A new tool-box for hadronic studies: Optics and Self-Organizing Networks

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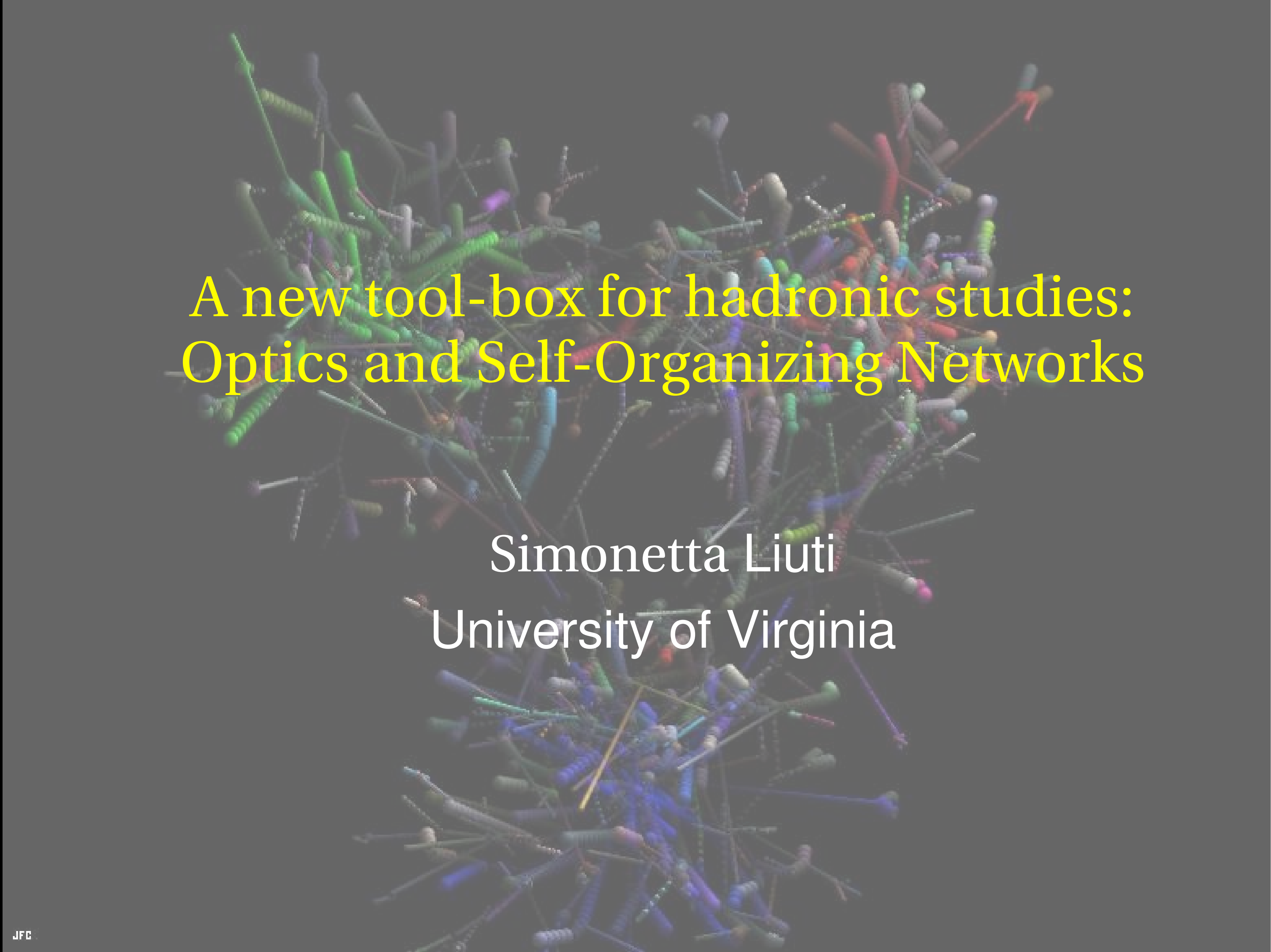

Collaborators

- Saeed Ahmad (graduate student, physics)
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- Heli Honkanen (post-doc, physics)
- Paul Reynolds (Co-PI, CS)
- Swadhin Taneja (graduate student, physics)

Outline

- An overview of the proton
- Outstanding Questions
- New experimental tools: Deeply virtual exclusive experiments and “femtoimages”
- Extracting femtoimages from experiment: Self-Organizing Map (SOM) technique
- Conclusions and Outlook

1. Overview of the proton

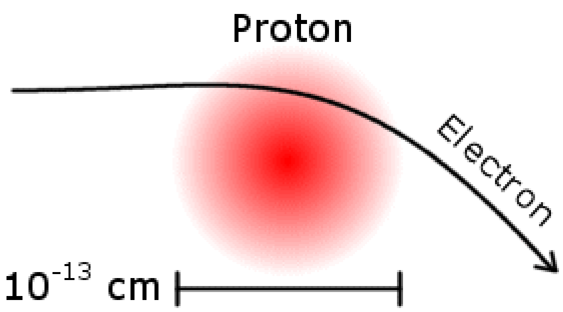
What we know ...

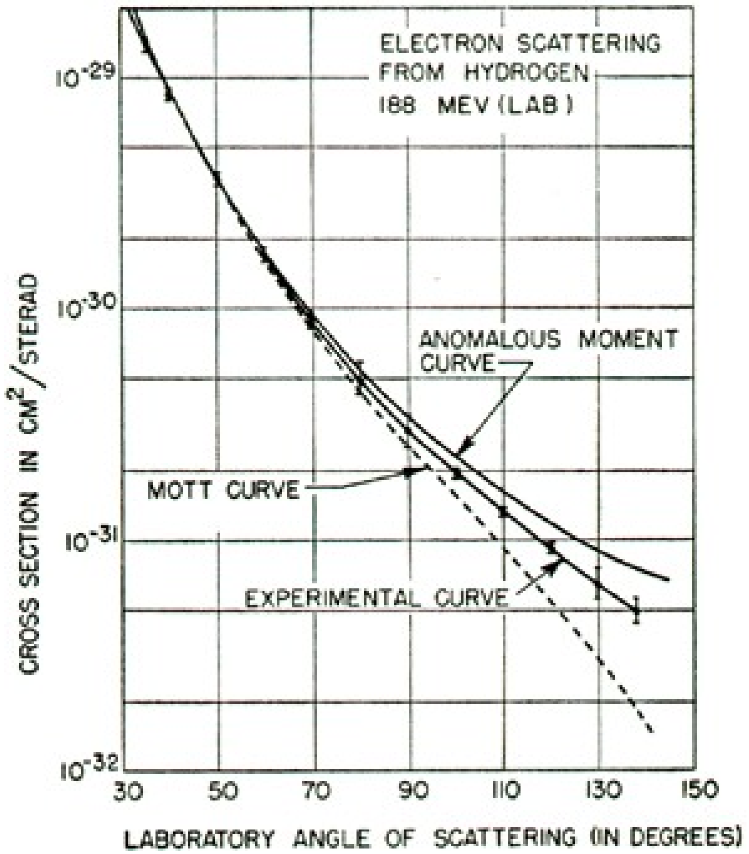
- Quantum Numbers: $1/2^+$
- Charge: +1
- Mass: 938.27 MeV
- Magnetic Moment: 2.79 μ_N

⇒ Measured by O. Stern (1933)
Proton has an internal Structure!

What we know ...

- Charge radius:
  $\approx$ 0.86 fm

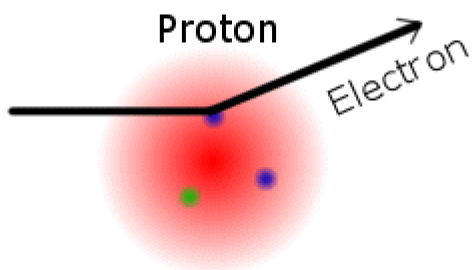


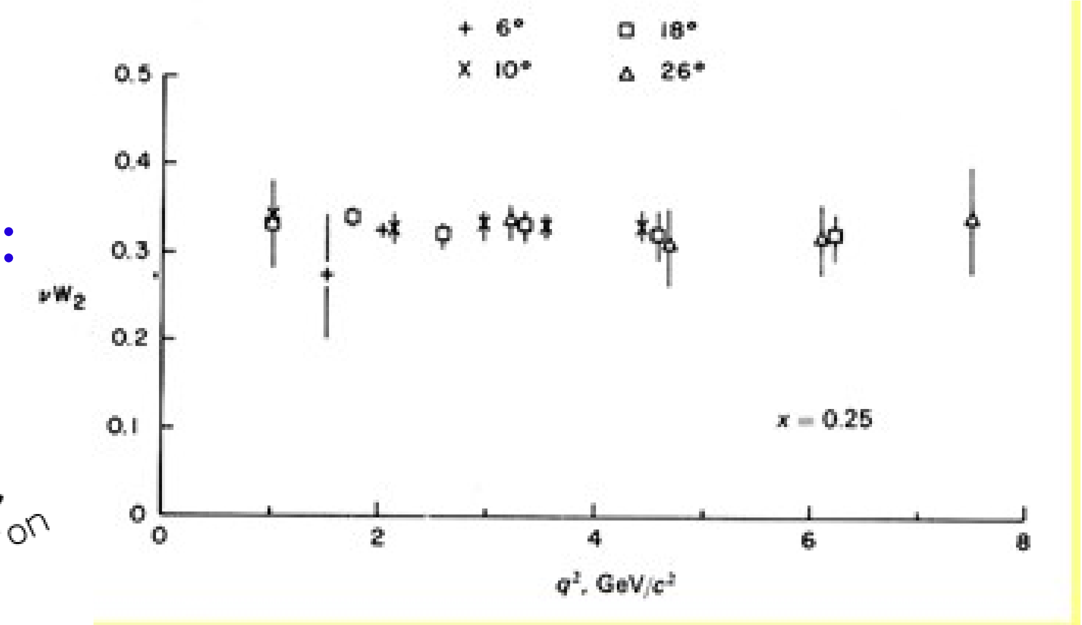


Measured by R. Hofstadter (1955)
<u>Inclusive</u>, <u>elastic</u> ep scattering!

What we know ...

- Proton is made of pointlike constituents: "*partons*"





Direct evidence observed by Friedman, Kendall and Taylor (1969): Inclusive, deep inelastic ep scattering!

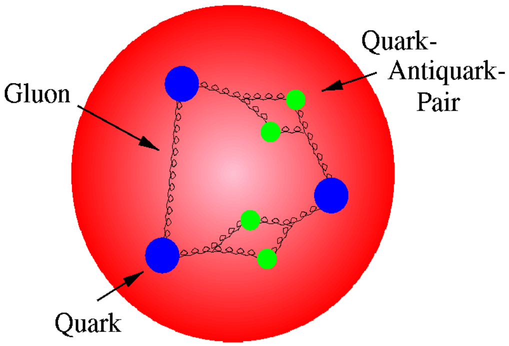
Proton Dynamics: QCD

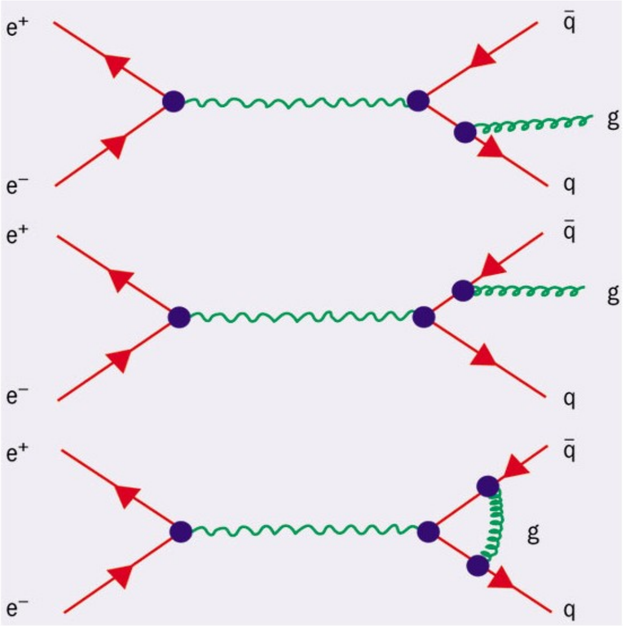
Quarks:

u,d,s,c,...; 3 colors; spin 1/2; $m_{u,d..} \ll m_p$

Gluons:

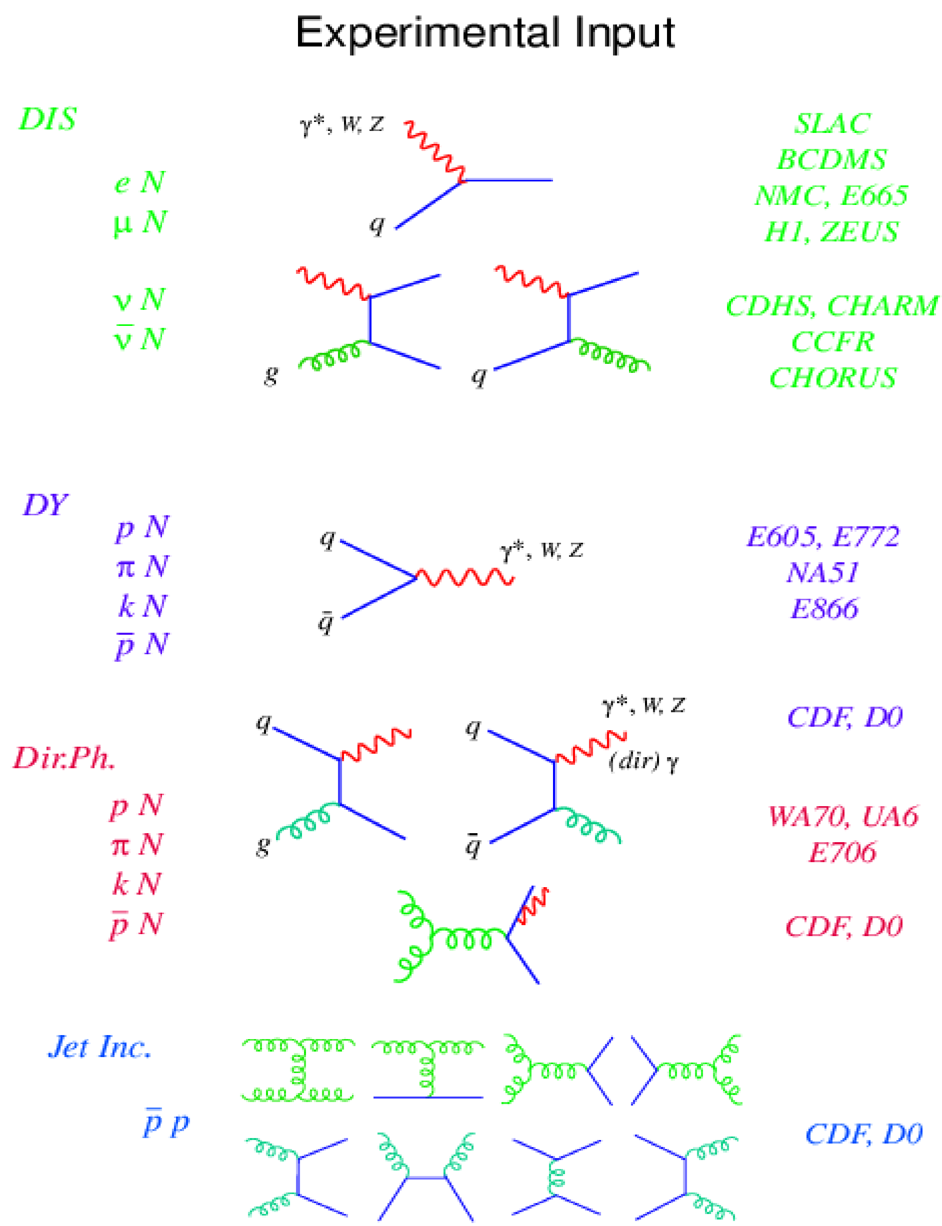
no charge; 8 “color types”; spin 1; $m_g = 0$





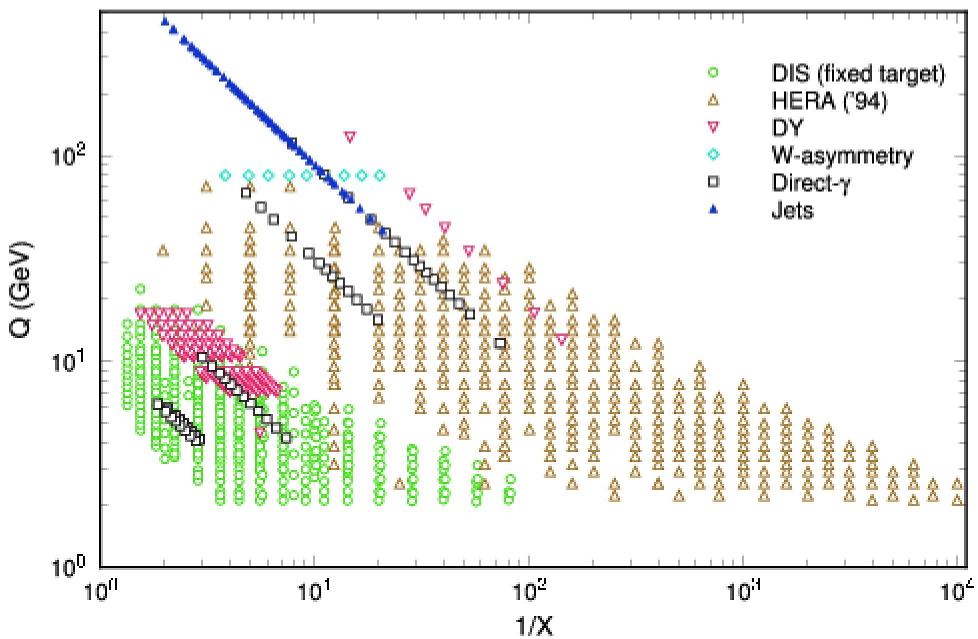
G. Wolf, P. Söding, S.L. Wu, B. Wiik (1979)

Proton Dynamics: Parton Distribution Functions (1)



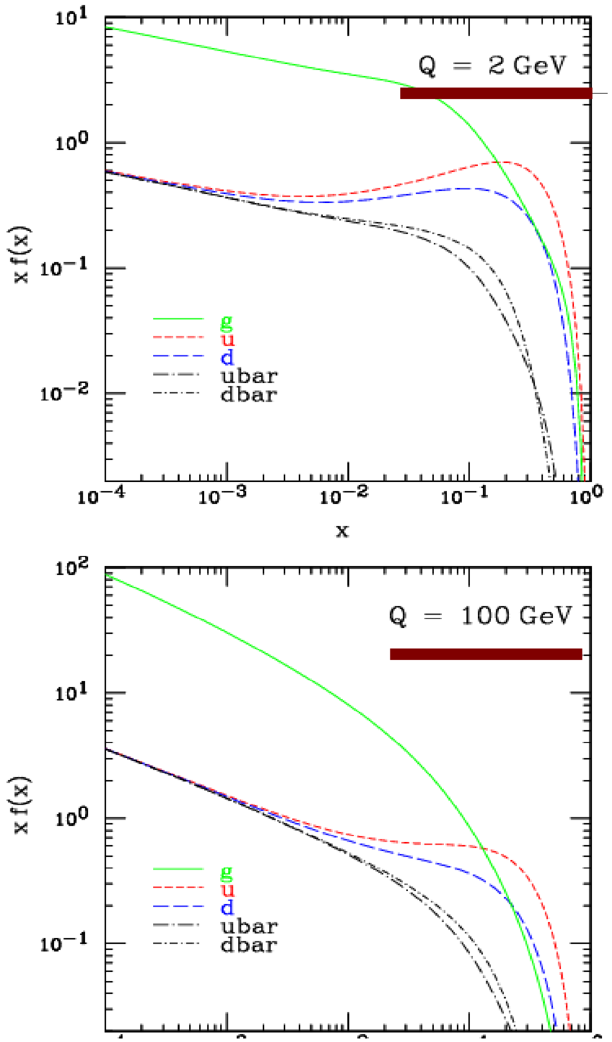
J. Pumplin's talk

Proton Dynamics: Parton Distribution Functions (2)



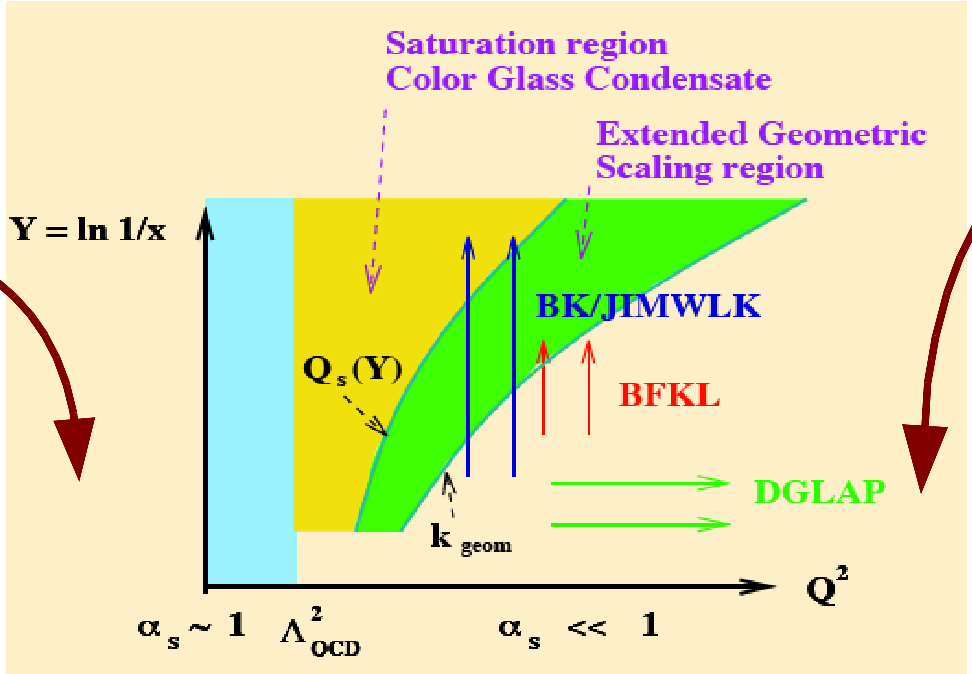
Kinematics covered by data

Parton distributions at $Q = 2$ and 100 GeV



Asymptotic Freedom

- <u>High momentum transfer</u> ⬌ Probe small distances "strong" coupling is "weak": perturbative QCD (DGLAP)
- <u>Low momentum transfer</u> ⬌ Low spatial resolution
  Relativistic, strongly coupled, many-body problem



How does one proceed? "A Flow Chart"

Theoretical tools:

- Lattice QCD
- EFT (Large N_c, χPT,..)
- AdS/CFT

Phenomenology

Devise observables and experimental probes

Experiments: measure and compare with theory

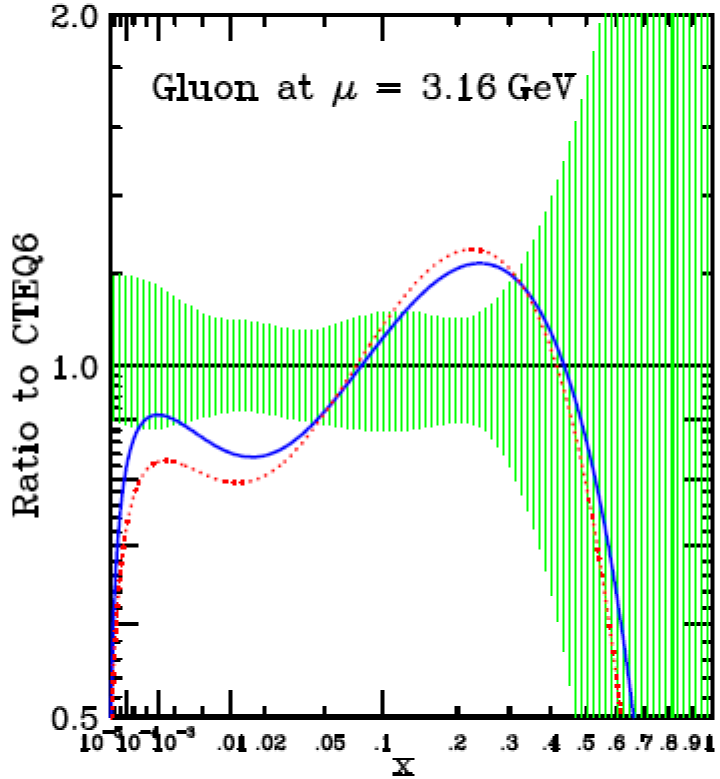
Validation

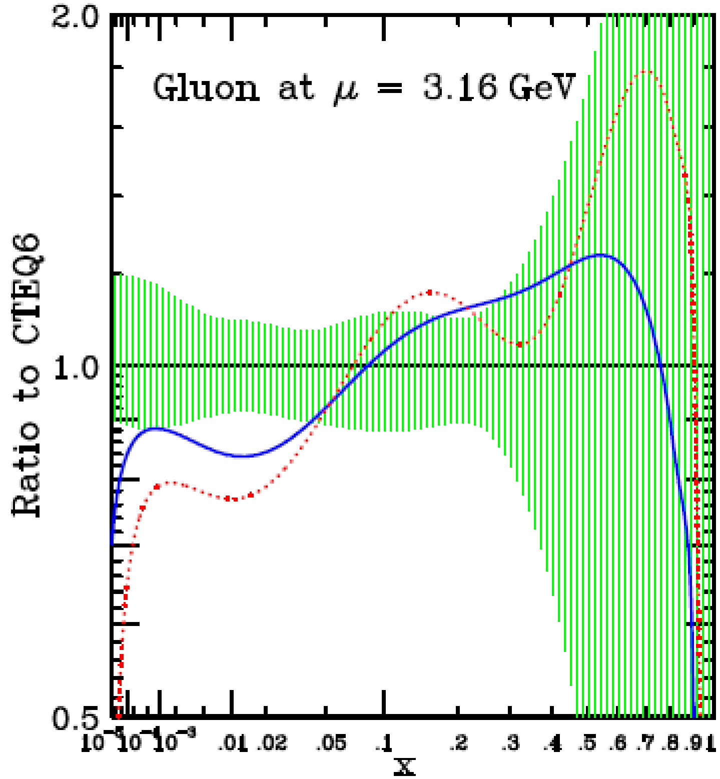
2. Outstanding Questions

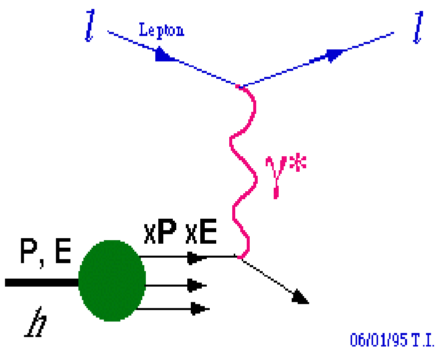
Outstanding Questions arise through Validation

- What are the momentum distributions of quarks, anti-quarks and gluons ?
- How is the flavor symmetry broken ? $\bar{u} \neq \bar{d}, \quad s \neq \bar{s}$
- How do partons carry spin ½ of proton ?
- Longitudinal vs. transverse spin difference?
- Spatial distribution of quarks?
- Transition from partonic to hadronic d.o.f.: how are quarks and gluons correlated?
- How do protons and neutrons form atomic nuclei ?

Example 1: momentum distributions







Fundamental input for the LHC !

Example 2: Proton's Spin Structure

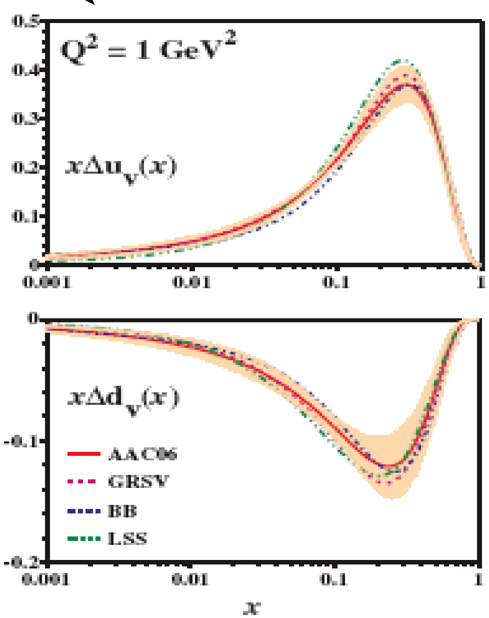
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

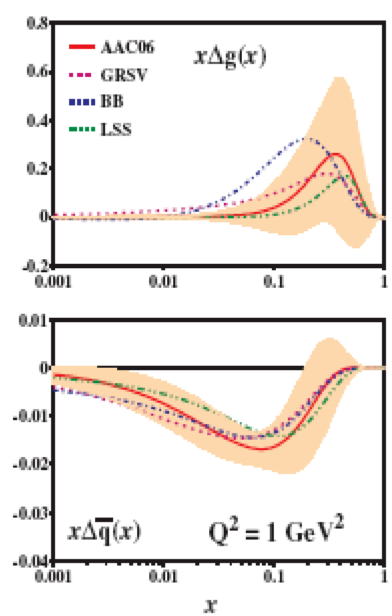
QUARKS SPIN

GLUONS SPIN

Q & G
ORBITAL ANGULAR MOM.

30 %





$\Delta g \approx 0$

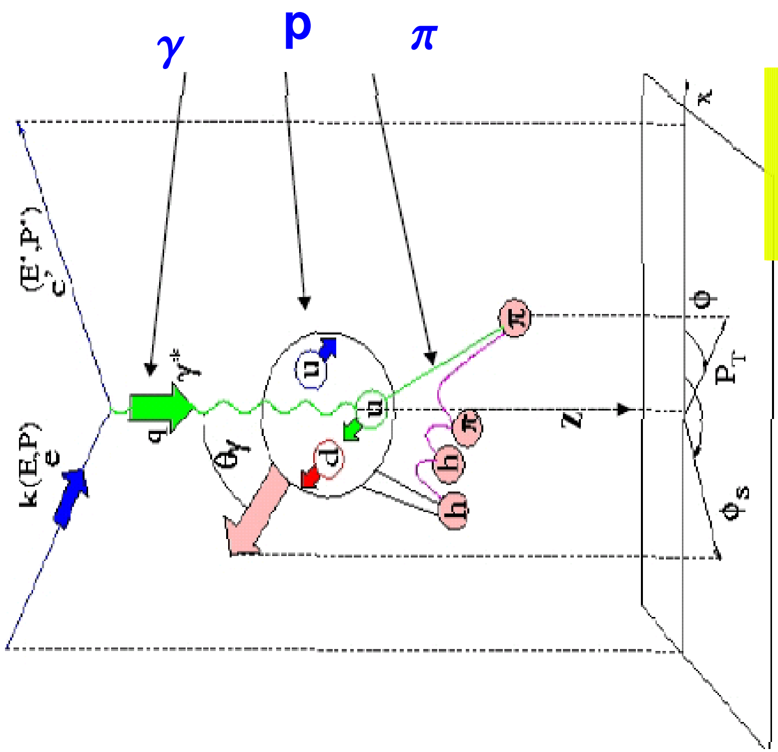
?

Hirai, Kumano, Saito, PRD 74(2006)

Proton Spin Crisis fostered searches for "different" observables

- Transversity (Goldstein & Moravscik, Jaffe, Ji, …)
- Orbital angular momentum (Ji, …)

"New type" of experiments: from inclusive to semi-inclusive …



Semi-inclusive DIS: Transversity

$$ep^{\uparrow} \to e'\pi X$$

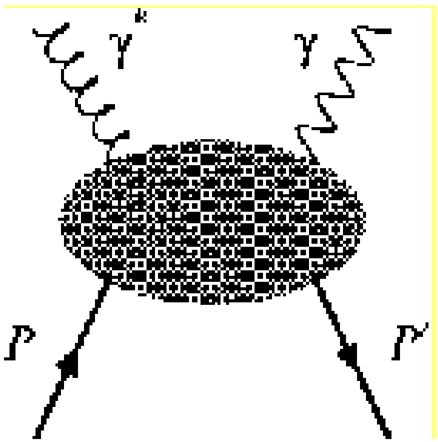
$$ep^{\uparrow} \to e'\Lambda' X$$

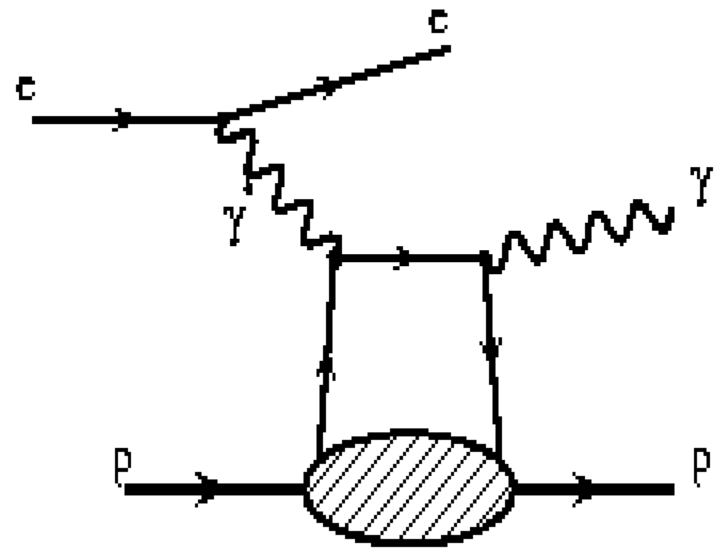
$$ep^{\uparrow} \to e'\pi\pi X$$

Observable =

azymuthal dist. of pions

... *and from* *semi-inclusive* *to* *exclusive* ...





a)

Virtual Compton Scattering

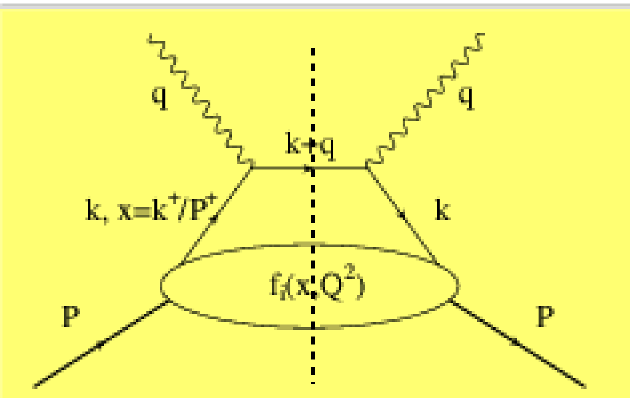
Deeply Virtual Compton Scattering

Bjorken limit

3. DVCS: new dimensions in proton studies

DVCS and Generalized Parton Distributions 1

Deep inelastic Scattering

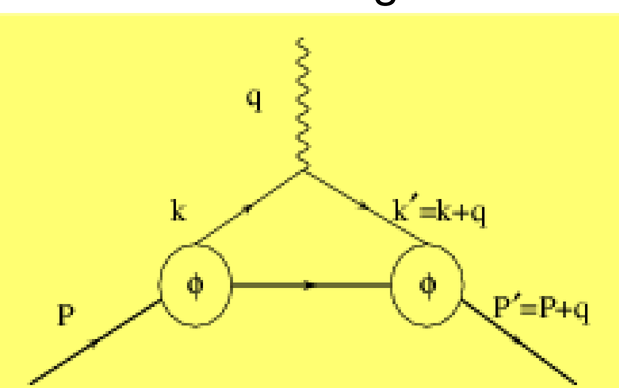


$$P^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P,S \mid \psi\left(-\frac{\xi^-}{2}\right)\gamma^+\psi\left(\frac{\xi^-}{2}\right) \mid P,S\rangle = \bar{u}(P,S)\gamma^+ u(P,S) f(x)$$

Extract Parton Distribution

Elastic Scattering

Extract Form Factors



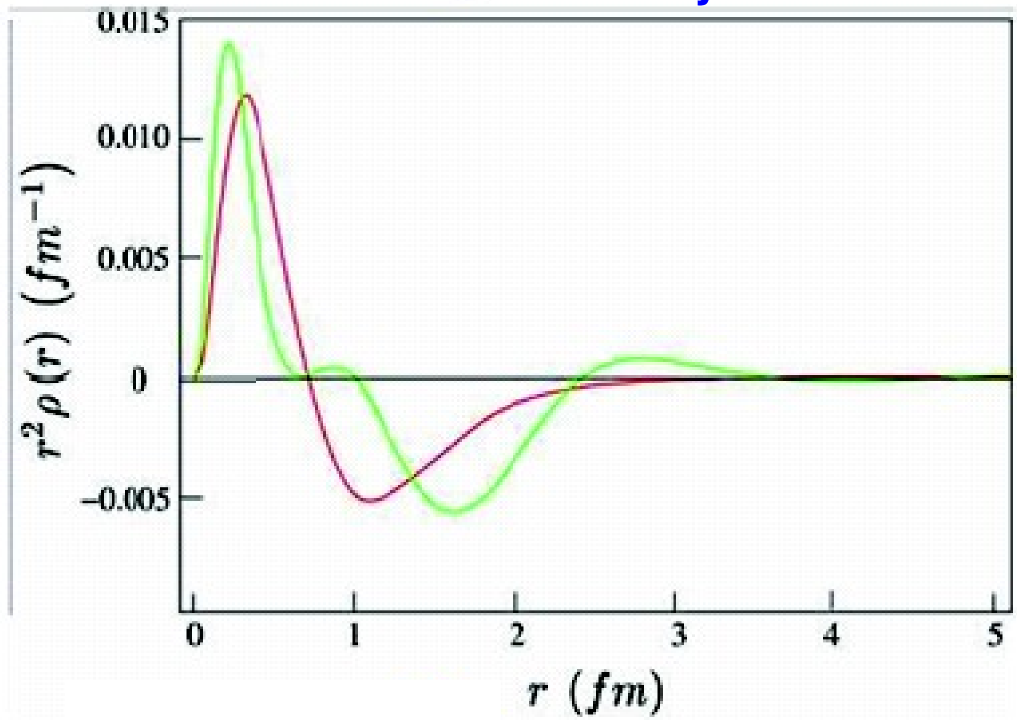
$$\langle P',S' \mid \psi(0)\gamma^+\psi(0) \mid P,S\rangle = \bar{u}(P',S')\left[\gamma^+ F_1(Q^2) + \frac{i\sigma^{+\nu}q_\nu}{2M} F_2(Q^2)\right] u(P,S)$$

DVCS and Generalized Parton Distributions 2

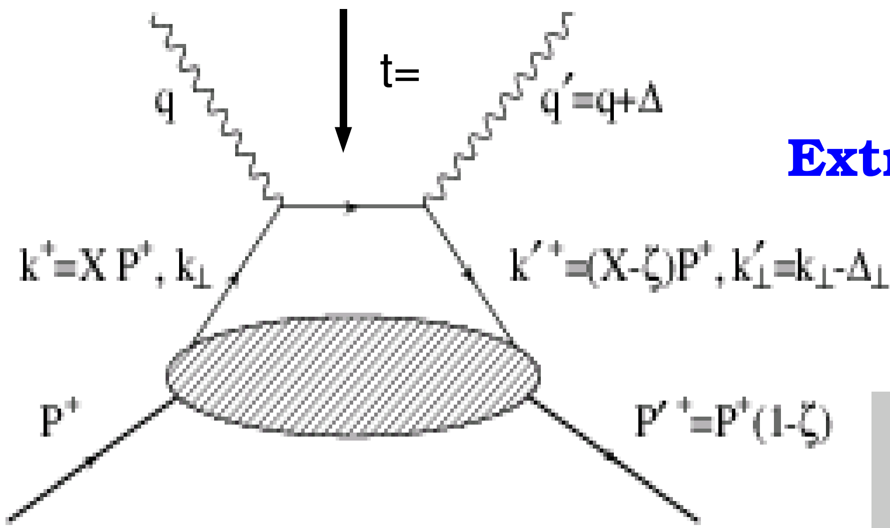
Breit Frame interpretation of the form factor (Sachs):

In Breit Frame a “non relativistic” picture is valid and the Fourier transform of the electric form factor gives the nucleon's density distribution

neutron density dist.



DVCS and Generalized Parton Distributions 3



Extract "Generalized Parton Distributions"

$$\bar{P}^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P', S' | \psi\left(-\frac{\xi^-}{2}\right) \gamma^+ \psi\left(\frac{\xi^-}{2}\right) | P, S \rangle =$$

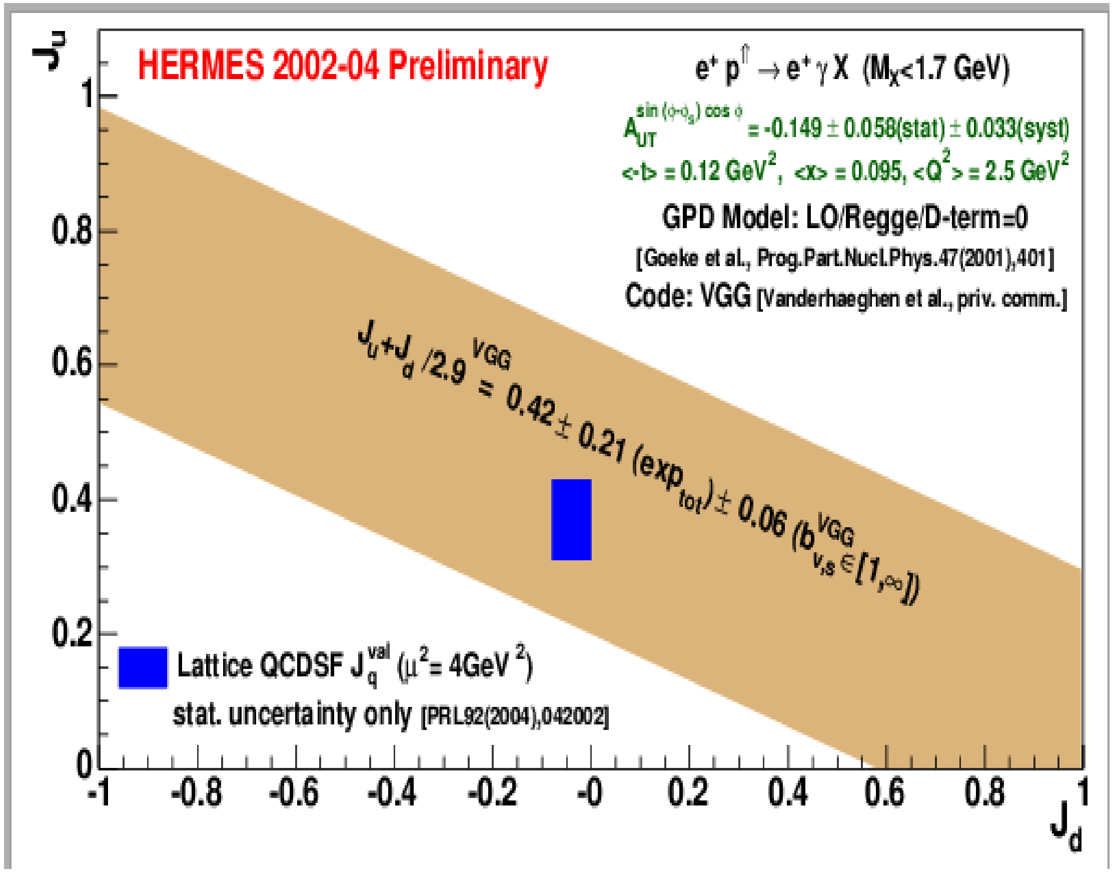
$$\bar{u}(P', S') \left[\gamma^+ H(x, \xi, -\Delta^2) + \frac{i\sigma^{+\nu} q_\nu}{2M} E(x, \xi, -\Delta^2) \right] u(P, S)$$

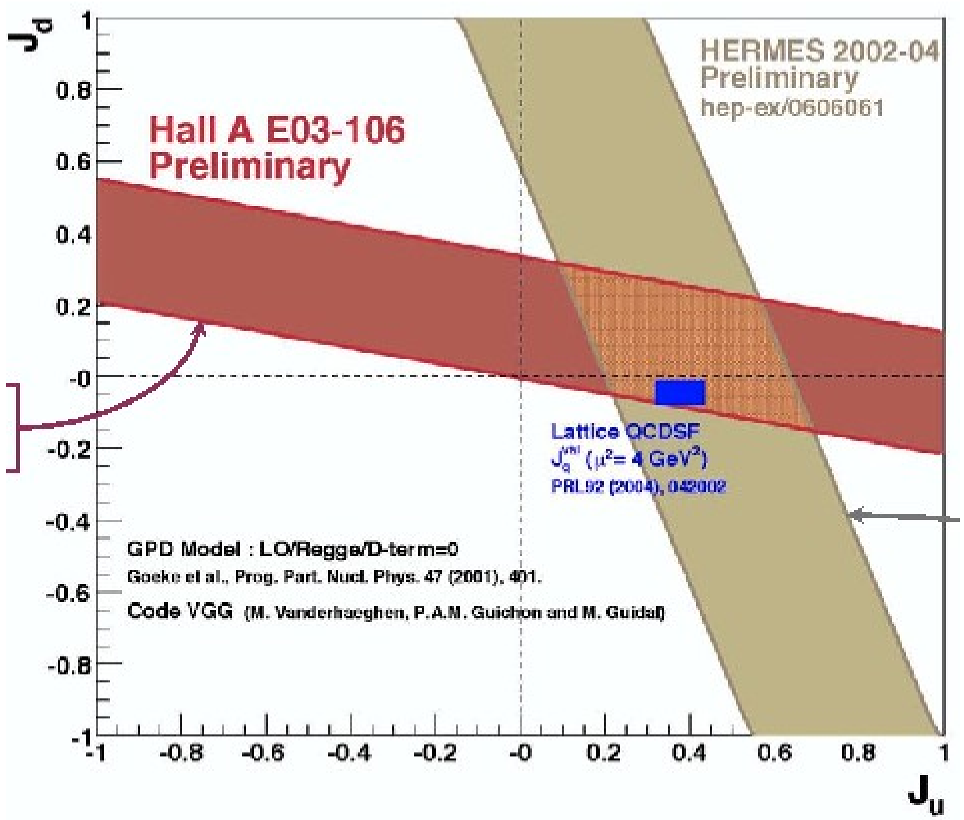
- **GPDs are hybrids of PDFs and FFs: describe simultaneously x and t-dependences !**
  - **GPDs give access to spatial d.o.f. of partons !**
    - **GPDs give access to orbital angular momentum of partons!**

$$\int dx H_q(x, \zeta, t) + E_q(x, \zeta, t) = 2J_q$$

X. Ji

Z. Ye, HERMES Collaboration, 2006





M. Mazouz, Ph.D. Thesis, Grenoble,2006

DVCS and Generalized Parton Distributions 4: Optics

Question #1: How do we interpret the spatial d.o.f. of partons?

Theoretical Ideas:

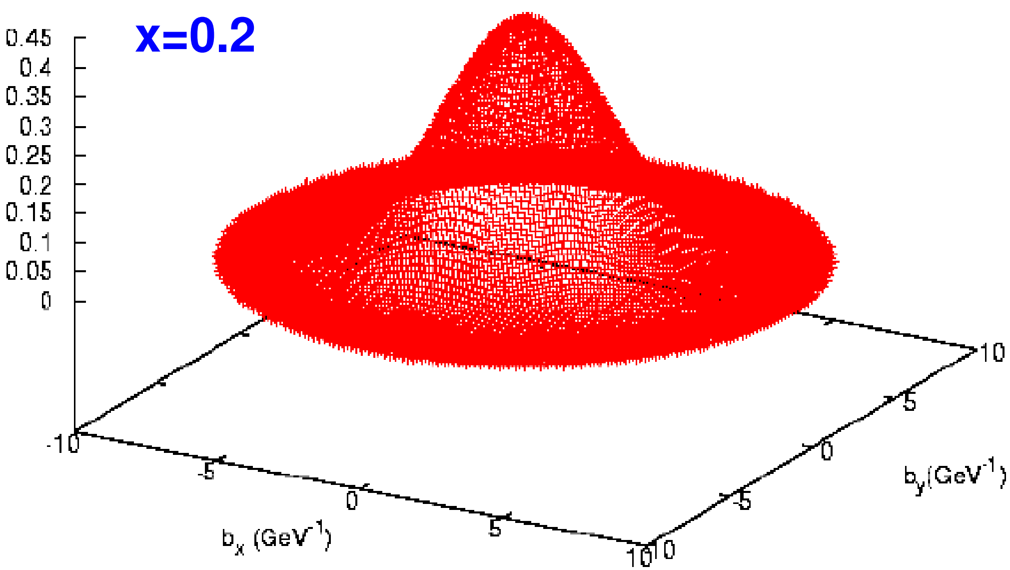
- Impact parameter dependent PDFs (M. Burkardt, 2000 $\longleftrightarrow$ D. Soper, 1977)
- Holography (Ralston and Pire, 2000)
- Interference patterns (Brodsky et al., 2006)
- Wigner Distributions (Belitsky, Ji, Yuan, 2004)

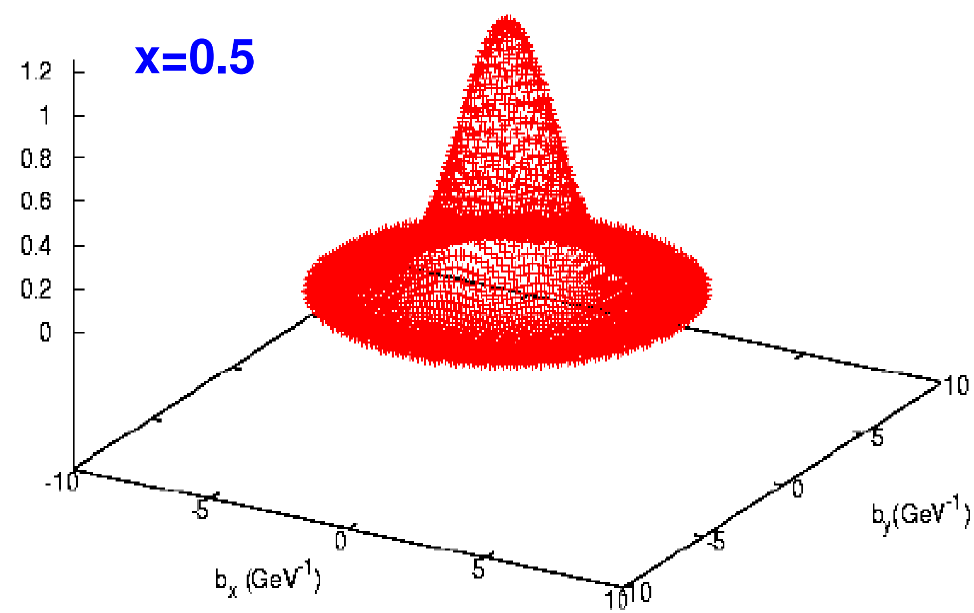
DVCS and Generalized Parton Distributions 5: IPPDFs

$\zeta = 0$

$$q(x, \mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{\Delta}} H_q(x, 0, -\mathbf{\Delta}^2)$$

$$\langle \mathbf{b}^2(x) \rangle = \mathcal{N}_b \int d^2\mathbf{b}\, q(x, \mathbf{b})\, \mathbf{b}^2$$





H_q (x,t) from Ahmad, Honkanen, S.L., Taneja (2006)

DVCS and GPDs 6: Wigner Distributions

$\zeta \neq 0$

"Quantum Phase-Space" distributions (Wigner, 1932): f(**p**,**r**)

Not positive-definite because of uncertainty principle

Become positive in classical limit

Vast literature – observable (!) in atomic systems

How do we generalize to relativistic systems/physics on the light-cone?

Belitsky, Ji, Yuan: **Breit Frame**

$$\rho_+(\vec{r}, x) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} [H(x,\xi,t) - \tau E(x,\xi,t)]$$

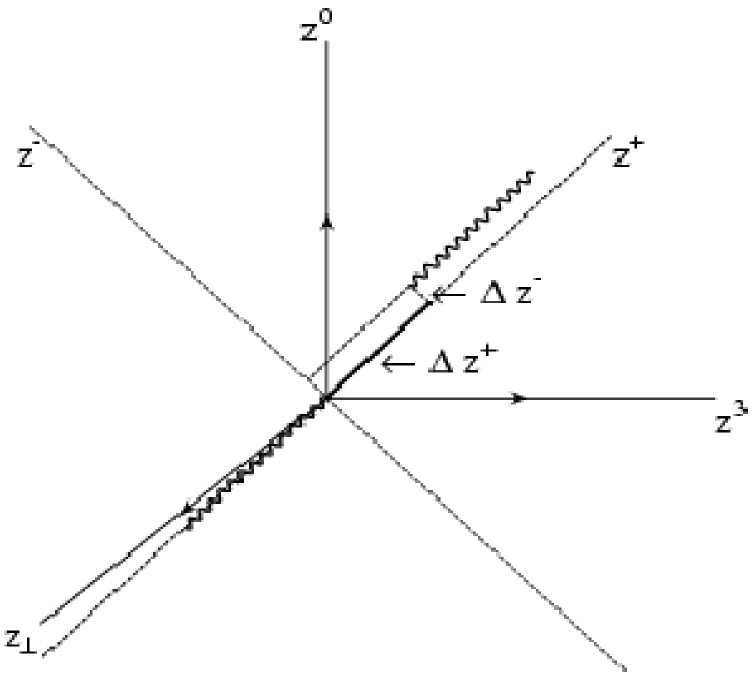
Phase-space Charge Density

$$j_+^z(\vec{r}, x) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} i[\vec{s}\times\vec{q}]^z \frac{1}{2M_N} [H(x,\xi,t) + E(x,\xi,t)]$$

Phase-space Convection Current

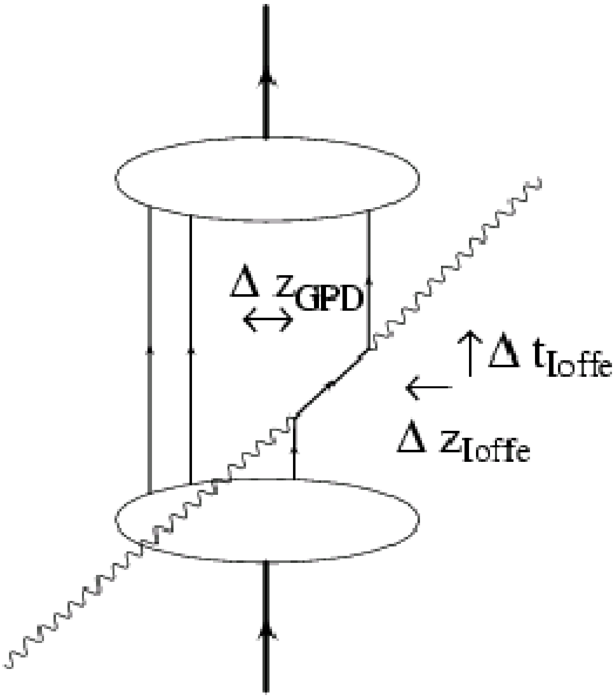
But: … "Ioffe time"

Deeply Virtual Compton Scattering in Coordinate Space



- Probe is local in $\perp$ direction:
  $\Delta z_\perp \approx 1/\sqrt{Q^2}$
- *Non-local* in $||$ direction:
  $\Delta z^+ \approx 1/M_N x_{Bj}$

Longitudinal variables



quark's *mobility* *(P. Hoyer)*

$$(\Delta z + \Delta t)_{\rm Ioffe} \equiv \Delta z^{+}$$

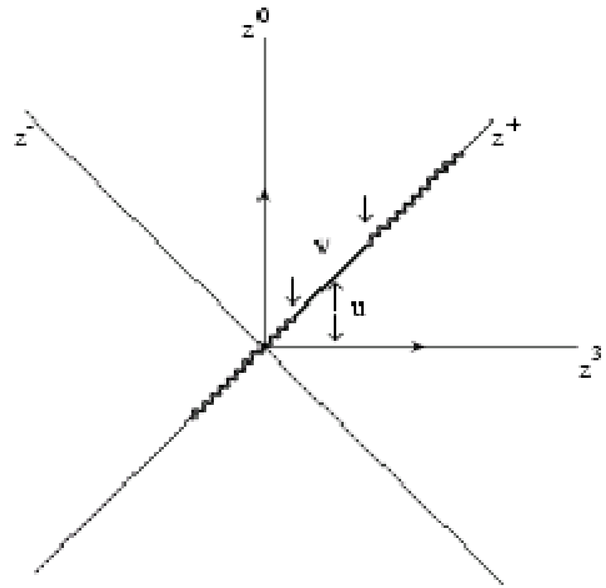
$$\Downarrow$$

We are aware of large Δz^{+} only through the observation of nuclear shadowing

$\Rightarrow$ Study the interplay between Δz^{+} and Δz_{GPD} in nuclei

Correlation Function in Coordinate Space

(work in progress...)



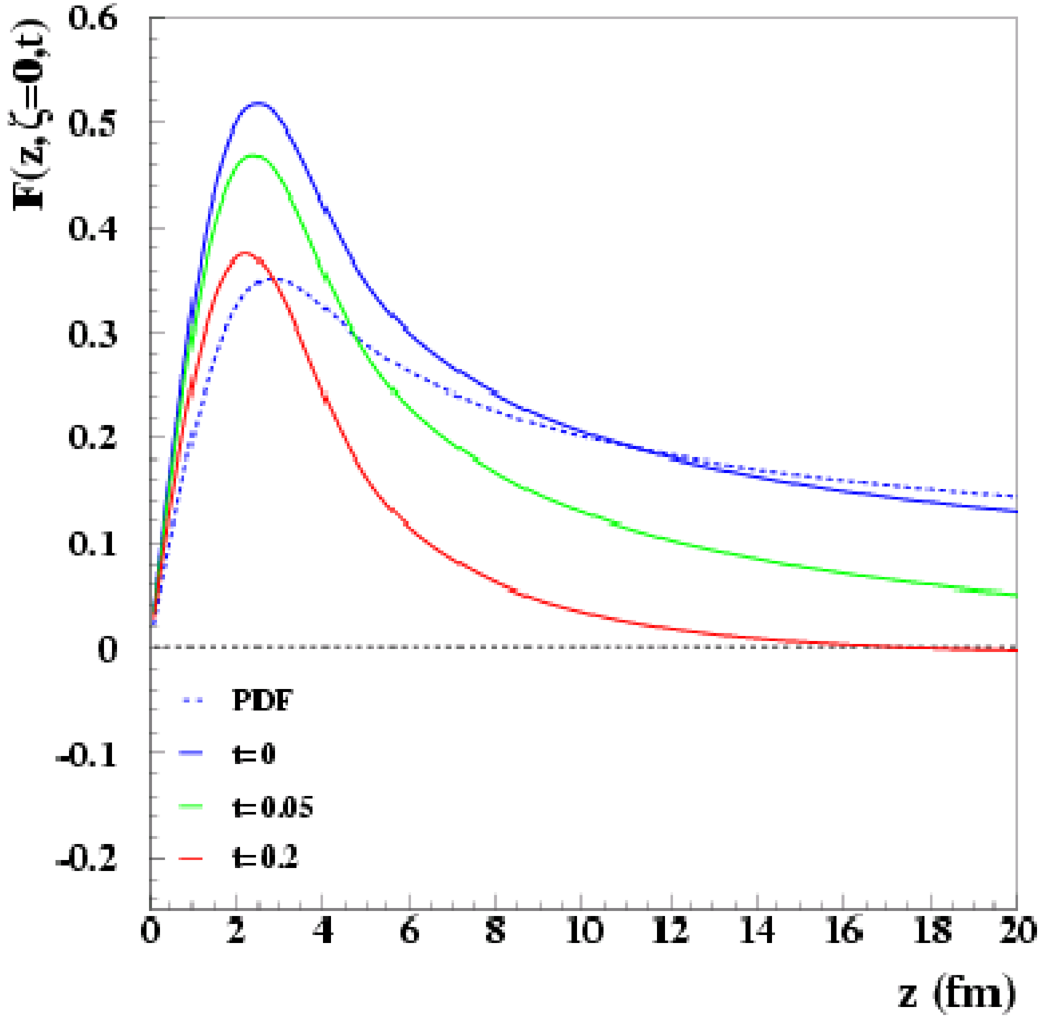
$$\langle P' \mid \overline{\Psi}\left(\frac{u+v}{2} z\right) \hat{z}\, \Psi\left(\frac{u-v}{2} z\right) \mid P\rangle_{z^2=0} = \overline{U}(P')\, \hat{z}\, U(P) \int d\zeta\, e^{iu\frac{\zeta}{2}(Pz)} F(vz, \zeta, t)$$

$$F(vz, \zeta, t) = \int dX\, H(X, \zeta, t) e^{ivX(Pz)}$$

$$\zeta = (\Delta z)/(Pz)$$

$$X = (kz)/(Pz)$$

Generalized Ioffe Time Distribution



S. Ahmad and S.L., preliminary

4. Extracting femtoimages requires new computational methods

DVCS and GPDs 7: Measurements

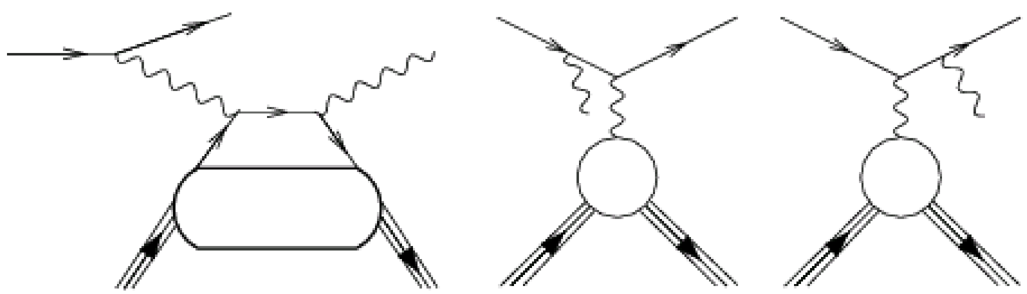
Question #2: How do we extract from exp. spatial d.o.f. of partons?

Direct measurements (BH interference)

- Observable given by Interference Term between DVCS (a) and BH(b):

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \propto sin\phi \left[F_1(\Delta^2)\mathcal{H} + \frac{x}{2-x}(F_1+F_2)\widetilde{\mathcal{H}} + \frac{\Delta^2}{M^2}F_2(\Delta^2)\mathcal{E} \right]$$

$$\mathcal{H} = \sum_q e_q^2 (H(\xi,\xi,\Delta^2) - H(-\xi,\xi,\Delta^2))$$



(a) (b)

A quantitative fitting procedure

Constraints from form factors

$$F_1(\Delta^2) = \int dx \sum_q e_q H_q(x, \xi, -\Delta^2 \equiv t)$$

Constraints from PDFs

$$q(x) = H_q(x, 0, 0)$$

Constraints from higher moments

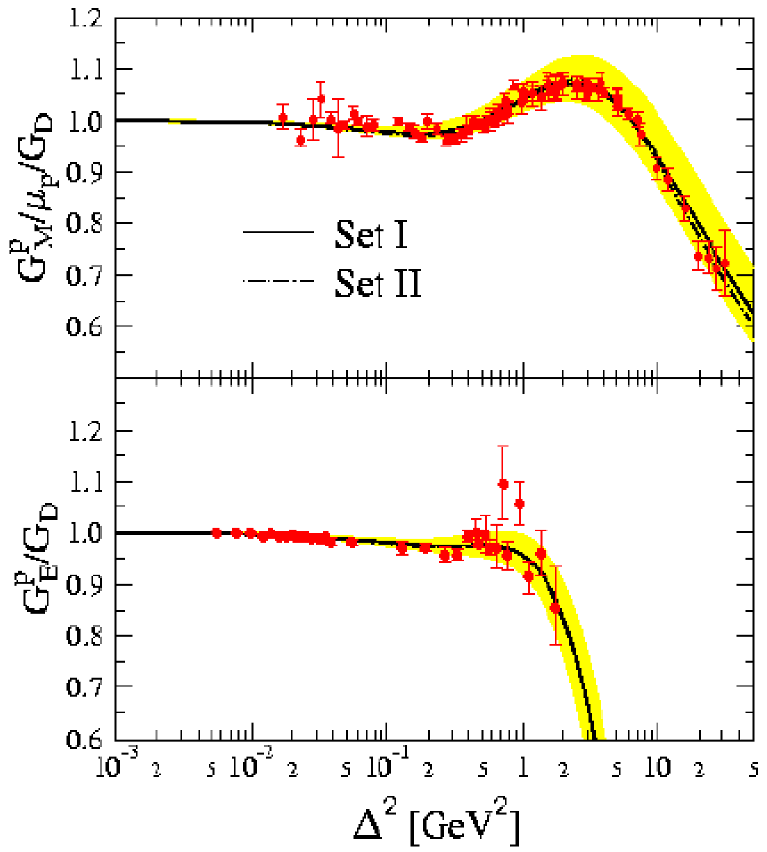
$$H_n^q(\zeta, t) = \int_0^1 dX X^{n-1} H^q(X, \zeta, t)$$

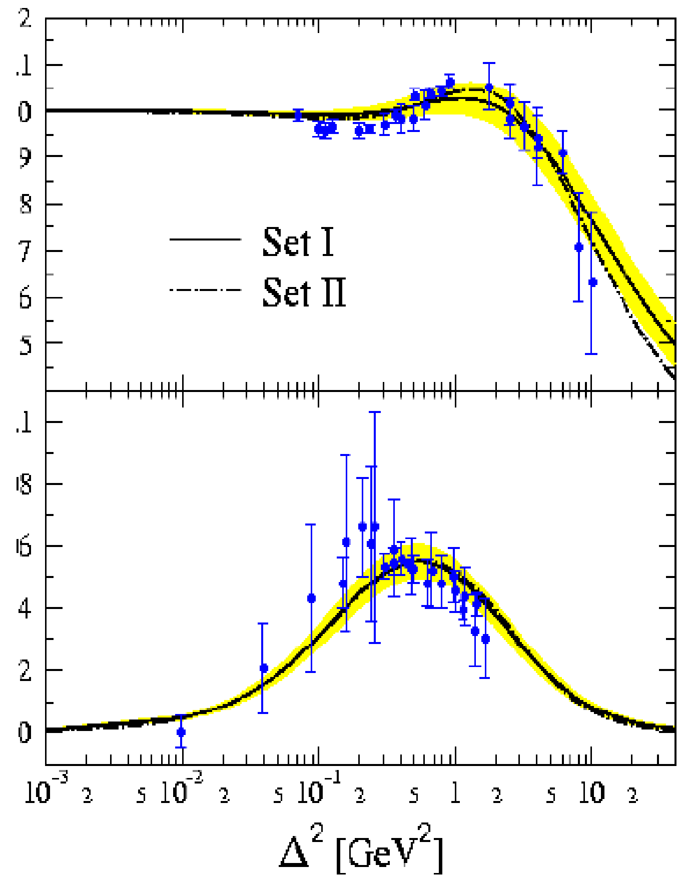
$$E_n^q(\zeta, t) = \int_0^1 dX X^{n-1} E^q(X, \zeta, t),$$

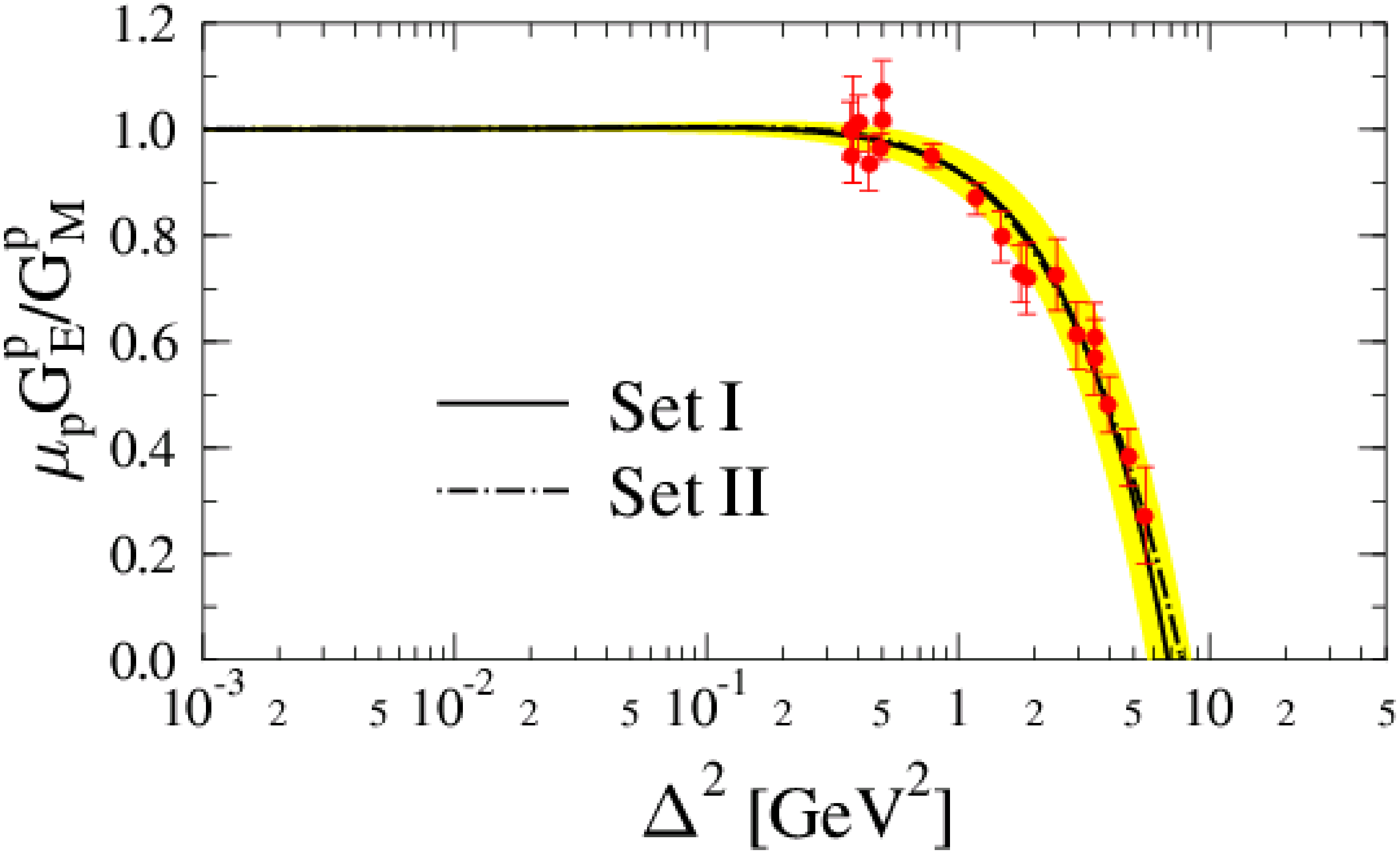
Perturbative evolution

GPDs from available data

$\zeta = 0$







$\mu_p G_E^p / G_M^p$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Set I
Set II
10^{-3} 2 5 10^{-2} 2 5 10^{-1} 2 5 1 2 5 10 2 5
Δ^2 [GeV^2]

GPDs from available data 2

Parton Distribution Functions

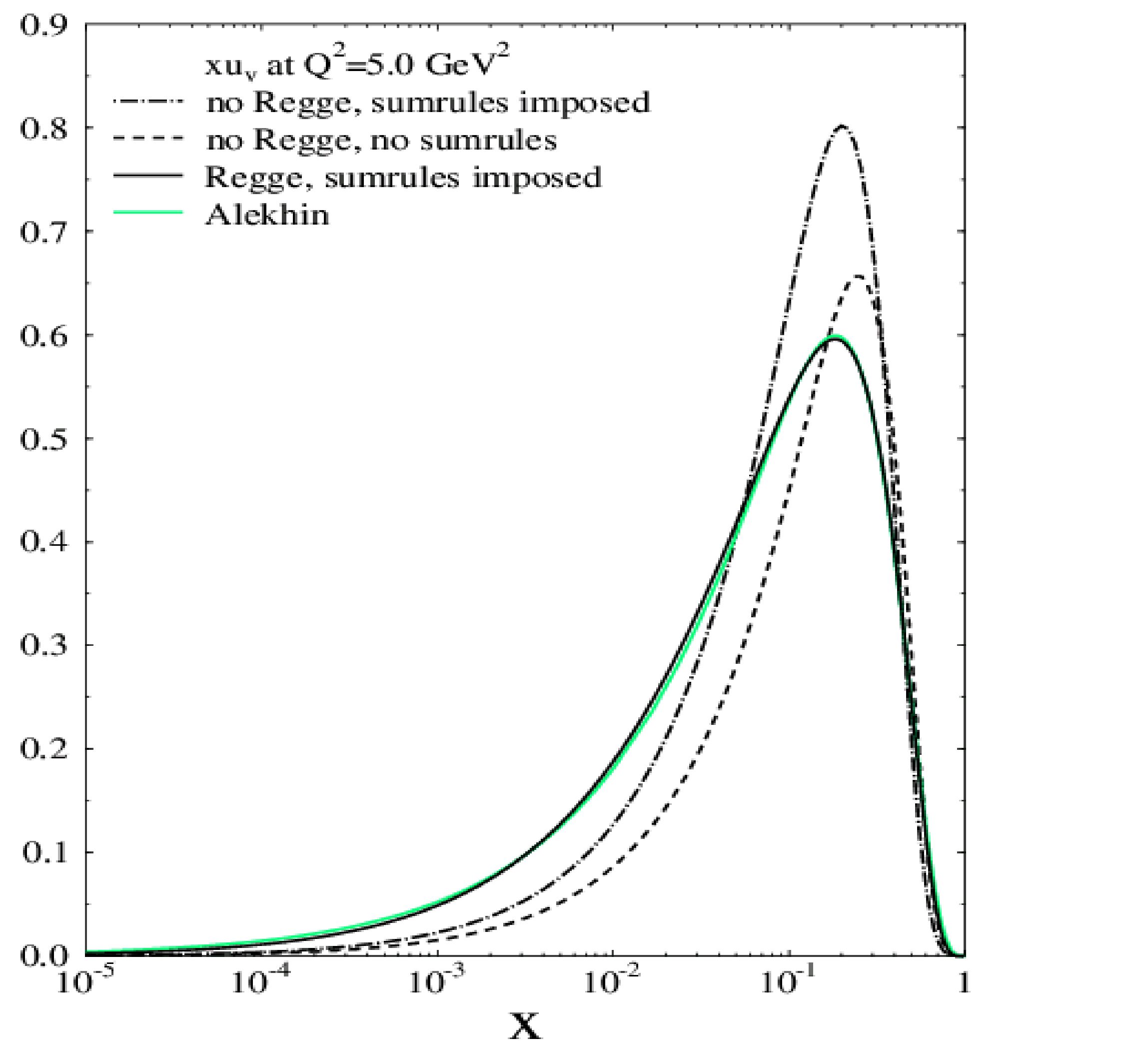
Notice! **GPD parametric form is given at $Q^2=\mu^2$ and evolved to Q^2 of data.**

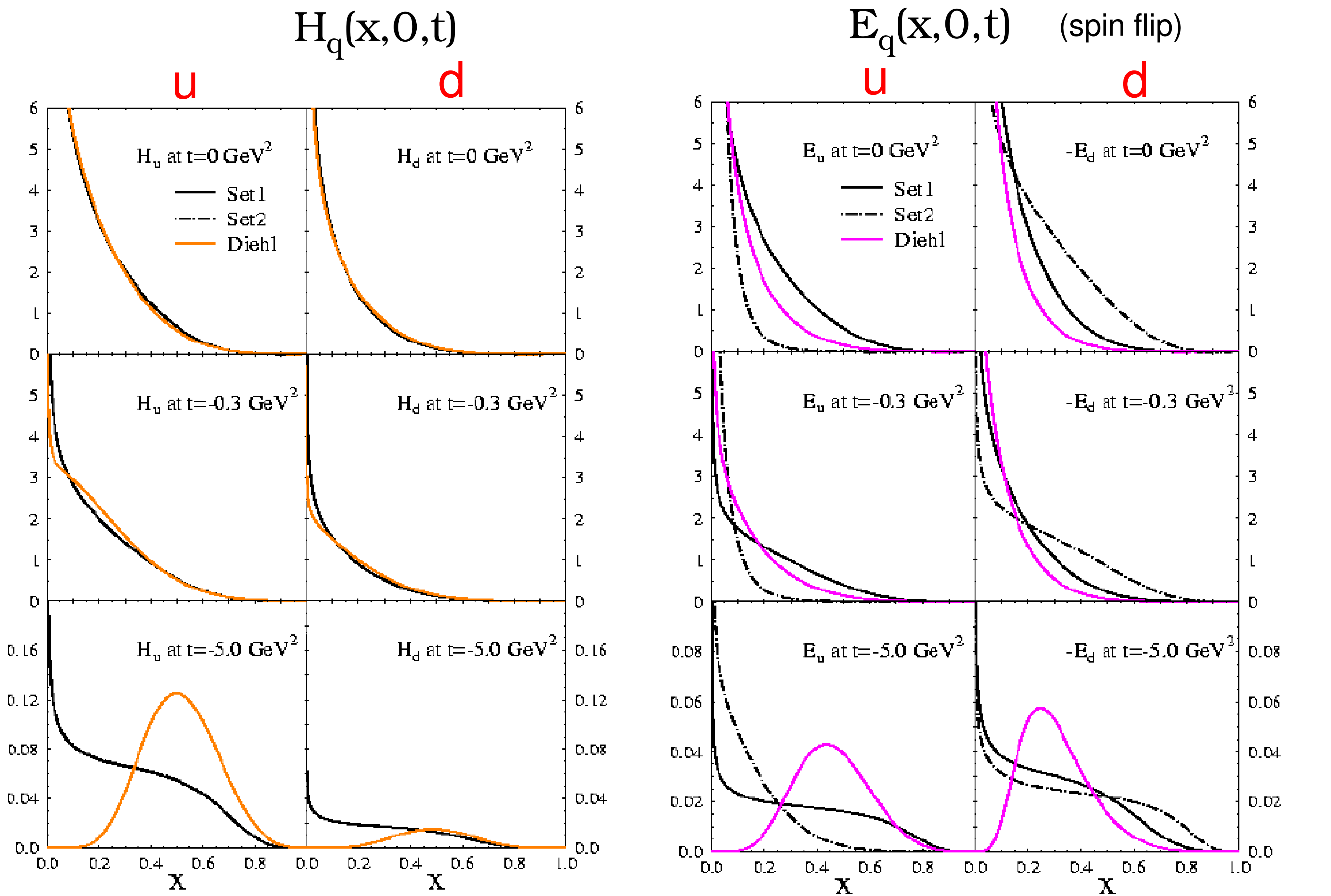
Notice! **We provide a parametrization for GPDs that simultaneously fits the PDFs:**

$$H_q(X,t) = R(X,t)\, G(X,t)$$

Regge

Quark-Diquark



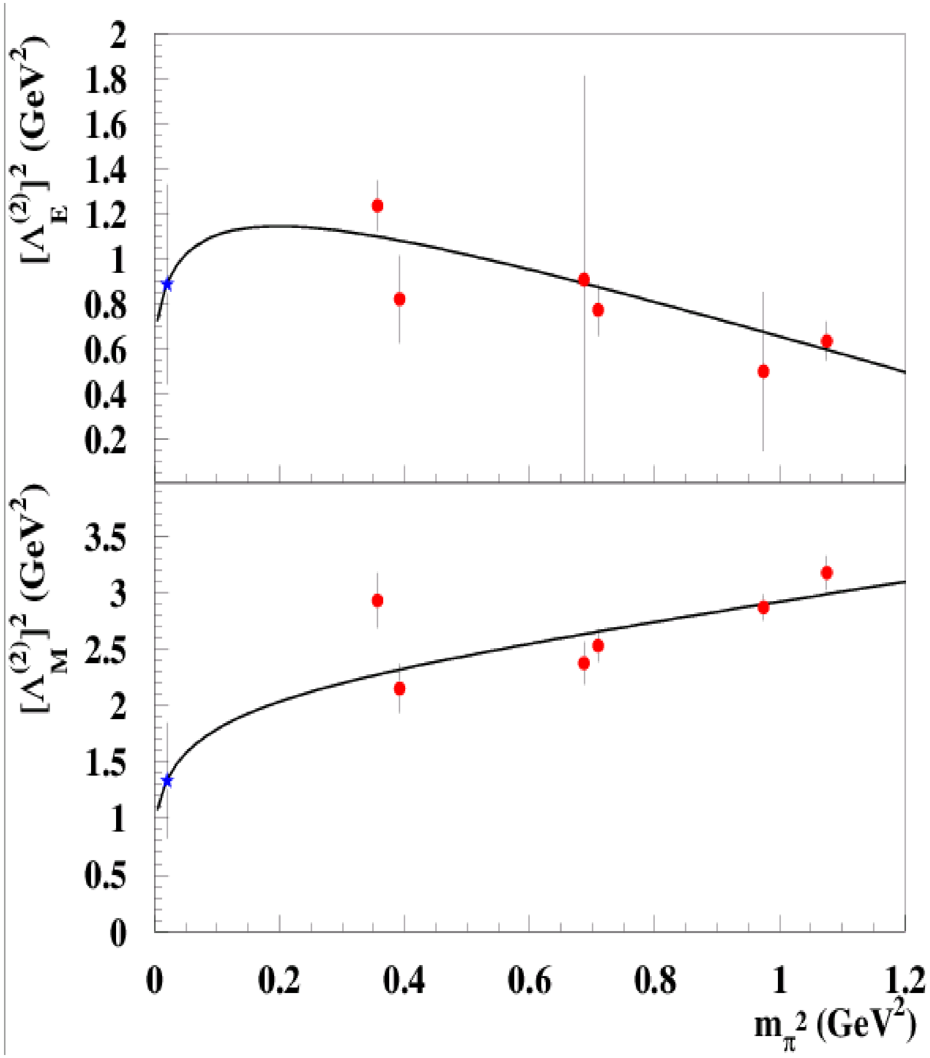


$H_q(x,0,t)$
$E_q(x,0,t)$ (spin flip)
u
d
u
d
H_u at t=0 GeV^2
H_d at t=0 GeV^2
H_u at t=-0.3 GeV^2
H_d at t=-0.3 GeV^2
H_u at t=-5.0 GeV^2
H_d at t=-5.0 GeV^2
E_u at t=0 GeV^2
$-E_d$ at t=0 GeV^2
E_u at t=-0.3 GeV^2
$-E_d$ at t=-0.3 GeV^2
E_u at t=-5.0 GeV^2
$-E_d$ at t=-5.0 GeV^2
Set1
Set2
Diehl
x

$\zeta \neq 0$

Use information from Lattice QCD GPD moments:

(1) chiral extrapolate dipole masses



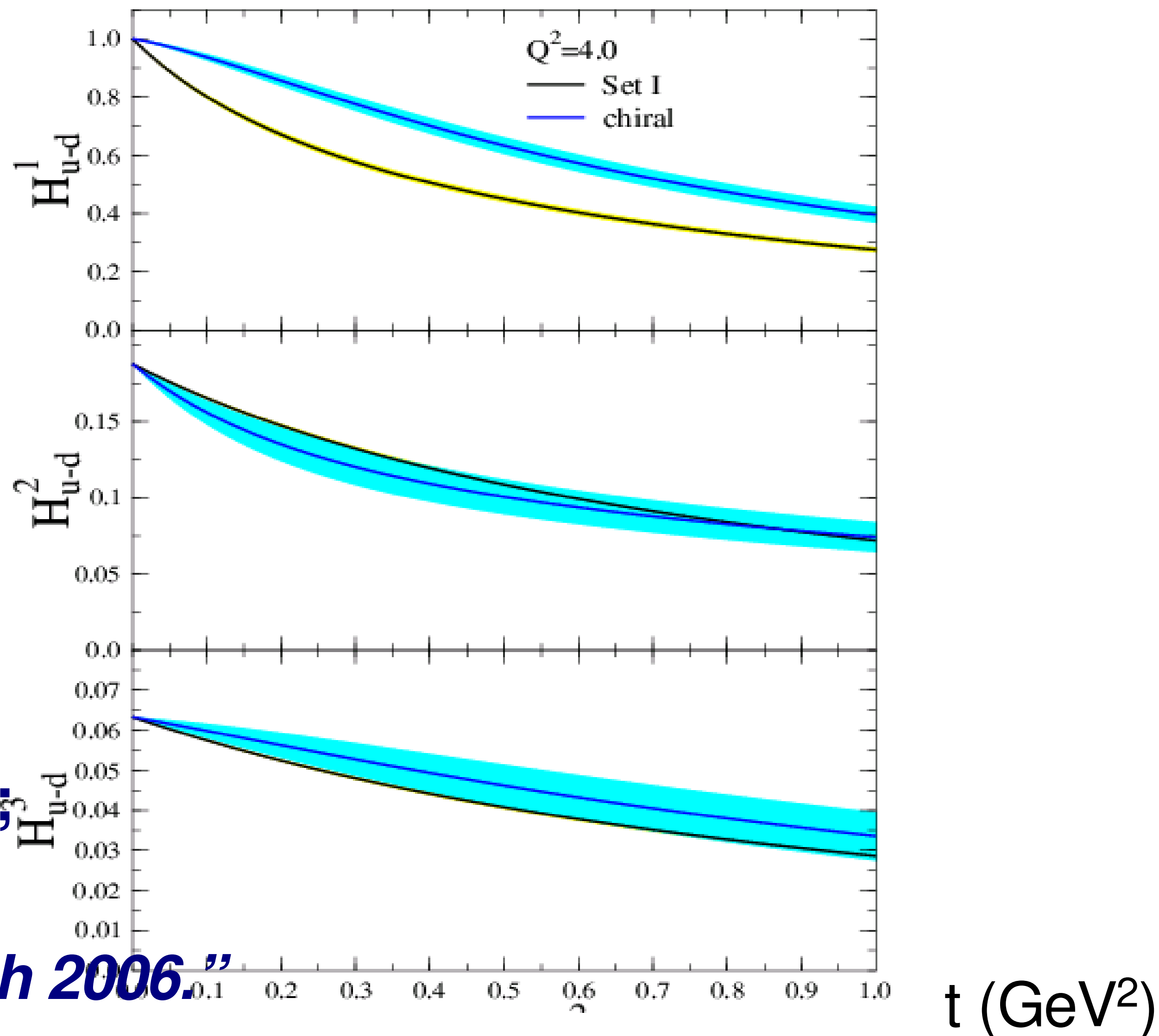
$\zeta \neq 0$

Use information from Lattice QCD:

(1) chiral extrapolate dipole masses

Dipole fit

$$G(Q^2) = \frac{G(0)}{(1 + Q^2/\Lambda^2)^2}$$



Lattice results from:

M. Gockeler et al. (2006);

J. Zanotti: hep-ph/0501209;

and *"44th Winter School in Schladming, Austria, March 2006."*

Isovector moments

n=1

$$H_1^{u-d} \equiv \int dX (H^u - H^d) = \frac{\tau G_M^V + G_E^V}{1+\tau}$$

$$E_1^{u-d} \equiv \int dX (E^u - E^d) = \frac{G_M^V - G_E^V}{1+\tau}.$$

"any" n

$$H_n^{u-d} \equiv \int dX X^{n-1} (H^u - H^d) = \frac{\tau (H_M^V)_n + (H_E^V)_n}{1+\tau}$$

$$E_n^{u-d} \equiv \int dX X^{n-1} (E^u - E^d) = \frac{(E_M^V)_n - (E_E^V)_n}{1+\tau},$$

Isovector dipole masses

(n=2)

$$\langle r^2\rangle_M^v \sim \frac{\chi_1}{m_\pi} + \chi_2 \ln\left(\frac{m_\pi}{\mu}\right).$$

$$\chi_1 = \frac{g_A^2 m_N}{8\pi f_\pi^2 \kappa_v},$$

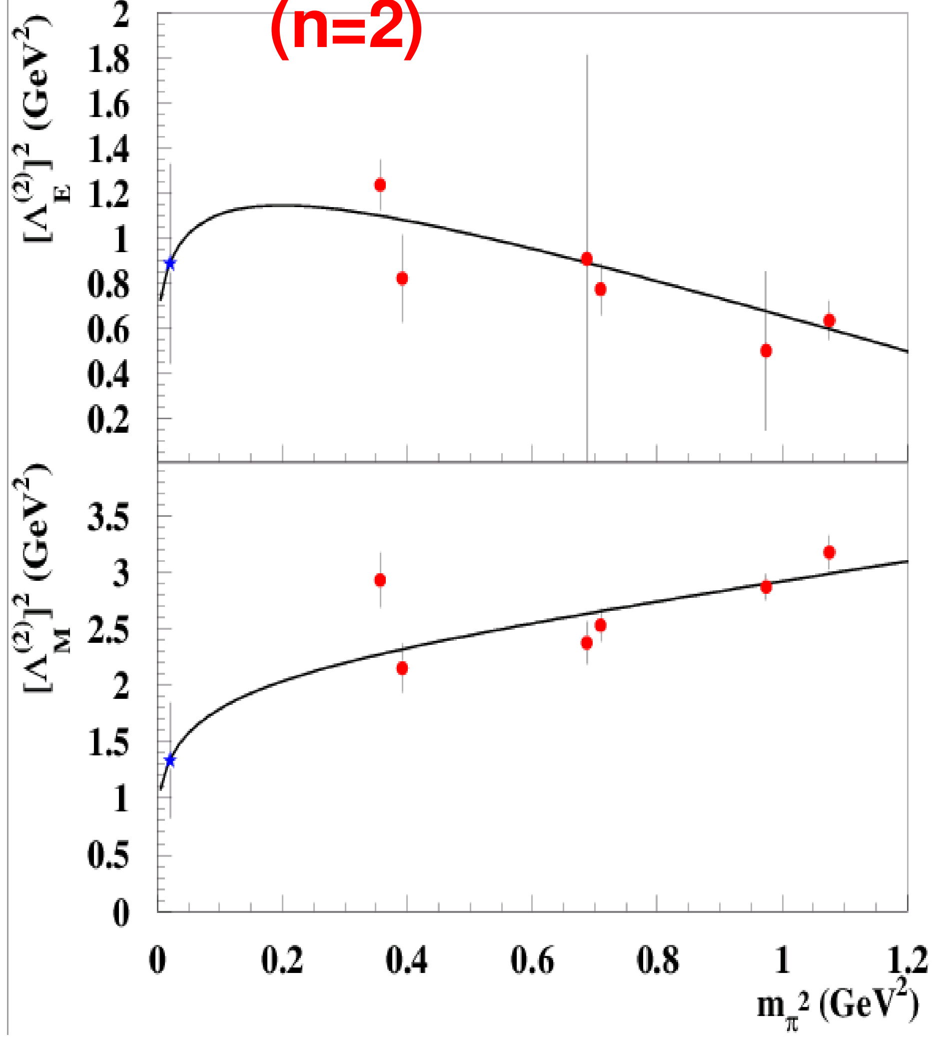
$$\chi_2 = -\frac{5g_A^2+1}{8\pi^2 f_\pi^2},$$

V. Bernard et al. (1995)

$$\langle r^2\rangle_M^v \sim \frac{\chi_1}{m_\pi}\frac{2}{\pi}\arctan(\mu/m_\pi) + \frac{\chi_2}{2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)$$

$$(\Lambda_M^v)^2 = \frac{12}{\langle r^2\rangle_M^v}.$$

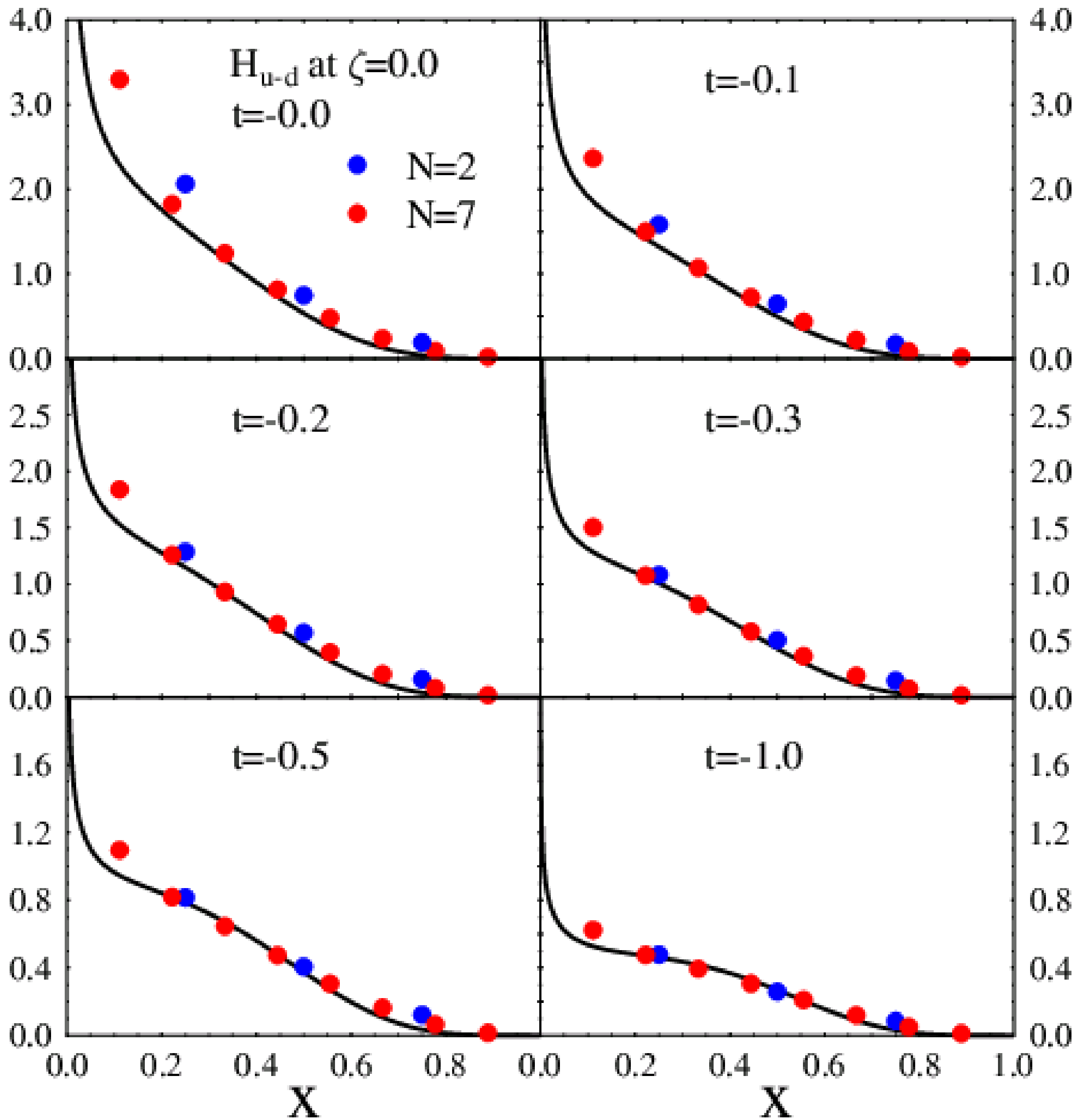
Ashley, Leinweber, Thomas, Young (2003)

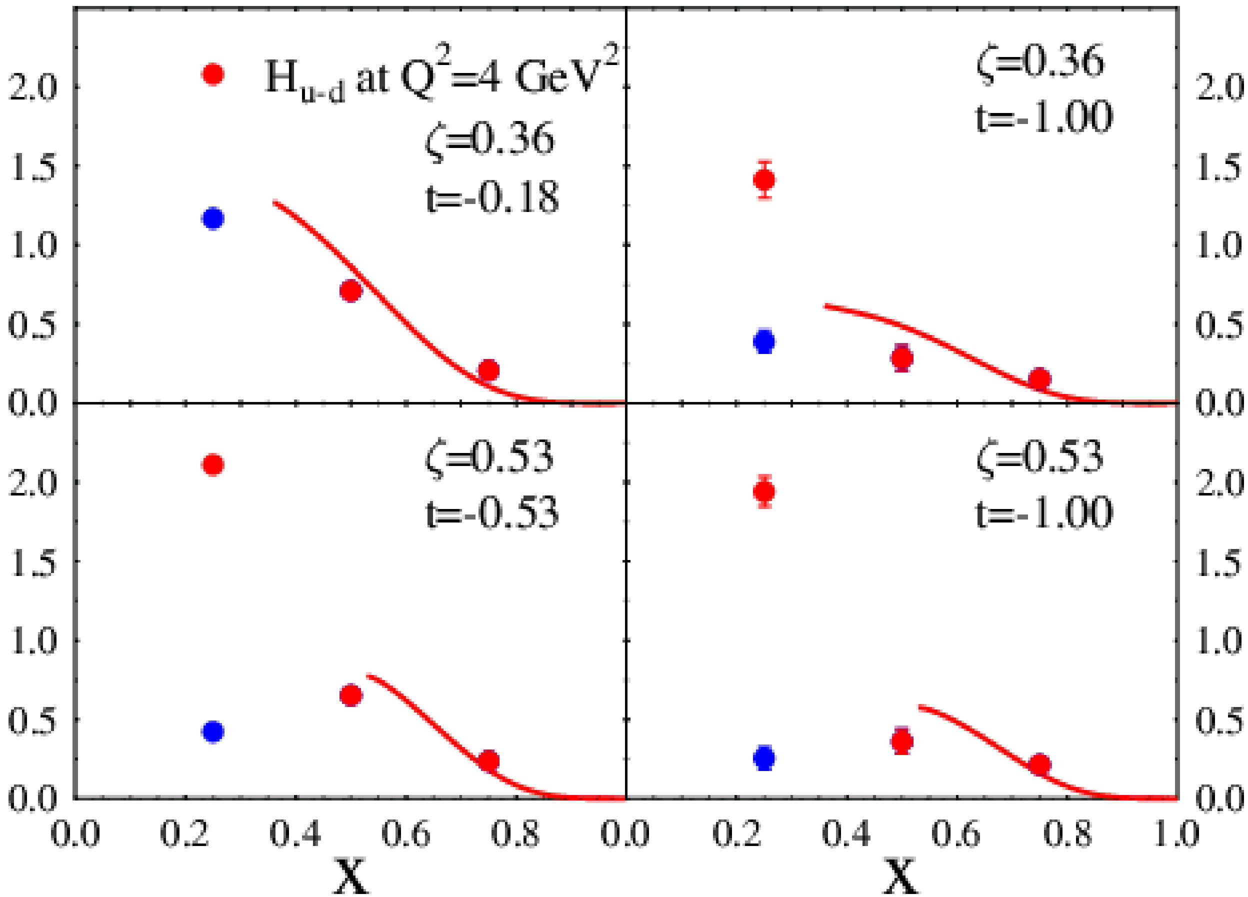


$$\left[(\Lambda^V_{M(E)})^2\right]_n = \frac{12(1+\alpha_n^{M(E)} m_\pi^2)}{\beta_n^{M(E)} + \gamma_n \ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)}$$

<u>Extension to higher moments</u>

(2) reconstruct GPD from its moments: Bernstein polynomials

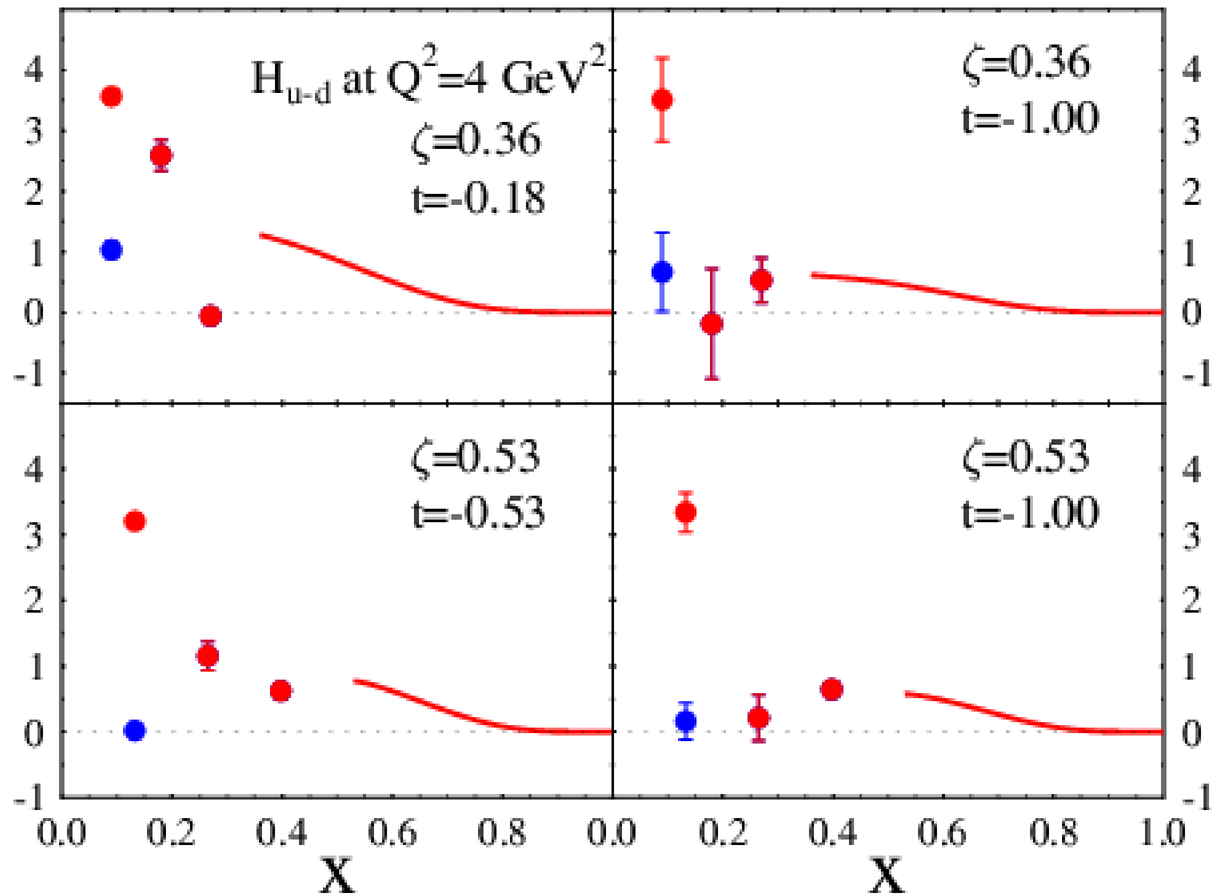




→ Reconstruct function in x=[0,1]

→ Kinematically Extend parameterization to x=[ζ,1]

→ Kinematic extension fits the moments well!

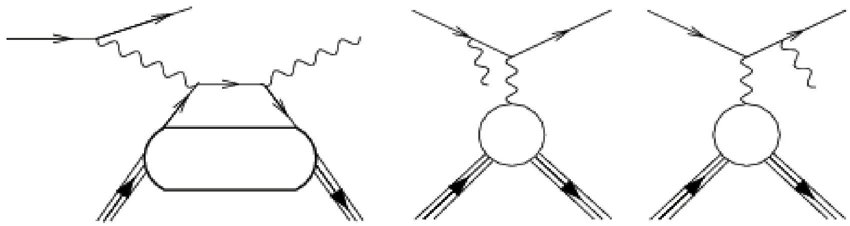


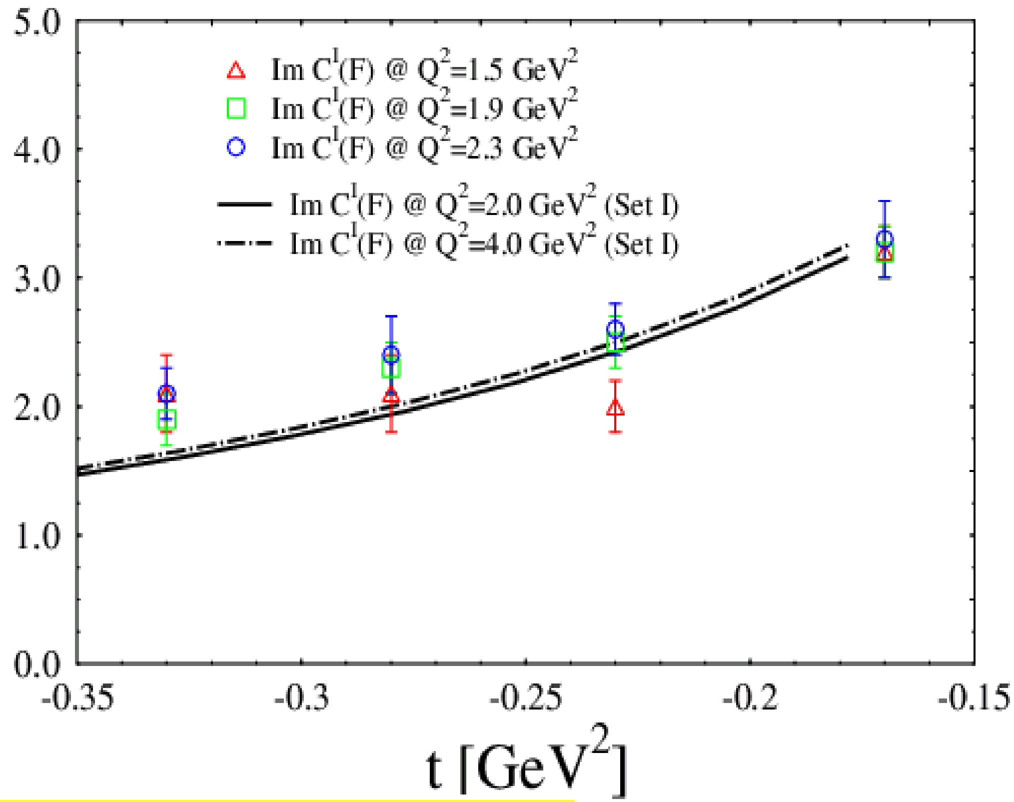
→ Form "subtracted" moments = [(total) - (x=[ζ,1] region)]

→ Reconstruct function in x=[0,ζ] !

→ **First model independent extraction of GPDs!!!**

Comparison with Jlab Hall A data



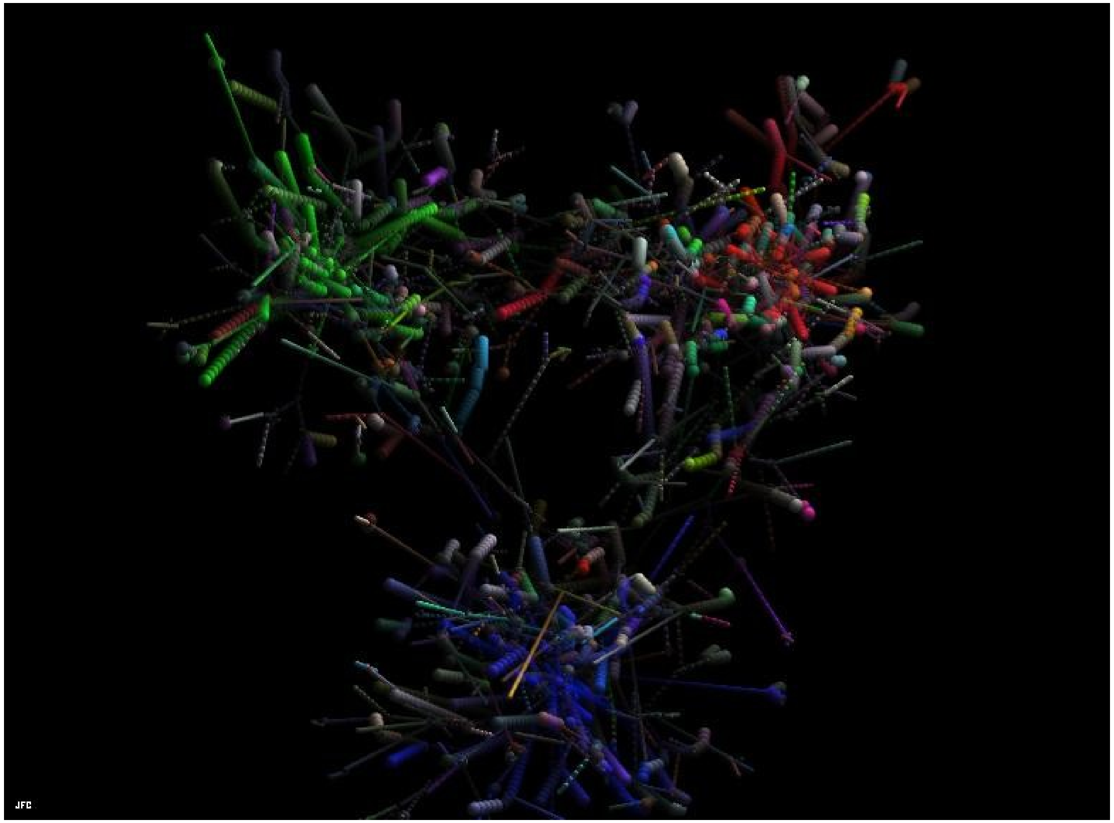


- Observable given by Interference Term between DVCS (a) and BH(b):

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \propto sin\phi \left[F_1(\Delta^2)\mathcal{H} + \frac{x}{2-x}(F_1+F_2)\tilde{\mathcal{H}} + \frac{\Delta^2}{M^2}F_2(\Delta^2)\mathcal{E} \right]$$

$$\mathcal{H} = \sum_q e_q^2 (H(\xi,\xi,\Delta^2) - H(-\xi,\xi,\Delta^2))$$

A new approach: Self-Organizing Maps



A rather large and diverse set of observations is produced that needs to be specifically detected, and compared to patterns predicted theoretically for different momentum, spin, and spatial configurations of the constituents.

Conventional approaches tend to explain such patterns in terms of miscroscopic properties of the theory → forces between two particles

Outline

- Introduction to Self-Organizing Maps
- Algorithm
- SOMPDFs: Results
- Comparison with conventional methods and NNPDFs
- Conclusions and Outlook

Self-Organizing Maps

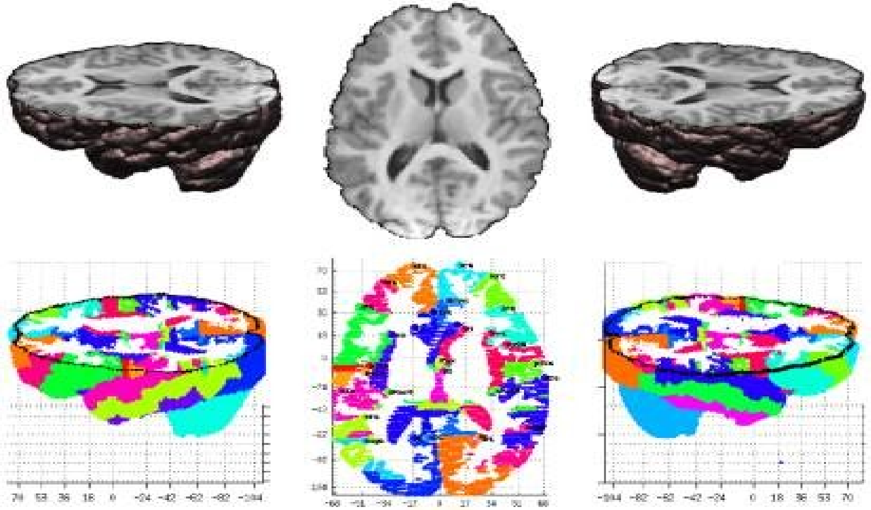
- Idea!   Attack the problem from a different perspective

- Study the behavior of multi-particle systems as they evolve from a large and varied number of initial conditions

- Goal can be reached with modern high performance computing

Self-Organizing Maps

Self-Organizing Maps (SOM) were derived as a mathematical model of these configurations (T. Kohonen, 1981)



Inspired by patterns in cerebral cortex: the detailed topographical order of the neural connections (synapses) form localized maps.

Brain maps are determined both genetically and by experience

“experience” = some projections – growth of axons of neural cells – are developed or stunted with respect to others, different cells are recruited for different tasks

Principles:

1) The neurons behave according to a form of
unsupervised self-organization

2) The representation of knowledge assumes the form of a map
geometrically organized over the brain so that
similar learning functions are associated to adjacent areas

2. Algorithm

Working of SOM

Each cell (neuron) is sensitized to a different domain of vectors: cell acts as decoder of domain

Initialization → Input vector of dimension "**n**" associated to cell "**i**":

$$V_i = \left[v_i^{(1)}, \ldots, v_i^{(n)}\right]$$

V_i is given spatial coordinates that define the geometry/topology of a 2D map

Training → Input data:

$$x = \left[\xi^{(1)}, \ldots, \xi^{(n)}\right]$$

x compared to V_i 's with "similarity" metric(L1):

$$|| x - m_i ||$$

(Aggawal et al., 2000)

Location of best match "winner" gives location of response (active cell, all others are passive)

Learning (updating) $\rightarrow$ cells $\mathbf{V_i}$ that are close up to a certain distance activate each other to "learn" from **x**

...in formulae:

$$V_i(t+1) = V_i(t) + h_{ci}(t)\left[x(t) - V_i(t)\right]$$

t = iteration

c = “winner” cell

i = cell

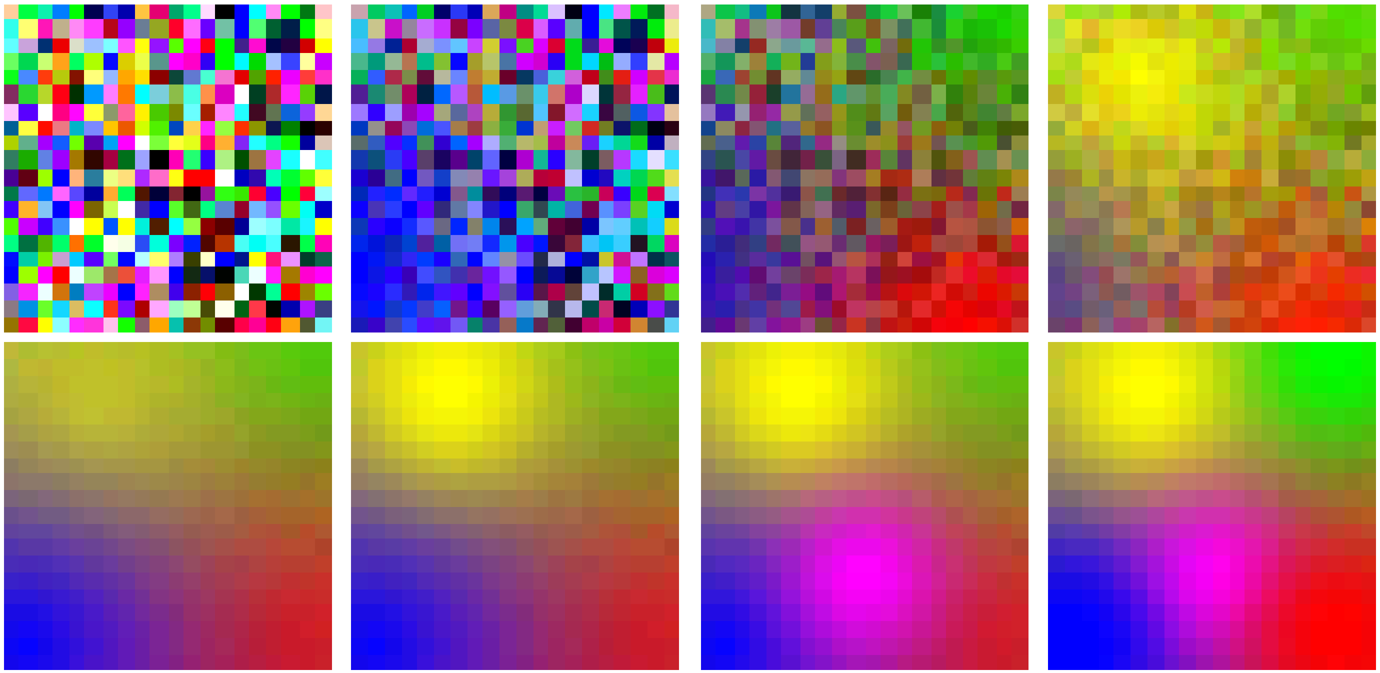
$$h_{ci}(t) \equiv h_{ci}(t, \| r_c - r_i \|)$$

$$h_{ci}(t) = w(t)e^{[-M(r_c,t)^2/r]}$$

0<w(t)<1

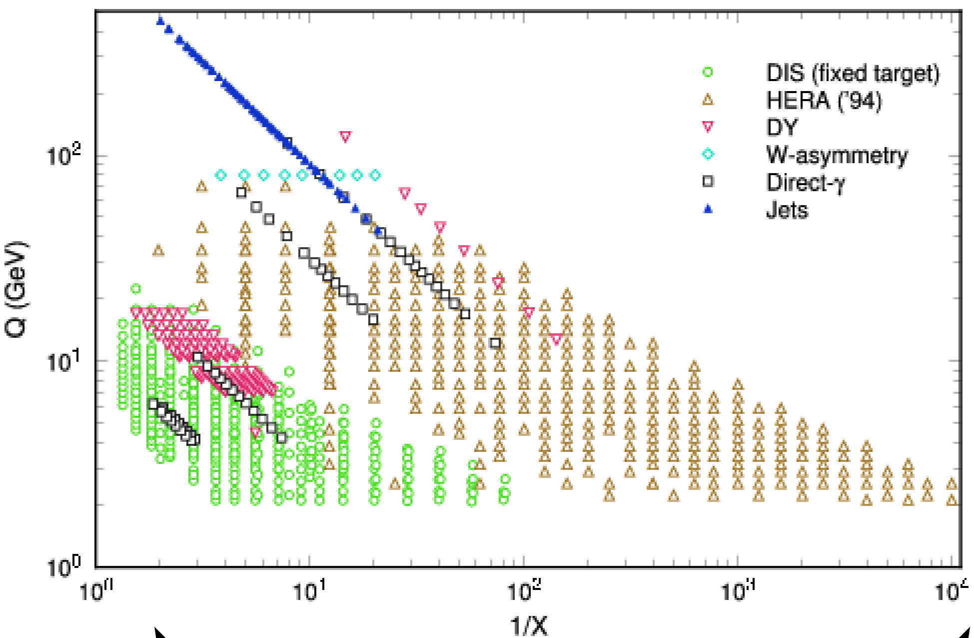
Self-Organizing Maps 6

"Colors" Example



Application to “test” problem: PDF fitting

Q^2

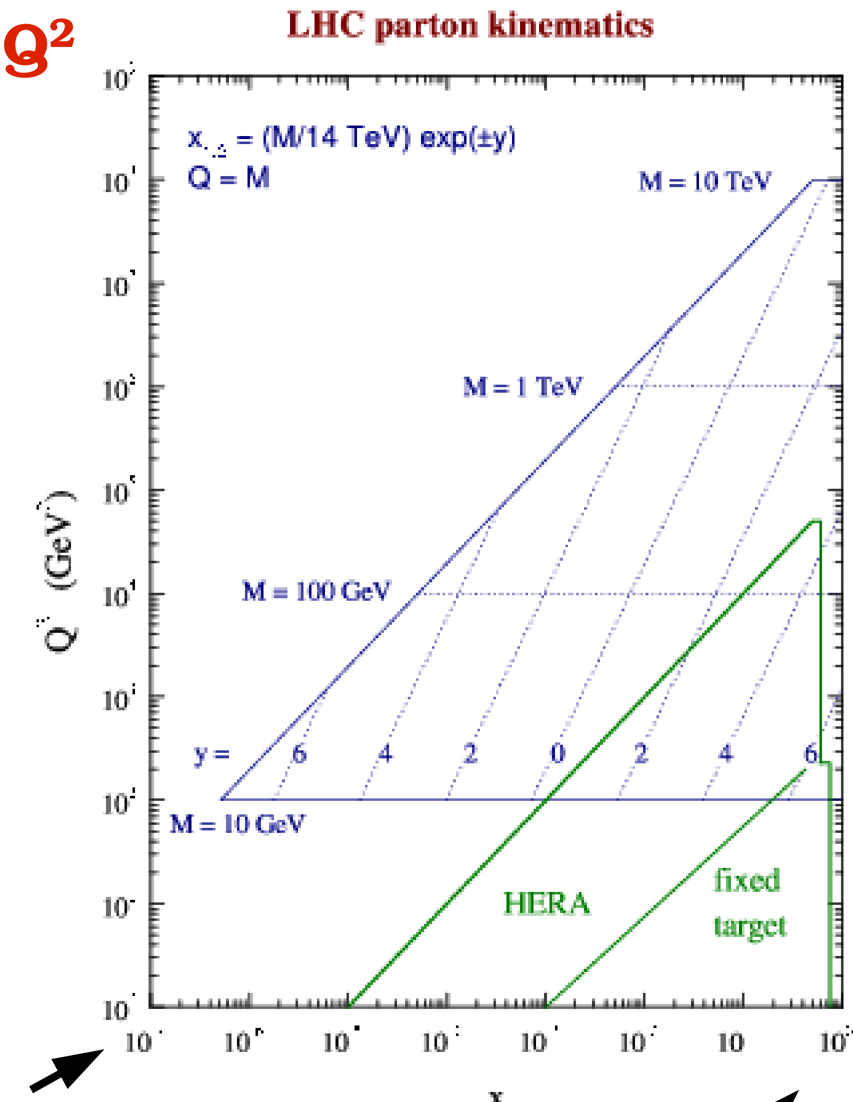


1

1/x

10^4

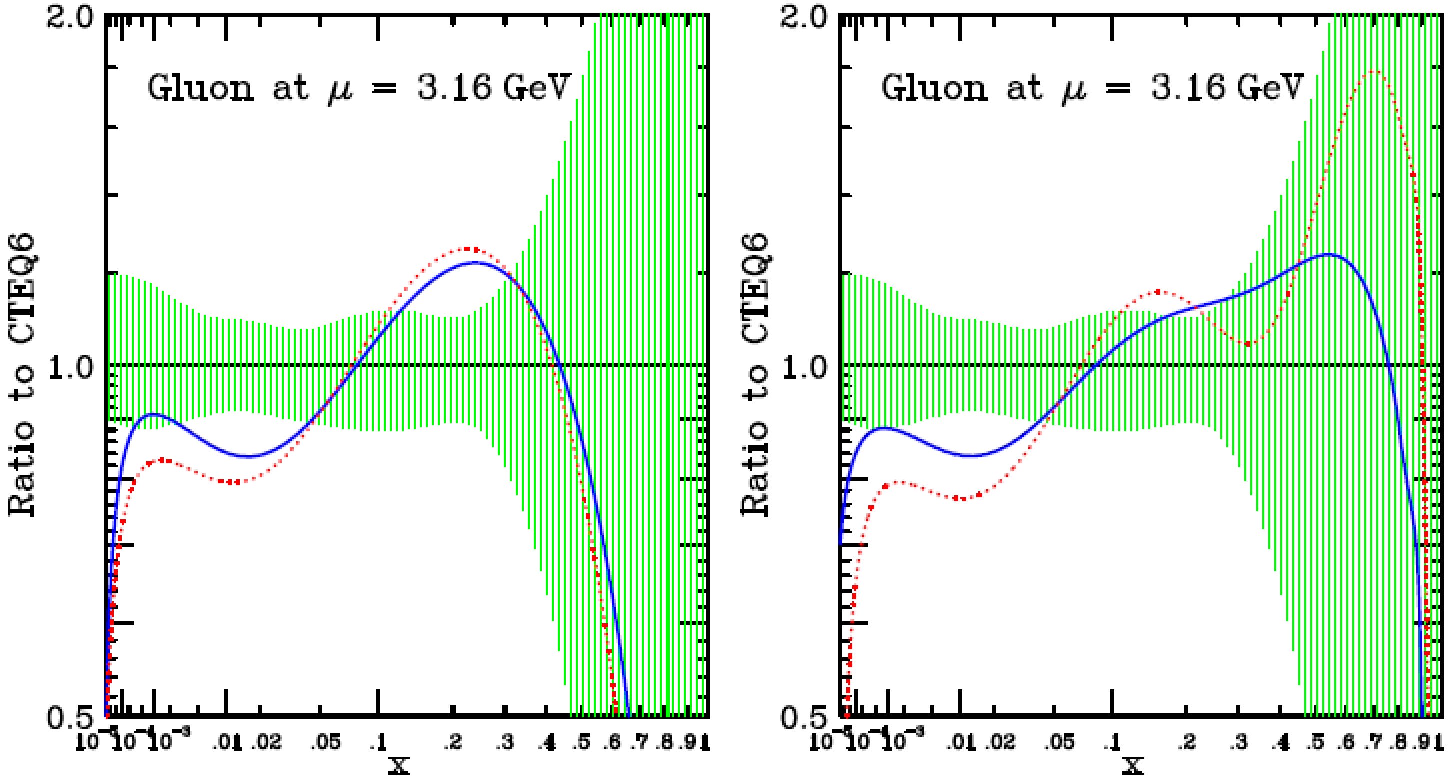
Q^2



10^{-7}

x

1



hep-ph/0201195

In a nutshell

(1) PDFs are clustered in a map according to similarity
(either among PDFs, or observables: structure functions)

(2) PDFs are identified with the code-vectors (<u>decoders</u>)

(3) Map vectors are updated by averaging the data samples
clustered within a neighborhood of the function to be updated

Advantages with respect to “conventional way”:

- Initial scale ansatz

$$F(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} P(x; A_3, ...)$$

- Evolve to higher scale
- Compute observables e.g. $F_2^p(x, Q^2)$
- Compare with the data e.g.

$$\chi^2(\{a\}) = \sum_{\text{expt.}} \left\{ \sum_{i=1}^{N_e} \frac{(D_i - T_i)^2}{\alpha_i^2} - \sum_{k,k'=1}^{K} B_k \left(A^{-1}\right)_{kk'} B_{k'} \right\}$$

where $B_k = \sum_{i=1}^{N_e} \frac{\beta_{ki}(D_i - T_i)}{\alpha_i^2}$, $\qquad A_{kk'} = \delta_{kk'} + \sum_{i=1}^{N_e} \frac{\beta_{ki}\beta_{k'i}}{\alpha_i^2}$

Similarly to NNPDFs we do not depend on a functional form,
the “initial bias”, and we can define a faithful estimate of the uncertainty

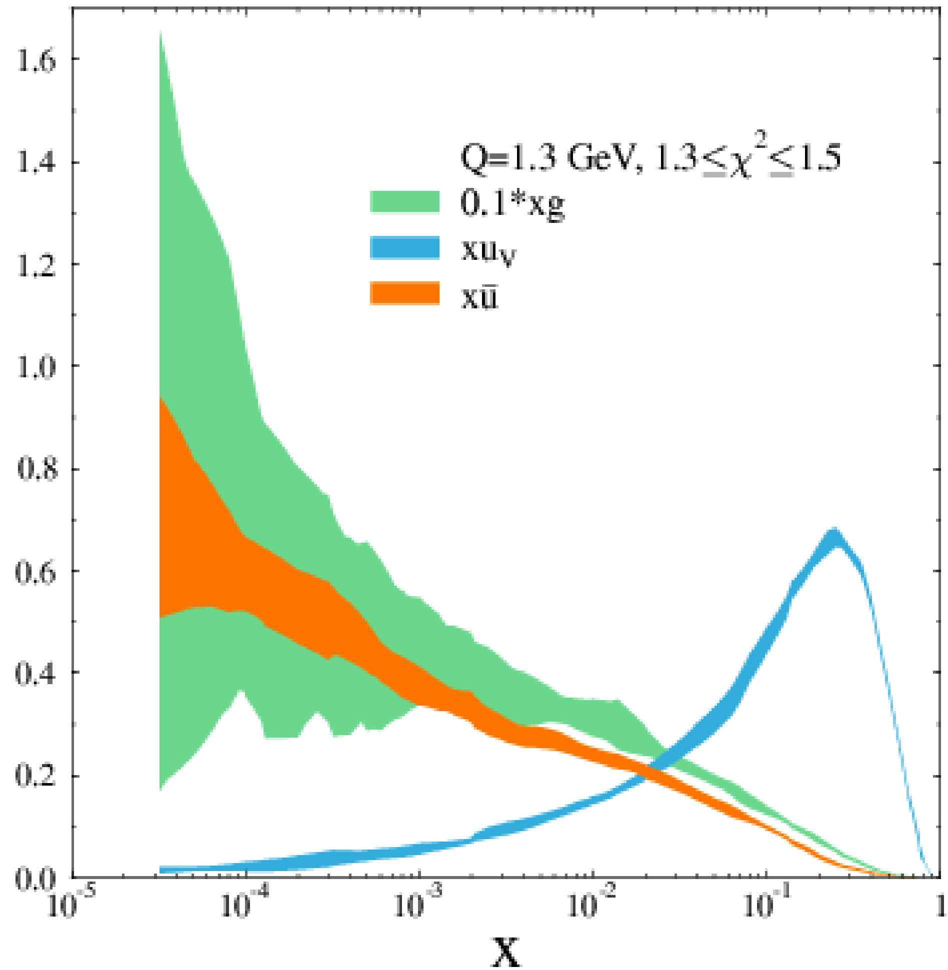
Advantages over NNPDFs

Mechanism responsible for the self-organization of the different representations of information: the response of the network changes in such a way that the location of the cell holding a given response corresponds to a specific input signal.

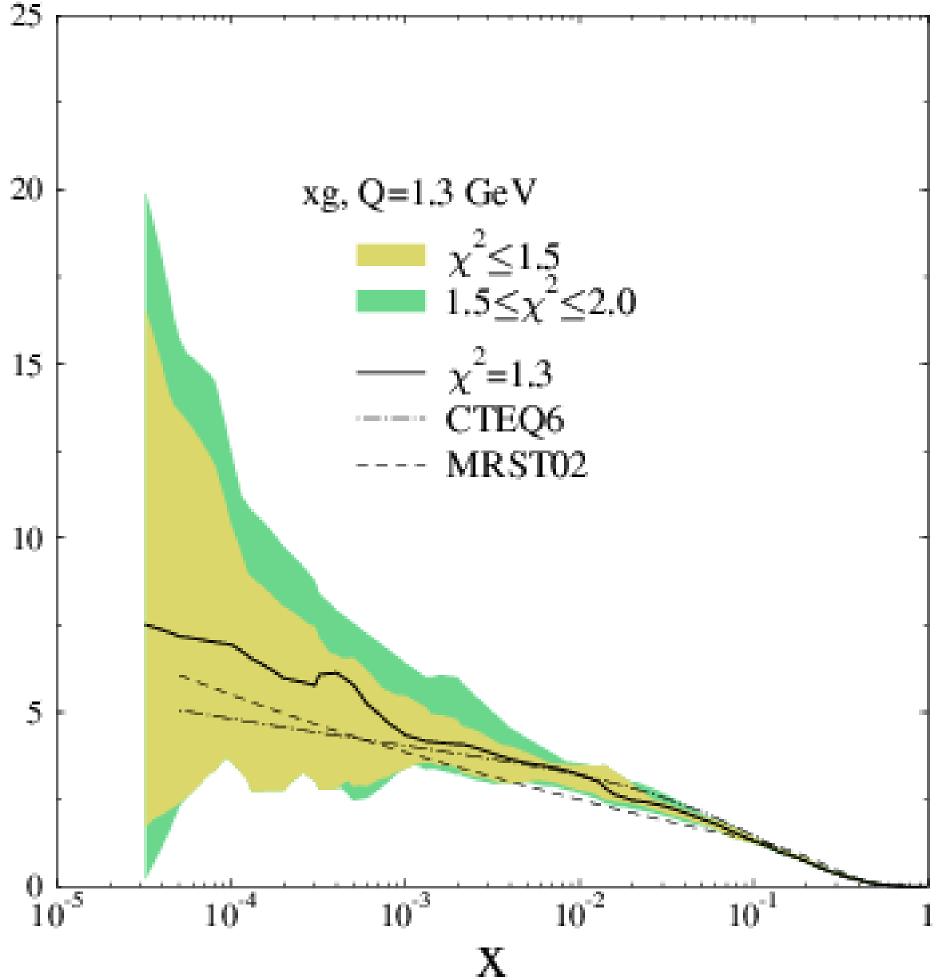
Geometrical arrangement of information is maintained during the training.

SOM work differently from ANN that do not keep track of the inter-connections among clustering of data at different stages of the network training.

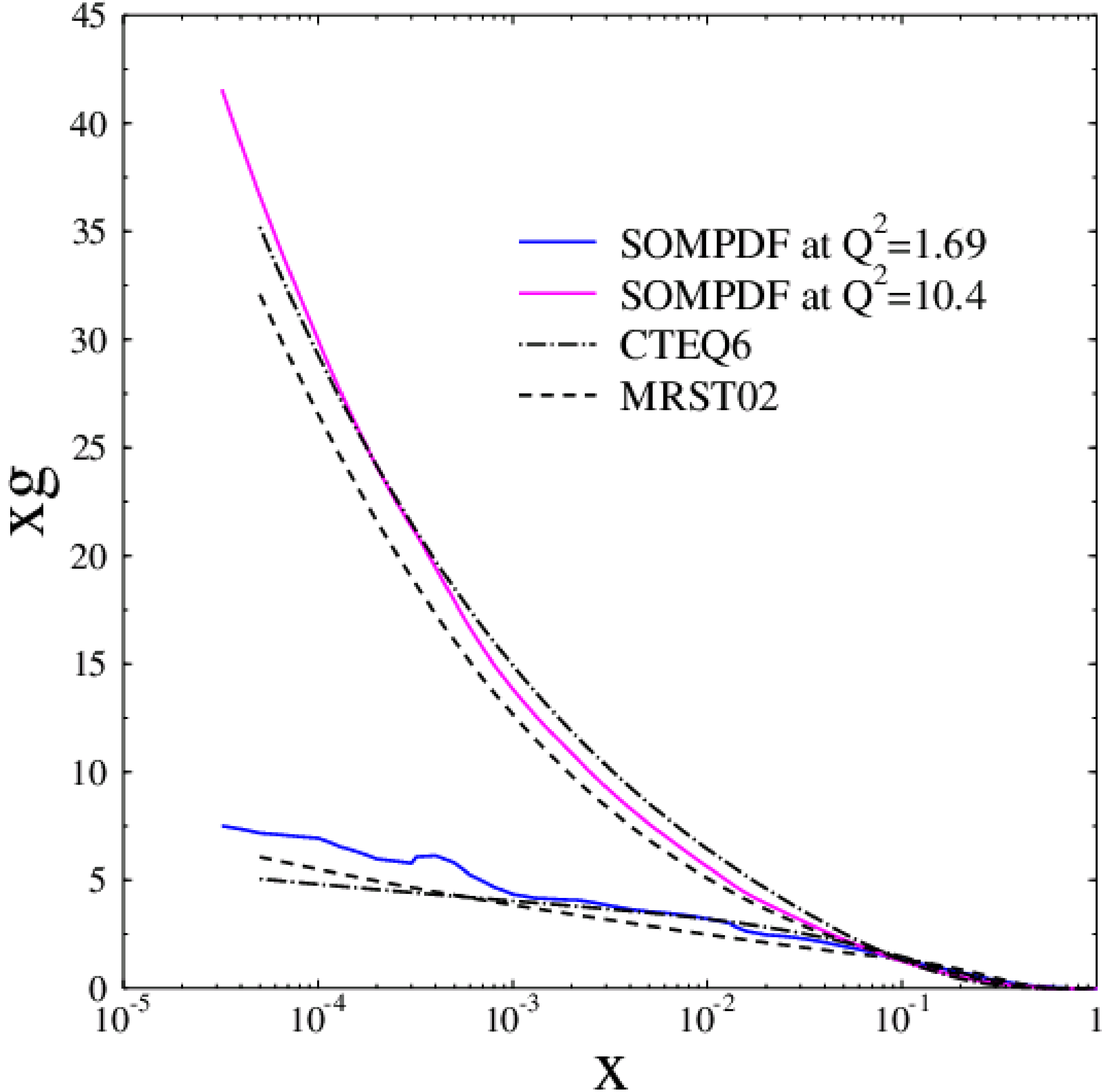
Important because it allows for "user/expert's" intervention: evaluate the impact of possibe theoretical input



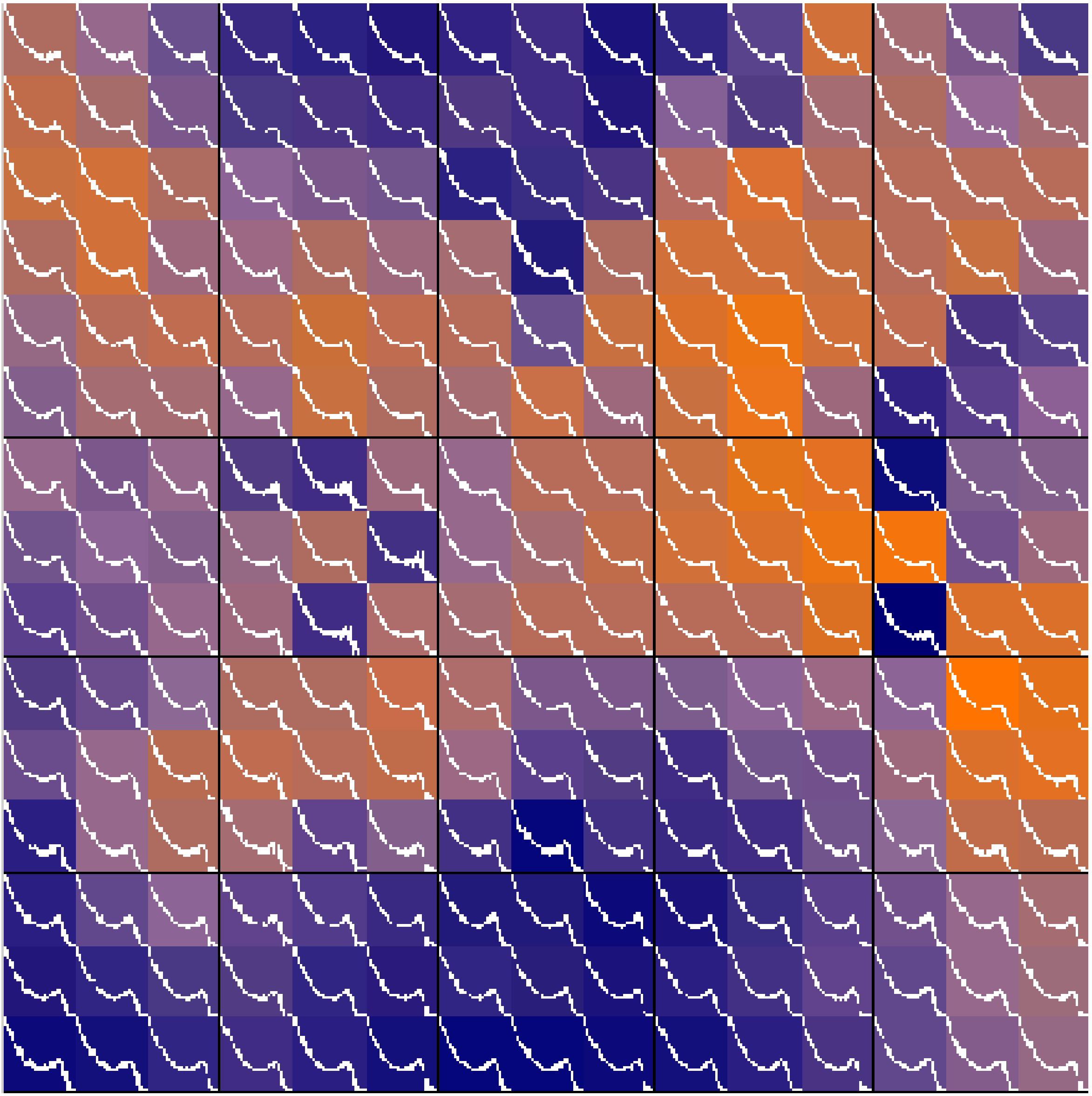
Q=1.3 GeV, 1.3≤χ²≤1.5
0.1*xg
xu_V
xū
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10^-5
10^-4
10^-3
10^-2
10^-1
1
X



xg, Q=1.3 GeV
$\chi^2 \leq 1.5$
$1.5 \leq \chi^2 \leq 2.0$
$\chi^2 = 1.3$
CTEQ6
MRST02
0
5
10
15
20
25
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x



45
40
35
30
25
20
15
10
5
0
xg
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
SOMPDF at Q^2=1.69
SOMPDF at Q^2=10.4
CTEQ6
MRST02



<u>Next step</u>. Study why the PDFs are arranged in a certain way in the map: introduce “flexbile points” in the analysis

Future: Connection with Complexity Theory

"<u>Critically interacting</u> components <u>self-organize</u> and form <u>evolving structures</u> exhibiting a <u>hierarchy of emergent system behaviors</u>"

5. Conclusions and Outlook

- Although we have learned a lot, there is still a lot to learn on hadron structure
- Outstanding questions lie ahead for the interpretation of exclusive and semi-inclusive experimental data: quark and gluons momentum, spin, spatial d.o.f, distributions can be accessed in principle but need to be mapped out with new methods.
- We overviewed new tools for phenomenology: GPDs, DVCS, IPPDFs.... towards forming images of hadrons
- We presented a new computational method: Self-Organizing Maps (SOM)that works well for proton PDFs

- Future: 1. Apply to nuclear PDFs, semi-inclusive...
- Future: 2. Connection with Complexity Theory?