Cavity optomechanics

Pierre Meystre

B2 Institute, Dept. of Physics and
College of Optical Sciences
The University of Arizona

Mishkat Bhattacharya
Omjyoti Dutta
Swati Singh

ARO NSF ONR

quantum metrology

- How can quantum resources be exploited with maximal robustness and minimal technical overhead?

- What are the inherent sensitivity-reliability tradeoffs in quantum sensors?

- What is the role of particle statistics in quantum metrology?

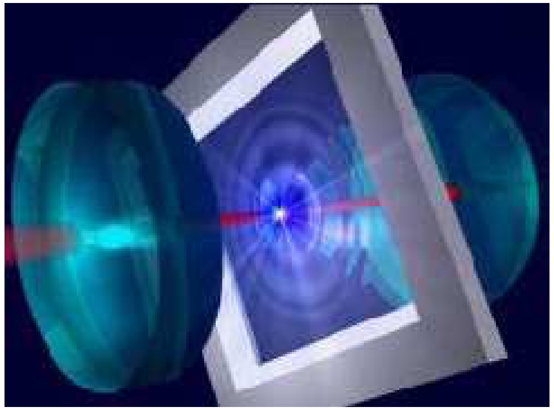
- How do decoherence mechanisms set fundamental limits to measurement precision?

This talk:

- Cooled nanoscale cantilevers for quantum control of AMO systems

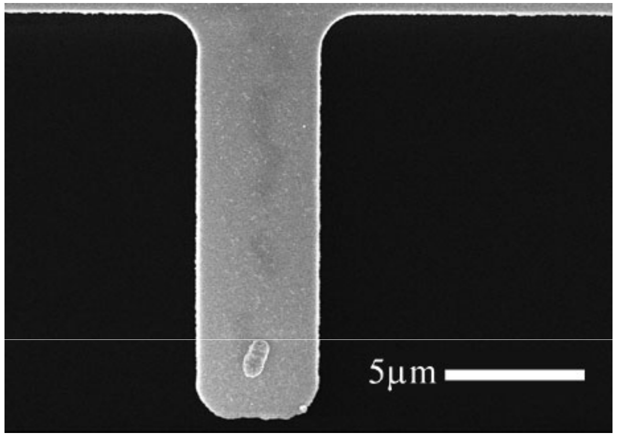
outline

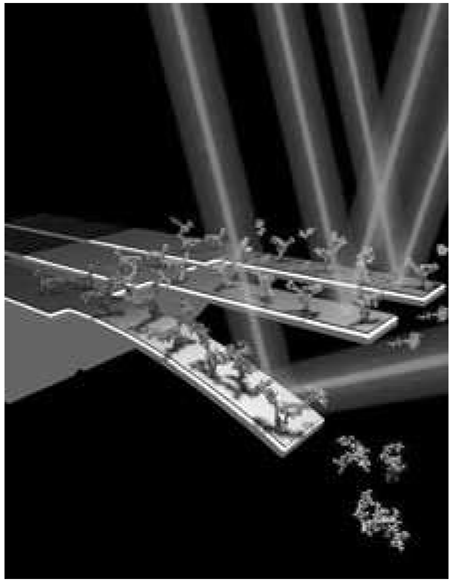
- Brief review of optomechanical mirror cooling
- Two- and three-mirror cavities
- Cantilever coupling to dipolar molecules
  - Single molecule
  - Phonon squeezing in molecular lattice
  - Coherent control
- Outlook



(J. Sankey, Yale University)

micromechanical cantilevers – single-particle detectors

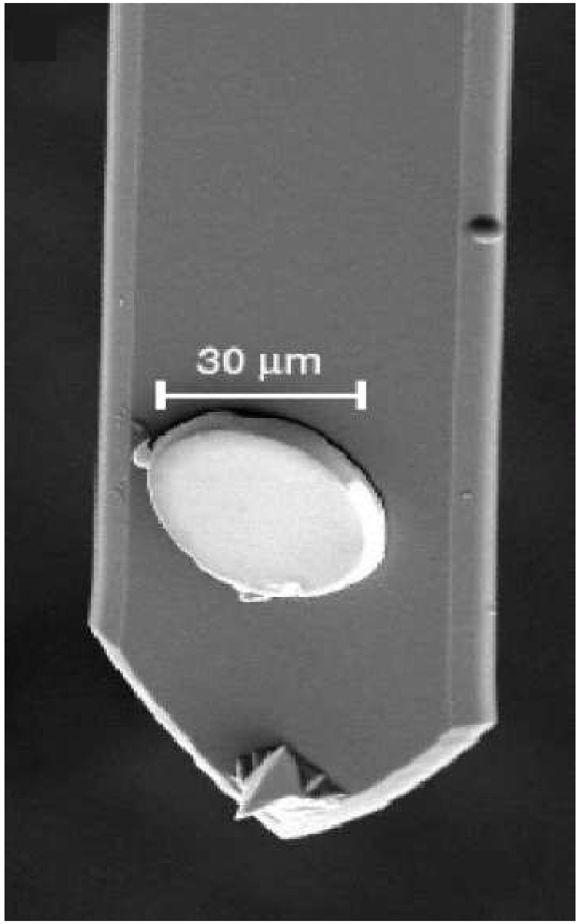




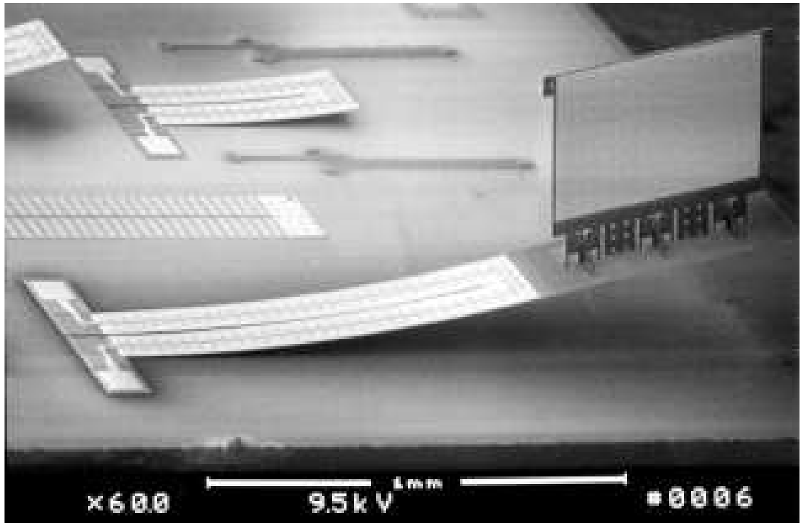
Micromechanical cantilever with a single E. Coli cell
A micromechanical cantilever with a single E. Coli cell attached. The cantilever is used to detect changes in mass due to selectively attached biological agents present in small quantities.
Craighead Research Group, Cornell University

Protein detection
Three cantilevers coated with antibodies to PSA, a prostate cancer marker found in the blood. The left cantilever bends as the protein PSA binds to the antibody. *Photo courtesy of Kenneth Hsu/UC Berkeley & the Protein Data Bank*

micromirrors



D. Kleckner & D. Bouwmeester, Nature **444**, 75 (2006)



SEM photograph of a vertical thermal actuator with integrated micromirror. Application of a current to the actuator arm produces vertical motion of the mirror, which can either reflect an optical beam or allow it to be transmitted.
(Southwest Research Institute, http://www.swri.org/3pubs)

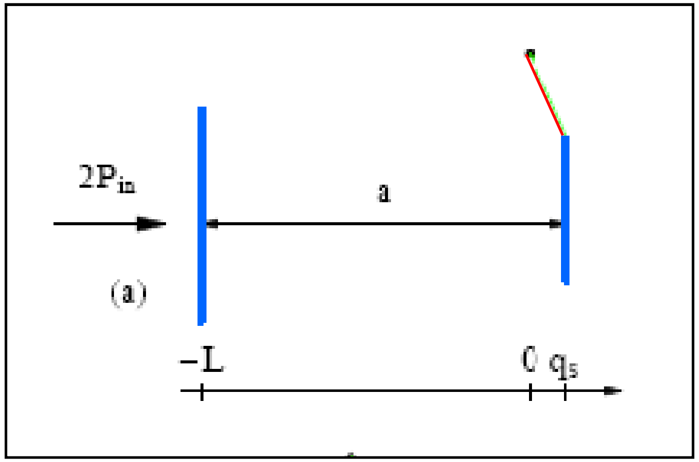
radiation pressure mirror cooling

Basic idea: change frequency and damping of a mirror mounted on a spring using radiation pressure

Number of quanta of vibrational motion:

$$\boxed{n = \frac{k_B T_{\mathrm{eff}}}{\hbar\Omega_{\mathrm{eff}}} \approx \frac{k_B T_{\mathrm{env}}}{\hbar\Omega_{\mathrm{eff}}}\left(\frac{\Gamma_M}{\Gamma_{\mathrm{eff}}}\right)}$$

[H. Metzger and K. Karrai, Nature **432**, 1002 (2004)]



- <u>Cold damping</u>: laser radiation increases mirror damping.
- Requires laser tuned below cavity resonance
- Little change in mirror frequency

- <u>Increase effective mirror frequency</u>
- Requires laser tuned above cavity resonance

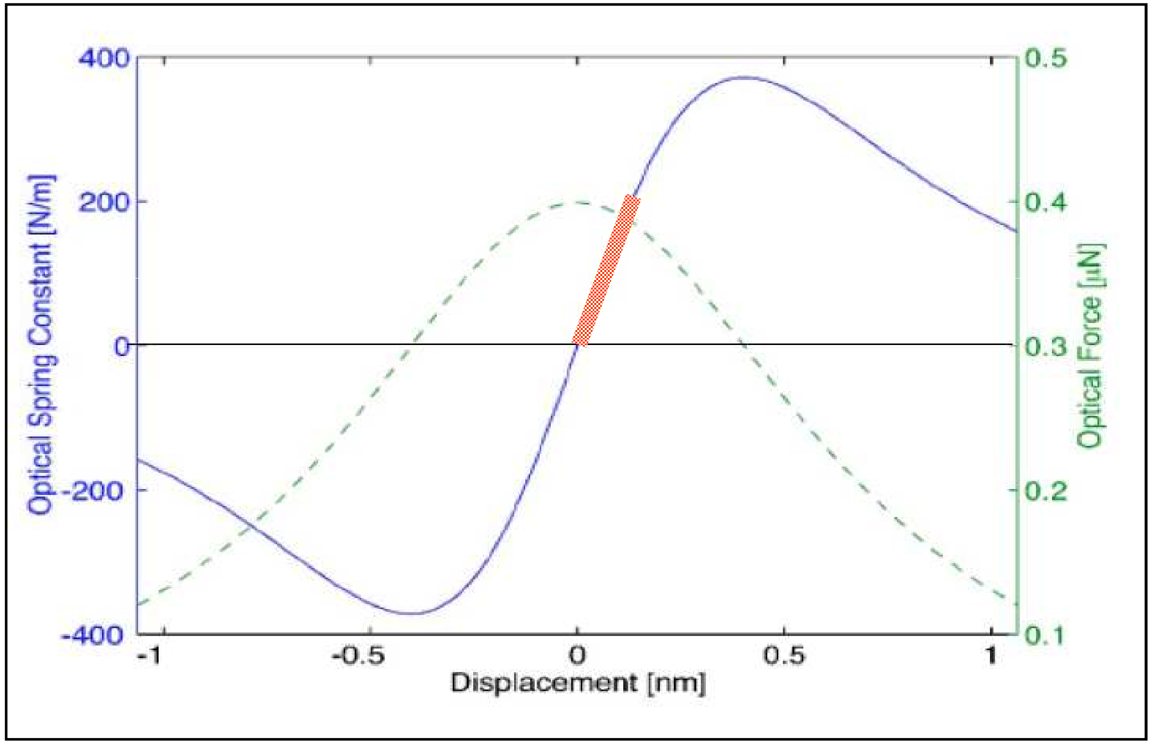
- Use two lasers!

[T. Corbitt *et al*, PRL **98**, 150802 (2007)]

basic optomechanics – cold damping

Goal: increasing Ω_{eff}

Trapping due to `optical spring': $k_{\mathrm{opt}} = -\frac{dF_{\mathrm{rp}}(x)}{dx}$



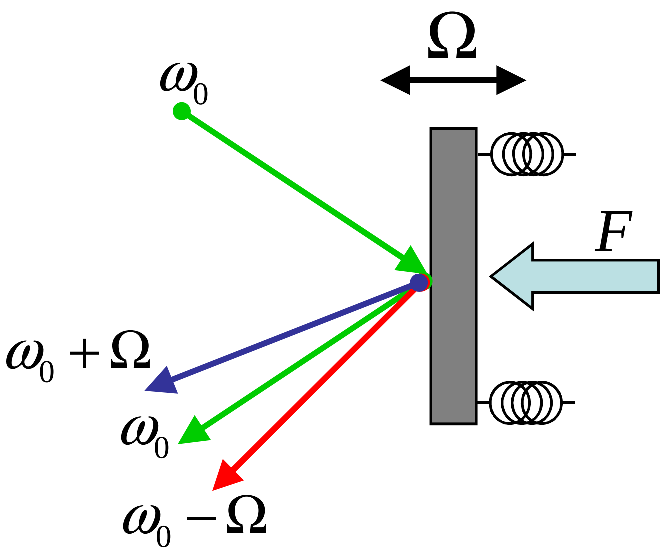
B. S. Sheard *et al.* PRA **69**, 051801(R) (2004)

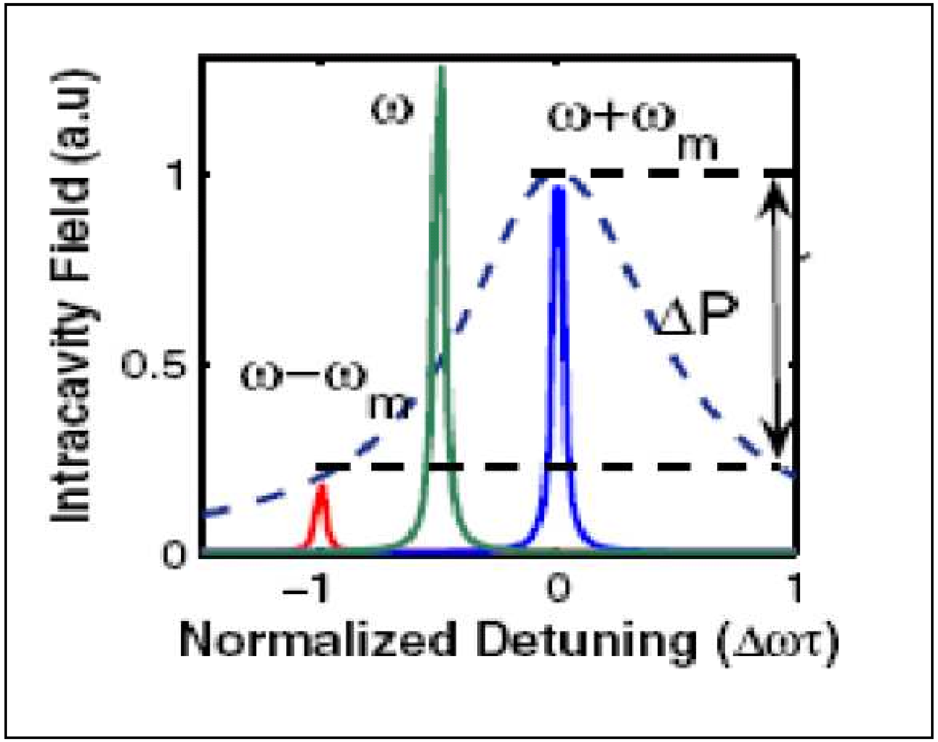
Need blue-detuned cavity

basic optomechanics – mirror cooling

Goal: decreasing $T_{\text{eff}} = T_{\text{env}} (\Gamma_M / \Gamma_{\text{eff}})$

Cooling due to asymmetric pumping of sidebands





M. Lucamarini *et al.*, PRA **74**, 063816 (2006)
A. Schliesser *et al.* PRL **97**, 243905 (2006)

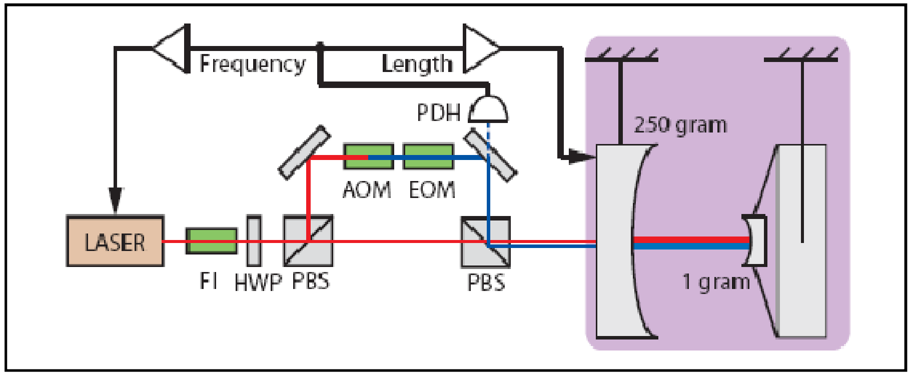
Need red-detuned cavity

competing requirements

- Blue detuned cavity
  - Restoring force (optical spring)
  - Antidamping

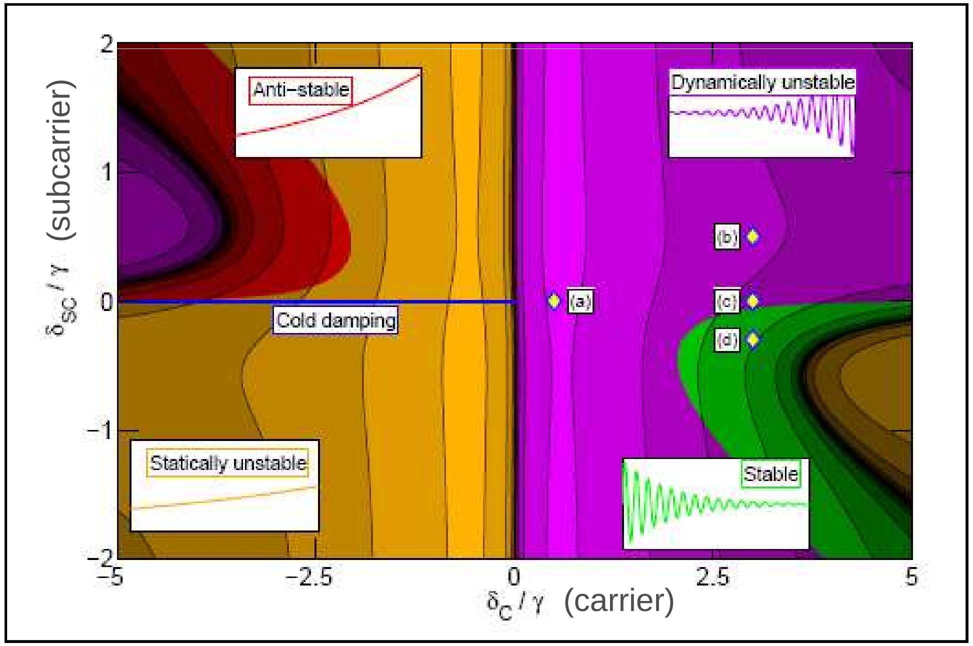
- Red detuned cavity
  - Anti-restoring force
  - Damping

two-wavelengths approach



T. Corbitt *et al*, PRL **98**, 150802 (2007)

Spring constant	damping	
$k>0$	$\Gamma>0$	Dynamically unstable
$k>0$	$\Gamma<0$	Stable
$k<0$	$\Gamma>0$	"anti-stable"
$k<0$	$\Gamma<0$	Statically unstable



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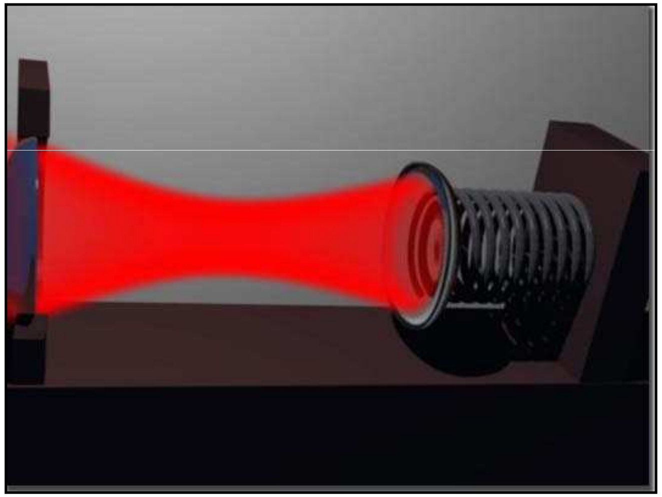
Prehistory

- V. Braginsky, “Measurement of weak forces in physics experiments” (Univ. of Chicago Press, Chicago, 1977).
- V.B. Braginsky and F.Y. Khalili, "Quantum measurement'' Cambridge University Press, Cambridge, 1992



- C. M. Caves, “Quantum mechanical noise in an interferometer,” Phys. Rev. **D23,** 1693 (1981).

prehistory

Optical Bistability and Mirror Confinement Induced by Radiation Pressure

A. Dorsel
Sektion Physik, Universität München, D-8046 Garching, West Germany

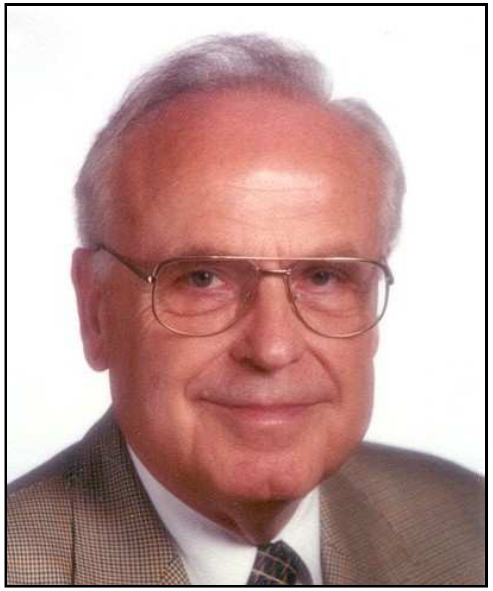
and

J. D. McCullen, P. Meystre, and E. Vignes
Max-Planck-Institut für Quantenoptik, D-8046 Garching, West Germany

and

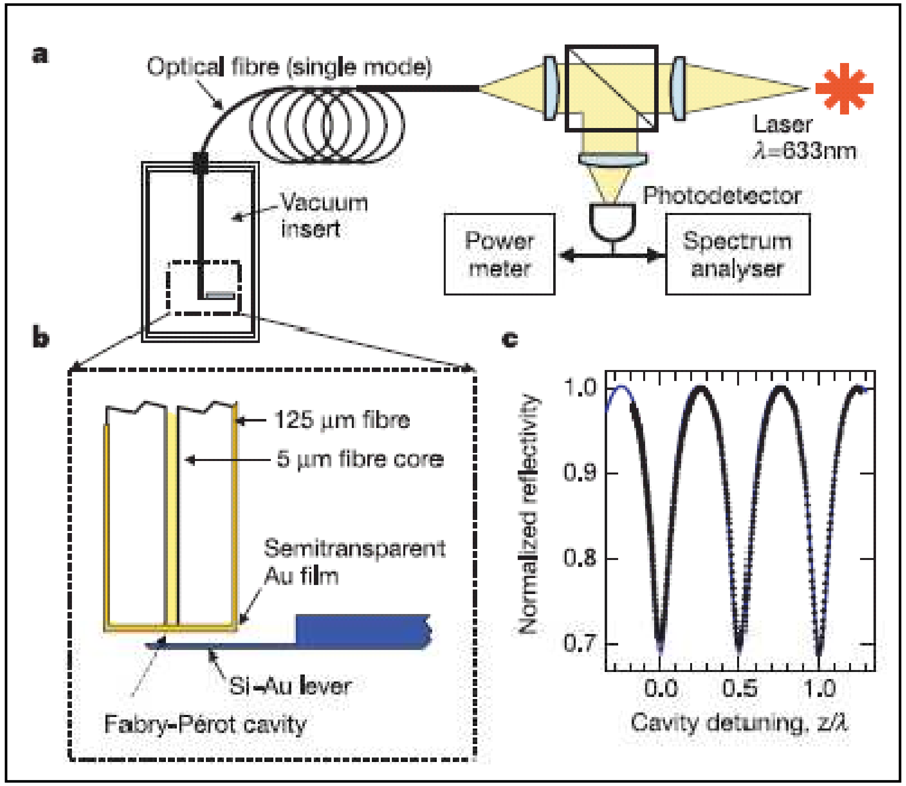
H. Walther
Sektion Physik, Universität München, D-8046 Garching, West Germany, and
Max-Planck-Institut für Quantenoptik, D-8046 Garching, West Germany
(Received 27 July 1983)

This paper reports the observation of optical bistability due to a radiation-pressure-induced change of the length of a Fabry-Perot resonator. In addition, for higher laser powers, a purely optical mechanism leading to the stabilization of the resonator has been observed.



early history

"Cavity cooling of a microlever," C. Höhberger-Metzger & K. Karrai, Nature **432**, 1002 (2004)



18K

recent history

S. Gigan *et al.* (Zeilinger group) “Self-cooling of a micromirror by radiation pressure”, Nature **444**, 67 (2006)

< 10K

O. Arcizet *et al*. (Pinard/Heidman group) “Radiation pressure cooling and optomechanical instability of a micromirror”, Nature **444**, 71 (2006)

10K

D. Kleckner *et al.* (Bouwmeester group) “Sub-Kelvin optical cooling of a micromechanical resonator”, Nature **444**, 75 (2006)

135 mK

A. Schliesser *et al*. (Vahala/Kippenberg groups) “Radiation pressure cooling of a micromechanical oscillator using dynamical back-action” PRL **97**, 243905 (2006)

11K

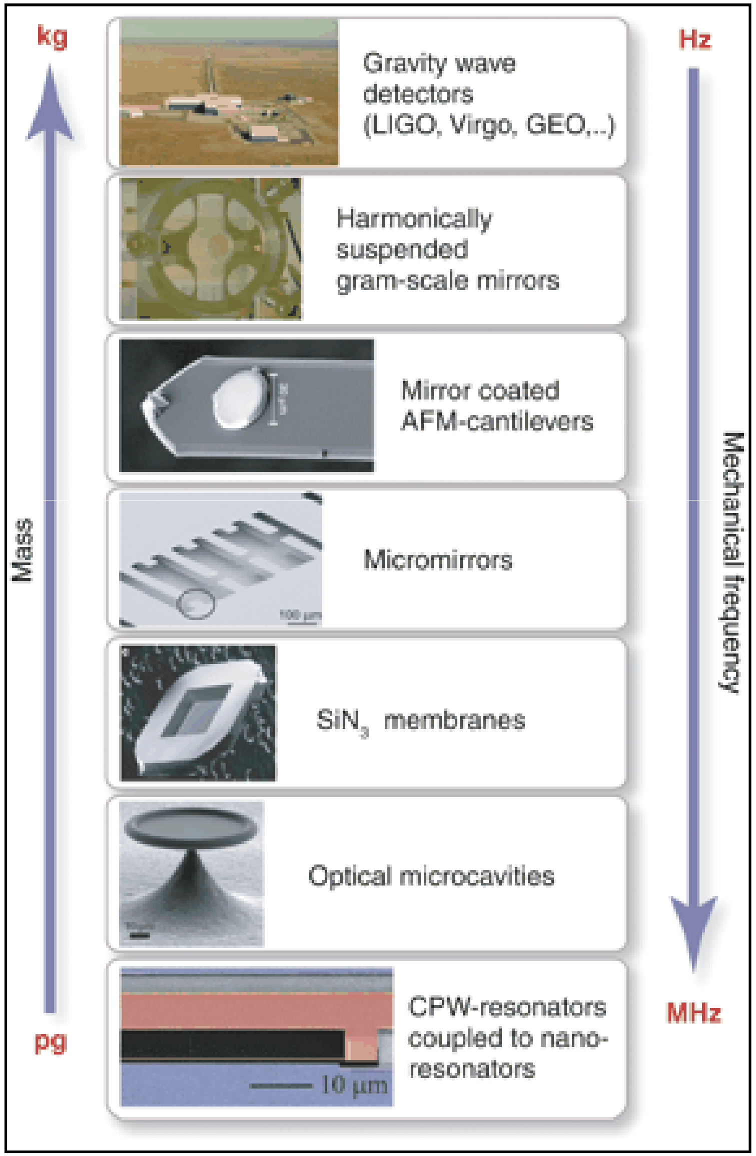
T. Corbitt *et al.* (LIGO) “An all-optical trap for a gram-scale mirror”, PRL **98**, 150802 (2007), “Optical dilution and feedback cooling of a gram-scale oscillator”, PRL **99**, 160801 (2007)

6.9 mK

A. Vinante *et al* (AURIGA gravitational wave detector mirror), M_{eff}= 1,100 Kg, PRL **101,** 033601 (2008)

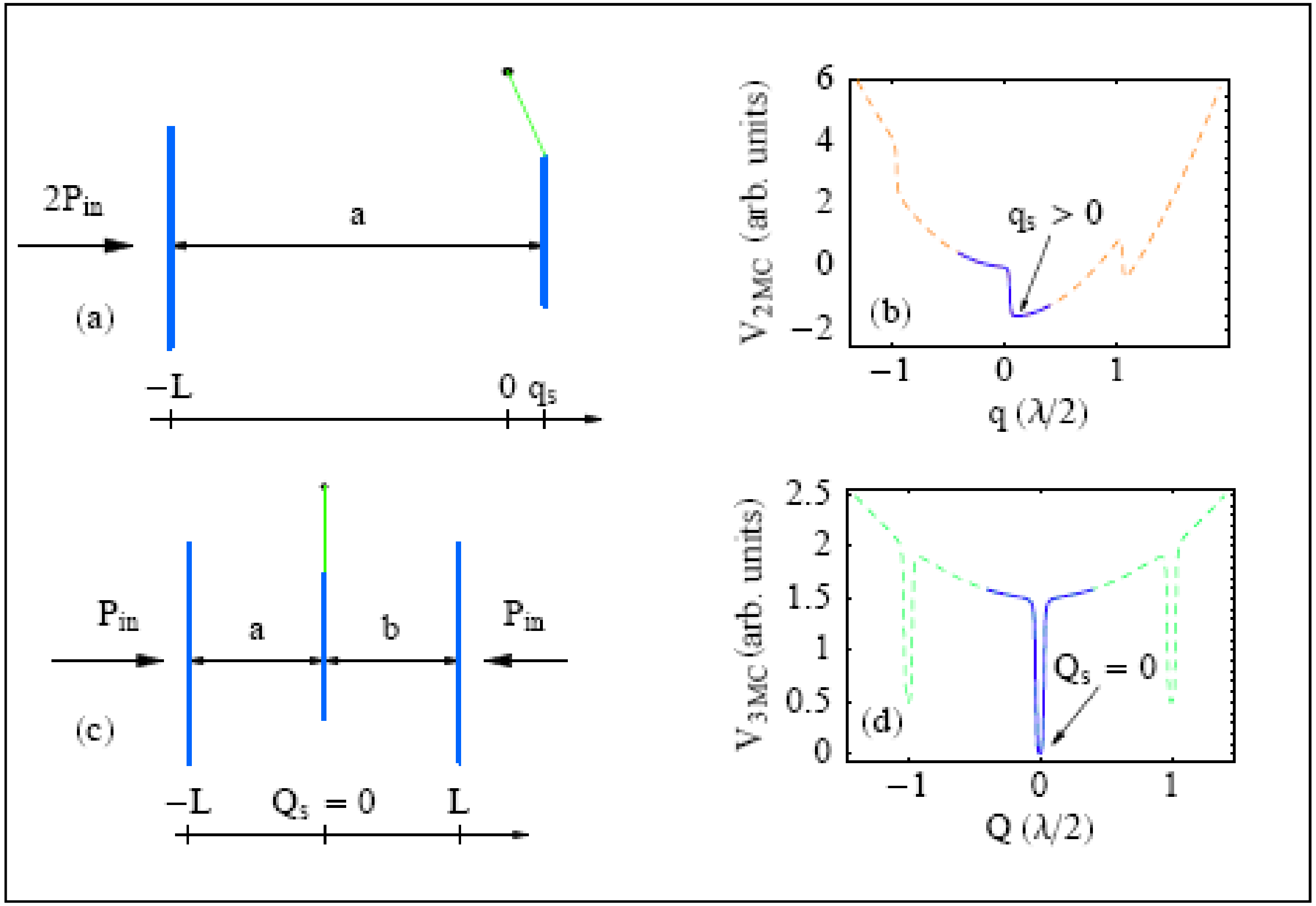
0.17 mK

systems



From T. Kippenberg and K. Vahala
Science **321**, 1172 (2008)

three-mirror geometry



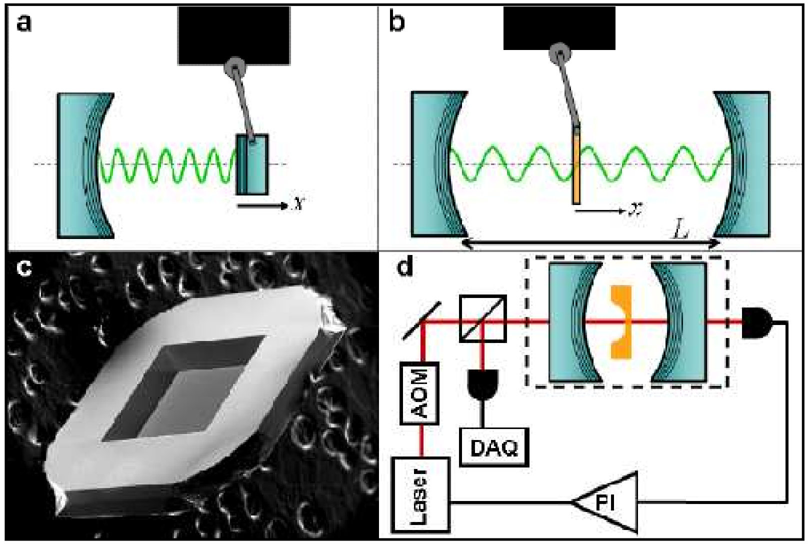
[A. Dorsel *et al.*, PRL 51, 1550 (1983); PM *et al*, JOSA B**11**, 1830 (1985); J. D. McCullen *et al.*, Optics Lett. **9**, 193 (1984)]

three-mirror cavity

Strong dispersive coupling of a high finesse cavity to a micromechanical membrane

J. D. Thompson[1], B. M. Zwickl[1], A. M. Jayich[1], Florian Marquardt[2], S. M. Girvin[1,3], & J. G. E. Harris[1,3] Nature **452**, 72 (2008)

… We demonstrate a cavity which is detuned by the motion of a 50 nm thick dielectric membrane placed between two macroscopic, rigid, high-finesse mirrors. This approach segregates optical and mechanical functionality to physically distinct structures and avoids compromising either. It also allows for direct measurement of the square of the membrane's displacement, and thus in principle the membrane's energy eigenstate. We estimate it should be practical to use this scheme to observe quantum jumps of a mechanical system, a major goal in the field of quantum measurement.



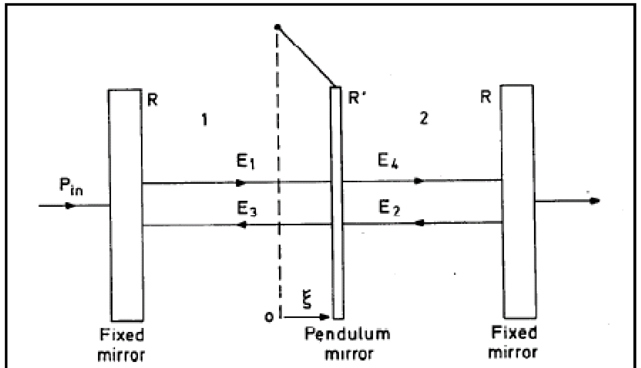
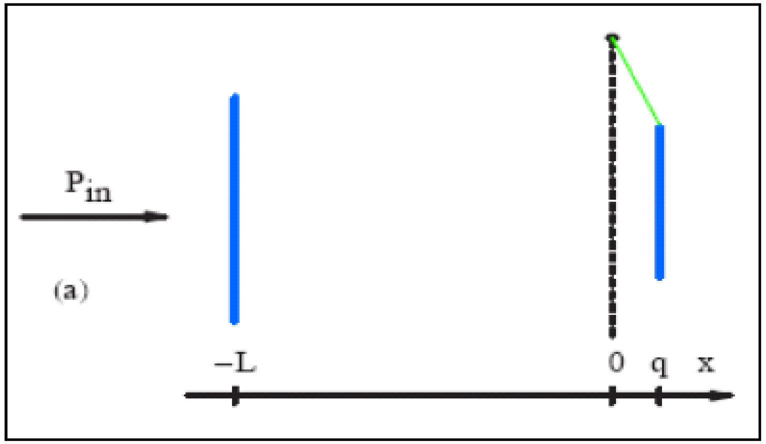


Fig. 6. Three-mirror system. The system comprises two subcavities 1 and 2 as marked.

See also: M. Bhattacharya and PM, PRL 99, 073601 (2007)
M. Bhattacharya *et al,* PRA **77**, 033819 (2008)

Early discussion: PM *et al*., JOSA **B2**, 1830 (1985)

basic optomechanics – quantum approach



$$H = \hbar\omega_n(q)a^\dagger a + \frac{p^2}{2m} + \frac{1}{2}m\omega_M^2 q^2$$

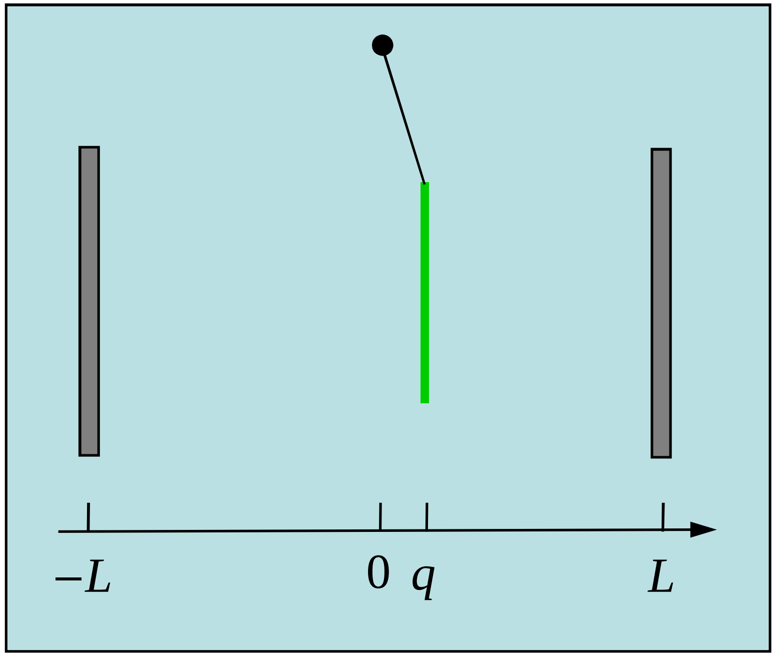
But:

$$\omega_n(q) = \frac{n\pi c}{L+q} = \omega_n\left(\frac{1}{1+q/L}\right) \approx \omega_n\left(1-\frac{q}{L}\right)$$

So:

$$\boxed{H \approx \hbar\omega_n a^\dagger a + \frac{p^2}{2m} + \frac{1}{2}m\omega_M^2 q^2 - \hbar\xi a^\dagger a q}$$

three-mirror cavity, $R=1$

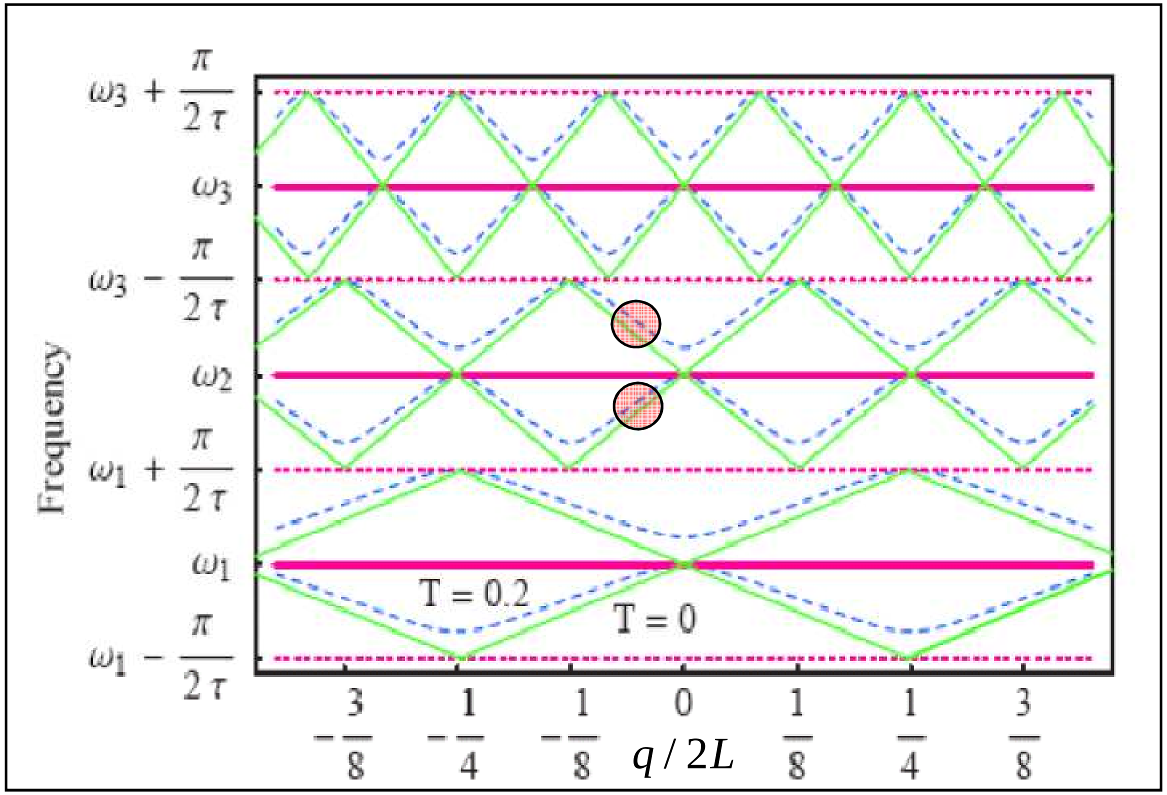


$$\omega_{n,l} \approx \omega_n (1 - q/L)$$

$$\omega_{n,r} \approx \omega_n (1 + q/L)$$

$$\boxed{H = \hbar\omega_n (a^\dagger a + b^\dagger b) + \frac{p^2}{2m} + \frac{1}{2} m\omega_M^2 q^2 - \hbar\xi (a^\dagger a - b^\dagger b) q}$$

three-mirror cavity, $R<1$



W. J. Fader, IEEE J. Quant. Electron. **21**, 1838 (1985)

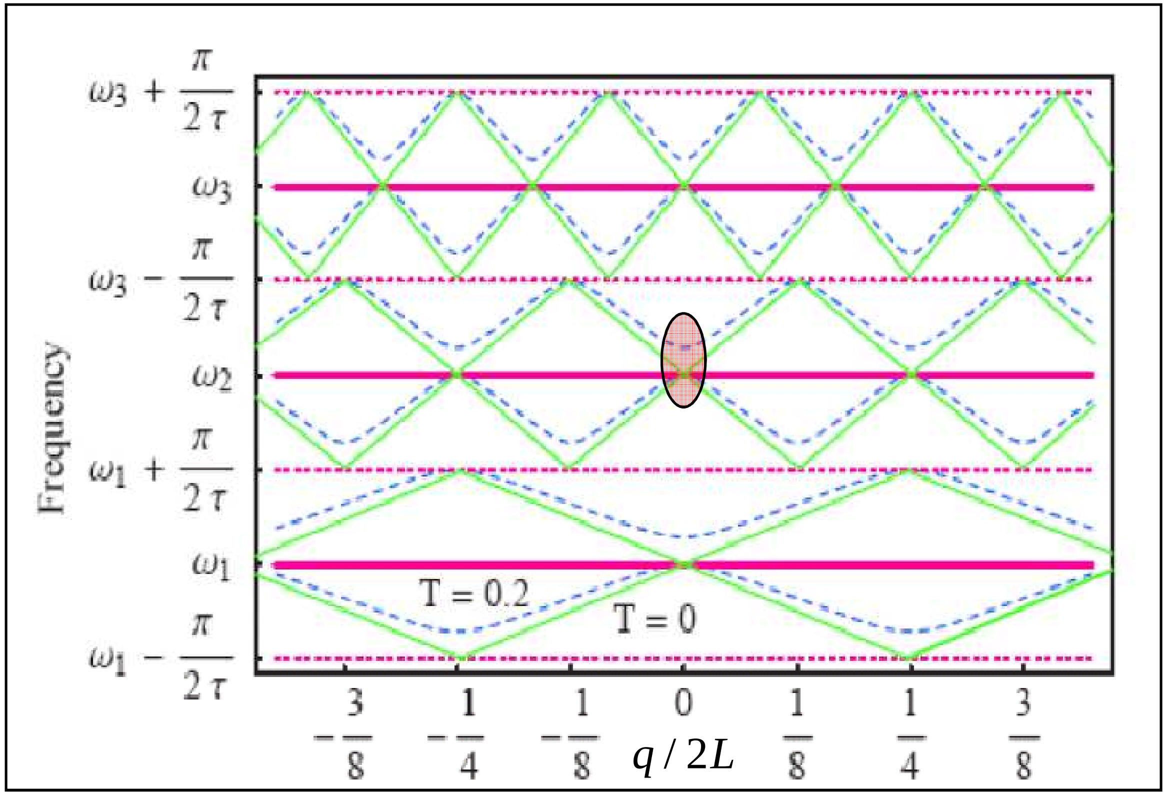
$$\boxed{q_0 \neq 0}$$

$$\omega_{n,e} \approx \omega_n - \delta_e - \xi_L(q - q_0)$$

$$\omega_{n,o} \approx \omega_n + \delta_o + \xi_L(q - q_0)$$

$$H = \hbar(\omega_n - \delta_e)a^\dagger a + \hbar(\omega_n + \delta_o)b^\dagger b + \frac{p^2}{2m} + \frac{1}{2}m\omega_M^2 q^2 - \hbar\xi_L(a^\dagger a - b^\dagger b)q$$

three-mirror cavity, *R*< 1



$$\boxed{q_0 \neq 0}$$

$$\omega_{n,e} \approx \omega_n - \xi_Q (q - q_0)^2$$

$$\omega_{n,o} \approx \omega_n + \Delta_o + \xi_Q (q - q_0)^2$$

$$H = \hbar(\omega_n - \delta_e) a^\dagger a + \hbar(\omega_n + \delta_o) b^\dagger b + \frac{p^2}{2m} + \frac{1}{2} m \omega_M^2 q^2 - \hbar \xi_Q (a^\dagger a - b^\dagger b) q^2$$

allows preparation of energy eigenstates
& observation of quantum jumps

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linear vs. quadratic coupling

Linear coupling

- back-action: mirror motion $\rightarrow$ changed cavity frequency $\rightarrow$ changed intracavity power $\rightarrow$ changed radiation pressure
- Important when storage time of light comparable to inverse mirror oscillation frequency
- Requires asymmetry in frequency change for two directions of mirror motion

Quadratic coupling

- Quadratic opto-mechanical coupling simply modifies the effective frequency of oscillating mirror
- Purely dispersive

a flavor of the theory (R=1)

$$H=\hbar\omega_c\left(a^\dagger a+b^\dagger b\right)+\frac{p^2}{2m}+\frac{1}{2}m\Omega_M^2 q^2-\hbar\xi\left(a^\dagger a-b^\dagger b\right)q$$

Quantum Langevin equations of motion, input-output formalism:

$$\dot{a}=-\left[i\left(\Delta-\xi Q\right)+\gamma/2\right]a+\sqrt{\gamma}a_{\text{in}}$$

$$\dot{b}=-\left[i\left(\Delta-\xi Q\right)+\gamma/2\right]b+\sqrt{\gamma}b_{\text{in}}$$

$$\dot{q}=p/m$$

$$\dot{p}=-m\Omega_M^2 q^2+\hbar\xi\left(a^\dagger a-b^\dagger b\right)-\left(\Gamma_M/2m\right)p+\varepsilon_{\text{in}}$$

Noise operators:

$$a_{\text{in}}(t)=\left\langle a_{\text{in},s}\right\rangle+\delta a_{\text{in}}(t) \qquad \left[\delta a_{\text{in}}(t),\delta a_{\text{in}}^\dagger(t')\right]=\delta(t-t')$$

$$\left\langle\varepsilon_{\text{in}}(t)\right\rangle=0 \qquad \left\langle\varepsilon_{\text{in}}(t)\varepsilon_{\text{in}}(t')\right\rangle=\frac{\Gamma_M}{2m}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\hbar\omega\left[1+\coth\left(\frac{\hbar\omega}{2k_BT}\right)\right]$$

steady-state – linear coupling

- Effective frequency: $$\Omega_{\text{eff}}^2 \approx \Omega_M^2 - \delta\left(\frac{4\xi\gamma P_{\text{in}}}{ML}\right)\frac{1}{[(\gamma/2)^2+\delta^2]^2}$$

- Cooling: $$\Gamma_{\text{eff}} \approx \Gamma_M + \delta\left(\frac{4\xi\gamma^2 P_{\text{in}}}{ML}\right)\frac{1}{[(\gamma/2)^2+\delta^2]^3}$$

“stiffening” field: $\delta \rightarrow \delta_s > 0 \quad P_{\text{in}} \rightarrow P_s$

“cooling” field: $\delta \rightarrow \delta_c < 0 \quad P_{\text{in}} \rightarrow P_c$

M. Bhattacharya and PM, PRL **99**, 073601 (2007)
M. Bhattacharya, H. Uys and PM, PRA **77,** 033819 (2008)

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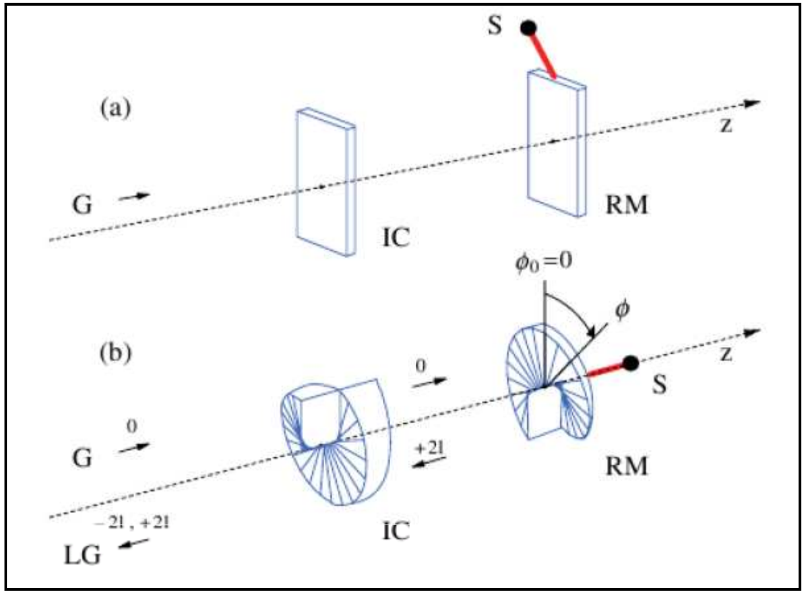
steady-state – quadratic coupling

- Effective frequency: $\Omega_{\text{eff}}^2 \approx \Omega_M^2 + \left(\dfrac{2\xi_Q \gamma P_{\text{in}}}{M\,\omega_n}\right)\dfrac{1}{(\gamma/2)^2+\delta^2}$

- Effective stiffness: $\Gamma_{\text{eff}} \approx \Gamma_M$

maximized on resonance

cooling of rotational motion



Spiral phase elements with opposite windings on both sides

$$H = \hbar\omega_c a^\dagger a + \frac{L_z^2}{2I} + \frac{1}{2} I\omega_\phi^2\phi^2 - \hbar\xi_\phi a^\dagger a\phi$$

$$[L_z, \phi] = -i\hbar$$

$$I = MR^2/2$$

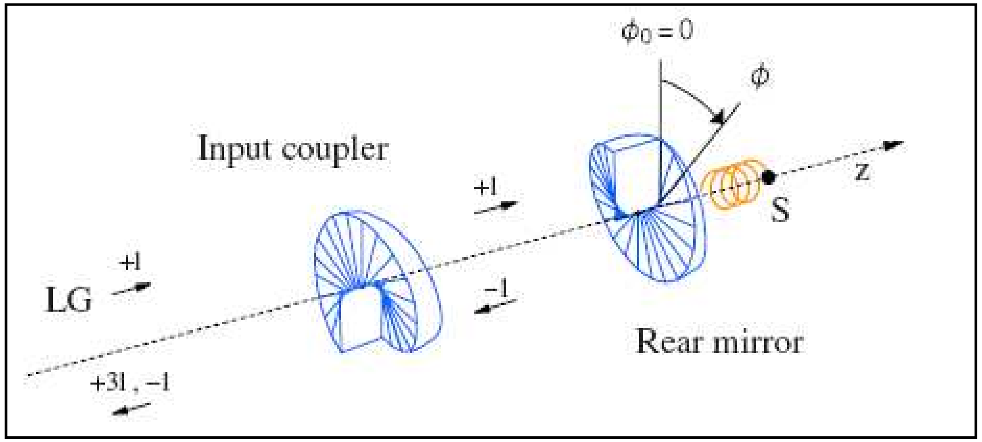
$$\omega_{\text{eff}}^2 \approx \omega_\phi^2 - \frac{2\xi_\phi^2\gamma P_{\text{in}}}{I\omega_c}\frac{\Delta}{[\Delta^2+(\gamma/2)^2]^2}$$

$$D_{\text{eff}} \approx D_\phi + \frac{2\xi_\phi^2\gamma^2 P_{\text{in}}}{I\omega_c}\frac{\Delta}{[\Delta^2+(\gamma/2)^2]^3}$$

$$\Delta = \omega_c - \omega_\phi - \hbar\xi_\phi\frac{|a_s|^2}{I\omega_\phi^2}$$

M. Bhattacharya & PM, PRL **99**, 153603 (2007)

rovibrational entanglement



scaling: $z : \sqrt{\hbar / M\omega_z}$ $\quad p_z : \sqrt{\hbar M \omega_z}$

$\phi : \sqrt{\hbar / I\omega_\phi}$ $\quad L_z : \sqrt{\hbar I \omega_\phi}$

Hamiltonian (dimensionless form)

$$H = \hbar\omega_c a^\dagger a + \frac{\hbar\omega_z}{2}(p_z^2 + z^2) + \frac{\hbar\omega_\phi}{2}(L_z^2 + \phi^2) - \hbar g_z a^\dagger a z + \hbar g_\phi a^\dagger a \phi$$

$$g_z = \frac{\omega_c}{L}\sqrt{\frac{\hbar}{M\omega_z}} \qquad g_\phi = \frac{c\ell}{L}\sqrt{\frac{\hbar}{I\omega_\phi}}$$

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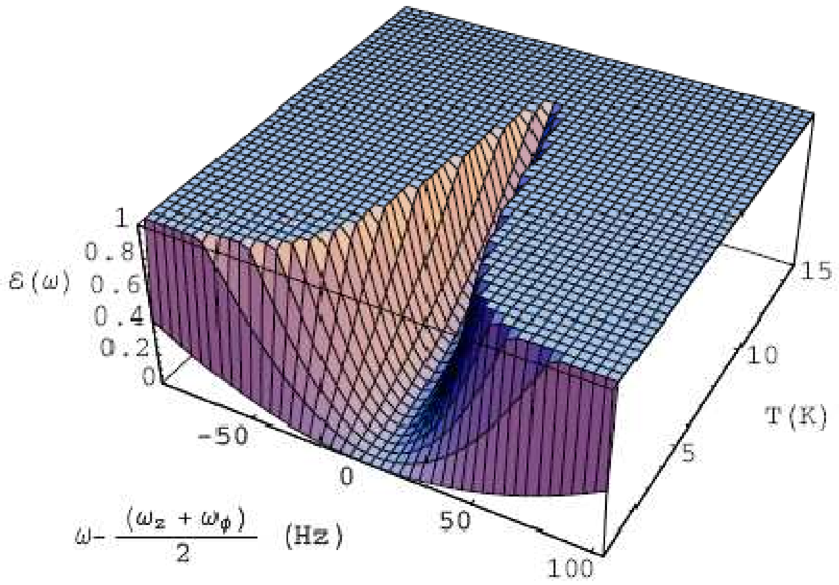
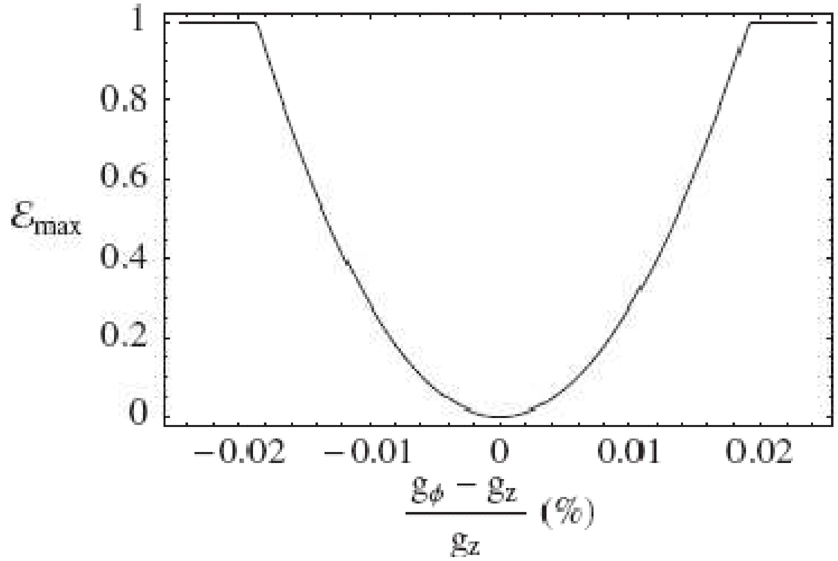
rovibrational entanglement





Entanglement measure:

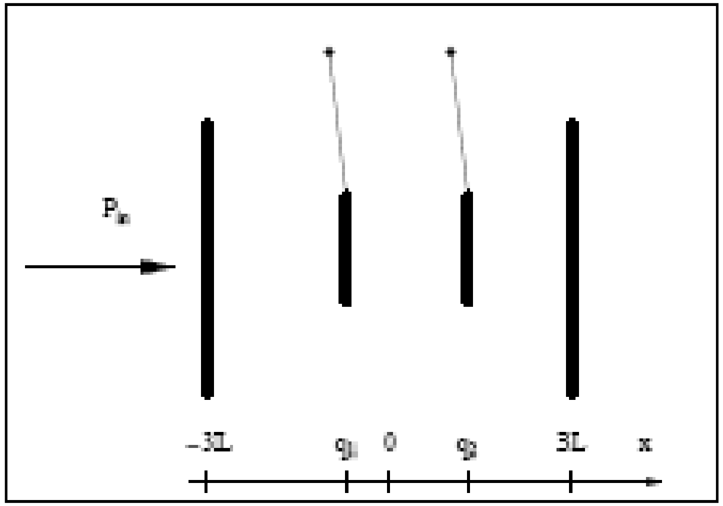
$$\mathcal{E}(\omega) \equiv \frac{\langle R_u^2(\omega)\rangle\langle R_v^2(\omega)\rangle}{|\langle [R_z(\omega), R_{p_z}(\omega)]\rangle|^2}$$

Rovibrational entanglement for $\mathcal{E}(\omega) < 1$

$$\delta u = \delta z - \delta\phi$$
$$\delta v = \delta p_z + \delta L_z$$
$$R_u = [\delta u(\omega) + \delta u(-\omega)]/2$$

M. Bhattacharya P.-L Giscard and PM, PRA **77**, 030303(R) (2008)

"Schrödinger drum"



Relative motion:	Center-of-mass motion:
$q = q_1 - q_2$ $m_{\text{eff}} = m/2$ $\omega_{\text{eff}} = \sqrt{3}\omega_m$	$Q = (q_1 + q_2)/2$ $M_{\text{eff}} = 2m$ $\Omega_{\text{eff}} = \omega_m$

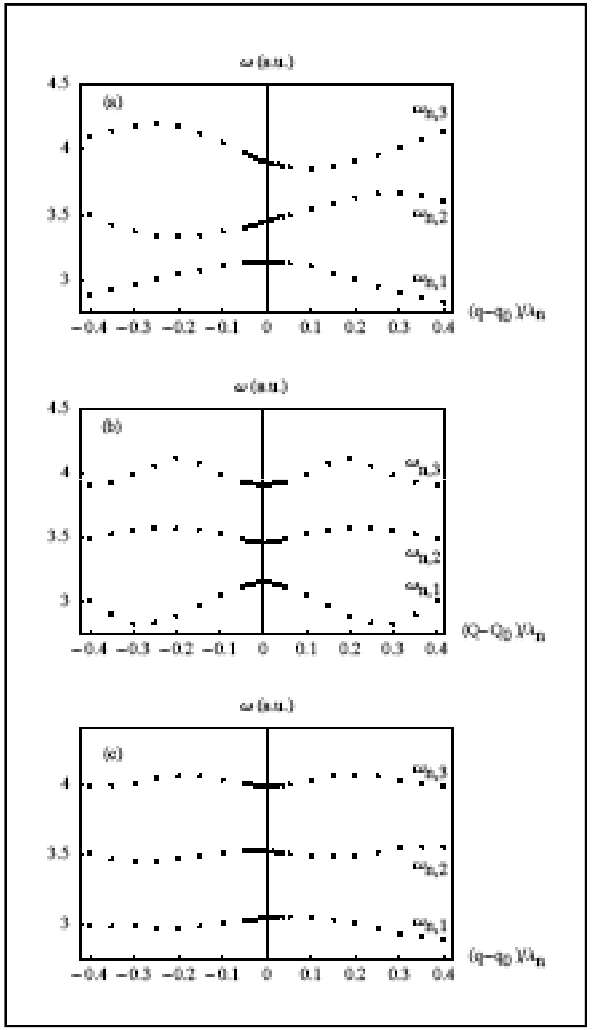
- Cavity spectrum (allowed values of *k*)

$$\sin 2(\theta + 3kL) + \sin^2\theta \sin 2k(3L - q) = 2\sin\theta\cos(\theta + kq)\cos 2kQ$$

$$\sin\theta = \sqrt{R}$$

M. Bhattacharya and PM, PRA**78,** 041801(R) (2008)

cavity spectrum

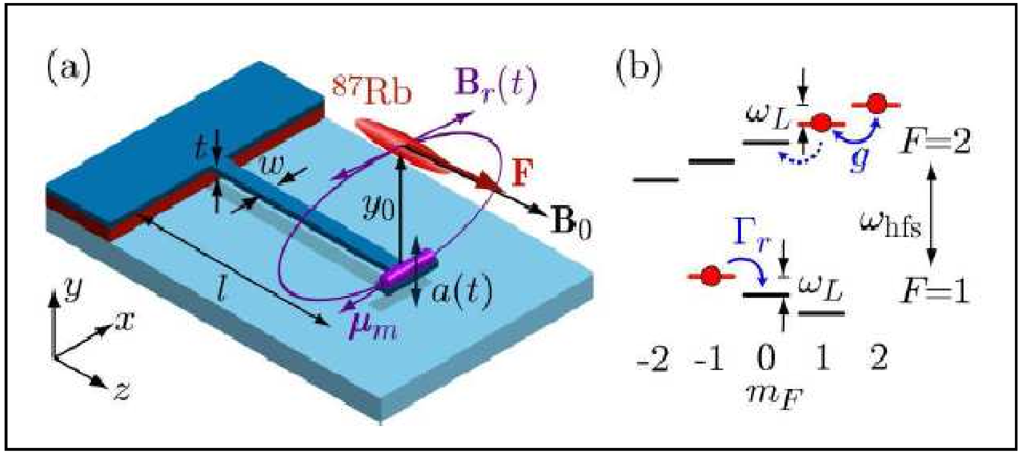


$$\begin{aligned}\omega_{n,i}(q,Q) = \Delta_{n,i} &+ B_{n,i}(q-q_0) + M_{n,i}(q-q_o)^2 \\ &+ B'_{n,i}(Q-Q_0) + M'_{n,i}(Q-Q_0)^2 \\ &+ P_{n,i}(q-q_0)(Q-Q_0) + \ldots\end{aligned}$$

- Can cool normal modes separately
- can perform QND energy measurements on normal modes separately
- Can use mode coupling to control one mode with the other
- Can generate quantum entanglement of modes (Hartmann and Plenio, PRL to be published)

cantilevers for quantum detection and control

- Pushing quantum mechanics to truly macroscopic systems
- Quantum superpositions and entanglement in macroscopic systems
- Novel detectors
- Coherent control



Single-domain ferromagnet with oscillatory component B(t) couples to atomic spin **F**

Described by Tavis-Cummings Hamiltonian

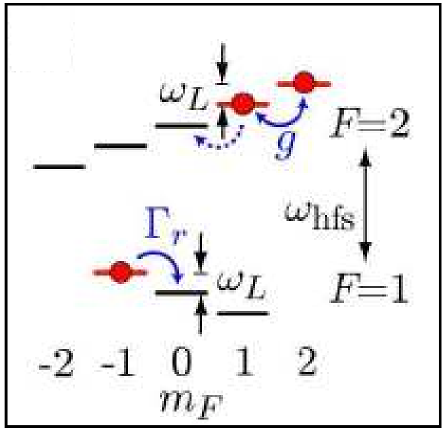
[P. Treutlein *et al.*, PRL 99, 140403 (2007)]

Tavis-Cummings Hamiltonian

Magnetic field at center of microtrap: $\mathbf{B}_r(t) = G_m a(t)\hat{\mathbf{e}}_x$

Cantilever-condensate interaction: $H = -\mu \cdot \mathbf{B}_r(t) = \mu_b g_F F_x\, a(t)$

Detuning: $\delta = \omega_L - \omega_c = \dfrac{\mu_B |g_F| B_0}{\hbar} - \omega_c$



Hamiltonian: $H = H_{\rm BEC} + H_r + V = \hbar\omega_L S_z + \hbar\omega_c a^\dagger a + \hbar g(S^+ a + S_- a^\dagger)$

spin $N/2$

$$g = \frac{\mu_B G_m}{2\hbar}\sqrt{\frac{\hbar}{2m\omega_r}}$$

[P. Treutlein *et al.*, PRL **99**, 140403 (2007)]

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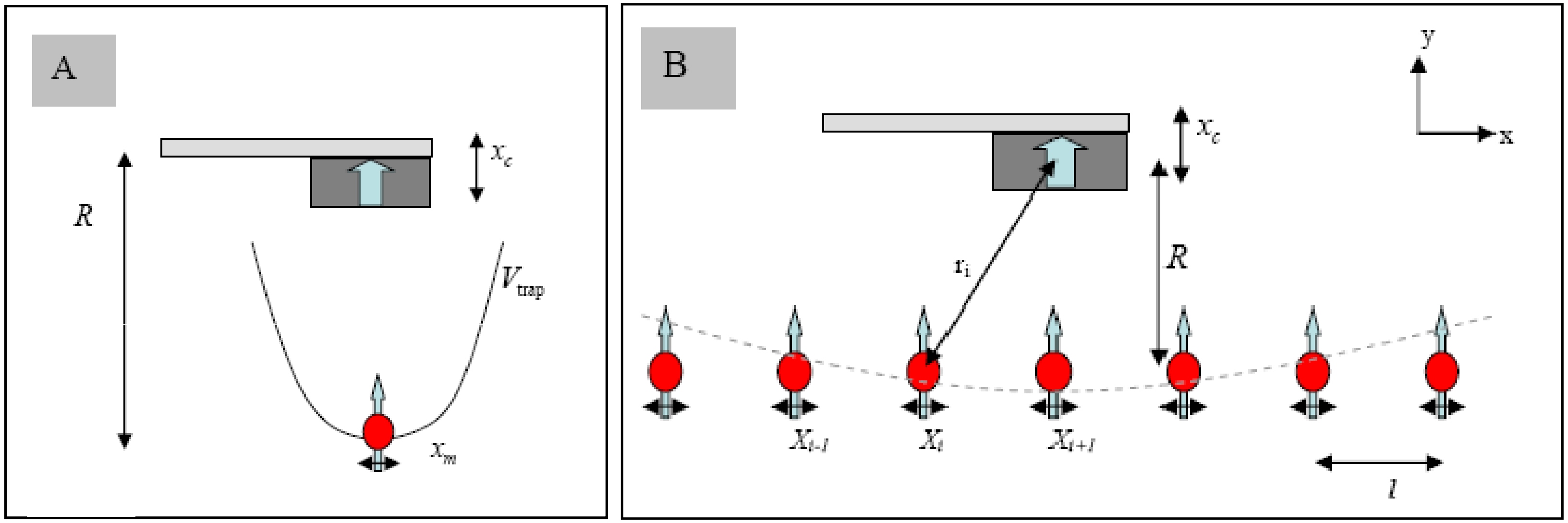
cantilever coupling to dipolar molecules



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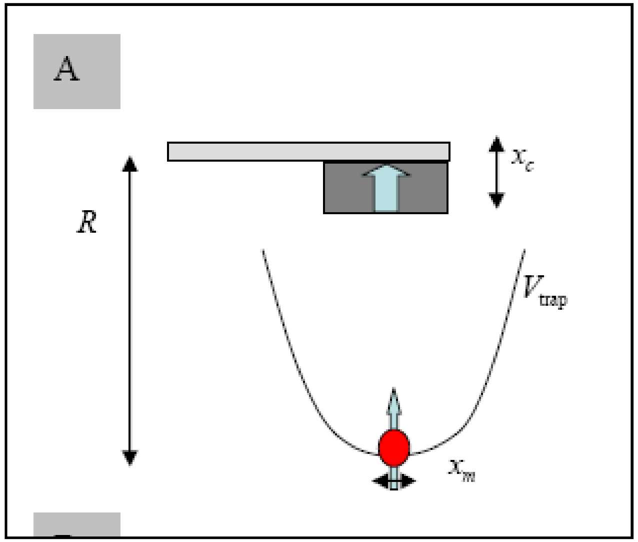
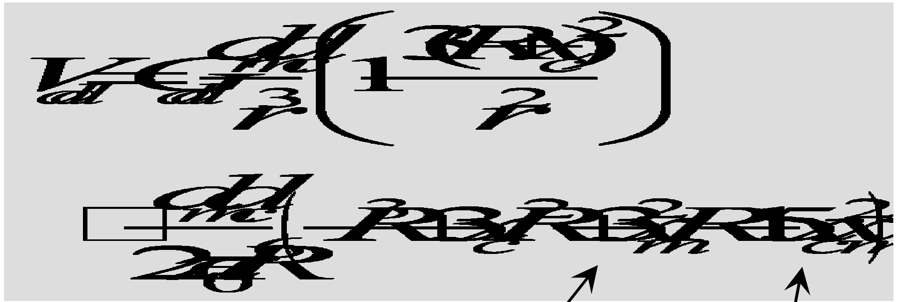
single molecule



Used in Magnetic Force Microscopy



Frequency shift

squeezing

$$\omega_{t'} = \left[\omega_t^2 + \frac{3d_m d_c}{\pi\epsilon_0 m R^5} \right]^{1/2}$$

$$G_{dd} = \frac{1}{4\pi\varepsilon_0}$$ Electric dipoles

$$G_{dd} = \frac{\mu_0}{4\pi}$$ Magnetic dipoles

$$r = \left[(R + x_c)^2 + x_m^2 \right]^{1/2}$$

motional squeezing

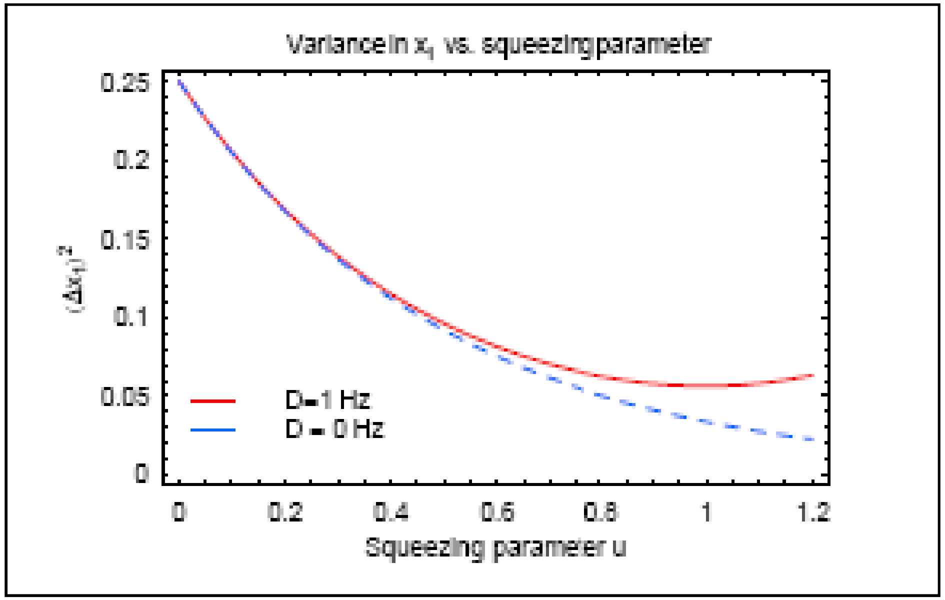
Classical cantilever motion, $\omega_c = 2\omega_{t'}$

$$\Rightarrow \boxed{V_I = -\hbar C\left(b^2 + b^{\dagger 2}\right)}$$

$$C = \sqrt{\bar{N}}\,\frac{15 d_m d_c}{4\pi\epsilon_o m\omega_{t'} R^6}\left(\frac{\hbar}{2m_c\omega_c}\right)^{1/2}$$

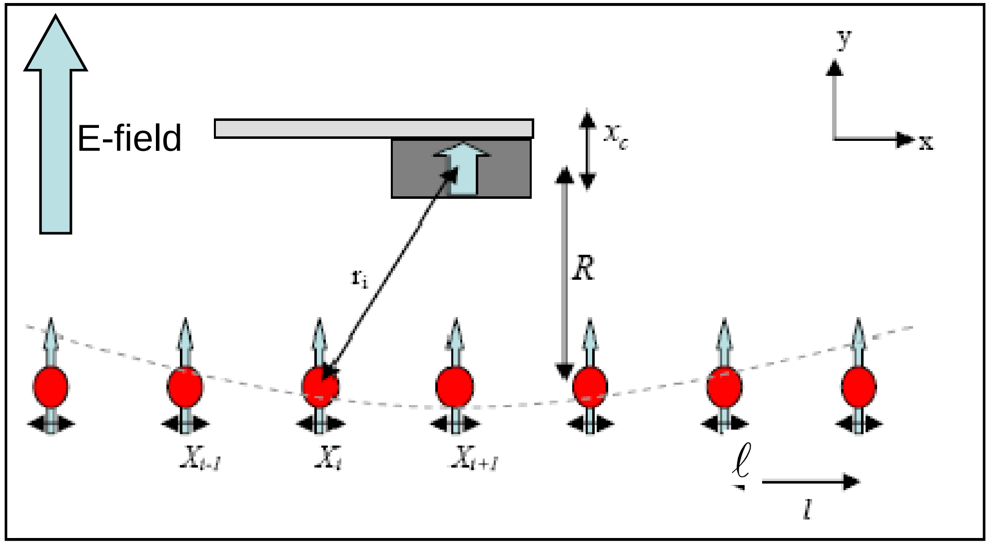
$$\bar{N} = k_B T_c / \hbar\omega_c$$

$$u = 2Ct$$



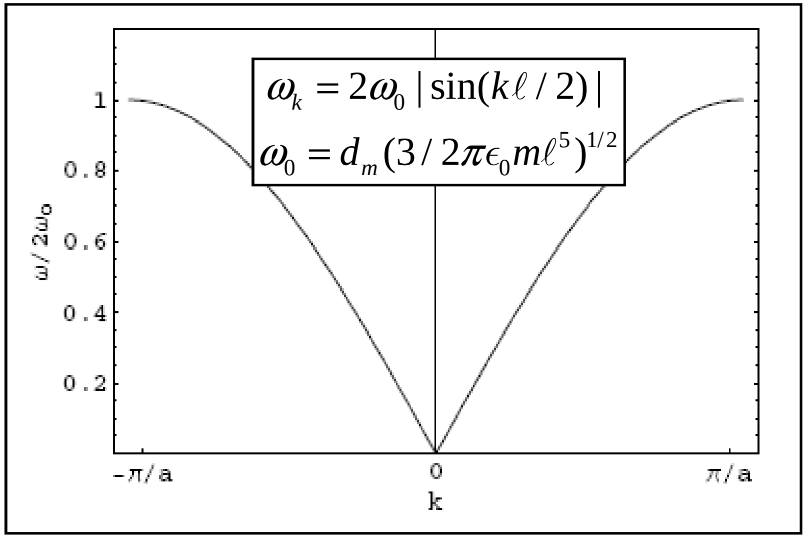
S. Singh, M. Bhattacharya, O. Dutta, PM, arXiv:0805.3735

lattice of molecular dipoles



One-dimensional molecular chain:

$$H_p = \sum_i^N \frac{p_i^2}{2m} + \frac{d_m^2}{4\pi\epsilon_o} \sum_{i<j}^N \frac{1}{|x_i - x_j|^3} + V_t$$



- Small displacements:

Acoustic phonons $\boxed{H_p = \sum_k \hbar\omega_k \left(b_k^\dagger b_k + \frac{1}{2} \right)}$

P. Rabl and P. Zoller PRA **76** 04230 (2007)

coupling to micromechanical oscillator

$$H_c = \sum_k \hbar\omega_c \left(a^\dagger a + \frac{1}{2} \right)$$

$$V_I = \sum_i \frac{d_m d_c}{r_i^3} \left[1 - \frac{3(R + x_c)^2}{r_i^2} \right]$$

- Lengths hierarchy: $q_i \ll \ell$
  $N\ell \ll R$

Squeezing!

- ❑ Frequency shift: $\omega_k \to \omega_k + \dfrac{1}{4\pi\epsilon_0} \dfrac{6 d_m d_c}{m\omega_k R^5}$

- ❑ Phonons-oscillator interaction $V_I = -\sum_k \hbar C_k \left(a + a^\dagger \right)\left(b_k b_{-k} + b_k^\dagger b_{-k}^\dagger + b_k^\dagger b_k + b_{-k} b_{-k}^\dagger \right)$

$$C_k = -\frac{3 d_m d_c}{2\pi\epsilon_o m\omega_{k'} R^6} \left(\frac{\hbar}{2 m_c \omega_c} \right)^{1/2}$$

phonon squeezing

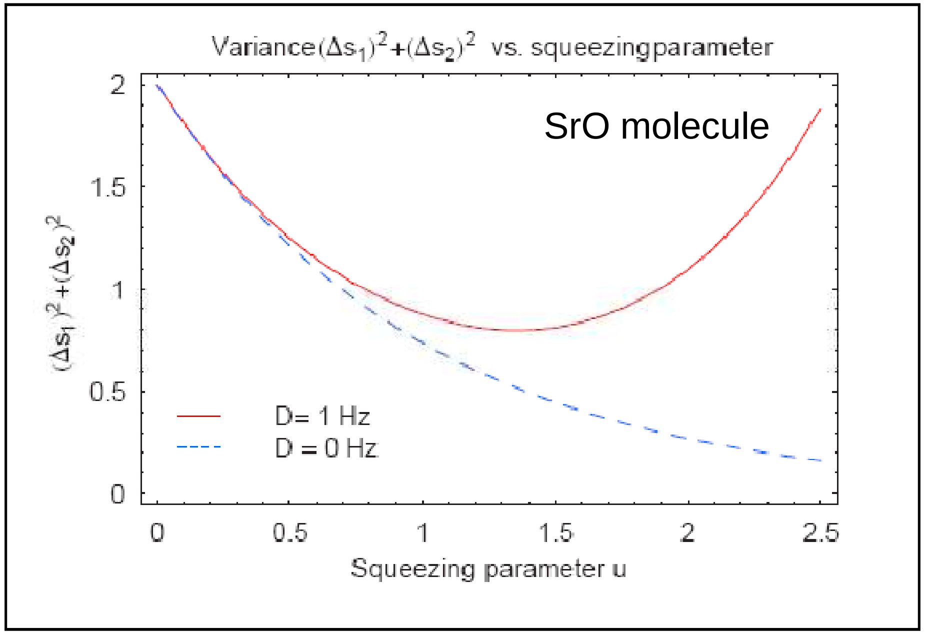
Classical cantilever motion, RWA,…

$$V_I = -\hbar\sqrt{\bar{N}}C_k\left(b_k b_{-k} + b_{-k}^\dagger b_k^\dagger\right)$$

Two-mode squeezing:

$$s_1 = \frac{1}{\sqrt{2}}(b_k + b_{-k} + b_k^\dagger + b_{-k}^\dagger)$$

$$s_2 = \frac{1}{\sqrt{2}i}(b_k - b_k^\dagger - b_{-k} + b_{-k}^\dagger)$$



$$\omega_c = 2\text{MHz}$$

$$m_c = 10^{-16}\,\text{Kg}$$

$$\bar{N} = 100$$

$$d_c = 10^{-21}\ \text{Cm}$$

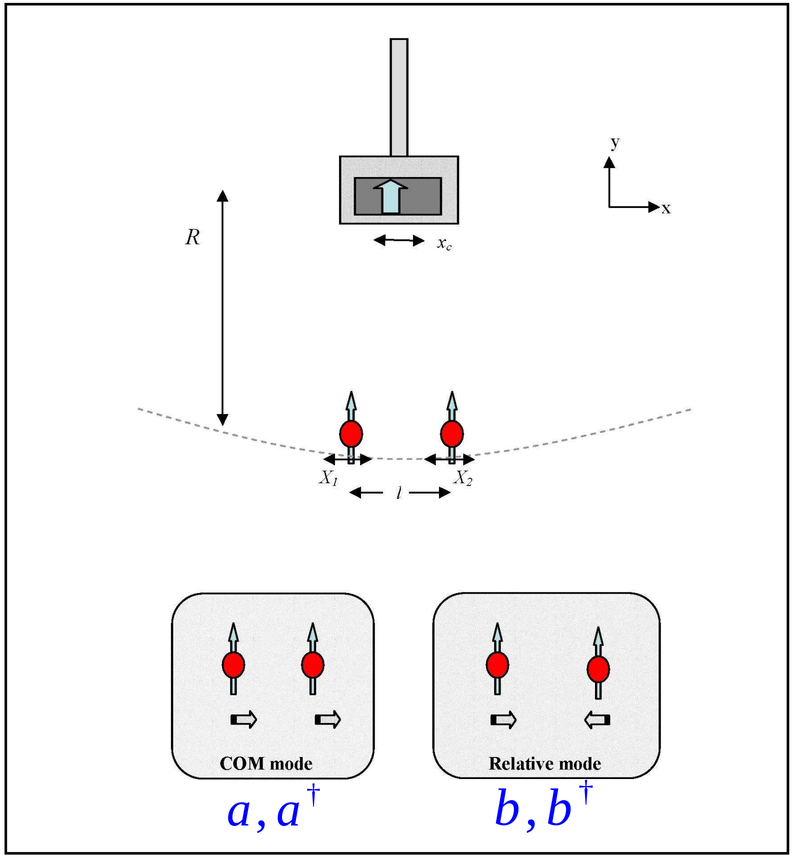
$$R = 2\mu\text{m}$$

$$\ell = 200\ \text{nm}$$

$$\text{C}_k = 1.08$$

$$\text{u} = 2\text{C}_k\sqrt{\bar{N}}t$$

coherent control



$$V = \frac{d_m d_c}{8\pi\epsilon_0 R^7}[24R^2(x_{\mathrm{com}}^2 + x_{\mathrm{rel}}^2 + x_c^2)$$

$$-48R^2 x_c x_{\mathrm{com}} - 48R^2 x_c x_{\mathrm{com}} - 140\ell x_{\mathrm{rel}}^3 + 60\ell x_{\mathrm{rel}} x_{\mathrm{com}}^2$$

$$-60\ell x_{\mathrm{rel}} x_c^2 + 120\ell x_c x_{\mathrm{rel}} x_{\mathrm{com}}]$$

- Center-of-mass and relative modes of motion, classical cantilever

$$V \propto L_c\, ab^\dagger \exp[i(\omega_{\mathrm{rel}} - \omega_{\mathrm{com}} - \omega_c(t))] + h.c.$$

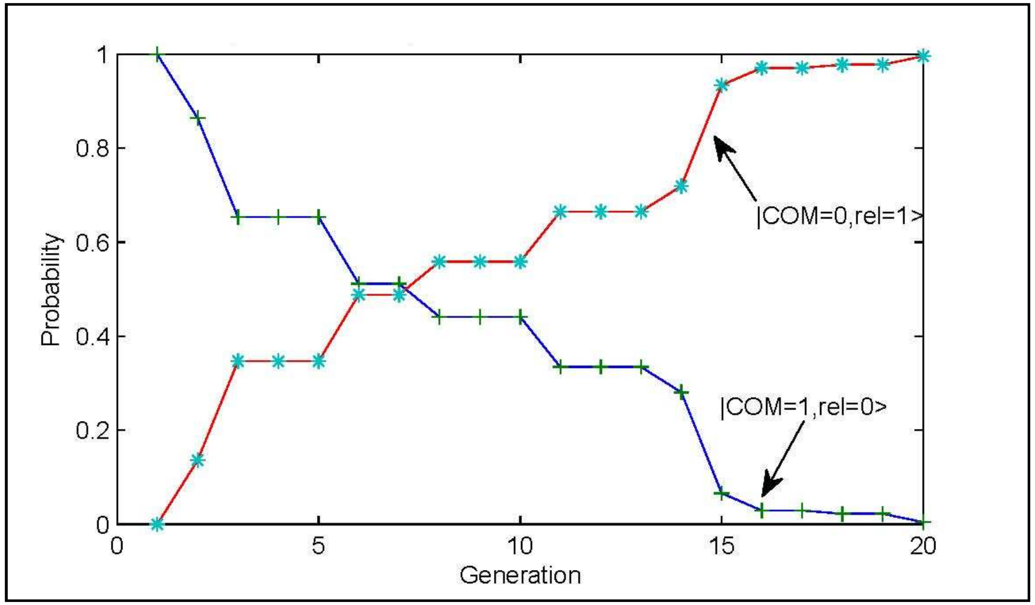
$$L_c = \sqrt{\bar{N}}\left(\frac{\hbar}{2m_c\omega_c}\right)^{1/2}$$

Varied for coherent control

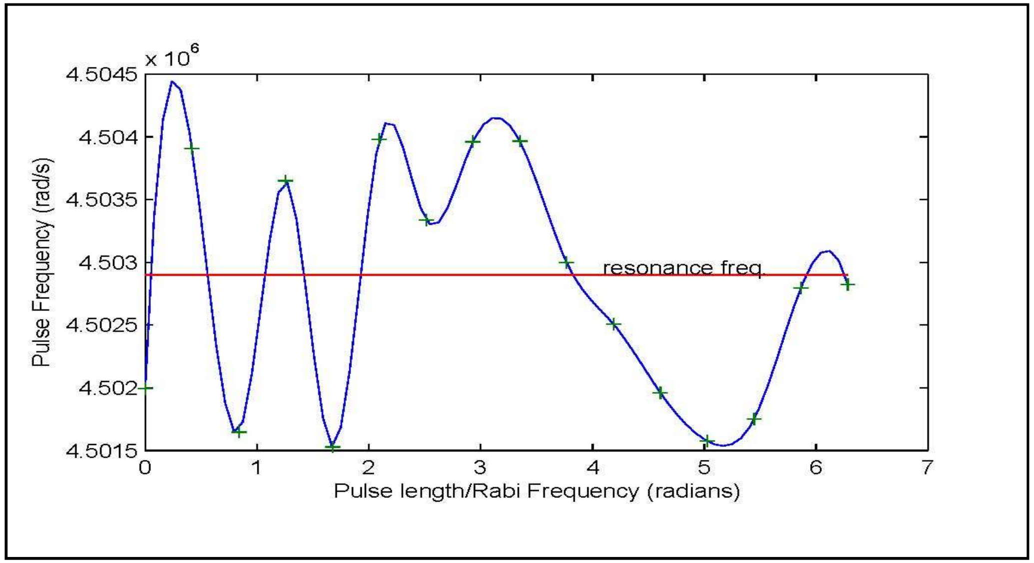
simple case

$$|\psi(0)\rangle = |1_{\mathrm{com}}, 0_{\mathrm{rel}}\rangle \rightarrow$$

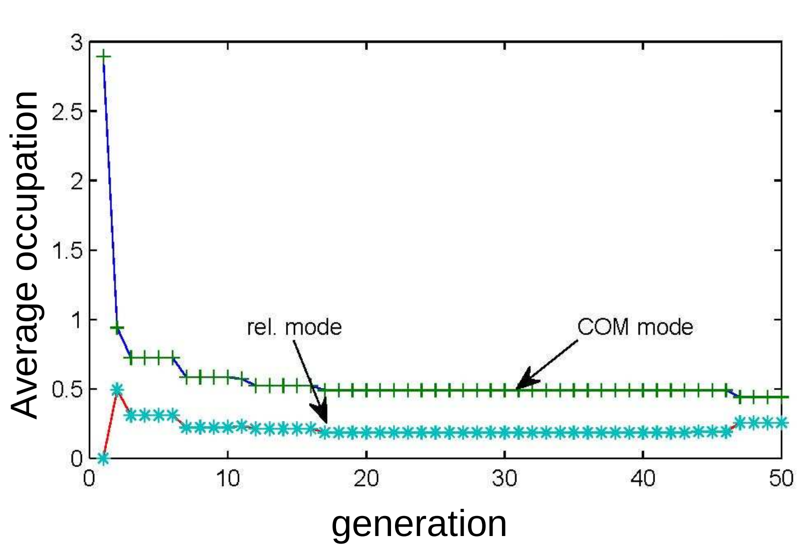
$$|\psi(t)\rangle = \alpha\,|1_{\mathrm{com}}, 0_{\mathrm{rel}}\rangle + \beta\,|0_{\mathrm{com}}, 1_{\mathrm{rel}}\rangle$$

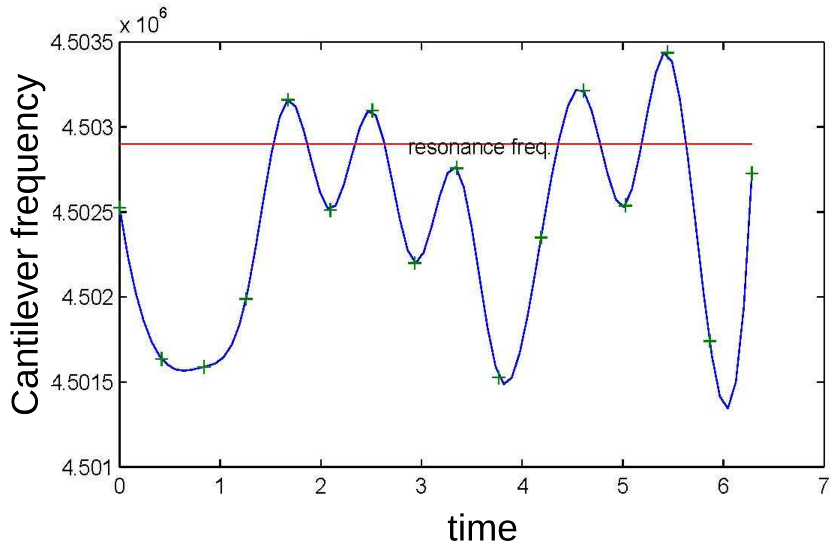


Cantilever frequency dependence – last generation

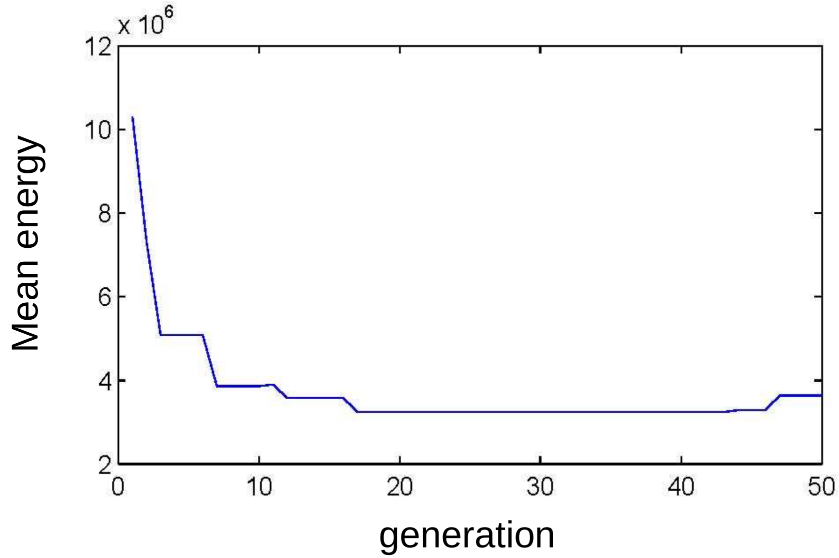


More interesting – start from a thermal state



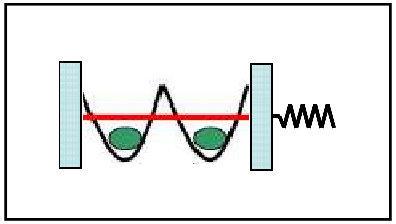


(Preliminary results)

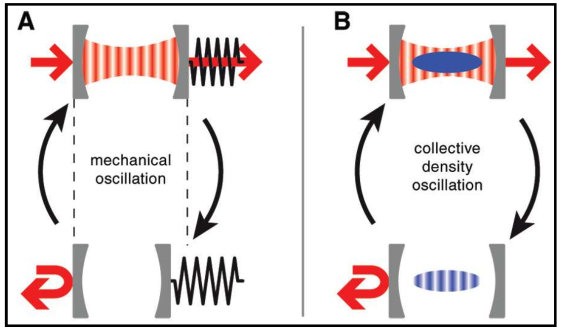


outlook

- Limits of cooling
- Preparation of exotic states
- Quantized cantilever motion
- Quantum phase transitions
  - Wenzhou Chen & PM (under construction)



- Condensates in high-Q cavities
  - Esslinger et al, Science **322**, 235 (2008)
  - Stamper-Kurn et al, Nature Physics **4**, 561 (2008)



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