Lattice hadron physics

William Detmold
University of Washington

University of Virginia, March 15 2007

- Strong interactions and lattice QCD
- External field techniques in lattice QCD
  Two examples:

  1. Hadron polarisabilities
  2. EMC effect: nuclear modification of parton distributions

Strong interactions

- What is strongly interacting QCD useful for?

Strong interactions

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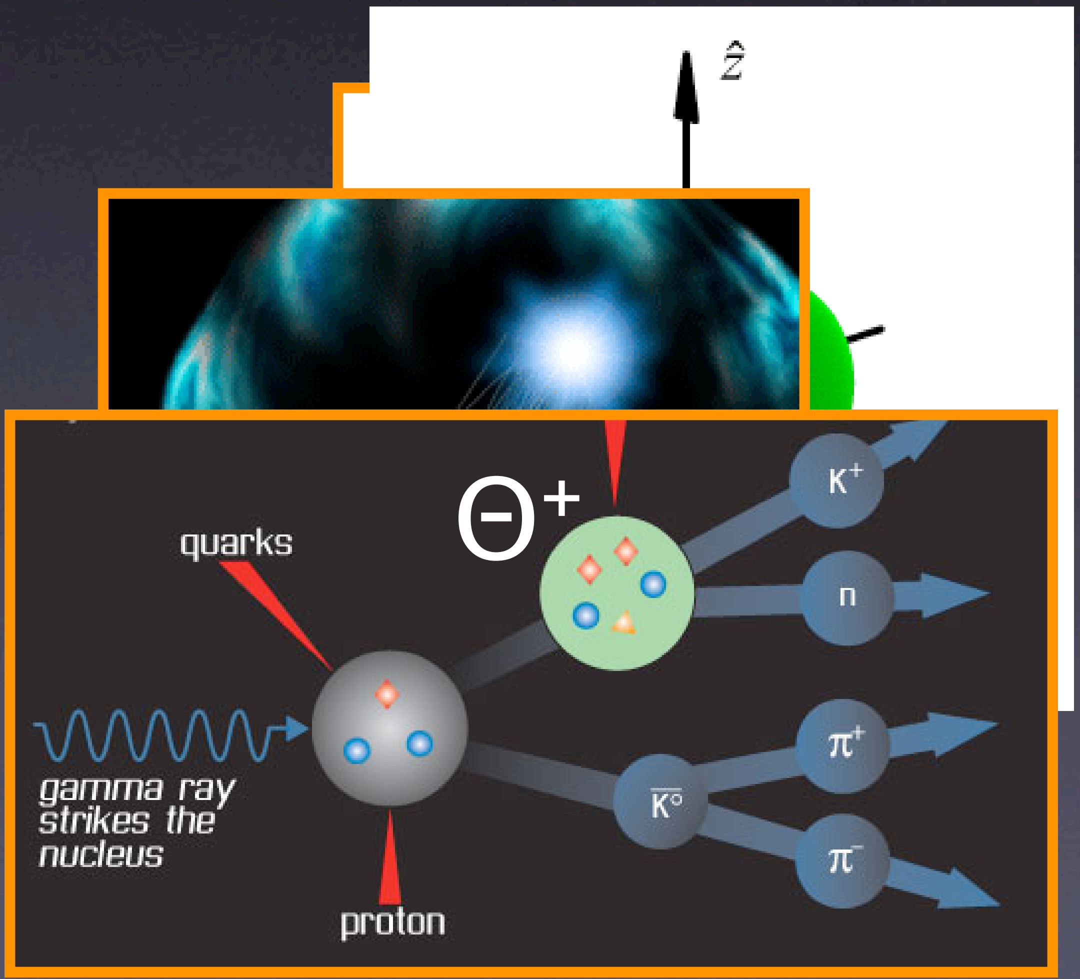
  - *Searches for physics beyond the Standard Model*
    - *Proton decay*
    - *CP violation: eg* $B^0 - \overline{B}^0$
    - *DAs for hard processes*
    - *Neutron EDM*

Strong interactions

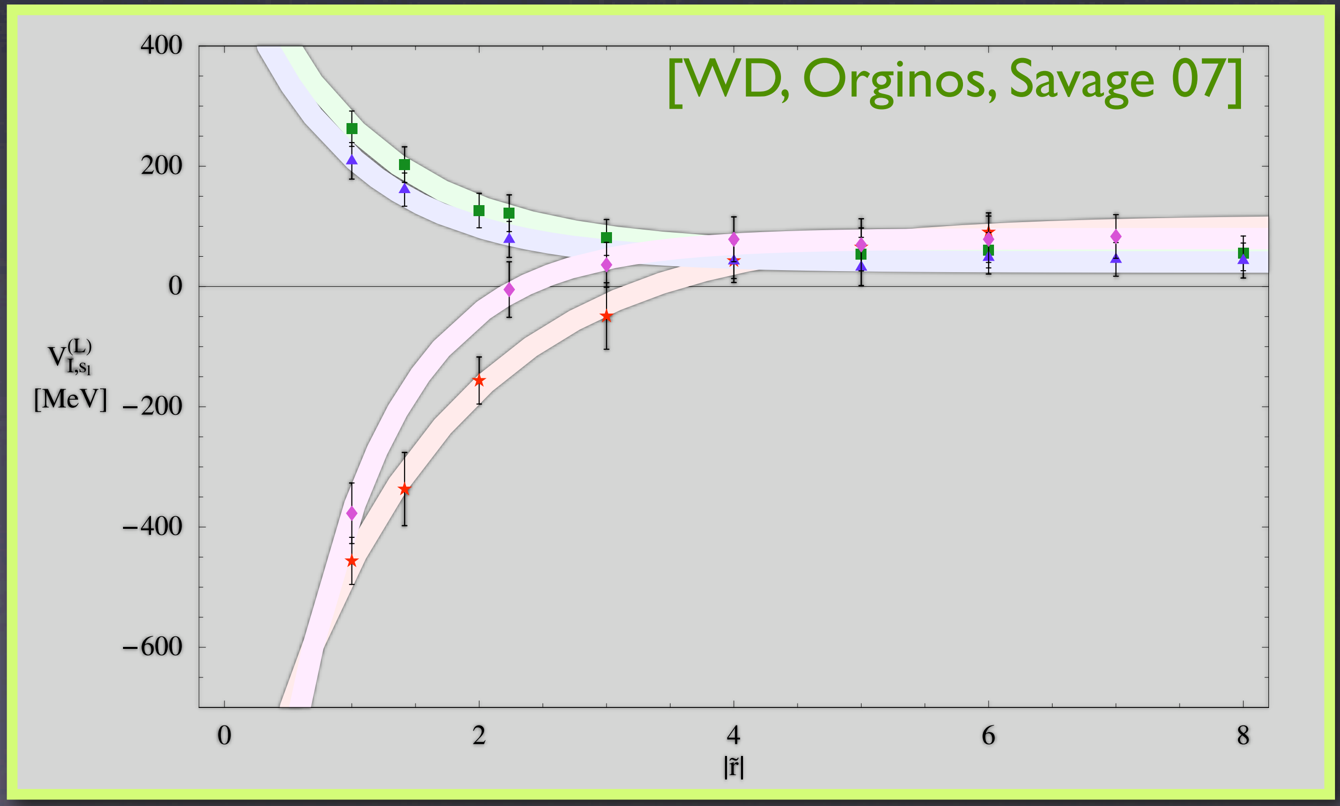
- What is strongly interacting QCD useful for?
  - *Searches for physics beyond the Standard Model*
  - *Hadron spectra and structure*
    - *Parton distributions/GPDs*
    - *Hadron polarisabilites*
    - *EM and weak form factors*
    - *Excited states, glueball and exotic spectra*



[Most pretty pictures from JLab]

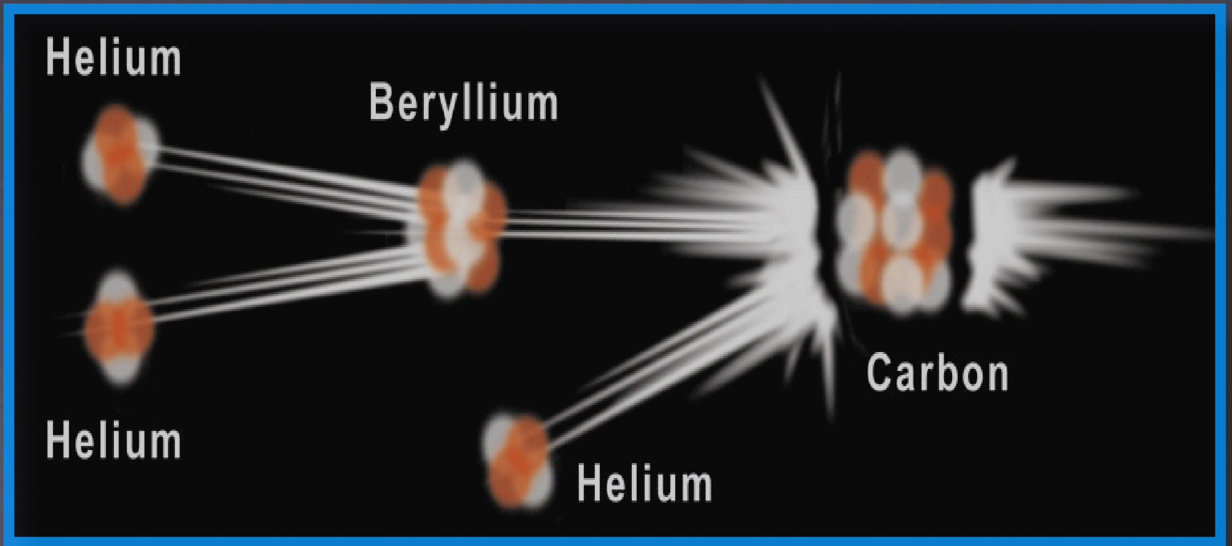
Strong interactions

- What is strongly interacting QCD useful for?
  - *Searches for physics beyond the Standard Model*
  - *Hadron spectra and structure*
  - *Nuclear structure and interactions*
    - *EMC effect*
    - *Electroweak: $\vec{n}p \rightarrow d\gamma$, $d\nu \rightarrow np$*
    - *$\pi\pi$, NN, YN, BB*



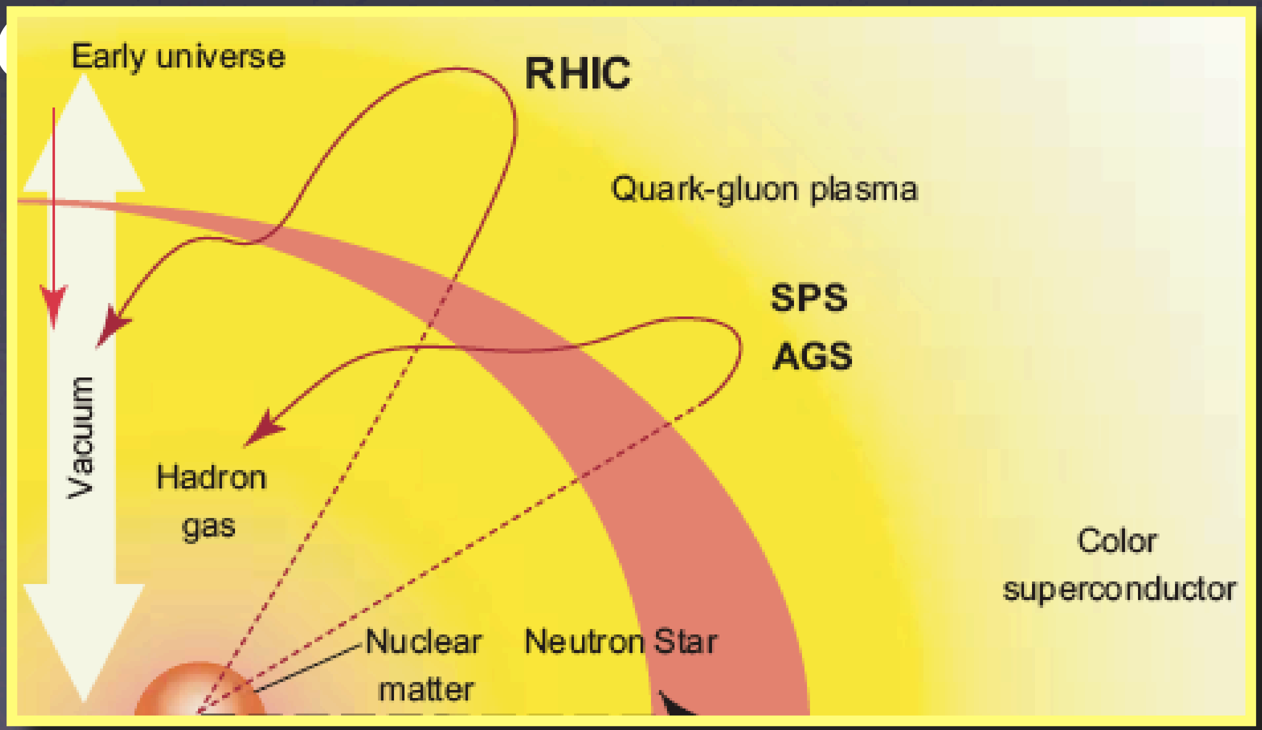
Strong interactions

- What is strongly interacting QCD useful for?
  - *Searches for physics beyond the Standard Model*
  - *Hadron spectra and structure*
  - *Nuclear structure and interactions*
  - *Sensitivity of nature to QCD parameters*
    - *ε_d dependence on m_q*
    - *Fine tunings: stellar carbon production*



Strong interactions

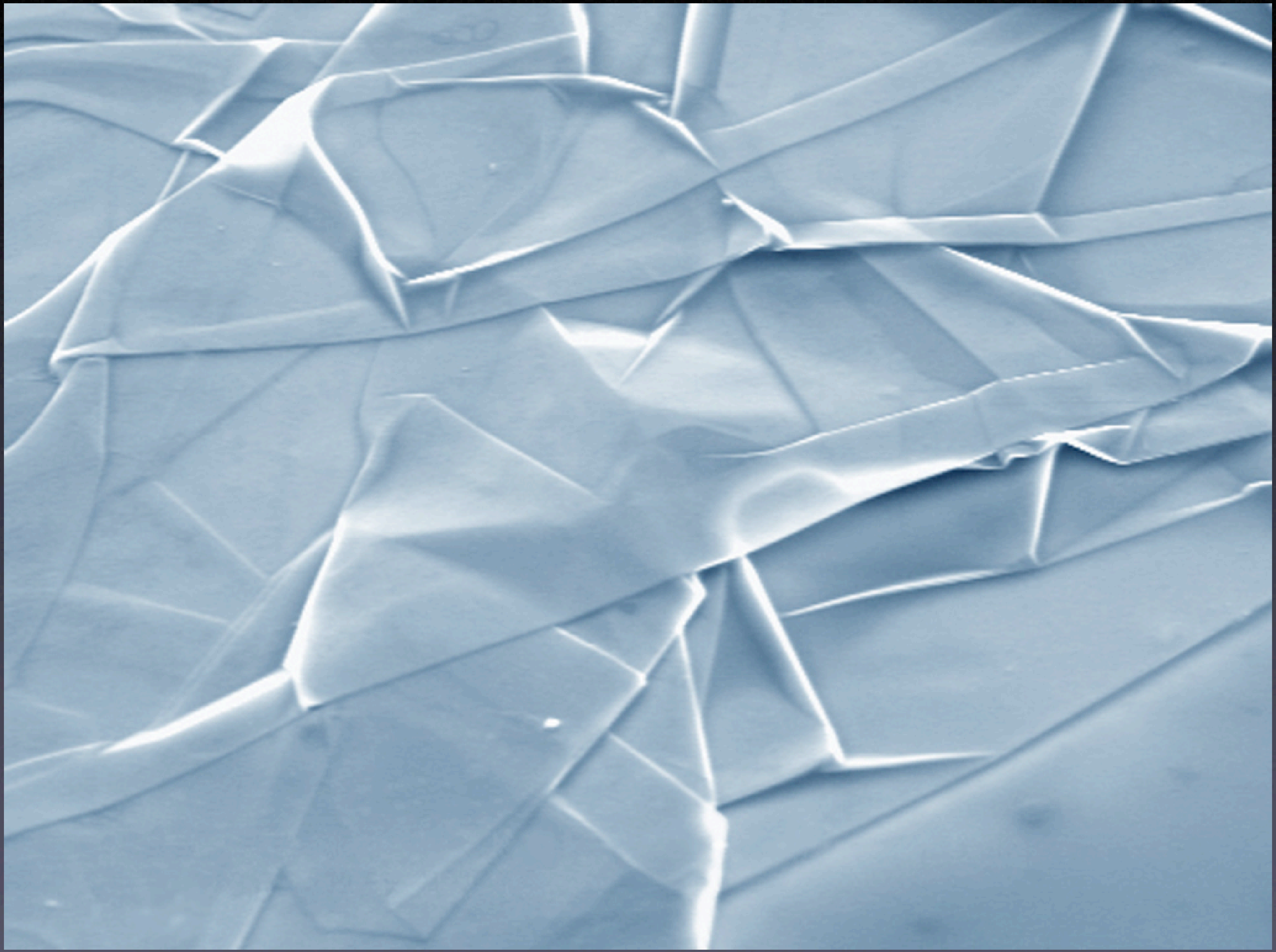
- What is strongly interacting QCD useful for?
  - *Searches for physics beyond the Standard Model*
  - *Hadron spectra and structure*
  - *Nuclear structure and interactions*
  - *Sensitivity of nature to Q*
  - *New forms of matter*
    - *Non-zero T, μ*



Strong interactions

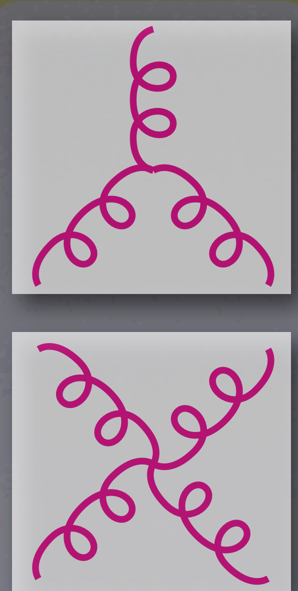
- Exotic hadrons within QCD and beyond
  - higher $SU(3)_c$ representation quarks
  - hadrons containing gluinos (SUSY) [Farrar-Fayet]
  - decays and signals at LHC
- Non-perturbative physics in theories beyond the SM: little Higgs models, “hidden valleys”,...
- Low dimensional systems (eg graphene)

Graphene



Simple equations ...

- Field equations are easy to write down



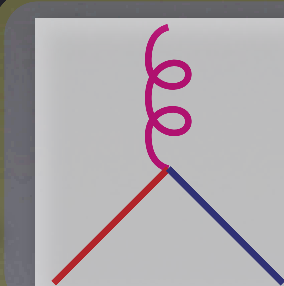
$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \overline{\psi}_f \left(i \not{D} + m_f \right) \psi_f$$

Quarks

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$$

Gluons

$$\not{D} = \gamma_\mu \left(\partial^\mu + i g A^\mu \right)$$



- ... but unsolved ... ($1M Clay Prize)

The running coupling

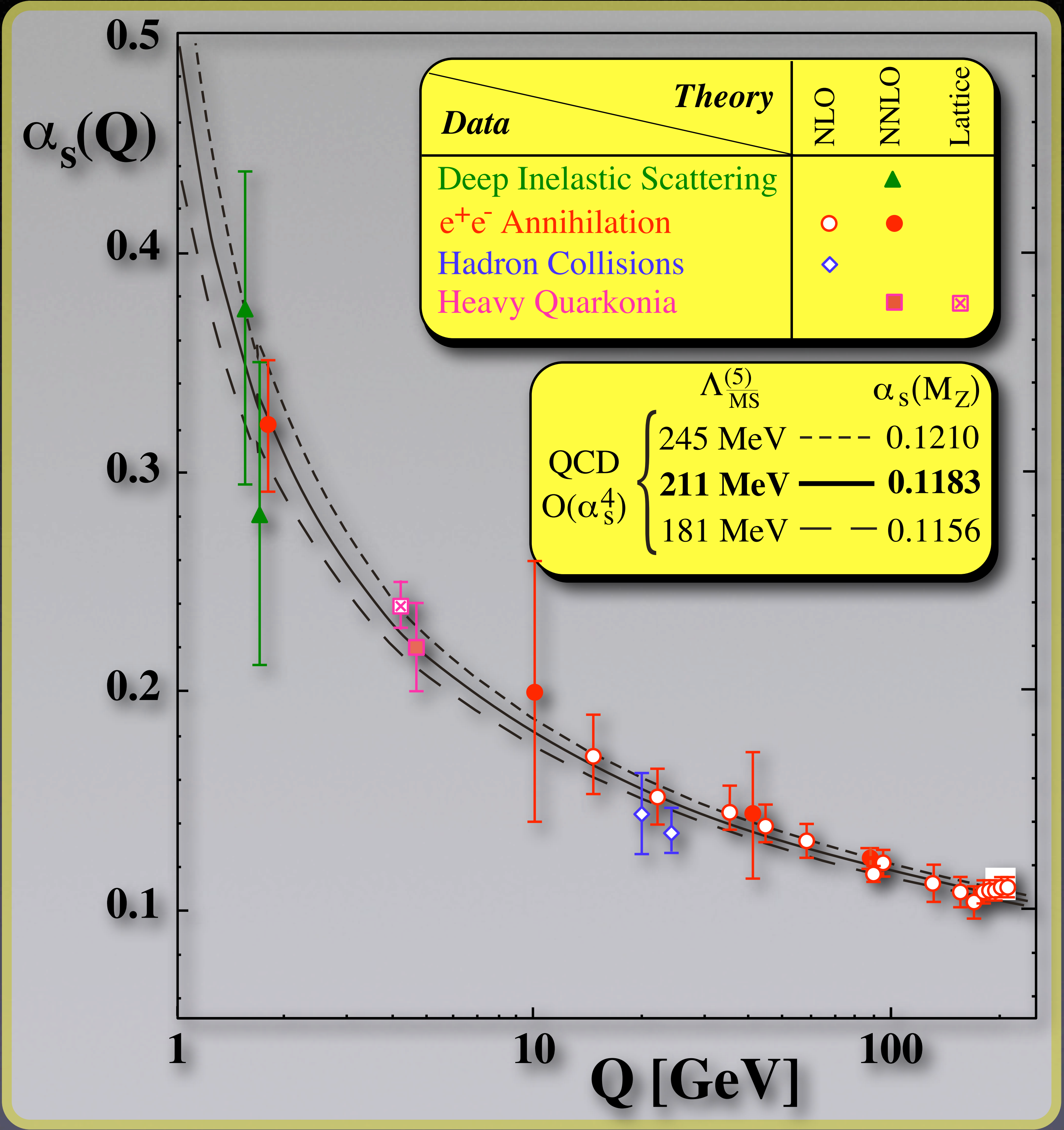
- Gross, Politzer and Wilczek showed that QCD has a negative β function

$$\alpha_s(Q^2) = \frac{g^2}{4\pi} = \frac{4\pi}{(11 - \frac{2}{3}N_f)\log\left[Q^2/\Lambda^2_{\rm QCD}\right]}$$

- Two important consequences
  - Asymptotic freedom
  - Confinement

Gross, Politzer & Wilczek
2004

Asymptotic Freedom

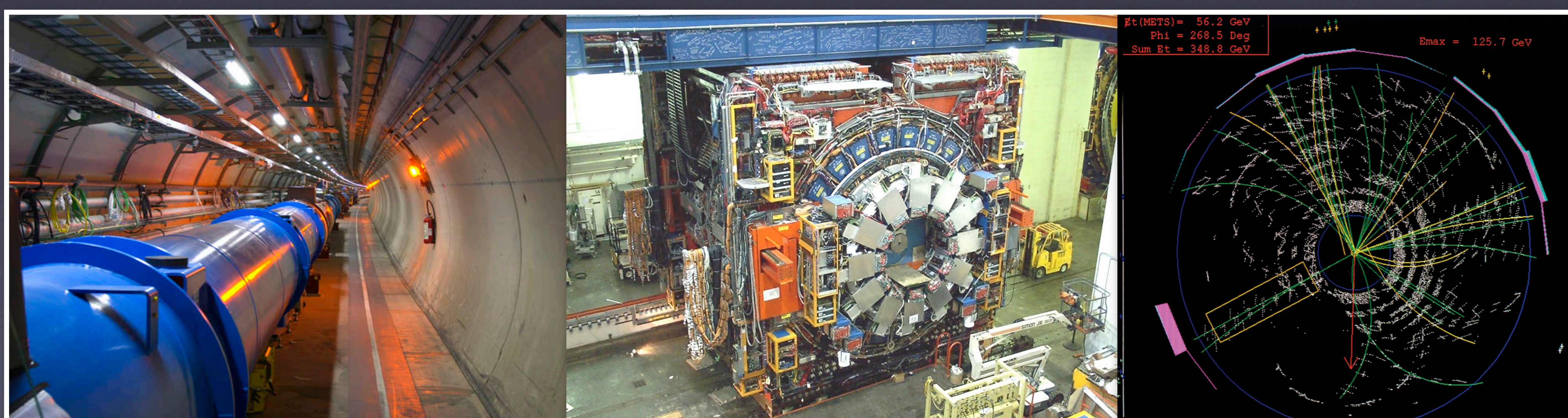


Expansion in powers of α_s becomes better and better

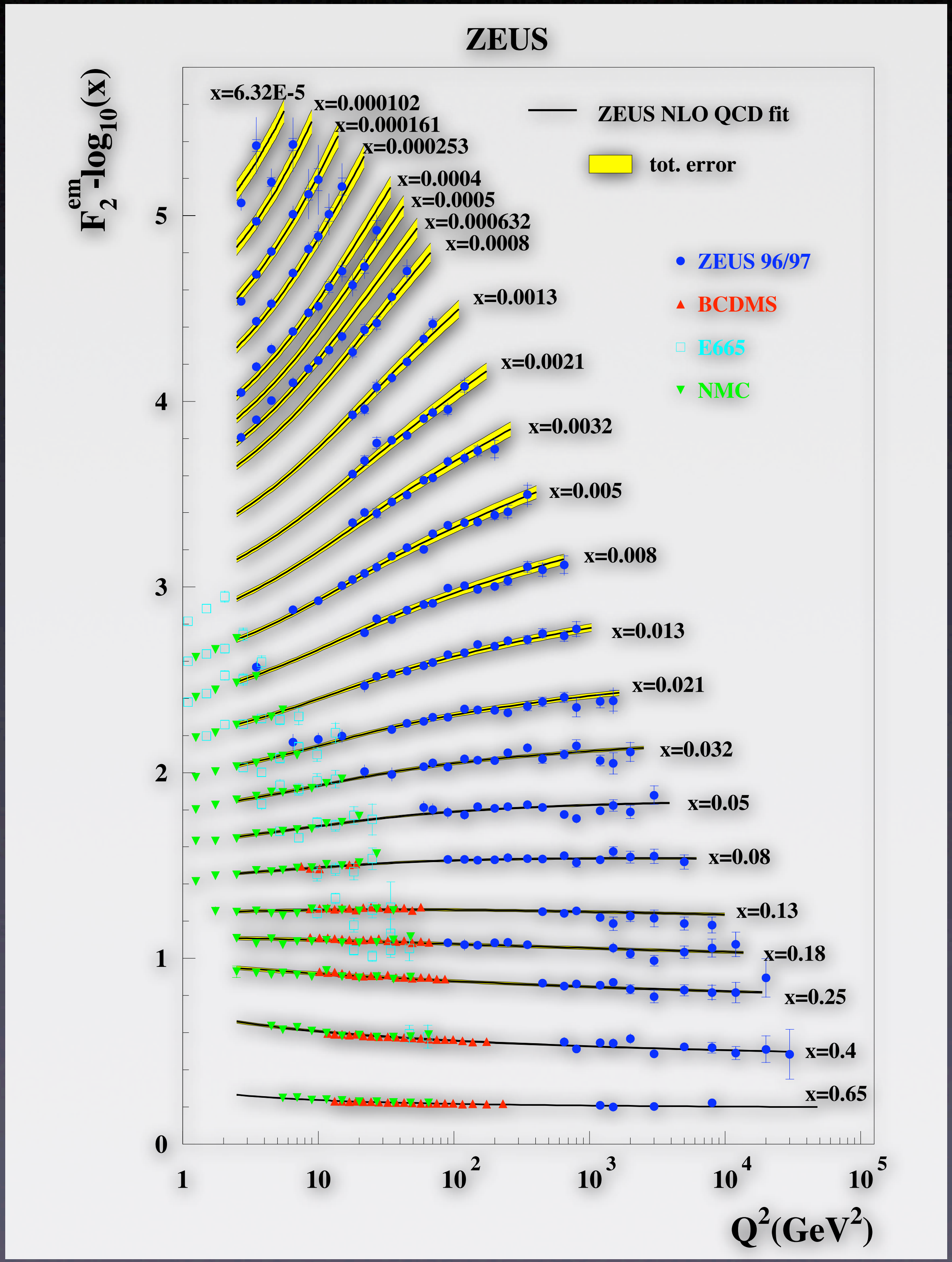
[Bethke 02]

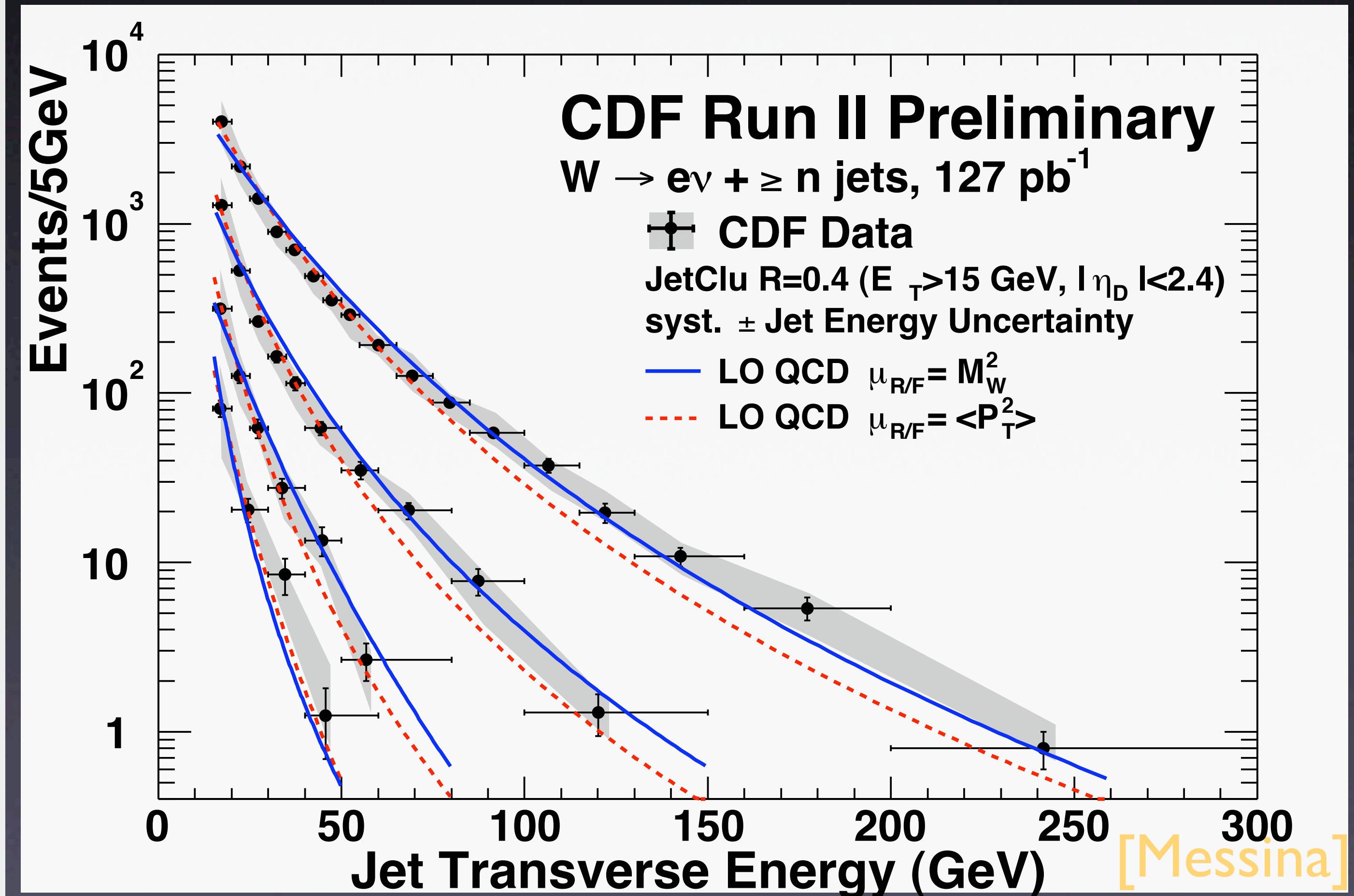
High energies

- We believe in QCD because high energy calculations beautifully reproduce data
- Major experiments (TeVatron, RHIC, LHC) collide hadron beams at very high energy



QCD vs Experiment

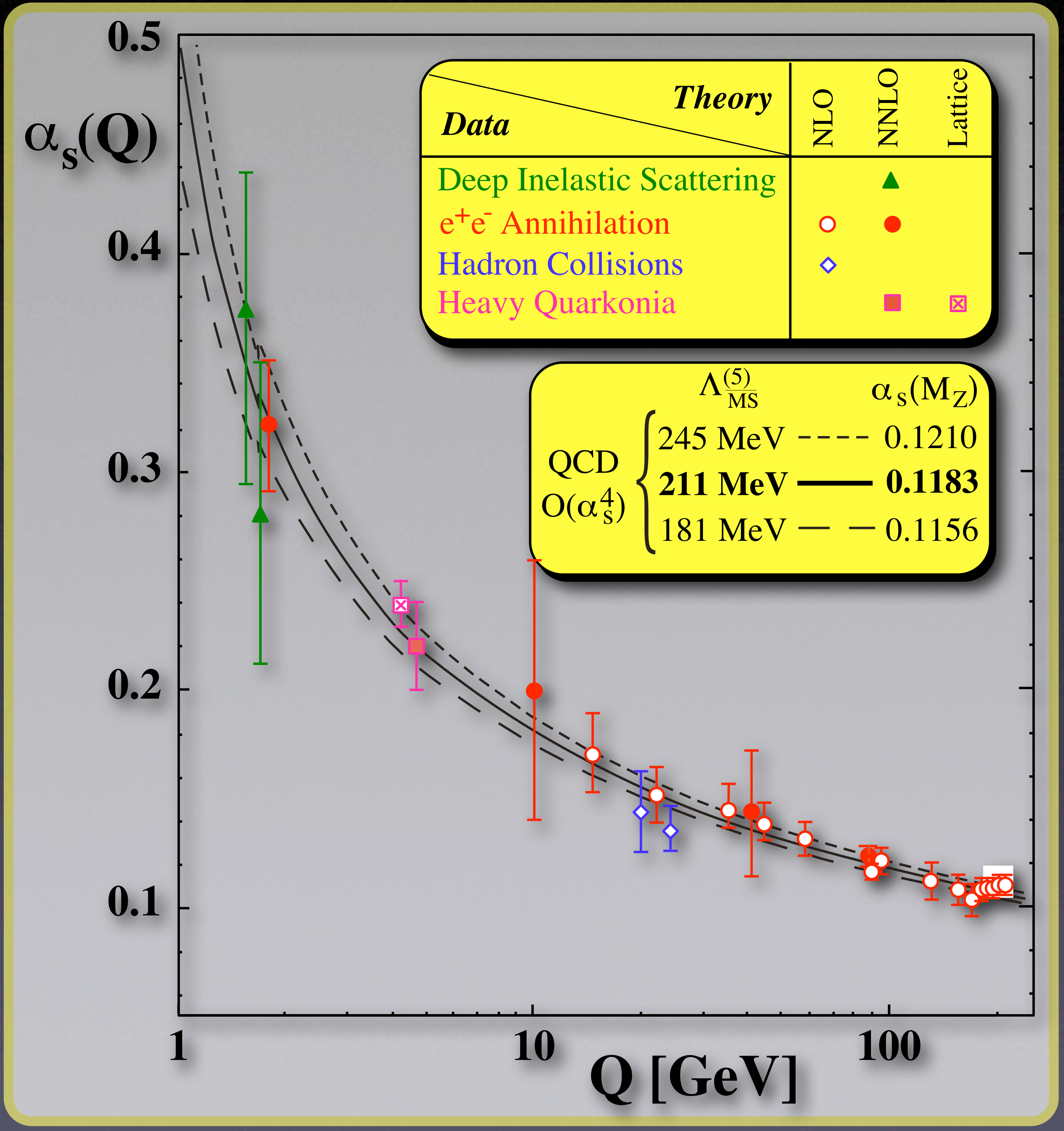




[Messina]

Confinement

and up
and up ...



[Bethke 02]

Why lattice QCD?

- Typical energies outside of colliders, exploding stars and the Early Universe

$$\boxed{E \sim M_{\text{proton}}}$$

- Perturbation theory breaks down

$$\boxed{\alpha_s\left(M_{\text{proton}}\right) \sim 1}$$

  - Cannot calculate the proton mass
- Need a non-perturbative method

Lattice QCD

- Numerical solution of QCD field equations
- QCD partition function

$$\mathcal{Z} \sim \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\overline{\psi} e^{-S_{QCD}[A,\psi,\overline{\psi}]}$$

  - Quark functional integral done exactly
- Observable

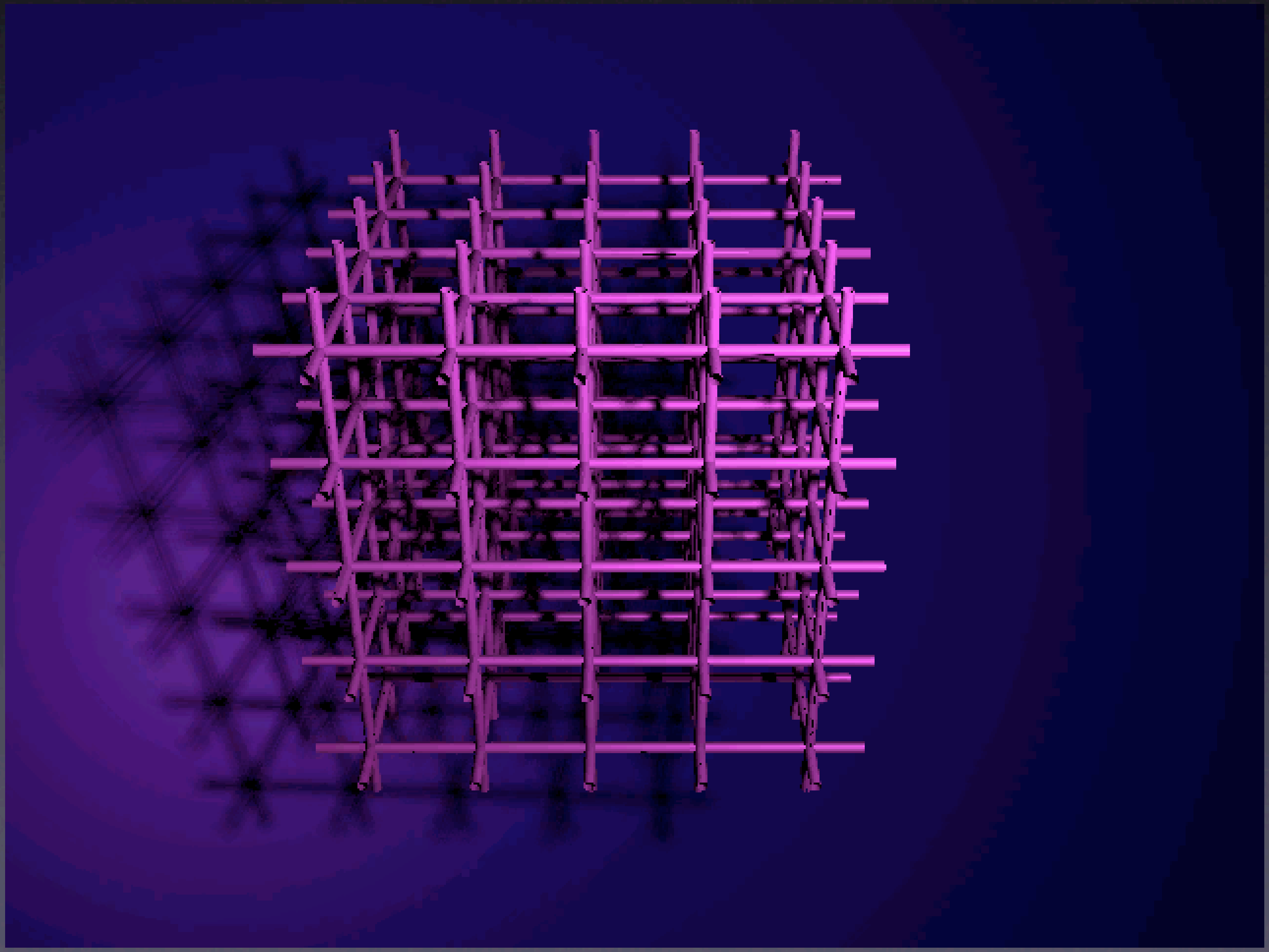
$$\langle \mathcal{O} \rangle \sim \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu \det[\mathcal{M}[A]] \mathcal{O}[A, \mathcal{M}] e^{-S_{QCD}[A]}$$

- Euclidean space

[K Wilson]

Lattice QCD

- Discretise and compactify space-time
  - Lattice spacing *a*, volume L^3xT



Lattice QCD Simplify

- Quarks live on lattice sites, gluons on links
- Functional integral is finite dimensional but still too many integrals (>10^7 !) to do
- Use Monte Carlo techniques to estimate
  - Configurations $\{\phi_i\}$ generated according to Boltzmann weight $\det[\mathcal{M}]\exp(-S_{\mathrm{QCD}})$
  - Observable: $\langle\mathcal{O}\rangle \to \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}[\phi_i]$
  - Errors $\sim 1/\sqrt{N}$

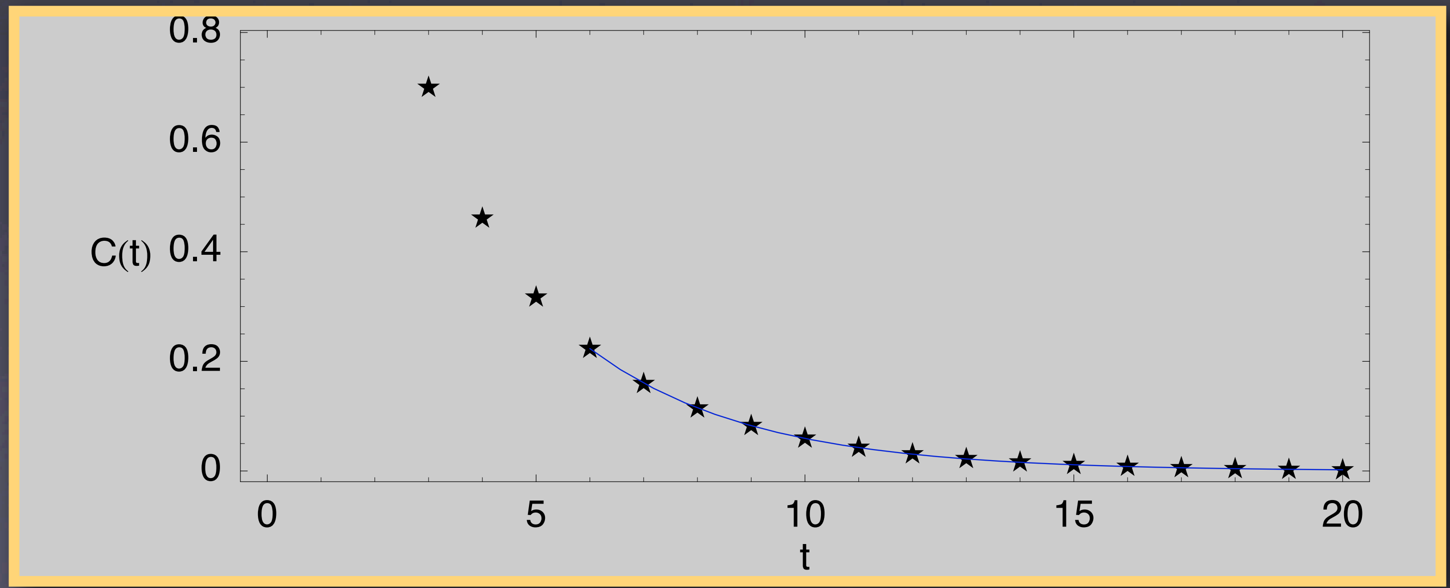
Ex: energy spectrum

- Measure correlator (χ = source with q# of hadron)

$$G_2(\mathbf{p},t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(0,0)|0\rangle$$

- Long times: only ground state survives

$$\xrightarrow{t\to\infty} e^{-E_0(\mathbf{p})t} \langle 0|\chi(0,0)|E_0,\mathbf{p}\rangle\langle E_0,\mathbf{p}|\overline{\chi}(0,0)|0\rangle$$



Lattice QCD in reality

- Light quark masses are numerically difficult
- *Partially-quenched QCD:* valence and sea quark masses different (sea quarks are more expensive)

$$\langle \mathcal{O} \rangle \sim \frac{1}{Z} \int \mathcal{D}A_\mu \det[\mathcal{M}_s[A]]\mathcal{O}[A, \mathcal{M}_v]e^{-S_{QCD}[A]}$$

  - Has QCD as a limit
- *Quenched QCD* (ignore vacuum polarisation) has *no connection to QCD*

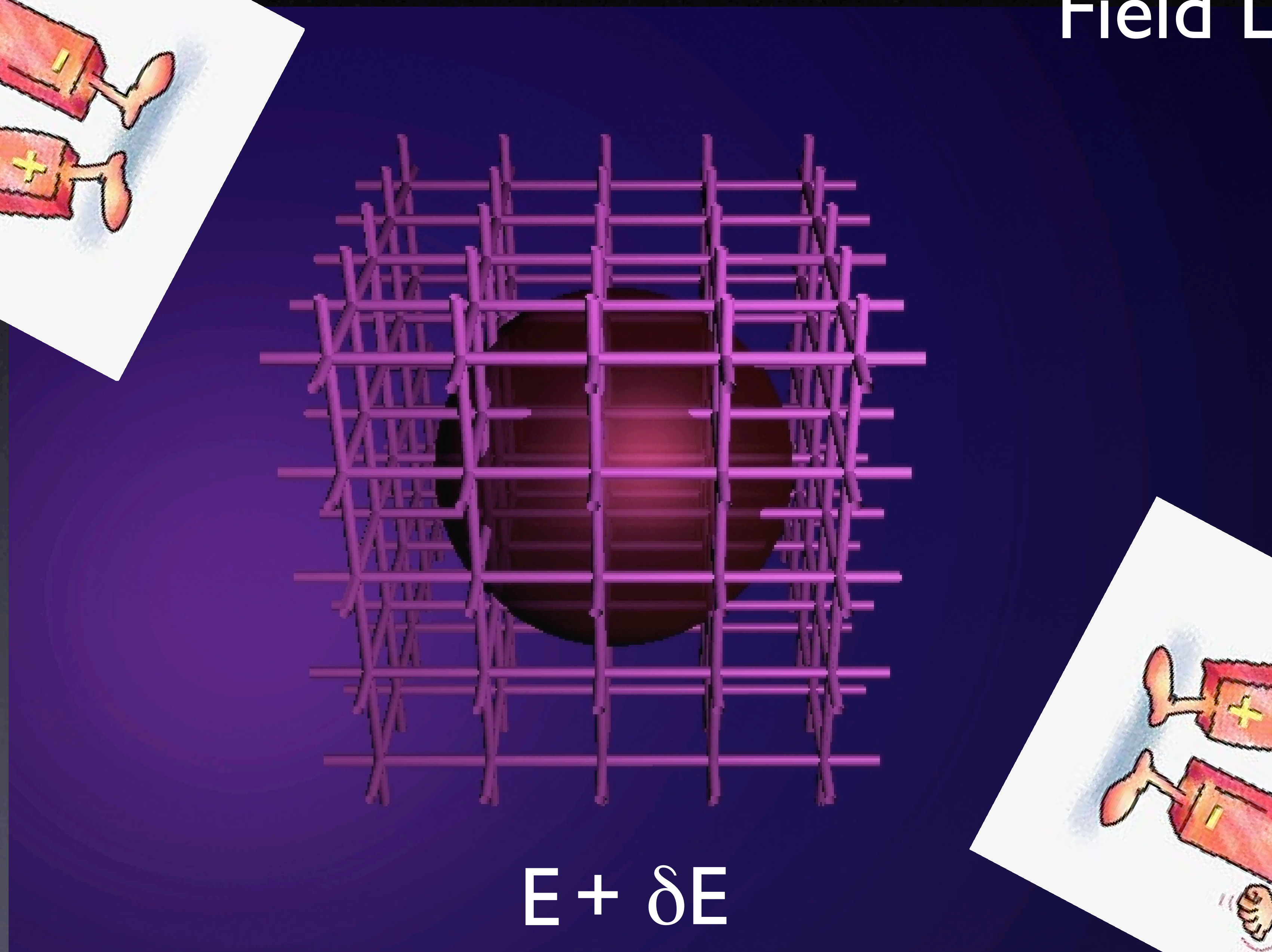
Extrapolations

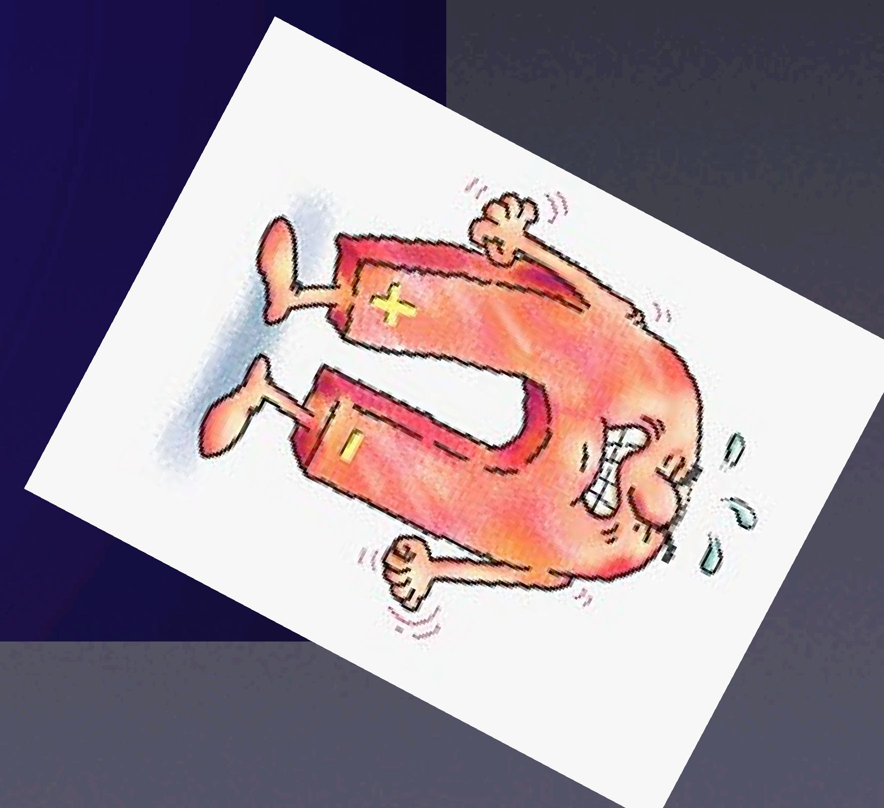
- To get real world physics from the lattice calculations we need to take: PICTURE
  - Lattice spacing to zero
  - Lattice volume to infinity
  - Quark masses to their physical values
    - *Calculations at the physical masses are too demanding for current computers*
- *All can be addressed in chiral perturbation theory*

External Fields

- The basic idea: compute lattice QCD correlation functions in the presence of controlled external sources (classical fields)
- The response to variation of the strength of the field determines the physics

External fields





Polarisabilities

Hadron polarisabilities

- Hadron polarisabilities describe the deformation of a particle in an external (EM) field
- Quadratic energy shifts from effective Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 - \vec{\mu} \cdot \vec{B} - 2\pi \alpha |\vec{E}|^2 - 2\pi \beta |\vec{H}|^2 - 2\pi \gamma_1 \vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \ldots$$

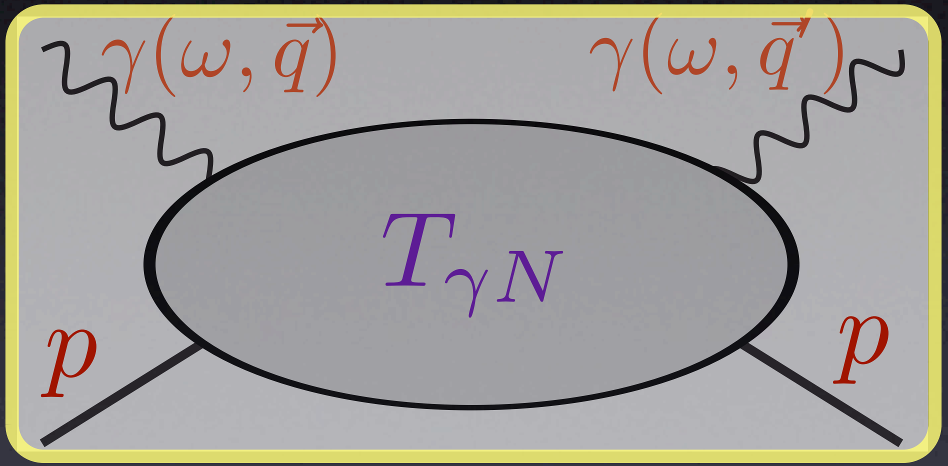
Magnetic moment Electric pol Magnetic pol First spin pol

- Electric and magnetic polarisabilities: ability to align with or against the applied field
- Spin and higher order polarisabilities: more detailed view of EM structure

[WD, A Walker-Loud, B Tiburzi 06]

Compton scattering

- Experimentally measured in the low frequency limit of real Compton scattering



- Thomson limit and Low–Gell-Mann–Goldberger LET determined by Born terms (charge and magnetic moment)

$$T_{\gamma N} = f(\underbrace{\omega, \vec{q}, \vec{q}', \vec{\epsilon}, \vec{\epsilon}', \vec{\sigma}}_{\text{Kinematics}}; Z, \mu, \underbrace{\alpha, \beta}, \underbrace{\gamma_{1,\ldots,4}}) + \mathcal{O}(\omega^4)$$

- Next order given in terms EM and spin polarisabilities

Experiment

- MAMI, Saskatoon, JLab, OOPS, ELSA, HIγS
- EM and 2 combinations of spin polarisabilities are measurable for the proton but *difficult* experiments
- Neutron accessed via (quasi-)elastic Compton scattering on the deuteron - *even more difficult*

$$\alpha_p = 12.0(6), \quad \beta_p = 1.9(6), \quad \alpha_n = 13(2), \quad \beta_n = 3(2) \quad 10^{-4}\,\mathrm{fm}^3$$

$$\gamma_\pi^{(p)} = -39(2), \quad \gamma_0^{(p)} = -1.0(1), \quad \gamma_\pi^{(n)} = 59(4), \quad 10^{-4}\,\mathrm{fm}^4$$

[de Jaeger & Hyde-Wright 05]

- Sign and small size of polarisabilities indicates tightly bound diamagnetic system - hard to deform

Lattice approaches

1. Four point correlators: $\langle 0|\chi(x_1)J^\mu(y_1)J^\nu(y_2)\overline{\chi}(x_2)|0\rangle$
   - Analogous to experimental measurement
   - Difficult - many disconnected contractions
   - Require $\omega \to 0$ extrapolation
2. Energy shifts in two point correlators in external U(1) field
   - *Quenched QCD*: external field can be added after gauge configurations are generated
   - *QCD*: external field must be known during gauge field generation - multipurpose but costly

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External field method

- Quenched external EM fields simple to apply:

$$U^a_\mu(x) \to U^a_\mu(x) \cdot U^{\text{ext}}_\mu(x) \qquad U^{(a)}_\mu(x) = e^{i\,a\,g\,A^{(a)}_\mu(x)}$$

- E.g.: magnetic field $\vec{B} = (0, 0, B)$

Quantised for periodic links

$$U^{\text{ext}}_0 = U^{\text{ext}}_2 = U^{\text{ext}}_3 = 1,\ U_1(x) = e^{ieBx_2}$$

- Look for shift in energy quadratic in |B|

$$C_{\uparrow\uparrow}(\tau, B) = \sum_{\vec{x}} \langle 0|\chi_\uparrow(\vec{x}, t)\overline{\chi}_\uparrow(0)|0\rangle$$

Magnetic polarisability

$$= \exp\left[-(M - \mu|B| + 2\pi\beta|B|^2)\tau\right] + \mathcal{O}(|B|^3)$$

Also Landau level contribution

Magnetic moment

Field constraints

- Field values are restricted by a number of constraints
  - Perturbative in EFT: $|eB|,\ |eE| < m_\pi^2$
  - For periodicity of box: e.g. magnetic field

$$\boxed{\begin{gathered} U_\mu(x + L\hat{\nu}) = U_\mu(x) \\ a^2|eB| = \frac{2\pi n}{L}, \quad n \in Z \end{gathered}}$$

  - Landau levels well represented

External field method

- Can study more than energy shifts - hadronic correlator analysis ≡ effective field theory

- E.g.: charged particle in constant electric field

Acceleration of proton at large times

$$C_{ss'}(\tau;E) = \sum_{\vec{x}} \langle 0|\chi_s(\vec{x},t)\overline{\chi}_{s'}(0)|0\rangle_E$$
$$= \delta_{s,s'} \exp\left[-(M+2\pi\alpha|E|^2)\tau - \boxed{\frac{q^2|E|^2}{6M}\tau^3}\right] + \ldots$$

Electric polarisability

- Valid for $L^{-1} < m_\pi$, $|eE| < m_\pi^2$

External field method

- All six polarisabilities can be calculated
  - utilise all information in hadron correlators including spin-flip matrix elements
  - spin polarisabilities require space/time varying U(1) fields: E.g. γ_{E1E1}

$$U_\mu^{\text{ext}} = e^{iaeA_\mu(x)},\; A_\mu(x) = \left(-\frac{a_6 t^2}{2a}, \frac{-ib_6 t}{2}, 0, 0\right)$$

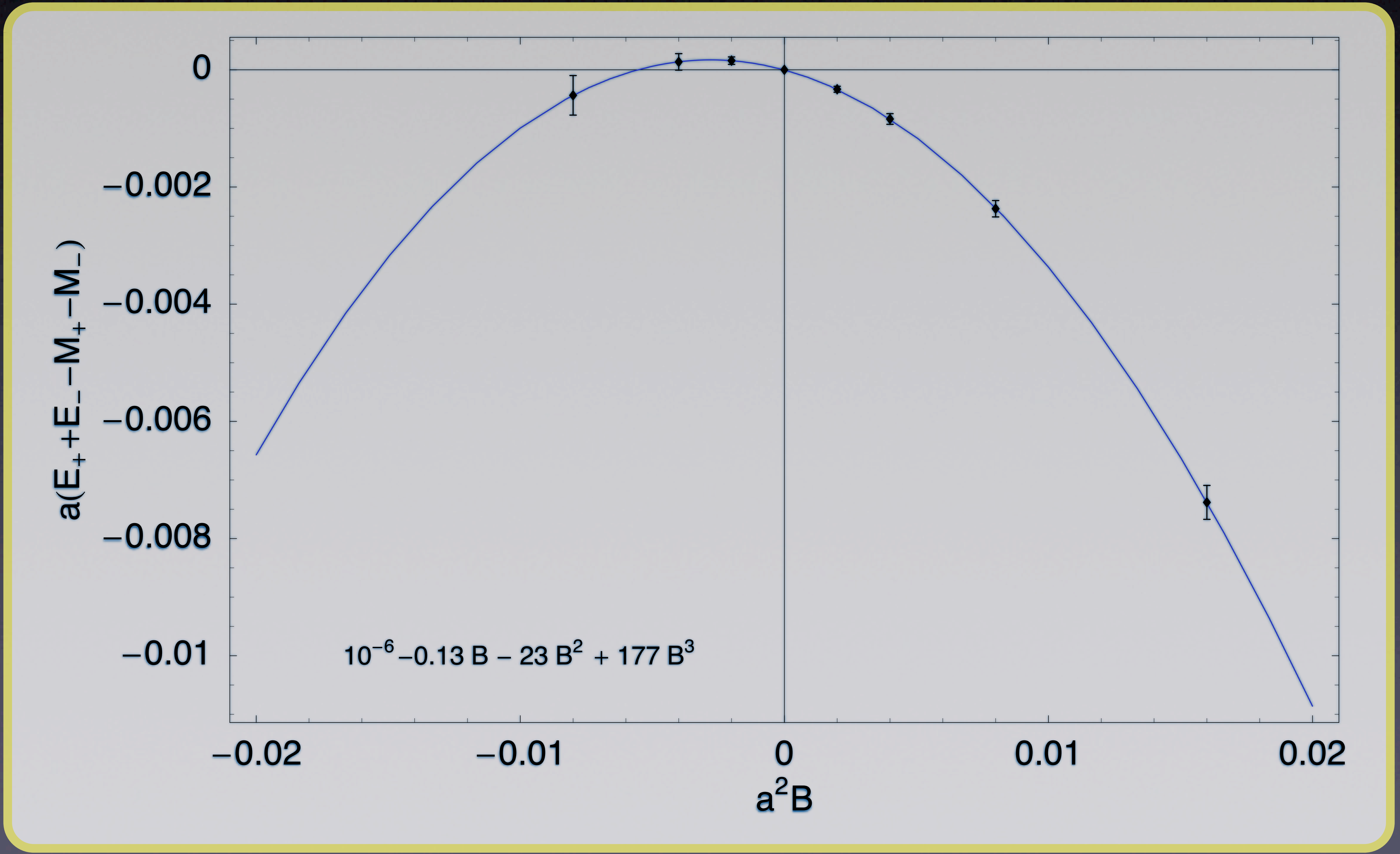
$$\frac{C_{\uparrow\uparrow}(\vec{p},\tau;A)}{C_{\downarrow\downarrow}(\vec{p},\tau;A)} = \exp\left[\frac{2\pi}{a} a_6\, b_6\, \gamma_{E_1E_1}\, \tau\right] + \ldots$$

Quenched polarisabilities

- External field calculations of magnetic moments and EM polarisabilities have a long history
  - Martinelli *et al.*, Bernard *et al.*: μ for n, p, Δ [83]
  - Fiebig *et al.*: α for neutron [89]
  - Christensen *et al.*: α for uncharged particles [05]
  - Lee *et al.*: μ for baryons [05]
  - Lee *et al.*: β for many baryons and mesons [05]
  - Preliminary work on spin polarisabilities

Quenched magnetic polarisabilities

- Use eight "weak" field values

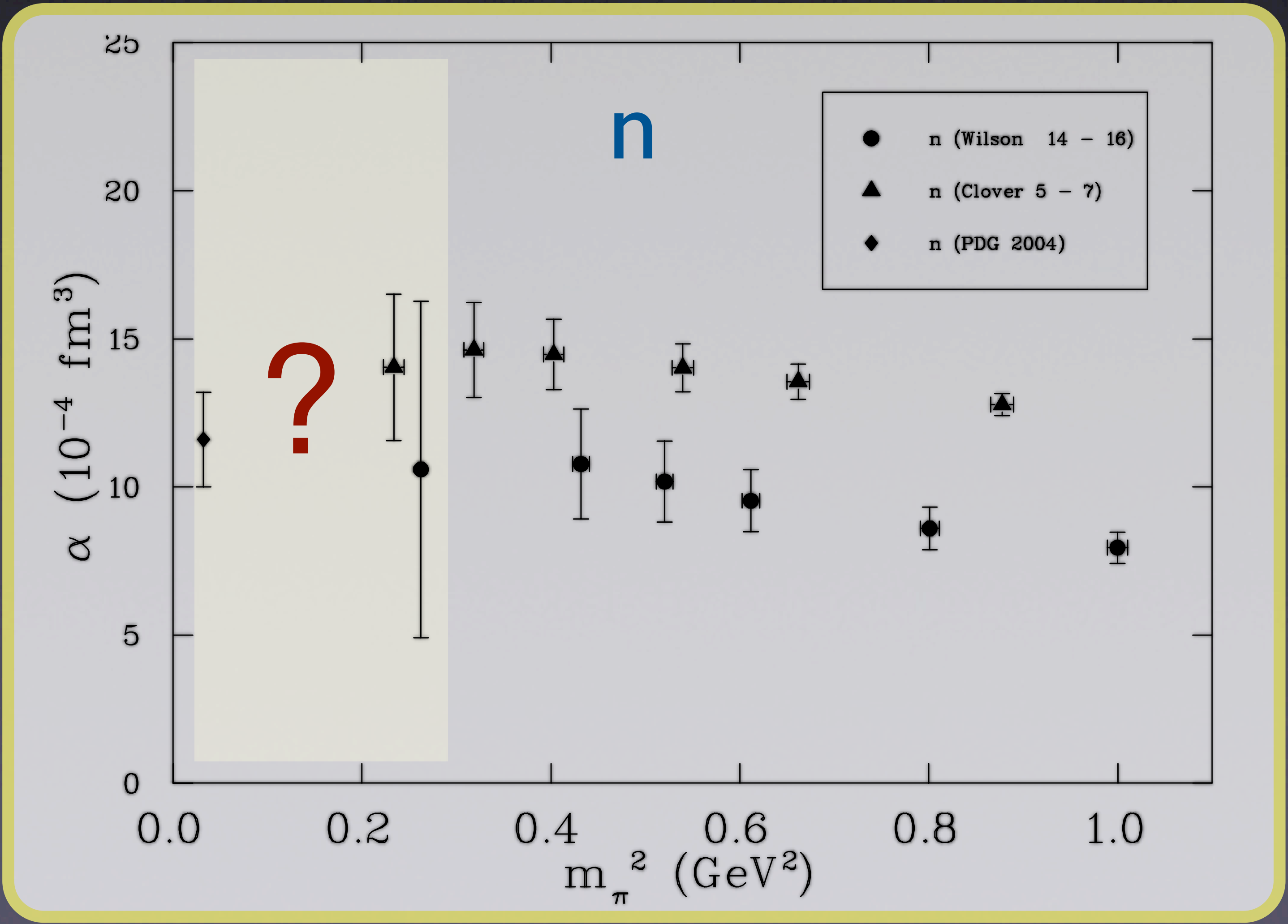


[WD, A Walker-Loud, B Tiburzi 07]

Quenched electric polarisabilities

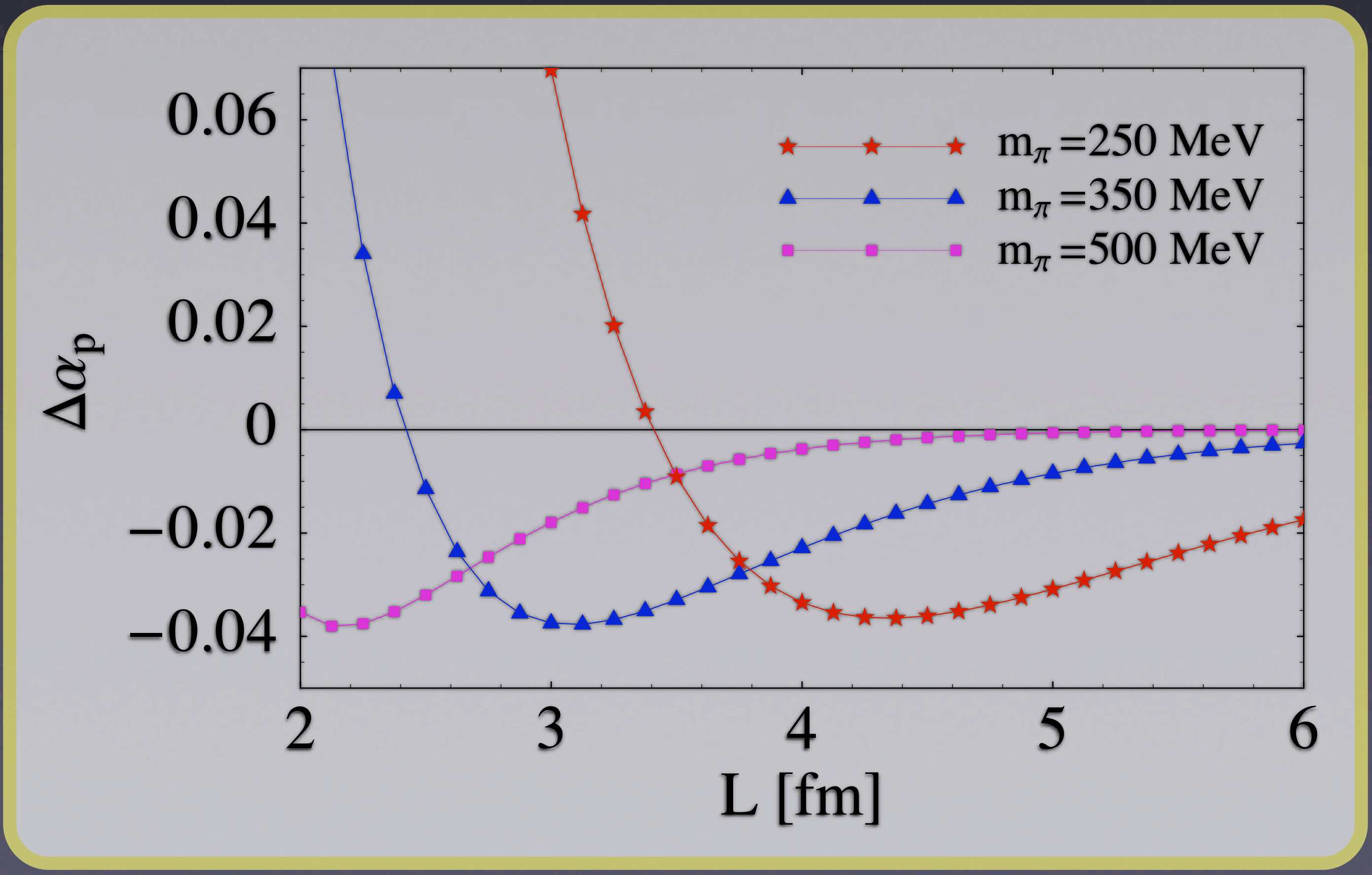
[Christensen *et al.*, hep-lat/0408024]

- Calculations with four field values (pos/neg)
- Neutral particles n, $\Sigma^0, \Xi^0, \Delta^0\, \Sigma^{*0}, \Xi^{*0}, \pi^0, K^0, \rho^0, K^{*0}$



Chiral perturbation theory

- Many studies of nucleon polarisabilities in the context of chiral perturbation theory
- Extended to partially-quenched χPT at finite volume
- Very IR sensitive
- Expect large FV effects



[WD, A Walker-Loud, B Tiburzi 06]

Unquenched calculations

- Full QCD results from unphysical calculations!
- PQQCD: ghost quarks cancel valence loops
- Electric charge matrix:

$$\mathrm{diag}\{q_u, q_d, q_s\}$$

$$\mathrm{diag}\{q_u, q_d, q_s, 0, 0, 0, q_u, q_d, q_s\}$$

$$\mathrm{diag}\{q_u, q_d, q_s, q_j, q_l, q_r, q_u, q_d, q_s\}$$

- Turn sea charges off (unphysical hadrons)
- Use chiral perturbation theory to reconstruct (some!!) physical hadrons (take QCD limit)

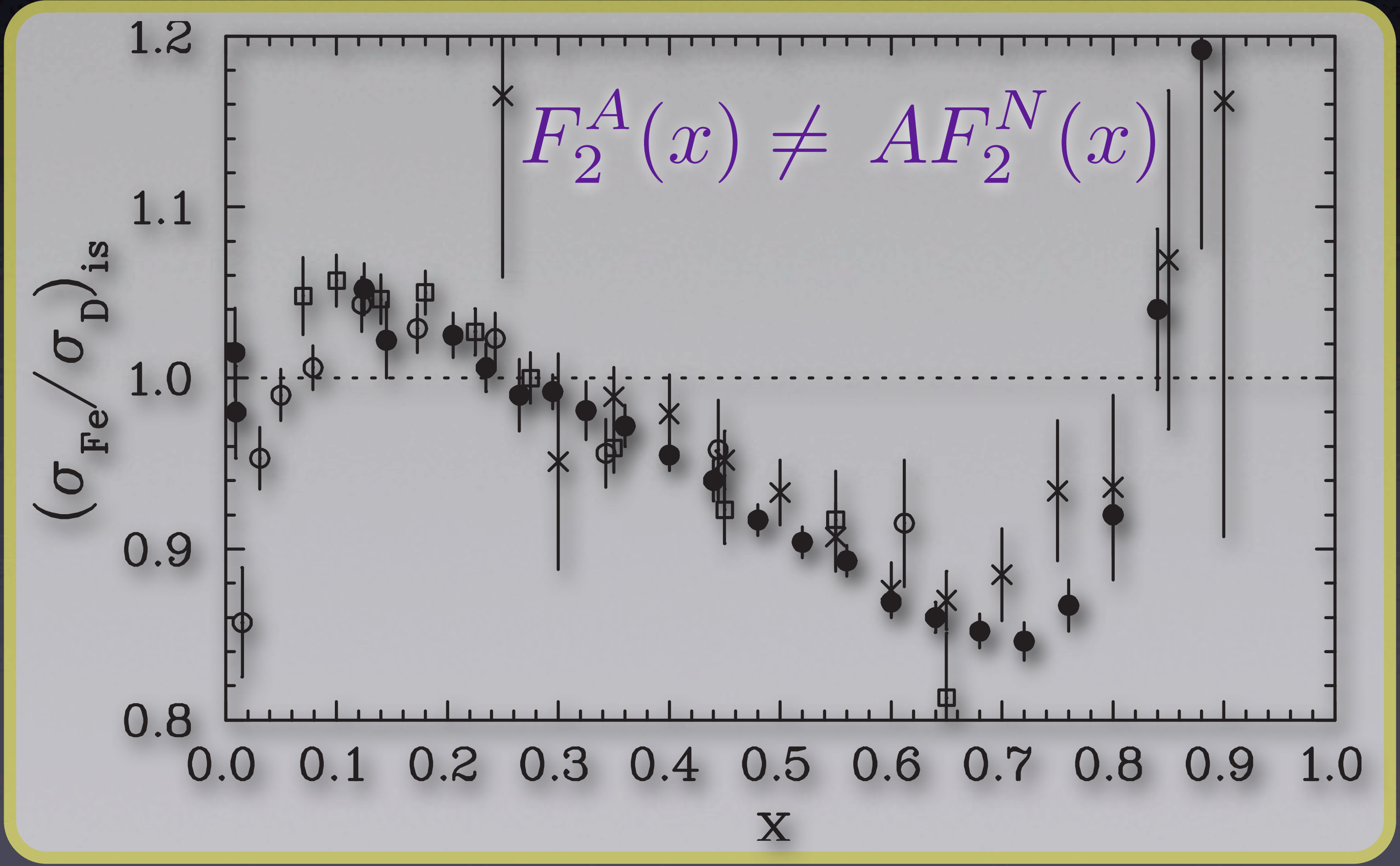
Lattice polarisabilities

- All EM and spin polarisabilities can be measured with external fields
- Preliminary lattice calculations underway for spin polarisabilities
- Large volume effects and strong mass dependence require large volumes and small masses!
- Higher order and generalised polarisabilities [(doubly)-virtual Compton scattering] are also measurable
- Unquenched? *First calculations through USQCD this year*

Nuclear structure:
EMC effect

EMC effect

- EMC 1983: Modification of PDFs in nuclei



- Large effect was a surprise since $\epsilon/M \sim 1\%$
- Can be investigated using lattice QCD

EMC on the lattice

- Simplest manifestation:

$$R^d(x,Q^2) = \frac{F_2^d(x,Q^2)}{F_2^p(x,Q^2)+F_2^n(x,Q^2)} \neq 1$$

- LC OPE: DIS structure $\Rightarrow$ twist two operators

$$\langle H|\overline{\psi}\gamma^{\{\mu_1}D^{\mu_2}\dots D^{\mu_n\}}\psi|H\rangle = \langle x^n\rangle_H p^{\mu_1}\dots p^{\mu_n}$$
$$\langle x^n\rangle_H = \int_0^1 dx\, x^{n-1} q^H(x) \sim M_2^H(n)$$

- Shift in moments from nuclear EFT [JW Chen & WD 04]

$$\langle x^n\rangle_d = 2\langle x^n\rangle_N + \alpha_n \langle d|(N^\dagger N)^2|d\rangle + \dots$$

EMC on the lattice

- Lattice methods combined with EFT can be used to investigate the EMC effect
- Measure shifts in two-particle energy levels in external field coupled to twist-two operators

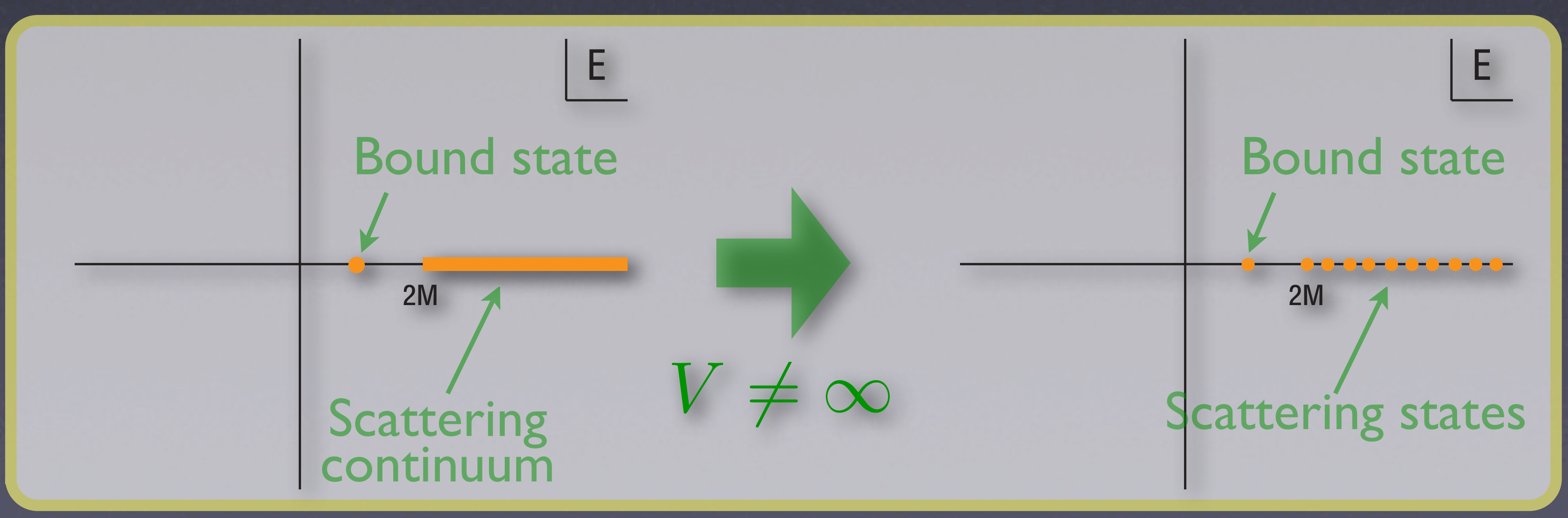
$$S_{QCD} \longrightarrow S_{QCD} + \int d^4x \, \underbrace{\Omega_{\mu_1 \ldots \mu_n}(x)}_{\text{external field}} \, \underbrace{\overline{\psi}(x)\gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}}\psi(x)}_{\text{twist-2 operator}}$$

- Determines two body coefficient α_n
  - Leading medium modification of moments of PDFs
  - Extend to larger A using nuclear EFT

[WD Phys Rev D71 054506]

Finite volume energies

- Maiani-Testa: *impossible to get Minkowski space S-matrix elements from infinite volume Euclidean space Monte-Carlo calculations*
- Lüscher [86]: *two-particle energy levels at finite volume related to scattering amplitude*



Finite volume energies

- Energies satisfy eigenvalue equation [Lüscher 86]

$$p \cot \delta(p) - \frac{1}{\pi L} S\left(\frac{L^2 p^2}{4\pi^2}\right) = 0$$

$$S(x) = \sum_{\vec{n}}^{\Lambda} \frac{1}{|\vec{n}|^2 - x} - 4\pi\Lambda$$

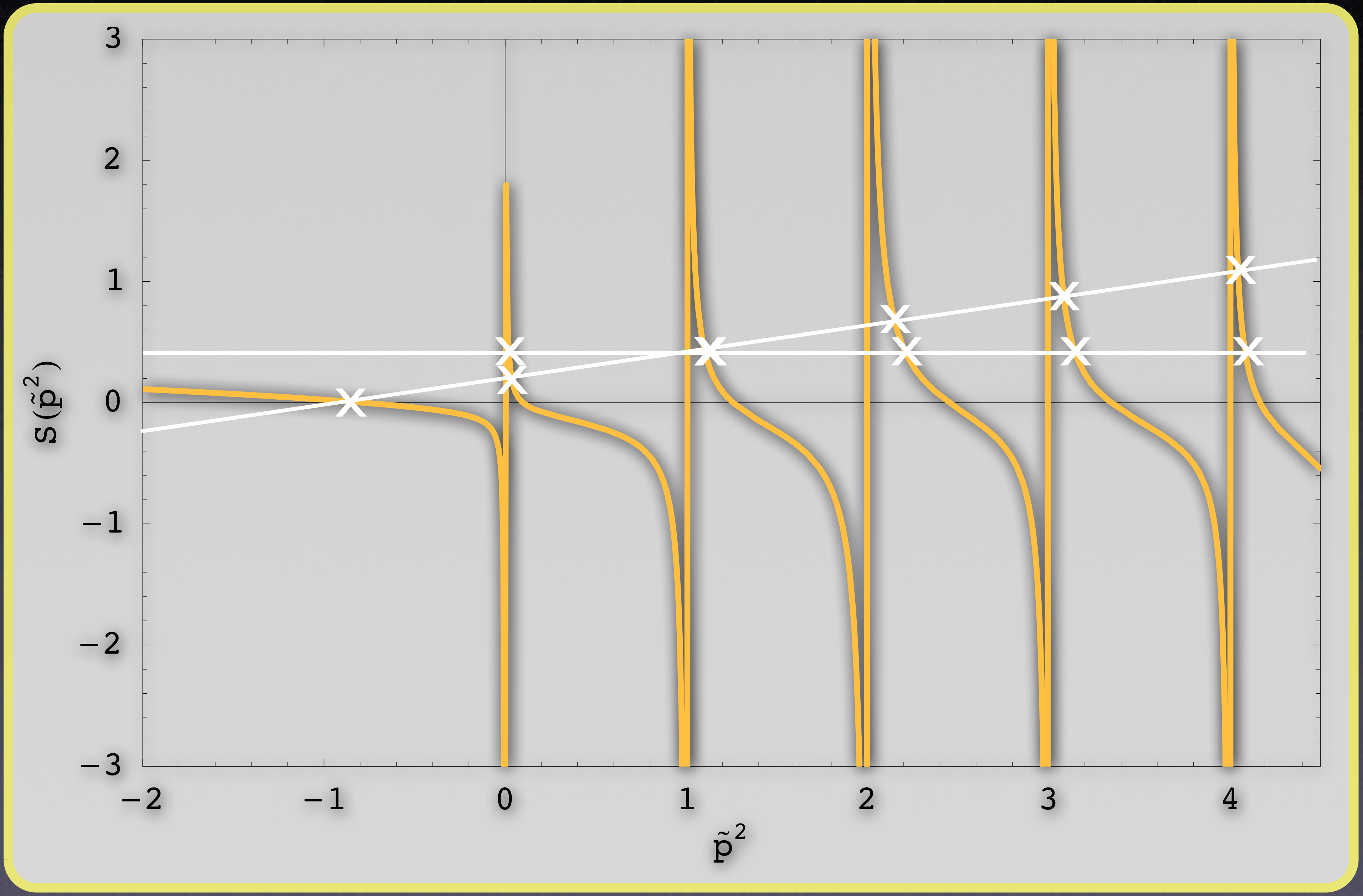
- Eg: lowest energy level (zero rel. mom.)

known coefficients

$$E_0 = \frac{4\pi a}{M L^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \ldots\right]$$

- Calculation of energy levels on the lattice determines scattering parameters

Two particle energies



Energy levels in BF

- Background field modifies eigenvalue equation

$$p \cot \delta - \frac{1}{\pi L} S\left(\frac{L^2}{4\pi^2} \left[p^2 \pm eB\kappa_0 \right] \right) \mp \frac{eB}{2}(L_2 - r_3 \kappa_0) = 0$$

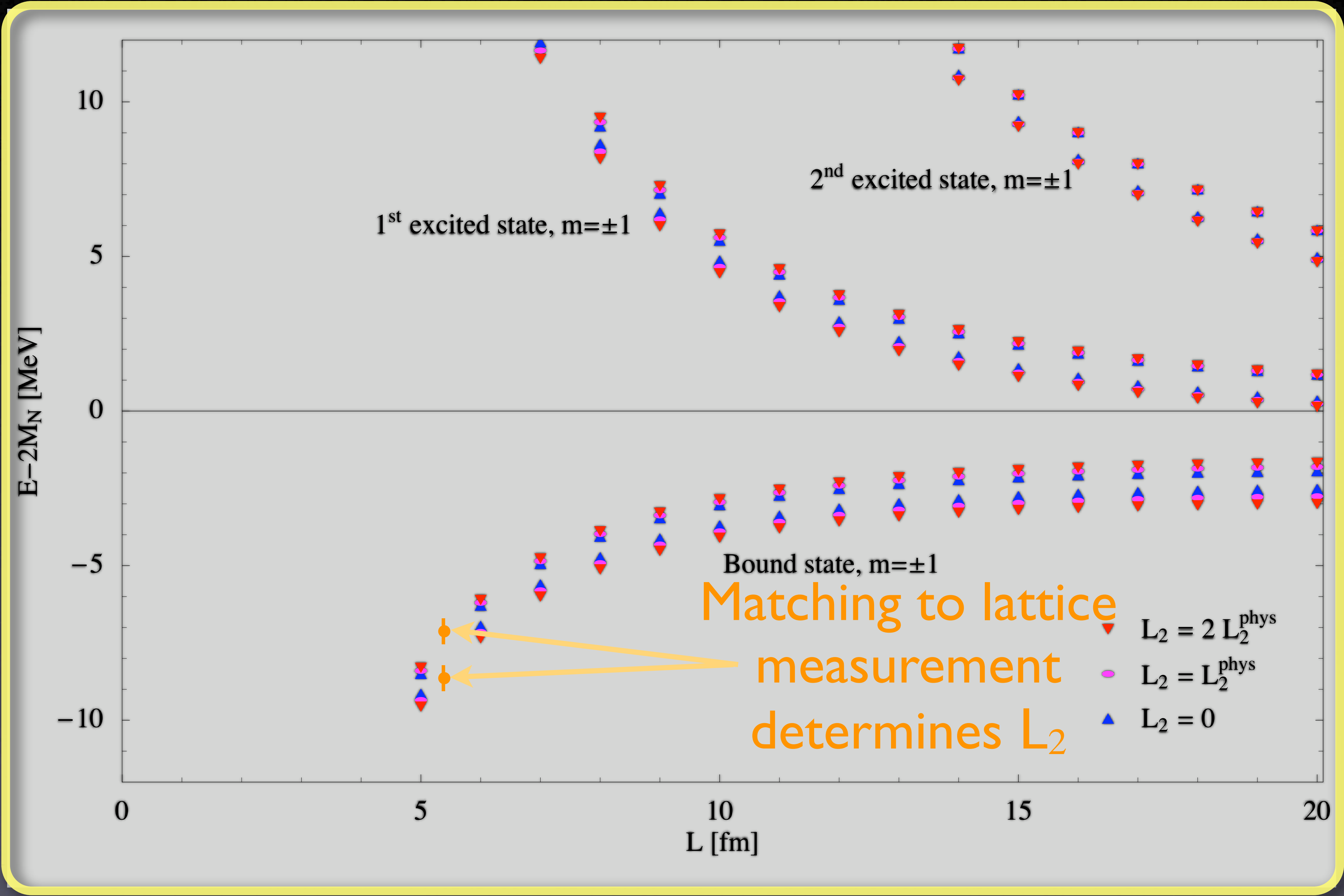
- Asymptotic expansion of lowest scattering level

$$E_0^{(m=\pm 1)} = \mp \frac{eB_0}{M}\kappa_0 + \frac{4\pi A_3}{ML^3}\left[1 - c_1 \frac{A_3}{L} + c_2 \left(\frac{A_3}{L} \right)^2 + \dots \right]$$

where $$\frac{1}{A_3} = \frac{1}{a_3} \pm \frac{eB_0}{2} L_2$$

[WD & MJ Savage Nucl Phys A 743, 170]

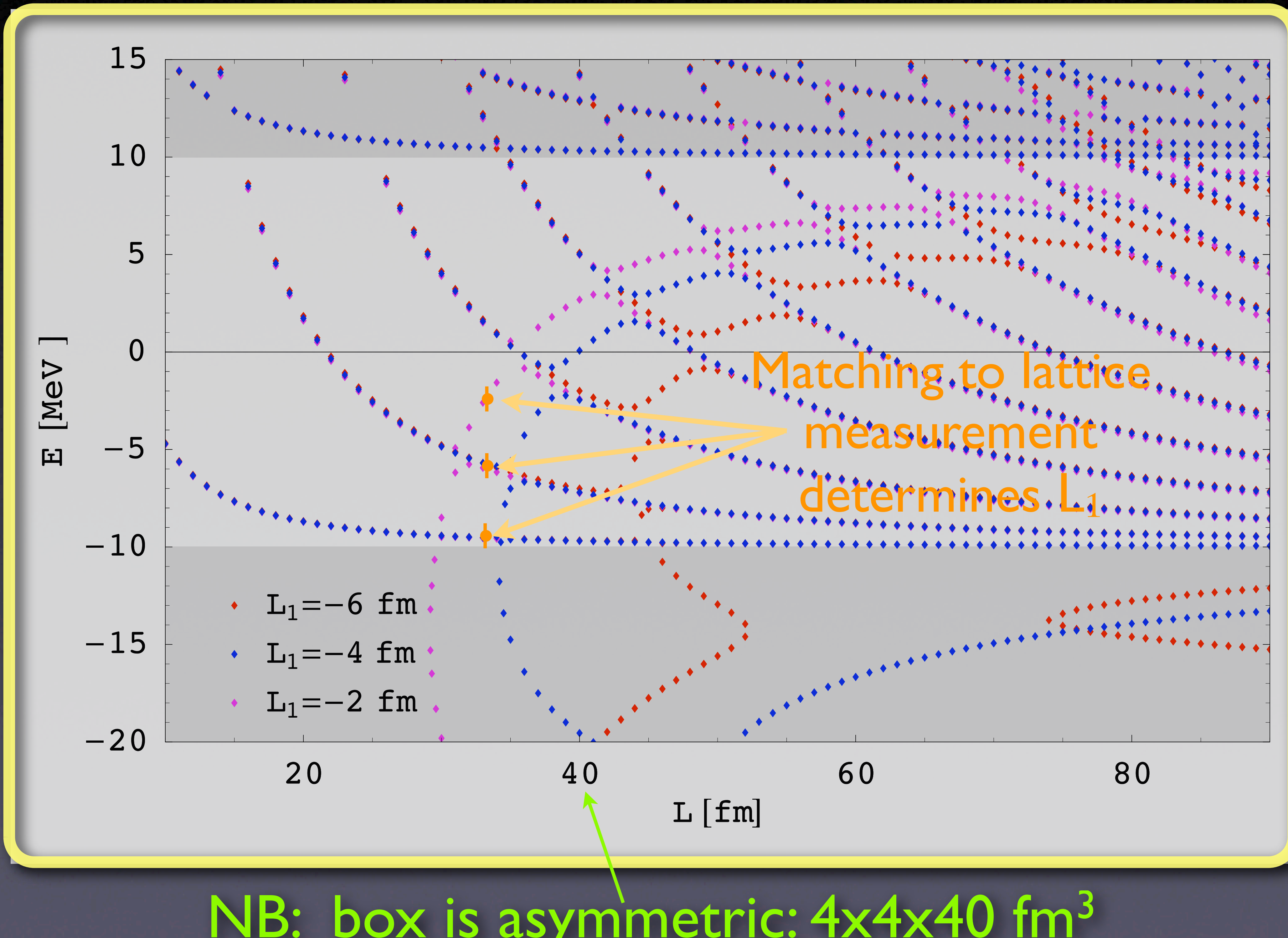
Energy levels in B field



|e B| = 1000 MeV²

EFT prediction for behaviour of *m*=±1 energy levels

Energy levels in B field



$|e B| = 1000\ MeV^2$

Reality sets in

- Are these measurements feasible? ✓?
- NPLQCD Collab. 2006 measurement of lowest $\Delta E = E_{2N} - 2E_N$ in $^3S_1$ and $^1S_0$ NN channels: PRL.97,012001
  - roughly $\Delta E = 40 \pm 5$ MeV (error 10x too large)
- Lighter quark masses $\Rightarrow$ bigger volumes
- Technical advances: anisotropic lattices, Lüscher-Wolff, *algorithms for multiple external fields?* ...

EMC effect

- Can also consider polarised/transversity cases
- Modified gauge fields also give
  - ➡ Moments of singlet quark PDFs/GPDs
  - ➡ J_q in the proton (Ji's sum rule)
- Extension to A>2 on lattice should be possible
- *Long term*: combine with nuclear EFT to assess nuclear effects in NuTeV anomaly: $F_3^A(x)$

Lattice nuclear physics

- Similar techniques allow investigation of
  - Cross-section for neutrino breakup of the deuteron (relevant for calibration of SNO)
  - ε_d vs quark masses (isovector can be done now)
  - Deuteron polarisabilites
  - Far future: $0\nu-\beta\beta$ decay nuclear matrix elements

More external fields

- Electric dipole moments **[Shintani et al. hep-lat/0611032]**
- Hadronic parity violation in baryons
  - PV NN interactions: $h_{\pi NN}$
  - Anapole moments of nucleon **[WD, D O'Connell in progress]**

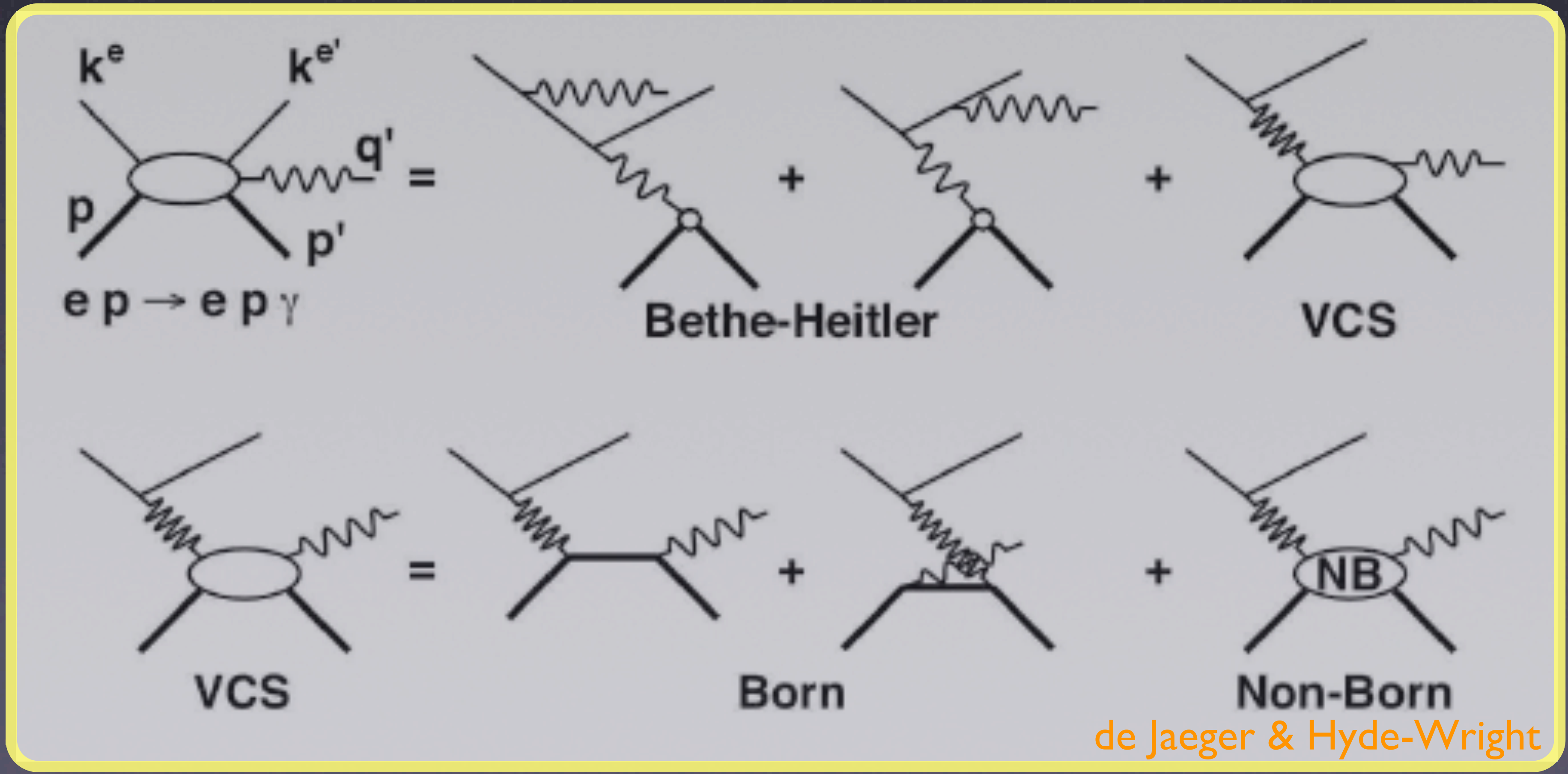
Summary

- Strong interaction physics is a vital field
- Lattice field theory is the tool of choice
- *External fields in lattice QCD provide a novel technique for investigating strongly interacting aspects of the SM and beyond*
- Examples
  - Structure of the proton: polarisabilities
  - Structure of the nucleus: EMC effect

The End.

Further polarisabilities

- Higher orders in the frequency expansion gives higher order polarisabilities [Holstein *et al.* ‘99]
- Virtual and doubly virtual Compton scattering leads to generalised polarisabilities [Guichon, Liu & Thomas ‘95]



To be more specific...

$$T_{\gamma N} = A_1(\omega,\theta)\,\vec{\epsilon}^{\,\prime}\cdot\vec{\epsilon} + A_2(\omega,\theta)\,\vec{\epsilon}^{\,\prime}\cdot\hat{k}\,\vec{\epsilon}\cdot\hat{k}' + i\,A_3(\omega,\theta)\,\vec{\sigma}\cdot(\vec{\epsilon}^{\,\prime}\times\vec{\epsilon}) + i\,A_4(\omega,\theta)\,\vec{\sigma}\cdot(\hat{k}'\times\hat{k})\,\vec{\epsilon}^{\,\prime}\cdot\vec{\epsilon}$$

$$+i\,A_5(\omega,\theta)\,\vec{\sigma}\cdot\left[(\vec{\epsilon}^{\,\prime}\times\hat{k})\,\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k}')\,\vec{\epsilon}^{\,\prime}\cdot\hat{k}\right] + i\,A_6(\omega,\theta)\,\vec{\sigma}\cdot\left[(\vec{\epsilon}^{\,\prime}\times\hat{k}')\,\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k})\,\vec{\epsilon}^{\,\prime}\cdot\hat{k}\right]$$

$$A_1(\omega,\theta) = -Z^2\frac{e^2}{M_N} + \frac{e^2}{4M_N^3}\left(\mu^2(1+\cos\theta) - Z^2\right)(1-\cos\theta)\,\omega^2 + 4\pi(\alpha+\beta\,\cos\theta)\omega^2 + \mathcal{O}(\omega^4)$$

$$A_2(\omega,\theta) = \frac{e^2}{4M_N^3}(\mu^2 - Z^2)\omega^2\cos\theta - 4\pi\beta\omega^2 + \mathcal{O}(\omega^4)$$

$$A_3(\omega,\theta) = \frac{e^2\omega}{2M_N^2}\left(Z(2\mu - Z) - \mu^2\cos\theta\right) + 4\pi\omega^3(\gamma_1 - (\gamma_2 + 2\gamma_4)\,\cos\theta) + \mathcal{O}(\omega^5)$$

$$A_4(\omega,\theta) = -\frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_2 + \mathcal{O}(\omega^5)$$

$$A_5(\omega,\theta) = \frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_4 + \mathcal{O}(\omega^5)$$

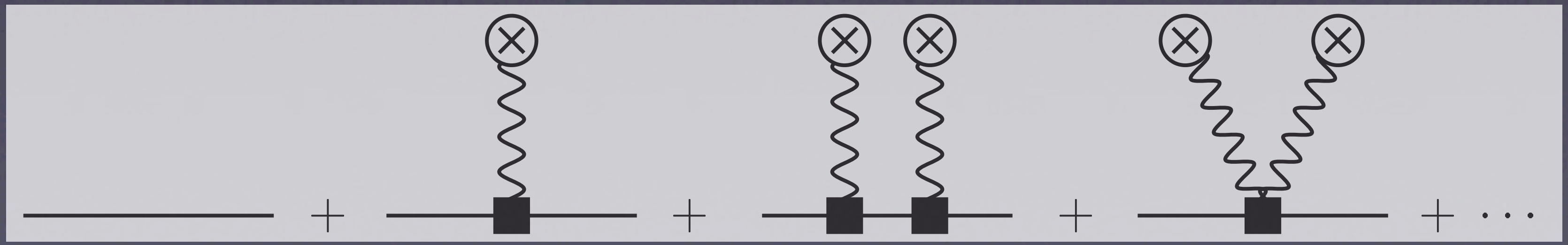
$$A_6(\omega,\theta) = -\frac{e^2\omega}{2M_N^2}Z\mu + 4\pi\omega^3\gamma_3 + \mathcal{O}(\omega^5)$$

EFT correlators

- Pionless effective field theory: cutoff $p < m_\pi$
- Lagrangian

$$\mathcal{L}_{\text{eff}}(\vec{x},\tau;A) = \Psi^\dagger(\vec{x},\tau)\Bigg[\left(\frac{\partial}{\partial\tau} + i\,q\,A_4\right) + \frac{(-i\vec{\nabla} - q\,\vec{A})^2}{2M} - \mu\,\vec{\sigma}\cdot\vec{H}$$
$$+2\pi\left(\alpha\,\vec{E}^2 - \beta\,\vec{H}^2\right) - 2\pi i\Big(- \gamma_{E_1E_1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}}$$
$$+\gamma_{M_1M_1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} + \gamma_{M_1E_2}\sigma^i E^{ij} H^j + \gamma_{E_1M_2}\sigma^i H^{ij} E^j\Big)\Bigg]\Psi(\vec{x},\tau) + \ldots$$

- Resum interactions with external field

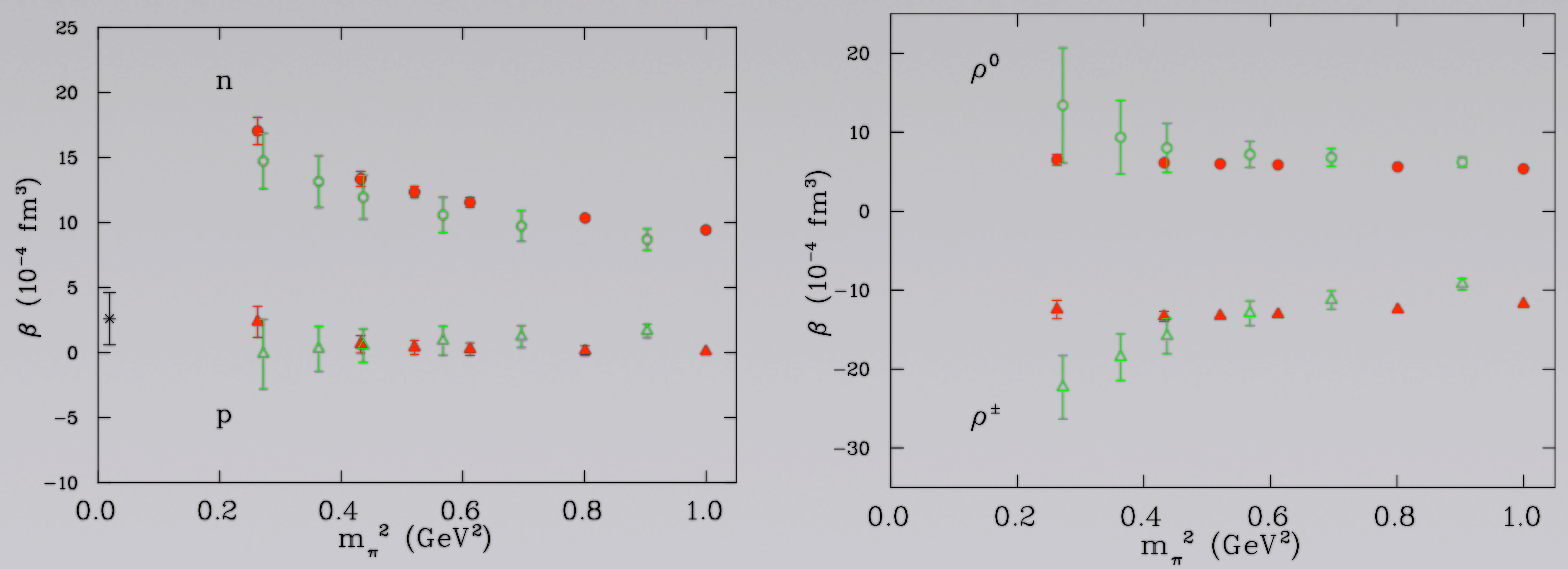


Quenched magnetic polarisabilities

[Lee *et al.*, hep-lat/0509065]

- Calculated for many hadrons

$n,\ p,\ \Sigma^{\pm,0},\ \Xi^{0,-},\ \Delta^{++,\pm,0}\ \Sigma^{*\pm,0},\ \Xi^{*0,-},\ \Omega,\ \pi^{\pm,0},\ K^{\pm,0},\ \rho^{\pm,0},\ K^{*\pm,0}$



Asymptotic expansions

- Other energy levels

$$E_0 = \frac{4\pi a}{ML^3}\left[1 - c_1\left(\frac{a}{L}\right) + c_2\left(\frac{a}{L}\right)^2 + \ldots\right]$$

$$E_1 = \frac{4\pi^2}{ML^2} - \frac{12\tan\delta_0}{ML^2}\left[1 + c_1'\tan\delta_0 + c_2'\tan\delta_0 + \ldots\right]$$

$$E_{-1}^{(^3S_1)} = -\frac{\gamma_0^2}{M}\left[1 + \frac{12}{\gamma_0 L(1-\gamma_0 r_3)}e^{-\gamma_0 L} + \ldots\right]$$

- Expansions also for L/a [Beane et al.]

Landau levels

- Infinite volume: transverse single-particle modes are LL

$$\hat{H} = \frac{|\hat{\mathbf{p}}|^2}{2M} + \frac{1}{2}M\omega^2(\hat{x}^2 + \hat{y}^2) + \frac{eB_0}{2M}\,\hat{l}_z \qquad \omega = |\frac{eB_0}{2M}|$$

$$E_{p\uparrow}^{(n)}(B_0) = M + \frac{|eB_0|}{M}\left(n + \frac{1}{2}\right) + \frac{p_z^2}{2M} + \mu_p B_0 + 4\pi\frac{\beta_p}{2}B_0^2$$

- Finite volume: potential bounded & perturbative
  - Require weak field: $|eB_0| \ll \dfrac{8\sqrt{3}\pi}{L_\perp^2}$
- Large z direction: low energy levels longitudinal

➡ Asymmetric boxes

Asymmetric boxes

- Asymmetry suppresses non-plane-wave-arity
- Eigenvalue equation modified

$$S(\tilde{p}^2) \longrightarrow S(\tilde{p}^2; \eta_1, \eta_2) = \frac{1}{\eta_1 \eta_2} \sum_{\tilde{\mathbf{n}}}^{\Lambda_n} \frac{1}{|\tilde{\mathbf{n}}|^2 - \tilde{p}^2} - 4\pi\Lambda_n$$

$$\tilde{\mathbf{n}} = (\frac{1}{\eta_1} n_1, \frac{1}{\eta_2} n_2, n_3)$$

- Asymptotic expansion

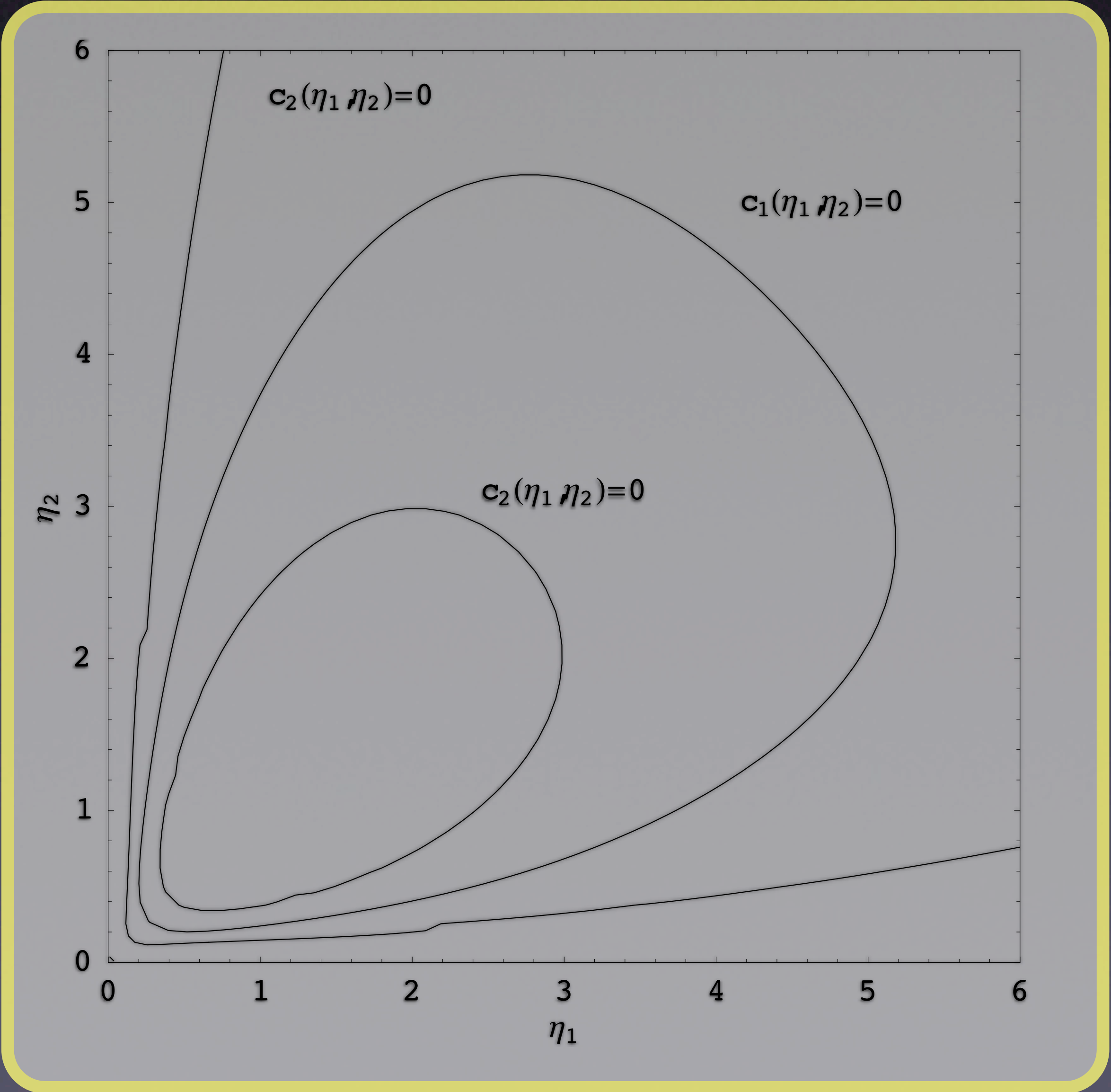
$$E_0 = \frac{4\pi a}{\eta_1 \eta_2 M L^3} \left[1 - c_1(\eta_1, \eta_2) \left(\frac{a}{L}\right) + c_2(\eta_1, \eta_2) \left(\frac{a}{L}\right)^2 + \ldots \right]$$

- Geometric coefficinets

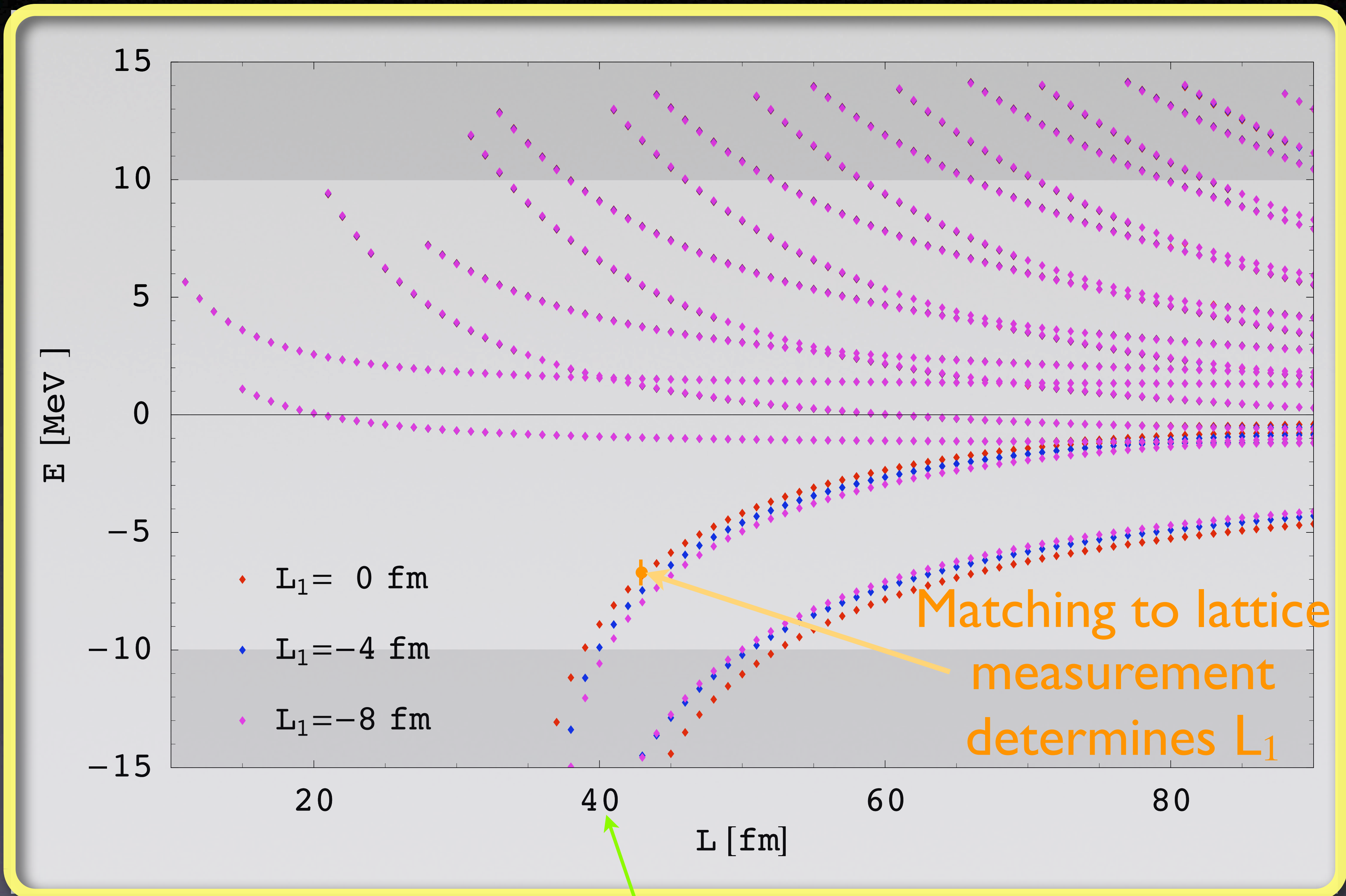
$$c_1(\eta_1, \eta_2) = \frac{1}{\pi} \left(\frac{1}{\eta_1 \eta_2} \sum_{\tilde{\mathbf{n}} \neq 0}^{\Lambda_n} \frac{1}{|\tilde{\mathbf{n}}|^2} - 4\pi\Lambda_n \right)$$

Magic boxes

- Asymmetries exist where subleading FV effects are suppressed: $c_i(\eta_1,\eta_2)=0$



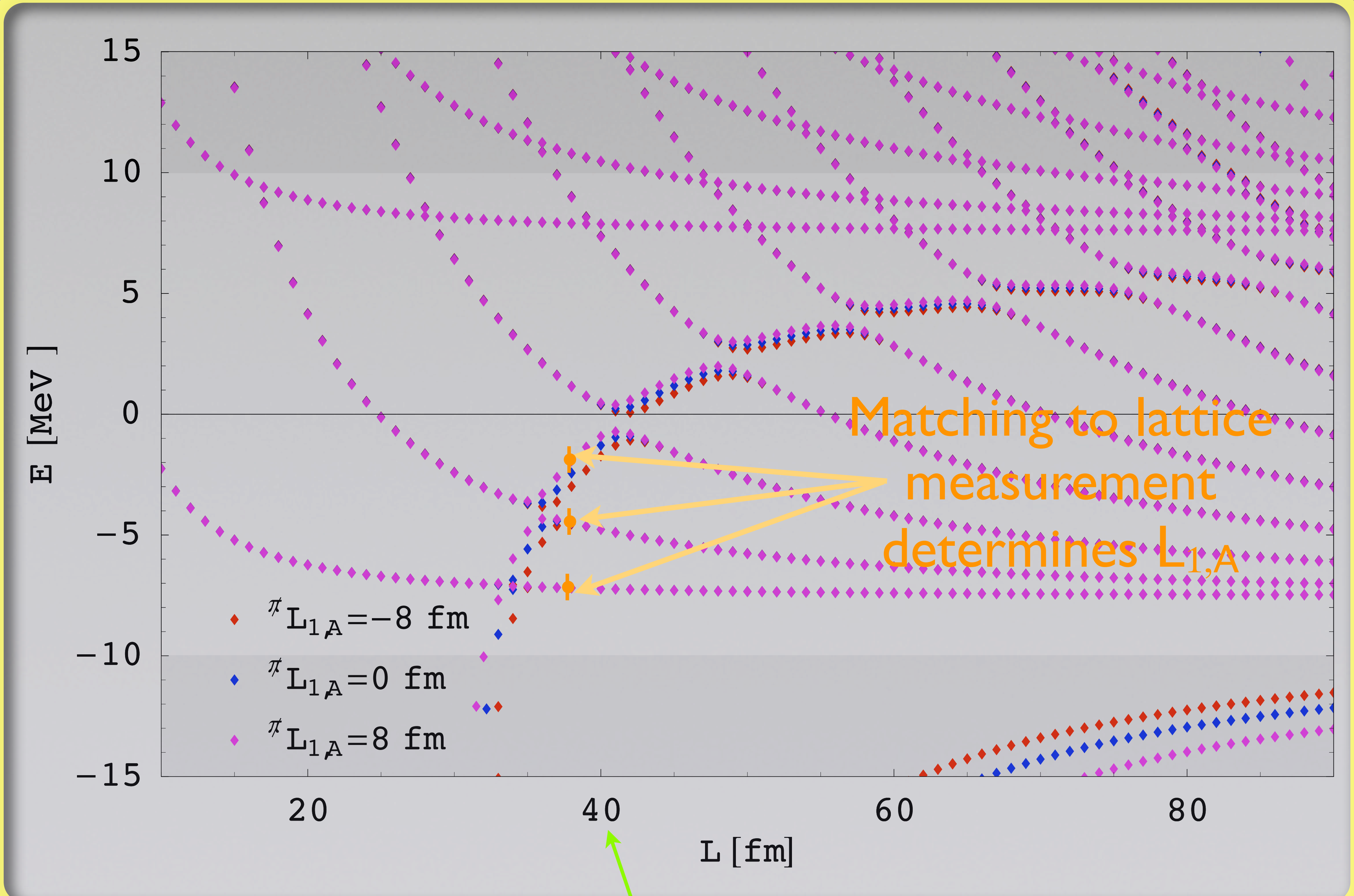
np→dγ: $^3S_1$–$^1S_0$ m=0



|e B| = 500 MeV^2

NB: box is asymmetric: 4x4x40 fm^3

$\nu d \to np$: EW BF



$|g_W| = 6$ MeV

NB: box is asymmetric: 4x4x40 fm^3

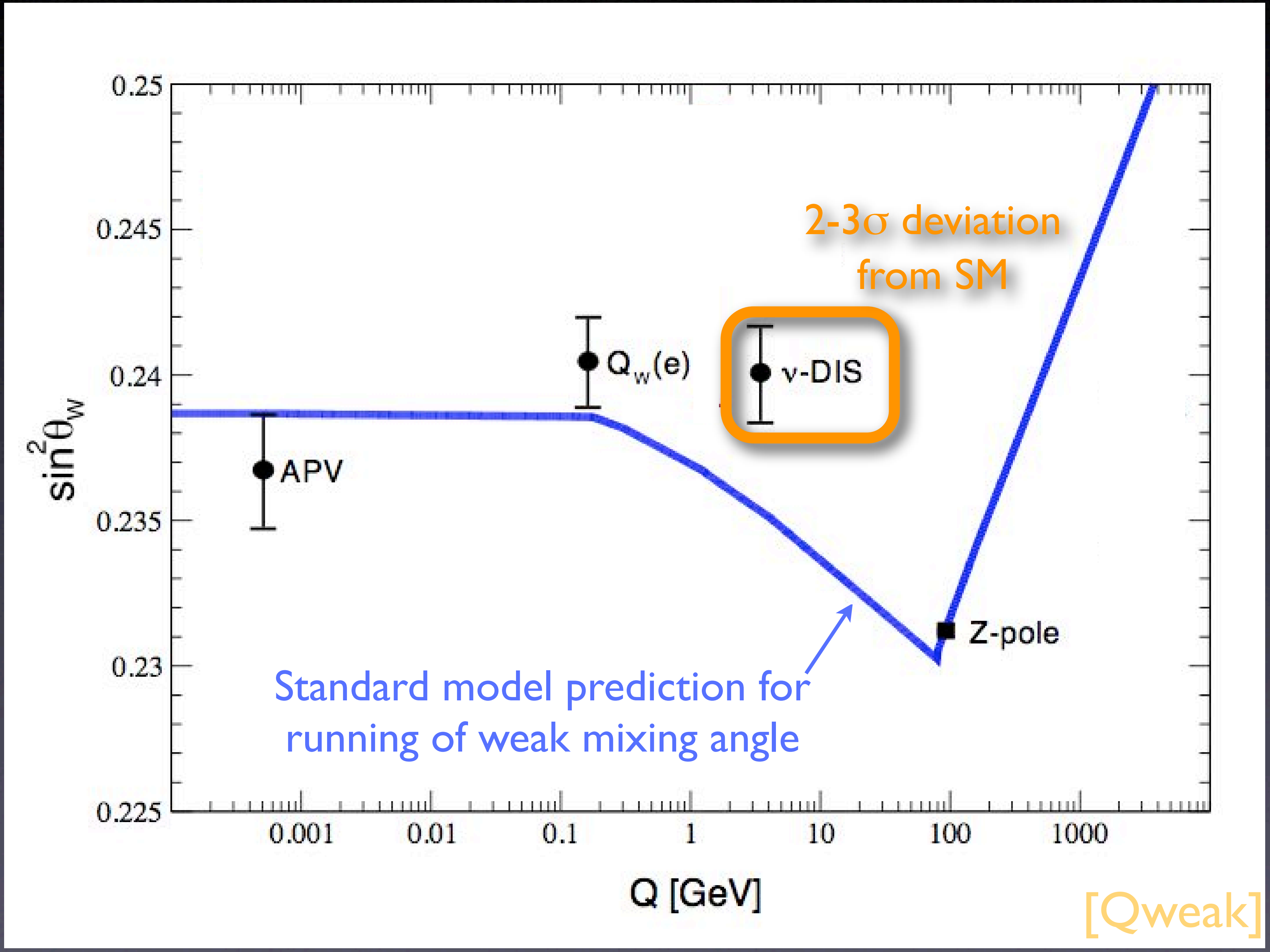
NuTeV anomaly: $\sin^2\theta_W$

- Neutrino deep-inelastic scattering
- Paschos-Wolfenstein relation:

$$R^- = \frac{\sigma_{NC}^{\nu N} - \sigma_{NC}^{\overline{\nu} N}}{\sigma_{CC}^{\nu N} - \sigma_{CC}^{\overline{\nu} N}} = \frac{1}{2} - \sin^2\theta_W$$

- NuTeV measure CC and NC neutrino scattering on steel target at Fermilab
- Extract the weak mixing angle

NuTeV anomaly: $\sin^2\theta_W$



NuTeV anomaly: $\sin^2\theta_W$

- Corrected Paschos-Wolfenstein relation:

$$R_A^- = \frac{\sigma_{NC}^{\nu A} - \sigma_{NC}^{\overline{\nu} A}}{\sigma_{CC}^{\nu A} - \sigma_{CC}^{\overline{\nu} A}} = \frac{1}{2} - \sin^2\theta_W + \epsilon_v + \epsilon_n + \epsilon_s + \epsilon_c$$

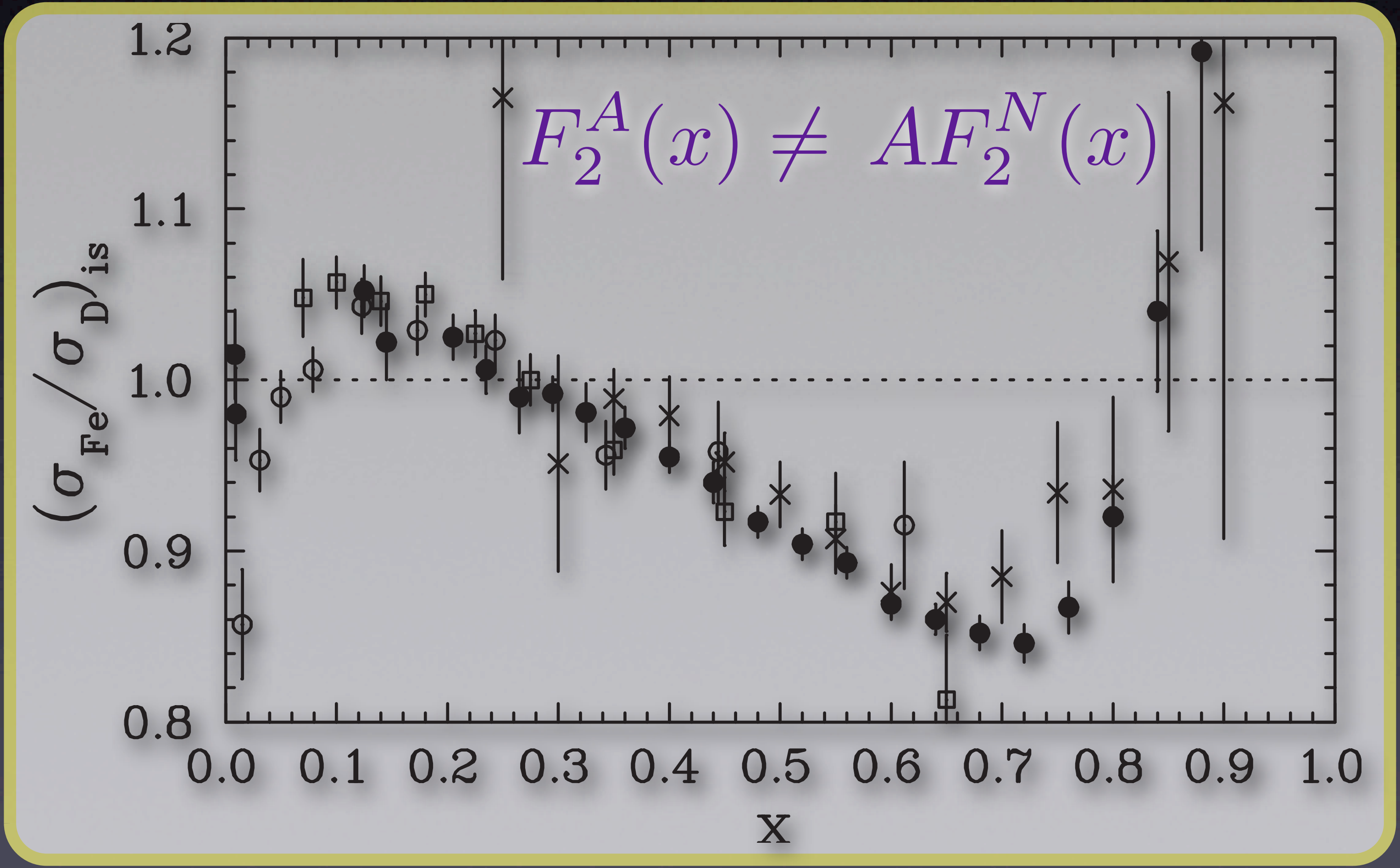
Nuclear modifications
Non-isoscalarity
Strange quarks
Charm quarks

- NuTeV take some of this into account
- Many authors find significant reduction in NuTeV significance from hadronic physics

Nuclear structure:
EMC effect

EMC effect

- EMC 1983: Modification of PDFs in nuclei



- Large effect was a surprise since $\epsilon/M \sim 1\%$
- Can be investigated using lattice QCD

Two nucleon states

- Brief introduction to two particles in LQCD
- Two particle scattering phase shift (ERE)

$$S = e^{2i\,\delta(p)} \qquad p\cot\delta(p) = -\frac{1}{a} + rp^2 + \ldots$$

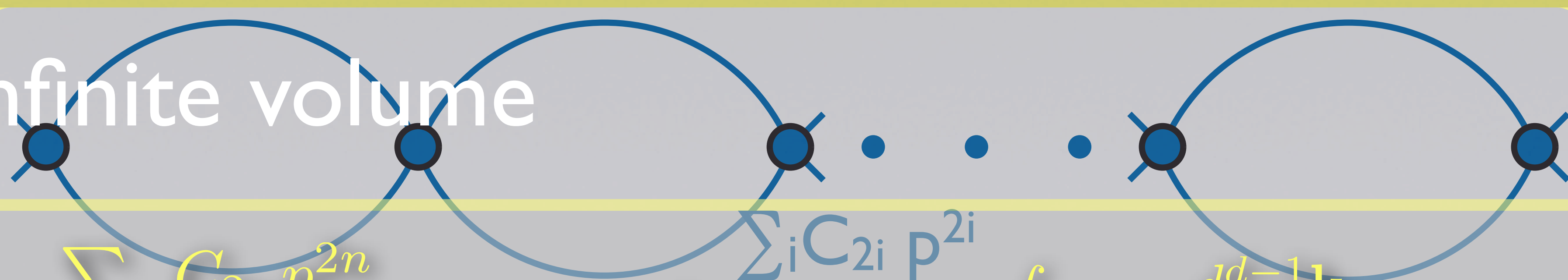
- $^1S_0$ channel: $a_1 = -23.7\,\mathrm{fm} \quad r_1 = 2.7\,\mathrm{fm}$
- $^3S_1(^3D_1)$ channel: $a_3 = 5.4\,\mathrm{fm} \quad r_3 = 1.8\,\mathrm{fm}$
  - Deuteron: $B = 2.2\,\mathrm{MeV}$

NN in EFT

- Extension of χPT to two nucleon systems well developed [Weinberg 90, ...]
- For |p| << m_π, pions can be integrated out

  ➡ theory of contact interactions amongst nucleons (similar to eff. range expansion)

  EFT($\not\pi$)
- Scattering amplitude given by bubble sum

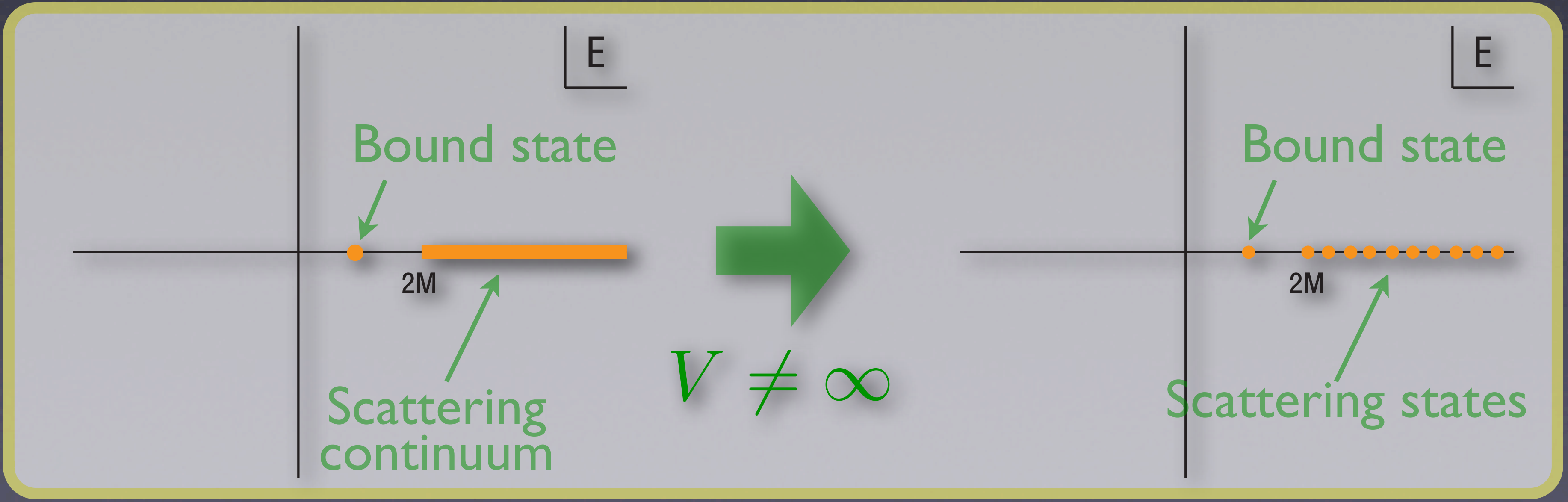


- Infinite volume

$$\mathcal{A} = \frac{\sum_n C_{2n} p^{2n}}{1 - I_0(p) \sum_n C_{2n} p^{2n}} \qquad I_0^{pds}(p) = \mu^{4-d} \int \frac{d^{d-1}\mathbf{k}}{E - |\mathbf{k}|^2/M + i\epsilon} = -\frac{M}{4\pi}(\mu + ip)$$

Finite volume energies

- Maiani-Testa: *impossible to get Minkowski space S-matrix elements from infinite volume Euclidean space Monte-Carlo calculations*
- Lüscher [86]: *two-particle energy levels at finite volume related to scattering amplitude*



Finite volume energies

$$\mathcal{A}(L) = \frac{\sum_n C_{2n} p^{2n}}{1 - I_0(L) \sum_n C_{2n} p^{2n}}$$

$$p \cot\delta(p) = \frac{4\pi}{M} \frac{1}{\sum_n C_{2n} p^{2n}} + \mu$$

$$0 = \mathcal{A}^{-1}(L) = p \cot\delta(p) - \frac{M\mu}{4\pi} - I_0^{pds}(L)$$

$$I_0^{pds}(L) = \frac{1}{L^3} \sum_{\mathbf{k}}^{pds} \frac{1}{E - |\mathbf{k}|^2/M}$$
$$= \frac{1}{L^3} \sum_{\mathbf{k}}^{\Lambda} \frac{1}{E - |\mathbf{k}|^2/M} + \int^{\Lambda} \frac{d^3\mathbf{k}/(2\pi)^3}{|\mathbf{k}|^2/M} - \int_{pds} \frac{d^3\mathbf{k}/(2\pi)^3}{|\mathbf{k}|^2/M}$$
$$= \frac{M}{4\pi} \left[-\frac{1}{\pi L} \sum_{\mathbf{n}}^{\Lambda} \frac{1}{|\mathbf{n}|^2 - \frac{L^2 E M}{4\pi^2}} - 4\frac{\Lambda}{L} - \mu \right]$$

Finite volume energies

- Energies satisfy eigenvalue equation [Lüscher 86]

$$p\cot\delta(p) - \frac{1}{\pi L} S\left(\frac{L^2 p^2}{4\pi^2}\right) = 0$$

$$S(x) = \sum_{\vec{n}}^{\Lambda} \frac{1}{|\vec{n}|^2 - x} - 4\pi\Lambda$$

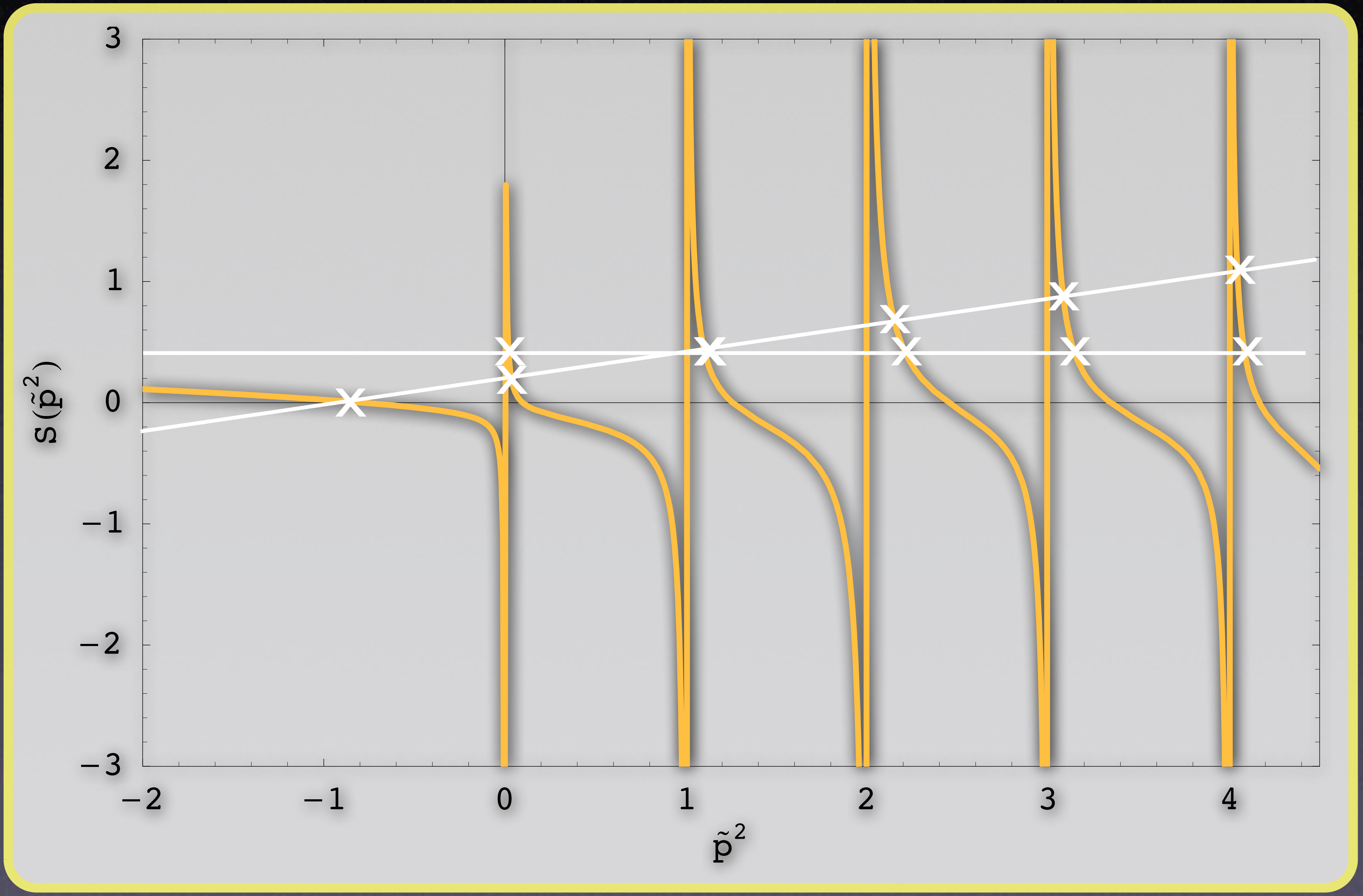
- Eg: lowest energy level (zero rel. mom.)

known coefficients

$$E_0 = \frac{4\pi a}{ML^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \ldots\right]$$

- Calculation of energy levels on the lattice determines scattering parameters

Two particle energies



EMC on the lattice

- Simplest manifestation:

$$R^d(x,Q^2) = \frac{F_2^d(x,Q^2)}{F_2^p(x,Q^2) + F_2^n(x,Q^2)} \neq 1$$

- LC OPE: DIS structure $\Rightarrow$ twist two operators

$$\langle H|\overline{\psi}\gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}}\psi|H\rangle = \langle x^n\rangle_H p^{\mu_1} \ldots p^{\mu_n}$$
$$\langle x^n\rangle_H = \int_0^1 dx\, x^{n-1} q^H(x) \sim M_2^H(n)$$

- Lattice methods combined with EFT *can* be used to investigate the EMC effect
  - Look for shift of two particle energy levels in external field coupled to twist two operators

[WD Phys Rev D71 054506

Background fields

- Demonstrate technique with electromagnetic fields
- Magnetic moment of the deuteron...

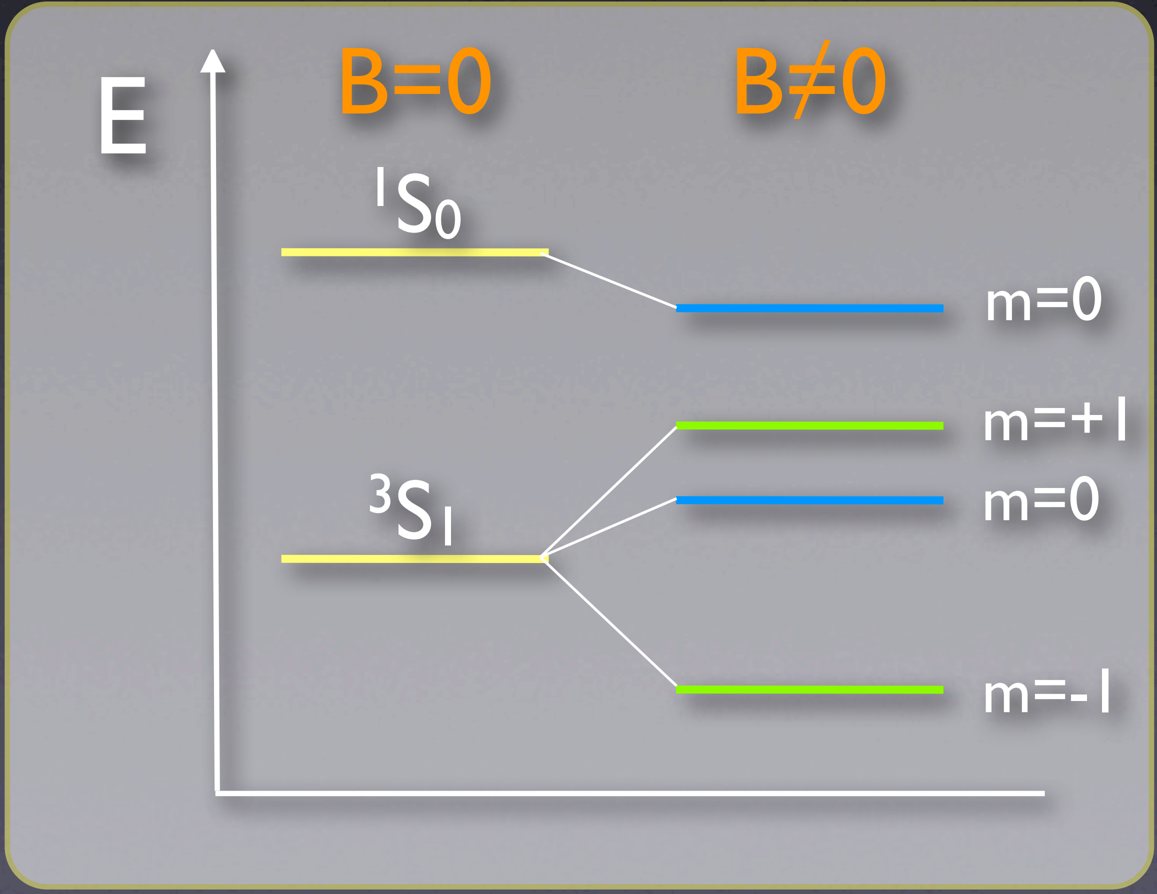
$$\mu_d = 0.8574382329(92)\mu_N$$

Background fields

- Constant magnetic fields shift spin-1/2 particle masses

$$\boxed{M_{\uparrow\downarrow} = M_0 \pm \mu|B| + 4\pi\beta|B|^2}$$

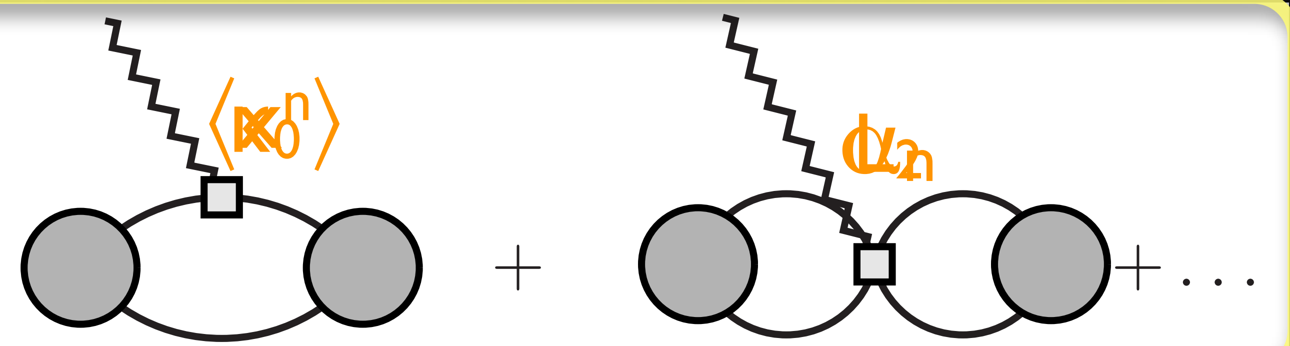
- Two nucleon states
  - Levels split and mix
  - Landau levels: consider asymmetric boxes
- Similar for electro-weak fields and twist-two fields



EFT two-body currents

- Two-body contributions

$$\langle d|\mathcal{O}|d\rangle =$$



- Magnetic moment: two body modification L_2

$$\mu_d = \frac{2}{1-\gamma r_3}(\gamma L_2 + \kappa_0)$$

- Twist-two current: leading EMC effect α_n (more complicated as necessary to include pions)

$$\langle x^n\rangle_d = 2\langle x^n\rangle_N + \alpha_n\langle d|(N^\dagger N)^2|d\rangle + \ldots$$

[JW Chen, & WD PLB 625, 165]

Energy levels in BF

- Background field modifies eigenvalue equation

$$p \cot\delta - \frac{1}{\pi L} S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm eB\kappa_0\right]\right) \mp \frac{eB}{2}(L_2 - r_3\kappa_0) = 0$$

- Asymptotic expansion of lowest scattering level

$$E_0^{(m=\pm 1)} = \mp \frac{eB_0}{M}\kappa_0 + \frac{4\pi A_3}{ML^3}\left[1 - c_1\frac{A_3}{L} + c_2\left(\frac{A_3}{L}\right)^2 + \dots\right]$$

where $$\frac{1}{A_3} = \frac{1}{a_3} \pm \frac{eB_0}{2} L_2$$

[WD & MJ Savage Nucl Phys A 743, 170]

Energy levels in BF

- Background field modifies eigenvalue equation

$$p\cot\delta - \frac{1}{\pi L} S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm eB\kappa_0\right]\right) \mp \frac{eB}{2}(L_2 - r_3\kappa_0) = 0$$

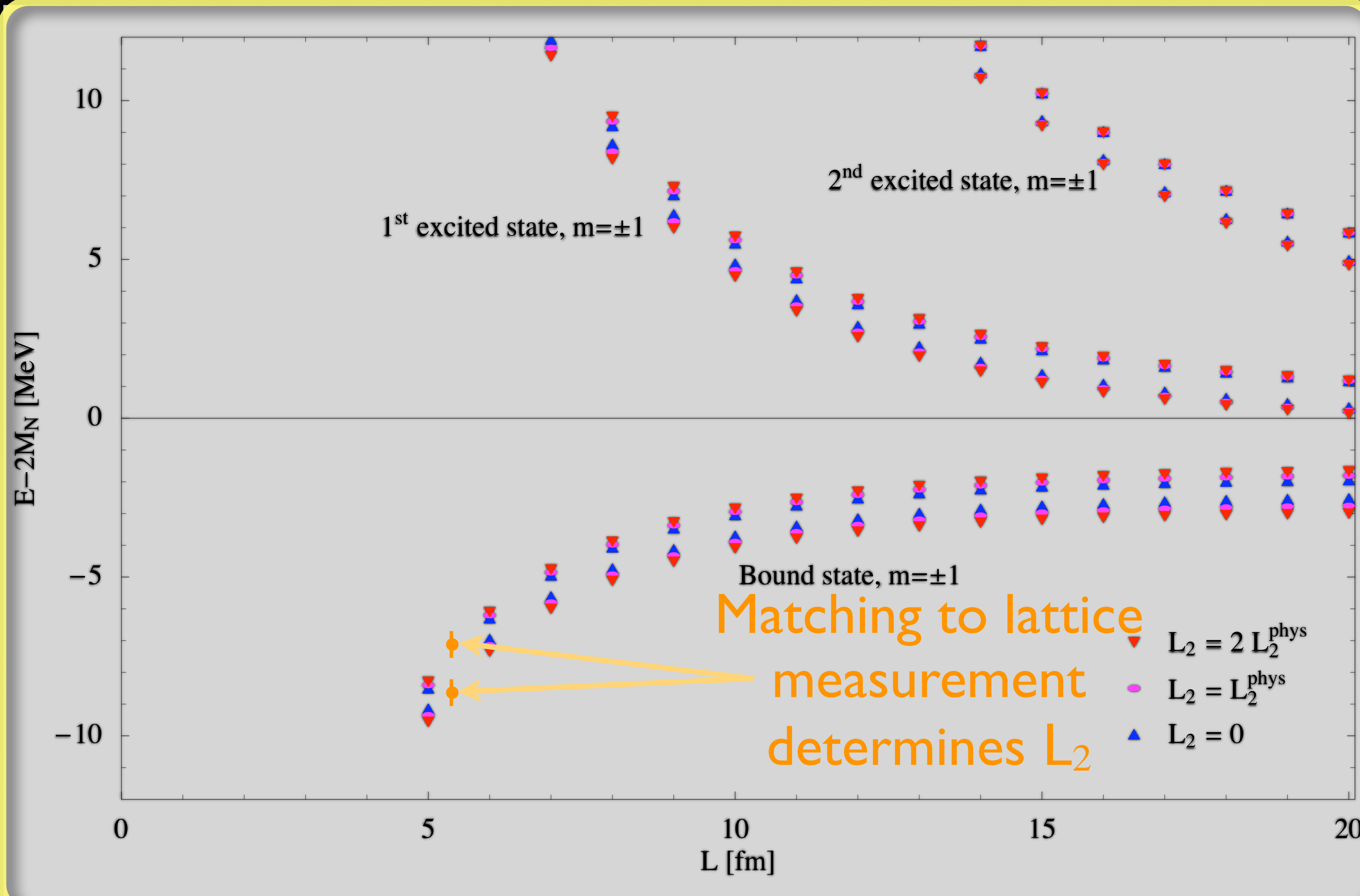
- Mixes $^1S_0$ and $^3S_1$ m=0 states (coupled channels)

$$\left[p\cot\delta_1 - \frac{S_+ + S_-}{2\pi L}\right]\left[p\cot\delta_3 - \frac{S_+ + S_-}{2\pi L}\right] = \left[\frac{eBL_1}{2} + \frac{S_+ - S_-}{2\pi L}\right]^2$$

where $S_\pm = S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm eB\kappa_1\right]\right)$

[WD & MJ Savage Nucl Phys A 743, 170]

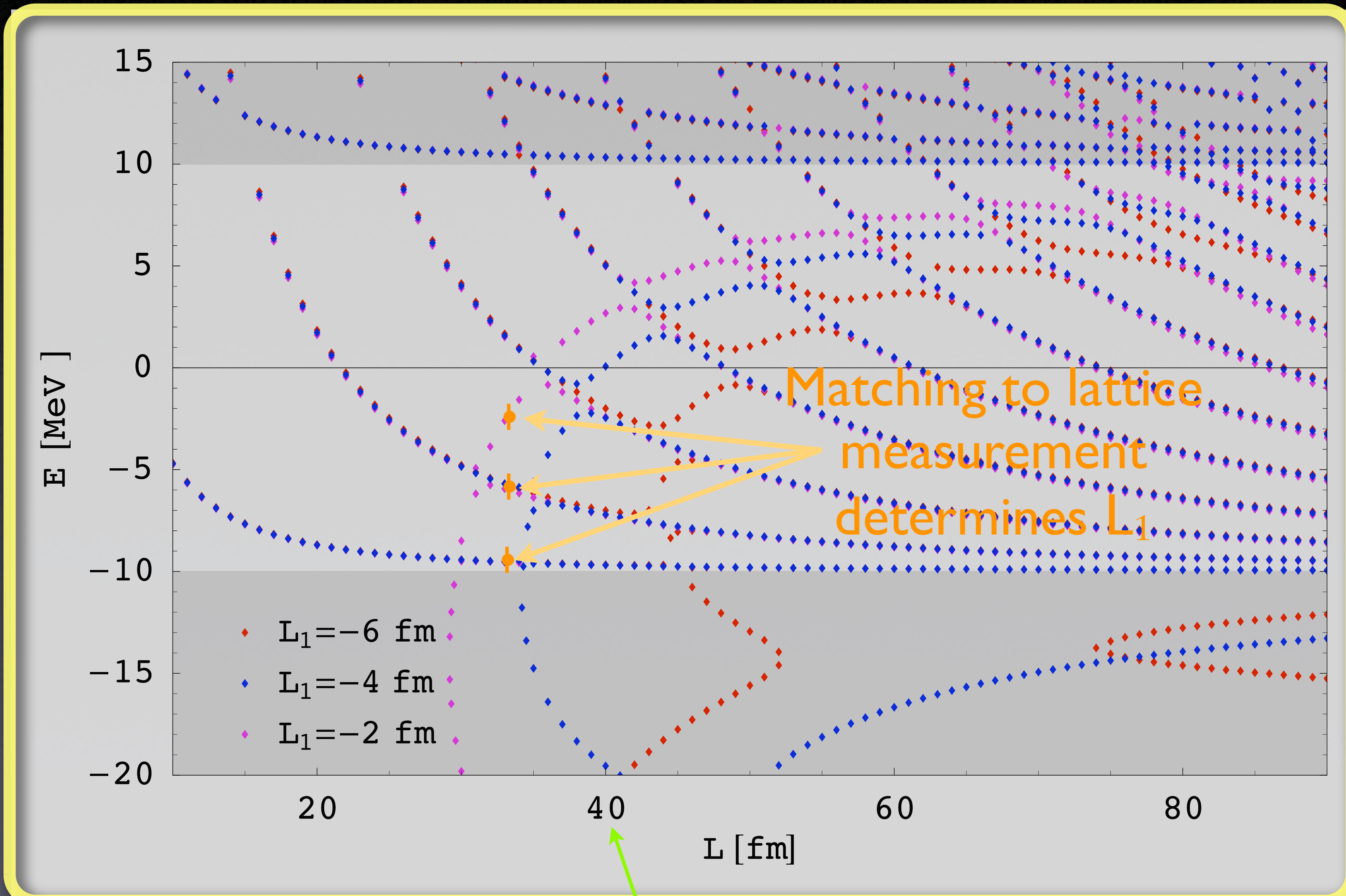
Energy levels in B field



|e B| = 1000 MeV^2

EFT prediction for behaviour of $m=\pm 1$ energy levels

np→dγ: $^3S_1$–$^1S_0$ m=0



$|e\,B| = 1000\ \mathrm{MeV}^2$

NB: box is asymmetric: 4x4x40 fm^3

EMC on the lattice

- Lattice methods combined with EFT can be used to investigate the EMC effect
- Measure shifts in two-particle energy levels in external field coupled to twist-two operators

$$S_{QCD} \longrightarrow S_{QCD} + \int d^4x \, \underbrace{\Omega_{\mu_1 \ldots \mu_n}(x)}_{\text{external field}} \, \underbrace{\overline{\psi}(x)\gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}}\psi(x)}_{\text{twist-2 operator}}$$

- Determines two body coefficient α_n
  - Leading medium modification of moments of PDFs
  - Extend to larger A using nuclear EFT

[WD Phys Rev D71 054506]

Reality sets in

- Are these measurements feasible? ✓?
- NPLQCD Collab. 2006 measurement of lowest $\Delta E = E_{2N} - 2E_N$ in $^3S_1$ and $^1S_0$ NN channels: PRL.97,012001
  - roughly $\Delta E = 40 \pm 5$ MeV (error 10x too large)
- Lighter quark masses* $\Rightarrow$ bigger volumes
- Technical advances: anisotropic lattices, Lüscher-Wolff, *algorithms for multiple external fields? ...*
- Moore's Law is a powerful thing

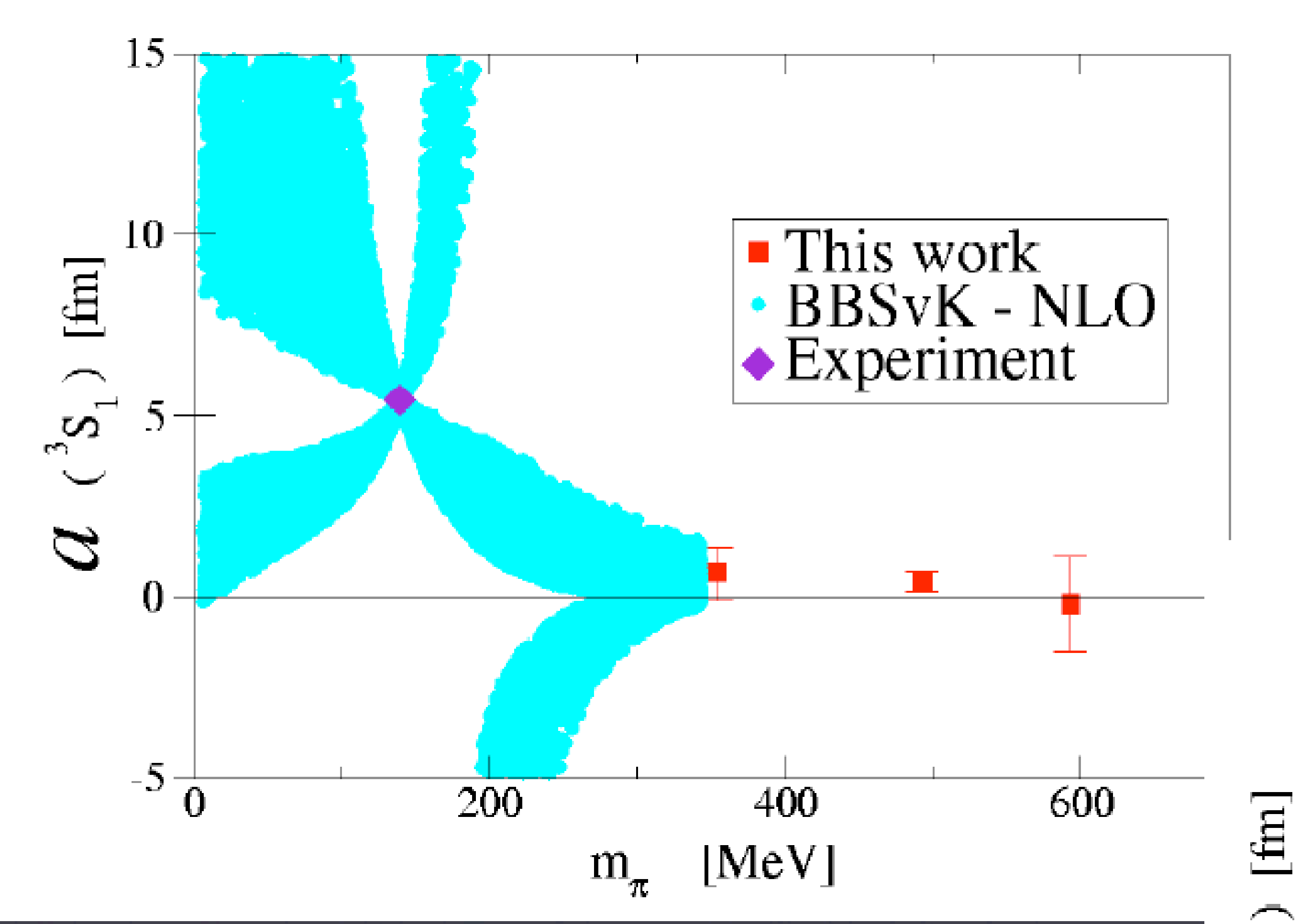
EMC effect

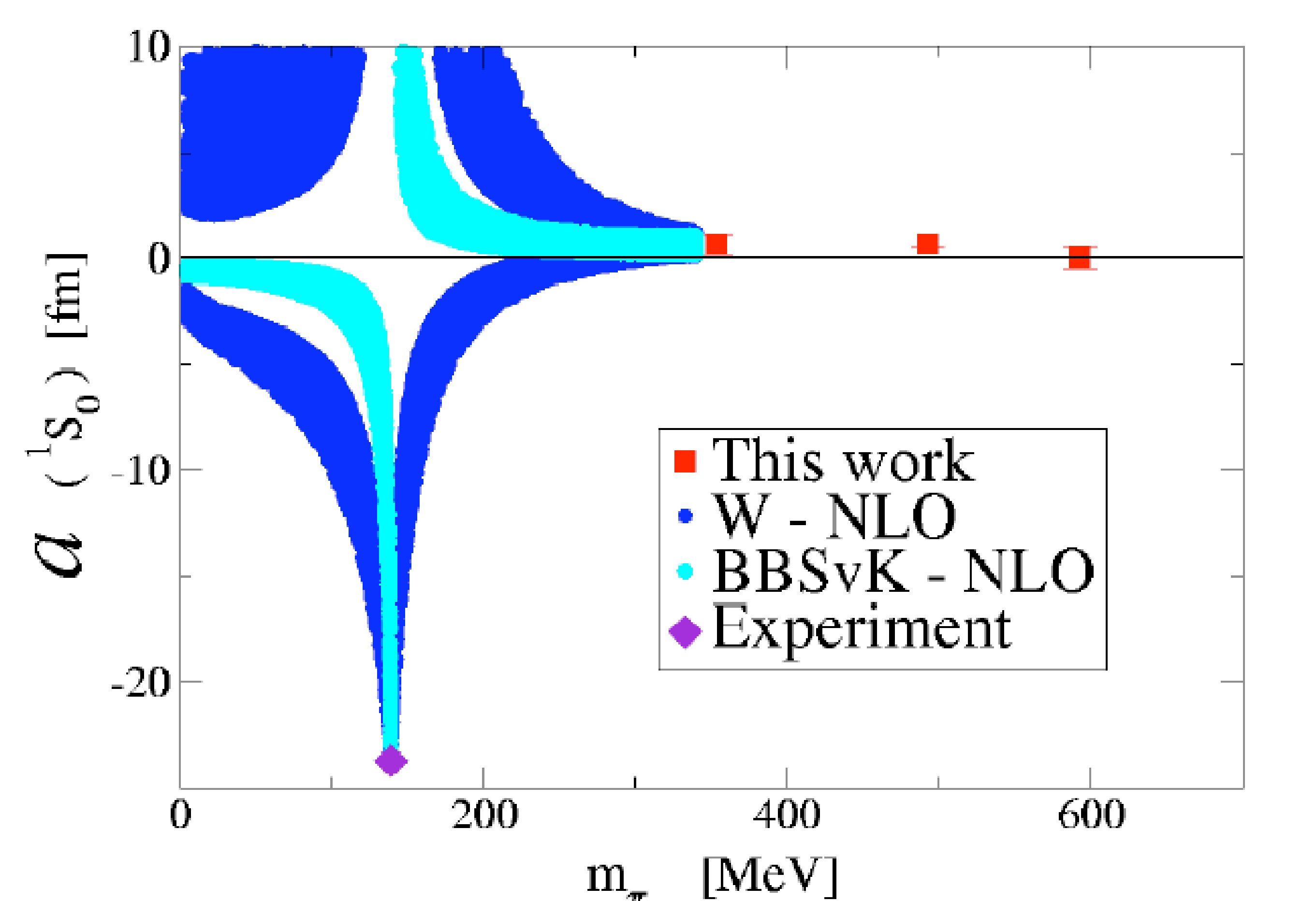
- Can also consider polarised/transversity cases
- Modified gauge fields also give
  - ➡ Moments of singlet quark PDFs/GPDs
  - ➡ Quark contribution to total angular momentum of the proton (Ji's sum rule)
- Extension to A>2 on lattice should be possible
  - Technical tools to develop for 3+ particles
  - *Long term*: combine with nuclear EFT to assess nuclear effects in NuTeV anomaly: $F_3^A(x)$

Lattice nuclear physics

- Similar techniques allow investigation of
  - Cross-section for neutrino breakup of the deuteron (relevant for calibration of SNO)
  - ε_d vs quark masses (isovector can be done now)
  - Deuteron polarisabilites
  - Far future: $0\nu-\beta\beta$ decay nuclear matrix elements

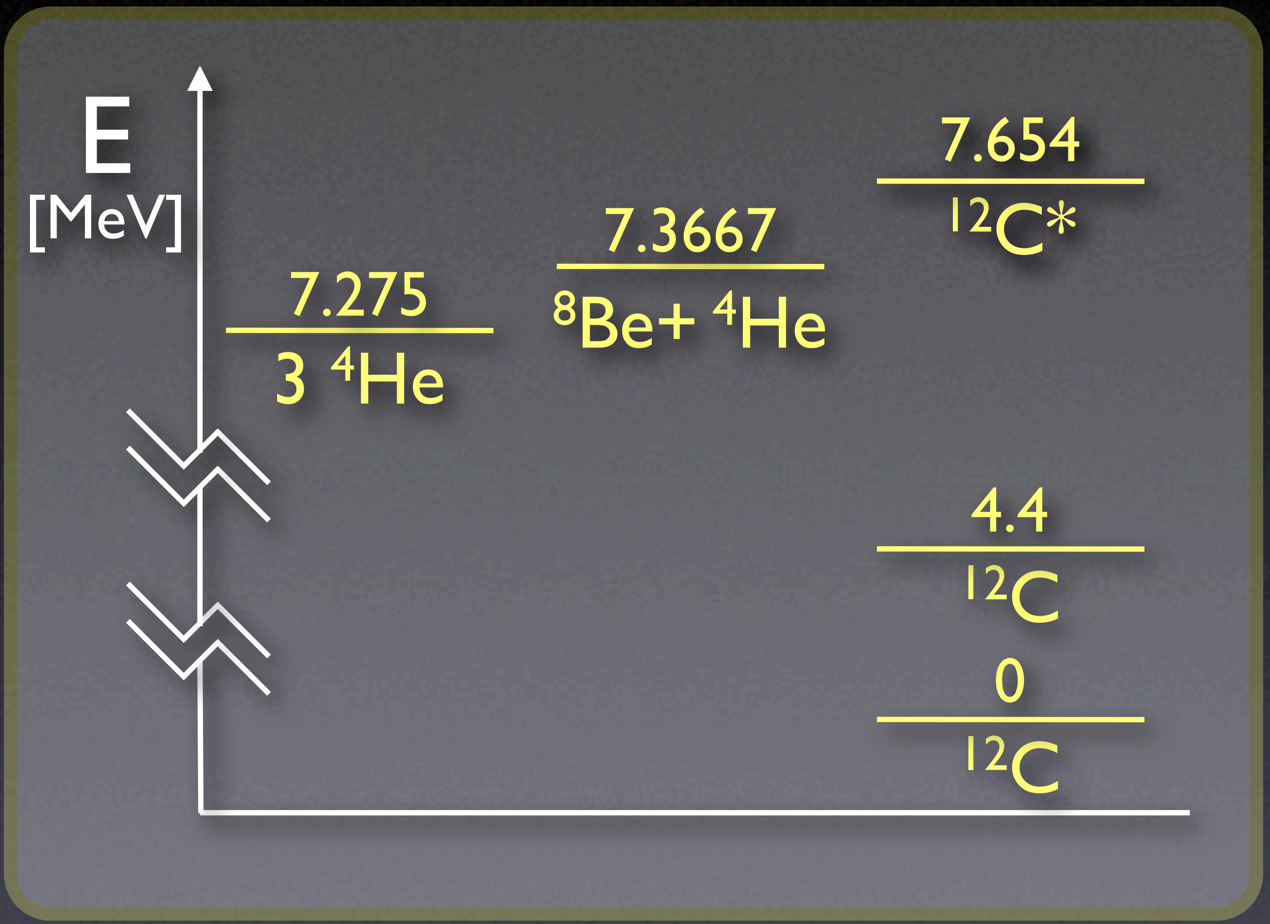
NN scattering lengths

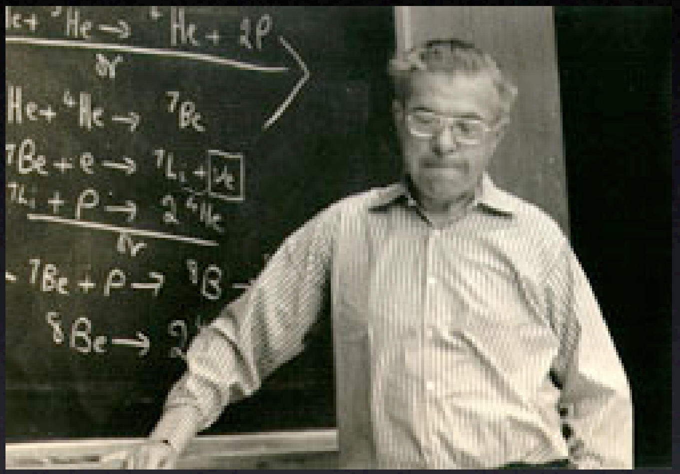


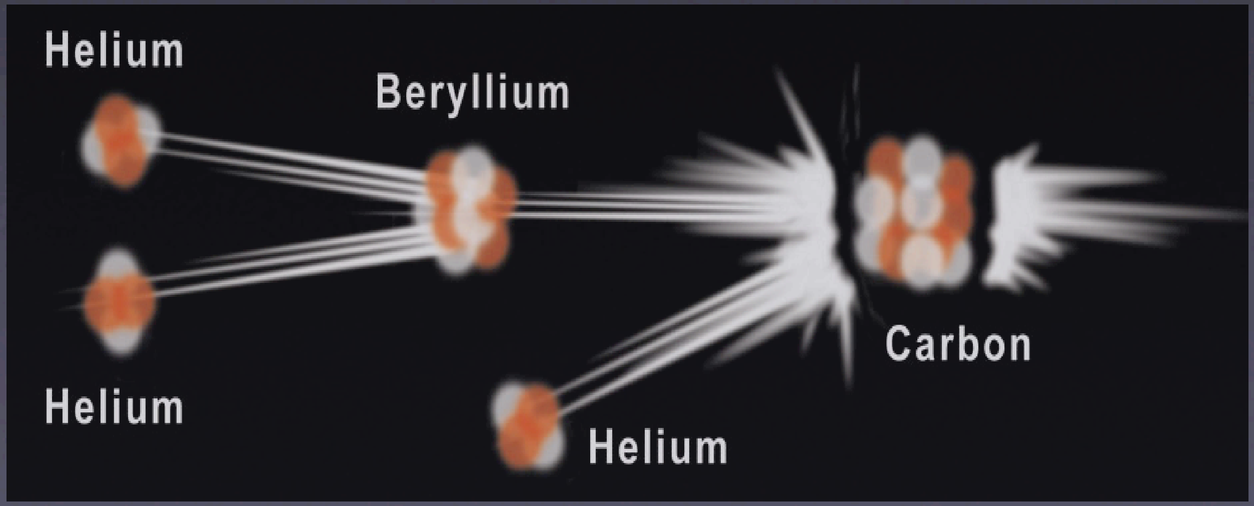


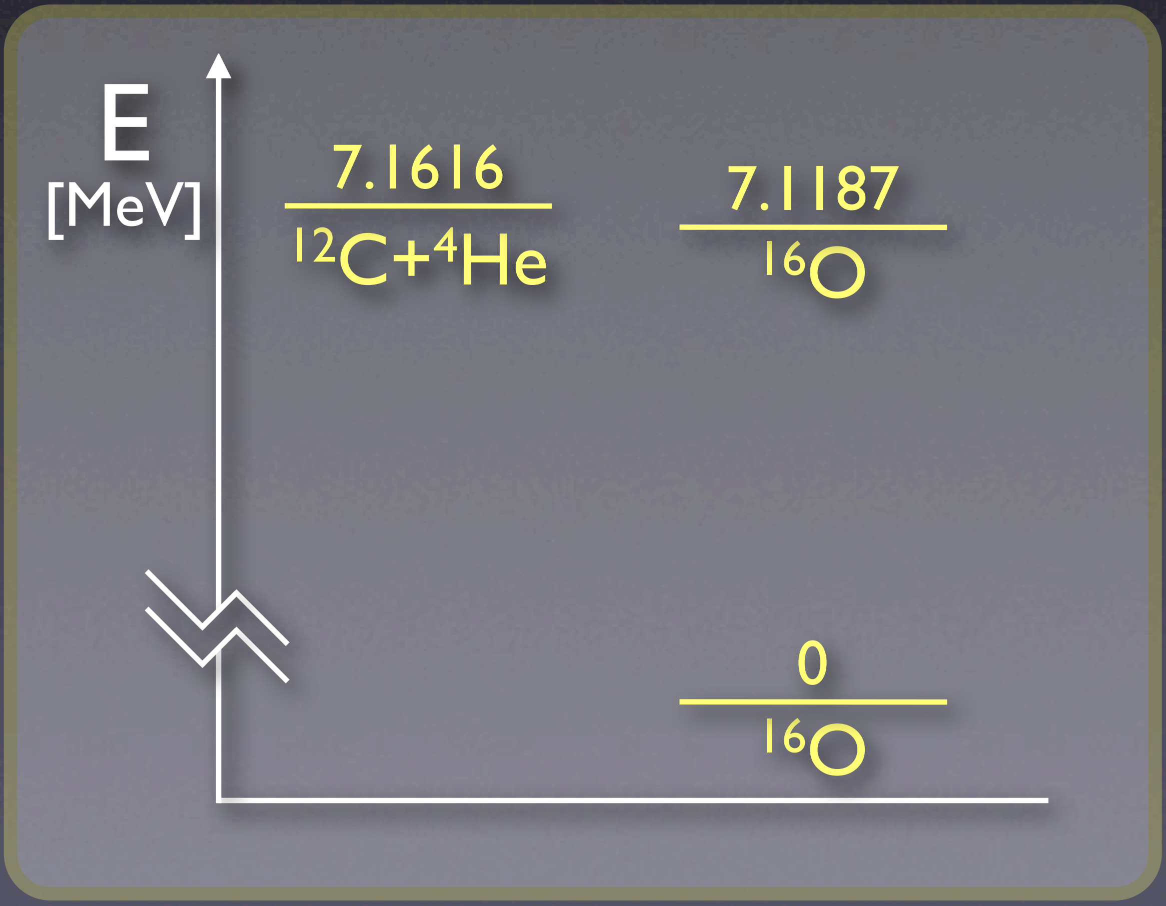
hep-lat/0602010

Stellar ^{12}C production





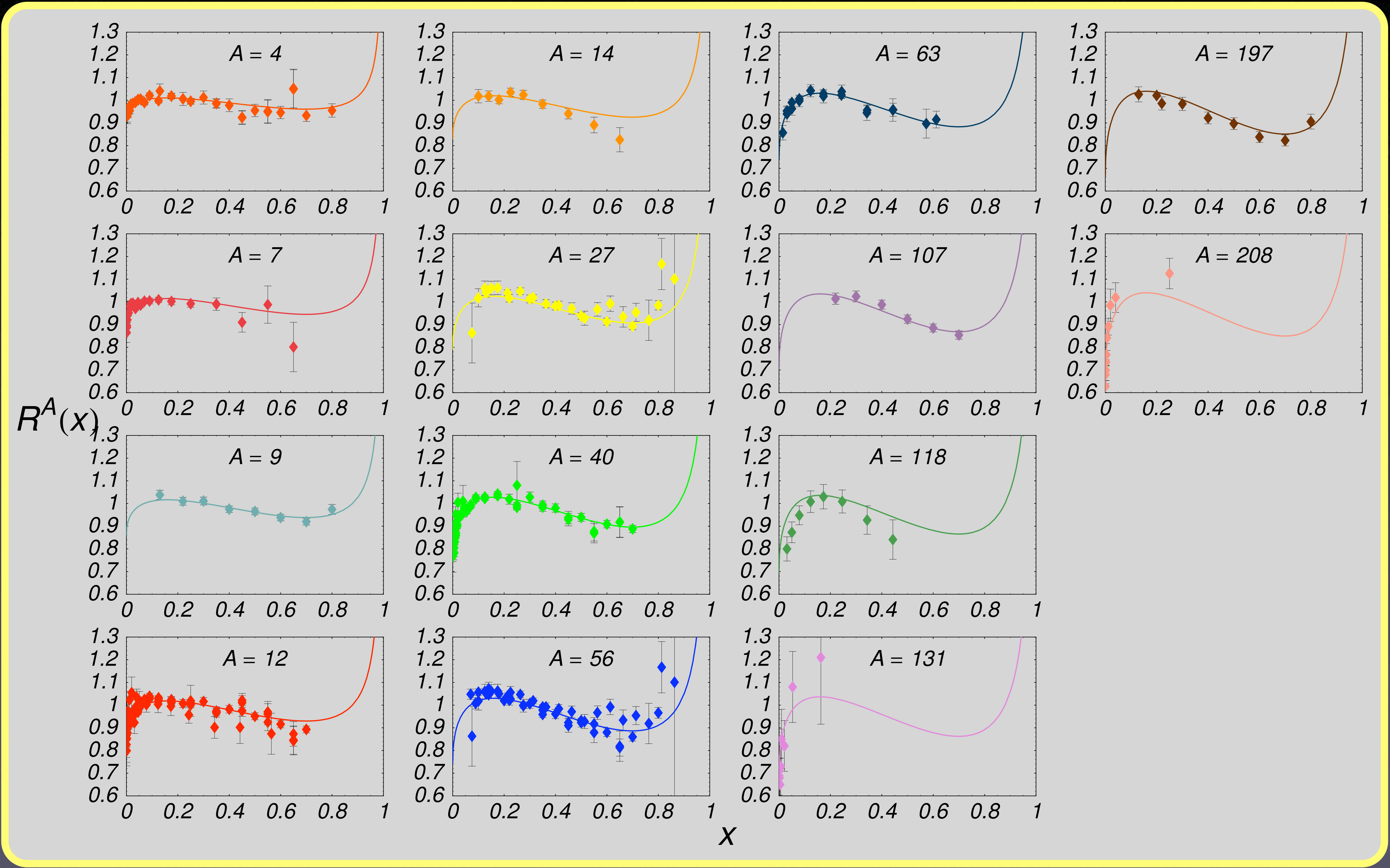




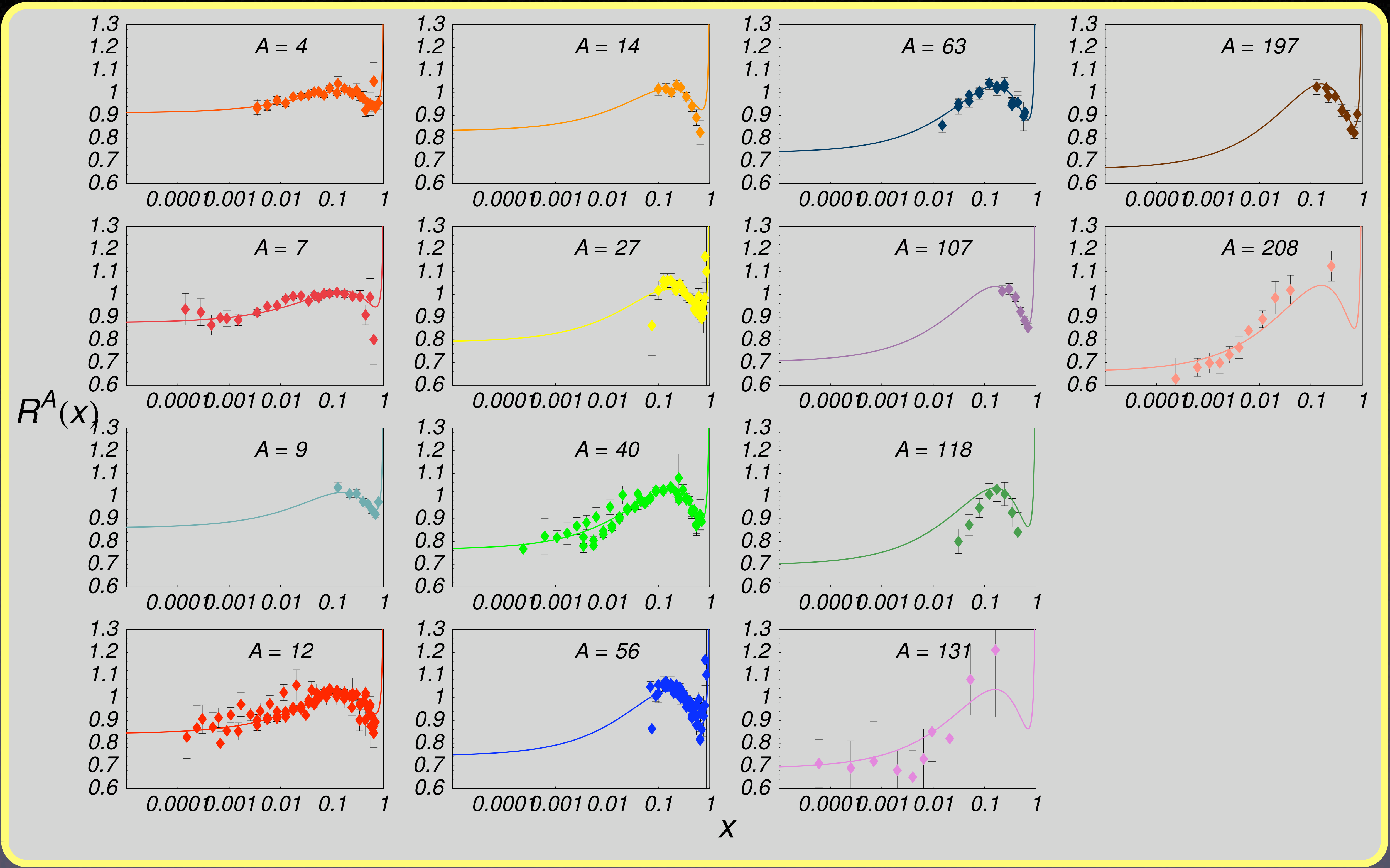
Stellar ^{12}C production

- *Why are these levels in exactly the right places for carbon-based life?*
  - The strong interaction must explain
  - How sensitive are they to fundamental parameters (quark masses, QCD scale)?
- Many other nuclear fine-tunings: deuteron binding energy, $M_n > M_p$,...

EMC fits



EMC fits



$R^A(x)$
x
A = 4
A = 14
A = 63
A = 197
A = 7
A = 27
A = 107
A = 208
A = 9
A = 40
A = 118
A = 12
A = 56
A = 131
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.0001
0.001
0.01
0.1
1

Chiral perturbation theory

- Pattern of chiral symmetry breaking in QCD

  ➡ pions (pseudo-Goldstone bosons) very light and dominate low energy observables

- Effective field theory of low energy dynamics
  - Integrate out modes above scale $\Lambda_\chi \sim 1$ GeV
  - Lagrangian: all operators consistent with χ symm. with short distance physics encoded in LECs
  - Power counting: expansion in $\frac{m_\pi}{\Lambda_\chi}, \; \frac{p}{\Lambda_\chi}$
- Heavy baryon χPT: nucleons and Δ resonances

Chiral perturbation theory

- Low energy observables have chiral expansion
- $\mathcal{O} \sim$ {terms analytic in m_q, p} + {chiral loops}
  - Chiral loops
    - Long range pion dynamics
    - Non-analytic mass dependence
    - Known coefficients
  - Analytic terms have unknown coefficients
    - Can be fit from experiment or lattice data
- Finite V and finite a effects can be included

Partially-quenched χPT

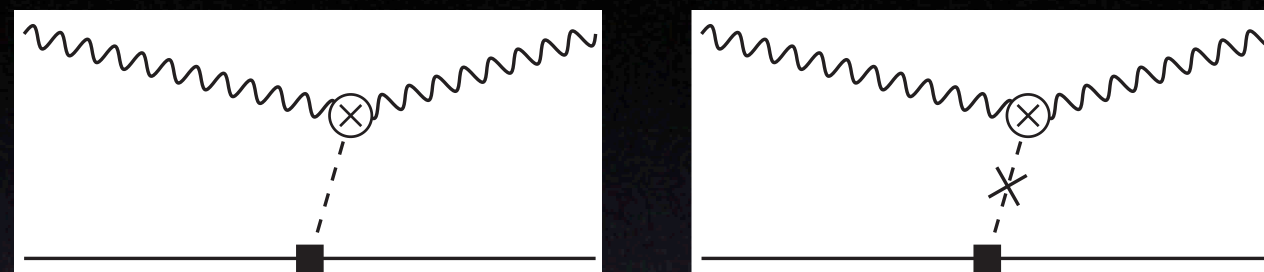
- Low energy effective theory for PQ-QCD
- Spontaneously broken (graded) symmetry

$$SU(2)_L \times SU(2)_R \longrightarrow SU(4|2)_L \times SU(4|2)_R$$

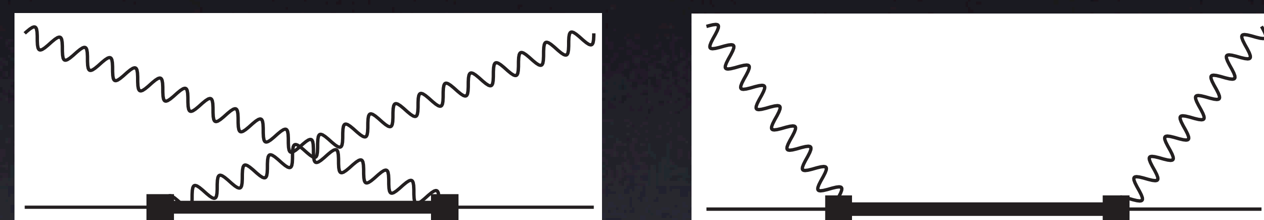
- Valence, sea and ghost quarks lead to both bosonic and fermionic Goldstone mesons
- PQχPT LECs are a superset of those in χPT

[Bernard & Golterman, Sharpe]

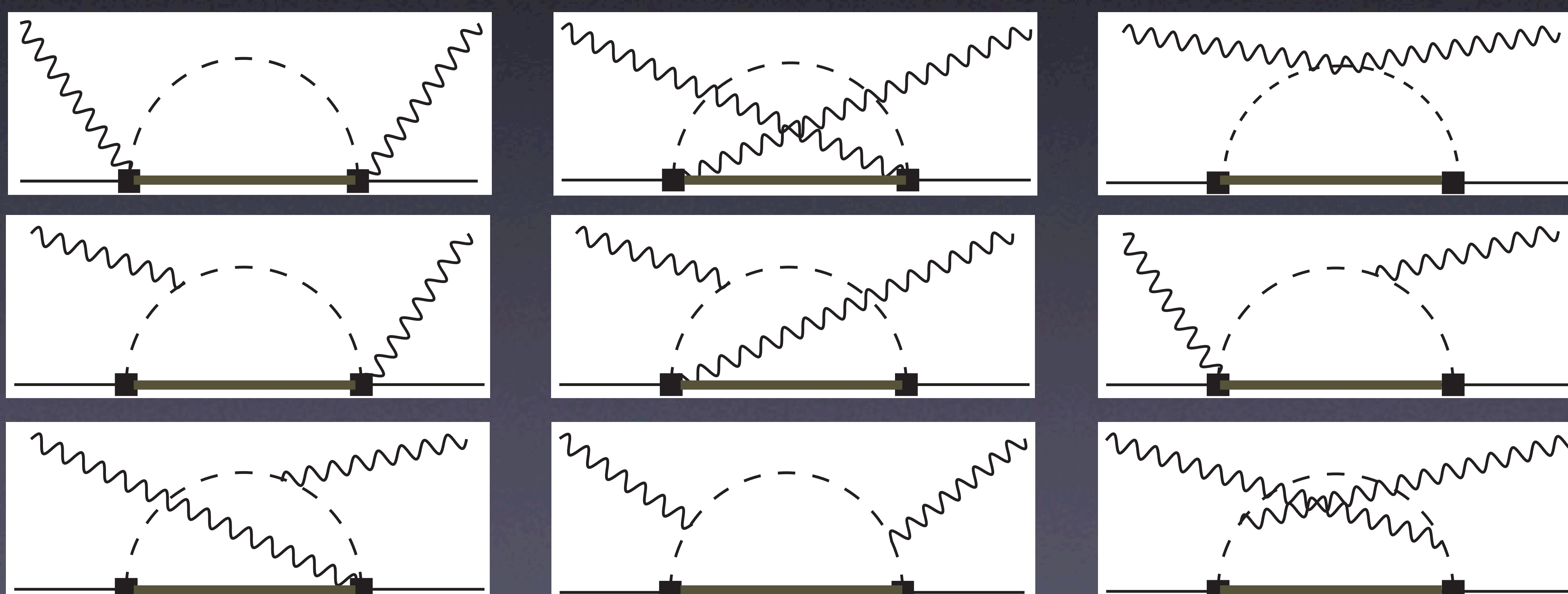
PQχPT contributions



Anomalous $\pi^0 \to \gamma\gamma$



Δ-pole graphs



LOOPS

Wess-Zumino-Witten

- Chiral anomaly contributes through $\pi^0 \to \gamma\gamma$

$$\mathcal{L}_{\pi^0\gamma\gamma} = -\frac{3e^2}{16\pi^2 f}\mathrm{tr}\left[\Phi Q^2\right]\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Extension to partially quenched QCD non-trivial

$$\mathcal{L}^{PQ}_{\pi^0\gamma\gamma} \propto \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\Big[a_1\,\mathrm{str}\left[\Phi Q^2\right] + a_2\,\mathrm{str}\left[\Phi Q\right]\mathrm{str}\left[Q\right] + a_3\,\mathrm{str}\left[\Phi\right]\mathrm{str}\left[Q\right]^2 + a_4\,\mathrm{str}\left[\Phi\right]\mathrm{str}\left[Q^2\right]\Big]$$

$$\mathcal{L}^{PQ}_{\pi^0\gamma\gamma} = -\frac{3e^2}{16\pi^2 f}\mathrm{str}\left[\Phi Q^2\right]\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- No need to extend Witten's global quantisation construction to graded Lie groups

Infinite volume results

- Proton electric polarisability

Involve axial couplings and quark charges

$$\alpha_p = \frac{e^2}{4\pi f^2}\left[\frac{5G_B}{192\pi}\boxed{\frac{1}{m_\pi}} + \frac{5G'_B}{192\pi}\frac{1}{m_{uj}} + \frac{G_T}{72\pi^2}F_\alpha(m_\pi) + \frac{G'_T}{72\pi^2}F_\alpha(m_{uj})\right]$$

Singular in chiral limit

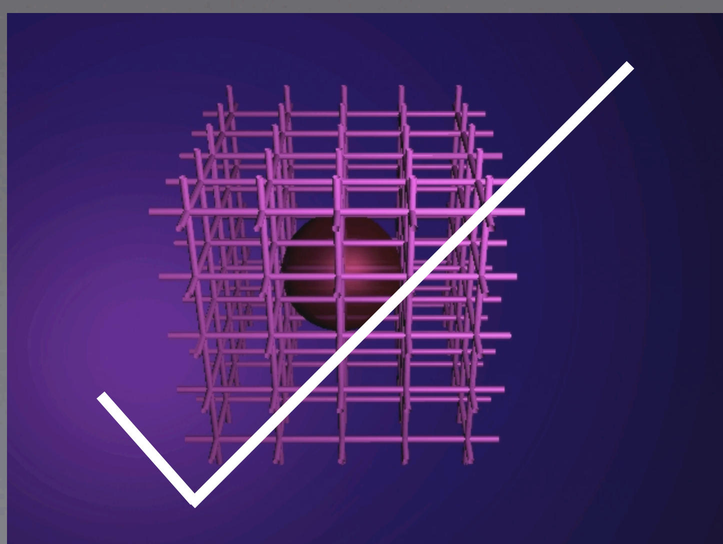
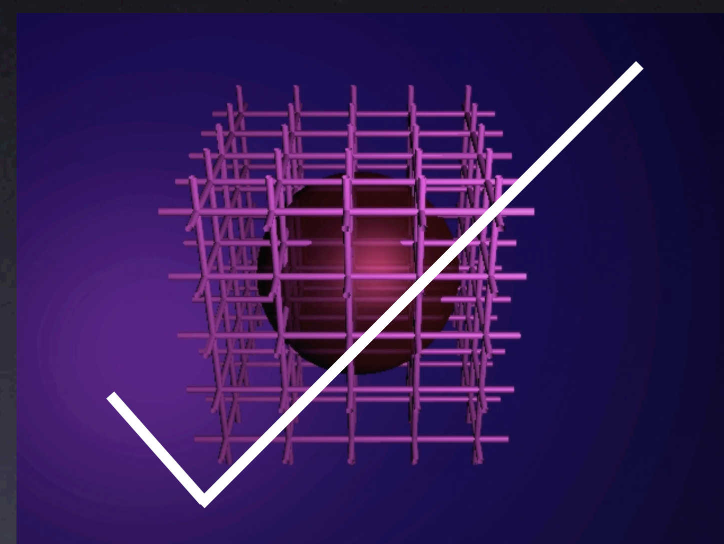
Non-analytic function involving Δ isobar

$$F_\alpha(m) = \frac{9\Delta}{\Delta^2 - m^2} - \frac{\Delta^2 - 10m^2}{2(\Delta^2 - m^2)^{3/2}}\ln\left[\frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}\right]$$

- Results for other polarisabilities similar but also have contributions from anomaly and Δ poles

χPT at finite volume

- Volume dependence can be incorporated depending on pion mass and volume

		
$m_\pi L \gg 1$	$\mu_{\rm had} L \gg 1$	$\mu_{\rm had} L \lesssim 1$
p-regime	ε-regime (pion zero modes become non-perturbative)	“Out of luck”-regime

Finite volume effects

- Polarisabilities are very sensitive to infrared scales
  - ➡ Expect large FV effects in lattice calculations
- Easily included in EFT for large volumes
  - Quantised momenta: $\vec{k} = \dfrac{2\pi}{L}\vec{n}$ for $n_i \in \mathbb{Z}$
  - Momentum integrals ⇒ mode sums

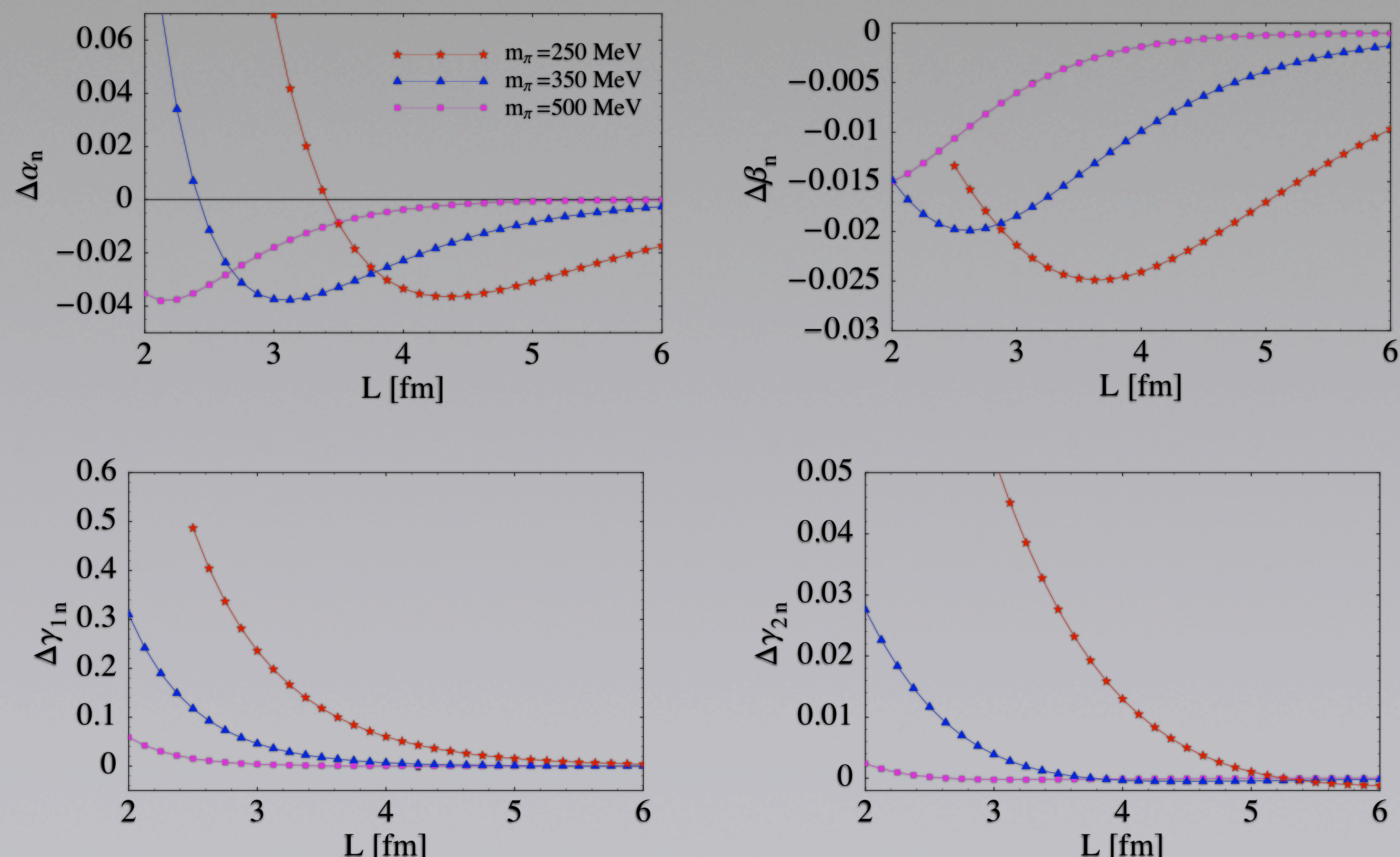
$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_\pi^2 + i\epsilon} \to \int \frac{dk_0}{2\pi L^3}\sum_{\vec{k}} \frac{1}{k_0^2 - |\vec{k}|^2 - m_\pi^2 + i\epsilon}$$

$$\to \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_\pi^2 + i\epsilon} + \frac{m_\pi^2}{4\pi^2}\sum_{\vec{n}\neq\vec{0}} \frac{1}{|\vec{n}|L} K_1(|\vec{n}| m_\pi L)$$

Poisson summation $\sum_{\vec{n}} \delta^{(3)}(\vec{y} - \vec{n}) = \sum_{\vec{m}} e^{2\pi i \vec{m}\cdot\vec{y}}$

$$\xrightarrow{m_\pi L \to \infty} \sqrt{m_\pi / 32\pi^3 L^3}\exp(-m_\pi L)$$

Volume Dependence: Neutron



FV Electric polarisability

$$\alpha(L) = \frac{e^2}{1152\pi f^2}\int_0^\infty d\lambda\Big[3G_B\mathcal{F}_\alpha(\mathcal{M}_{uu}) + 3G'_B\mathcal{F}_\alpha(\mathcal{M}_{uj}) \\ +8G_T\mathcal{F}_\alpha(\mathcal{M}^\Delta_{uu}) + 8G'_T\mathcal{F}_\alpha(\mathcal{M}^\Delta_{uj})\Big]$$

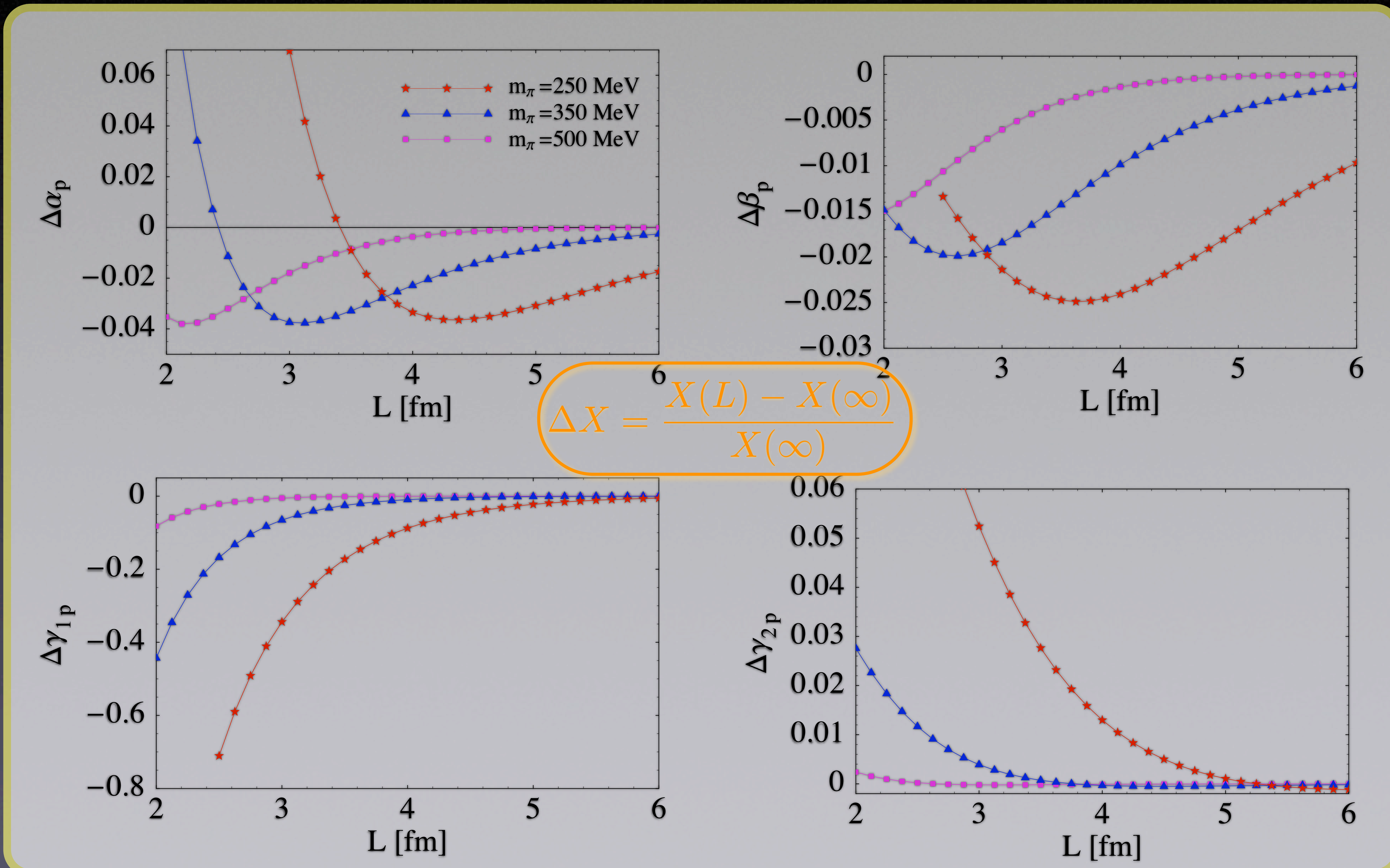
$$\mathcal{M}^\Delta_{ab} = \sqrt{m^2_{ab} + 2\lambda\Delta + \lambda^2}$$

$$\mathcal{F}_\alpha(m) = 180\lambda^2\mathcal{I}_{\frac{7}{2}}(m) + 190\mathcal{J}_{\frac{7}{2}}(m) - 280\lambda^2\mathcal{J}_{\frac{9}{2}}(m) - 455\mathcal{K}_{\frac{9}{2}}(m) \\ +315\lambda^2\mathcal{K}_{\frac{11}{2}}(m) + 252\mathcal{L}_{\frac{11}{2}}(m)$$

$$\mathcal{I}_\beta(M) = \sum_{\vec{n}}\frac{E_{1-\beta}(|\vec{n}|^2 + x^2)}{L^3\Gamma(\beta)} + \frac{\pi^{\frac{3}{2}}}{\Gamma(\beta)L^3}\int_0^1 dt\, t^{\beta-5/2}e^{-t\,x^2}\left[\sum_{\vec{n}\neq 0} e^{-\frac{\pi^2|\vec{n}|^2}{t}} + 1\right]$$

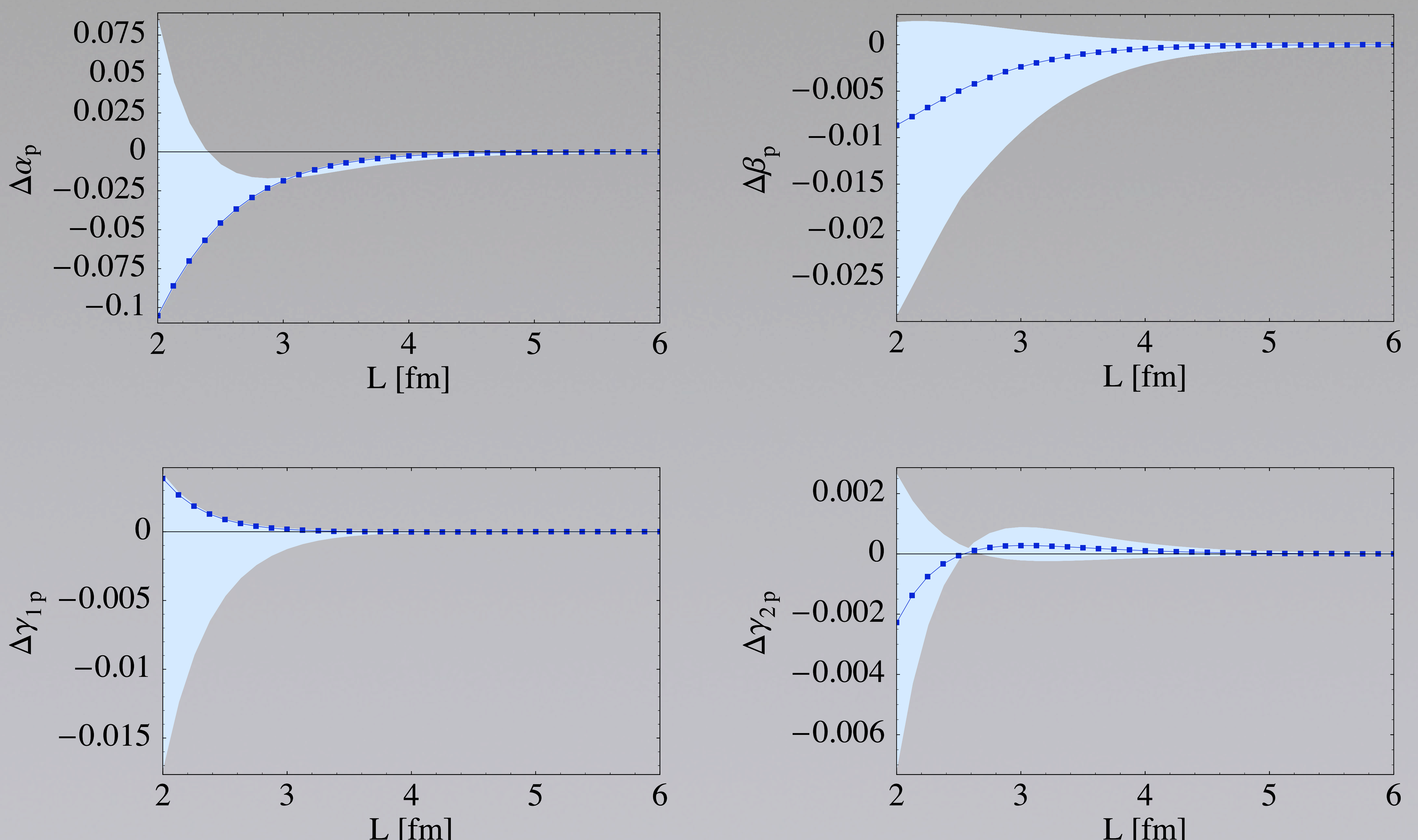
$$\mathcal{L}_\beta(M) = \mathcal{I}_{\beta-3}(M) - 3M^2\mathcal{I}_{\beta-2}(M) + 3M^4\mathcal{I}_{\beta-1}(M) - M^6\mathcal{I}_\beta(M)$$
$$\mathcal{K}_\beta(M) = \mathcal{I}_{\beta-2}(M) - 2M^2\mathcal{I}_{\beta-1}(M) + M^4\mathcal{I}_\beta(M)$$
$$\mathcal{J}_\beta(M) = \mathcal{I}_{\beta-1}(M) - M^2\mathcal{I}_\beta(M)$$

Volume Dependence: Proton



$$\Delta X = \frac{X(L) - X(\infty)}{X(\infty)}$$

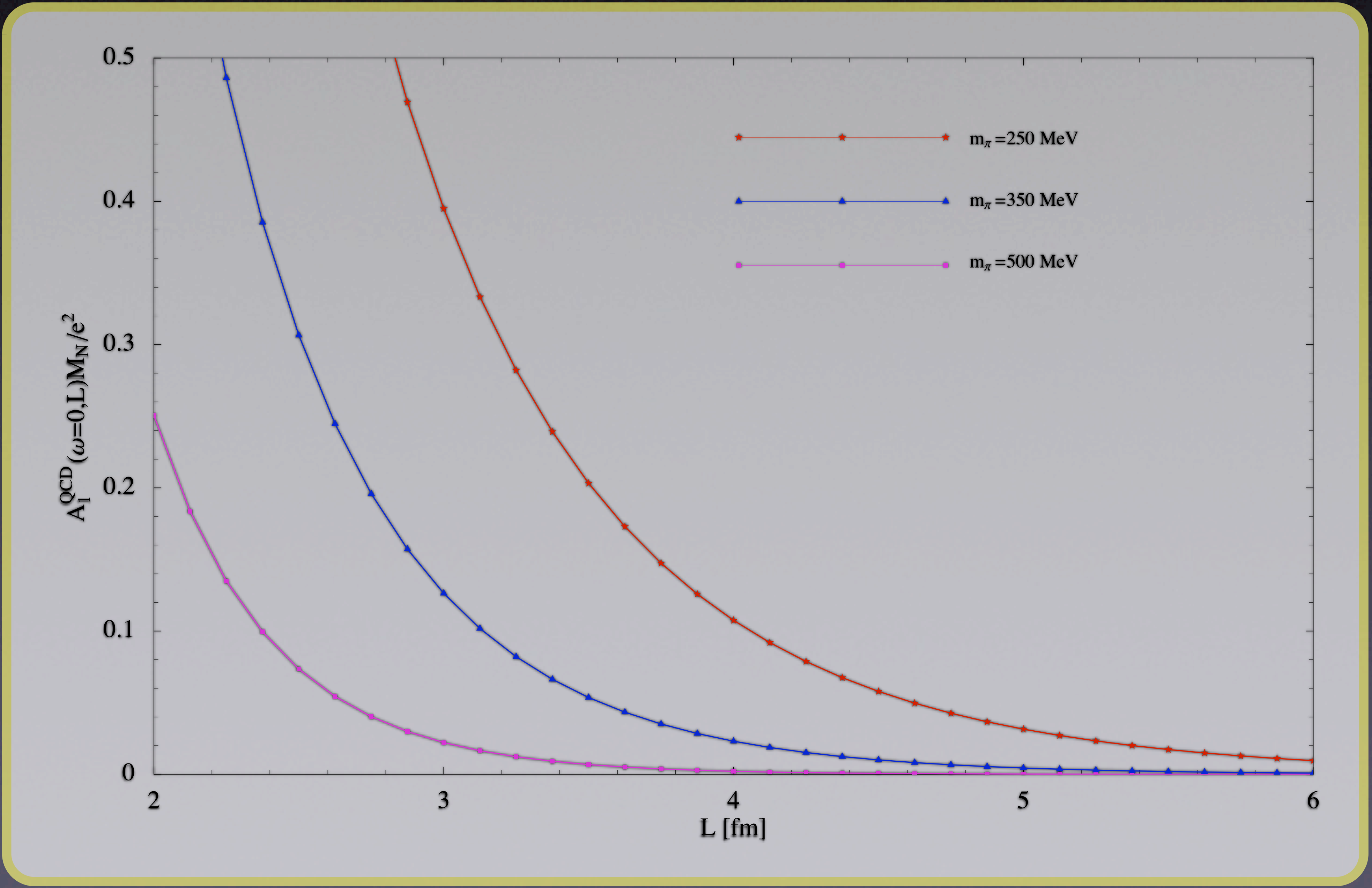
Volume Dependence: Quenched



m_π = 500 MeV

Thomson Limit ($\omega = 0$)

- Thomson limit for photon-neutron scattering
  ➡ Vanishes at infinite volume!



Hadronic parity violation

Hadronic PV

- Hadronic PV arises from W/Z exchange *within* the hadronic state
- CP violation in kaons, B^0, B_s
- Consequences for baryons:
  - P-odd NN interactions
  - P-odd anapole EM form-factor [Zeldovich 58]

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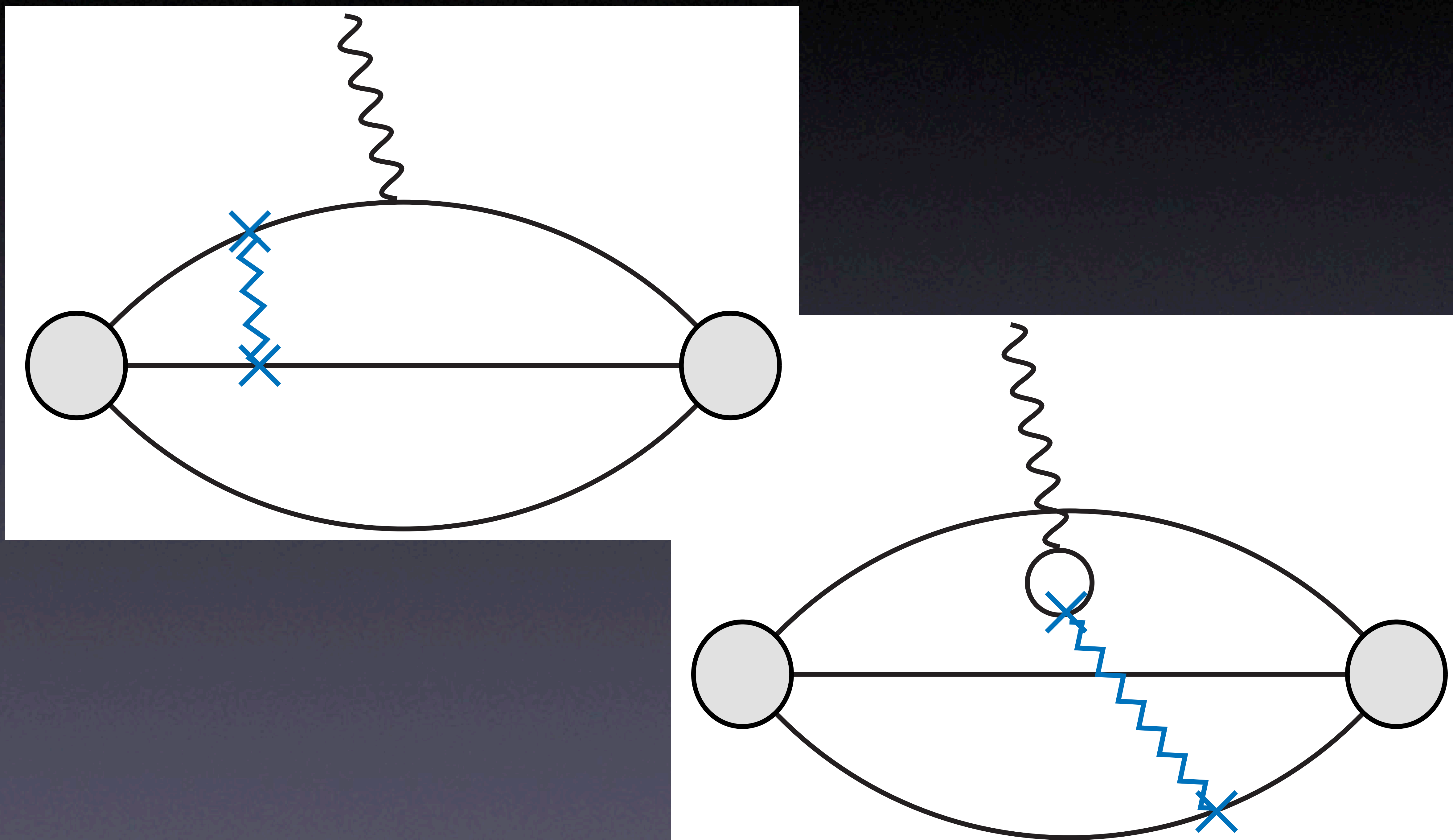
Anapole Form Factor

- Decomposition of hadronic EM current

$$\langle N(p')|J^\mu_{\rm em}|N(p)\rangle = \overline{u}(p')\Big[\gamma^\mu F_1 + \frac{\sigma^{\mu\nu}q_\nu}{2M}F_2 \\ +\sigma^{\mu\nu}q_\nu\gamma_5 F_D + \boxed{(q^2\gamma^\mu - \not{q}q^\mu)\gamma_5 F_A}\Big]u(p)$$

- PV: electric dipole and anapole form-factors
  - Weak interactions produce AFF
- $F_A(0)$ = Anapole Moment: magnetic field of toroidal winding carrying current
- Vanishes for real photons

Anapole Form Factor

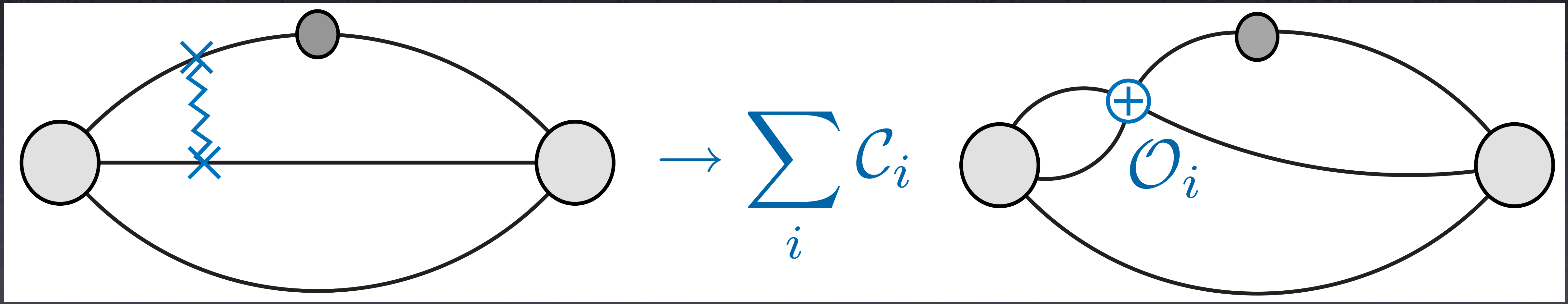


Anapole Form Factor

- P-odd, T-even contribution to EM processes
- Measured in ^{133}Cs [Wood... 97]: Z enhancement
- Significant source of uncertainty in PV ep scattering: SAMPLE, HAPPEX, G0
- Only estimate for AM from chiral PT: A_1=-0.11(44), A_0=0.02(26) [Zhu et al 00]
- Direct measurement in LQCD very useful

Anapole Form Factor

- Hadronic scales: integrate out W/Z
- Operator product expansion (effective H)



- AFF determined by matrix elements

$$\sum \mathcal{C}_i \langle N | T J^{\mu}_{\text{em}} \mathcal{O}_i | N \rangle$$

Anapole Form Factor

- Short distance physics encoded in Wilson coefficients $C_i(x, M_W, \mu)$
  - Perturbatively calcualable
- Leading operators are dimension-6 four quark operators (12 operators) eg:

$$\boxed{\begin{aligned} \mathcal{O}_1 &= (\bar{q}\gamma_\mu q)_{aa}(\bar{q}\gamma^\mu\gamma_5\tau_3 q)_{bb} \\ \mathcal{O}_4 &= (\bar{q}\gamma_\mu\gamma_5 q)_{ab}(\bar{q}\gamma^\mu\tau_3 q)_{ab} \\ \mathcal{O}_4^s &= (\bar{s}\gamma_\mu\gamma_5 s)_{ab}(\bar{q}\gamma^\mu\tau_3 q)_{ab} \end{aligned}}$$

- Other operators suppressed by $1/M_W$

[Kaplan & Savage 92]

Lattice AFF

- Computation of the anapole moment/FF very involved on the lattice
- *O*(20) Wick contractions per four quark operator (x12):possible but unmanagable
- Many (doubly) “disconnected contributions”: hard to determine

Lattice AFF

- Greatly simplified using external fields
- Two methods
  - 4Q matrix elements in external EM field
  - Current matrix element in “EW” field

Weak scales on the lattice

- Currently, lattice spacing $a \sim 0.1$ fm
- Physical M_W requires separations $x \ll a$
- In the OPE all short distance physics in Wilson coefficients (perturbation theory)
- *We are not limited to physical W/Z exchange!*

[WD & CJD Lin 05, Dawson et al 97]

Method

- Consider $S_{QCD} + \int_x \Omega^\mu(x)\overline{q}(x)\Gamma^\mu q(x)$ with
- $$\boxed{\Omega^\mu(x) = \omega\,\epsilon^\mu \exp\left[-M_X\sqrt{x^2}\right]}$$
-
-
-

Related stuff

- Technique also allows extraction of the PV pion-nucleon coupling $h_{\pi NN}$
  - Similar to $K \to \pi\pi$ decays
  - First place to start
- Electric dipole moment also accessed using external fields (electric) [Shintani et al. hep-lat/0611032]