

Interplay of interactions and disorder in two dimensions

Sergey Kravchenko



in collaboration with:

S. Anissimova, V.T. Dolgoplov, A. M. Finkelstein, T.M. Klapwijk,
A. Punnoose, A.A. Shashkin

9/28/2007

University of Virginia

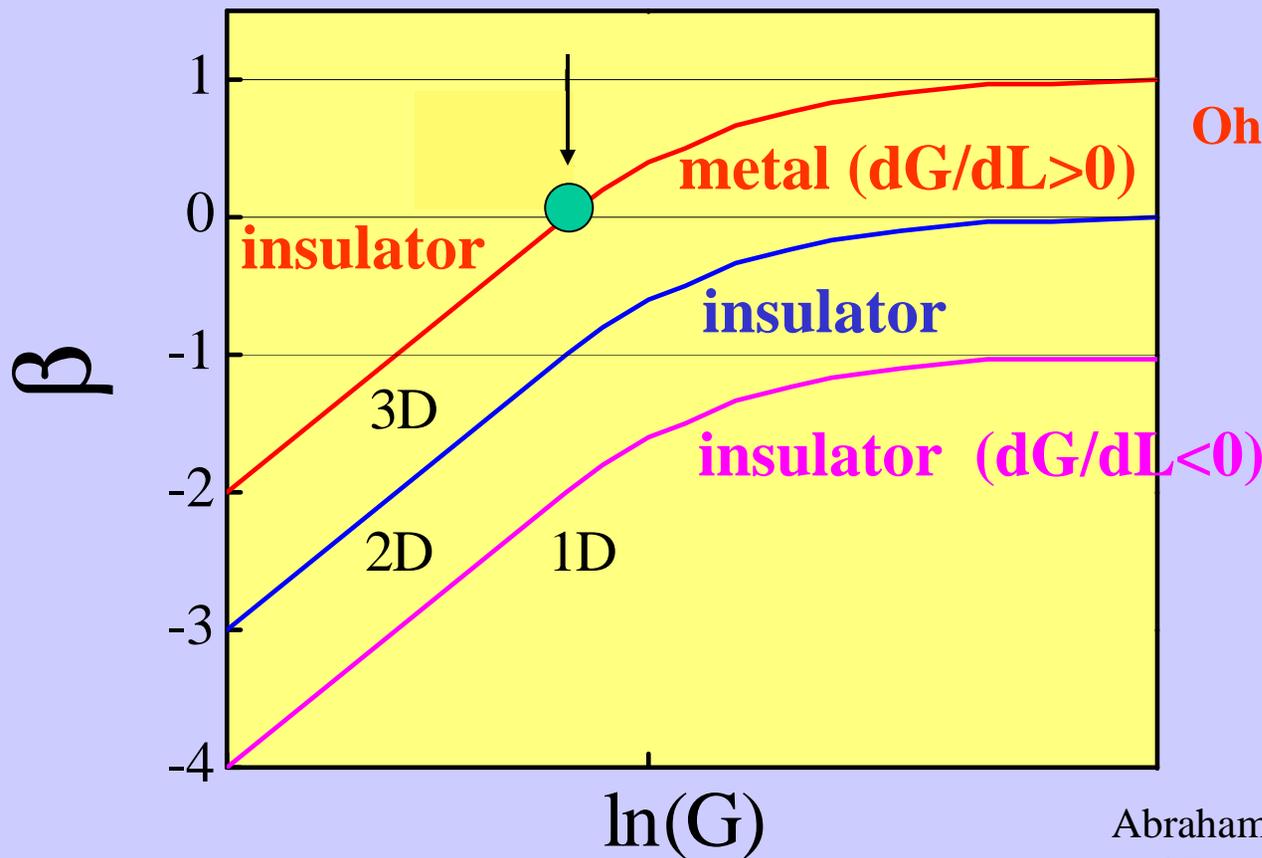
Outline

- **Scaling theory of localization: the origin of the common wisdom “all electron states are localized in 2D”**
- Samples
- What do experiments show?
- What do theorists have to say?
- Interplay between disorder and interactions: experimental test
- “Clean” regime: diverging spin susceptibility
- Summary

One-parameter scaling theory for non-interacting electrons: the origin of the common wisdom “all states are localized in 2D”

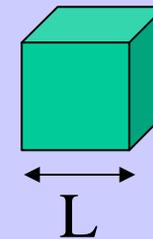
$$d(\ln G)/d(\ln L) = \beta(G)$$

$$G \sim L^{d-2} \exp(-L/L_{loc})$$



QM interference

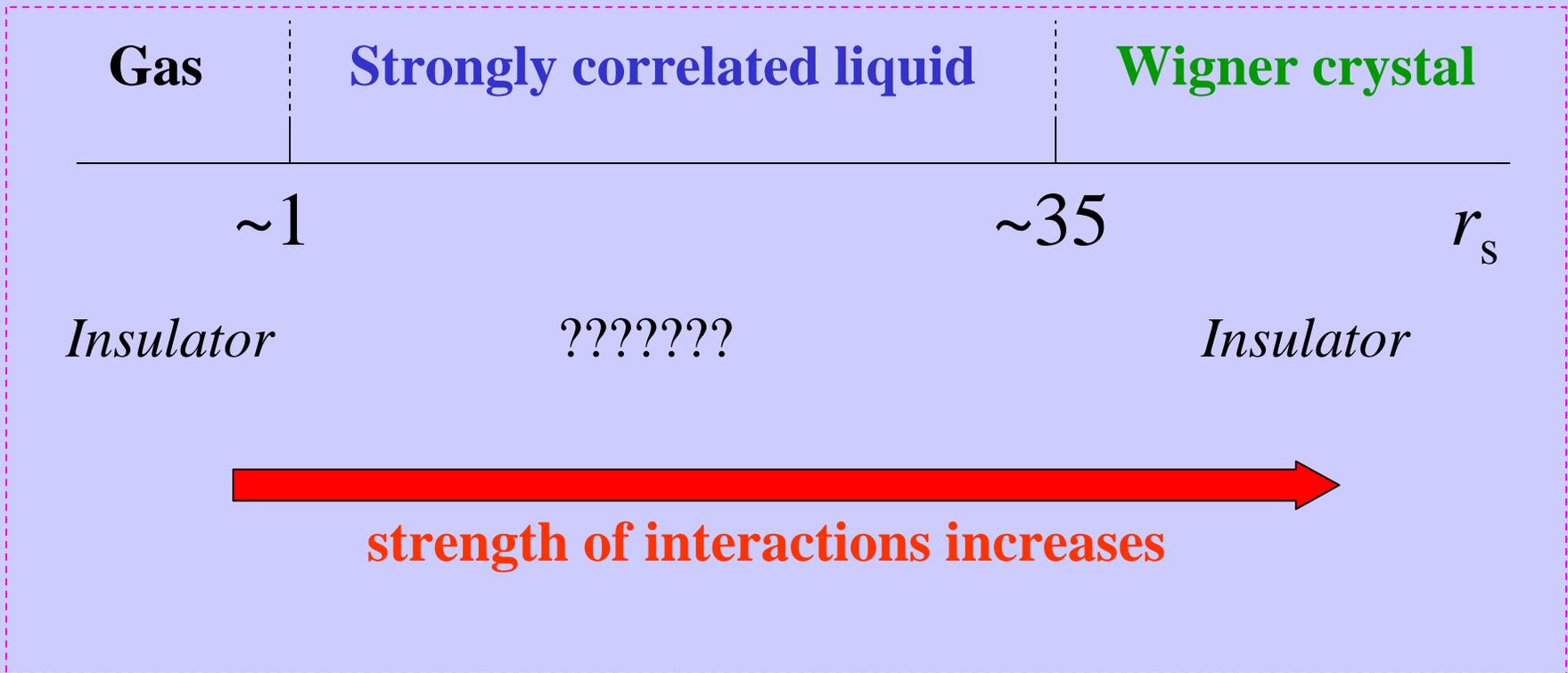
Ohm's law in d dimensions



$$G = 1/R$$

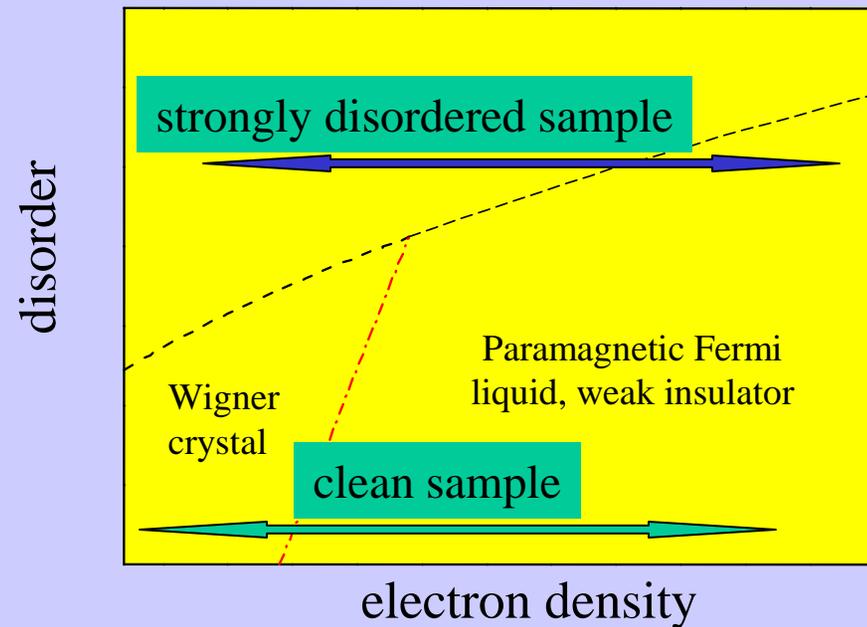
Abrahams, Anderson, Licciardello,
and Ramakrishnan, *PRL* 42, 673
(1979)

$$r_s = \frac{\text{Coulomb energy}}{\text{Fermi energy}}$$



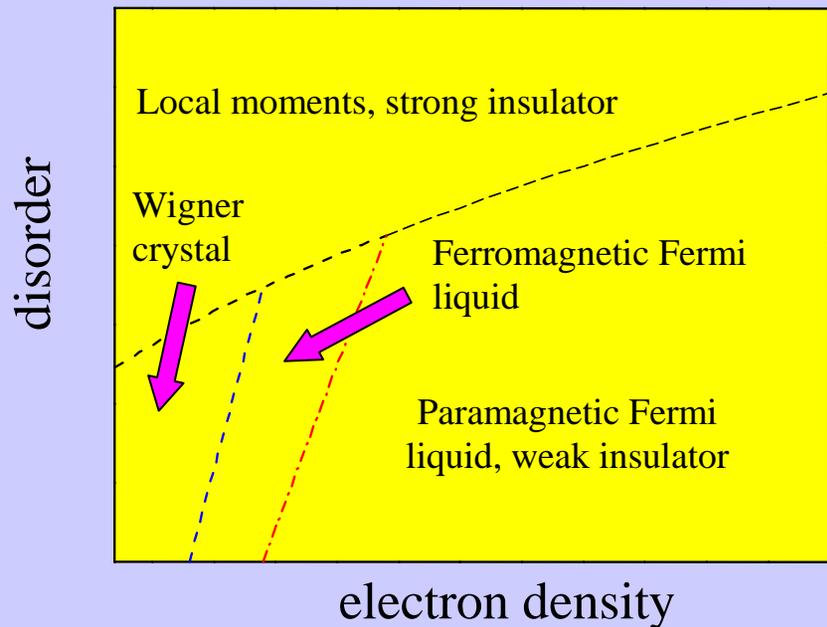
Suggested phase diagrams for strongly interacting electrons in two dimensions

Tanatar and Ceperley,
Phys. Rev. B **39**, 5005 (1989)



← strength of interactions increases

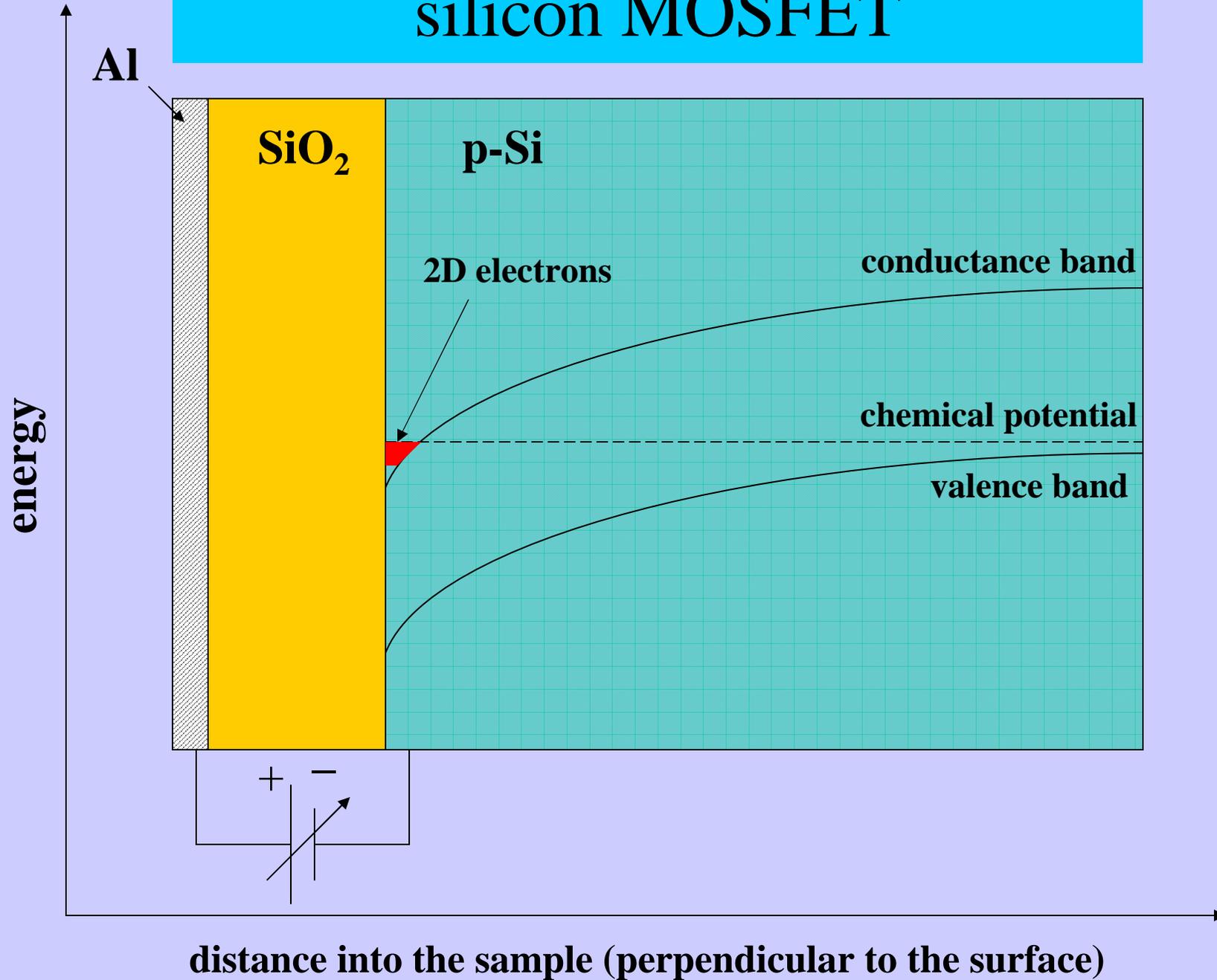
Attacalite *et al.*
Phys. Rev. Lett. **88**, 256601 (2002)



← strength of interactions increases

- Scaling theory of localization: the origin of the common wisdom “all electron states are localized in 2D”
- **Samples**
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- “Clean” regime: diverging spin susceptibility
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silicon MOSFET



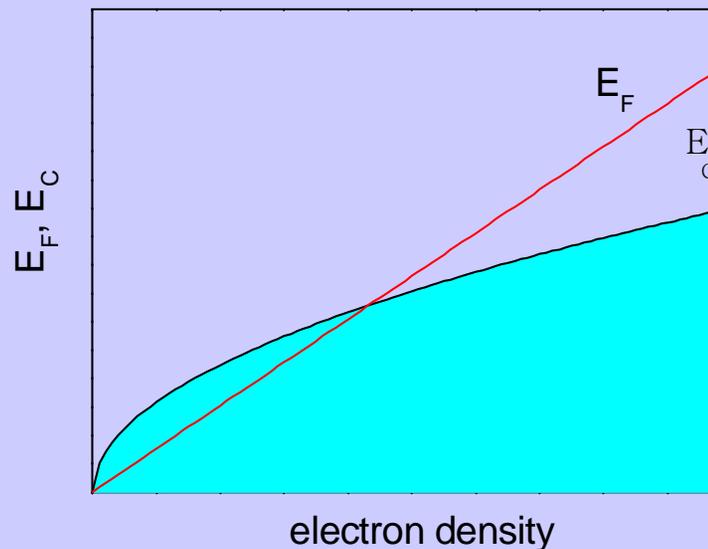
Why Si MOSFETs?

It turns out to be a very convenient 2D system to study strongly-interacting regime because of:

- large effective mass $m^* = 0.19 m_0$
- two valleys in the electronic spectrum
- low average dielectric constant $\epsilon = 7.7$

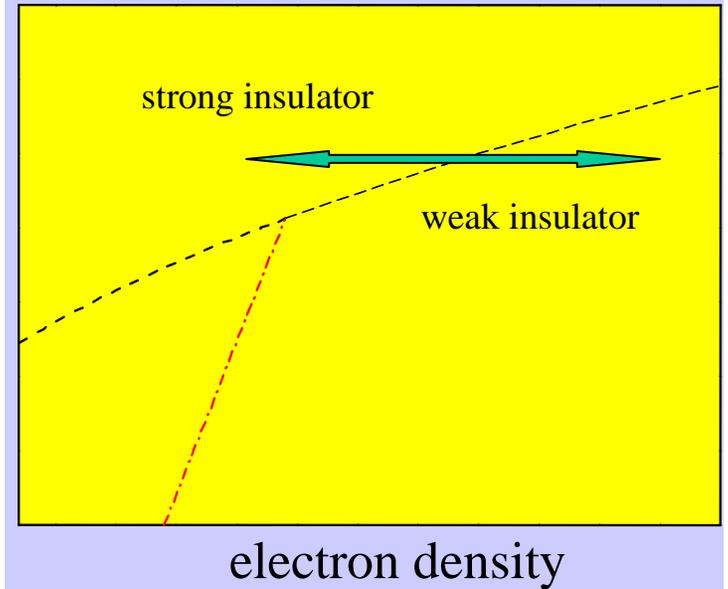
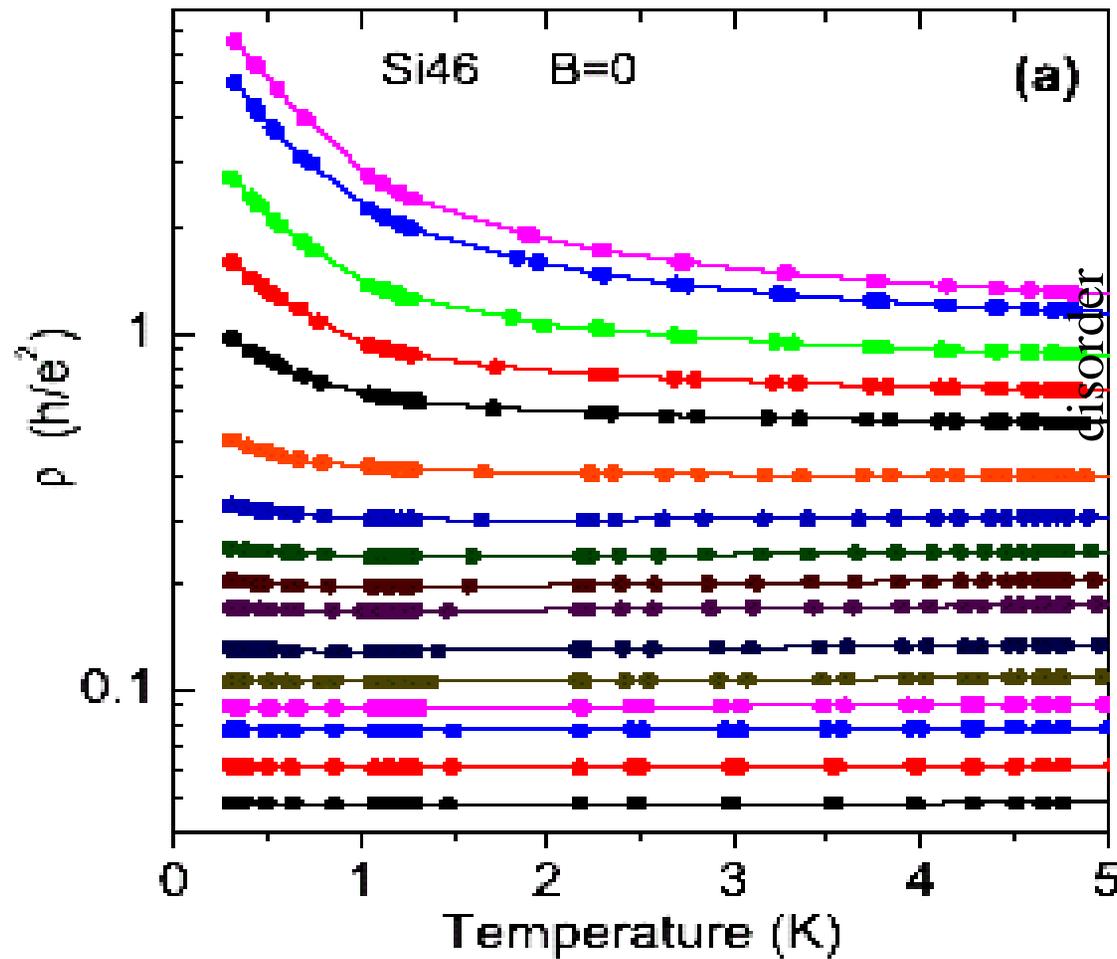
As a result, at low densities, Coulomb energy strongly exceeds Fermi energy: $E_C \gg E_F$

$r_s = E_C / E_F > 10$ can easily be reached in clean samples



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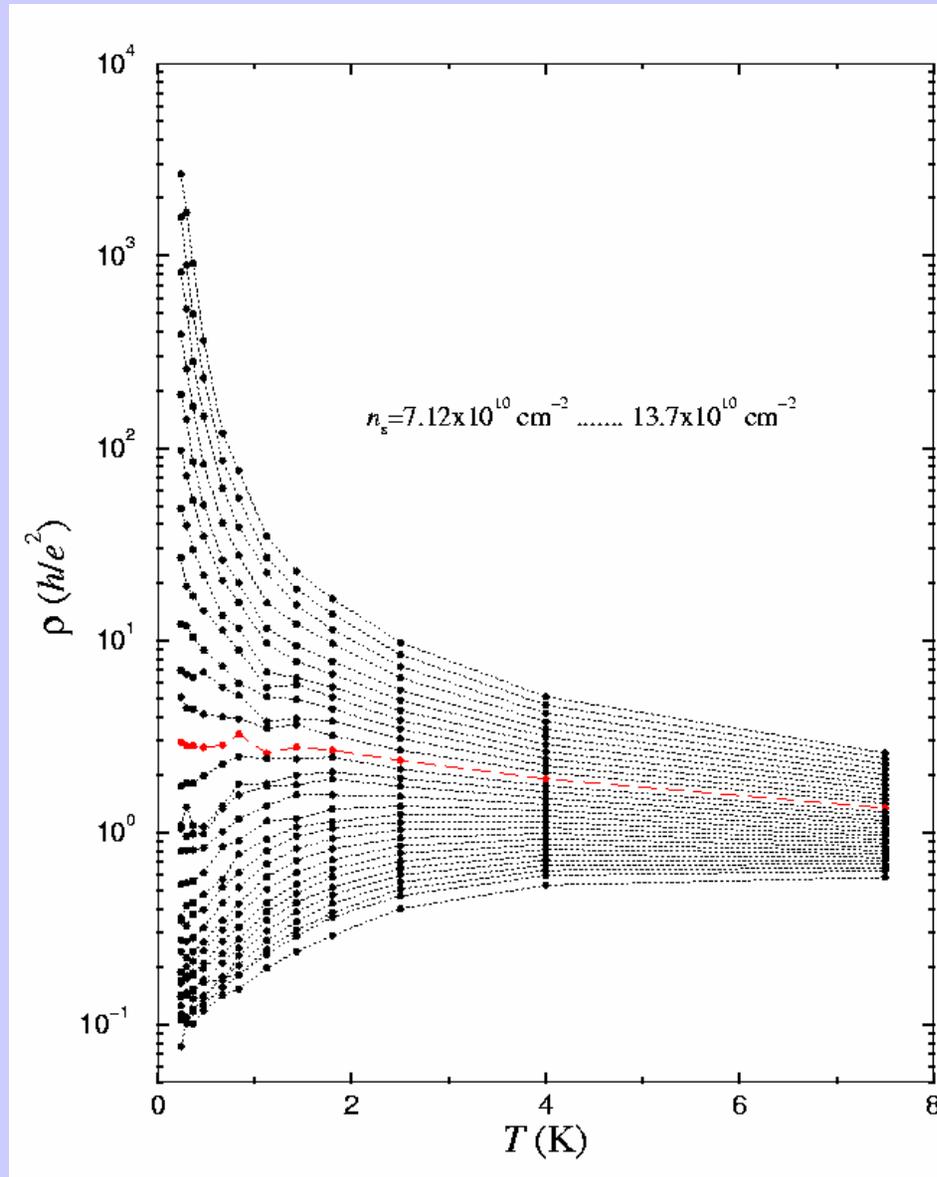
Strongly disordered Si MOSFET



(data of Pudalov *et al.*)

➤ Consistent (more or less) with the one-parameter scaling theory

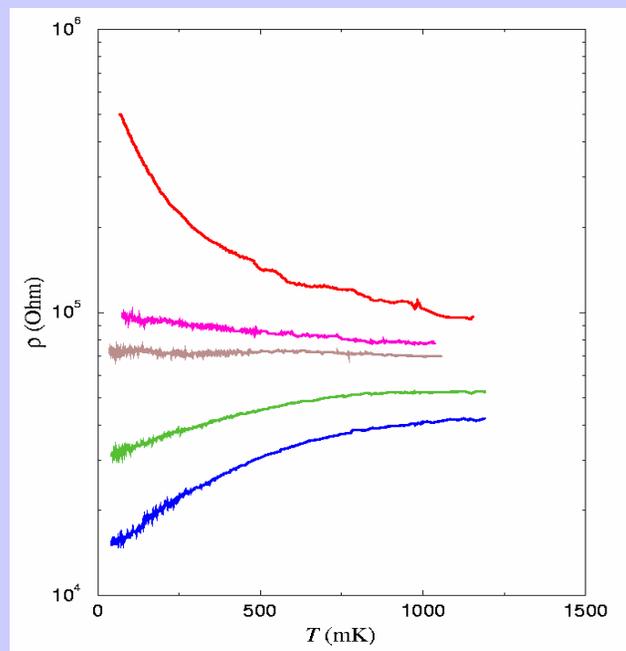
Clean sample, much lower electron densities



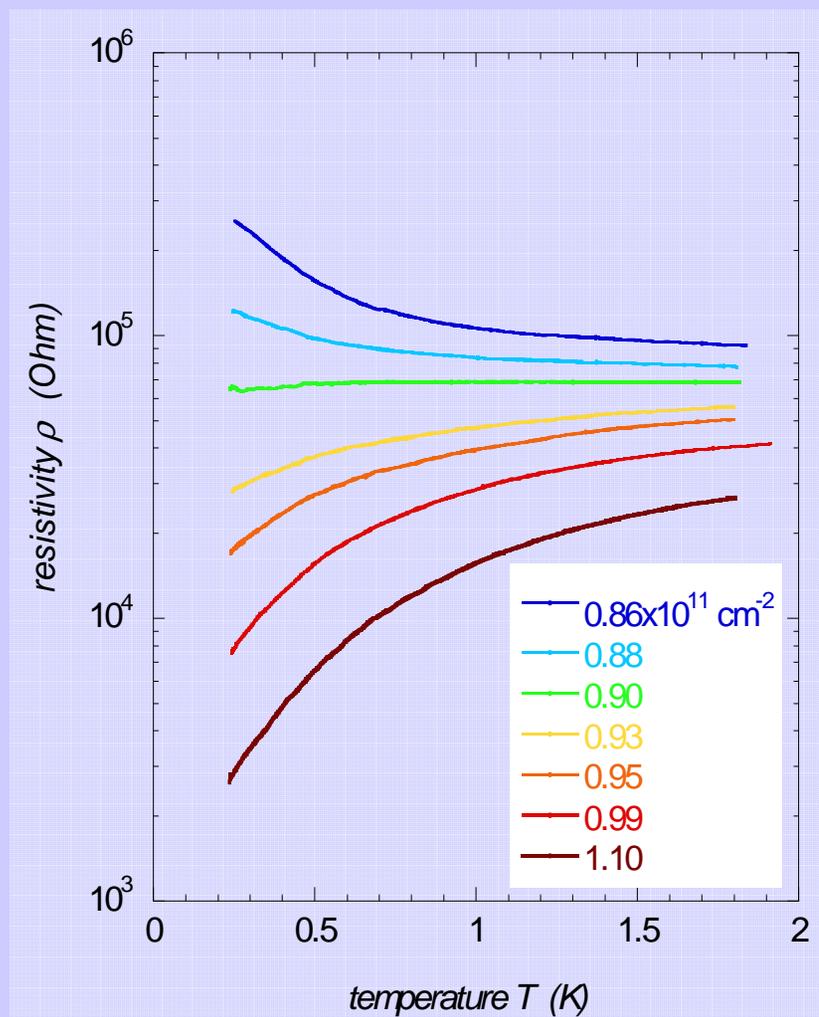
**Kravchenko, Mason, Bowker,
Furieux, Pudalov, and
D'Iorio, *PRB* 1995**

In very clean samples, the transition is practically universal:

Klapwijk's sample:



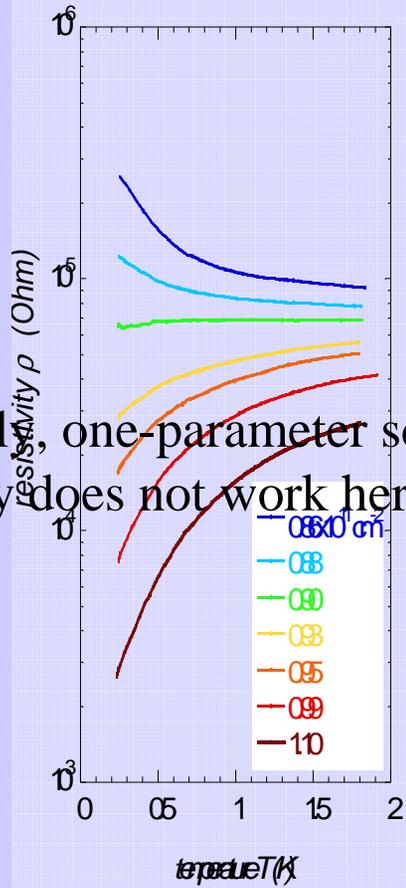
Pudalov's sample:



(Note: samples from different sources, measured in different labs)

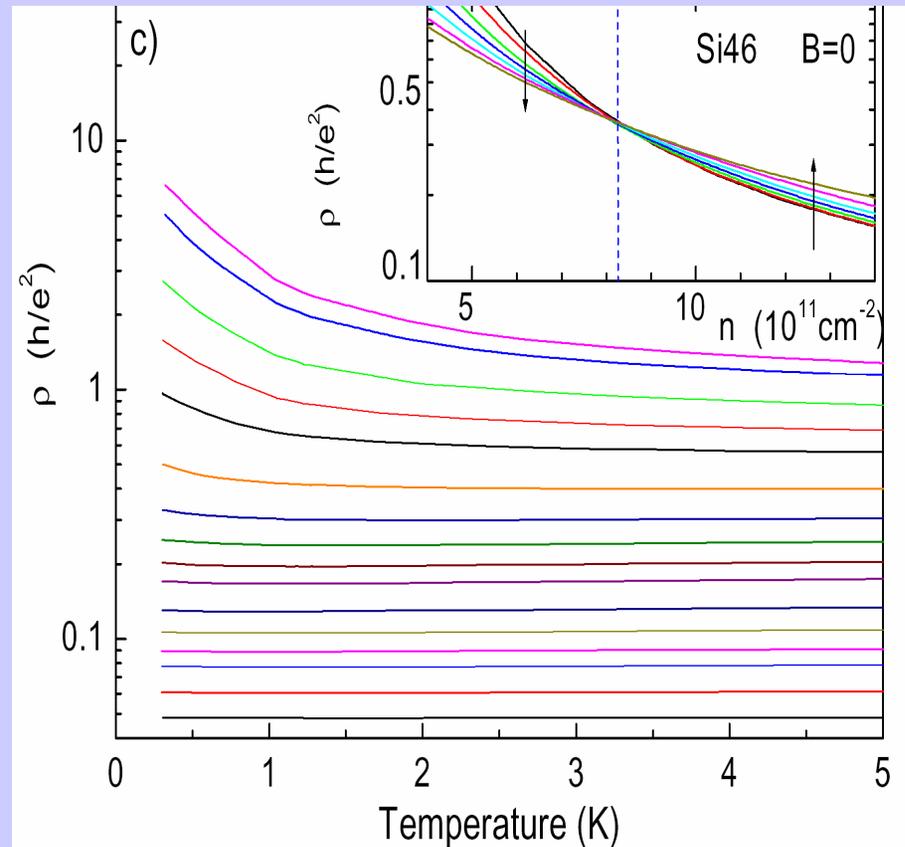
... in contrast to strongly disordered samples:

clean sample:

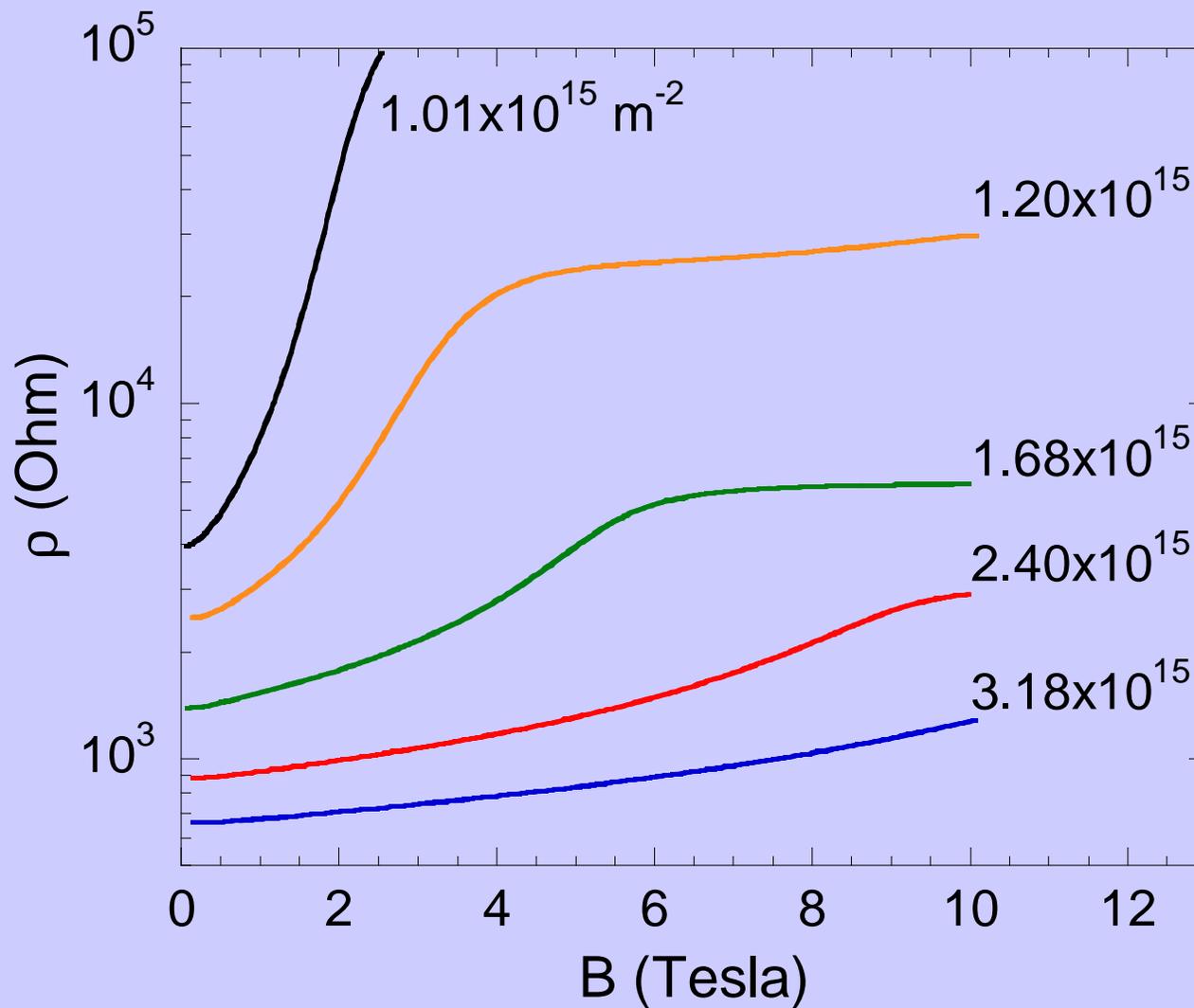


➤ Clearly, one-parameter scaling theory does not work here

disordered sample:



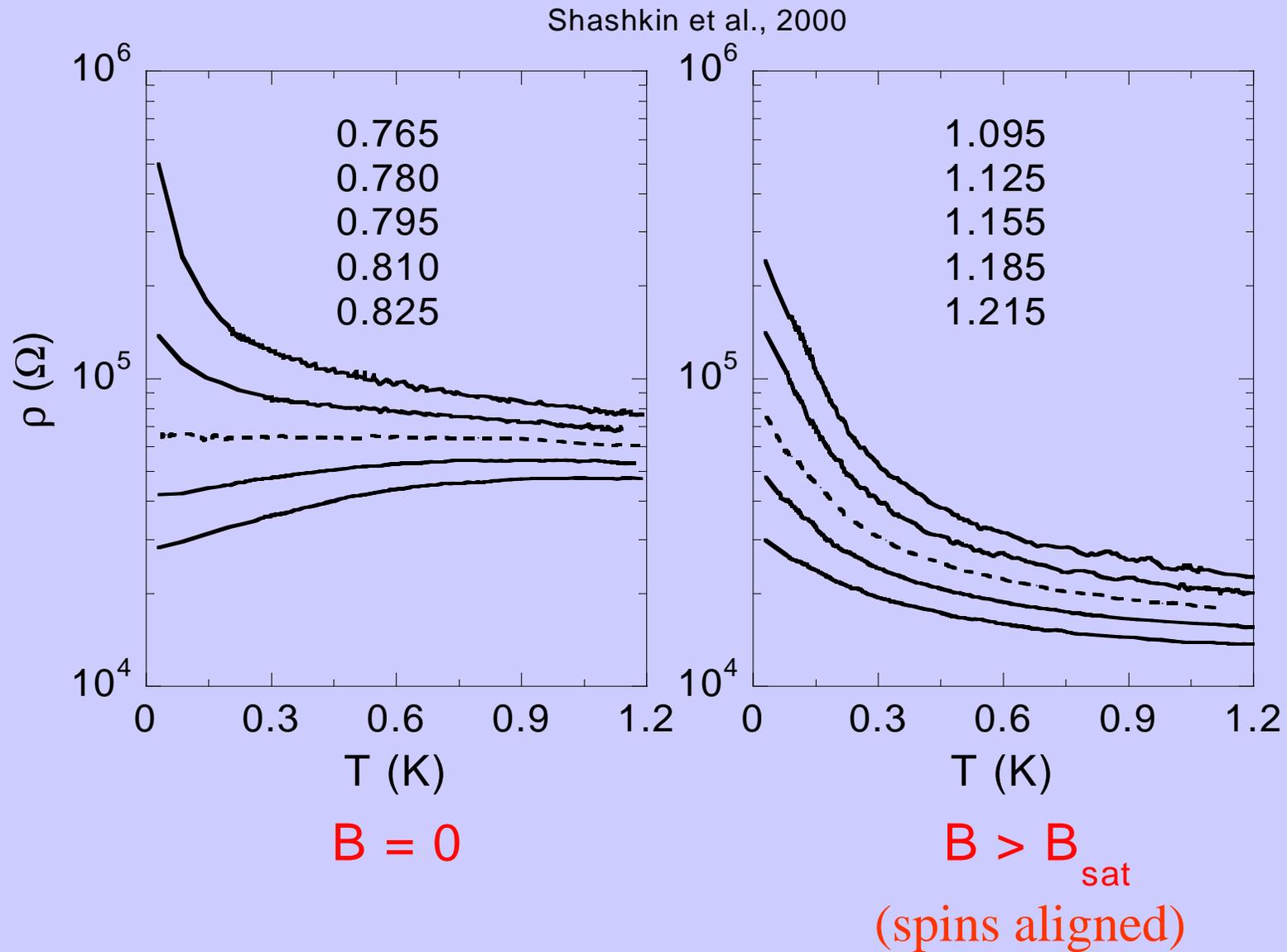
The effect of the *parallel* magnetic field:



$T = 30$ mK

**Shashkin, Kravchenko,
Dolgoplov, and
Klapwijk, *PRL* 2001**

Magnetic field, by aligning spins, changes metallic $R(T)$ to insulating:



Reaction of referees:

Referee A:

“The paper should not be published in PRL. Everyone knows there is no zero-temperature conductivity in 2-d.”

Referee B:

“The reported results are most intriguing, but they must be wrong. If there indeed were a metal-insulator transition in these systems, it would have been discovered years ago.”

Referee C:

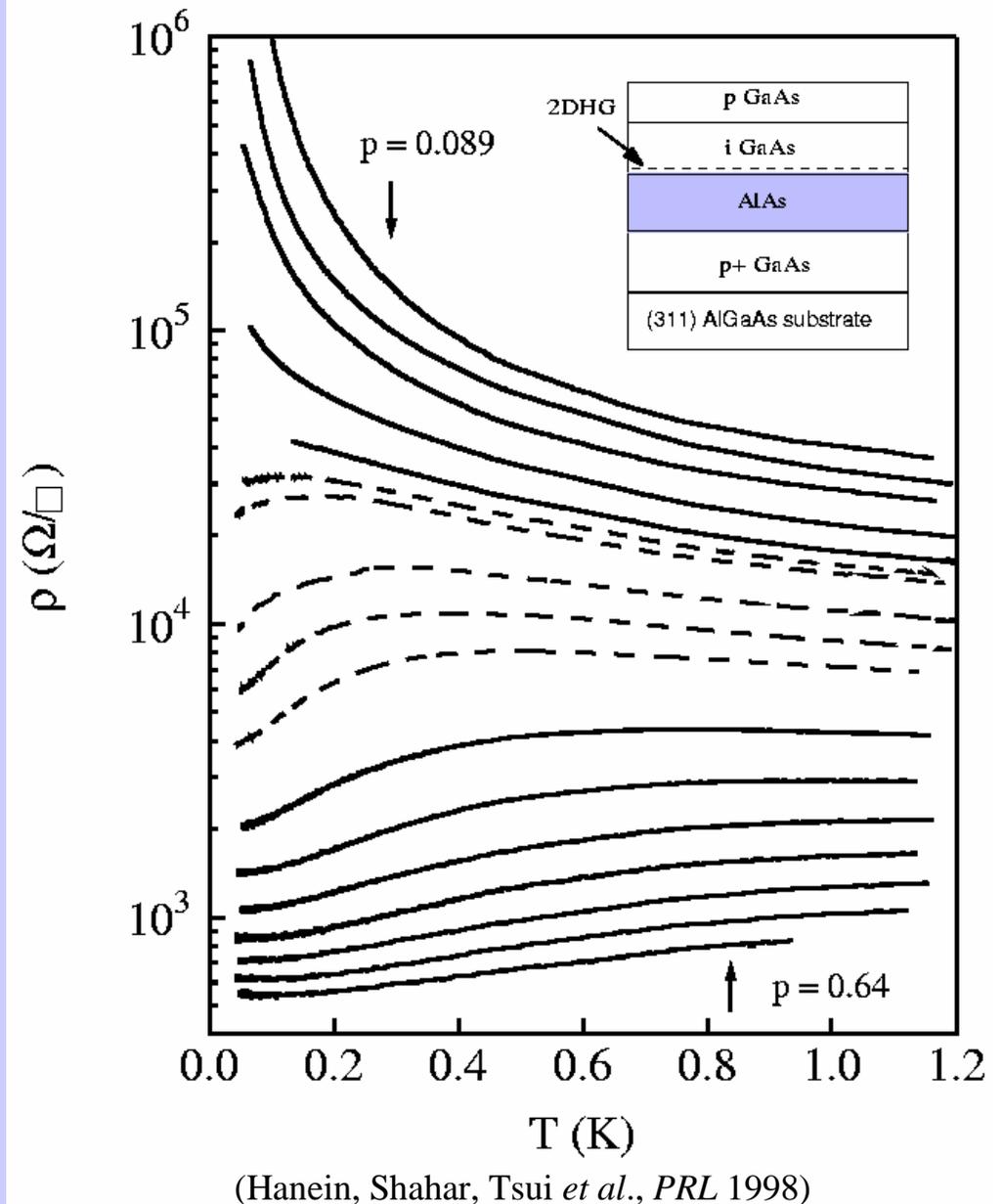
“I cannot explain the reported behavior offhand. Therefore, it must be an experimental error.”

... I remember being challenged over that well-known fact that all states were localized in two dimensions, something that made no sense at all in light of the experiments I had just shown.

R. B. Laughlin, Nobel Lecture

However, later similar transition has also been observed in other 2D structures:

- p-Si:Ge (Coleridge's group; Ensslin's group)
- p-GaAs/AlGaAs (Tsui's group, Boebinger's group)
- n-GaAs/AlGaAs (Tsui's group, Stormer's group, Eisenstein's group)
- n-Si:Ge (Okamoto's group, Tsui's group)
- p-AlAs (Shayegan's group)



Metal-Insulator Transition Unexpectedly Appears in a Two-Dimensional Electron System

For roughly the last two decades, it has generally been believed among those interested in two-dimensional disordered electron systems that, in zero magnetic field, such systems do not undergo a metal-insulator transi-

▶ When the temperature approaches 0 K, can a two-dimensional electron system become a metal? Some recent experiments suggest it can.

litions based on scattering of electrons from static impurities. They found that in two dimensions there was an enhancement of back scattering, which means that if you send an electron along the x axis, it'll start scattering

(*Physics Today* “Search and Discovery” July 1997)

Condensed-matter physics

Another surprise from two dimensions

T. Maurice Rice

Once again, after the discoveries of the quantum Hall effect and high-temperature superconductivity, it is a two-dimensional system that provides a real surprise in condensed-matter physics. For some years now there has been a general belief, bolstered by experiment, that the

furthermore, there are signs that the metallic ground state may be a perfect conductor, and an experiment reported last month³ shows that this effect disappears if the spins of the electrons are polarized — a clear sign that this is no ordinary metal.

The technology behind the semiconduc-

(*Nature* 389, “News and views” 30 October 1997)

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SCIENCE AND TECHNOLOGY

the approach predates him. The “Discovery” programme, which began two years before he got the job, epitomises what this means in practice: a reversal of the old system. Instead of deciding on big projects and seeking money for them, NASA managers set aside lots of small budgets and solicit proposals from people who think they can do something useful with the money.

NEAR and *Mars Pathfinder* were the first two to fly under Discovery’s aegis. Two more—*Lunar Prospector*, which will scan the moon from a low orbit, and *Stardust*, which will taste a comet’s tail—have been approved. *Mars Surveyor* was planned us-

took two years to design, build and launch. By contrast, *Cassini*—a monster probe to Saturn that is the last of the old guard—was begun in 1989, will take off in October, and cost \$1½ billion.

Less is lost, too, if a craft crashes or malfunctions. Previously, a scientist’s life’s work could disappear in a few seconds—as happened to the Russian-European *Mars 96* launch, which crashed into the sea last year with two dozen experiments on board. (The European Space Agency, ESA, decided last month to ape NASA’s approach by supporting more small missions.) Less is also lost to another kind of havoc—the kind

Superconductors

Silicon waves

THE scientist who cries superconductor is like the boy who cried wolf. Over the years, too many sightings of supposedly new superconductors (materials that lose all resistance to electrical current at low enough temperatures) have turned out to be experimental glitches. As a result, careful researchers often dare not breathe the word “superconductor”. This may explain why the recent discovery of something that

(*The Economist* “Science and Technology” July 1997)

Condensed-matter physics

Real metals, 2D or not 2D?

Mohelle Y. Simmons and Alex R. Hamilton

The distinction between metals and insulators appears simple — metals conduct electricity whereas insulators do not. Yet, for the past 25 years, arguments

insulating. The celebrated discovery of the quantum Hall effect in 1980 demonstrated that it is possible to have metallic states in a 2D system that persist to $T = 0$ by applying a

(*Nature* 400,
“News and views”
19 August 1999)

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non-Fermi liquid:

VOLUME 79, NUMBER 3

PHYSICAL REVIEW LETTERS

21 JULY 1997

Scaling Theory of Two-Dimensional Metal-Insulator Transitions

V. Dobrosavljević,¹ Elihu Abrahams,^{1,2} E. Miranda,¹ and Sudip Chakravarty³

¹*National High Magnetic Field Laboratory, Florida State University 1800 E. Paul Dirac Drive, Tallahassee, Florida 32306*

²*Serim Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855-0849*

³*Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-1547*

(Received 10 April 1997)

PHYSICAL REVIEW B

VOLUME 58, NUMBER 2

RAPID COMMUNICATIONS

1 JULY 1998-II

Interactions and scaling in a disordered two-dimensional metal

Sudip Chakravarty and Lan Yin

Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-1547

Elihu Abrahams

Serim Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855-0849

(Received 19 December 1997)

We show that a non-Fermi-liquid state of interacting electrons in two dimensions is stable in the presence of disorder and is a perfect conductor, provided the interactions are sufficiently strong. Otherwise, the disorder

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continued....

(superconductivity)

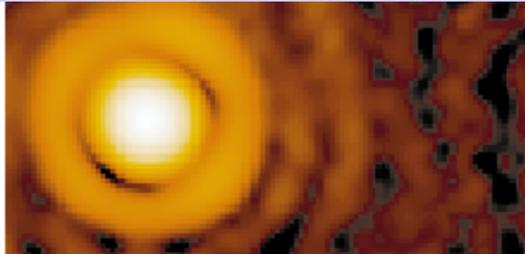


Figure 4 Stellar diffraction patterns obtained by subtracting the nulled images of Fig. 3 from the corresponding constructive images. For α Ori, a misalignment of the centroids by 0.3 arcsec was applied to obtain the symmetric diffraction pattern shown. The non-interfered images of unresolved stars, when similarly aligned and stacked, are not so perfect because they include the halo components from nebular emission or scattering due to higher-order wavefront aberrations. These subtract out in the difference image.

NATURE | VOL 395 | 17 SEPTEMBER 1998

Nature © Macmillan Publishers Ltd 1998

253

Superconductivity in a two-dimensional electron gas

Philip Phillips, Yi Wan, Ivar Martin, Sergey Knysch & Denis Dalkovich

Loomis Laboratory of Physics, University of Illinois at Urbana-Champaign, 1100 W. Green Street, Urbana, Illinois 61801-3080, USA

In a series of recent experiments, Kravchenko and colleagues^{1,2} observed unexpectedly that a two-dimensional electron gas in zero magnetic field can become conducting at low temperatures: the two-dimensionality was imposed by confining the electron gas to the interface between two semiconductors. The observation of

PHYSICAL REVIEW B

VOLUME 58, NUMBER 20

15 NOVEMBER 1998-II

Superconductivity in a correlated disordered two-dimensional electron gas

J. S. Thakur and D. Neilson

School of Physics, The University of New South Wales, Sydney 2052, Australia

(Received 22 May 1998)

9/28/2007

University of Virginia

continued....

(superconductivity)

PHYSICAL REVIEW B

VOLUME 58, NUMBER 13

1 OCTOBER 1998-I

Possible triplet superconductivity in MOSFETs

D. Belitz

Department of Physics and Materials Science Institute, University of Oregon, Eugene, Oregon 97403

T. R. Kirkpatrick

Institute for Physical Science and Technology, and Department of Physics, University of Maryland, College Park, Maryland 20742

(Received 4 February 1998; revised manuscript received 21 April 1998)

PHYSICAL REVIEW B, VOLUME 64, 245115

***P*-wave pairing and ferromagnetism in the metal-insulator transition in two dimensions**

Claudio Chamon,¹ Eduardo R. Mucciolo,² and A. H. Castro Neto^{1,*}

¹*Department of Physics, Boston University, Boston, Massachusetts 02215*

²*Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, Caixa Postal 38071,
22452-970 Rio de Janeiro, Brazil*

(Received 15 June 2001; published 10 December 2001)

continued....

(new quantum phase prior to Wigner crystallization)

VOLUME 83, NUMBER 9

PHYSICAL REVIEW LETTERS

30 AUGUST 1999

New Quantum Phase between the Fermi Glass and the Wigner Crystal in Two Dimensions

Giuliano Benenti, Xavier Waintal, and Jean-Louis Pichard

CEA, Service de Physique de l'Etat Condensé, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette, France

(Received 21 December 1998; revised manuscript received 7 April 1999)

VOLUME 83, NUMBER 22

PHYSICAL REVIEW LETTERS

29 NOVEMBER 1999

Conducting Phase in the Two-Dimensional Disordered Hubbard Model

P. J. H. Denteneer

Lorentz Institute, University of Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands

R. T. Scalettar

Physics Department, University of California, 1 Shields Avenue, Davis, California 95616

N. Trivedi

Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400-005, India

(Received 1 April 1999)

9/28/2007

University of Virginia

continued....

(spin-orbit interaction)

Unconventional metallic state in a two-dimensional system with broken inversion symmetry

V. M. Pudalov

Institute of High-Pressure Physics, 142092 Troitsk, Moscow Region, Russia

(Submitted 2 June 1997; resubmitted 2 July 1997)

Pis'ma Zh. Eksp. Teor. Fiz. **66**, No. 3, 168–172 (10 August 1997)

VOLUME 80, NUMBER 19

PHYSICAL REVIEW LETTERS

11 MAY 1998

Quantum Interference and Electron-Electron Interactions at Strong Spin-Orbit Coupling in Disordered Systems

Yuli Lyanda-Geller

Department of Physics, Materials Research Laboratory and Beckman Institute, University of Illinois, Urbana, Illinois 61801

(Received 25 September 1997)

Weak antilocalization in a 2D electron gas with the chiral splitting of the spectrum

M. A. Skvortsov

L. D. Landau Institute for Theoretical Physics, Moscow 117940, RUSSIA

(September 20, 2001)

continued....

(percolation)

VOLUME 80, NUMBER 15

PHYSICAL REVIEW LETTERS

13 APRIL 1998

New Liquid Phase and Metal-Insulator Transition in Si MOSFETS

Song He

Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974

X. C. Xie

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078

RAPID COMMUNICATIONS

PHYSICAL REVIEW B

VOLUME 60, NUMBER 20

15 NOVEMBER 1999-II

Droplet state in an interacting two-dimensional electron system

Juntan Shi

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078

Song He

Hexaa Laboratory, Warren, New Jersey 07059

X. C. Xie

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078

(Received 1 July 1999)

VOLUME 83, NUMBER 17

PHYSICAL REVIEW LETTERS

25 OCTOBER 1999

Percolation-Type Description of the Metal-Insulator Transition in Two Dimensions

Yigal Meir

Department of Physics, Ben-Gurion University, Beer Sheva 84105, Israel

(Received 27 April 1999)

continued....

Charging/discharging of interface traps:

VOLUME 82, NUMBER 1

PHYSICAL REVIEW LETTERS

4 JANUARY 1999

Theory of Metal-Insulator Transitions in Gated Semiconductors

Boris L. Altshuler^{1,2} and Dmitrii L. Maslov³

¹*NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540*

²*Physics Department, Princeton University, Princeton, New Jersey 08544*

³*Department of Physics, University of Florida, P.O. Box 118440 Gainesville, Florida 32611-8440*

(Received 25 August 1998)

Temperature-dependent screening:

VOLUME 83, NUMBER 1

PHYSICAL REVIEW LETTERS

5 JULY 1999

Charged Impurity-Scattering-Limited Low-Temperature Resistivity of Low-Density Silicon Inversion Layers

S. Das Sarma and E. H. Hwang

Department of Physics, University of Maryland, College Park, Maryland 20742-4111

(Received 14 December 1998)

Spin-orbit scattering:

VOLUME 84, NUMBER 21

PHYSICAL REVIEW LETTERS

22 MAY 2000

Interband Scattering and the "Metallic Phase" of Two-Dimensional Holes in GaAs/AlGaAs

Yuval Yaish,¹ Oleg Pcus,¹ Evgeny Buchstab,¹ Shye Shapira,² Gidi Ben Yoseph,¹ Uri Sivan,¹ and Ady Stern³

¹*Department of Physics and Solid State Institute, Technion-HT, Haifa 32000, Israel*

²*Cambridge Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom*

³*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

(Received 4 May 1999)

9/28/2007

University of Virginia

continued....

...and, finally, an unspecified mechanism:



Physica E 9 (2001) 209–225



Metal–insulator transition in 2D: resistance in the critical region

B.L. Altshuler^{a,b}, D.L. Maslov^c, V.M. Pudalov^{d,*}

^a*NEC Research Institute, 4 Independence Way, Princeton, NJ 08540, USA*

^b*Physics Department, Princeton University, Princeton, NJ 08544, USA*

^c*Department of Physics, University of Florida, P.O. Box 118440, Gainesville, FL 32611-8440, USA*

^d*P.N. Lebedev Physics Institute, Leninskii Prospekt 53, Moscow 117924, Russia*

Accepted 15 June 2000

Now, a more reasonable approach

Corrections to conductivity due to electron-electron interactions in the diffusive regime ($T\tau < 1$)

VOLUME 44, NUMBER 19

PHYSICAL REVIEW LETTERS

12 MAY 1980

Interaction Effects in Disordered Fermi Systems in Two Dimensions

B. L. Altshuler and A. G. Aronov

Leningrad Nuclear Physics Institute, Gatchina, Leningrad 188 350, U.S.S.R.

and

P. A. Lee

Bell Laboratories, Murray Hill, New Jersey 07974

(Received 11 February 1980)

Interaction effects in disordered Fermi systems are considered in the metallic regime. In two dimensions, logarithmic corrections are obtained for conductivity, density of states, specific heat, and Hall constant. These results are compared with a recent theory of localization as well as some experiments.

$$\delta\sigma = (e^2/4\pi^2\hbar)(2 - 2F) \ln(T\tau),$$

($0 < F < 1$, N.B.: F is *not* the Landau Fermi-liquid constant)

➤ always insulating behavior

However, later this prediction was shown to be incorrect

Zeitschrift fur Physik B (Condensed Matter) -- 1984 -- vol.56, no.3, pp. 189-96

Weak localization and Coulomb interaction in disordered systems

Finkel'stein, A.M.

L.D. Landau Inst. for Theoretical Phys., Acad. of Sci., Moscow, USSR

PHYSICAL REVIEW B

VOLUME 30, NUMBER 2

15 JULY 1984

Interaction-driven metal-insulator transitions in disordered fermion systems

C. Castellani

*Dipartimento di Fisica, Università degli Studi di Roma "La Sapienza," I-00185 Roma, Italy
and Istituto di Fisica, Università degli Studi dell'Aquila, I-67100 l'Aquila, Italy*

C. Di Castro

Dipartimento di Fisica, Università degli Studi di Roma "La Sapienza," I-00185 Roma, Italy

P. A. Lee and M. Ma

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 7 February 1984)

$$\delta\sigma = \frac{e^2}{2\pi^2\hbar} \cdot \ln(T\tau) \cdot \left[1 + 3 \cdot \left(1 - \frac{\ln(1 + F_0^\sigma)}{F_0^\sigma} \right) \right]$$

➤ Insulating behavior when interactions are weak

➤ Altshuler-Aronov-
Lee's result
➤ Metallic behavior when interactions are strong

➤ Effective strength of interactions grows as the temperature decreases

Finkelstein's & Castellani-
DiCastro-Lee-Ma's term

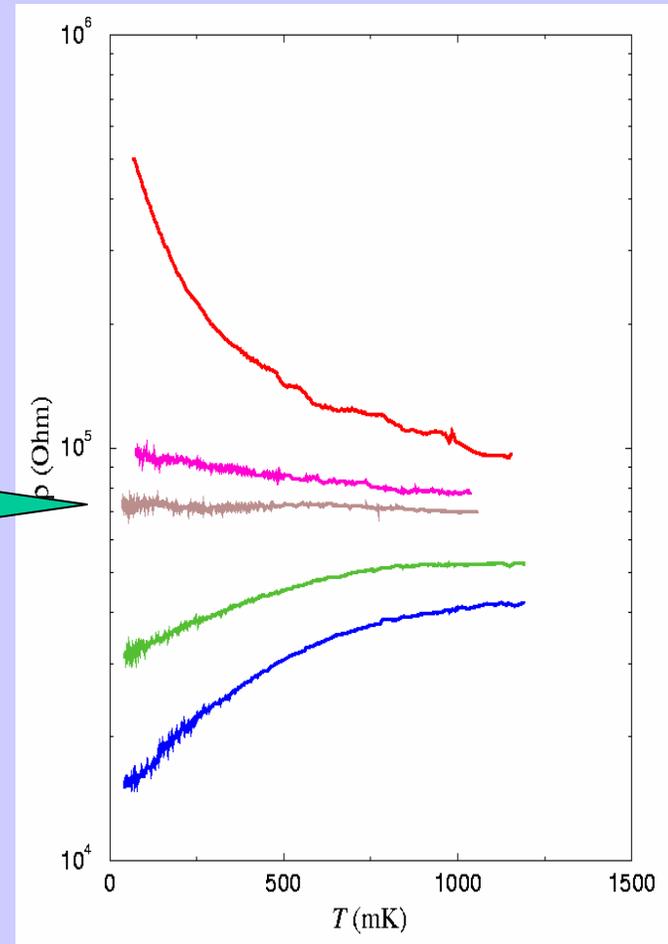
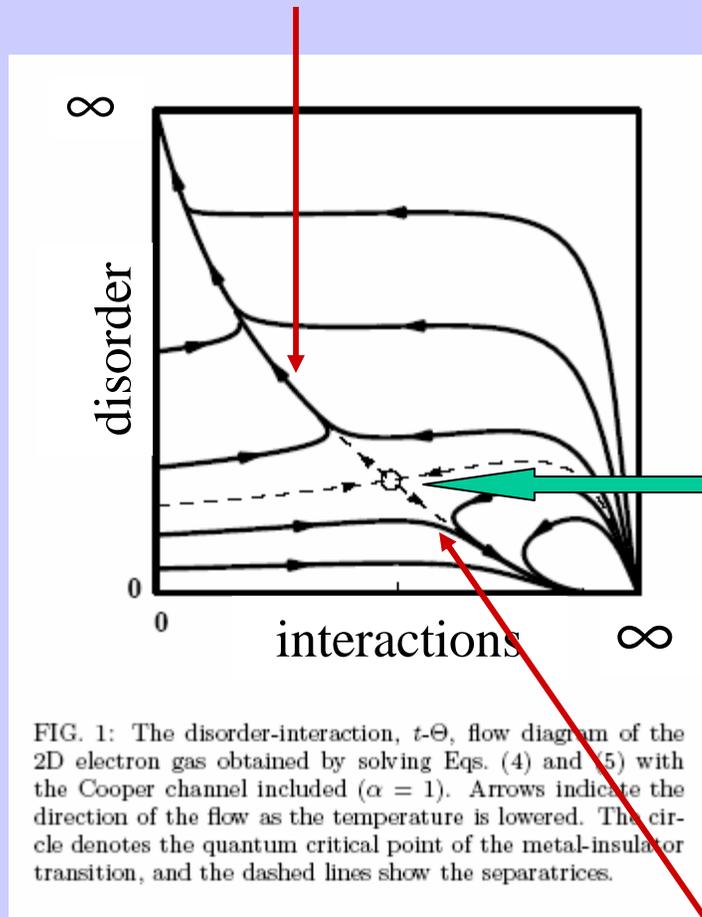
Recent theory of the MIT in 2D

Metal-Insulator Transition in Disordered Two-Dimensional Electron Systems

Alexander Punnoose^{1*} and Alexander M. Finkel'stein²

We present a theory of the metal-insulator transition in a disordered two-dimensional electron gas. A quantum critical point, separating the metallic phase, which is stabilized by electronic interactions, from the insulating phase, where disorder prevails over the electronic interactions, has been identified. The existence of the quantum critical point leads to a divergence in the density of states of the underlying collective modes at the transition, causing the thermodynamic properties to behave critically as the transition is approached. We show that the interplay of electron-electron interactions and disorder can explain the observed transport properties and the anomalous enhancement of the spin susceptibility near the metal-insulator transition.

disorder takes over



Punnoose and Finkelstein, *Science*
310, 289 (2005)

**metallic phase stabilized
by e - e interaction**

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First, one needs to ensure that the system is in the diffusive regime ($T\tau < 1$).

One can distinguish between diffusive and ballistic regimes by studying magnetoconductance:

$$\Delta \sigma (B, T) \propto \left(\frac{B}{T} \right)^2 \quad \text{- diffusive: low temperatures, higher disorder } (Tt < 1).$$

$$\Delta \sigma (B, T) \propto \frac{B^2}{T} \quad \text{- ballistic: low disorder, higher temperatures } (Tt > 1).$$

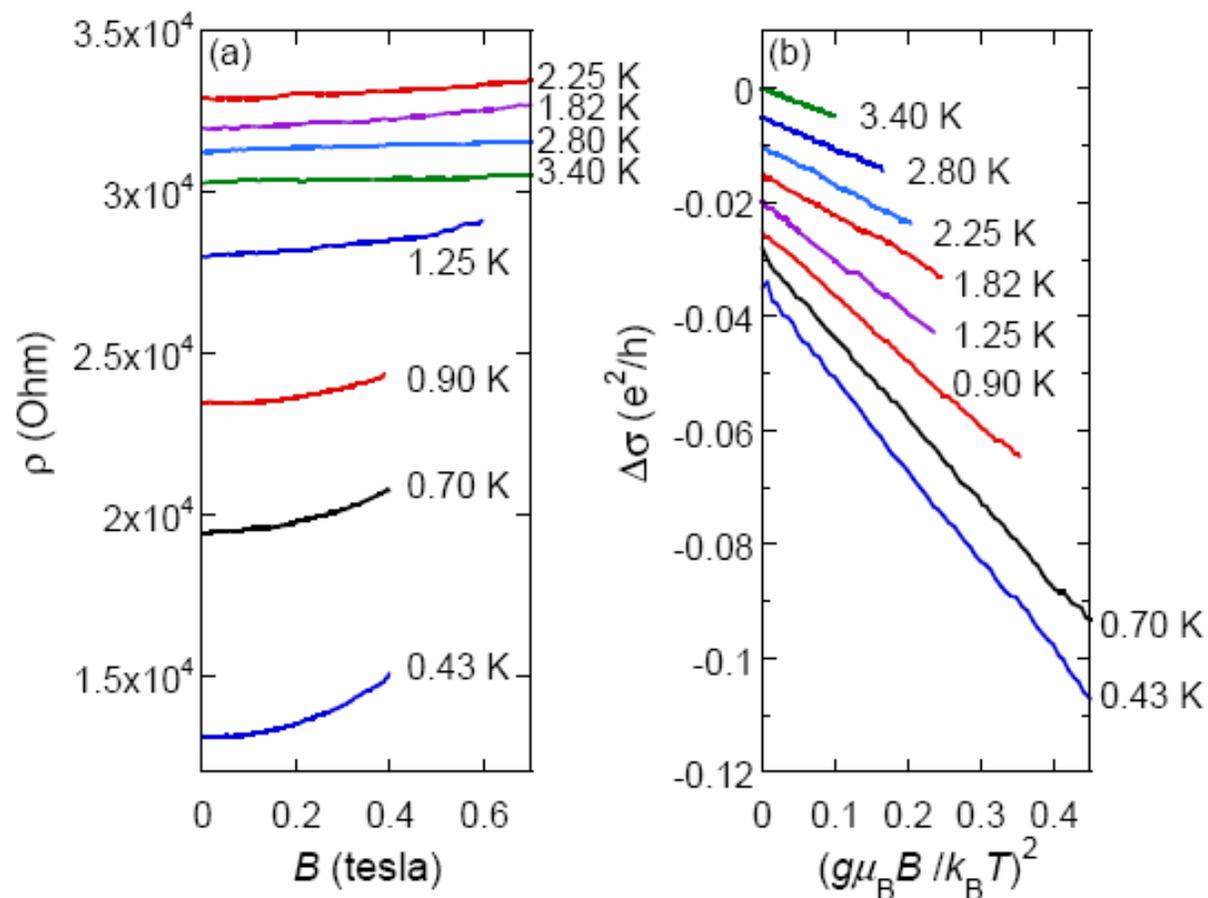
The exact formula for magnetoconductance (Lee and Ramakrishnan, 1982):

$$\Delta \sigma (B, T) = \underbrace{-4}_{\substack{\text{2 valleys} \\ \nearrow}} \left[\frac{0.091 e^2}{\pi \cdot h} \right] \cdot \underbrace{\gamma_2 (\gamma_2 + 1)}_{\nearrow} \cdot \left(\frac{g \mu_B}{k_B} \right)^2 \left(\frac{B}{T} \right)^2$$

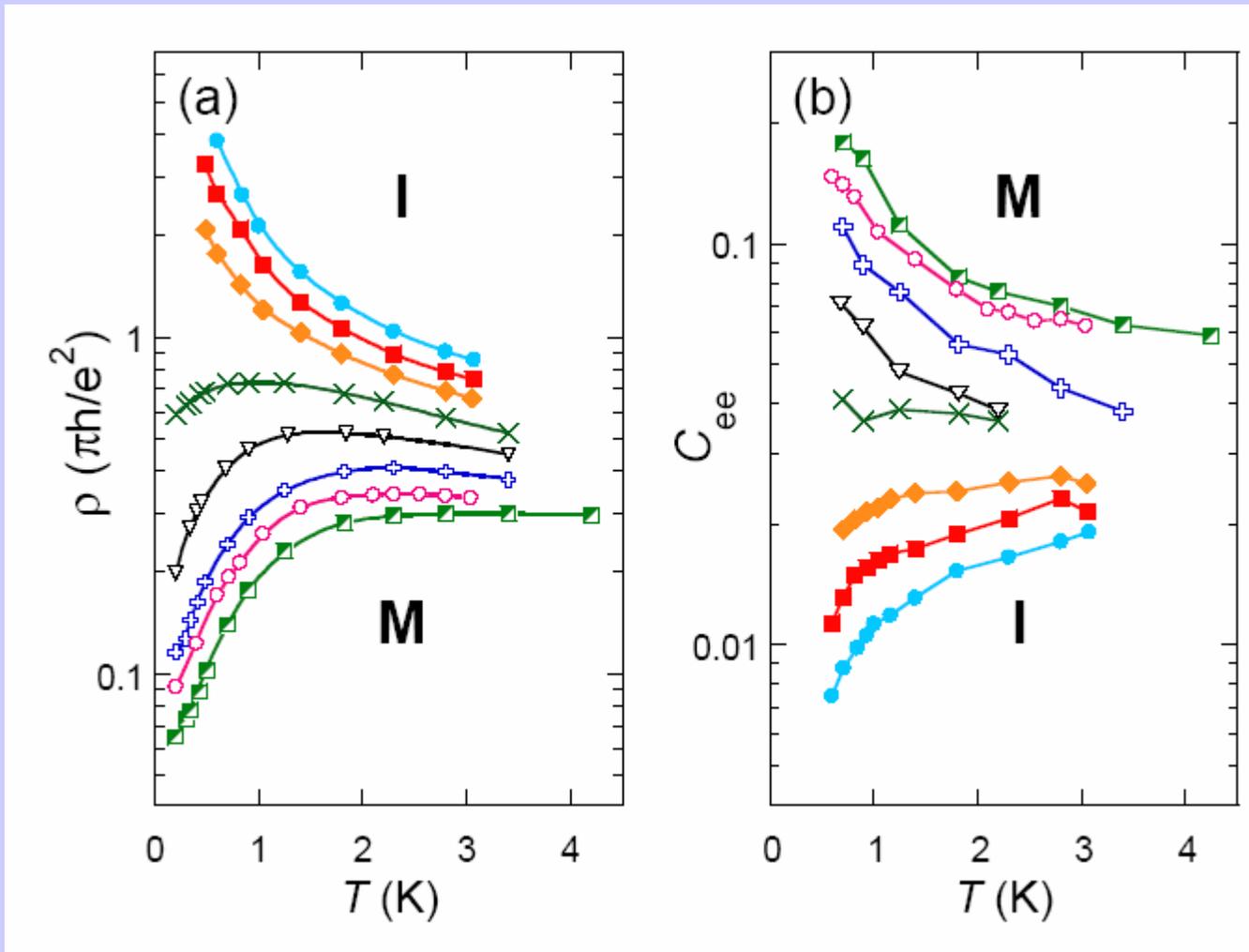
for $\left(\frac{g \mu_B B}{k_B T} \right)^2 \ll 1$

In standard Fermi-liquid notations, $\gamma_2 = - \frac{F_0^a}{1 + F_0^a}$

Experimental results (low-disordered Si MOSFETs;
“just metallic” regime; $n_s = 9.14 \times 10^{10} \text{ cm}^{-2}$):

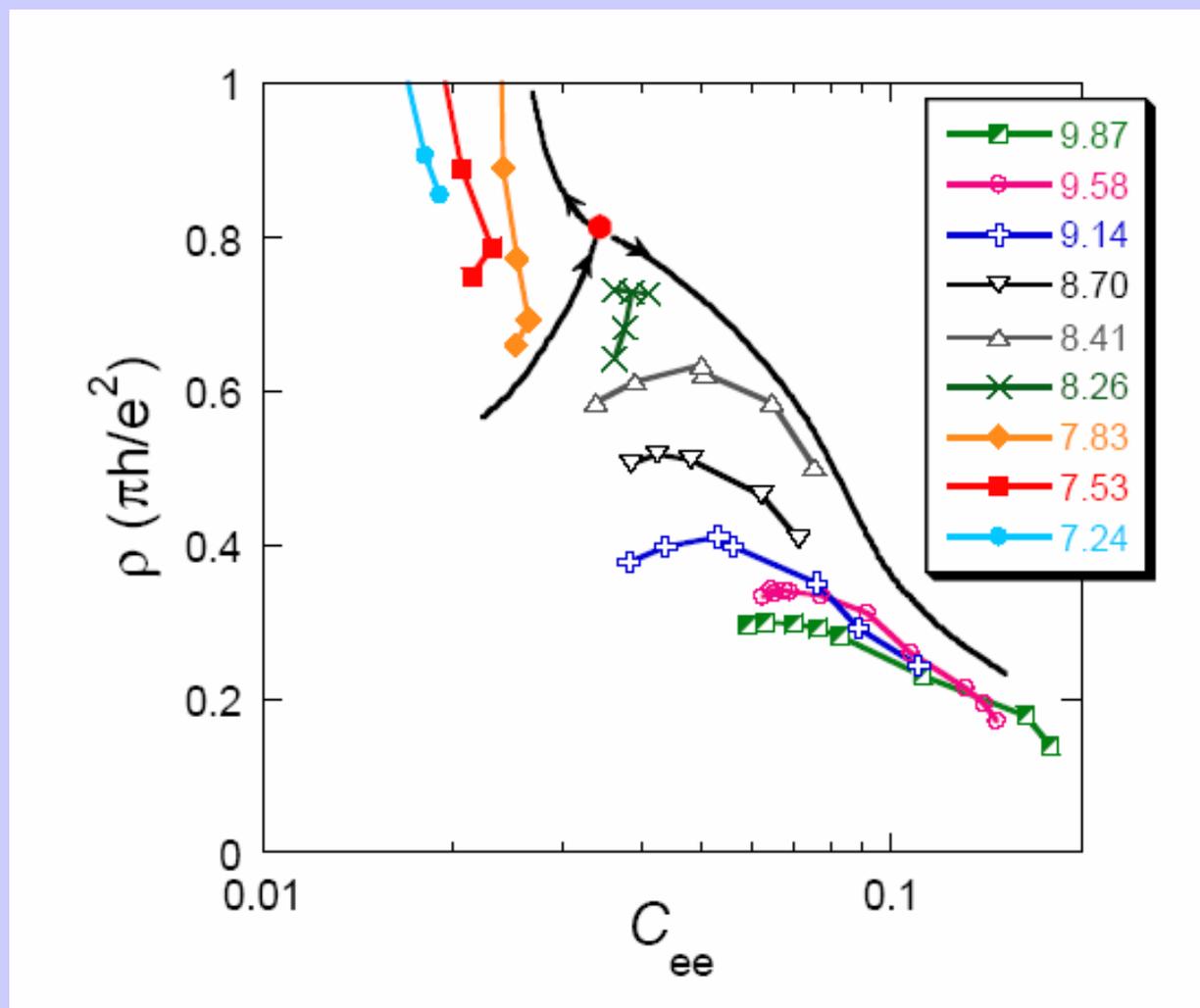


Temperature dependences of the
resistance (a) and strength of interactions (b)



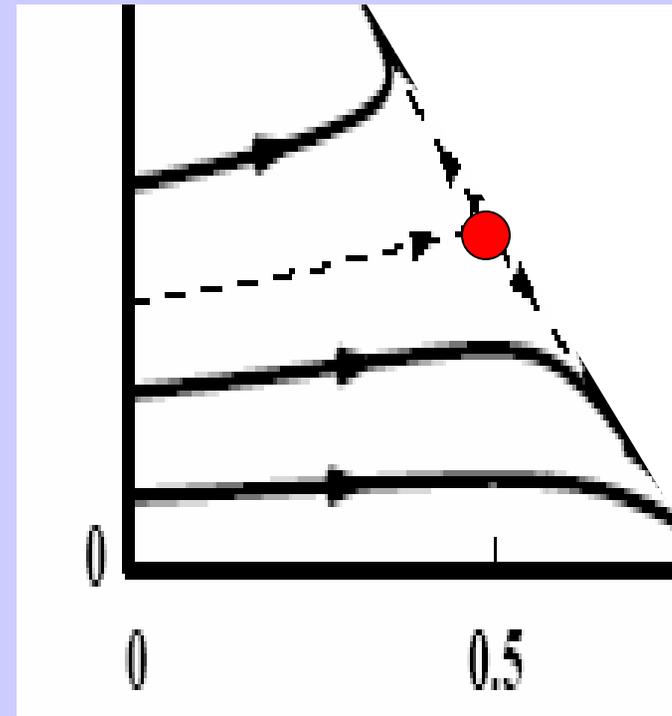
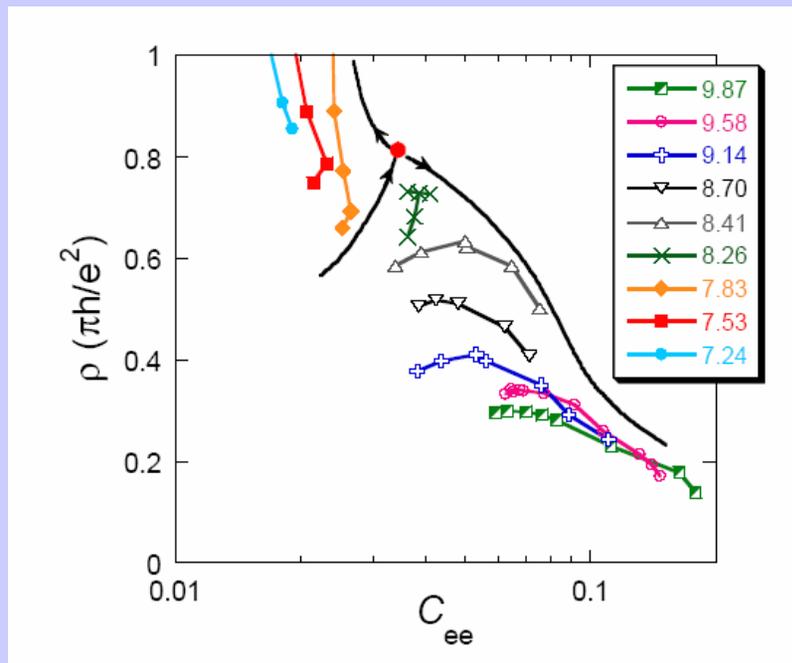
This is the first time effective strength of interactions
has been seen to depend on T

Experimental disorder-interaction flow diagram of the 2D electron liquid



Experimental vs. theoretical flow diagram

(qualitative comparison b/c the 2-loop theory was developed for multi-valley systems)



Quantitative predictions of the one-loop RG for 2-valley systems

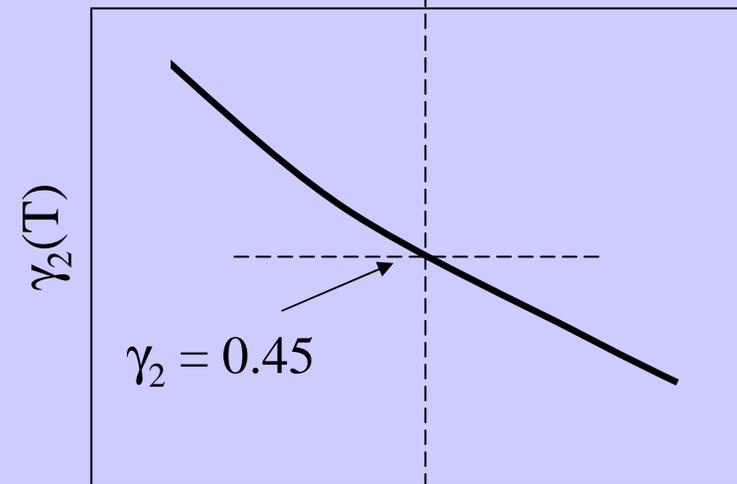
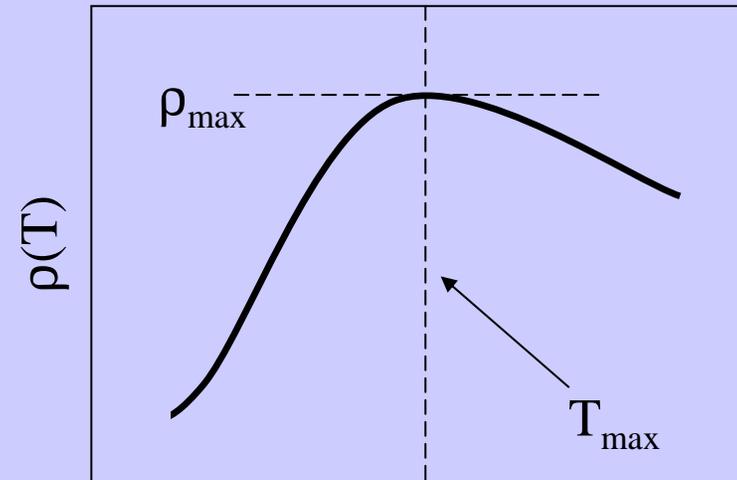
(Punnoose and Finkelstein, *Phys. Rev. Lett.* 2002)

Solutions of the RG-equations for $\rho \ll \pi h/e^2$:
a series of non-monotonic curves $\rho(T)$. After rescaling, the solutions are described by a *single* universal curve:

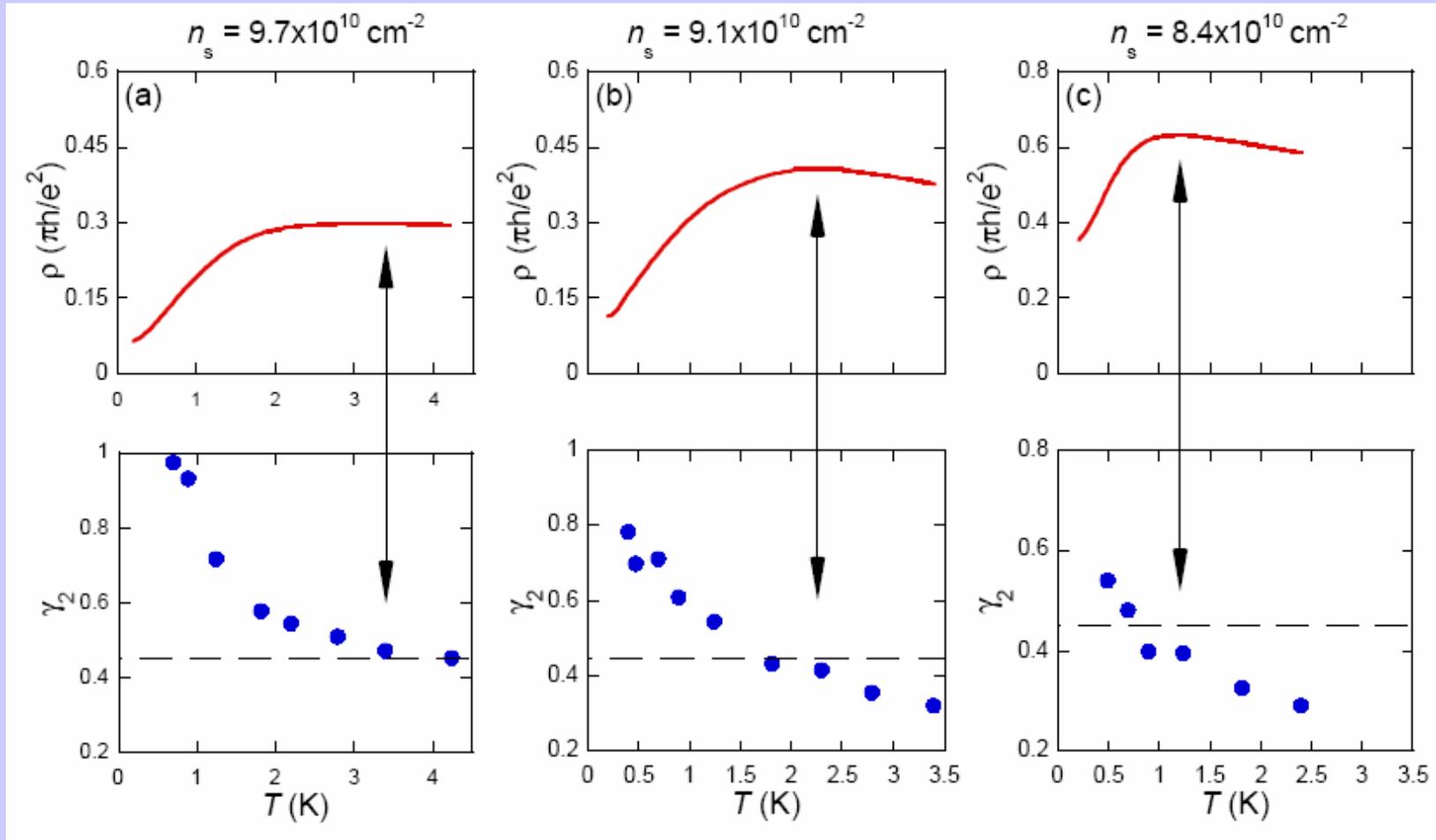
$$\rho(T) = \rho_{\max} R(\eta)$$

$$\eta = \rho_{\max} \ln(T_{\max}/T)$$

For a 2-valley system (like Si MOSFET),
metallic $\rho(T)$ sets in when $\gamma_2 > 0.45$

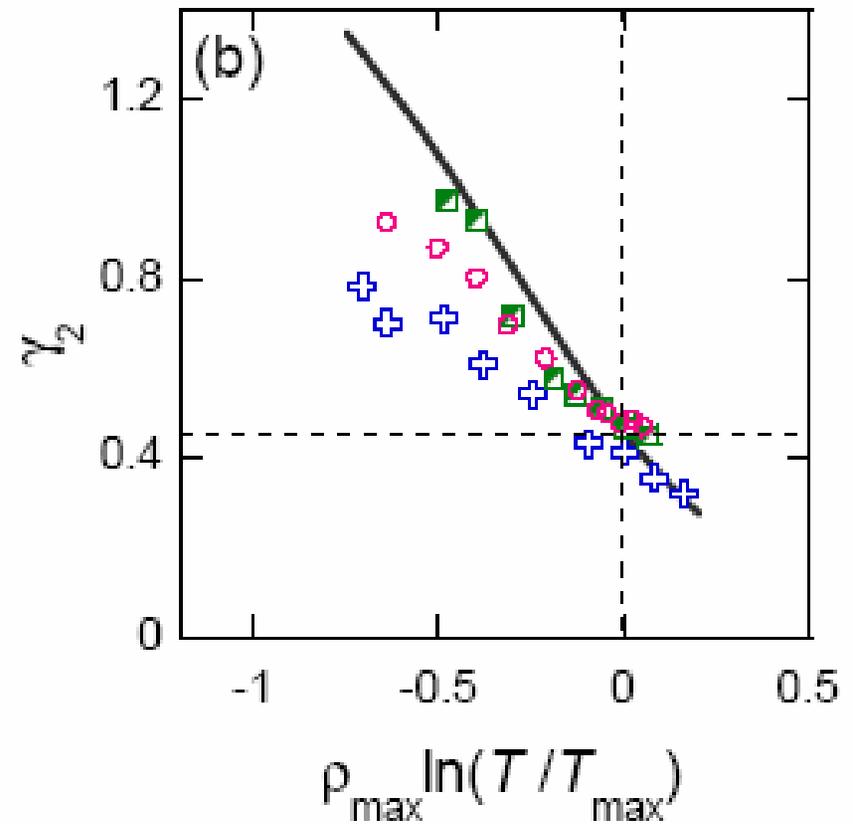
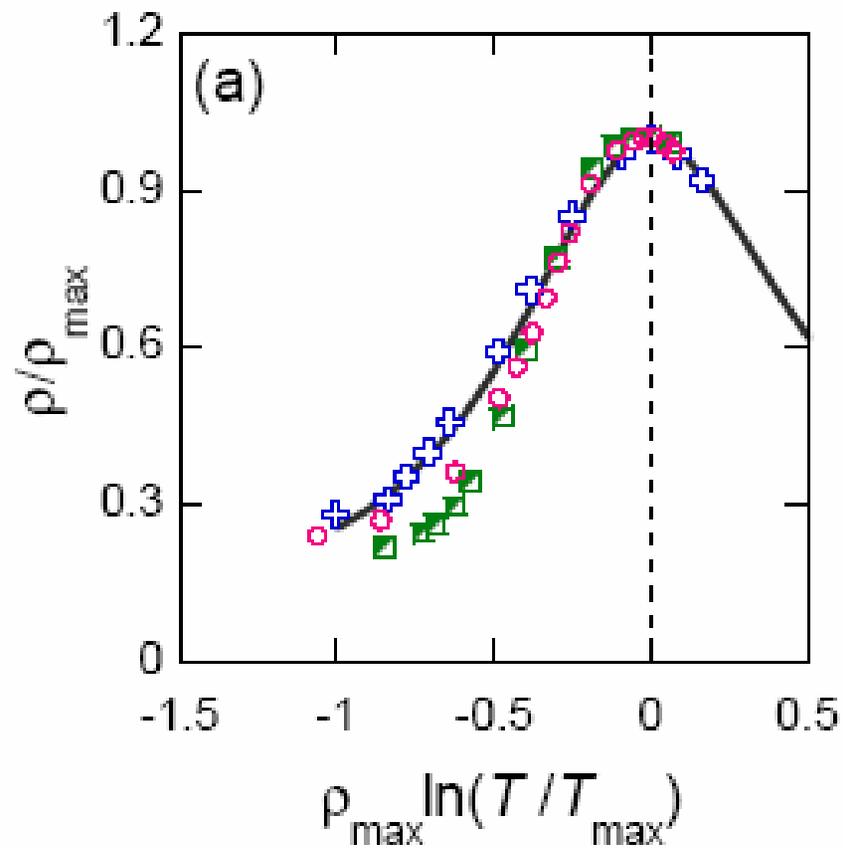


Resistance and interactions vs. T

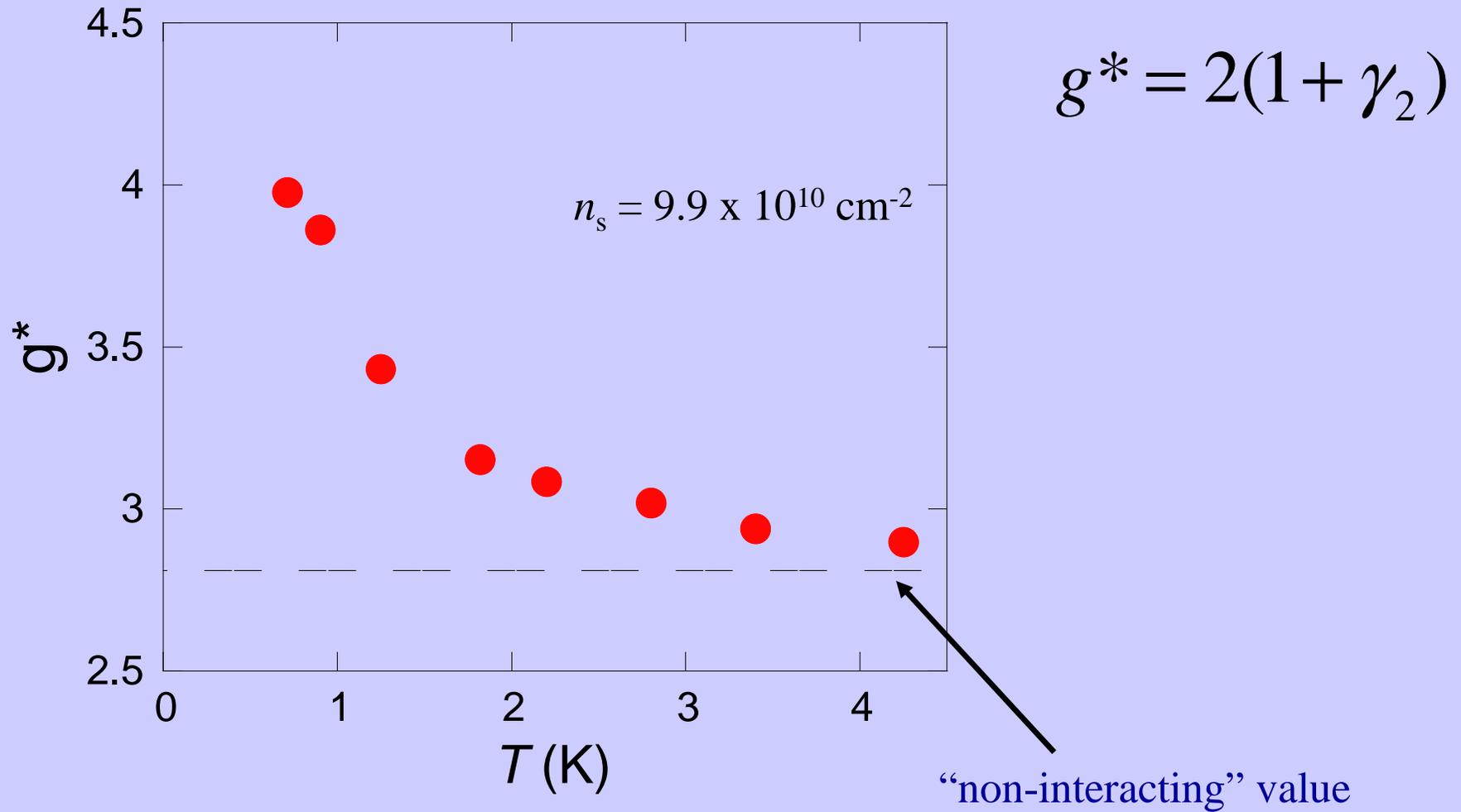


Note that the metallic behavior sets in when $\gamma_2 \sim 0.45$, exactly as predicted by the RG theory

Comparison between theory (lines) and experiment (symbols) (no adjustable parameters used!)



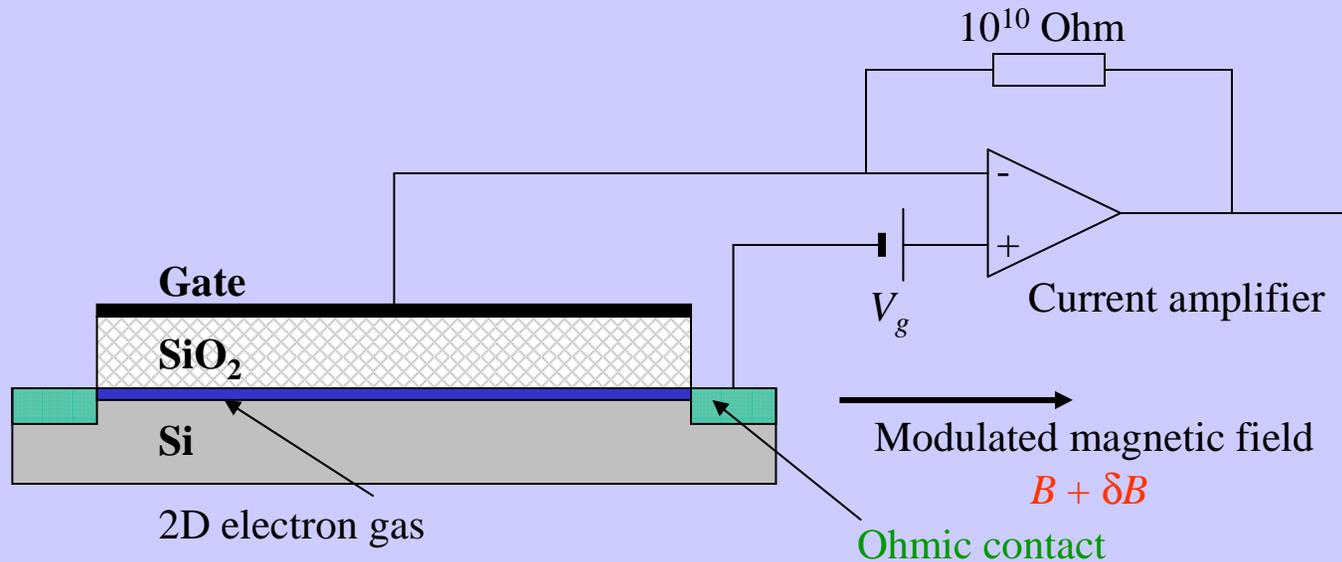
g-factor grows as T decreases



- Scaling theory of localization: the origin of the common wisdom “all electron states are localized in 2D”
- Samples
- What do experiments show?
- What do theorists have to say?
- Interplay between disorder and interactions: experimental test
- **“Clean” regime: diverging spin susceptibility**
- Summary

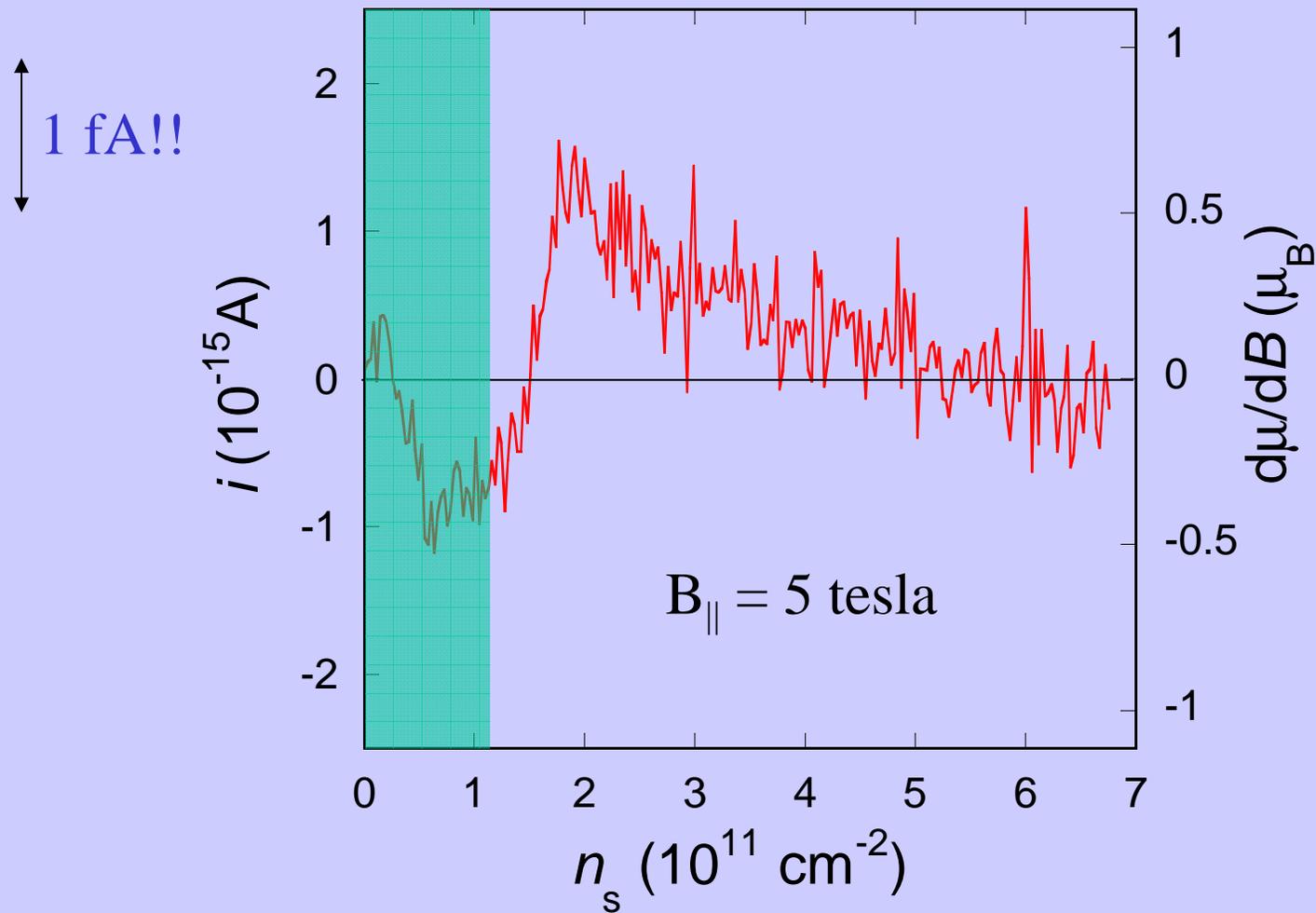
Magnetic measurements without magnetometer

suggested by B. I. Halperin (1998); first implemented by O. Prus, M. Reznikov, U. Sivan *et al.* (2002)

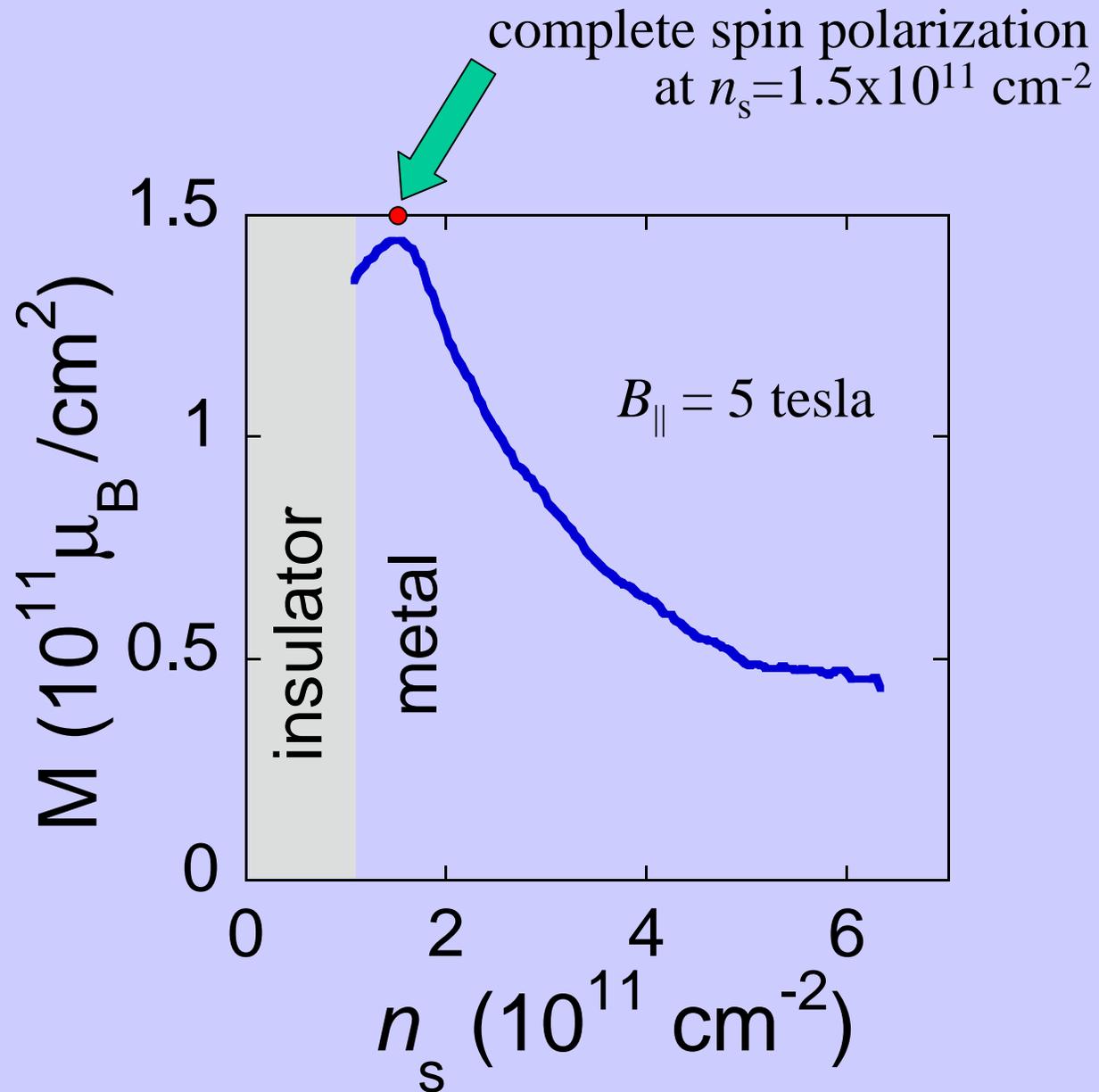


$$i \sim d\mu/dB = - dM/dn_s$$

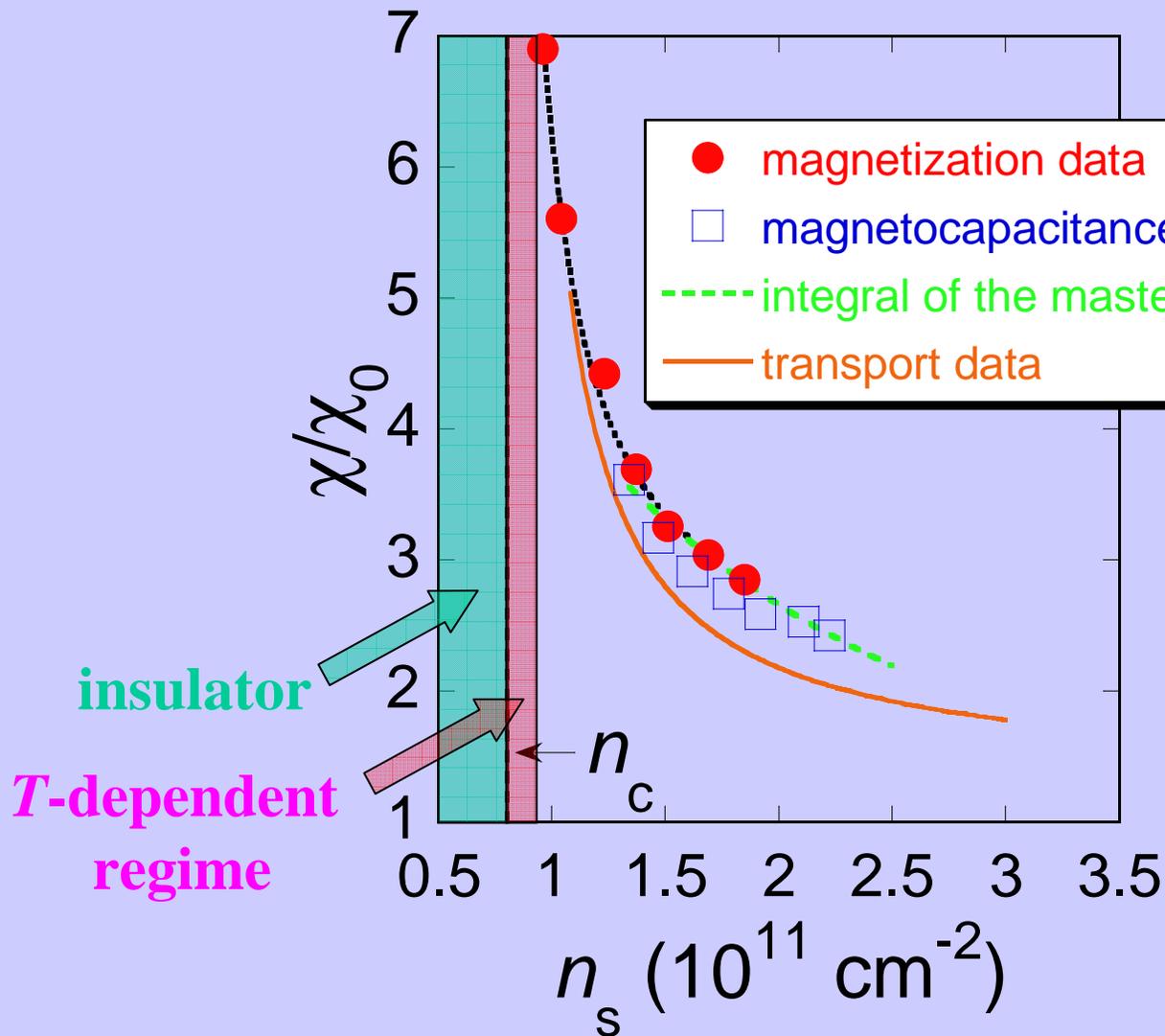
$$d\mu/dB = - dM/dn$$

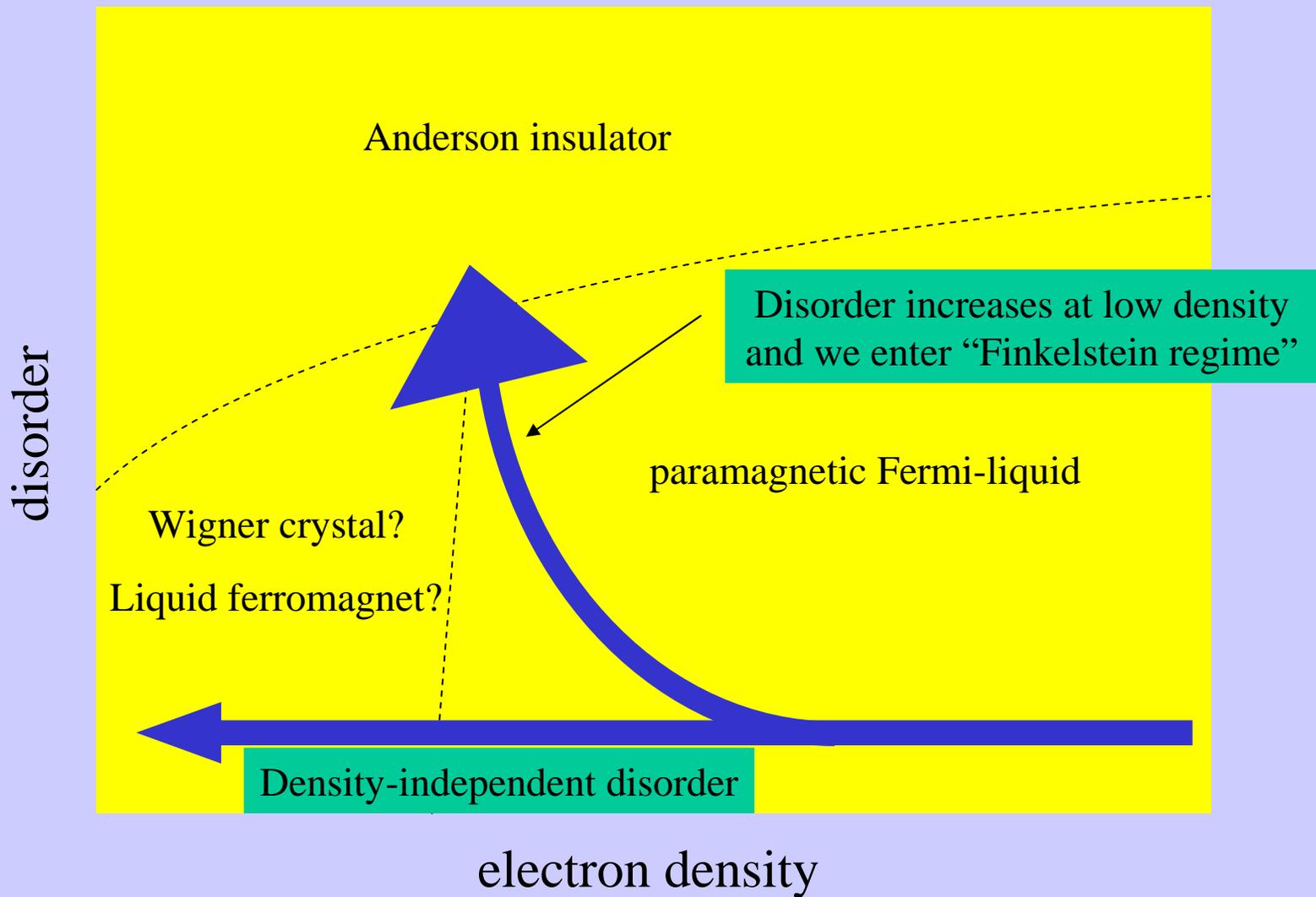


Integral of the previous slide gives $M(n_s)$:



Spin susceptibility exhibits critical behavior near the sample-independent critical density n_χ : $\chi \sim n_s / (n_s - n_\chi)$





SUMMARY:

- Competition between electron-electron interactions and disorder leads to the existence of the metal-insulator transition in two dimensions. The metallic state is stabilized by the electron-electron interactions. In the insulating state, the disorder takes over.
- **Modern renormalization-group theory** (Punnoose and Finkelstein, *Phys. Rev. Lett.* 2002; *Science* 2005) **gives quantitatively correct description of the metallic state without any fitting parameters.**