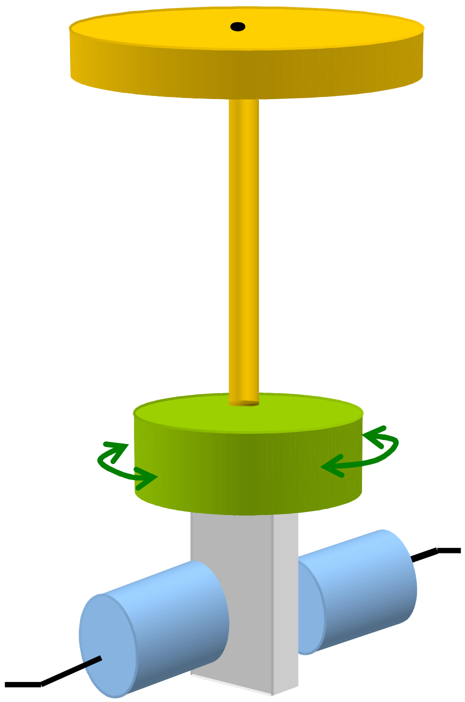


Can a solid be superfluid?

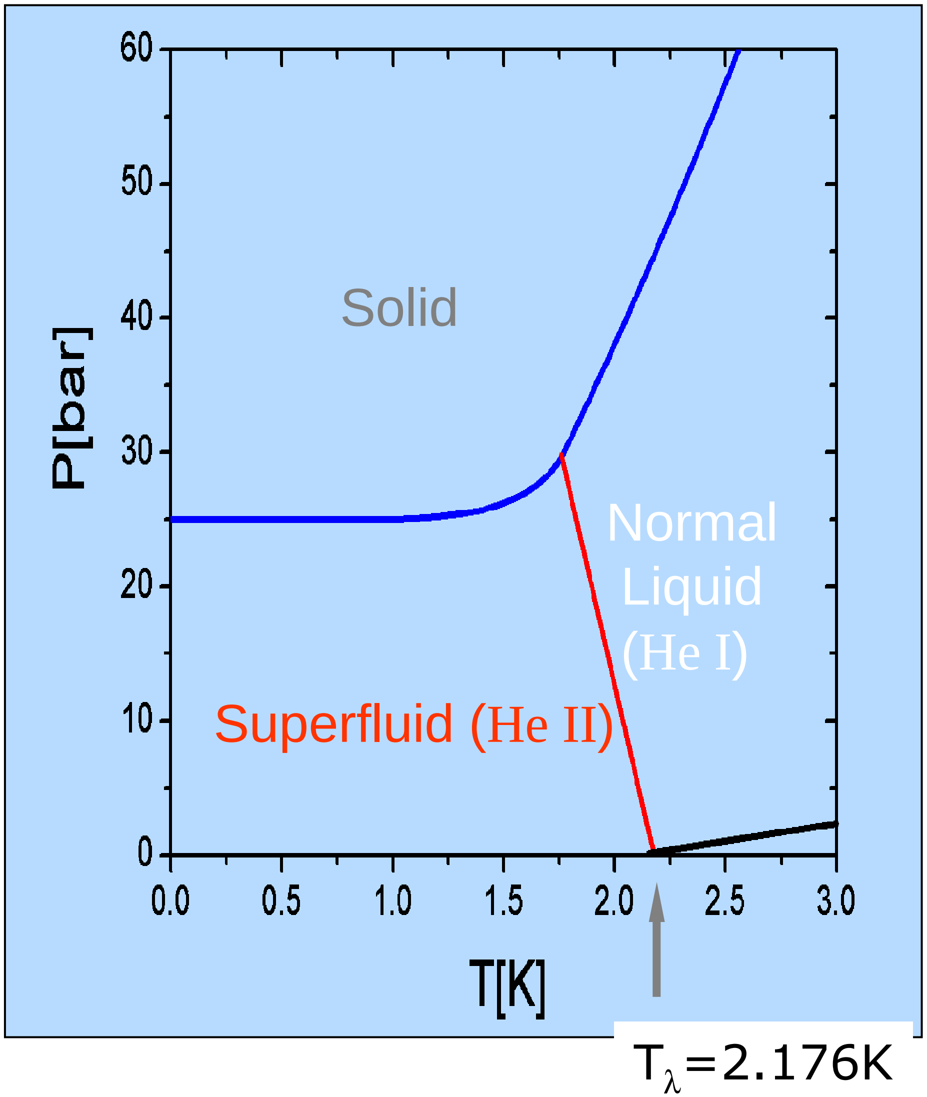
University of Virginia,
August 31, 2007

Moses Chan - Penn State

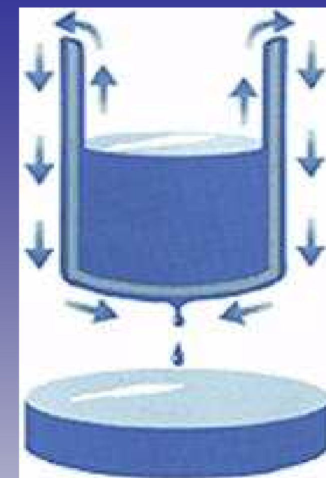
Outline

- Introduction
- Torsional oscillator measurements.
- Specific heat measurements

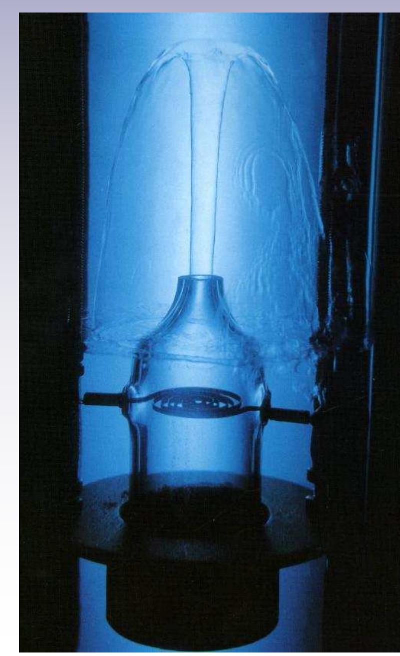
Superfluidity in liquid ^{4}He



- Superfluid helium film can flow up a wall



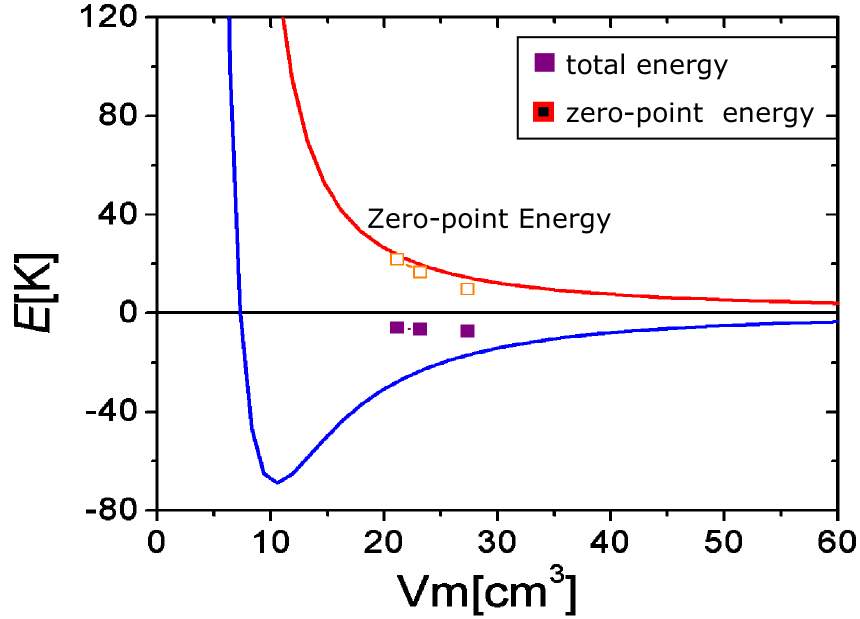
- Superfluid Fountain



- Lindemann Parameter

the ratio of the root mean square of the displacement of atoms to the interatomic distance (d_a)

$$\gamma_L = \frac{\sqrt{< u^2 >}}{d_a} = 0.26$$



A classical solid will melt if the Lindemann's parameter exceeds the critical value of ~0.1 .

- X-ray measurement of the Debye-Waller factor of solid helium at ~0.7K and near melting curve shows this ratio to be 0.262.

(Burns and Issacs, *Phys. Rev. B* **55,** 5767(1997))

- Theoretical ‘consensus’ in 1970s:
  Superfluidity in solid is not impossible!

- If solid ^{4}He can be described by a Jastraw-type wavefunction that is commonly used to describe liquid helium then crystalline order (with finite fraction of vacancies) and BEC can coexist.

G.V. Chester, *Lectures in Theoretical Physics* Vol XI-B(1969);
Phys. Rev. A **2**, 256 (1970)

J. Sarfatt, *Phys. Lett.* **30A**, 300 (1969)

L. Reatto, *Phys. Rev.* **183**, 334 (1969)

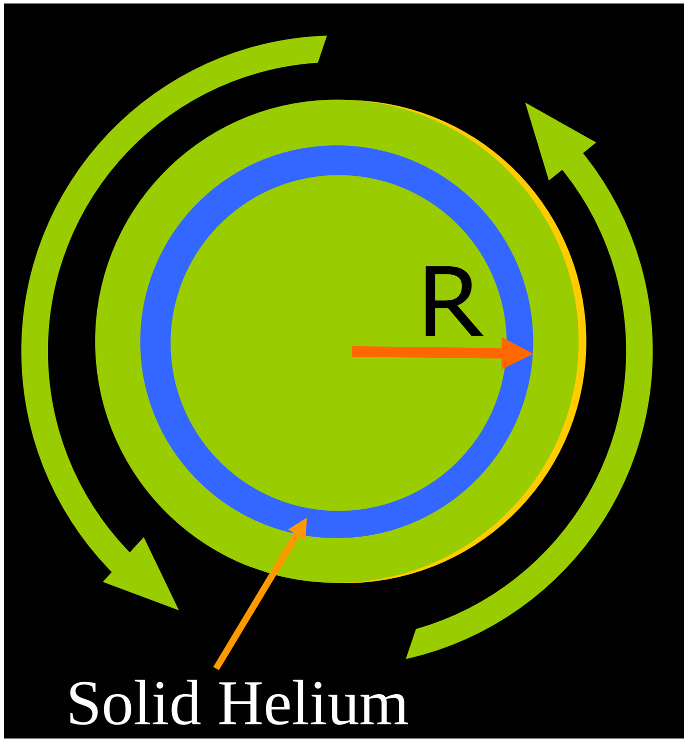
- Andreev and Liftshitz assume the specific scenario of zero-point vacancies and other defects (e.g. interstitial atoms) undergoing BEC and exhibit superfluidity.

Andreev & Liftshitz, *Zh.Eksp.Teor.Fiz.* **56**, 205 (1969).

The ideal method of detection of superfluidity is to subject solid to dc or ac rotation and look for evidence of nonclassical rotational inertia

A. J. Leggett, "Can a solid be superfluid?" PRL 25, 1543 (1970)

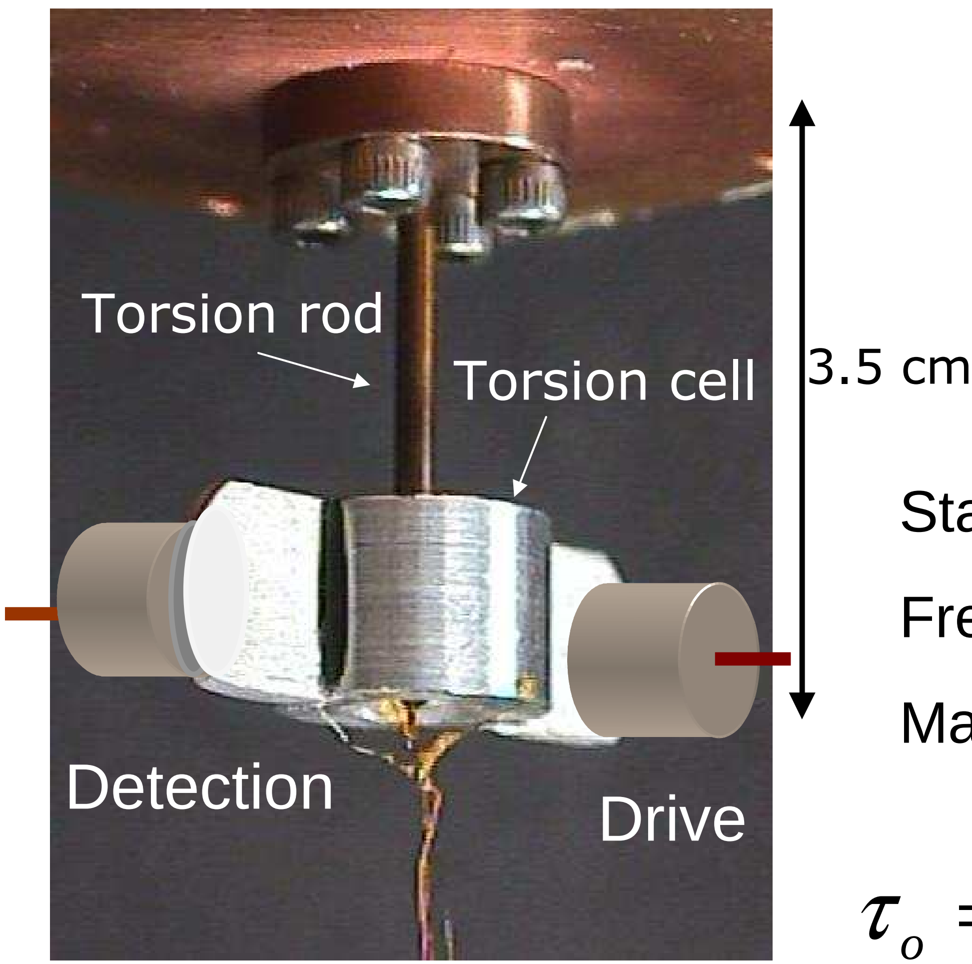
Quantum exchange of particles arranged in an annulus under rotation leads to a measured moment of inertia that is smaller than the classical value

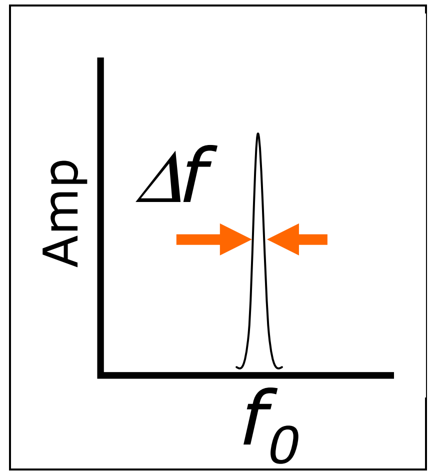


$$I(T)=I_{classical}[1-f_s(T)]$$

$f_s(T)$ is the supersolid fraction
Its upper limit is estimated by different theorists to range from 10^{-6} to 0.4; Leggett: 10^{-4}

Torsional Oscillator Technique is ideal for the detection of superfluidity





Quality Factor

Q= f_0 / Δf ~10^6

Stability in the period is ~0.1 ns

Frequency resolution of 1 part in 10^7

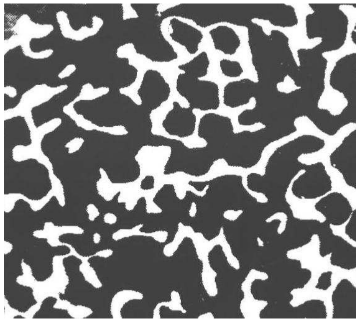
Mass sensitivity of ~10^{-7} g

$$\tau_o = 2\pi\sqrt{\frac{I}{K}}$$

f~ 1kHz

Torsional oscillator studies of superfluid films

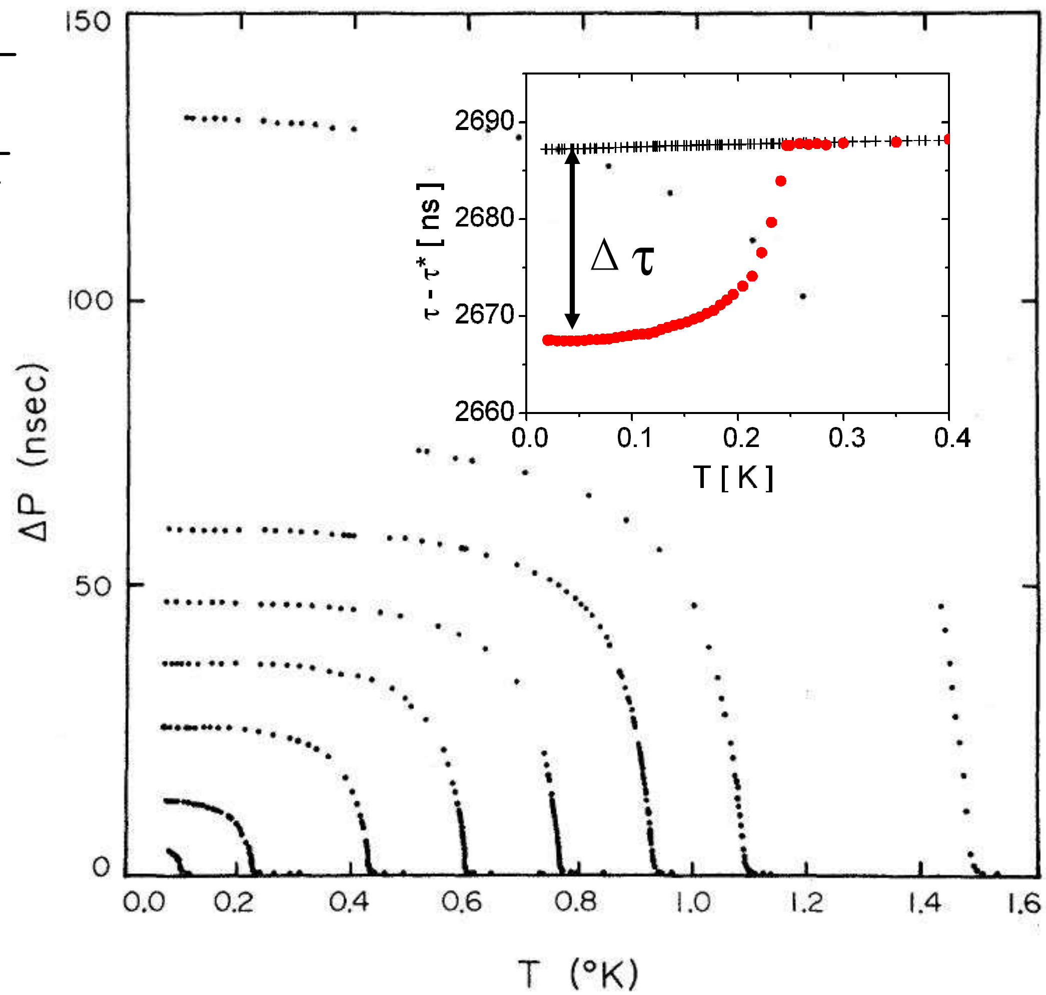
Vycor



$$\tau_o = 2\pi\sqrt{\frac{I}{K}}$$

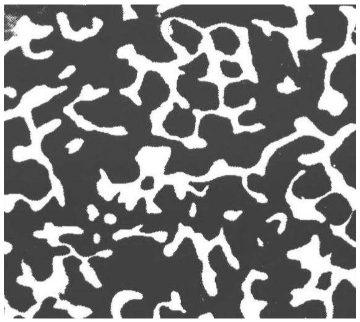
$\boldsymbol{I}_{total} = I_{cell} + I_{\text{helium film}},$

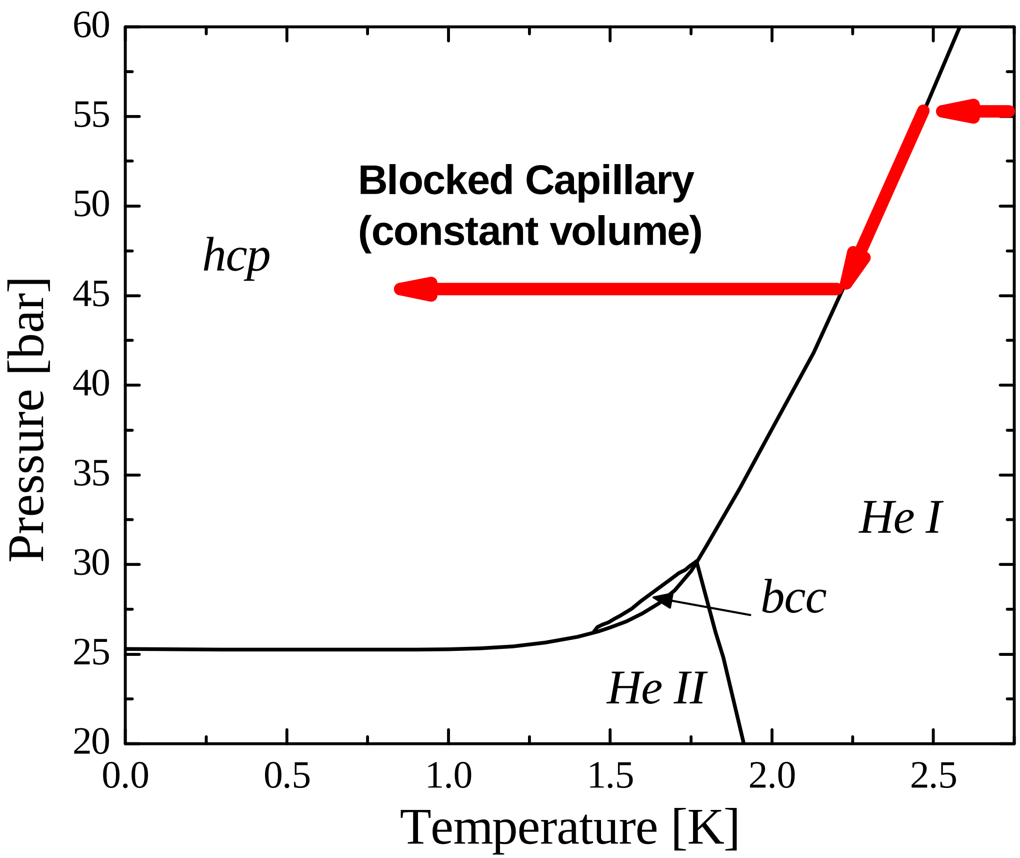
Above Tc the adsorbed normal liquid film behaves as solid and oscillates with the cell. In the superfluid phase, helium film decouples from oscillation. Hence I_{total} *and* τ *drops.*

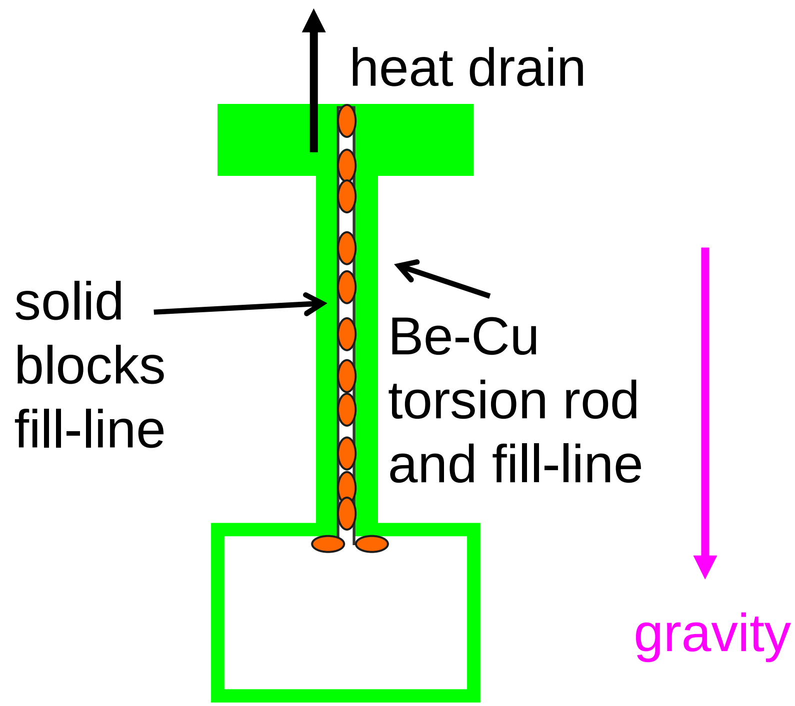


Berthold,Bishop, Reppy, *PRL* **39**,348(1977)

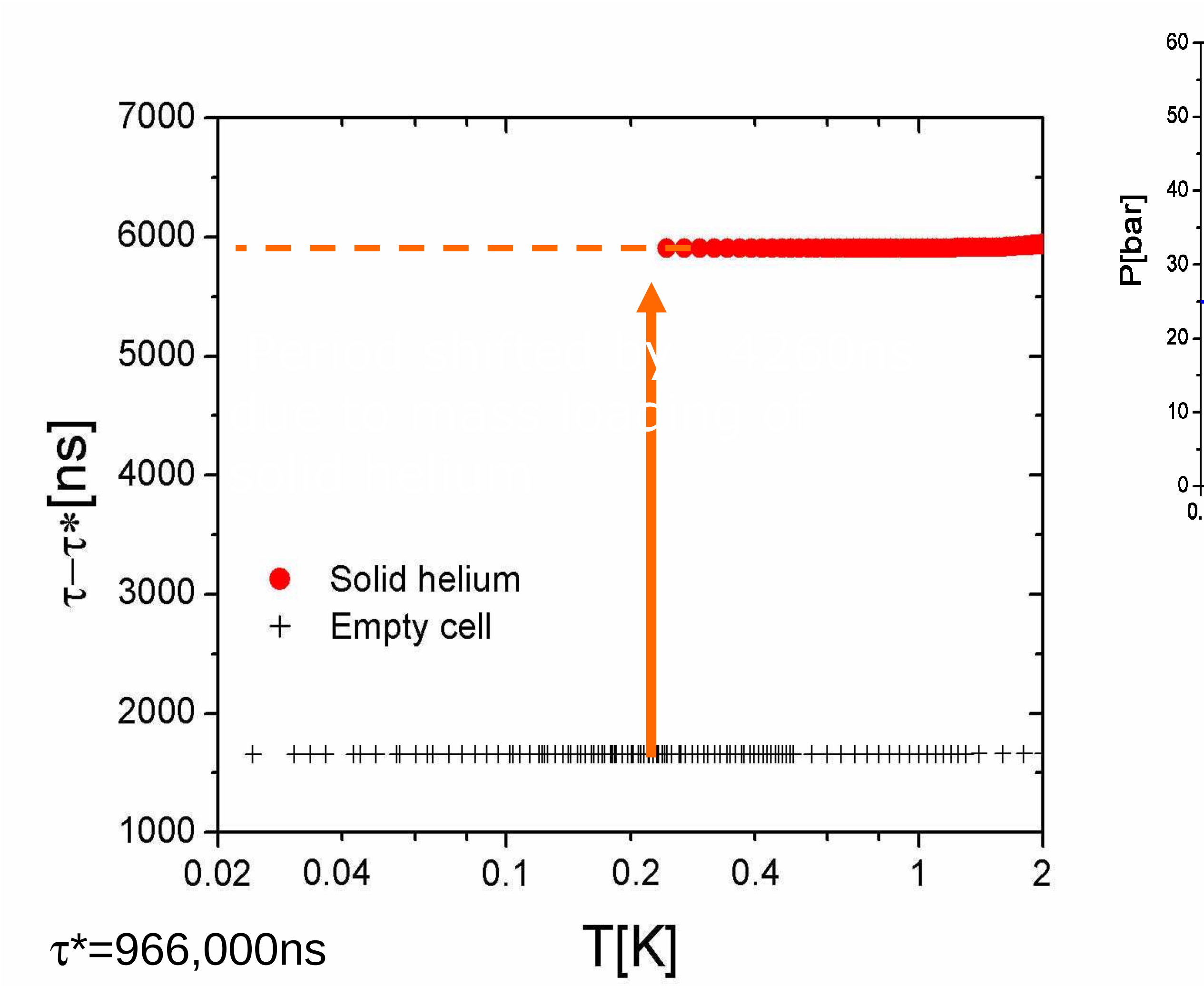
Blocked capillary (BC) method of growing solid samples

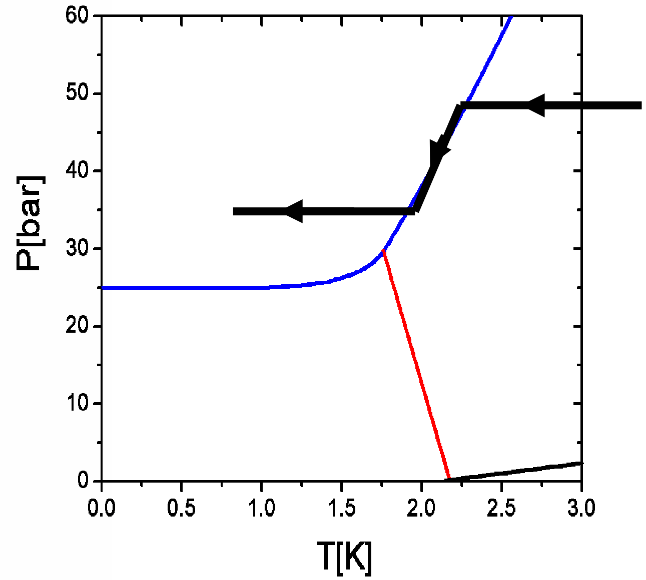




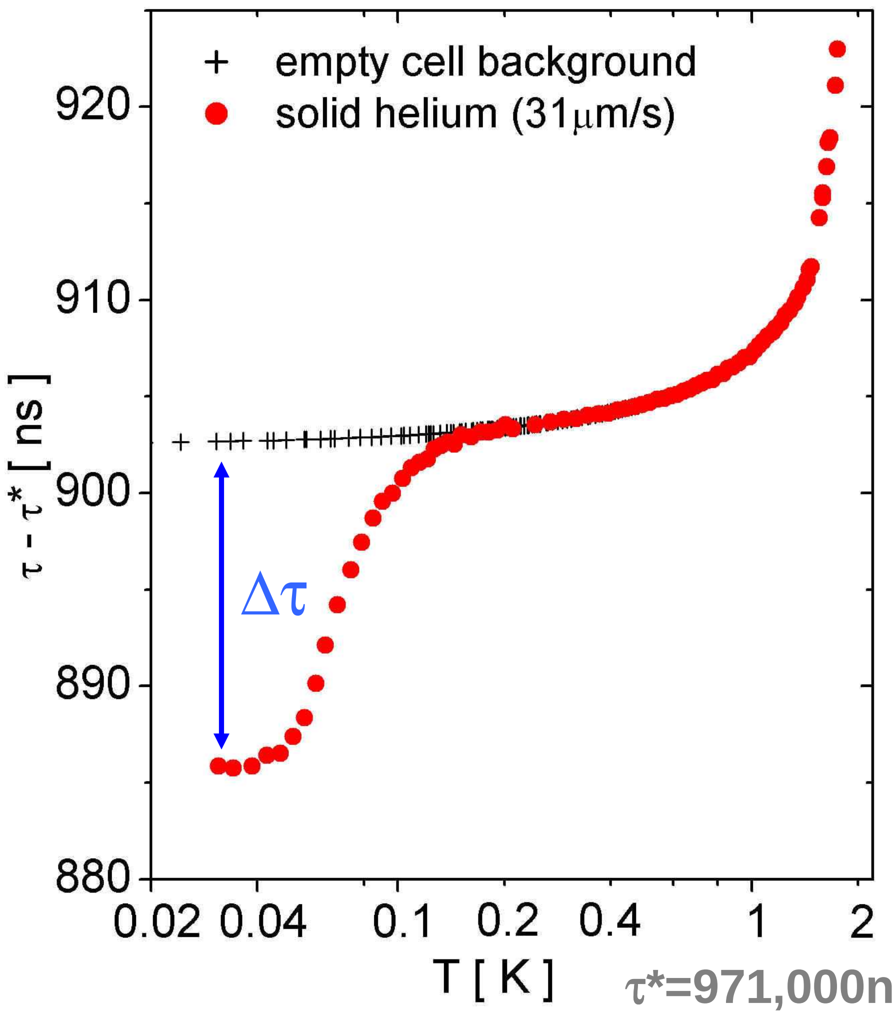


Solid ^{4}He at 62 bars in Vycor glass





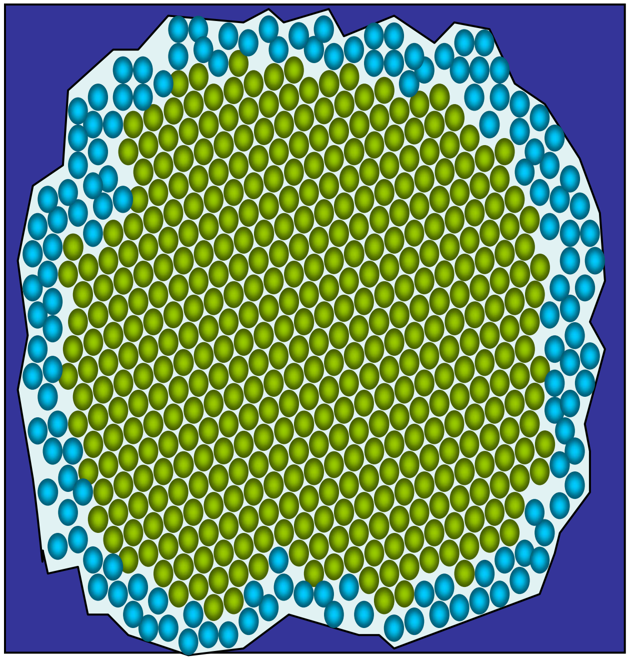
Supersolid response of helium in Vycor glass



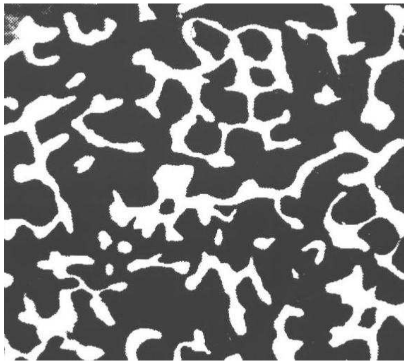
τ*=971,000ns

- Period drops at 175mK
  → appearance of non-classical rotational inertia (NCRI)

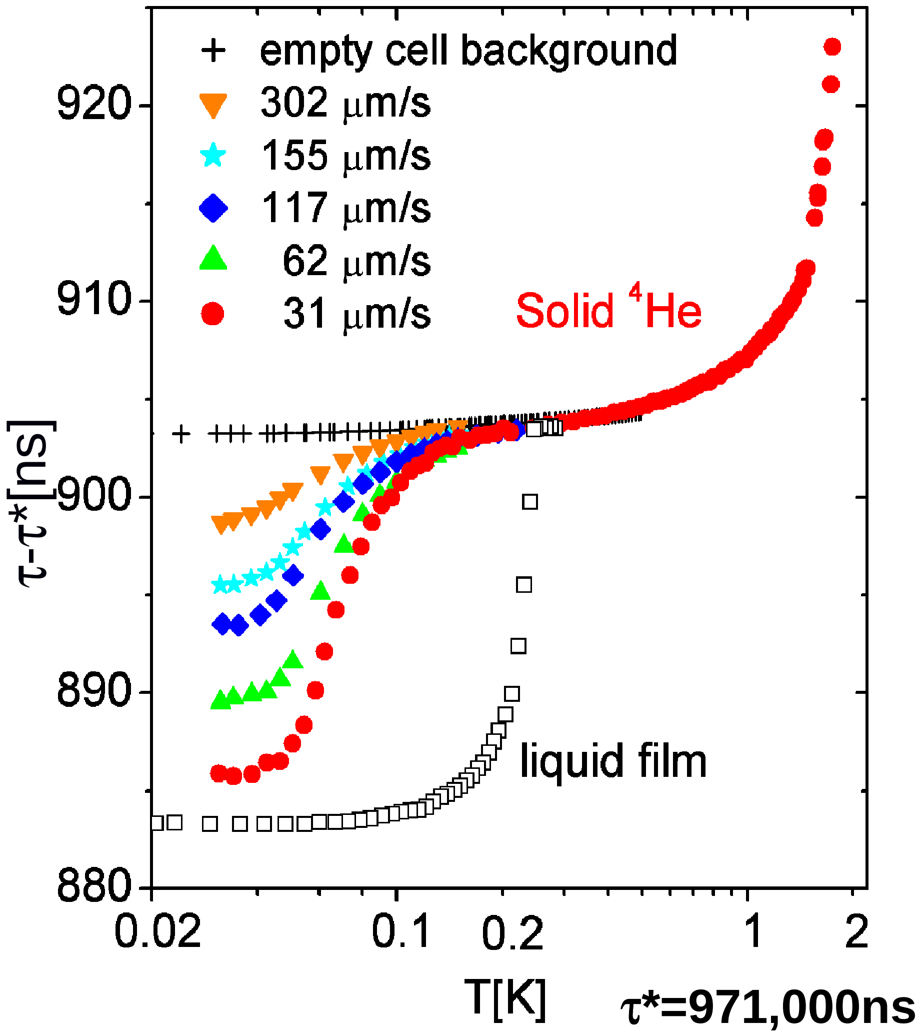
- size of period drop
  -Δτ ~17ns



Solid helium in Vycor glass



f_0=1024Hz



62bar

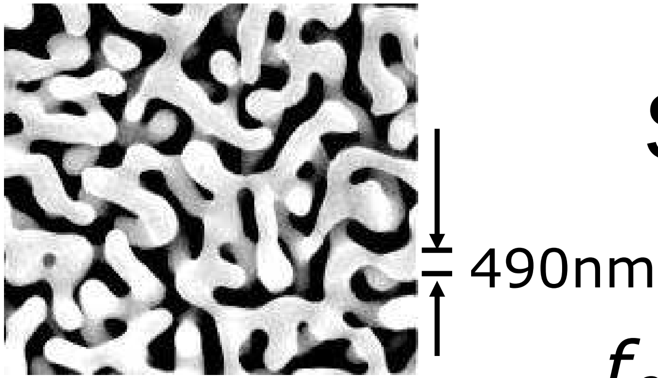
Total mass loading = 4260ns

Measured decoupling, $-\Delta\tau_o$=17ns

NCRI fraction, or NCRIF = 0.4%

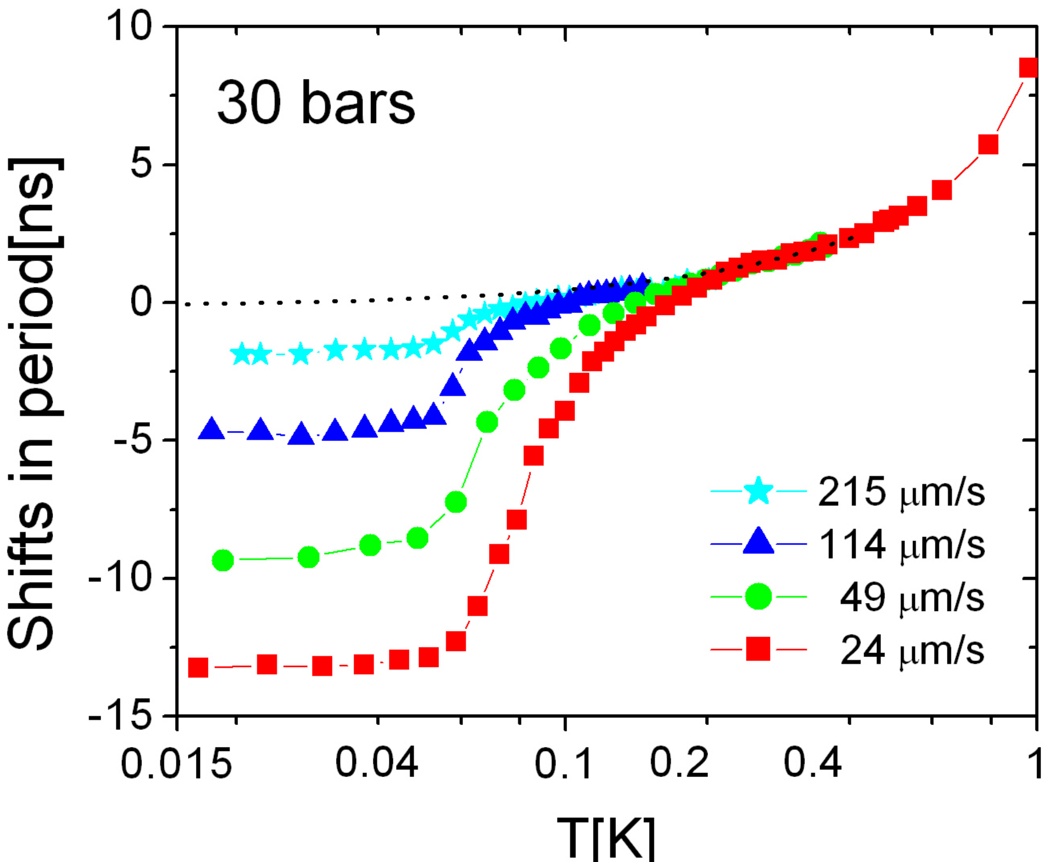
(with tortuosity, 2%)

E. Kim & M.H.W. Chan, Nature 427, 225 (2004).



Solid helium in porous gold

f_0=359Hz



27bar

Total mass loading = 1625ns

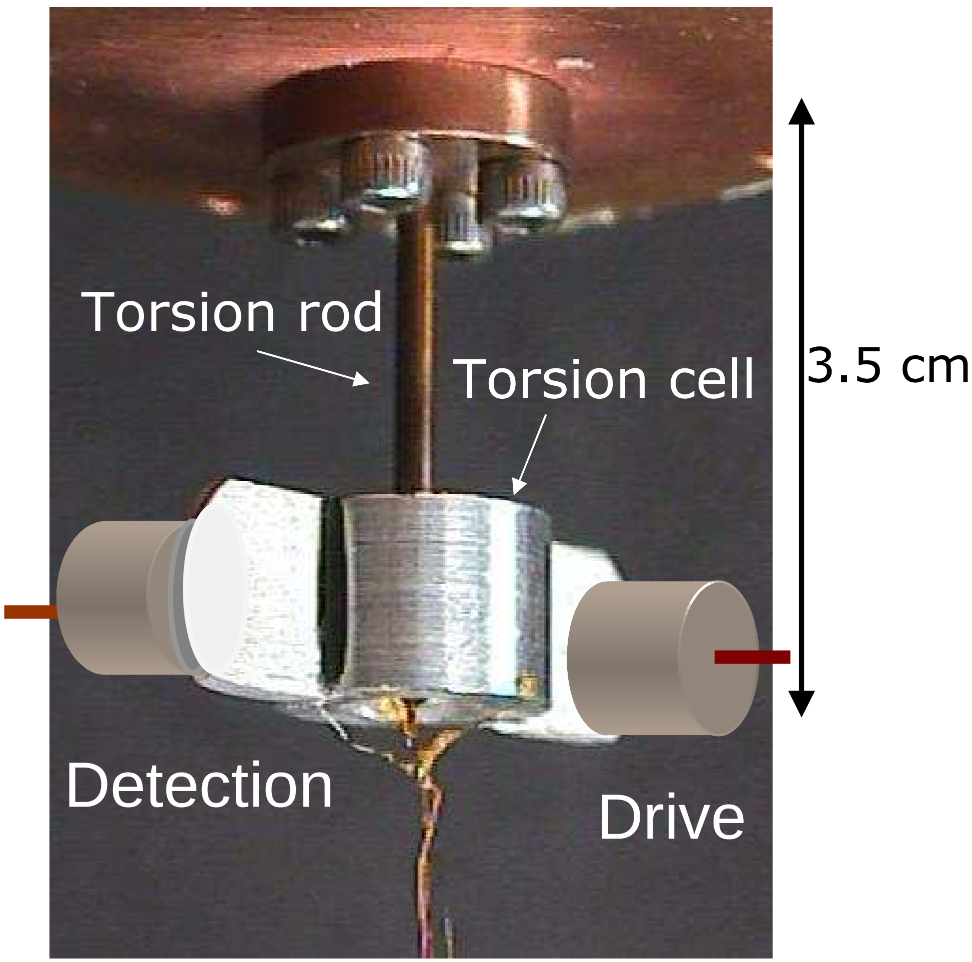
Measured decoupling, $-\Delta\tau_o$=13ns

NCRIF = 0.8%

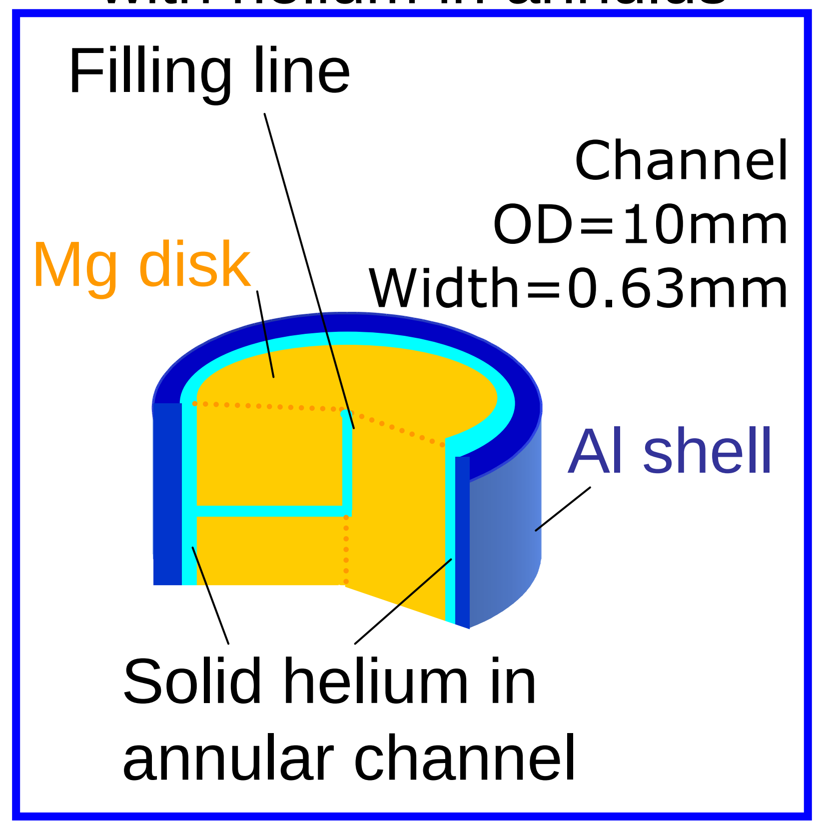
(with tortuosity, 1.2%)

E. Kim & M.H.W. Chan, JLTP 138, 859 (2005).

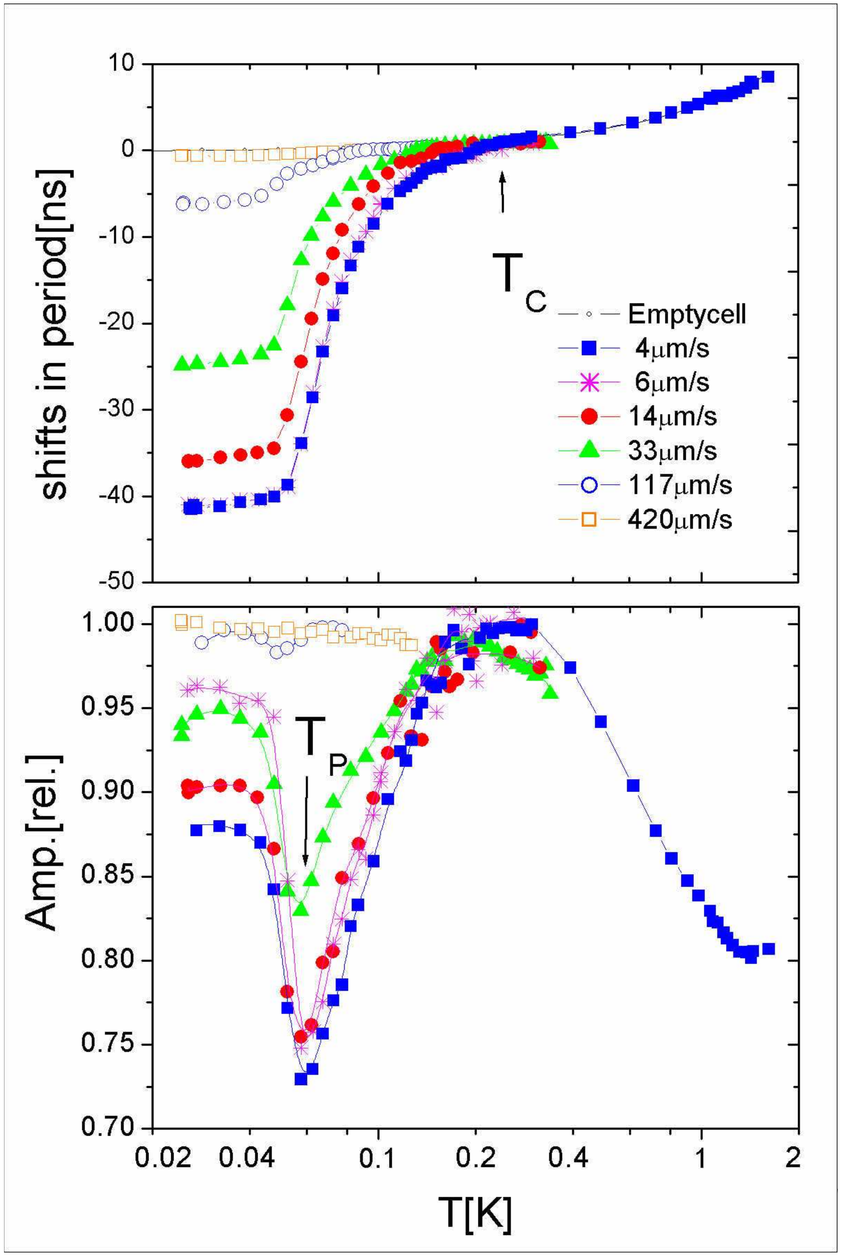
Bulk solid helium in annulus



Torsion cell
with helium in annulus



Bulk solid helium in annulus



f_0=912Hz

51bar

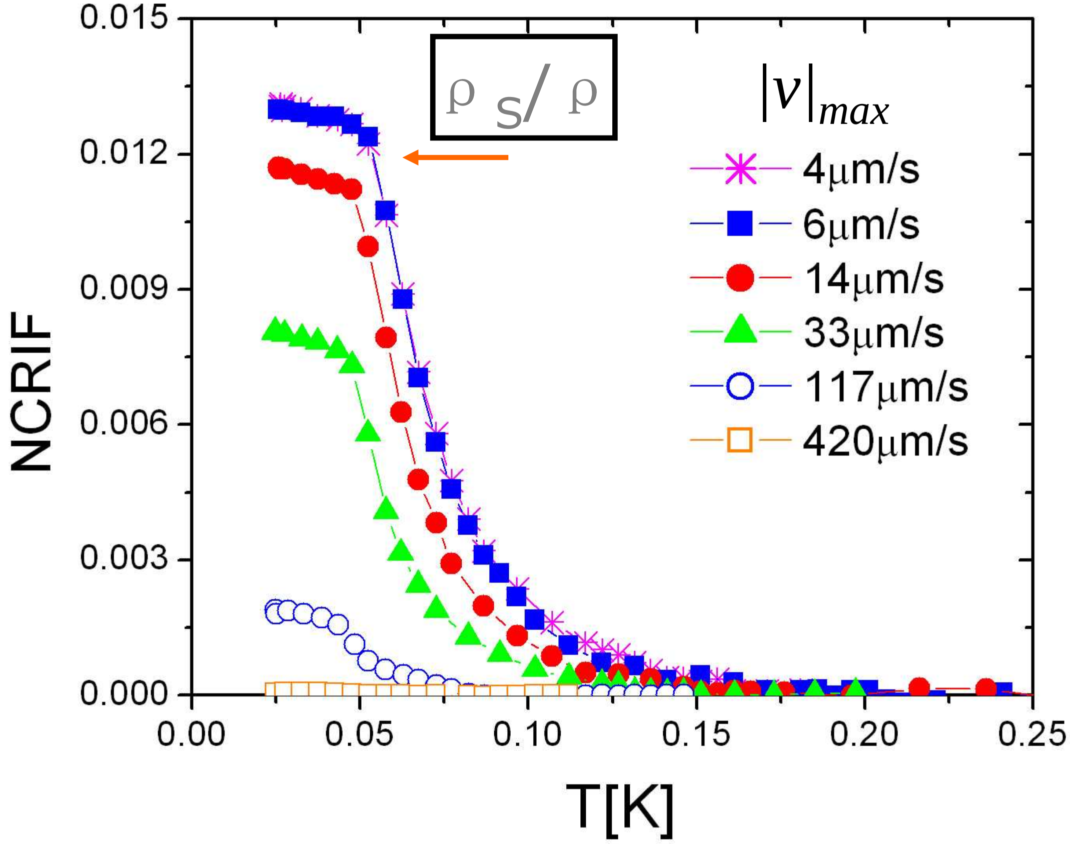
Total mass loading = 3012ns

Measured decoupling, -$\Delta\tau_o$=41ns

NCRIF = 1.4%

. Kim & M.H.W. Chan, Science 305, 1941 (2004)

Non-Classical Rotational Inertia Fraction

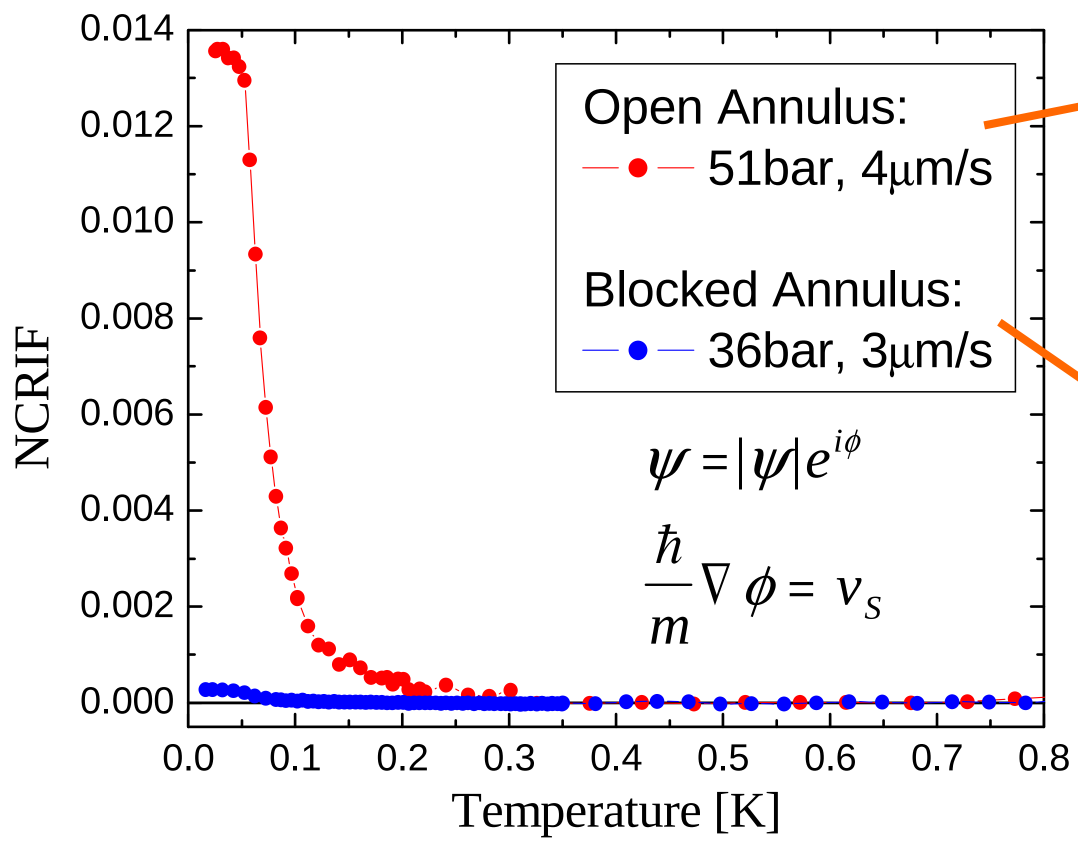


$$NCRIF = \frac{\Delta\tau}{\text{total mass loading}}$$

Total mass loading =3012ns at 51 bars

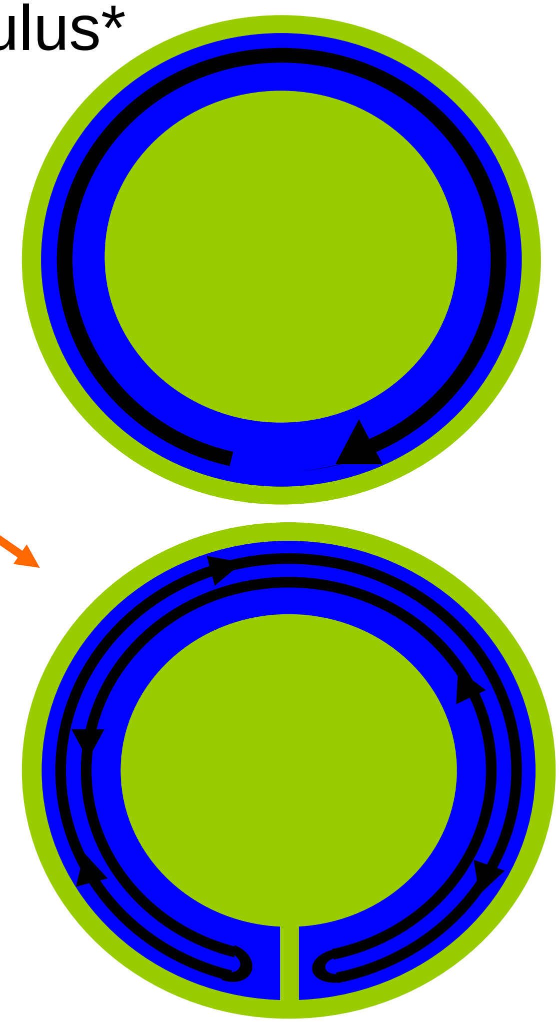
Irrotational Flow

- Superfluids exhibit potential (irrotational) flow
  - For our exact dimensions, NCRIF in the blocked cell should be about 1% that of the annulus*



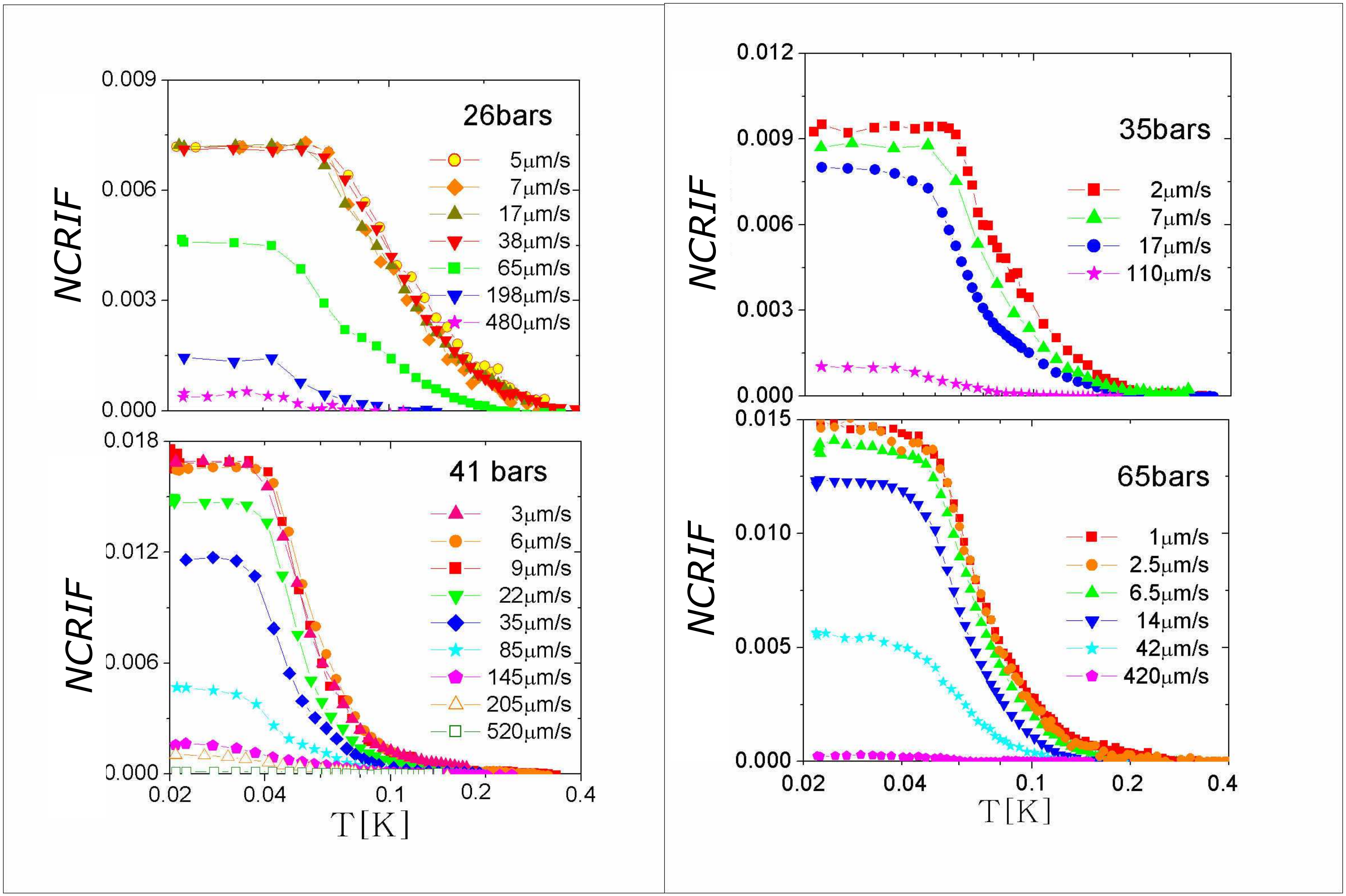
$$\psi = |\psi| e^{i\phi}$$

$$\frac{\hbar}{m} \nabla \phi = v_S$$

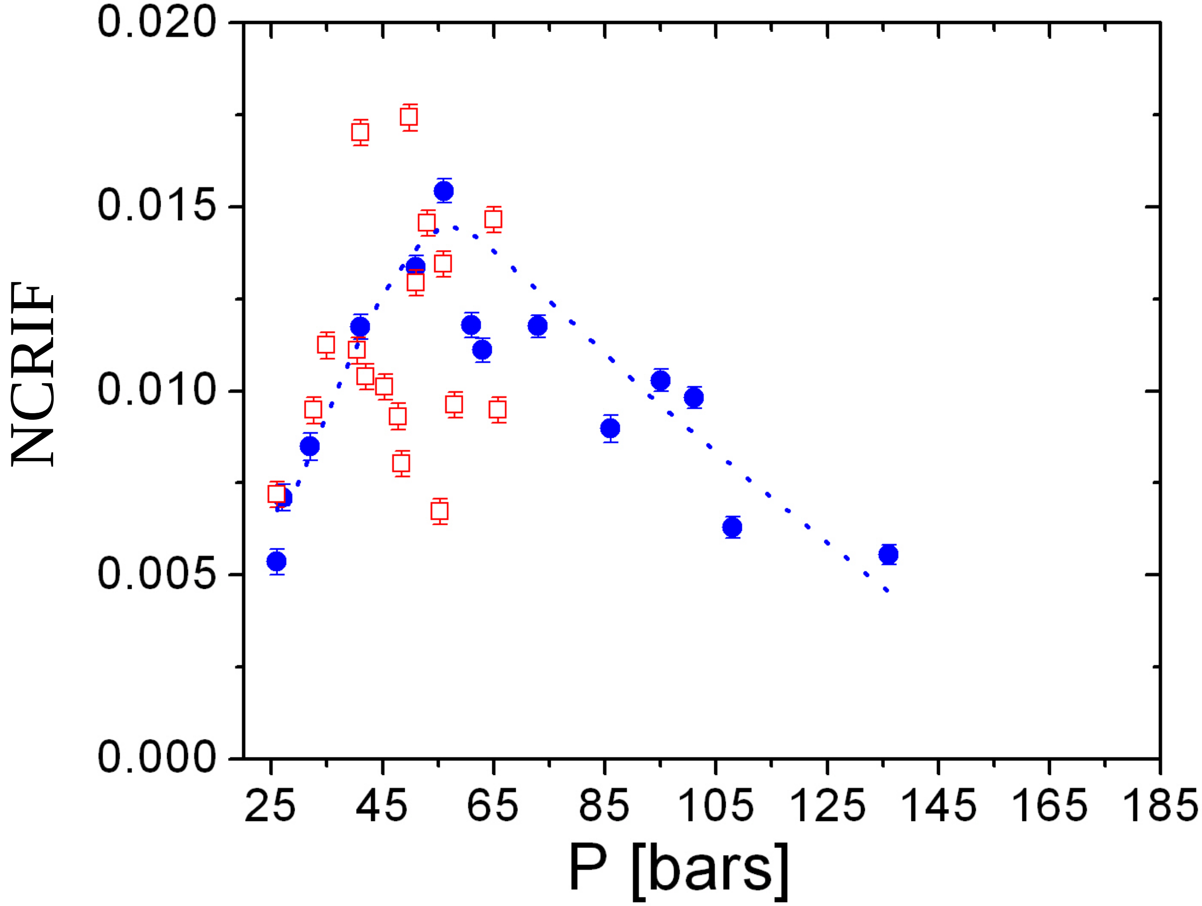


*E. Mueller, private communication.

Solid ^{4}He at various pressures show similar temperature dependence, but the measured supersolid fraction shows scatter with no obvious pressure dependence



Pressure dependence of supersolid fraction



Blue data points were obtained by seeding the solid helium samples from the bottom of the annulus.

What are the causes of the scatter in NCRIF?

Large number of experimental parameters.

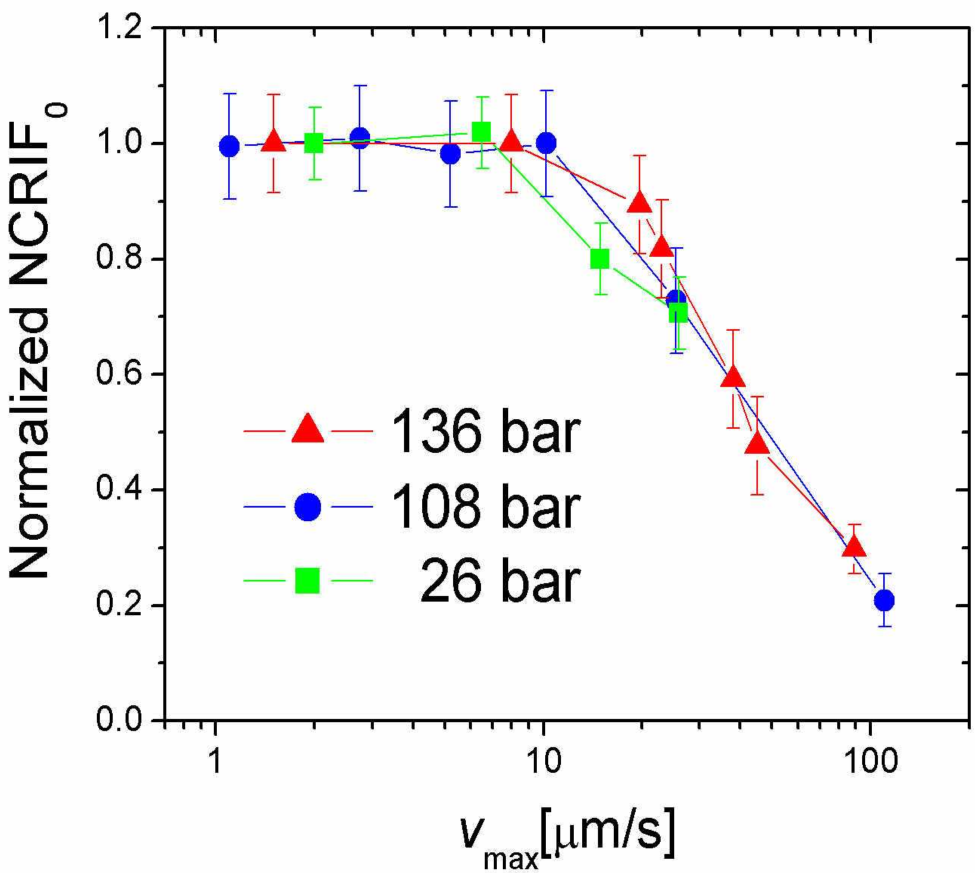
1. Pressure

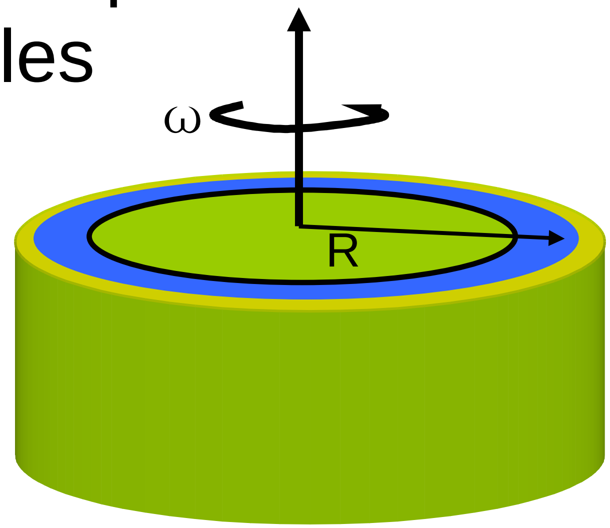
2. Oscillation speed.

3. ^{3}He concentration

4. Sample geometry/ crystal quality

Strong and ‘universal’ velocity dependence in all annular samples





v_C~ 10μm/s

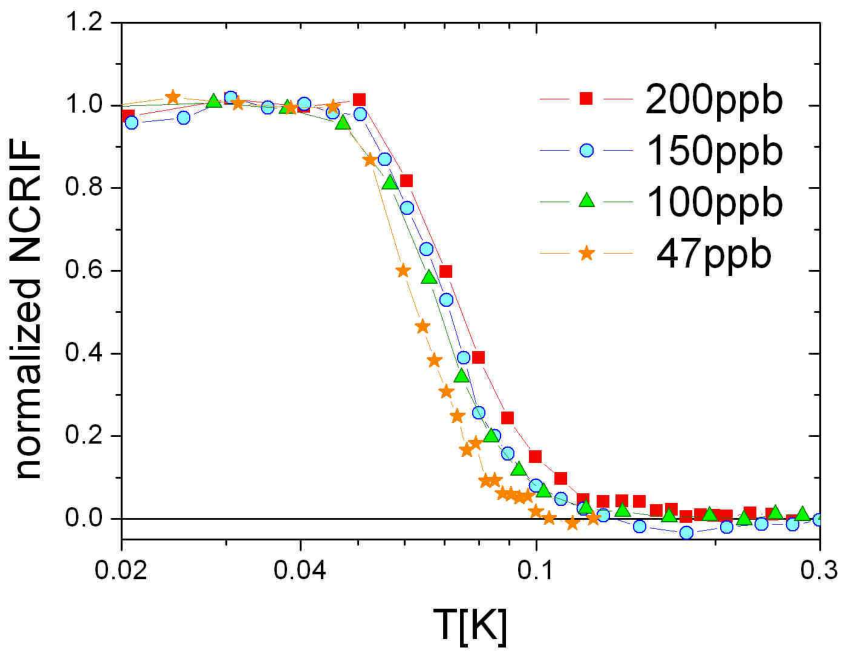
$$\oint v_s \, dl = \frac{h}{m} \cdot n$$

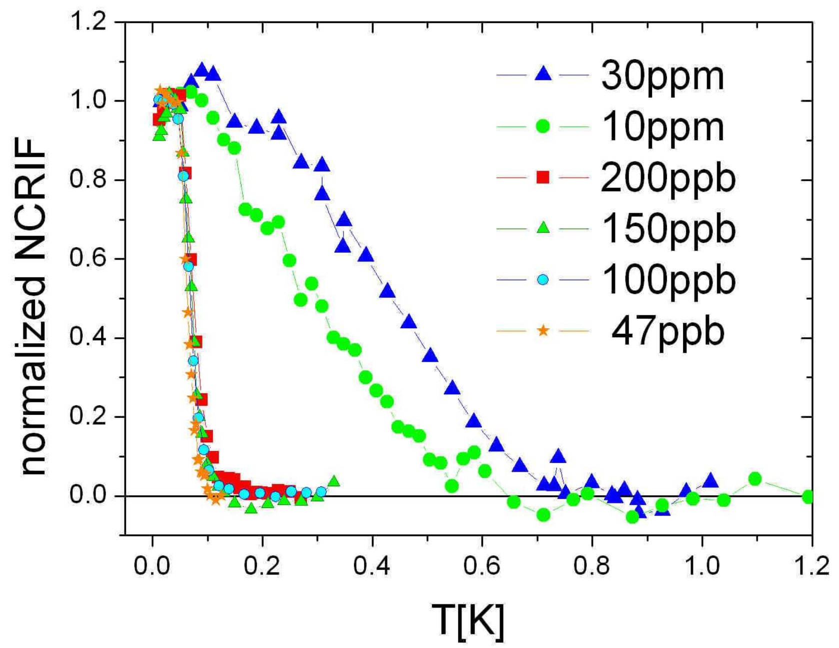
$$v_s = \frac{h}{2\pi R m} \cdot n$$

=3.16μm/s
for *n=1*

Vortices are important

^{3}He Effect





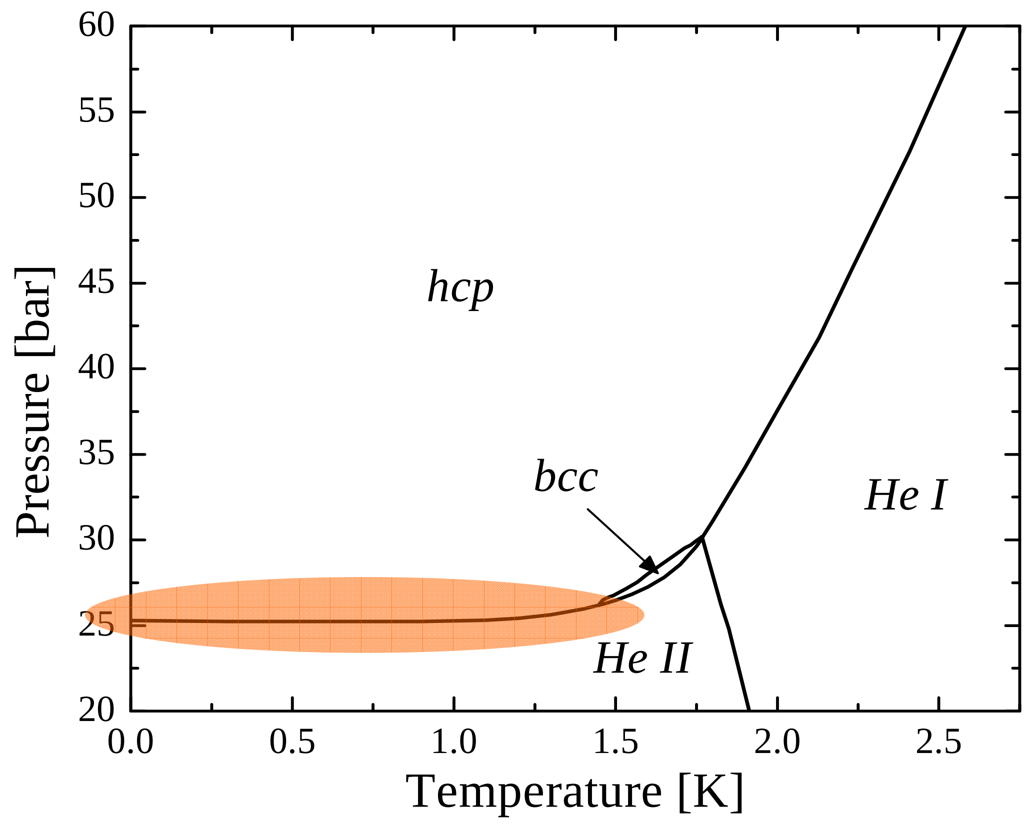
Eunseong Kim

E. Kim, J. S. Xia, J. T. West, X. Lin, and M. H. W. Chan, To be published.

- **Nonclassical rotational inertia results have been replicated in four other labs.**
- **The temperature dependence of NCRI is reproduced.**
- **However, the magnitude of NCRIF varies from 0.03% up to 20%(!!)**
- **It has been suggested that defects, vacancies, dislocation and grain boundaries in crystal are responsible for the observed NCRI and it has also been suggested the phenomena is due to superfluid film flowing along the grain boundaries.**

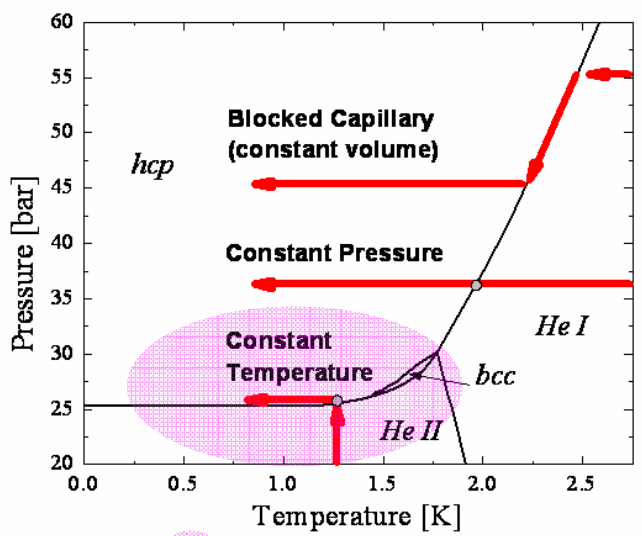
Crystal Growth

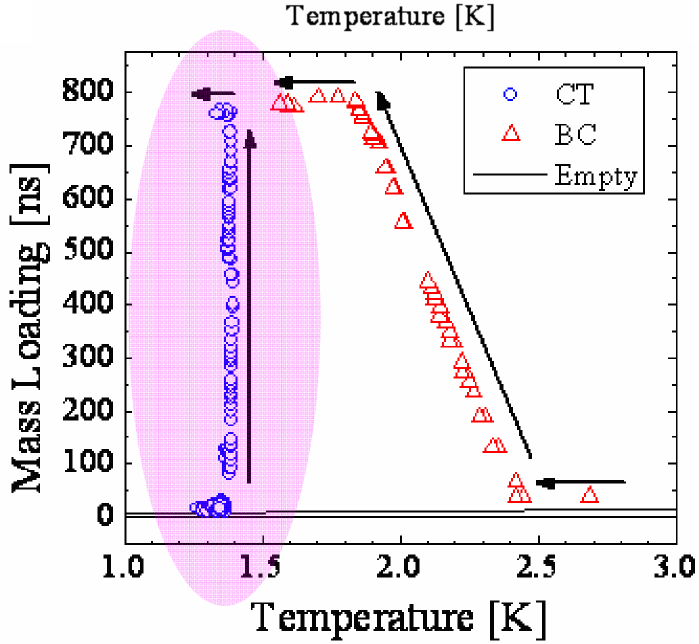
- High quality single crystals have been grown under constant temperature[1] and pressure[2]
- Best crystals grown in zero temperature limit

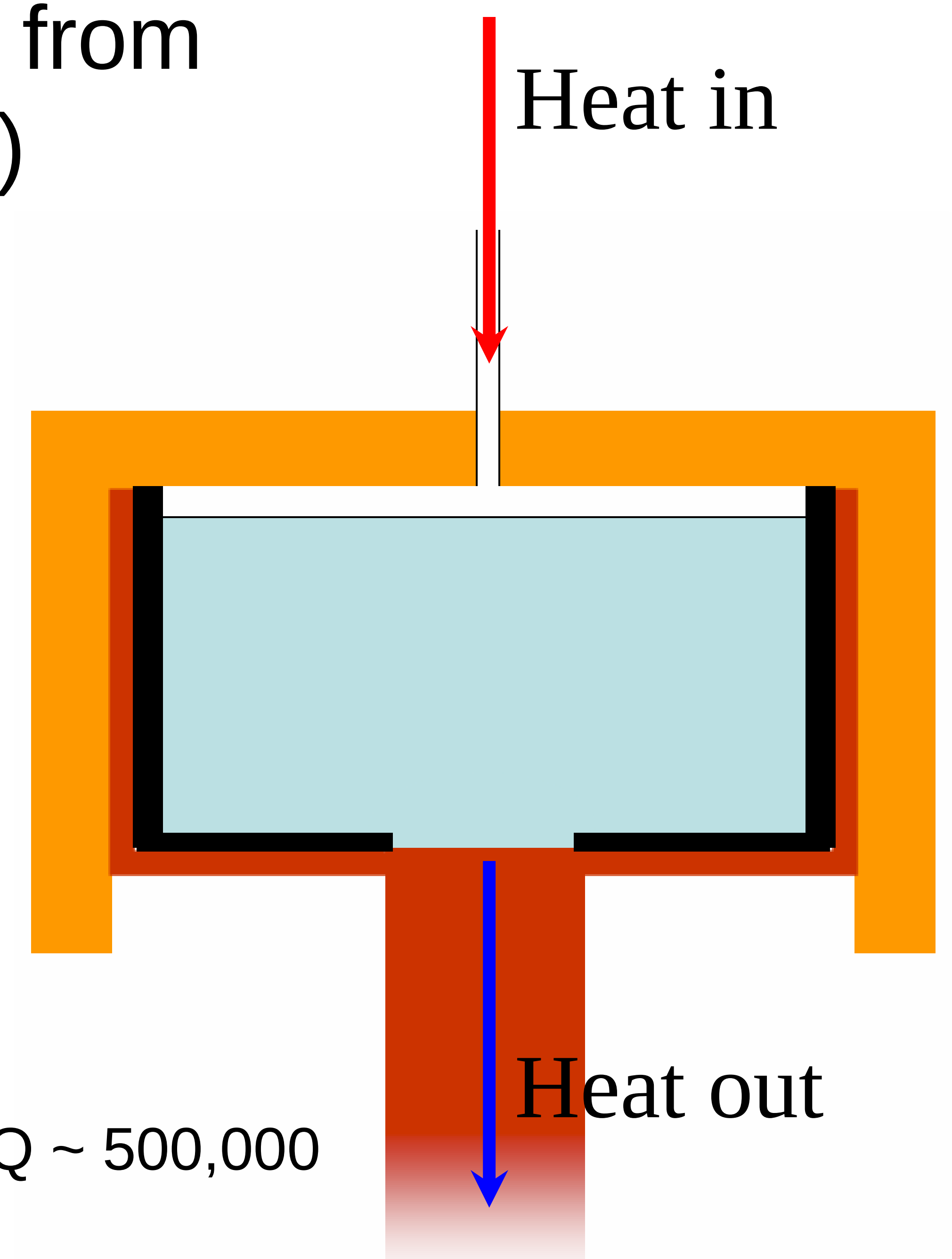


1. O.W. Heybey & D.M. Lee, PRL 19, 106 (1967); S. Balibar, H. Alles & A. Ya Parshin, Rev. Mod. Phys. 77, 317 (2005).
2. L.P. Mezhov-Deglin, Sov. Phys. JETP 22, 47 (1966); D.S. Greywall, PRA 3, 2106 (1971).

Constant T/P growth from superfluid (1ppb ^3He)



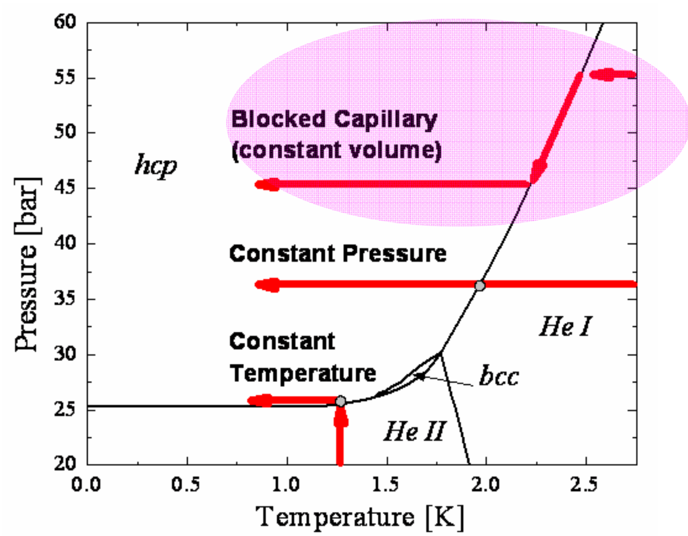


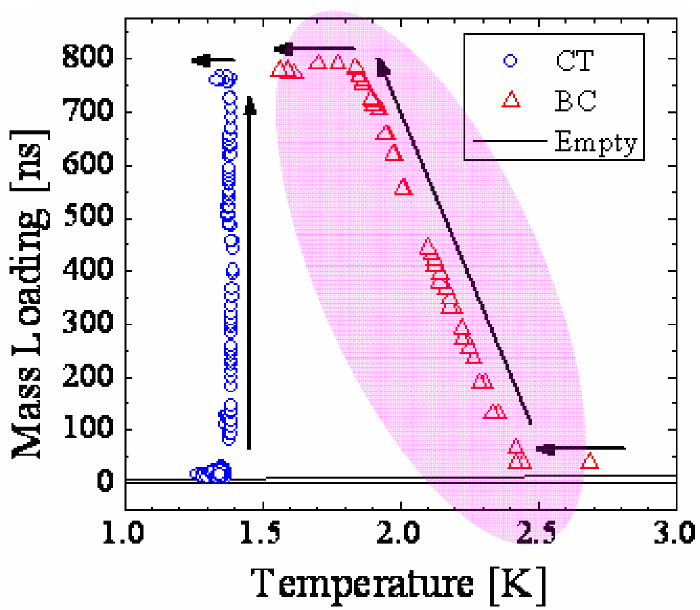


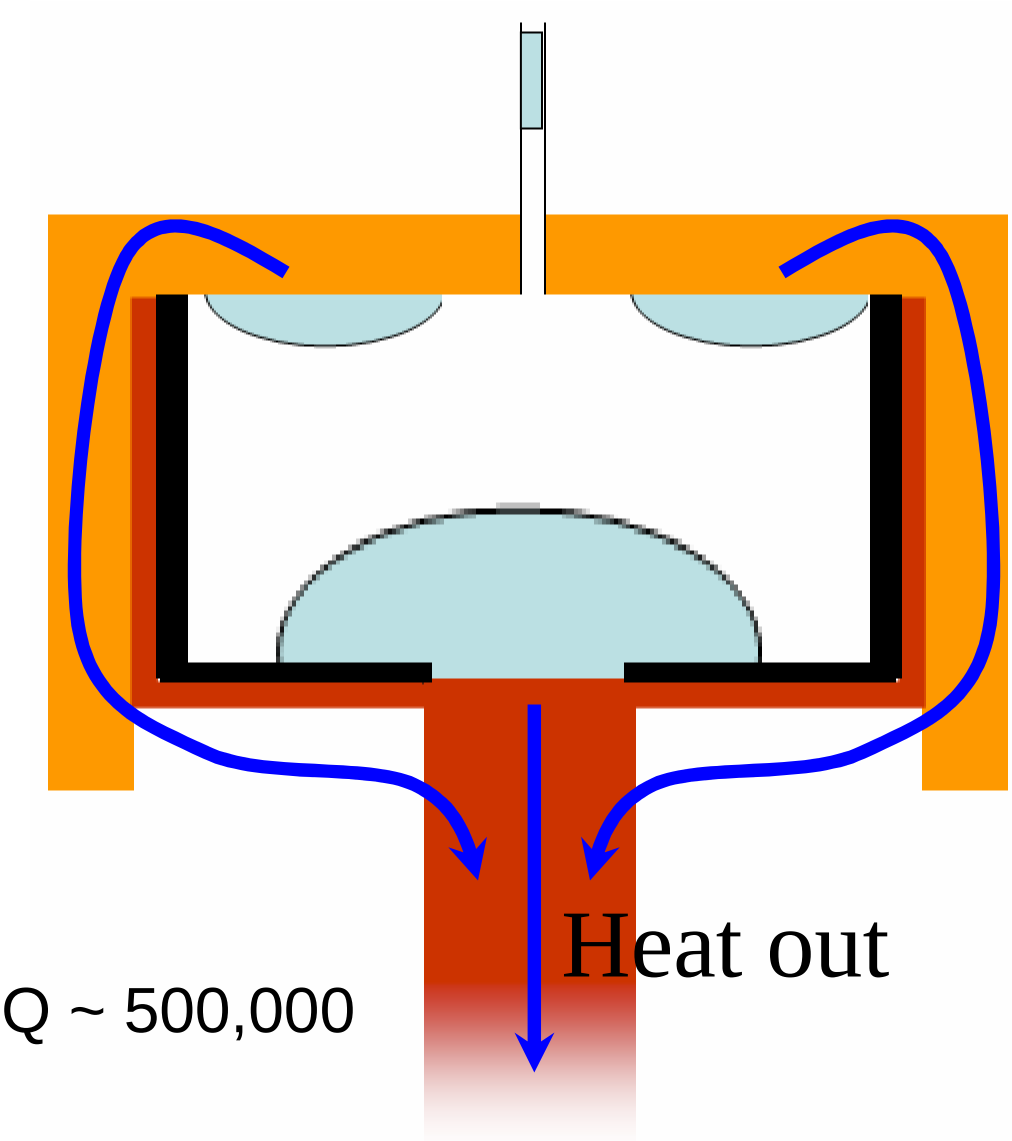
Q ~ 500,000

Tony Clark and Josh West

BC samples can also be grown





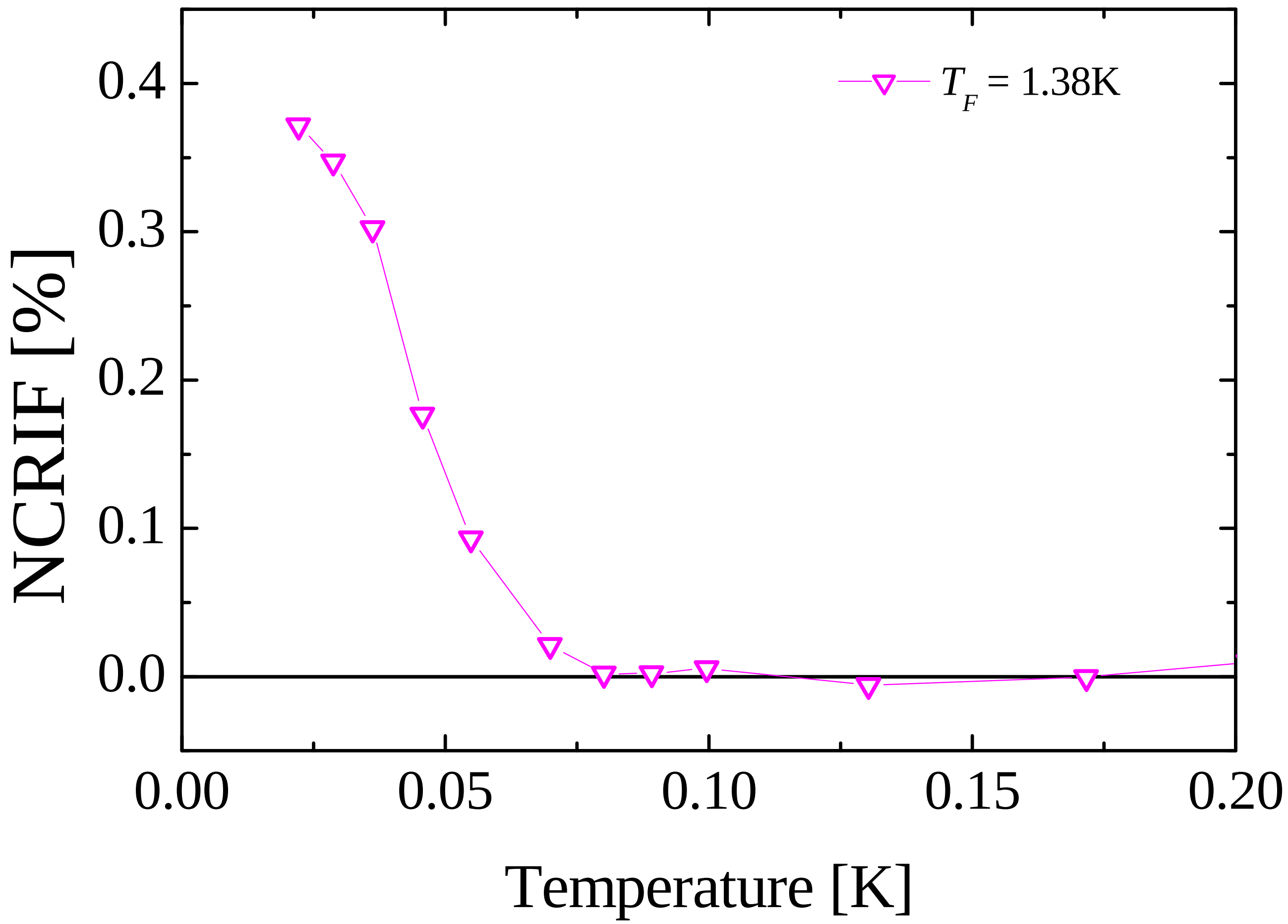


Q ~ 500,000

NCRI in solid helium (1ppb ^{3}He)

Samples grown carefully from superfluid collapse onto one curve for T > 40mK and share common onset temperature, T_C ~ 80mK

NCRIF ~ 0.3%



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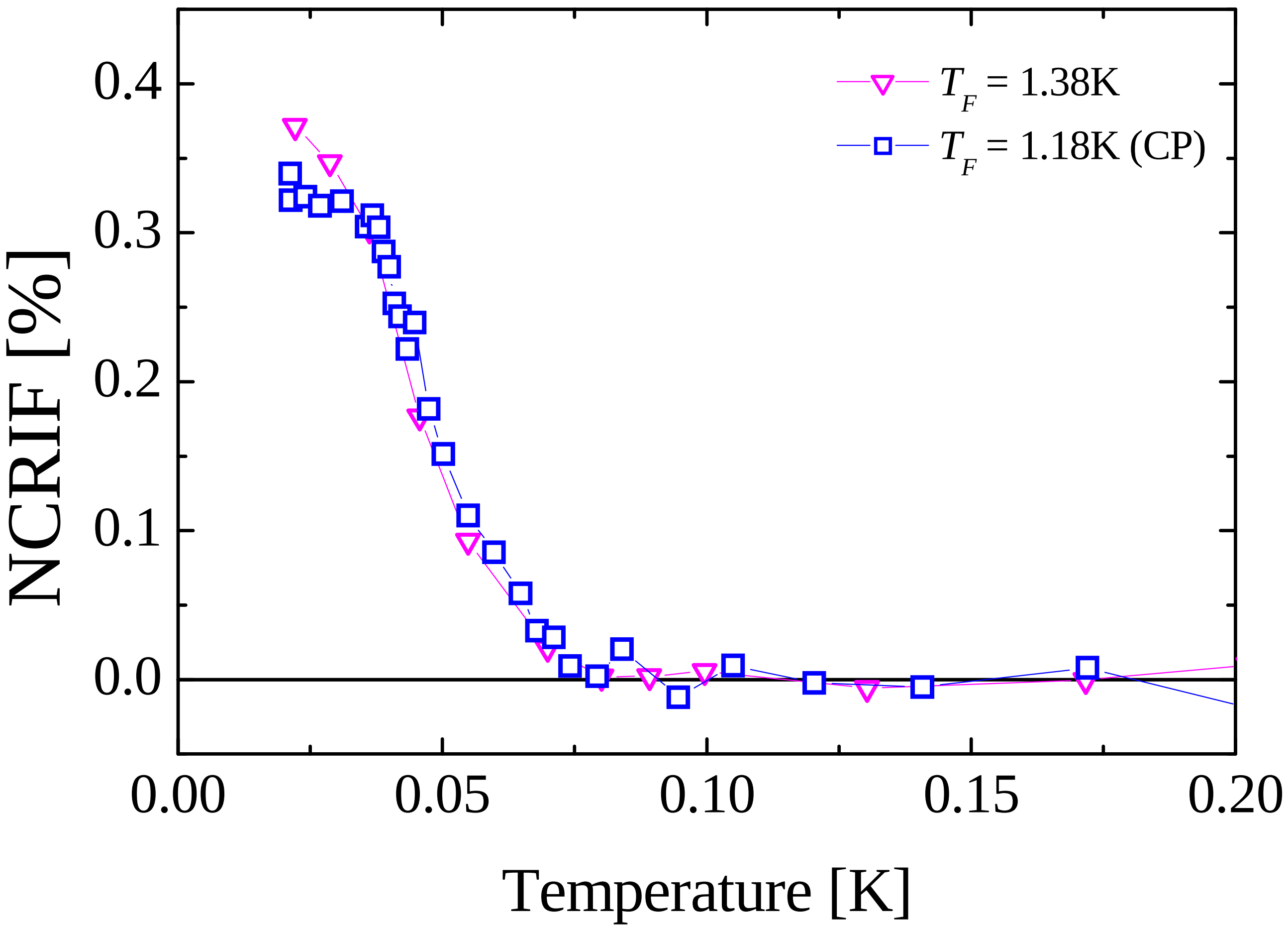

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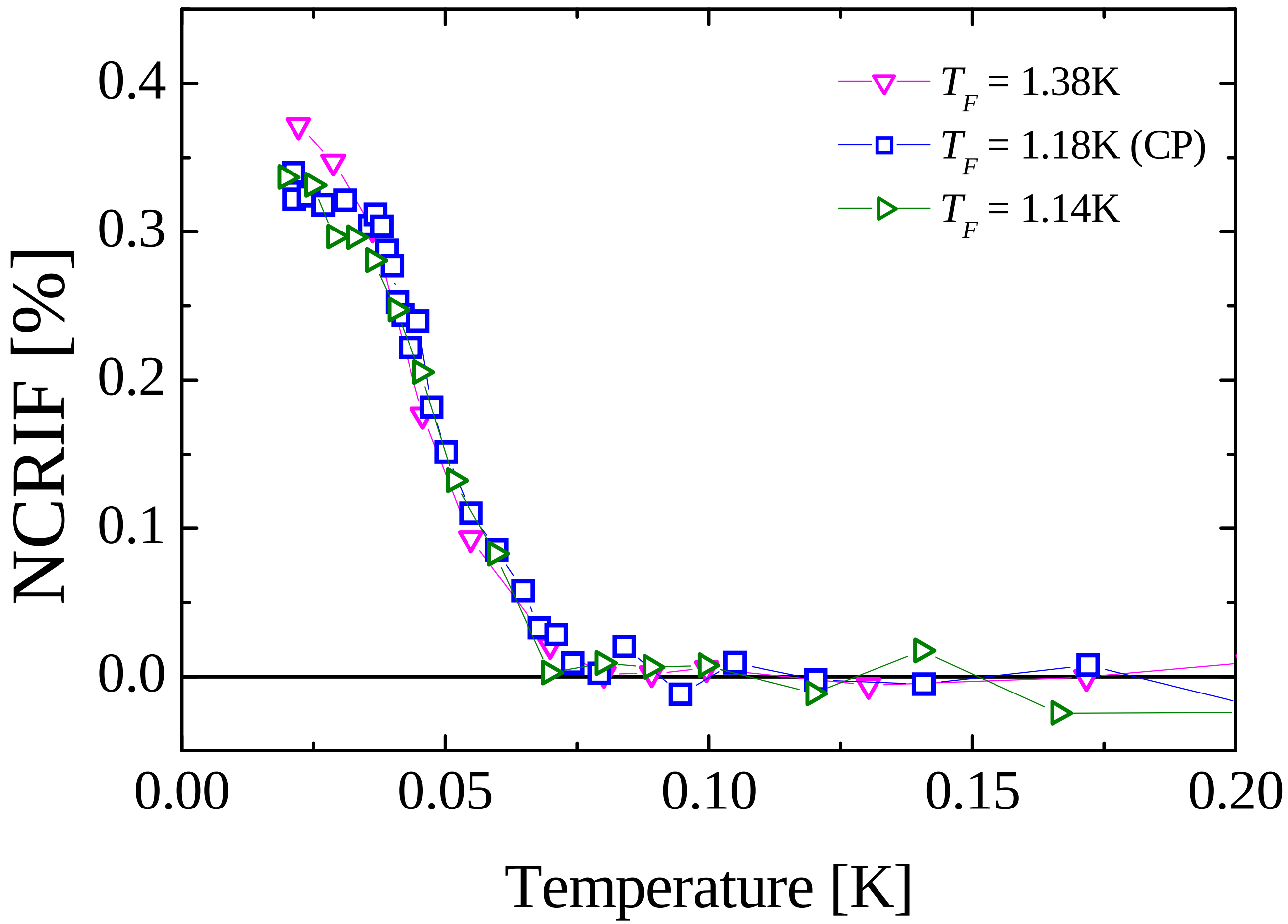

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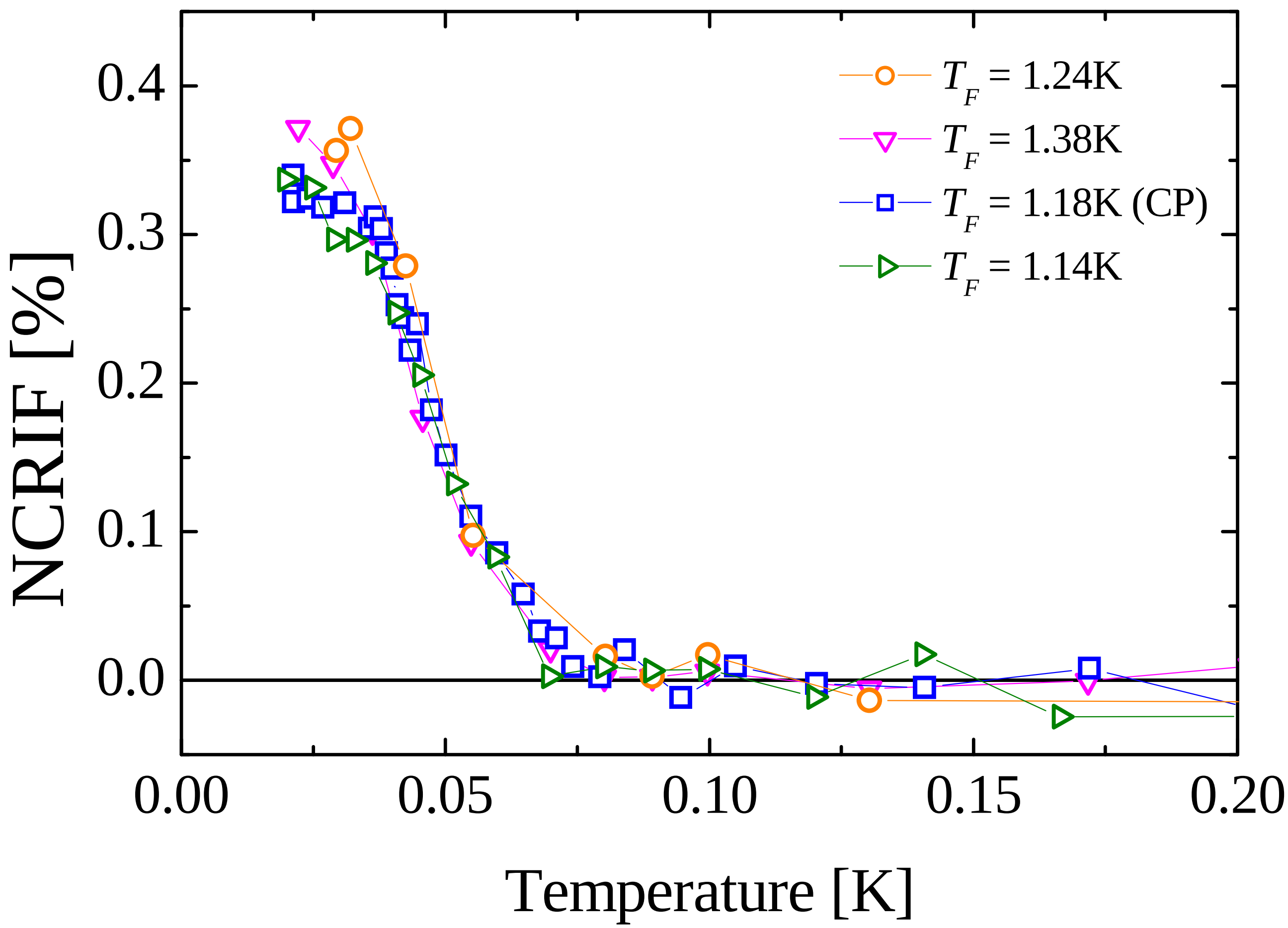

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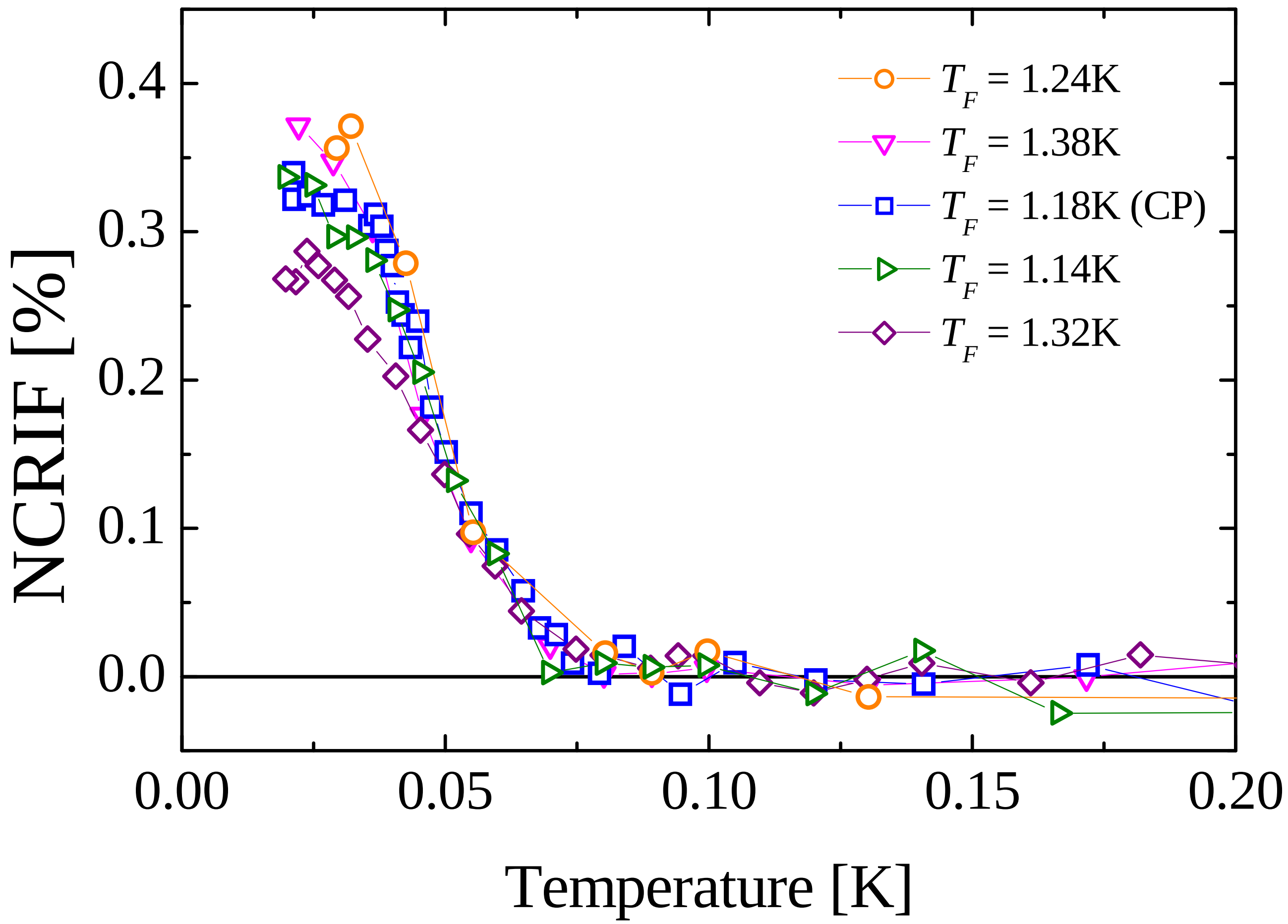

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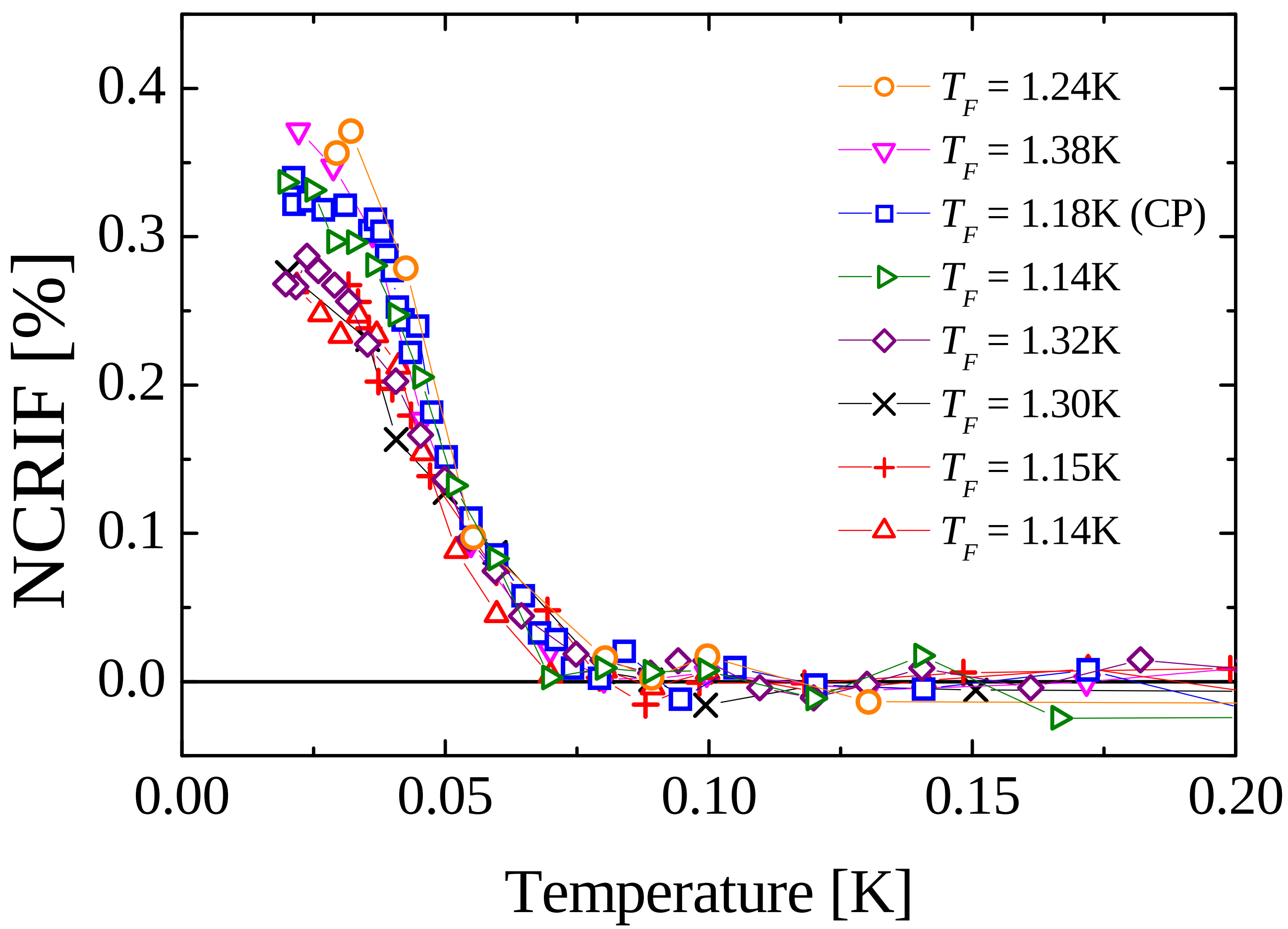

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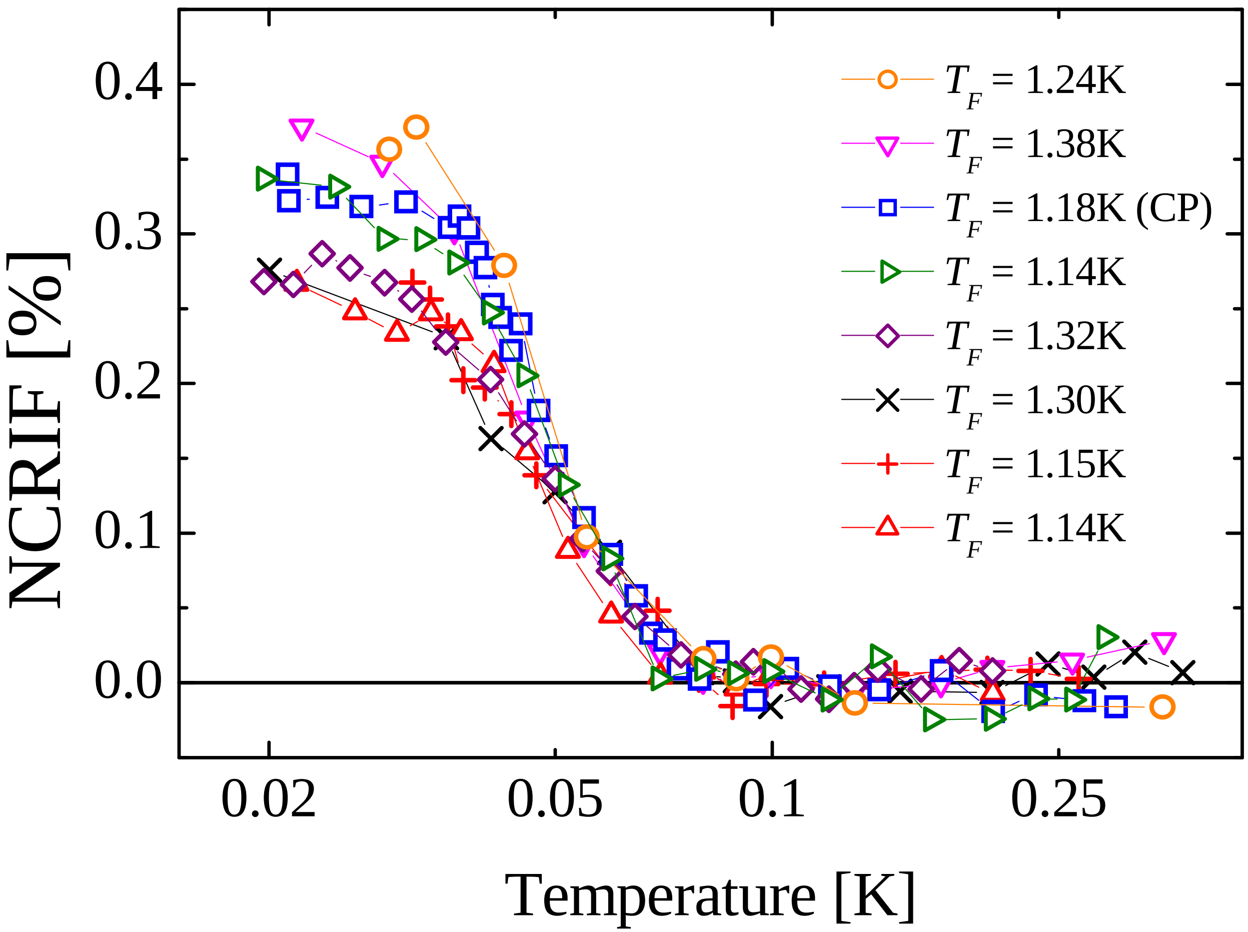
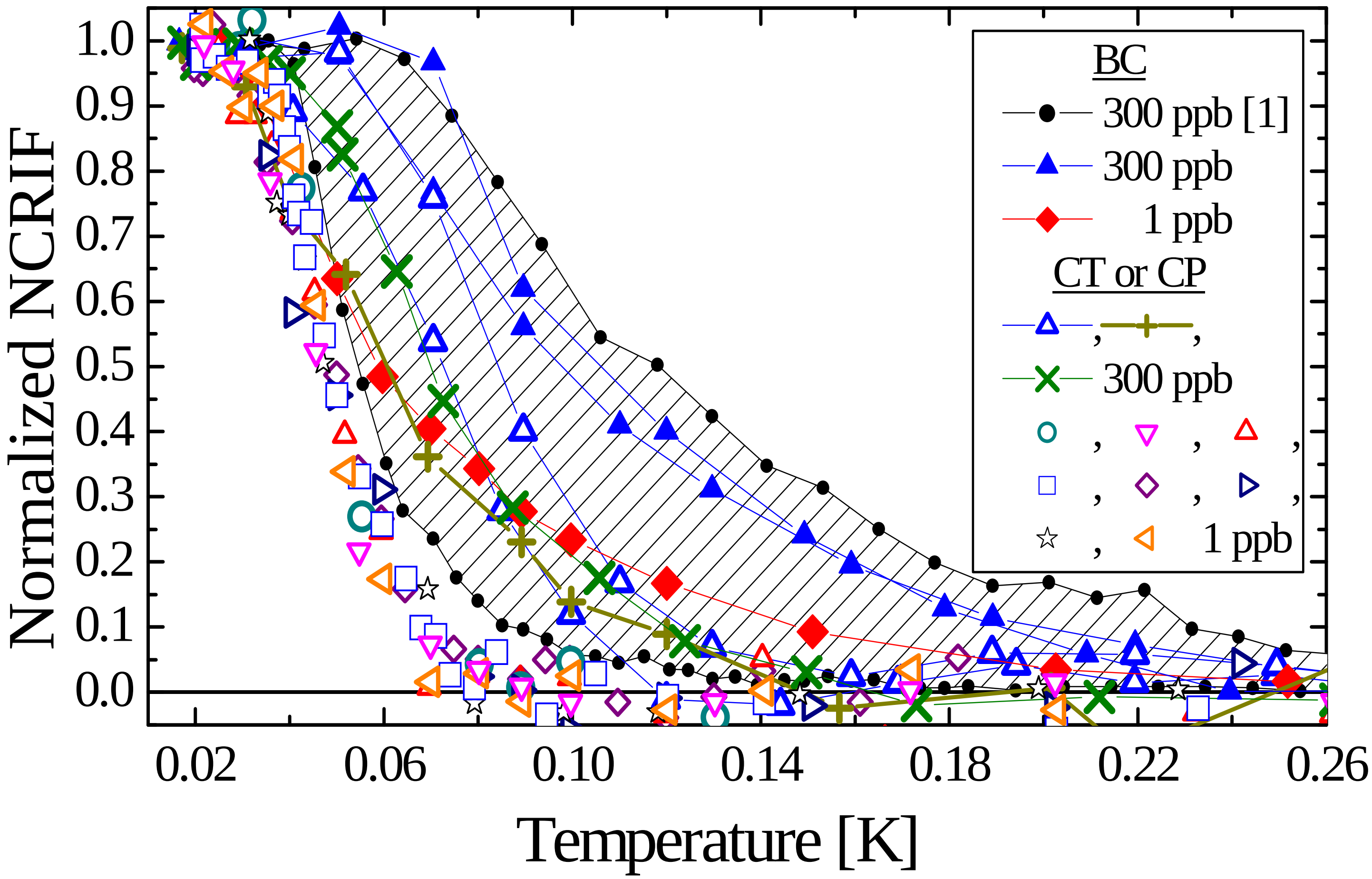


High temperature tail of NCRI

Transition broadened in BC samples (probably "polycrystalline") and by ^{3}He impurities



- Grain boundaries surely cannot be the sole mechanism.
- What then is the cause for variation in NCRI from cell to cell?
- Dislocation lines with density that ranges from 10^5 cm^{-2} to 10^9 cm^{-2} and in particular how the interaction of vortices and ^{3}He with dislocation lines are important.

Anderson's vortex liquid model

Just a few details:

-"Free" vortices (relative to time scale of oscillator = resonant period) can respond to motion of oscillator and screen supercurrents, reducing measured NCRIF

-NCRI related to susceptibility of vortices: NCRIF largest when they are "pinned"

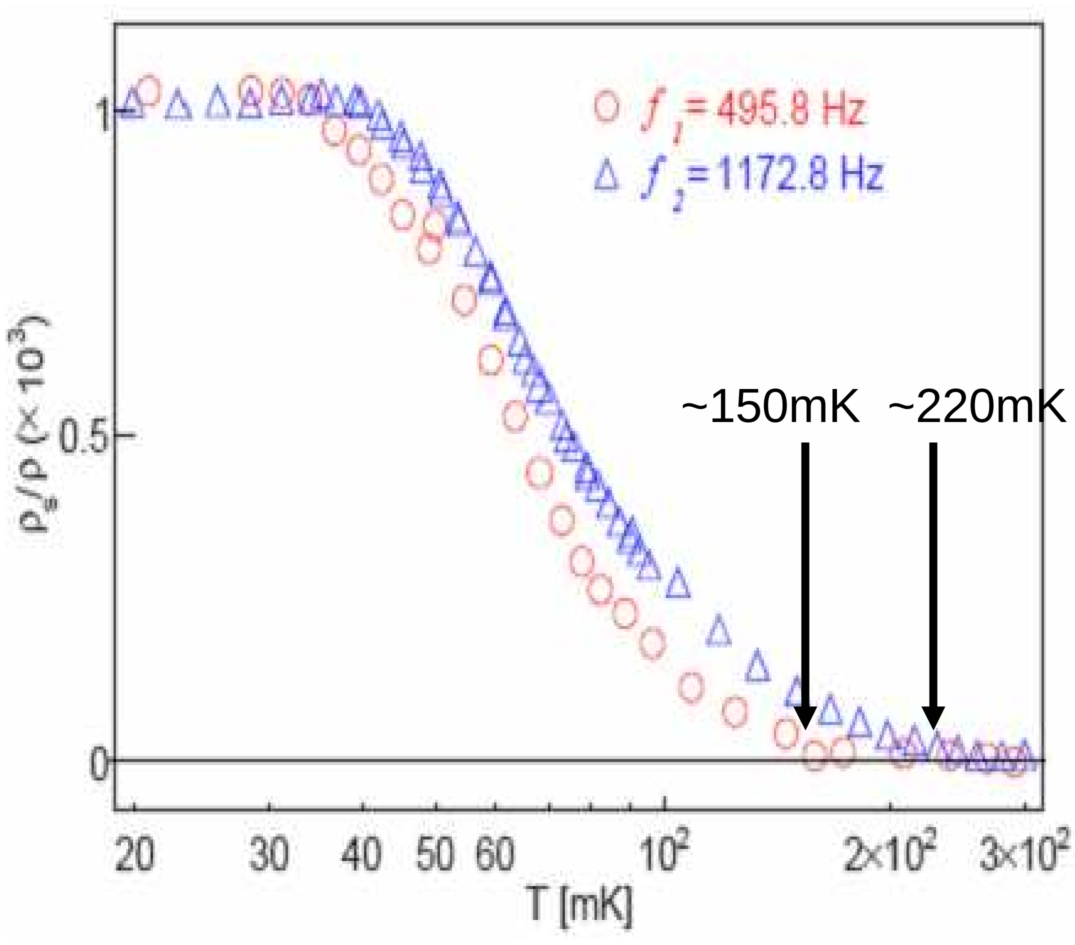
-^{3}He may attach to vortices and slow them down (higher T_O)

-Dissipation peak: vortex rate of motion ~ oscillator frequency (higher frequency, higher T_O)

P.W. Anderson, Nature Phys. 3, 160 (2007).

Frequency dependence

- T_O increases with frequency
- Low temperature NCRIF unchanged



Aoki, Graves & Kojima, PRL 99, 015301 (2007).

Heat capacity

Is there a thermodynamic signature of the transition?

Is NCRI due to a glassy phase or glassy regions in solid helium?

Previous solid ^{4}He heat capacity measurements below 1K

	Year	Low temperature limit
Swenson[1]	1962,1967	0.2K
Edwards[2]	1965	0.3K
Gardner[3]	1973	0.35K
Adams[4]	1975	0.13K
Hebral[5]	1980	0.1K
Clark[6]	2005	0.08K

They all observed T^3 phonon contribution.

Their sample cells used in these experiments were all constructed with heavy wall metal or epoxy which contribute significantly to the heat capacity at low temperature.

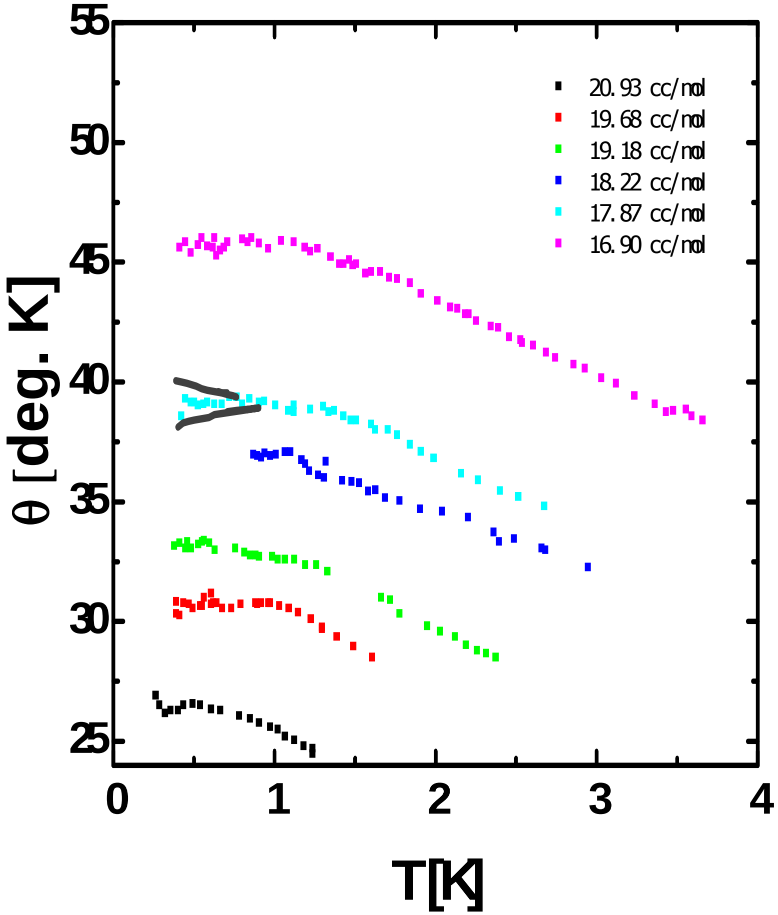
1. E. C. Heltemes and C. A. Swenson, *Phys. Rev*. **128**, 1512 (1962); H. H. Sample and C. A. Swenson, *Phys. Rev*. **158**, 188 (1967).
2. D. O. Edwards and R. C. Pandorf, *Phys. Rev*. **140**, A816 (1965).
3. W. R. Gardner et al., *Phys. Rev. A* **7**, 1029 (1973).
4. S. H. Castles and E. D. Adams, *J. Low Temp. Phys*. **19**, 397 (1975).
5. B. Hébral et al., *Phonons in Condensed Matter*, edited by H. J. Maris (Plenum, New York, 1980), pg. 169.
6. A. C. Clark and M. H. W. Chan, *J. Low Temp. Phys*. **138**, 853 (2005).

Results from Edwards and Hebral

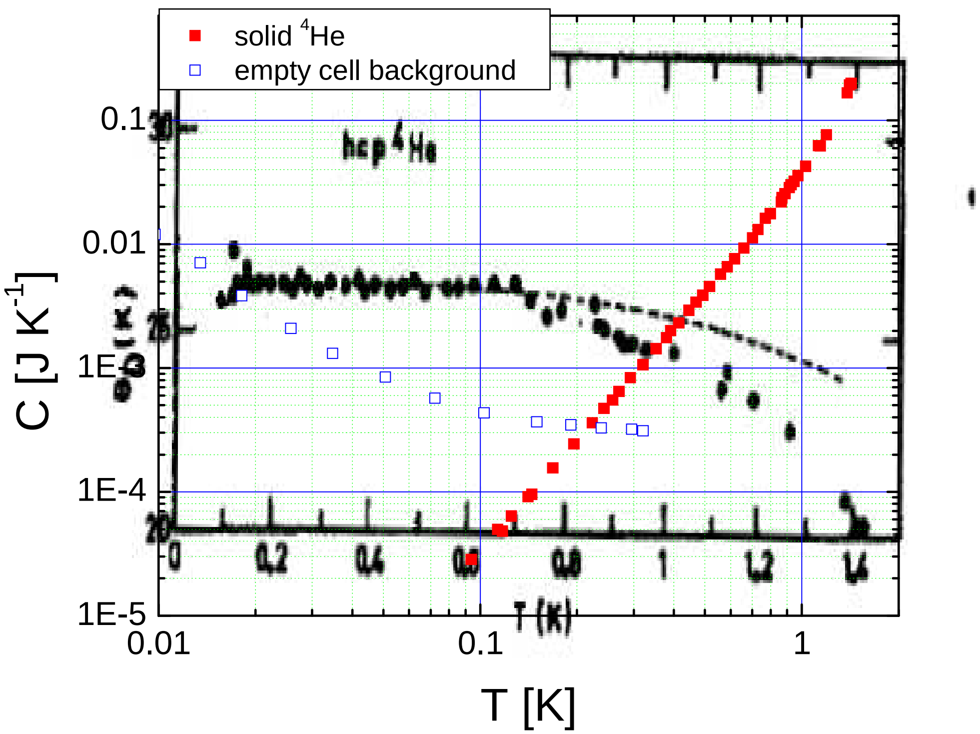
The background problem

Edwards

Effect of changing 1% of the empty cell



Hebral



$$C_v = \left(\frac{12\pi^4}{5}\right) N_a K_b \left(\frac{T}{\theta}\right)^3$$

Our experiment

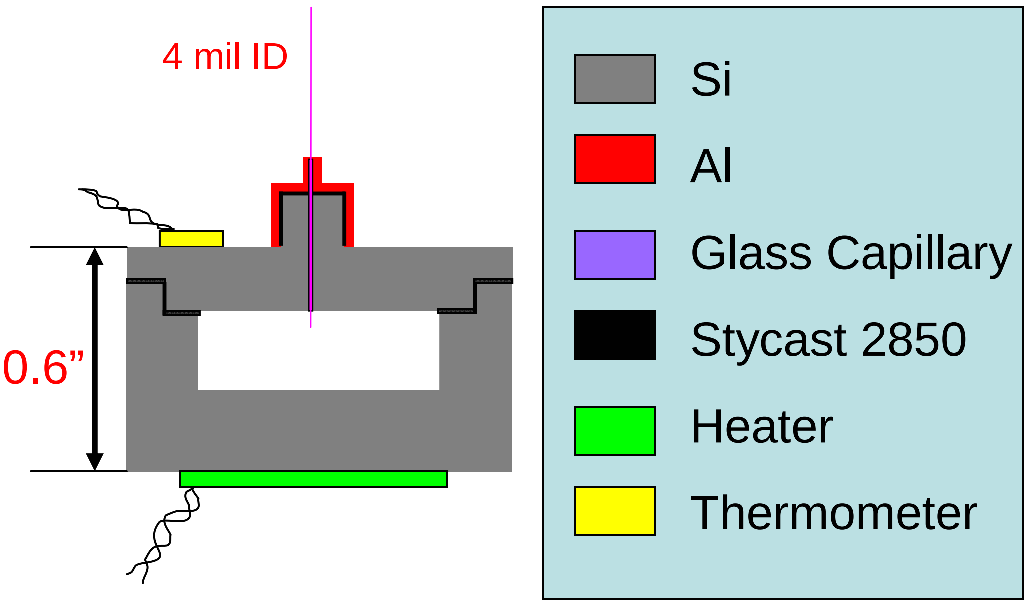
The Silicon cell

- Reasons for Si:

Low heat capacity:

High thermal conductivity:

Helium Volume= 0.926cc



At 0.1K	Si	Cu	He
Specific Heat [J mol^{-1} K^{-1}]	4x10^{-9}	7x10^{-5}	7x10^{-5}
Thermal conductivity [W cm^{-1} K^{-1}]	10^{-4*}	4x10^{-2}	4x10^{-3}

*** Using the value of quartz**

AC Calorimetry [1,2]

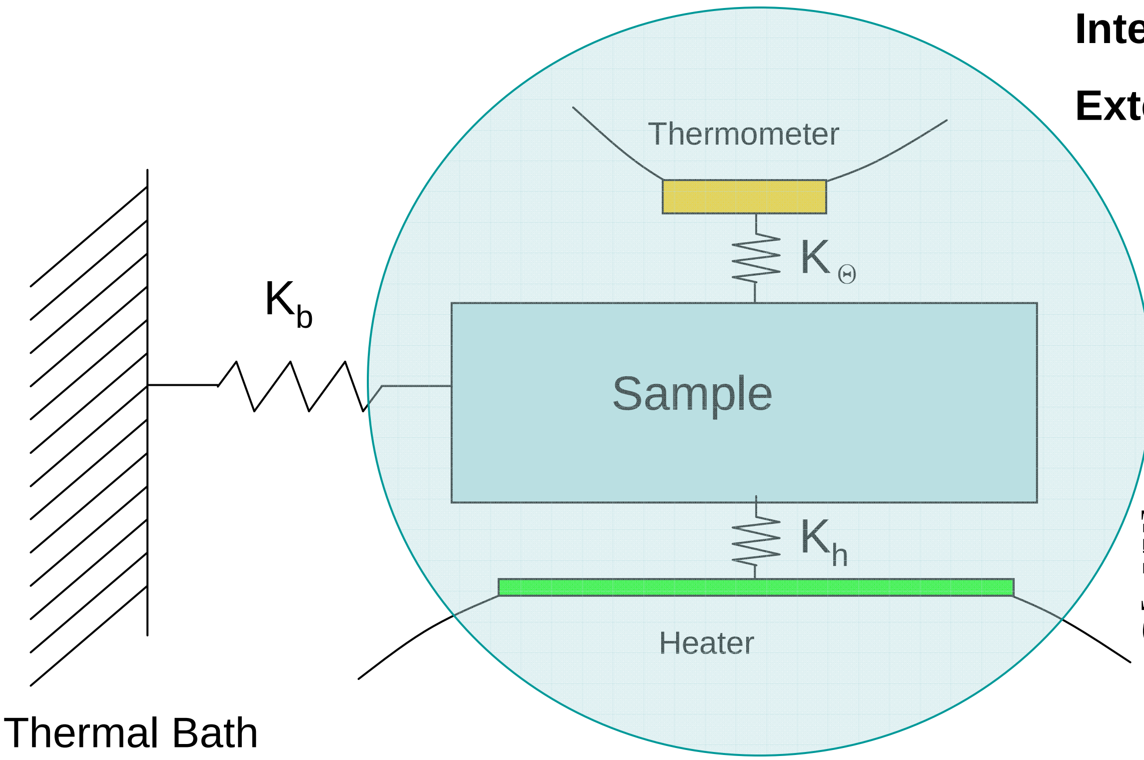
$$\dot{Q} = \dot{Q}_0 (\cos \tfrac{1}{2} \omega t)^2 \rightarrow \left|T_{ac}(L,t)\right| \approx \frac{\dot{Q}_0}{2\omega C_s}\left[1 + \frac{1}{\omega^2 \tau_s^2} + \omega^2 \tau_{\mathrm{int}}^2 + \frac{2\mathrm{K}_b}{3\mathrm{K}_s}\right]^{-1/2} \rightarrow$$

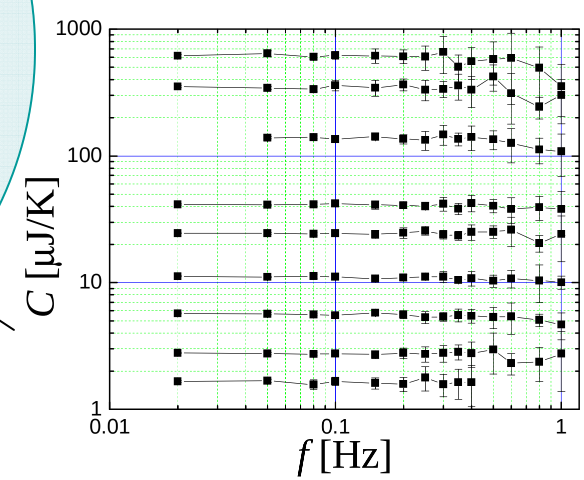
$$T_{ac} = \frac{\dot{Q}}{2\omega C}$$

$$C = \frac{\dot{Q}}{2\omega T_{ac}}$$

Internal time constant << 1/ ω

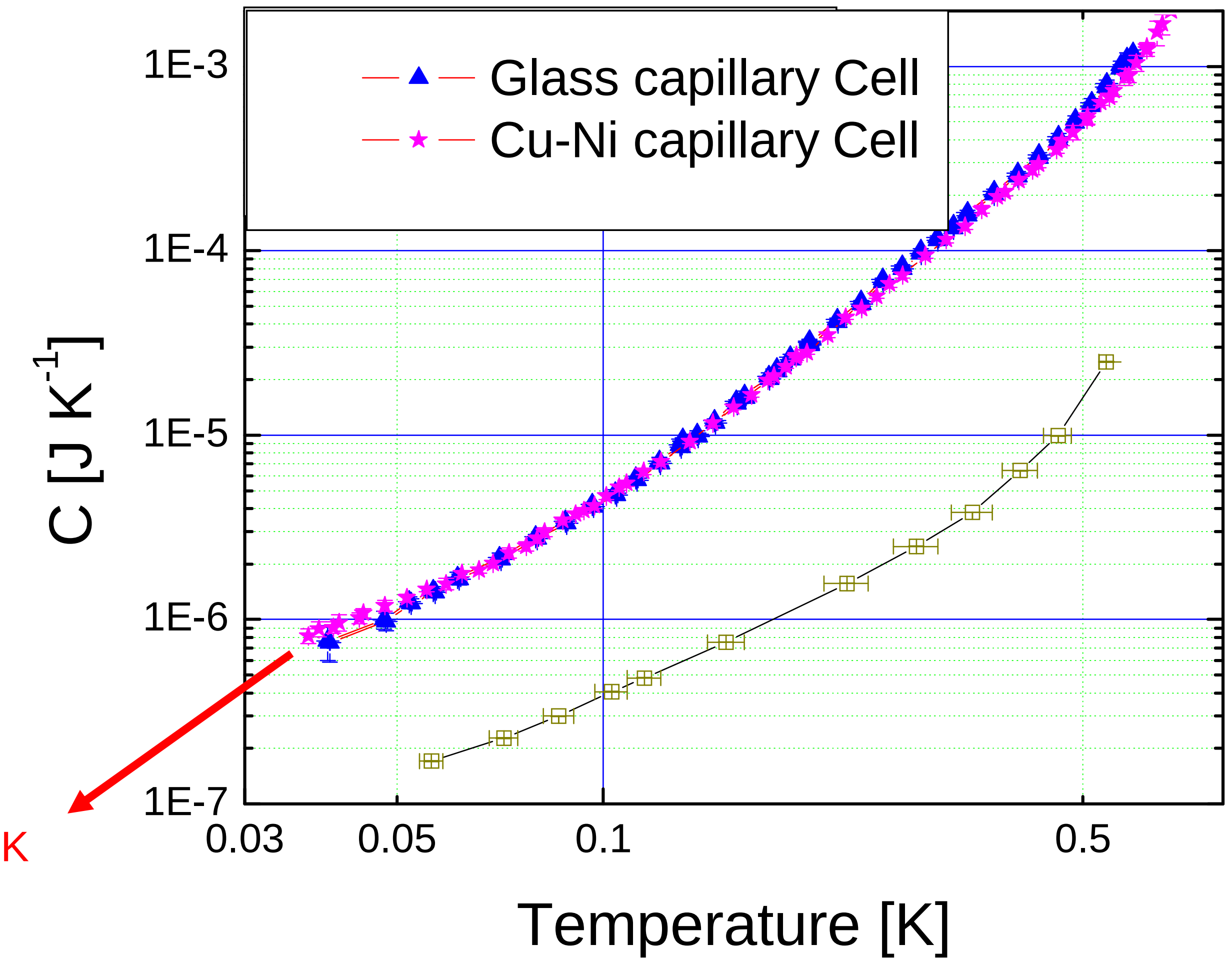
External time constant >> 1/ ω





1. Paul F. Sullivan, G. Seidel, *Phys Rev*. **173**, 679 (1968).
2. Yaakov Kraftmakher, *Physics Reports,* **356** (2002) 1-117.

Results: pure ^{4}He (0.3ppm)

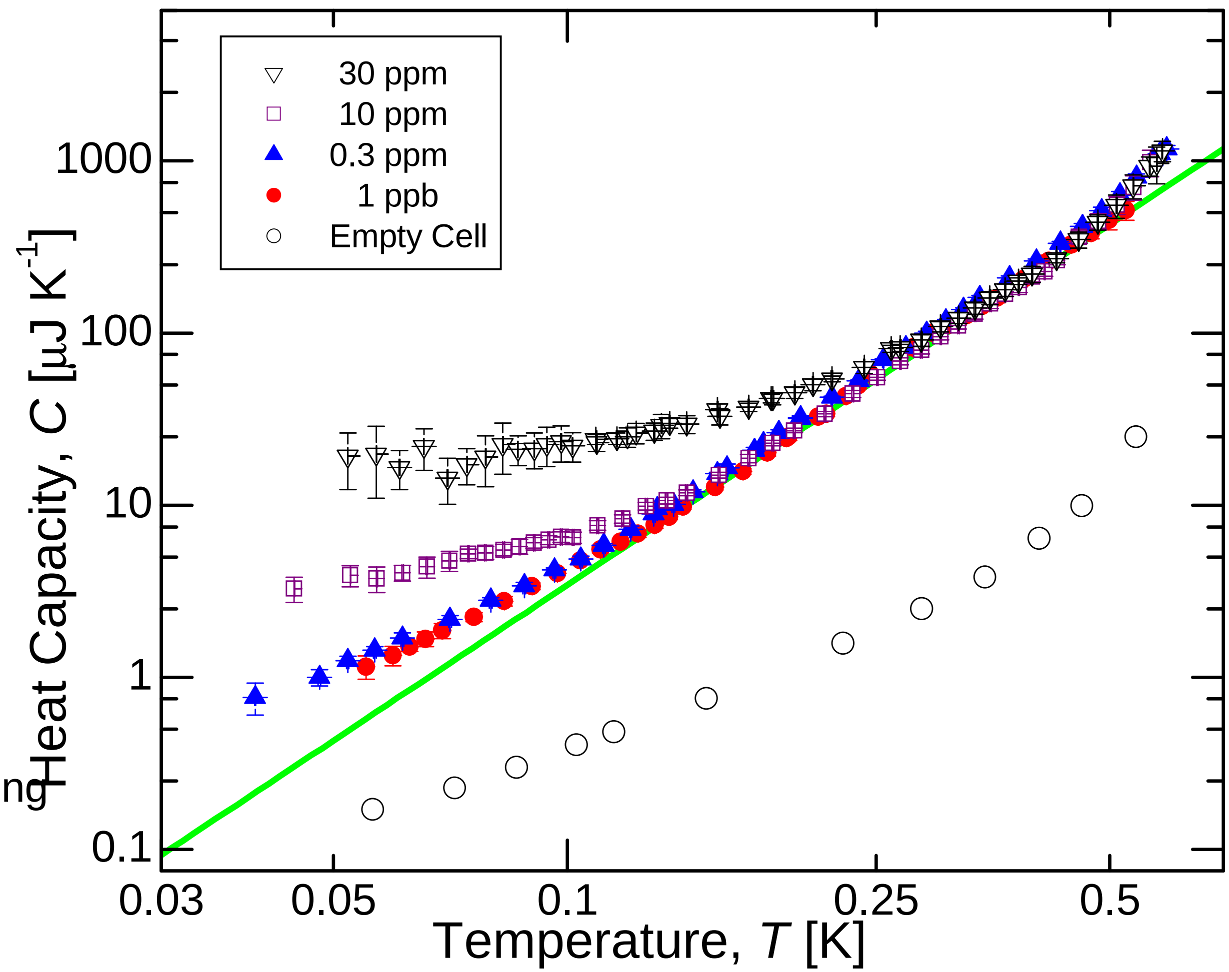


Minimum temperature 40mK

Temperature scale based on ^{3}He melting curve

Results: ^{4}He at different ^{3}He concentrations in glass capillary cell

Constant volume technique



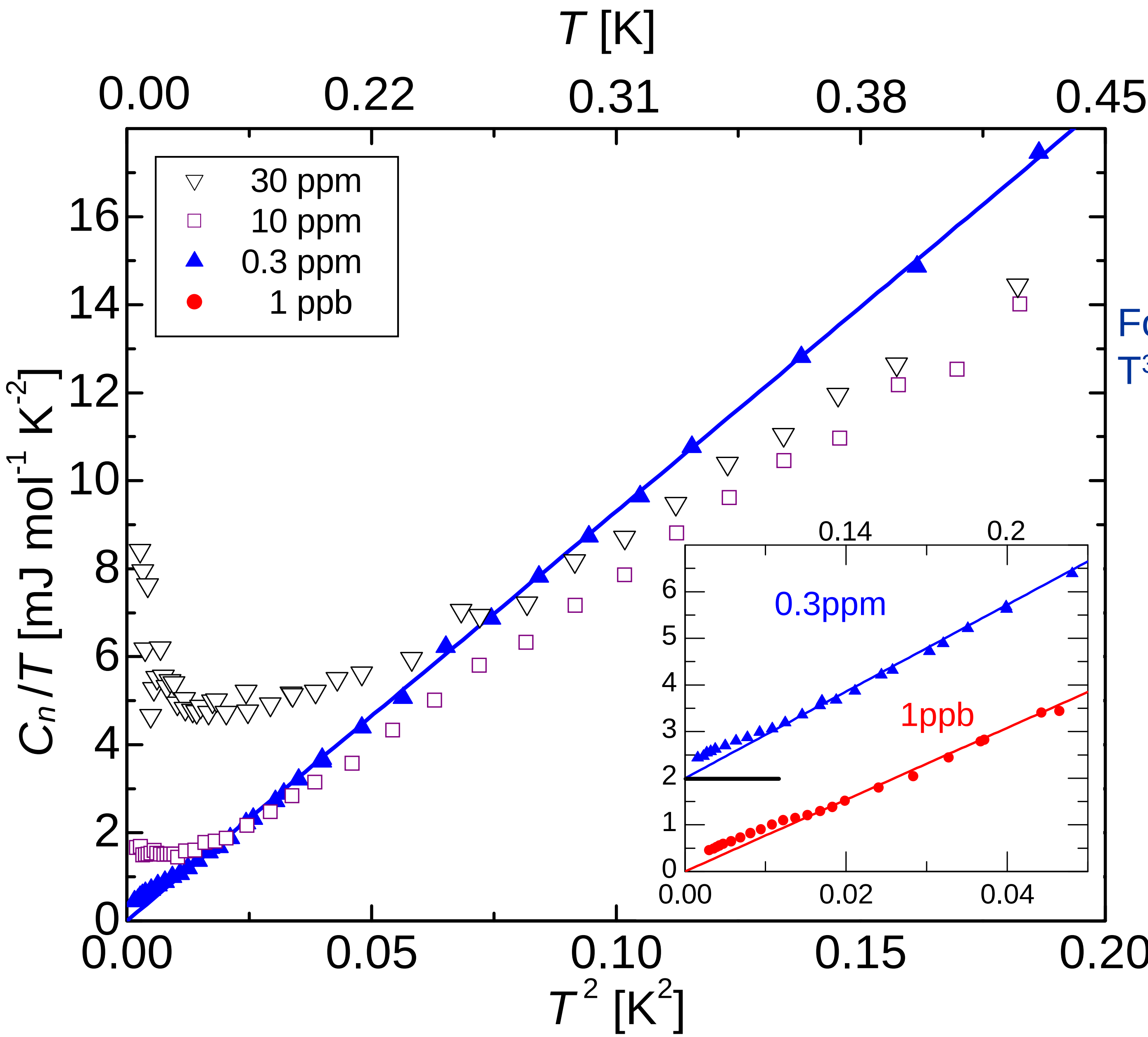
No long time constant.

No hysteresis

No change due to annealing

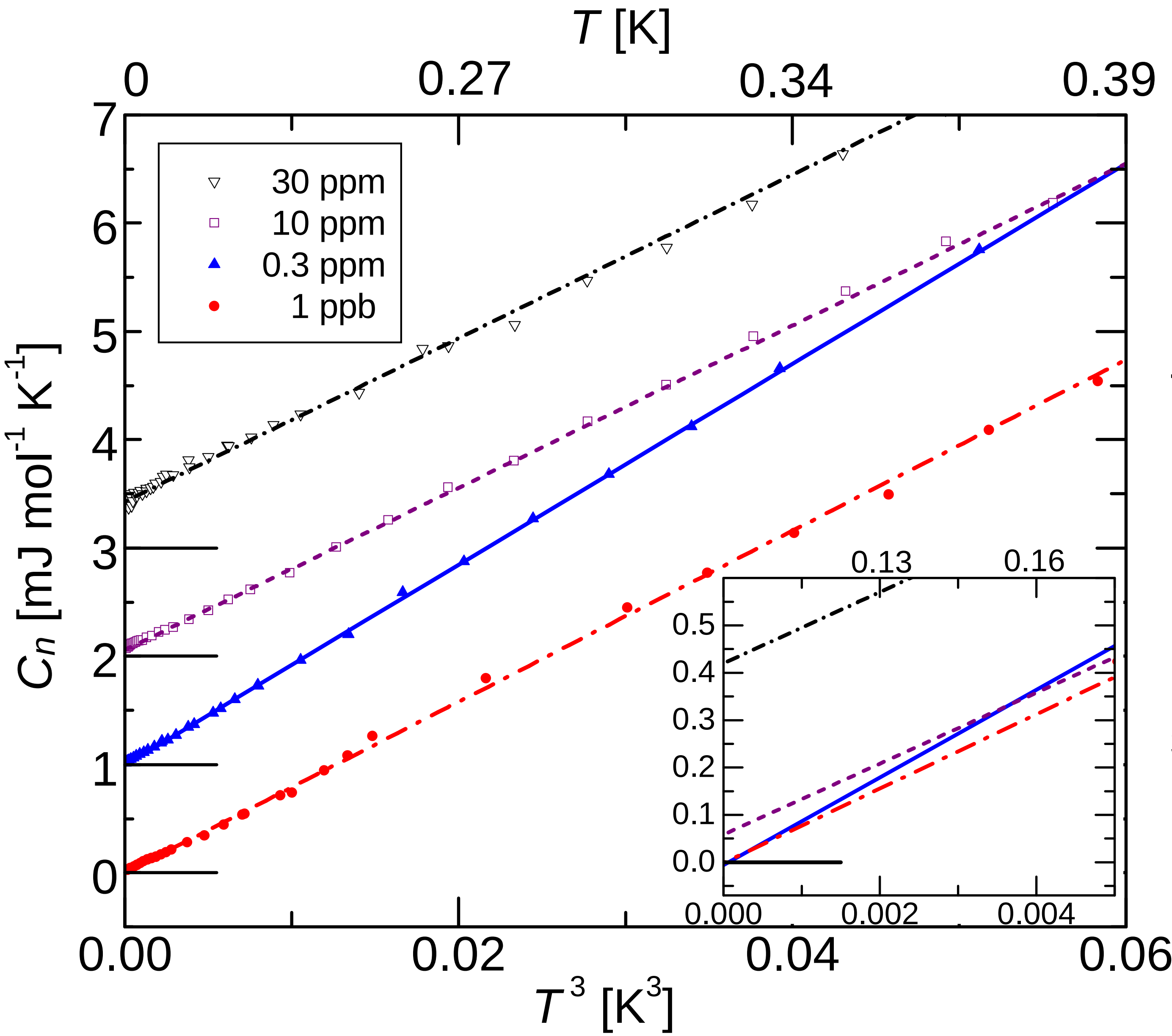
No thermal cycle effect

Is there a linear T term?



For 0.3ppm, T>0.14K
T^3 relation only

C vs T^3



Constant contribution from ^{3}He impurity

10ppm sample

0.7+/-0.2 k_B per ^{3}He

30ppm sample

1.7+/-0.3 k_B per ^{3}He

NMR measurement of spin (^{3}He) diffusion:

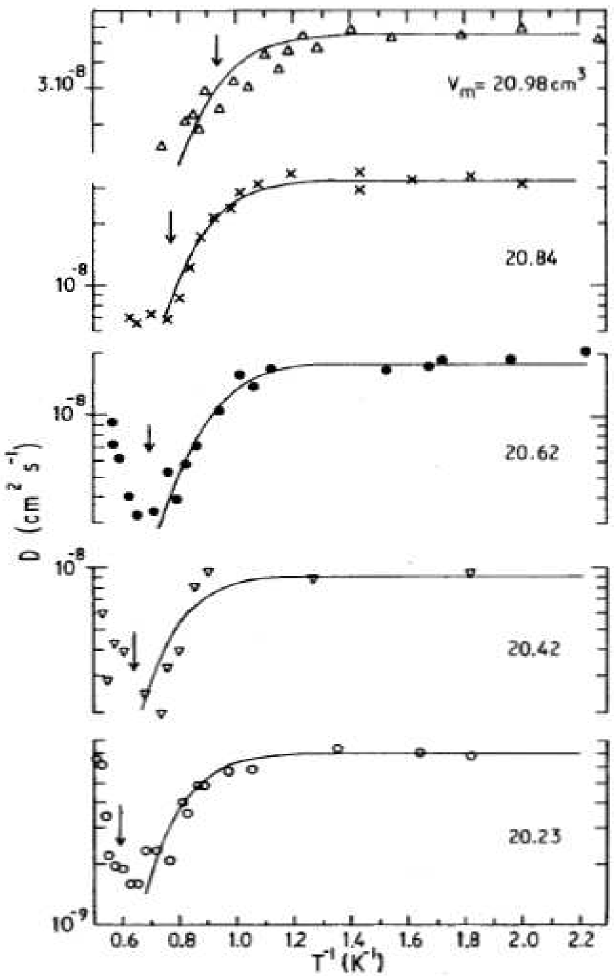


Fig. 1. Temperature dependence of the spin diffusion coefficient of ^{3}He of fractional concentration 5×10^{-4} in solid ^{4}He for five different labeled molar volumes. The solid lines are fits to Eq. (15) and the significance of the arrows is explained in the text following that equation.

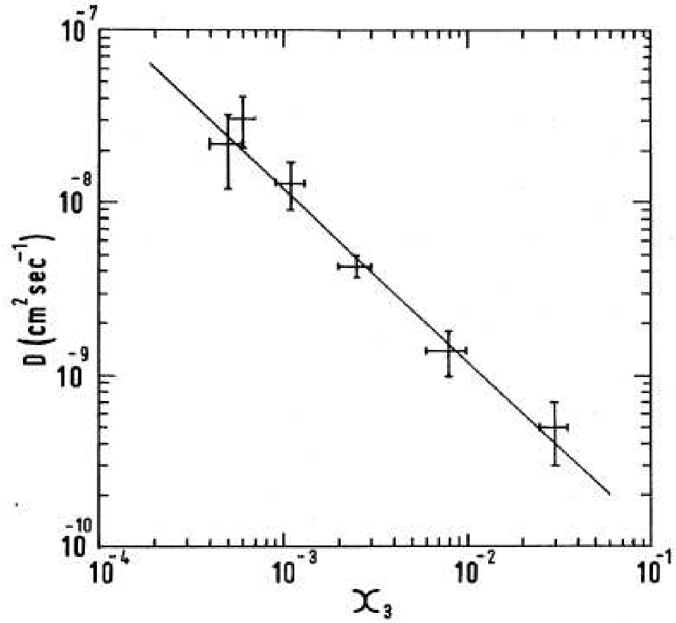
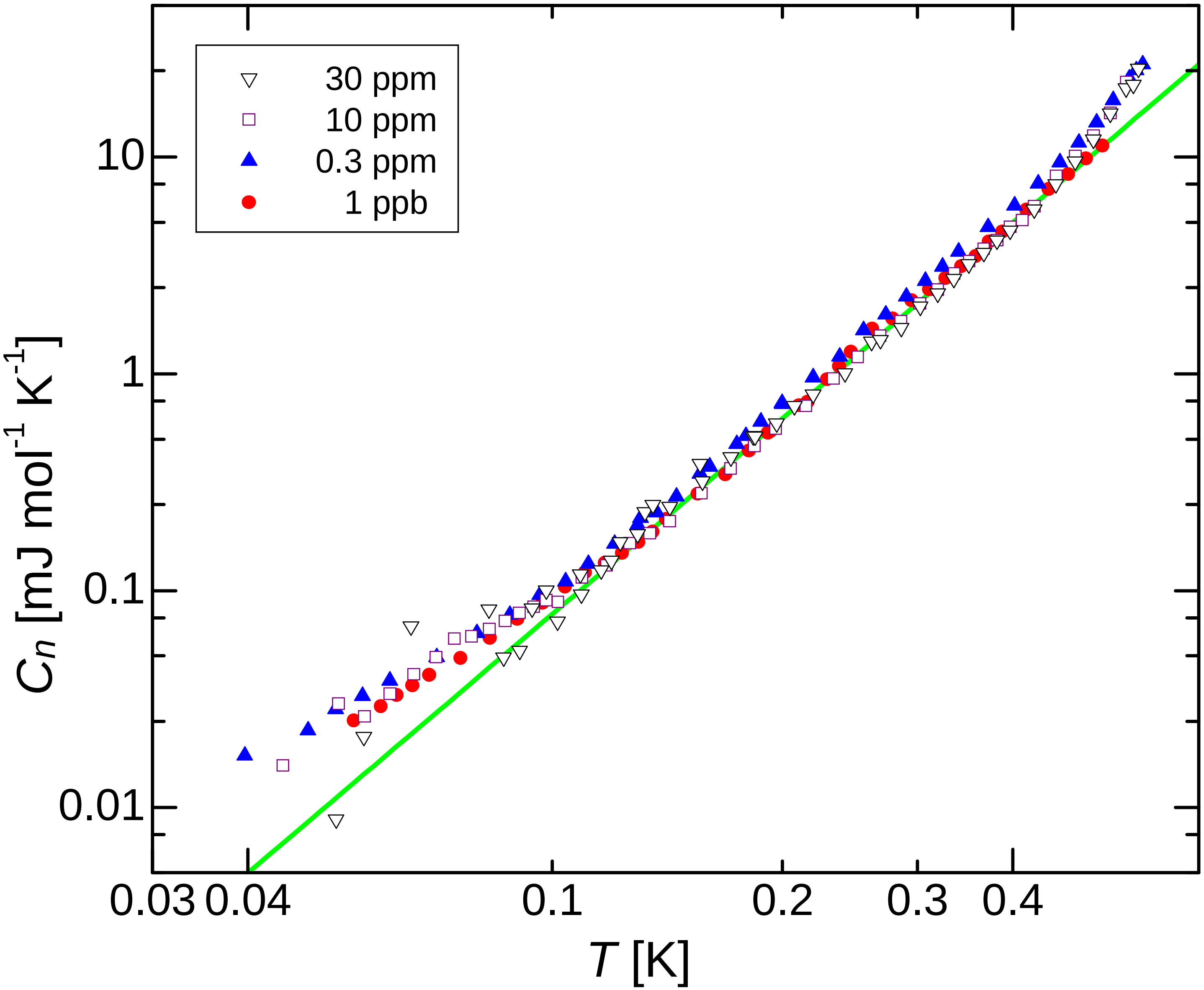


FIG. 1. Spin diffusion coefficient D of ^{3}He impurity in solid ^{4}He versus x_3, the mole fraction of ^{3}He. Temperature, 0.53 K; sample molar volume, 21 cm^3; operating frequency, 5 MHz. $D\propto 1/x_3$ is characteristic of the impuriton model. The line drawn is $Dx_3=1.2\times10^{-11}$ cm^2 sec^{-1}.

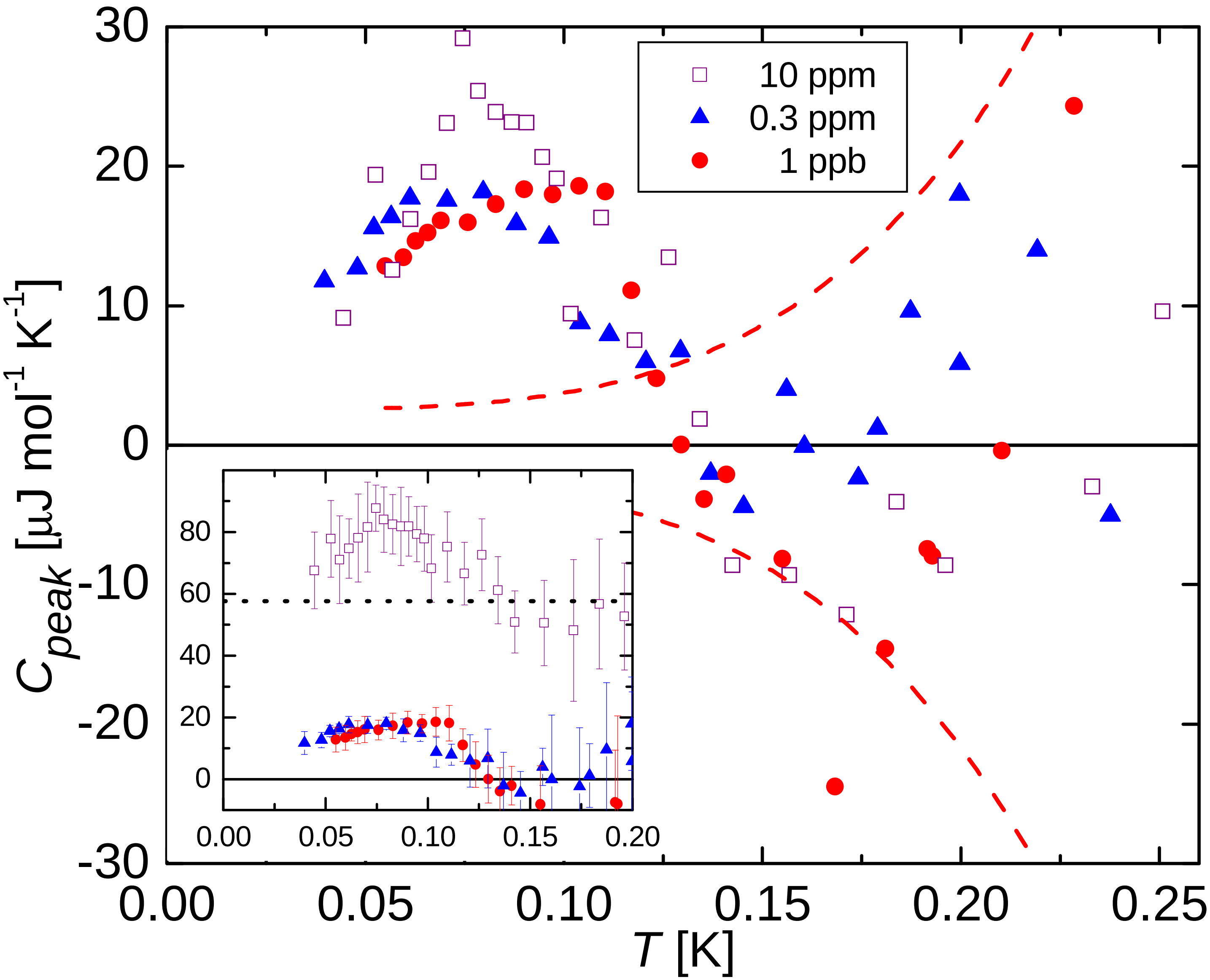
A. R. Allen, M. G. Richards & J. Schratter *J. Low Temp. Phys.* **47**, 289 (1982).

M. G. Richards, J. Pope & A. Widom, *Phys. Rev. Lett.* **29**, 708 (1972).

Specific heat with the temperature independent constant term subtracted



Specific heat peak is found when T^3 term subtracted
The peak is independent of ^{3}He concentration



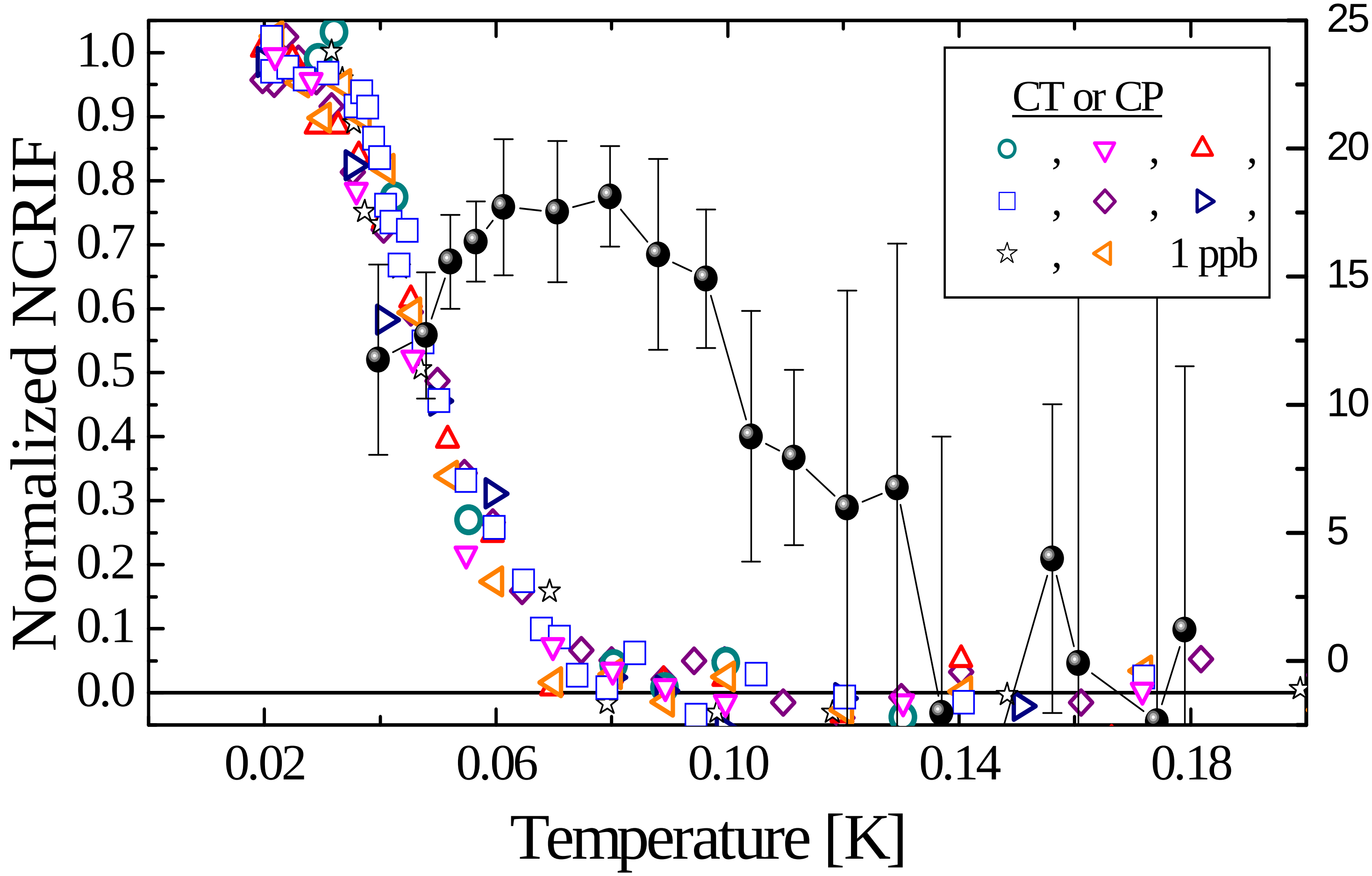
Peak height: 20 μ J mol^{-1} K^{-1}

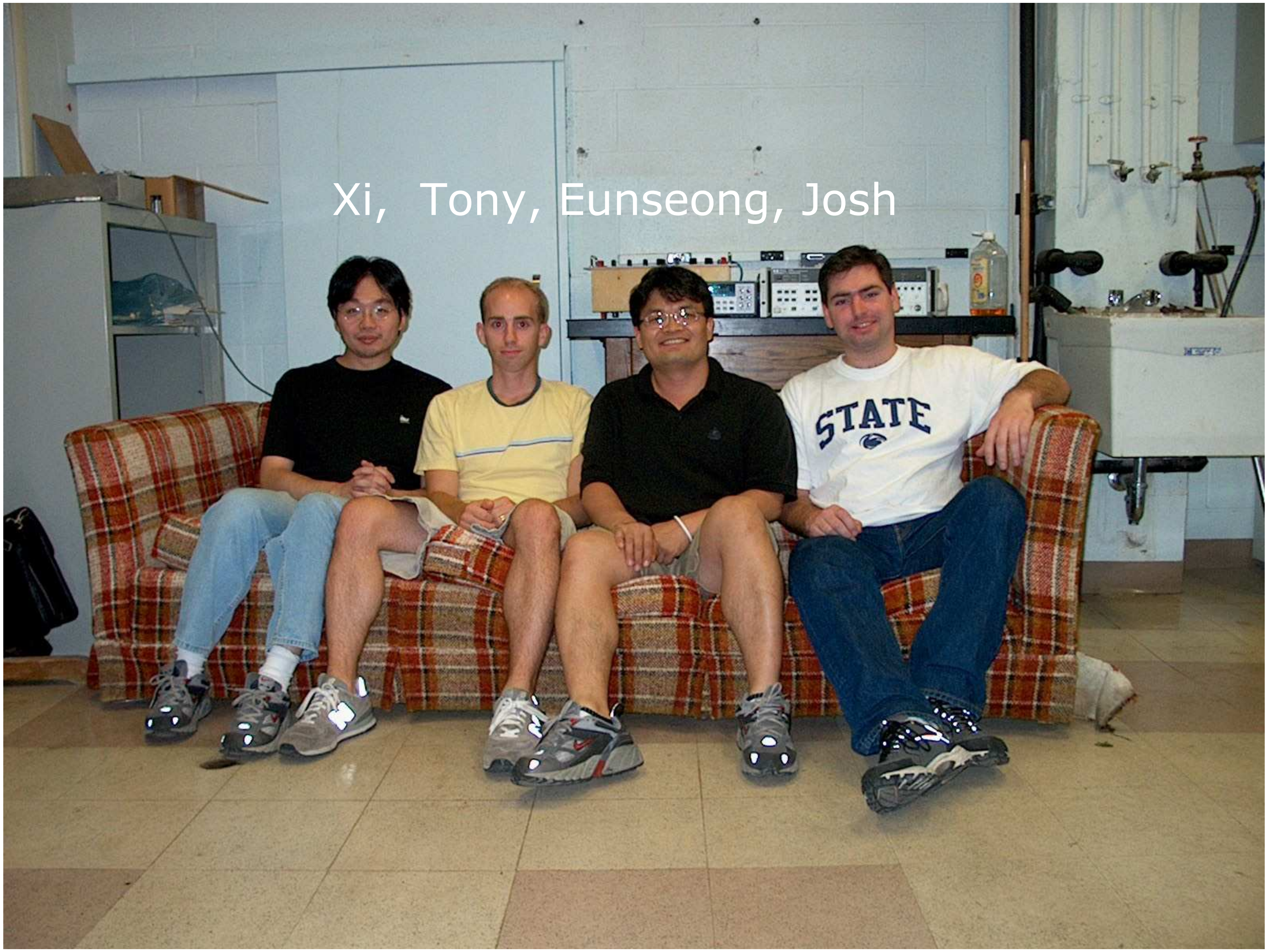
(2.5 x 10^{-6} k_B per ^{4}He atom)

Excess entropy: 28 μ J mol^{-1} K^{-1}

(3.5 x 10^{-6} k_B per ^{4}He atom)

Compare with torsional oscillator



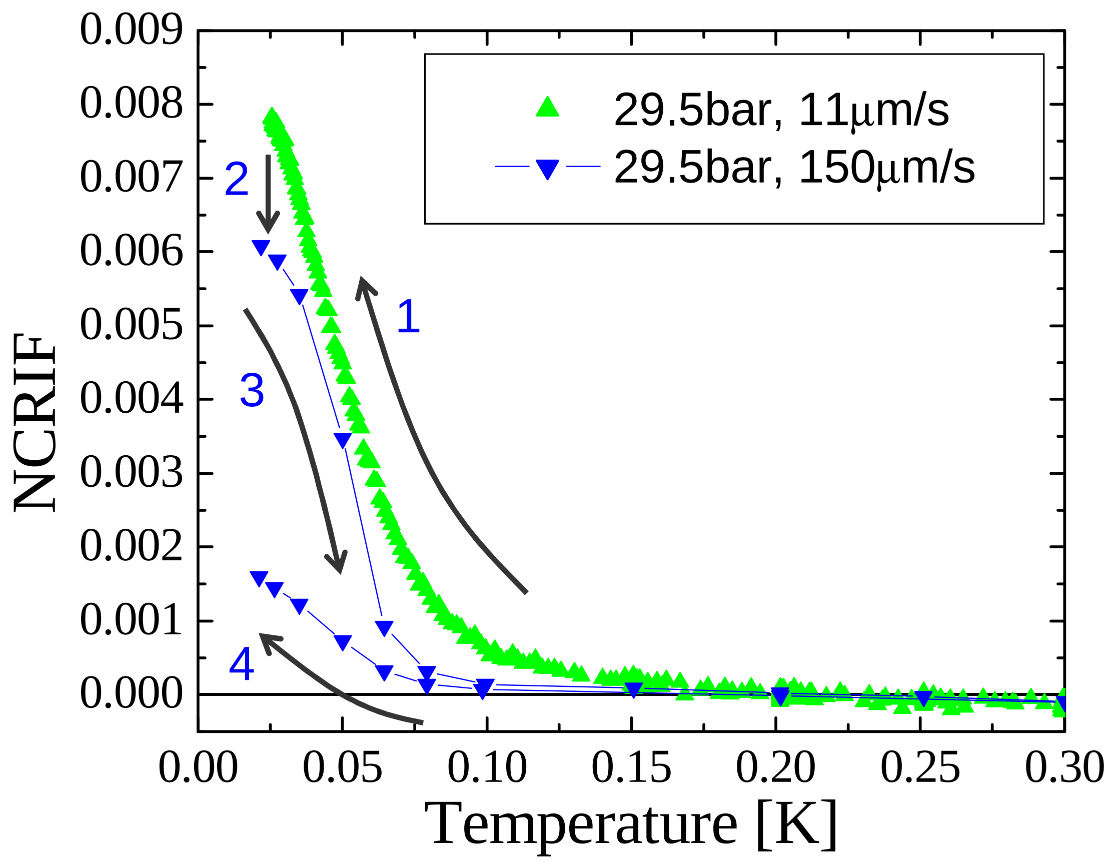

Xi, Tony, Eunseong, Josh
STATE

Thermal history of 1ppb samples

Velocity changes at low temperature lead to interesting behavior…

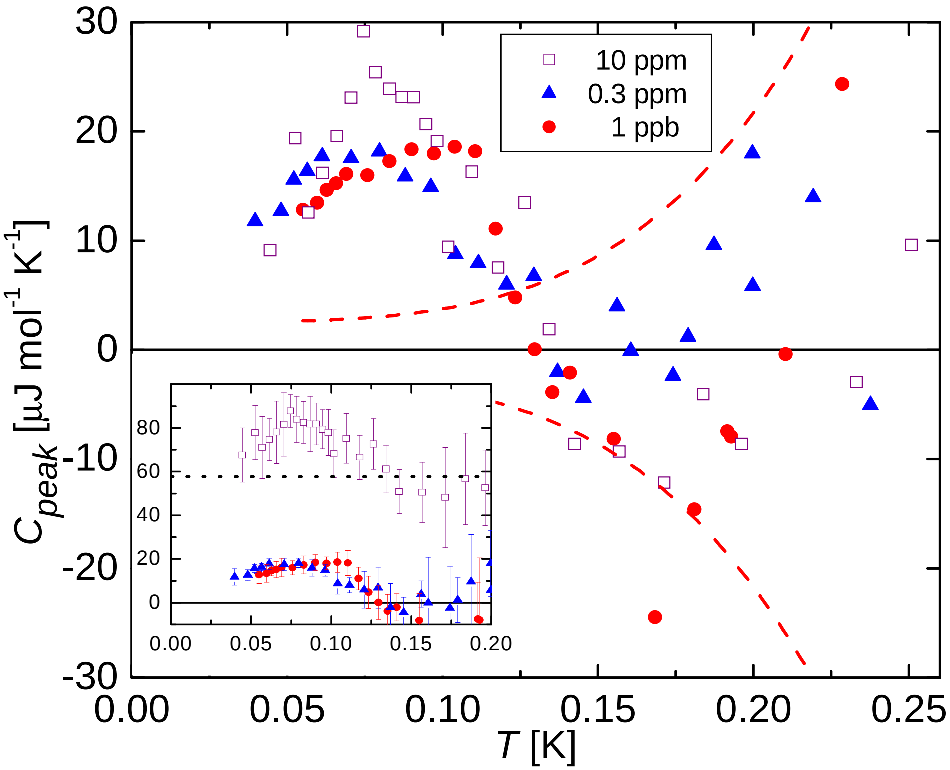
Protocol followed below:

(1) cooling, (2) velocity increase, (3) warming, (4) cooling



End

Specific heat with T^3 term subtracted



Peak height:
20 μ J mol^{-1} K^{-1} (2.5 x 10^{-6} k_B per ^{4}He atom)

Excess entropy:
28 μ J mol^{-1} K^{-1} (3.5 x 10^{-6} k_B per ^{4}He atom)

1. **Specific heat peak is independent of ^{3}He concentrations.**
2. **Assuming 3D-xy universality class (same as the lambda transition in liquid ^{4}He).**
3. **Use two-scale-factor universality hypothesis, ρ_s ~0.06%. 1ppb study of TO found this number lays between 0.03% and 0.3%.**

Hysteresis in Pressure measurement of phase separation

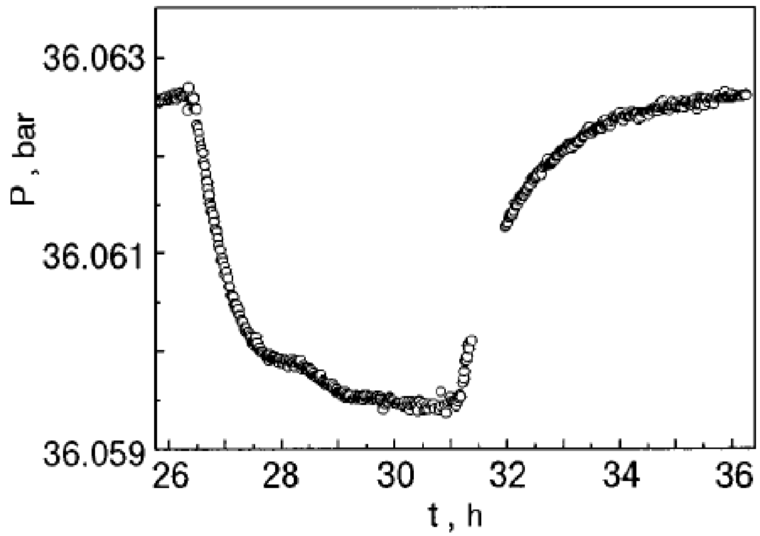
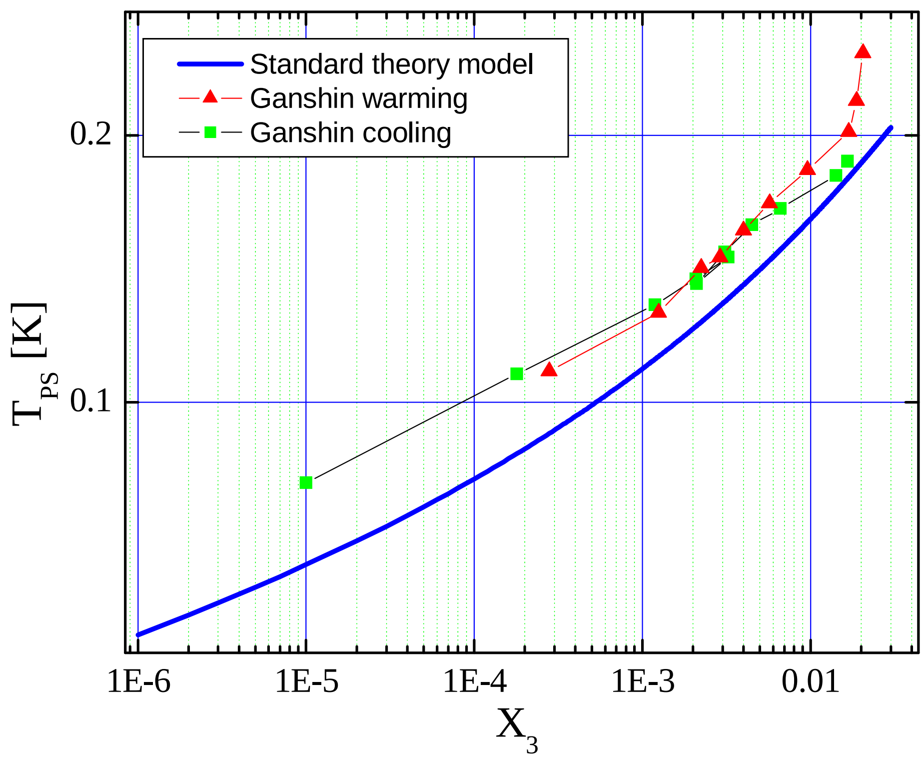


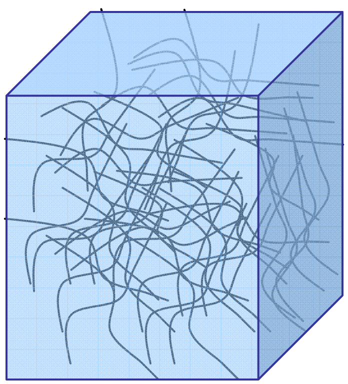
FIG. 1. Typical kinetics of the change in pressure in the sample over the course of a single temperature step. The two-phase crystal is initially heated from 81 mK (^{3}He concentration $x = 0.001\%$) to 108 mK ($x = 0.02\%$ ^{3}He) and then cooled from 108 to 81 mK.

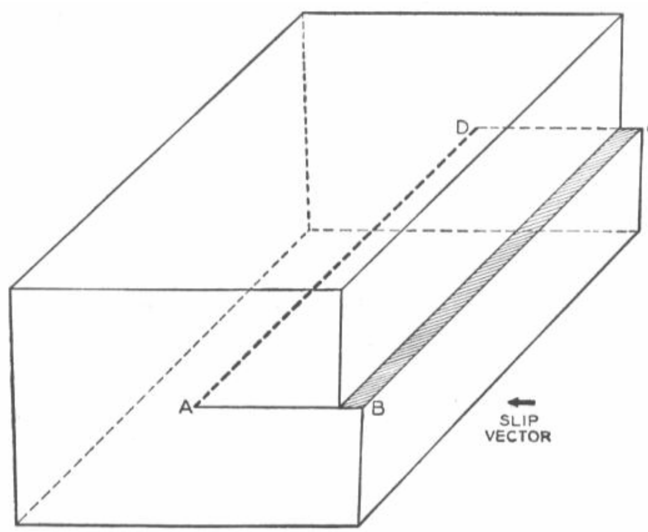


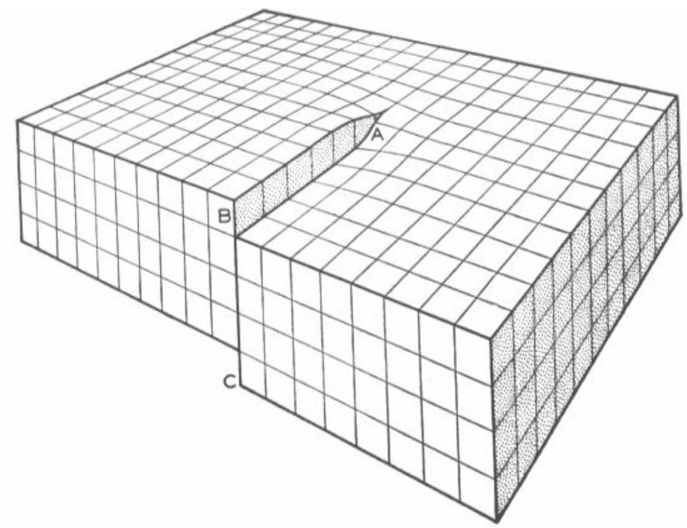
A.N.Gan'shin, V.N.Grigor'ev, V.A.Maidanov, N.F.Omelaenko, A.A.Penzev, É.Ya.Rudavskii, A.S.Rybalko., *Low Temp. Phys*. **26**, 869 (2000).

Dislocations

- Two of the common types: edge & screw





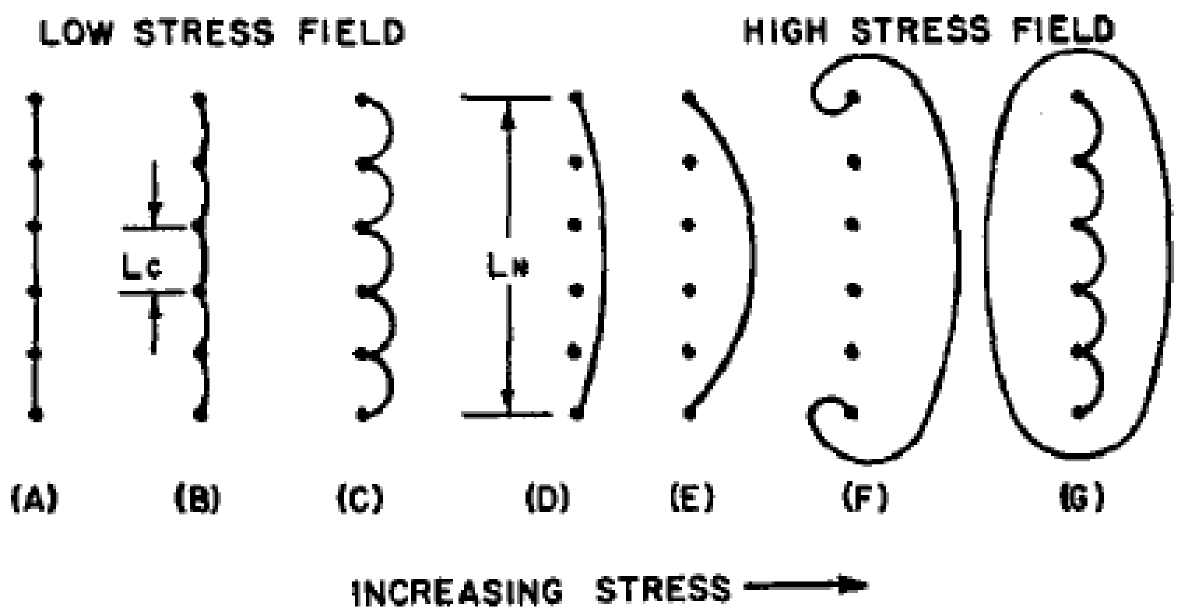


- Dislocation density, Λ = ~5 → <10^{10} cm^{-2}
  – 3-d network, L_N ~ 1 to 10 μm (Λ~10^5 to 10^7)

[L_N ~ 0.1 to 1 μm (Λ~10^9)]

Granato-Lucke applied to ^{4}He

- Dislocations intersect on a characteristic length scale of L_N ~ 1→ 5μm
- Dislocations can also be pinned by ^{3}He impurities
  - Distance between ^{3}He atoms (if uniformly distributed):
  - 1ppb → 1000a ~ 0.3μm
  - 0.3ppm → 150a ~ 45nm
  - 1% → 5a ~ 15nm



^{3}He-dislocation interaction

Actual ^{3}He concentration on dislocation line is thermally activated

$$x_3 = x_0 \exp\left[\frac{W_0}{T}\right]$$

*Typical binding energy, W_0 is 0.3K to 0.7K

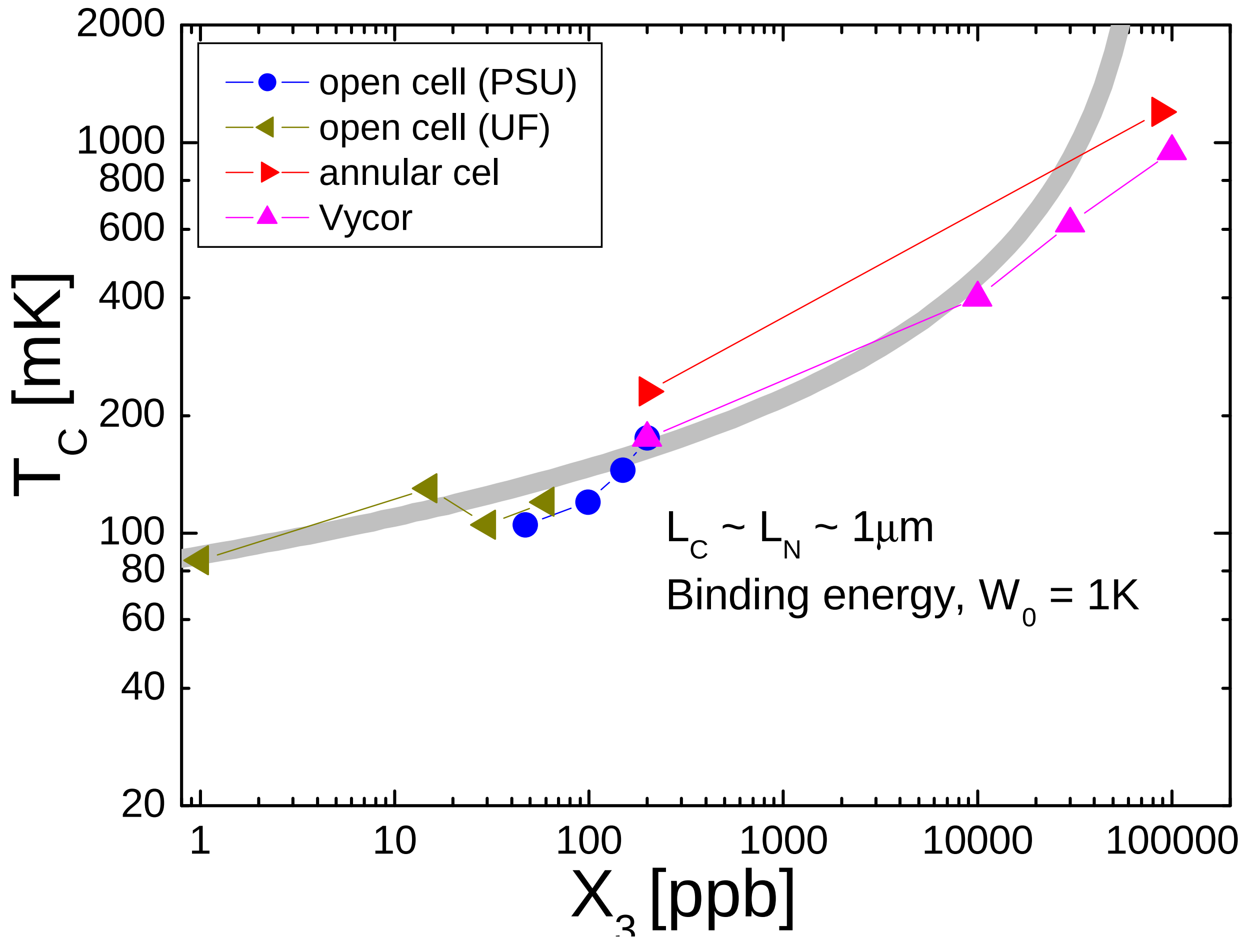
^{3}He-dislocations interaction

Line considered as crossover from network pinning to ^{3}He impurity pinning of dislocations
($L_{Network} \sim L_{3He\ spacing}$)

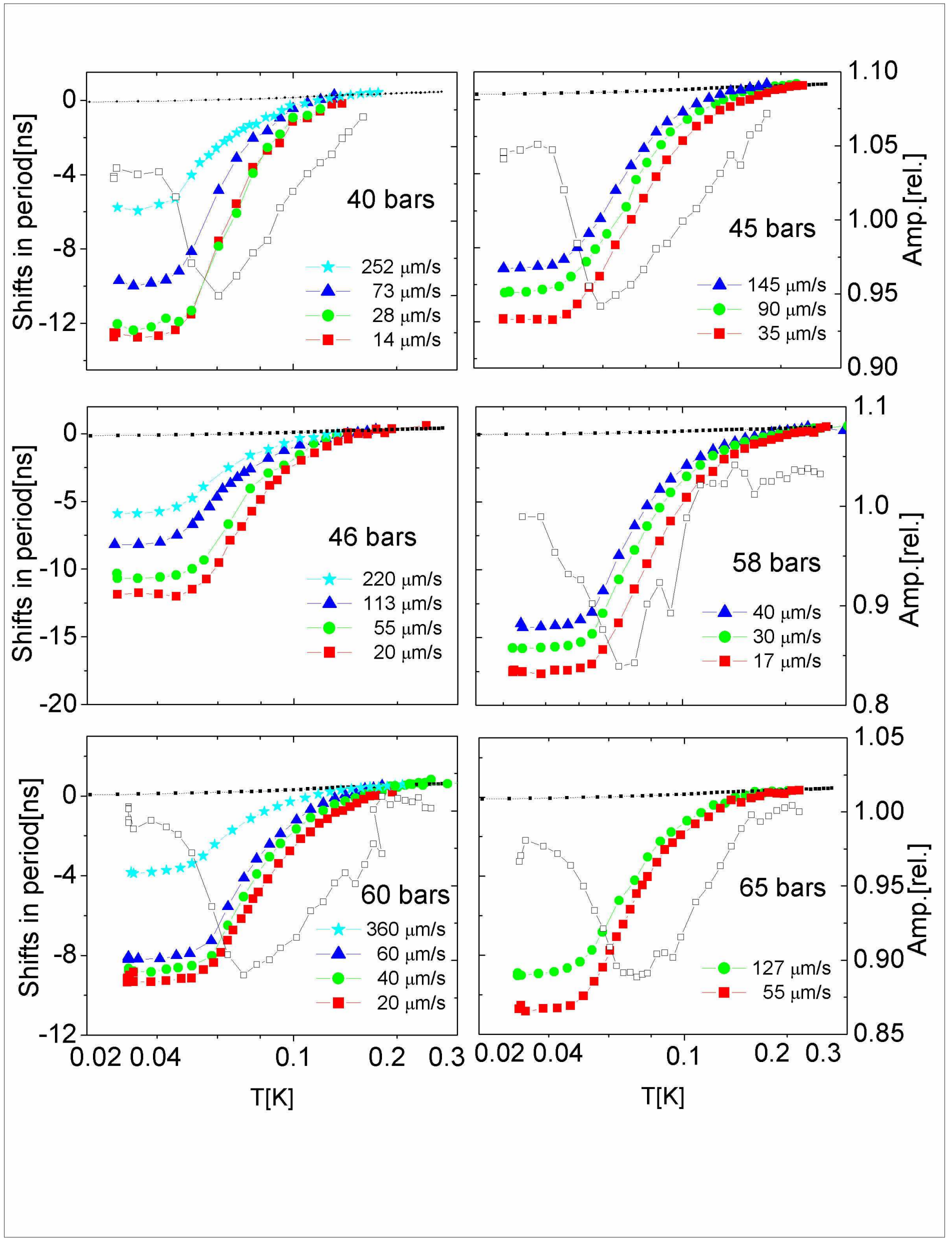
$$L_{3He} = AW_0^{-1/3} x_0^{-2/3} \exp\left[\frac{-2W_0}{3T}\right]$$

Average length
$L_{Network} \sim 1$ to $10\mu m$
For $\Lambda \sim 10^5$ to $10^6 cm^{-2}$

Smaller lengths
(< 1μm) are expected
for larger dislocation
densities



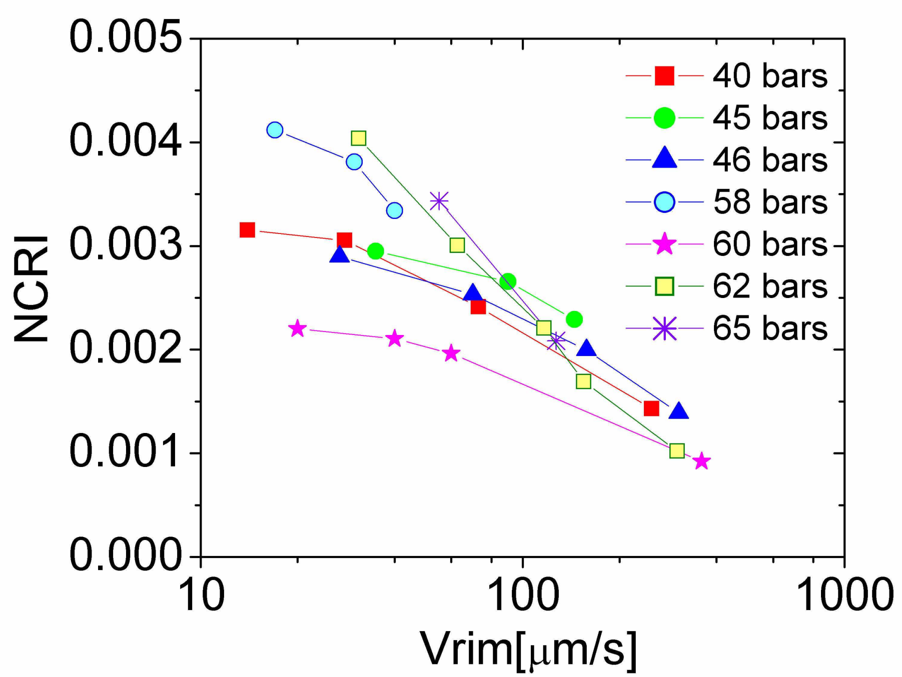
Solid helium in Vycor glass



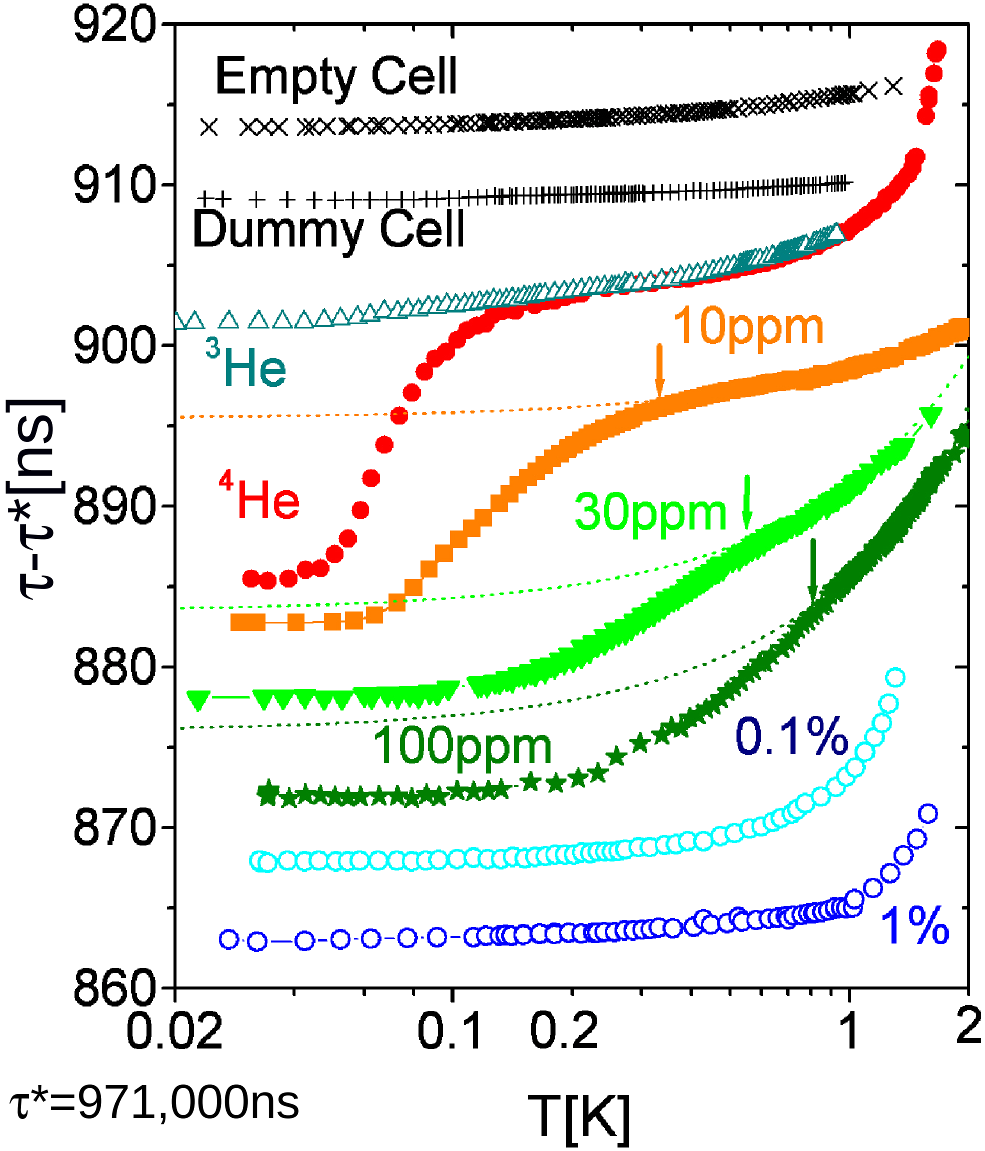
Weak pressure dependence…

from 40 to 65bar

Strong velocity dependence



^3He-^4He mixtures



Question: If there is a transition between the normal and supersolid phases, where is the transition temperature?

^3He-^4He mixtures