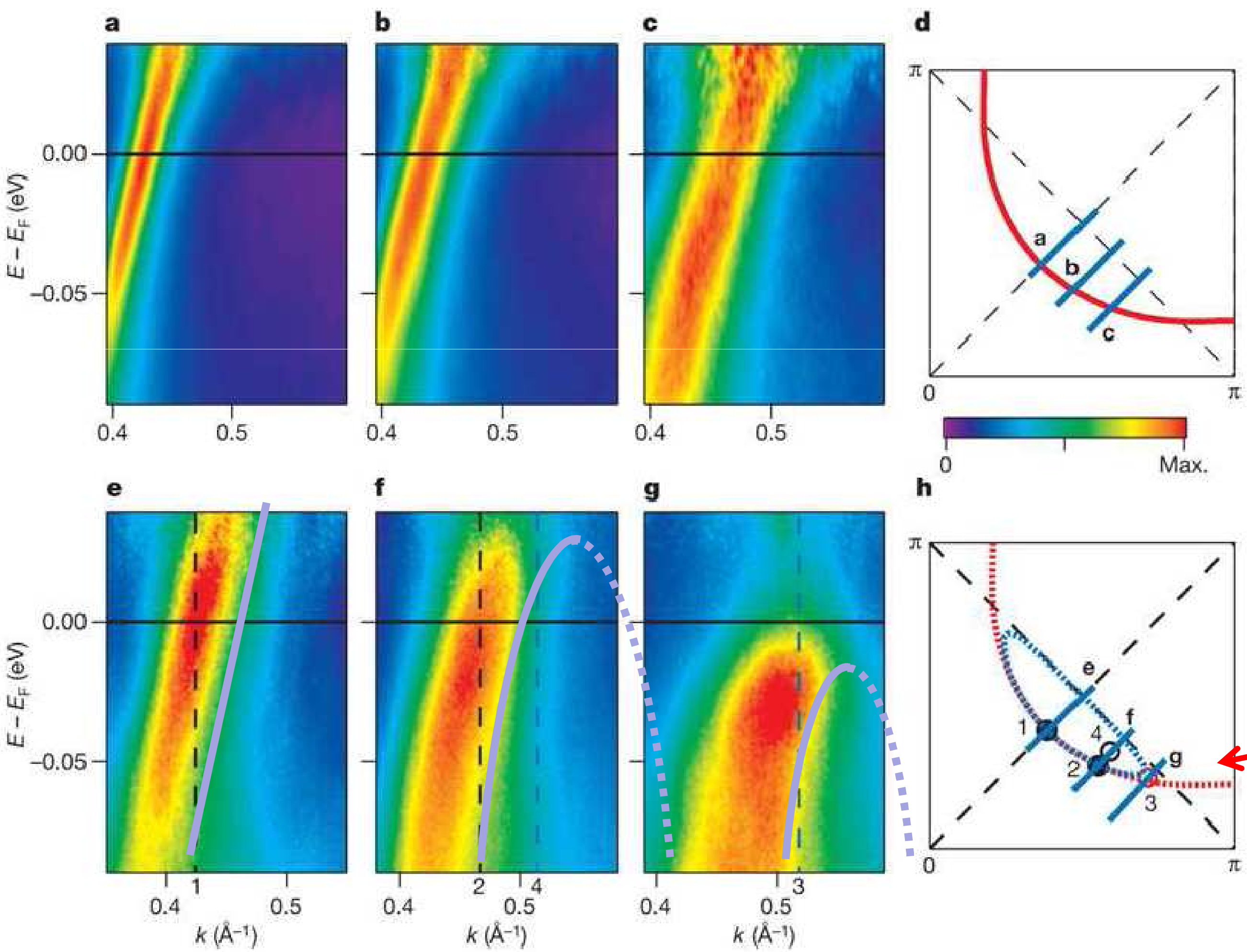






Motivation: ARPES (cont)

Nodal region Excited States at T>Tc (T = 140K)



T_c=91 K
(Optimal doping)

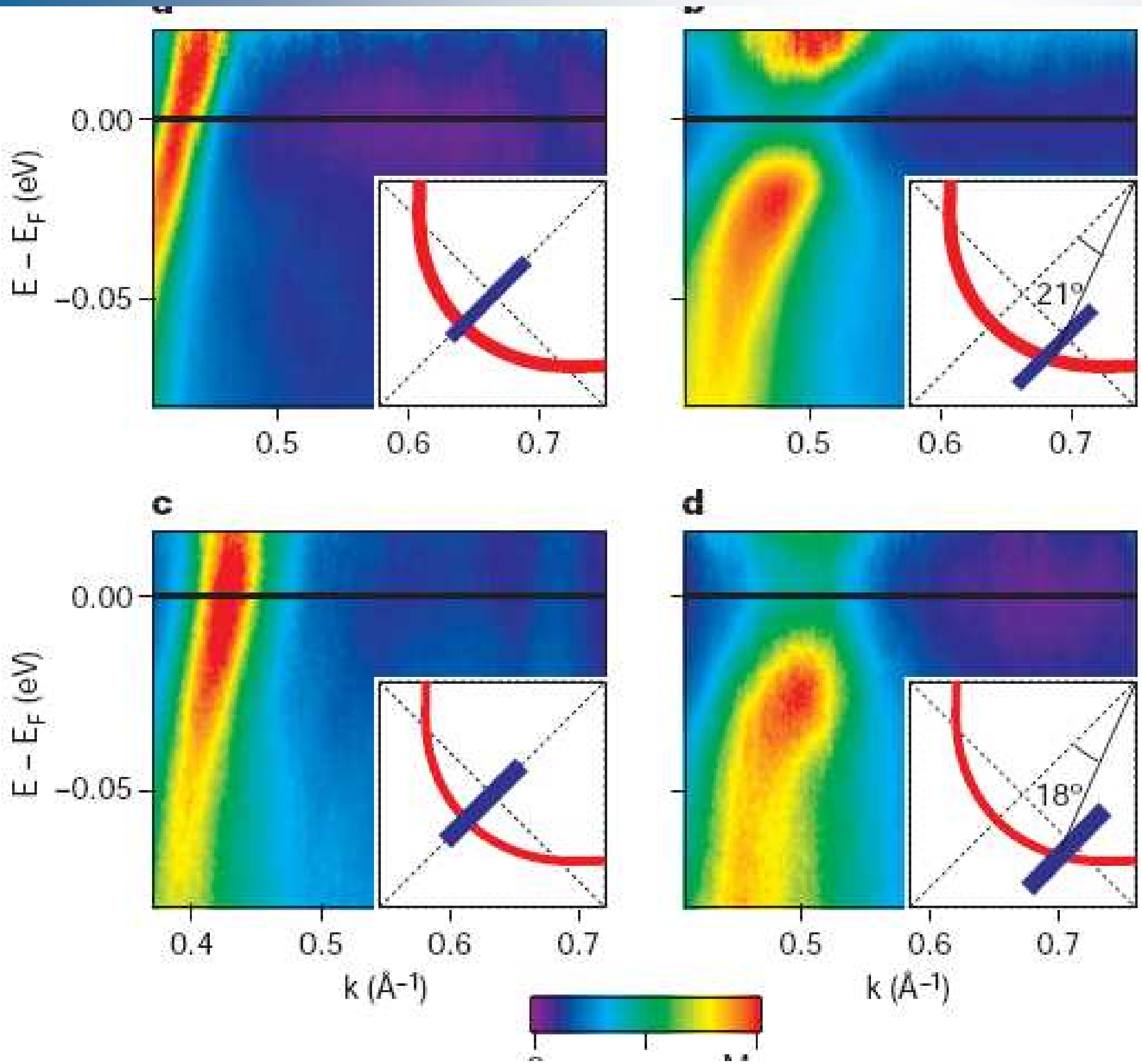
“Arc”

T_c=65 K
(under doping)

Area=doping!

“Pocket”

Motivation: ARPES (cont)



Superconducting State

T<Tc

d-wave symmetry

order parameter

Motivation: ARPES (cont)

What is measured?

$$I(k',\omega') \propto A(k,\omega) f(\omega) \otimes R(k',\omega' | k,\omega)$$

$$\boxed{A(k,\omega) = -\frac{1}{\pi}\operatorname{Im} G^R(k,\omega)}$$

BCS: $$A(k,\omega) = u_k^2 \delta(\omega - E_k) + v_k^2 \delta(\omega + E_k)$$

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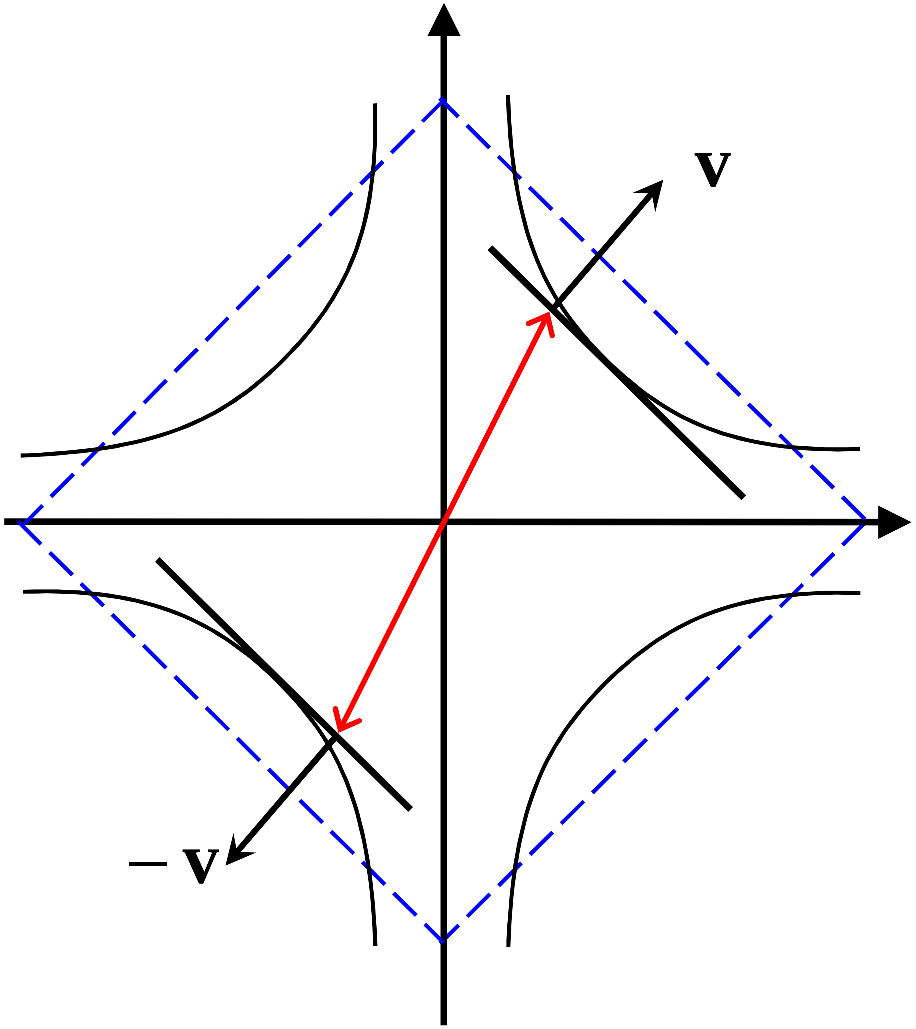
Model



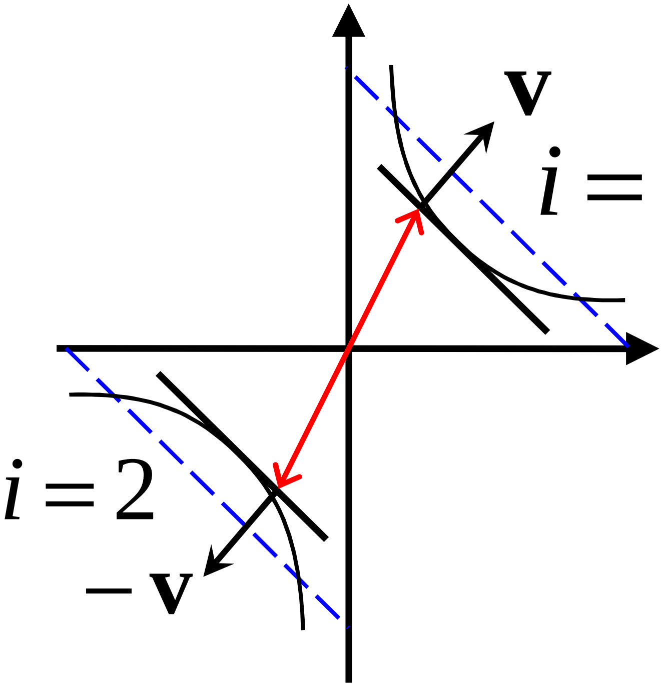
Quasi-particles:

- Anisotropic FS
- Effectively 1D

Fluctuations:

- Isotropic
- Classical

Model (cont)



Nambu-spinor:

$$\chi = (\psi_{\uparrow,1}, \psi^{+}_{\downarrow,2})^T$$

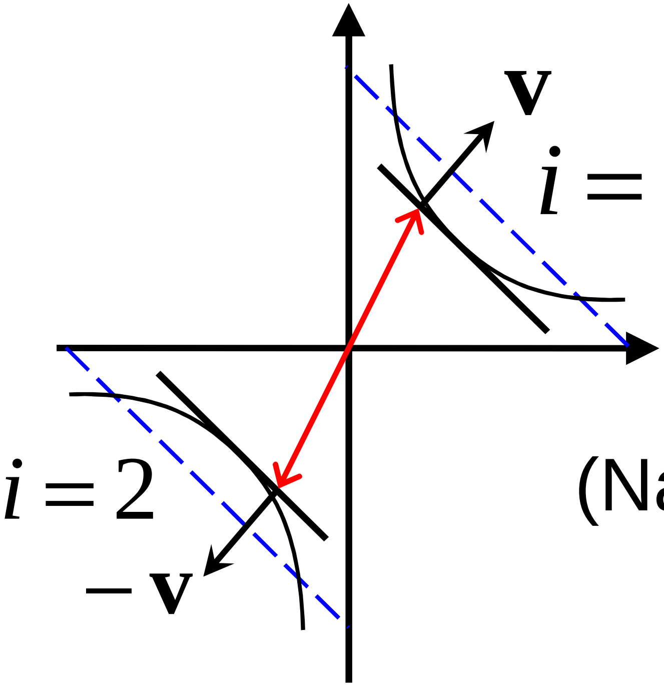
+

Classical phase fluctuations

$$\mathcal{L} = \bar{\chi}_\omega \left[-\mathrm{i}\omega_n \hat{1}_2 - \mathrm{i}v\partial_x \sigma^z + \Delta\sigma^+ + \Delta^*\sigma^- \right] \chi_\omega + F_\phi$$

$$\frac{F_\phi}{T} = \frac{\rho_s}{2T} \int dx dy \left[(\partial_x \phi)^2 + (\partial_y \phi)^2 \right]$$

Impurity Model



Nambu-type transformation:

$$[\psi_\uparrow, \psi_\downarrow^\dagger] = [\Psi_\uparrow, -\mathrm{i}\Psi_\downarrow^\dagger]$$

$\Updownarrow$

(Nambu) particle number conservation

$\Downarrow$

Effective Impurity model $\quad \tau^a = \Psi^+ \tau^a \Psi$

$$H = v^{-1}\{(i\omega - 0)\tau^3 + \Delta_q[\tau^+ \exp(-i\varphi) + \tau^- \exp(i\varphi)]\} + H_{bulk}$$

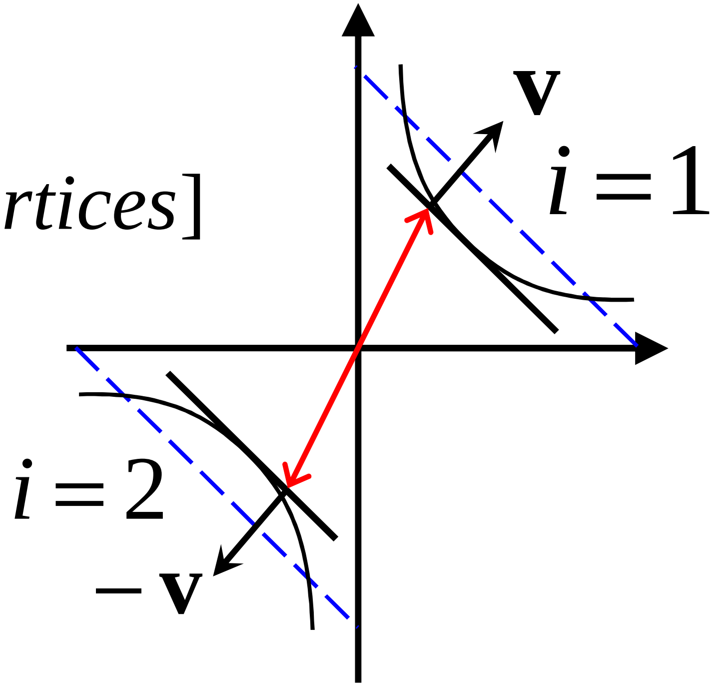
Model (cont)

$$\varphi = \varphi(x,0)$$

$$H_{bulk}[\varphi] = \frac{2\pi}{d}\int dy[\Pi^2 + (\partial_x\varphi)^2 + vortices]$$

$$[\varphi(y),\Pi(y')] = i\delta(y-y')$$

$$d = T/8T_{BKT}$$

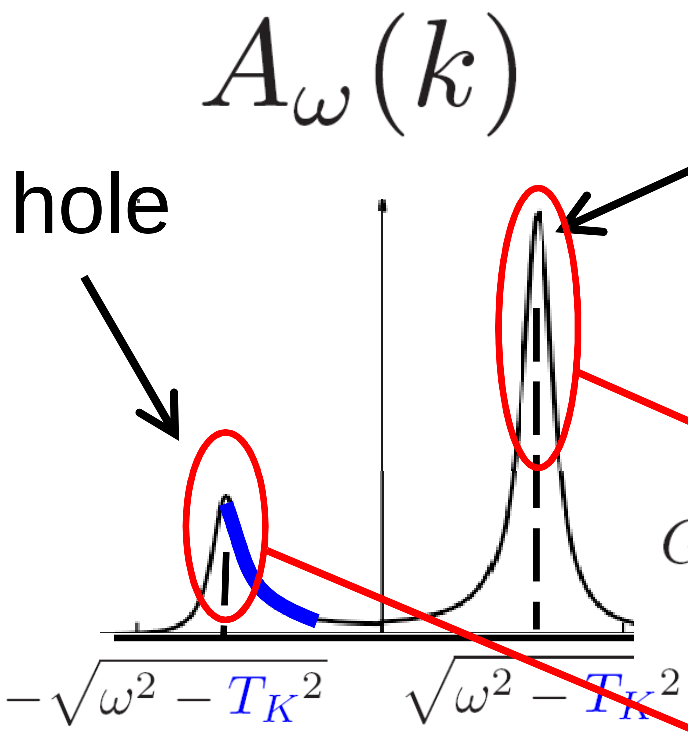


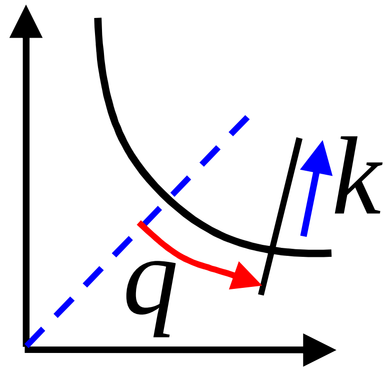
$$< e^{i\varphi(r)}e^{-i\varphi(r')} > = (a/\xi)^{2d}F(|\vec{r}-\vec{r'}|/\xi)$$

$$F(z \ll 1) = z^{-2d},$$

$$F(z > 1) \sim K_0(z)$$

Results: (T<Tc)





$$G_\omega(k) = \left[\omega - k - \frac{\Delta^2(q)a^{2d}}{(\omega+k)^{1-2d}}(1 - \mathrm{i}\pi d)\right]^{-1}$$

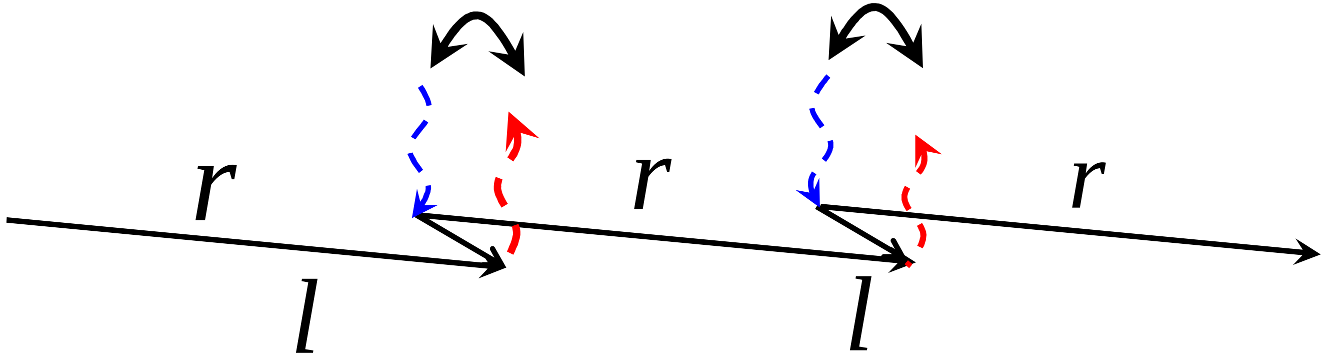
$$T_K = \Delta(q)\,[$$

$$G_\omega(k) \simeq \frac{1}{2\omega}\left[1 + \frac{\Delta^2(q)\mathrm{e}^{-\mathrm{i}\pi d}}{2\omega\left(\omega + k - \frac{\Delta^2(q)\mathrm{e}^{-\pi \mathrm{i} d}}{(2\omega)^{1-2d}}\right)^{1-2d}}\right]$$

Results: (T<Tc)

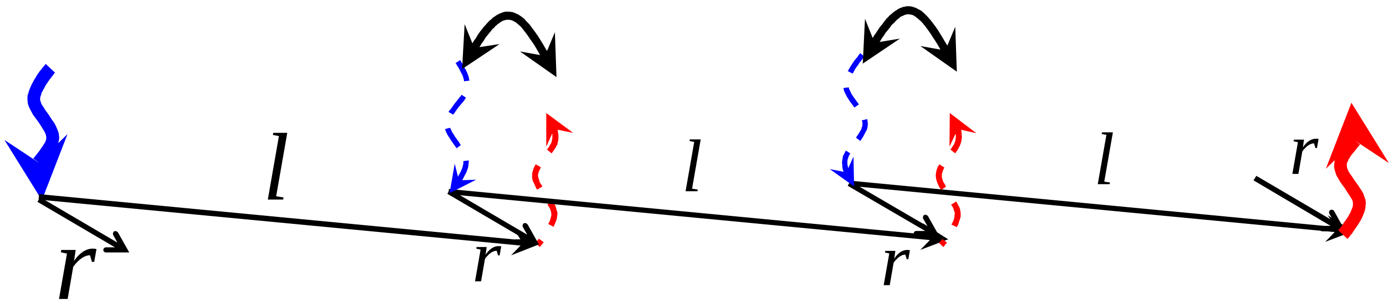
Mass Shell: $\omega - vk << \omega$

exponents fuse



Lorentzian

$\omega + vk << \omega$

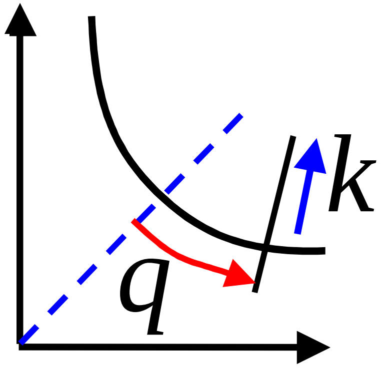


Lorentzian
+
Power Law

Orthogonality Catastrophe

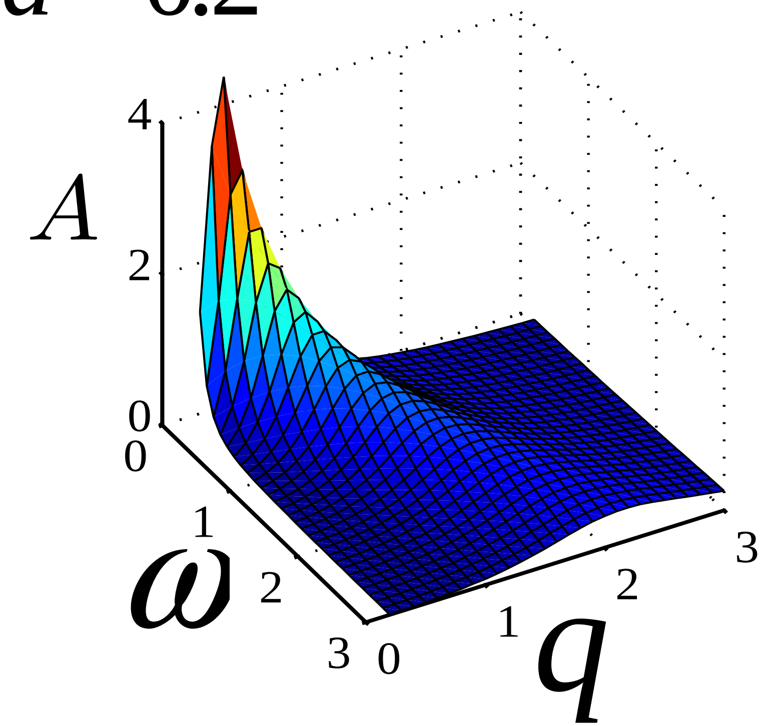
Results (T>Tc)

$$\Sigma^{(2)}(q,k,\omega) = \Delta^2(q)\xi(\xi/a)^{-2d}(1+(\xi(\omega+k))^2)^{-1/2} \times \left[\exp\left\{-i\pi d + 2d\sinh^{-1}[\xi(\omega+k)]\right\} - (1+(\xi(\omega+k))^2)^{-1/2}\right]$$

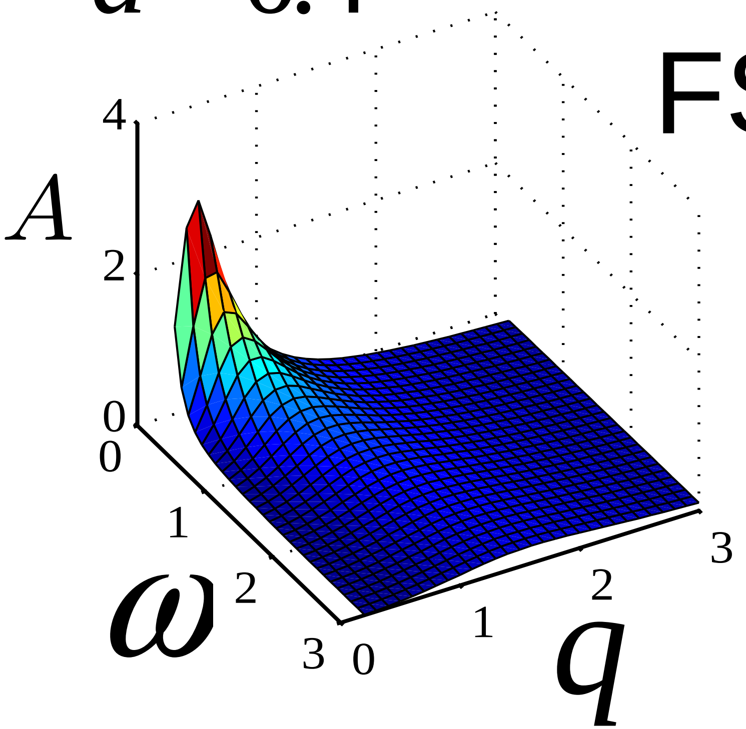


FS

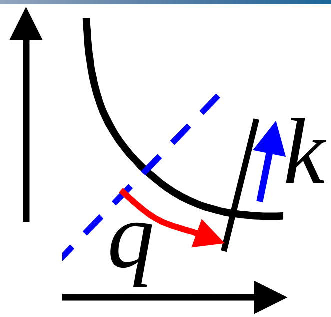
$d=0.2$



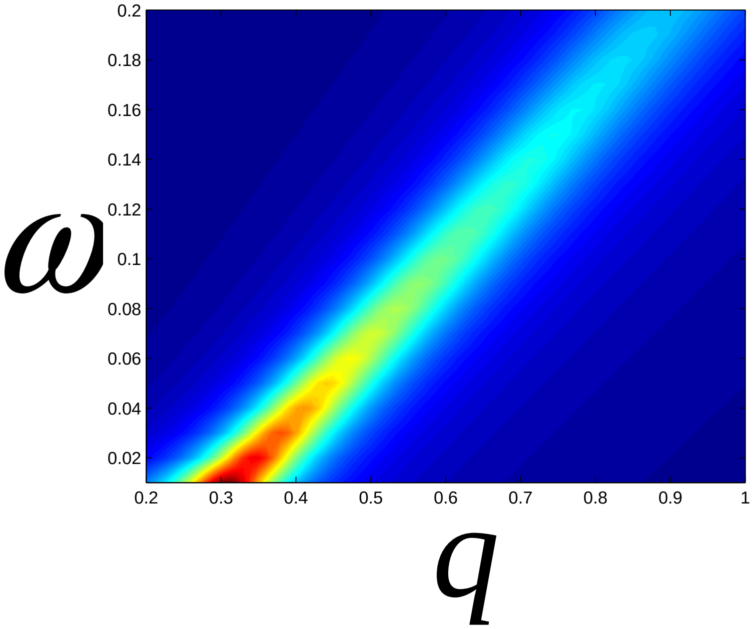
$d=0.4$



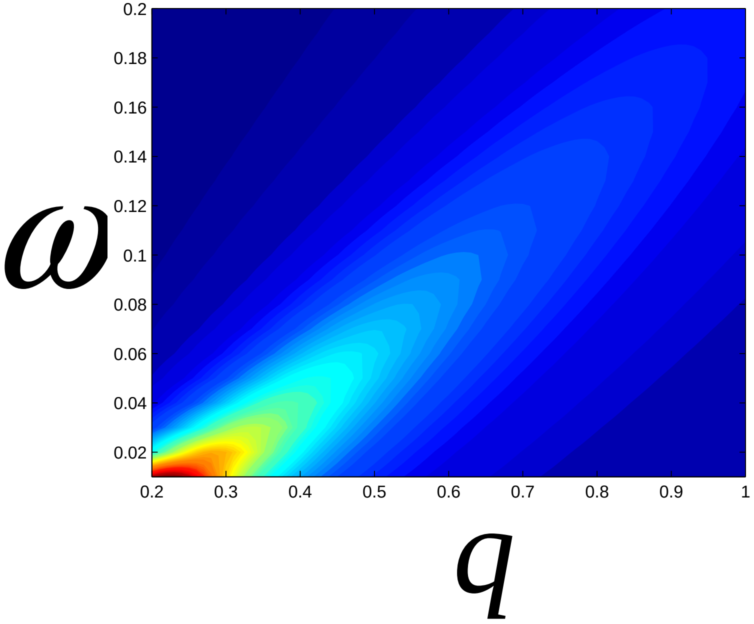
Results (T>Tc), (cont)



$d = 0.05$



$d = 0.15$



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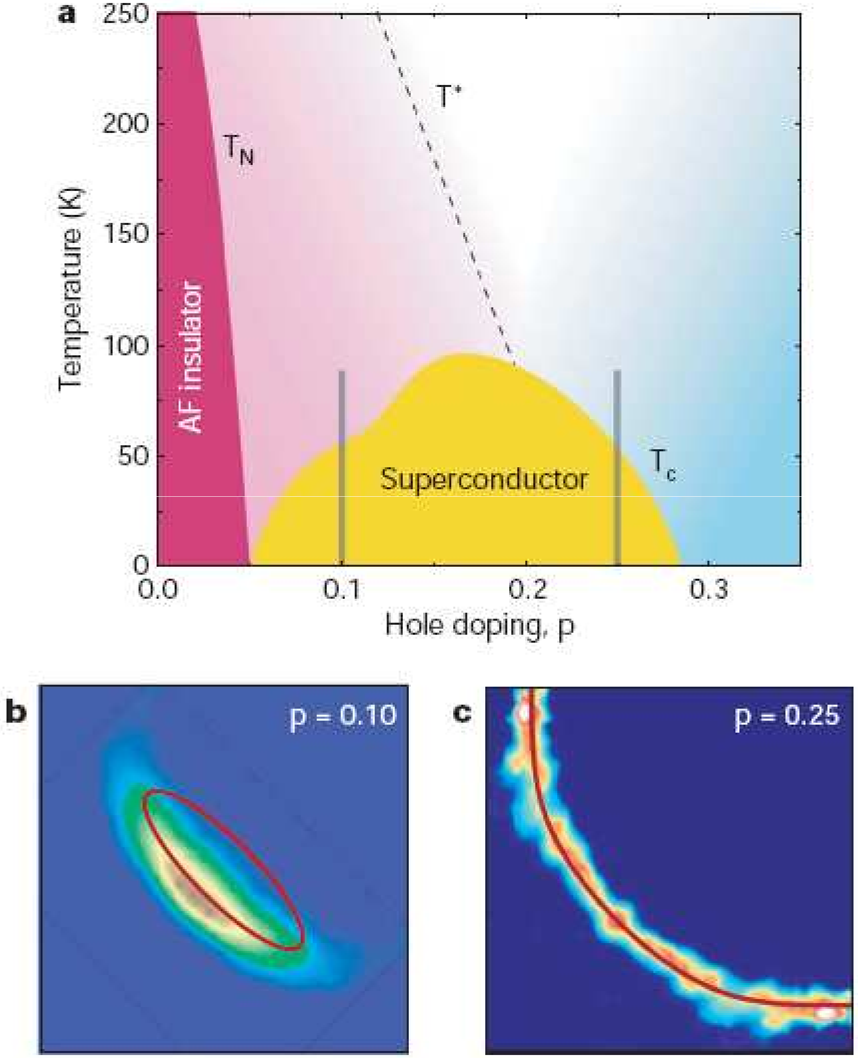
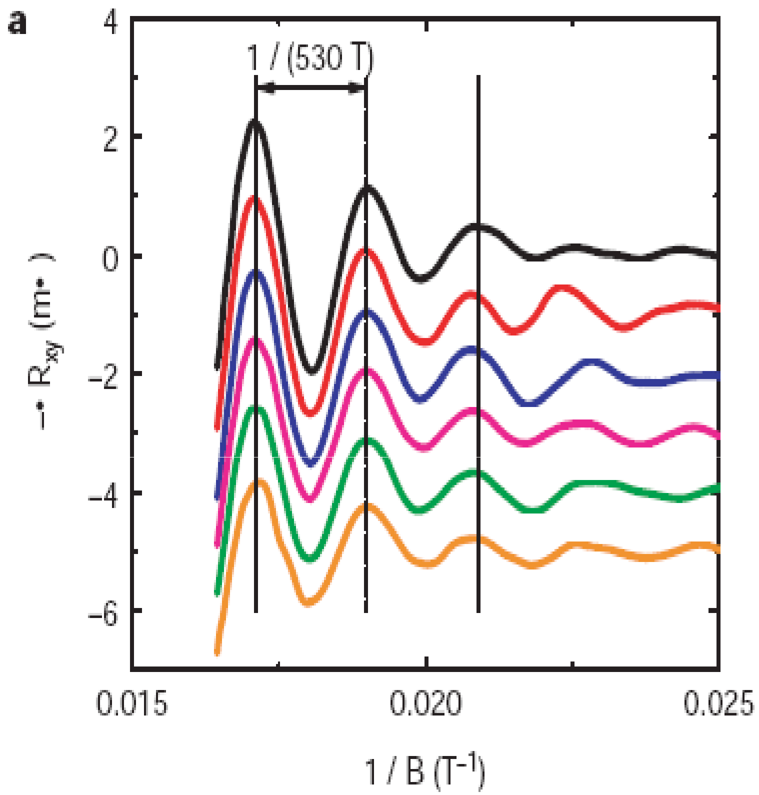
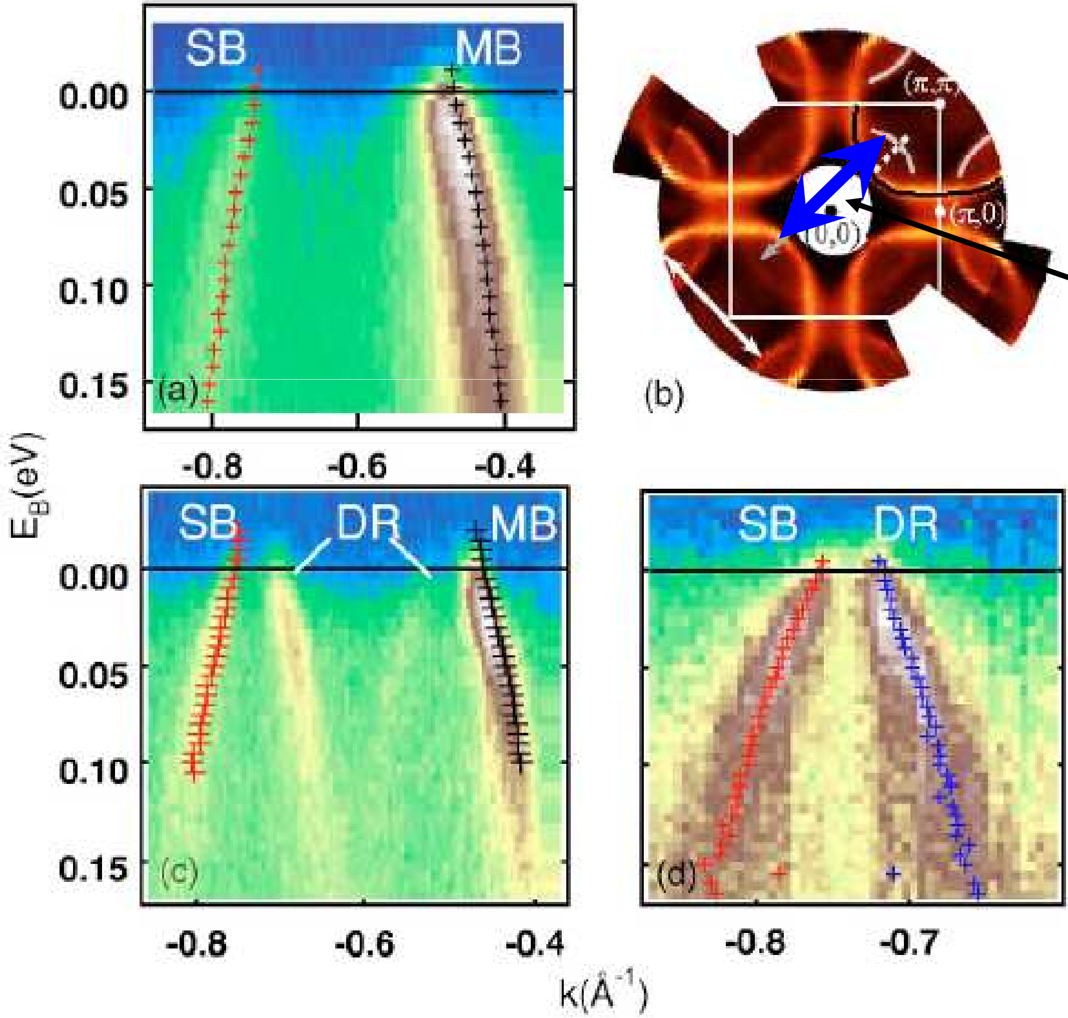
Fermi Surface Reconstruction





YBCO !

Fermi Surface Reconstruction (cont)



Shadow band

$$\mathbf{Q} = (\pi, \pi)$$

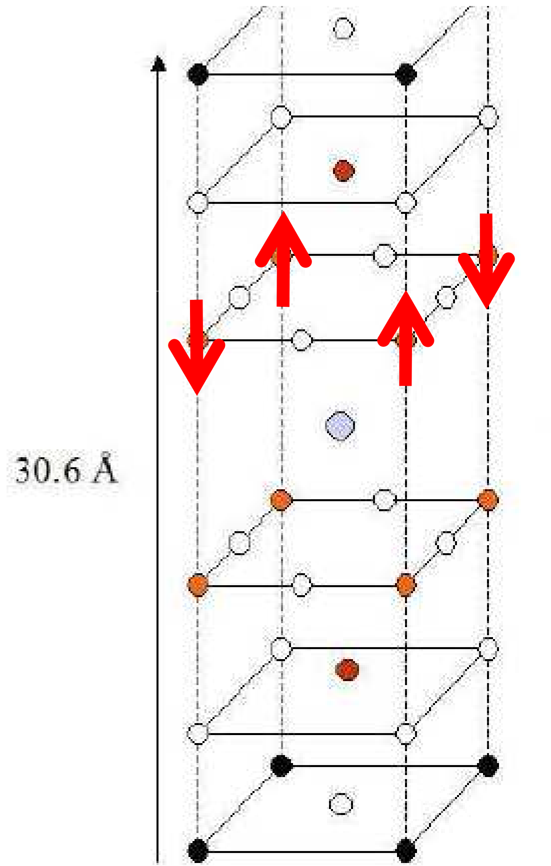
Main band

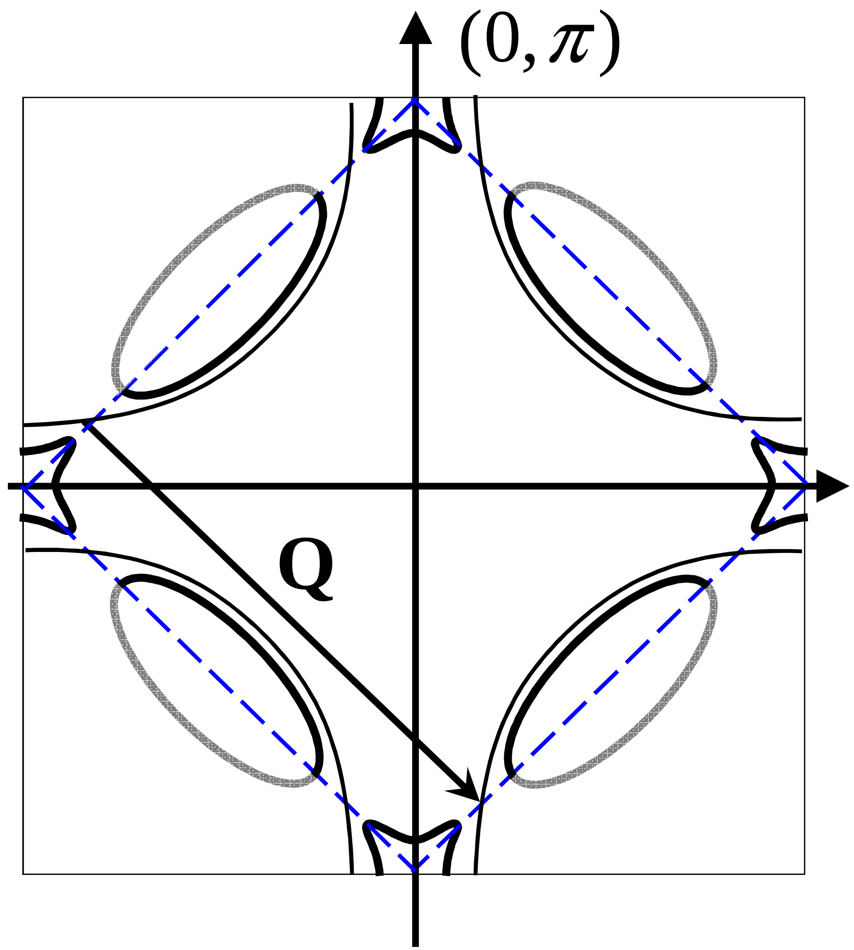
↓

Pockets

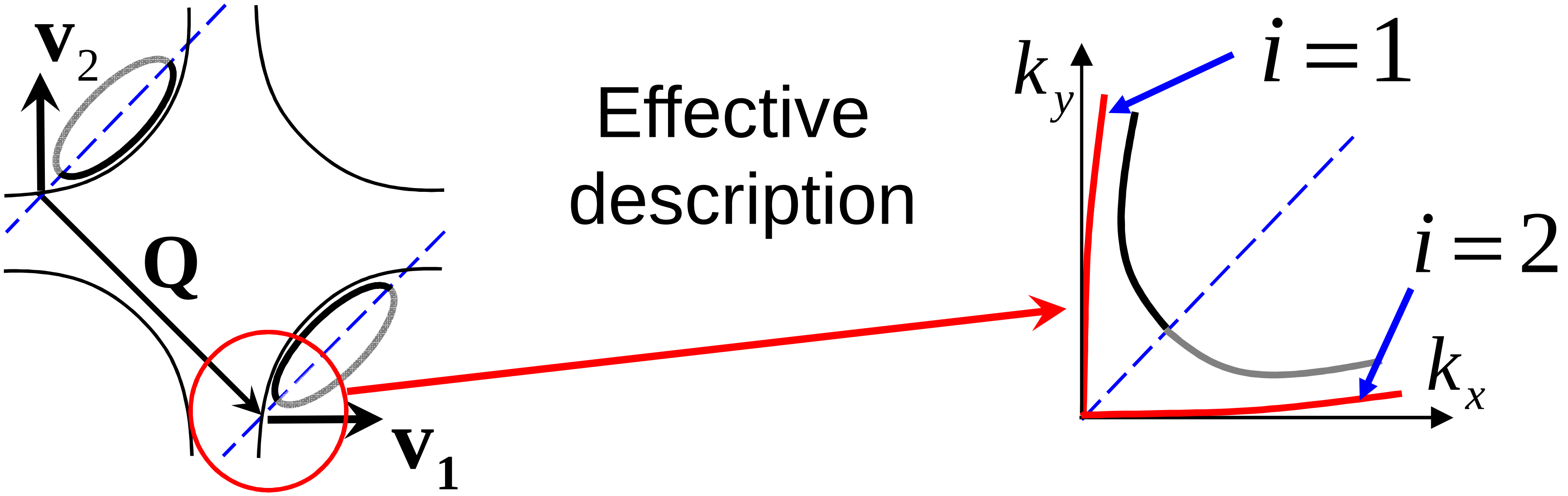
Fermi Surface Reconstruction (cont)

$$H = \sum_{\boldsymbol{k}\alpha} \xi(\boldsymbol{k}) \psi^\dagger_{\boldsymbol{k}\alpha} \psi_{\boldsymbol{k}\alpha} + J \sum_{\boldsymbol{k}} \boldsymbol{S} \psi^\dagger_{\boldsymbol{k}+\boldsymbol{Q}\alpha} \boldsymbol{\sigma}_{\alpha\beta} \psi_{\boldsymbol{k}\beta}$$





Fermi Surface Reconstruction (cont)



$$\mathcal{H} = K.E. + H_{coupl} + H\left[\boldsymbol{S}\right]$$

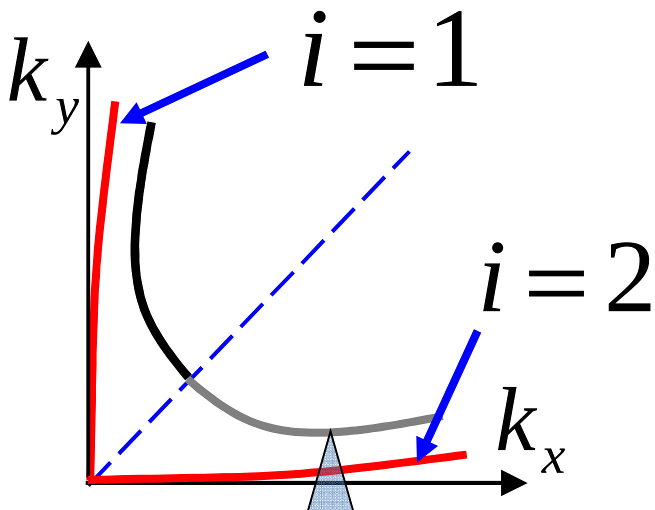
$$\xi_k \longrightarrow \boldsymbol{v}_{1,2}\boldsymbol{k}$$

Fermi Surface Reconstruction (cont)

$$H\left[\boldsymbol{S}\right] = \frac{2\pi}{d}\int d^2\boldsymbol{r}\left[\boldsymbol{\nabla}\phi(\boldsymbol{r})\right]^2$$

$$d = T/4\pi\rho_s$$

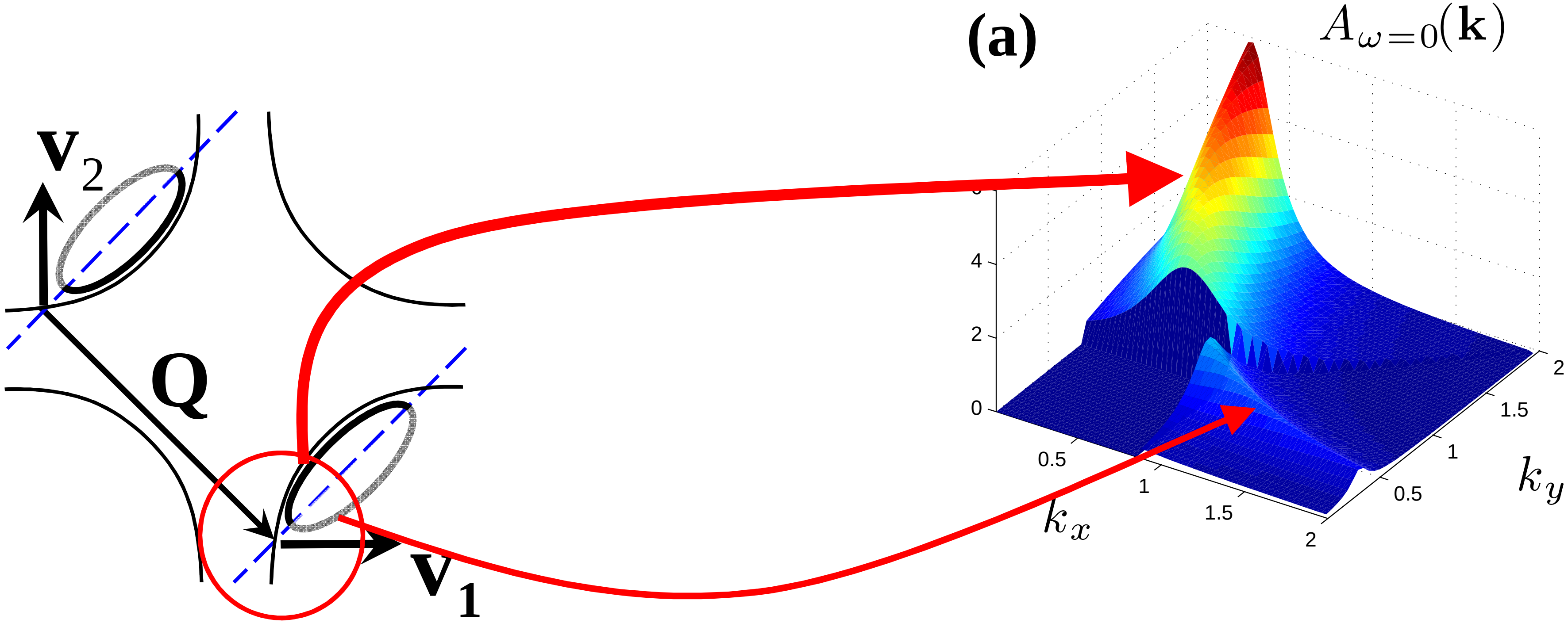
Easy plane



$$H_{coupl} = J\sum_{\alpha\beta} \mathrm{e}^{\mathrm{i}\phi}\bar{\psi}_{1,\alpha}\sigma^{-}_{\alpha\beta}\psi_{2,\beta}$$

$$\rho_s \longrightarrow \infty$$ Mean field

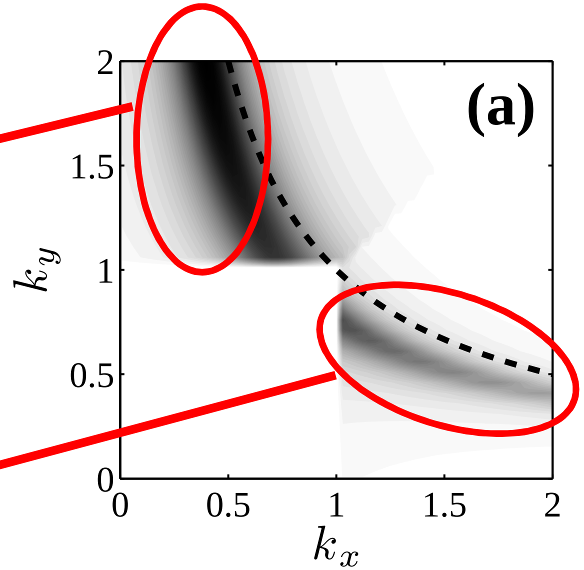
Fermi Surface $$v^2 k_x k_y - J^2 = 0$$



Higher T

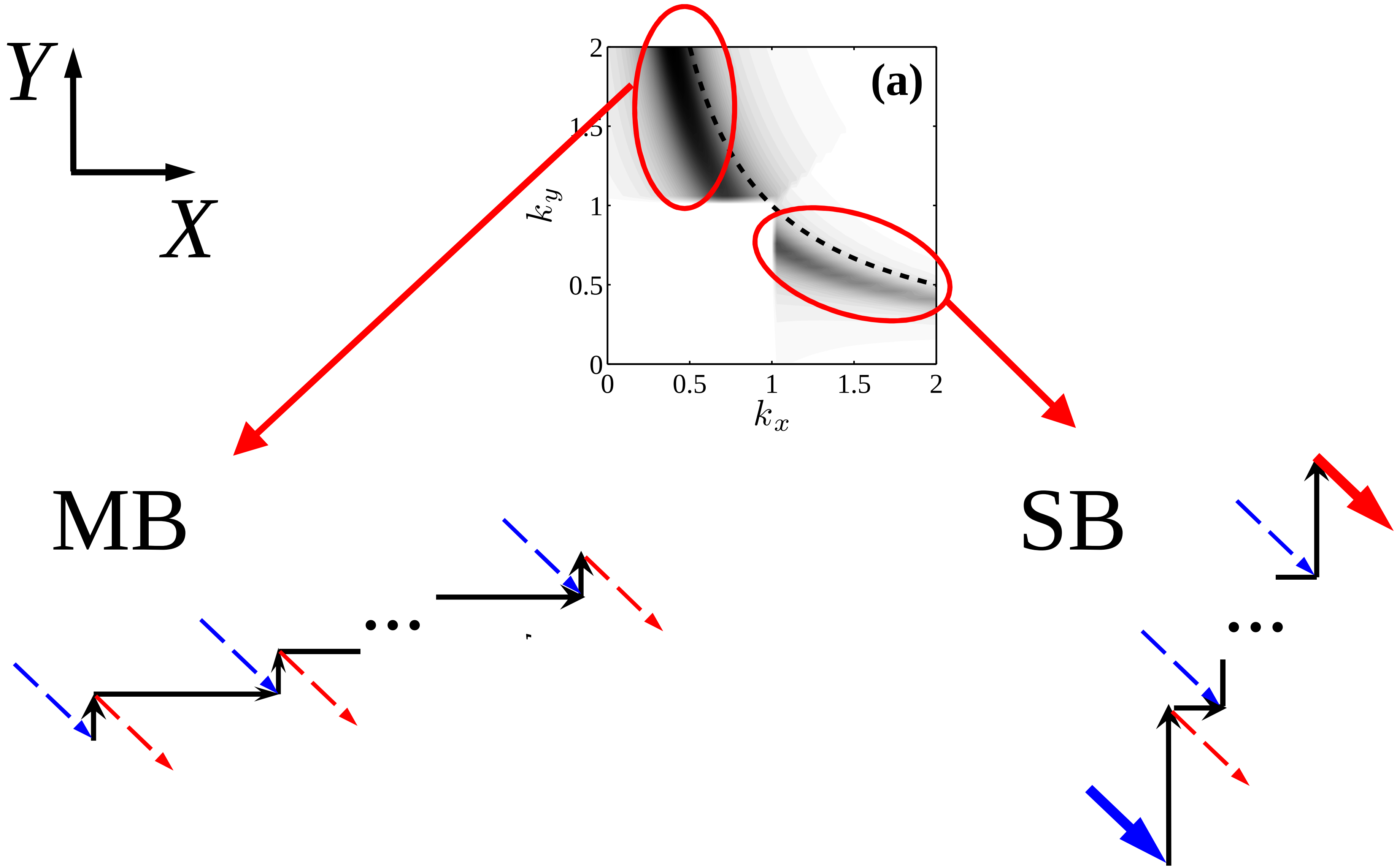
Results

$$G^{-1} = \omega - k_x + \frac{\mathrm{i}J^2 a^{2d}}{(-\mathrm{i}(\omega - k_y))^{1-2d}}$$



$$G = \frac{J^2 e^{\mathrm{i}\pi d}}{(\omega - k_x)^2}\left[\omega - k_y + \frac{\mathrm{i}J^2 a^{2d}}{(-\mathrm{i}(\omega - k_x))^{1-2d}}\right]^{-1+2d}$$

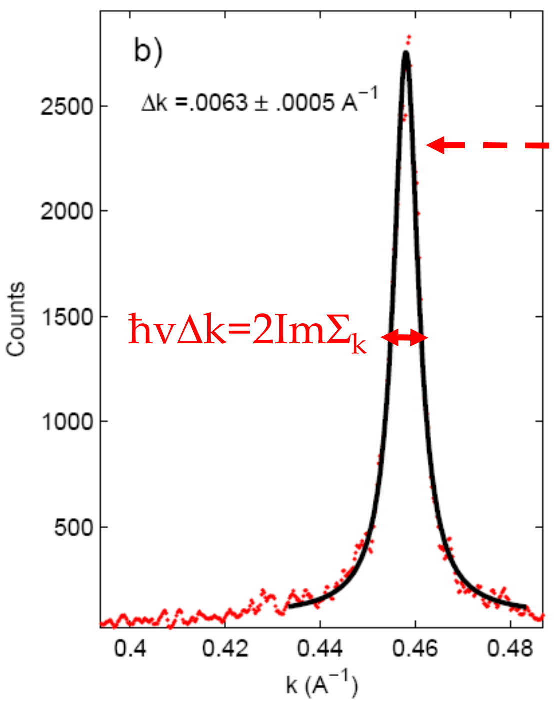
Results

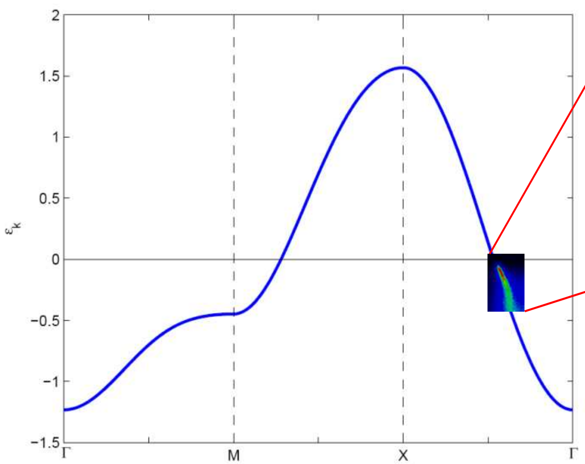


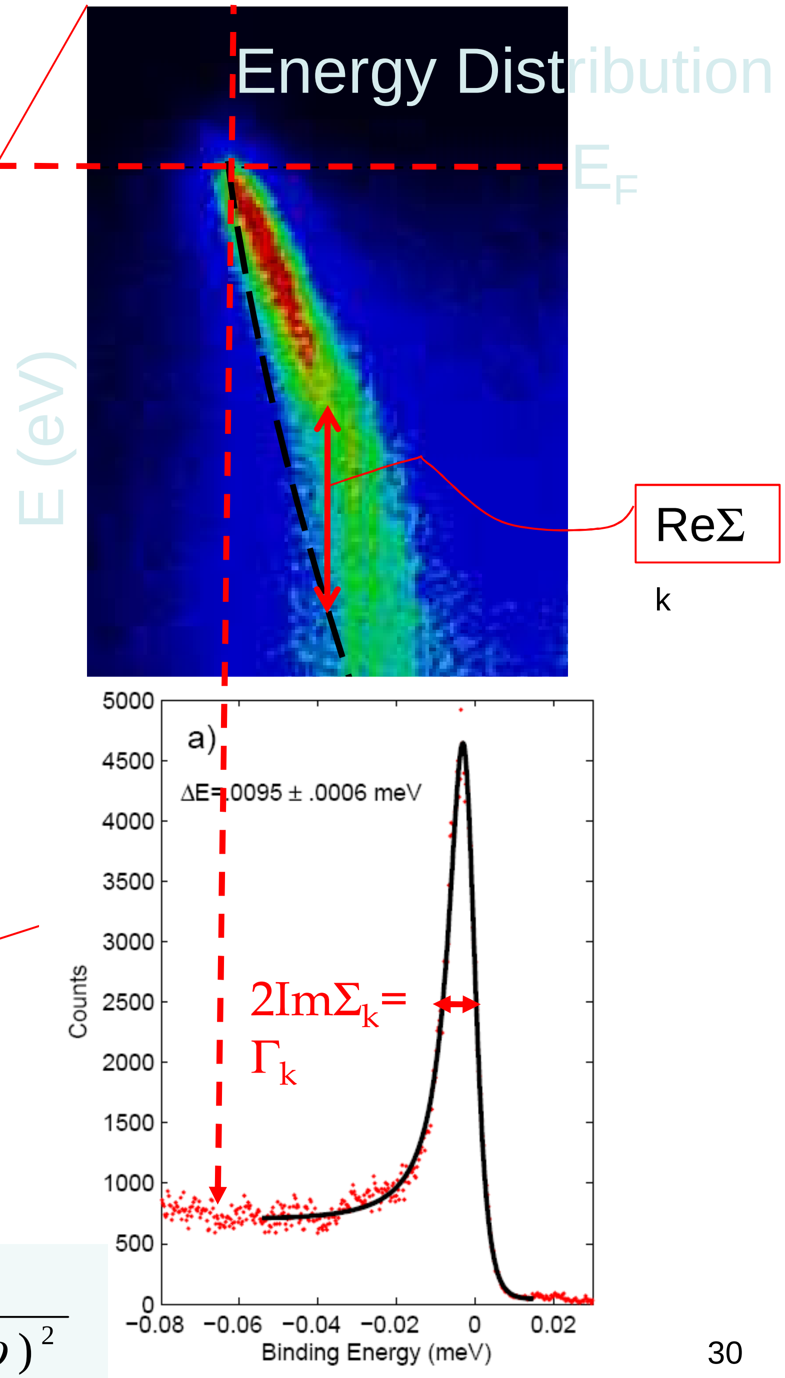
Conclusions

1. We have studied the spectral density in the presence of strong phase fluctuations
2. Closed expressions is obtained below Tc. Interpolation expression in the close proximity of the transition were derived as well.
3. The Fermi Surface topology changes with the doping. We have elucidated the role played by critical fluctuations of the order parameter in formation of shadow bands.

Thank you!





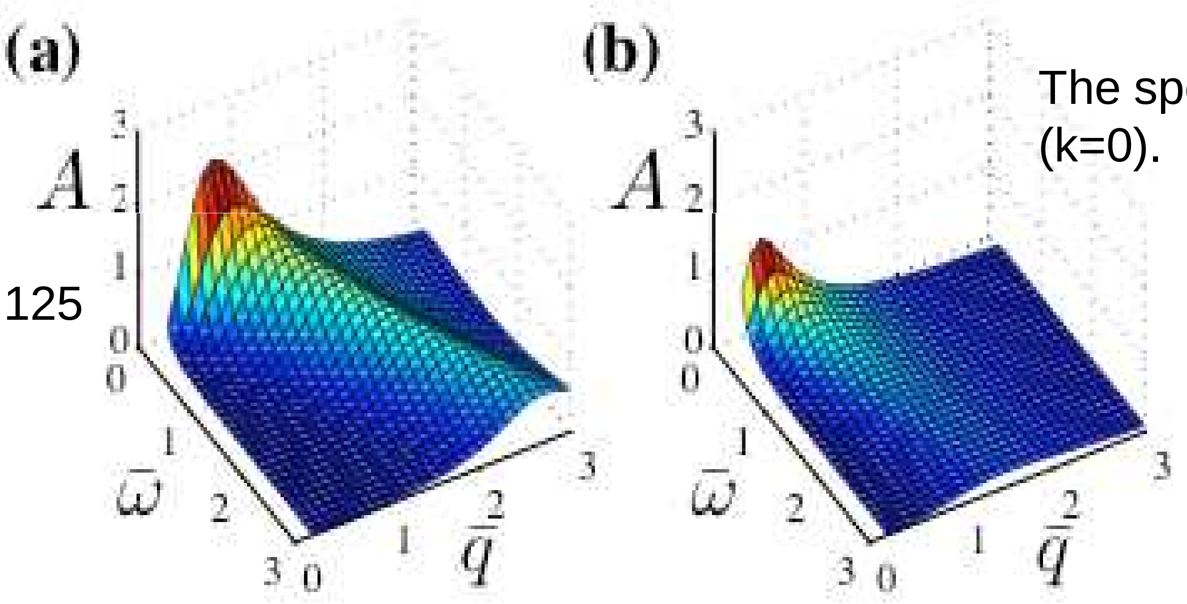


$$\tilde{A}(k,\omega) = \frac{1}{\pi}\frac{f(\omega)Z_k\ \mathrm{Im}\ \Sigma_k(\omega)}{(\omega-\varepsilon_k-\mathrm{Re}\ \Sigma_k(\omega))^2+\mathrm{Im}\ \Sigma_k(\omega)^2}$$

Results (T>Tc)

$$\Sigma(q,k,\omega)=\Delta^2(q)(\xi/a)^{-2d}\,\frac{e^{-i\pi d}\left[(k+\omega/v)\xi+\sqrt{(k+\omega/v)^2\xi^2+1}\right]^{2d}-\left[(k+\omega/v)^2\xi^2+1\right]^{-1/2}}{\left[(k+\omega/v)^2+\xi^{-2}\right]^{1/2}}$$

d=0.125



d=0.25

The spectral function along FS (k=0).

Away from the singularity we employ the extrapolation formula

$$\Sigma(q,k,\omega)=\Delta^2(q)(\xi/a)^{-2d}\,\frac{e^{-i\pi d}[(k+\omega/v)\xi+\sqrt{(k+\omega/v)^2\xi^2+1}]^{2d}-[(k+\omega/v)^2\xi^2+1]^{-1/2}}{[(k+\omega/v)^2+\xi^{-2}]^{1/2}}$$

which reproduces both limits $\xi=\infty$ and $\xi(\omega/v+k)<<1$

Effective Impurity model.

$$H = v^{-1}\{(i\omega - 0)\tau^3 + (a\Delta_q)[\tau^+ \exp(-i\varphi) + \tau^- \exp(i\varphi)]\} + H_{bulk}$$

X is dual to k and plays a role of Matsubara time,
real frequency is like an imaginary magnetic field.

$$\tau^a = \Psi^+ \tau^a \Psi,$$

$$\Psi = (\psi_\uparrow, \psi_\downarrow^+)^T$$

$$\sum_\sigma \psi_\sigma^+ \psi_\sigma = 1$$

$$\varphi = \varphi(x,0)$$

$$H_{bulk}[\varphi] = \frac{2\pi}{d}\int dy[\Pi^2 + (\partial_x \varphi)^2 + vortices]$$

$$[\varphi(y), \Pi(y')] = i\delta(y - y')$$

$$d = T/8T_{BKT}$$

The occupation number is fixed because the fermions are integrated out and hence there are no fermionic loops.

$$< e^{i\varphi(r)} e^{-i\varphi(r')} >= (a/\xi)^{2d} F(|\vec{r} - \vec{r'}|/\xi)$$

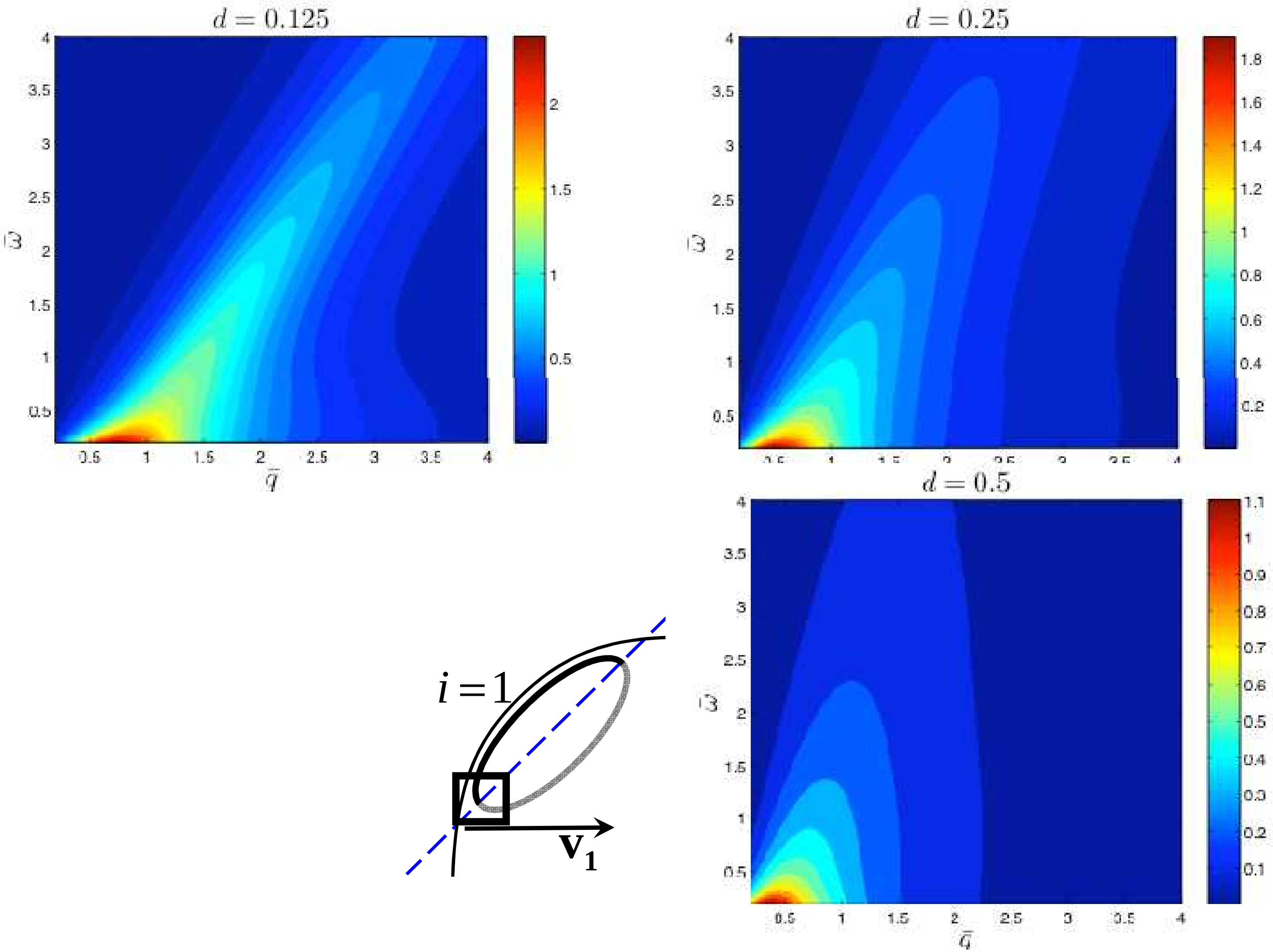
$$F(z << 1) = z^{-2d},$$

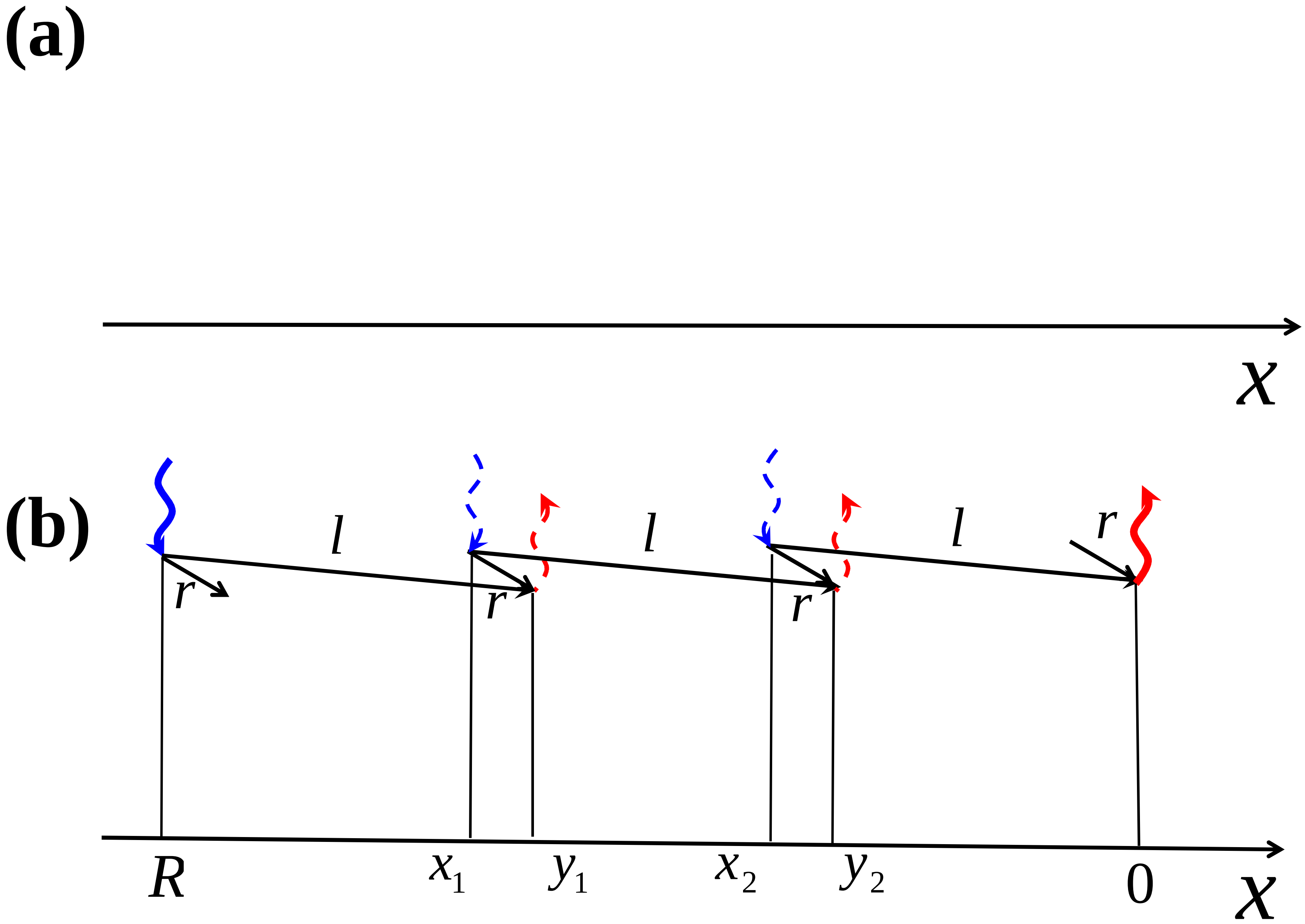
$$F(z > 1) \sim K_0(z)$$

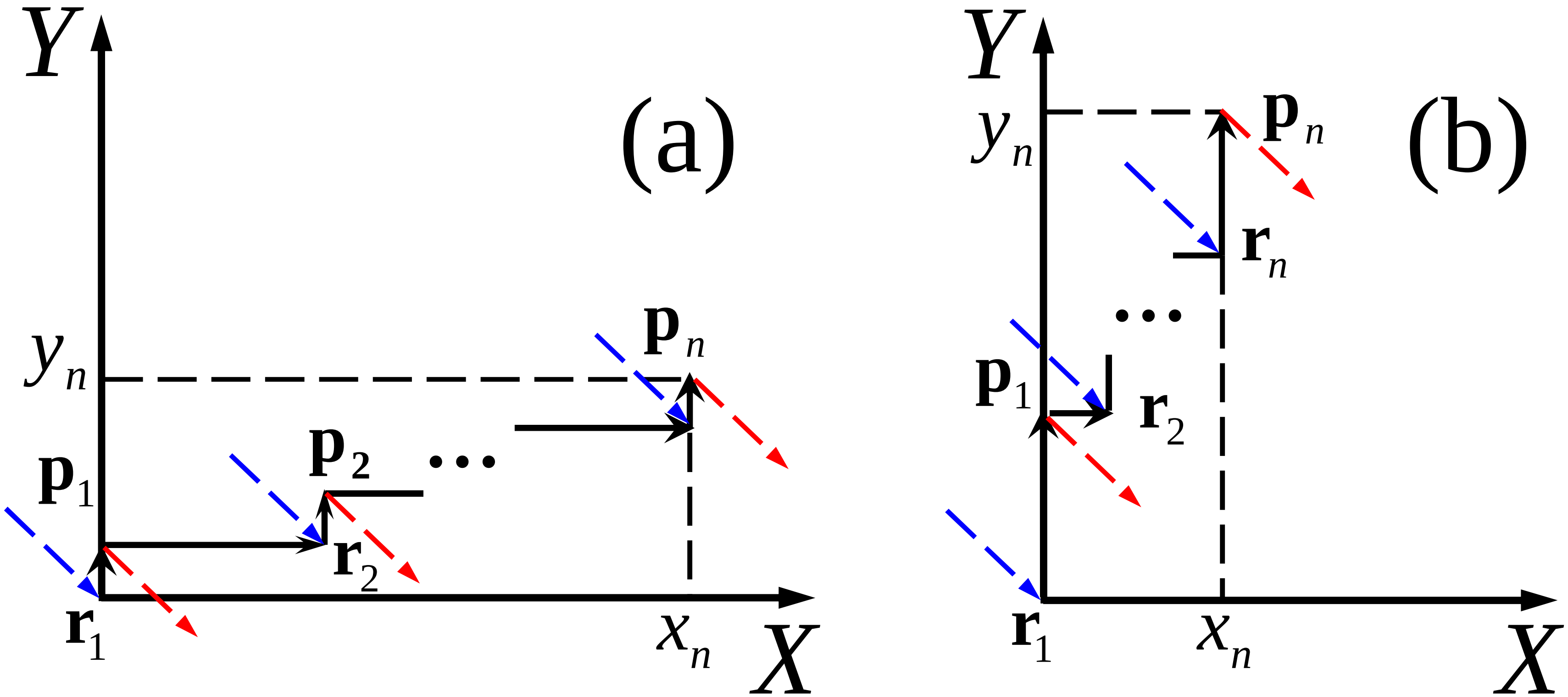
d=T/8T$_{BKT}$

Below T$_{BKT}$ the correlation length =
and the problem ~ to anisotropic Kondo
model in
imaginary magnetic field

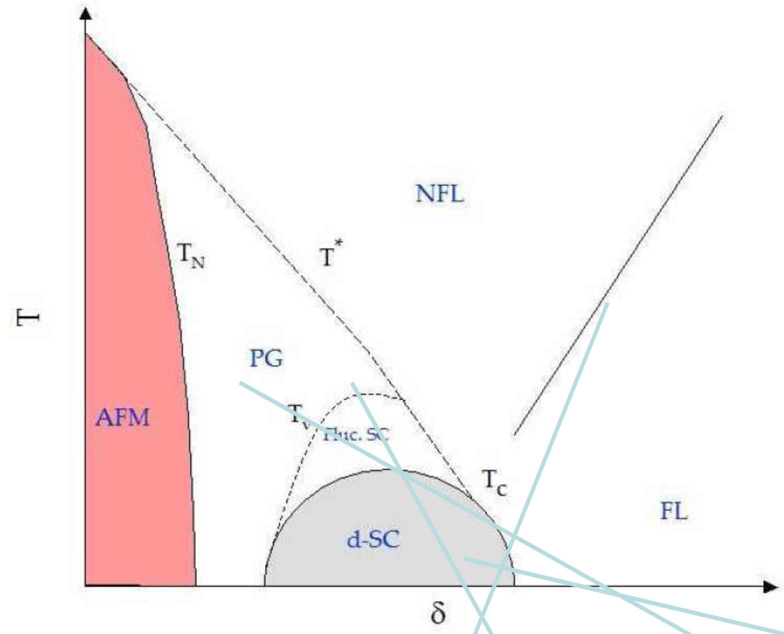
Mean Field Dispersion

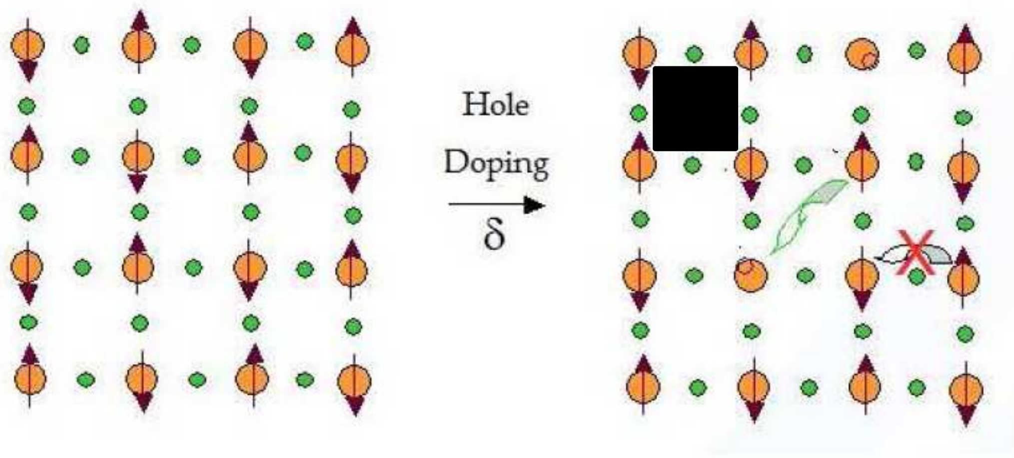



(a)
X
(b)
l
l
l
r
r
r
r
R
x1
y1
x2
y2
0
X

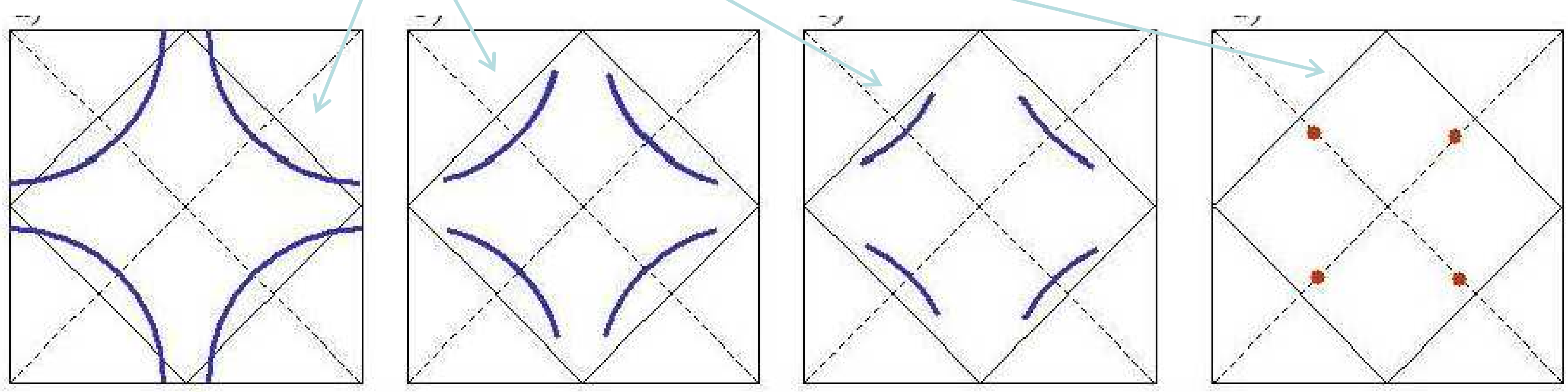


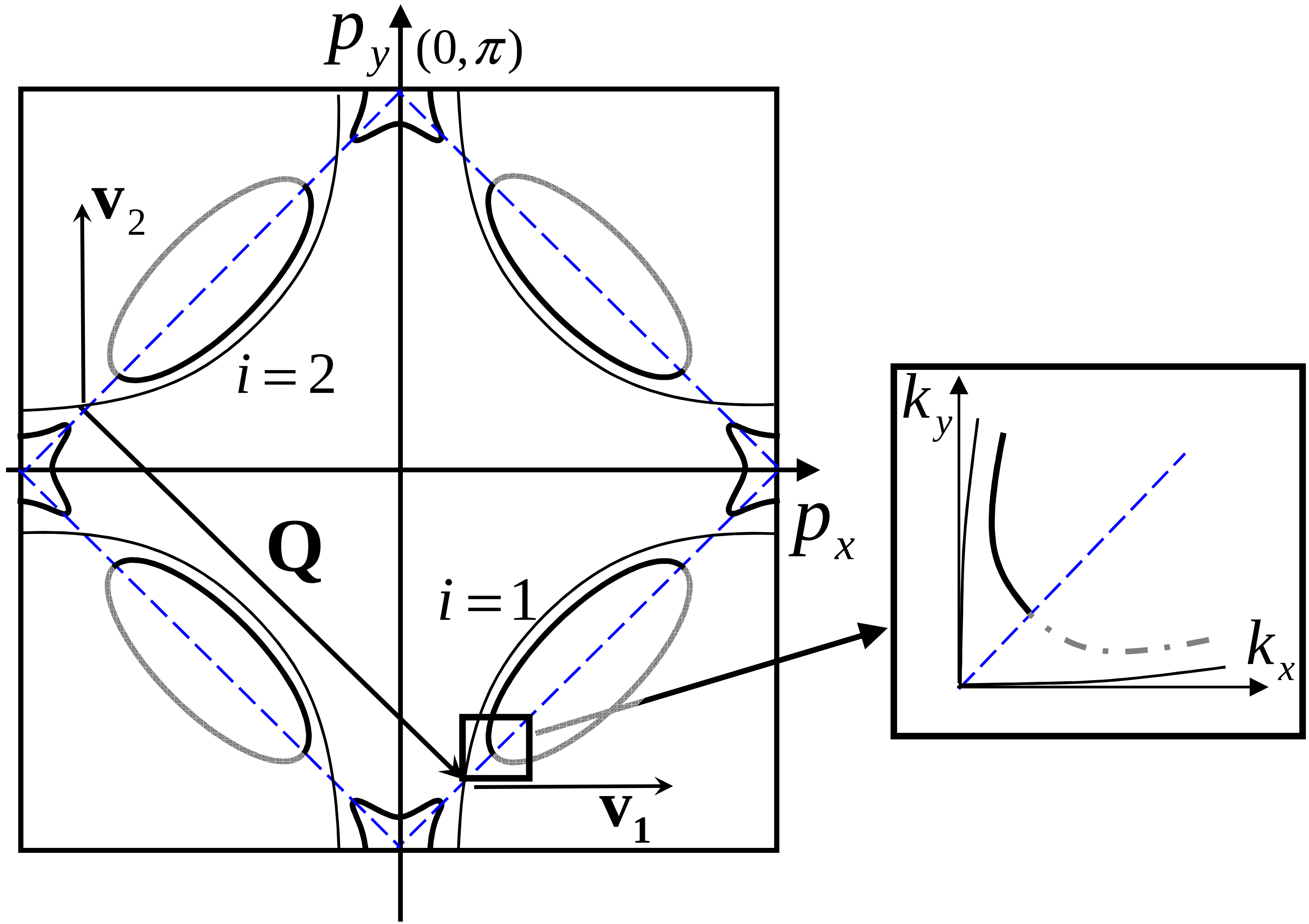
(a)
Y
X
y_n
x_n
$\mathbf{p}_1$
$\mathbf{p}_2$
$\mathbf{p}_n$
$\mathbf{r}_1$
$\mathbf{r}_2$
(b)
Y
X
y_n
x_n
$\mathbf{p}_1$
$\mathbf{p}_n$
$\mathbf{r}_1$
$\mathbf{r}_2$
$\mathbf{r}_n$





Doped Mott Insulator!





p_y
$(0,\pi)$
$\mathbf{v}_2$
$i=2$
p_x
$\mathbf{Q}$
$i=1$
$\mathbf{v}_1$
k_y
k_x

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3. Phase fluctuations and Spectral Density
4. Fermi Surface reconstruction
5. Spin Density Wave order parameter fluctuations
6. Results and Conclusions