Conditional Measurements in cavity QED

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Cavity QED:

Quantum electrodynamics for pedestrians (Haroche). No need to renormalize. Only one mode of the electromagnetic field:

ATOM(S) + CAVITY MODE

Perturbative: dissipation >> coupling (Purcell)
Non Perturbative: dissipation << coupling (Sanchez Mondragon, Eberly)

Coupling:

$$g = \frac{d \cdot E_v}{\hbar}$$

d depends on the radial and angular parts of the wavefunctions of the electrons. For the D_2 lines in alkali (Rb) it is a few times a_0 (el radio de Bohr radius) times the electric charge.

$$\vec{d} = e\left\langle 5S_{1/2}\middle|\vec{r}\middle|5P_{3/2}\right\rangle = 5.96a_0e$$

The field associated with a photon in a mode of the cavity with volume V_{eff} is:

$$E_{\nu} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

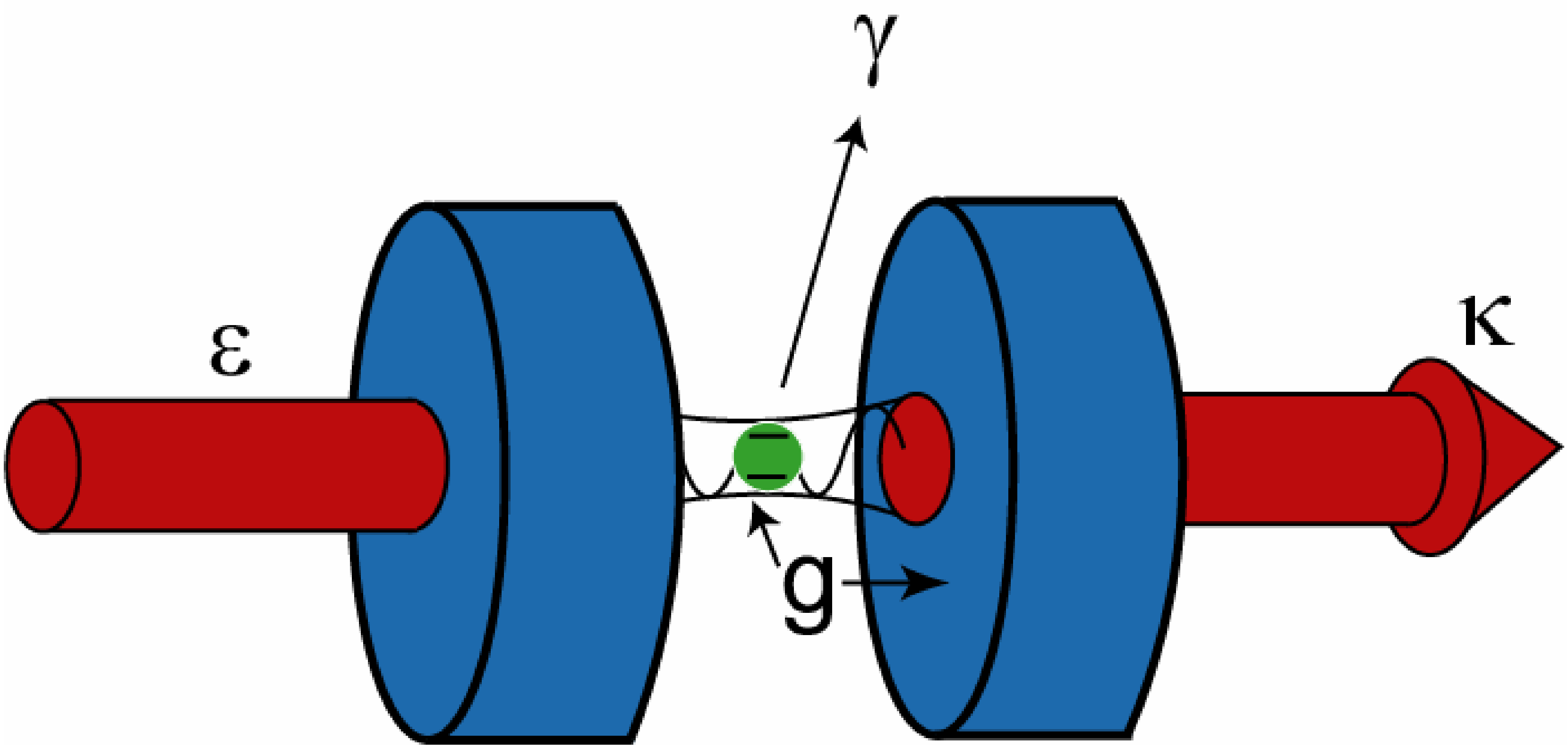
The square of the electric field is an energy density.

How large is the electric field of a photon in an optical cavity as those from Maryland, CalTech or Garching?

Electric field in a wall plug! ~ 100 V/cm

It is possible to measure it!

Cavity QED System



$$\left(\frac{\mathbf{g}}{2\pi}, \frac{\kappa}{2\pi}, \frac{\gamma/2}{2\pi}\right) \approx (2,\ 2.6,\ 3)\,\mathrm{MHz}$$

Jaynes-Cummings Model

How does a single atom interact with a single mode of the electromagnetic field?

$$H = \frac{1}{2}\hbar\omega_a\sigma_z + \hbar\omega_c a^+ a + \hbar\mathbf{g}(\sigma^- a^+ + \sigma^+ a)$$

No interaction term:

$|g,0\rangle$ ground state

$|e,0\rangle, |g,1\rangle$ degenerate excited states

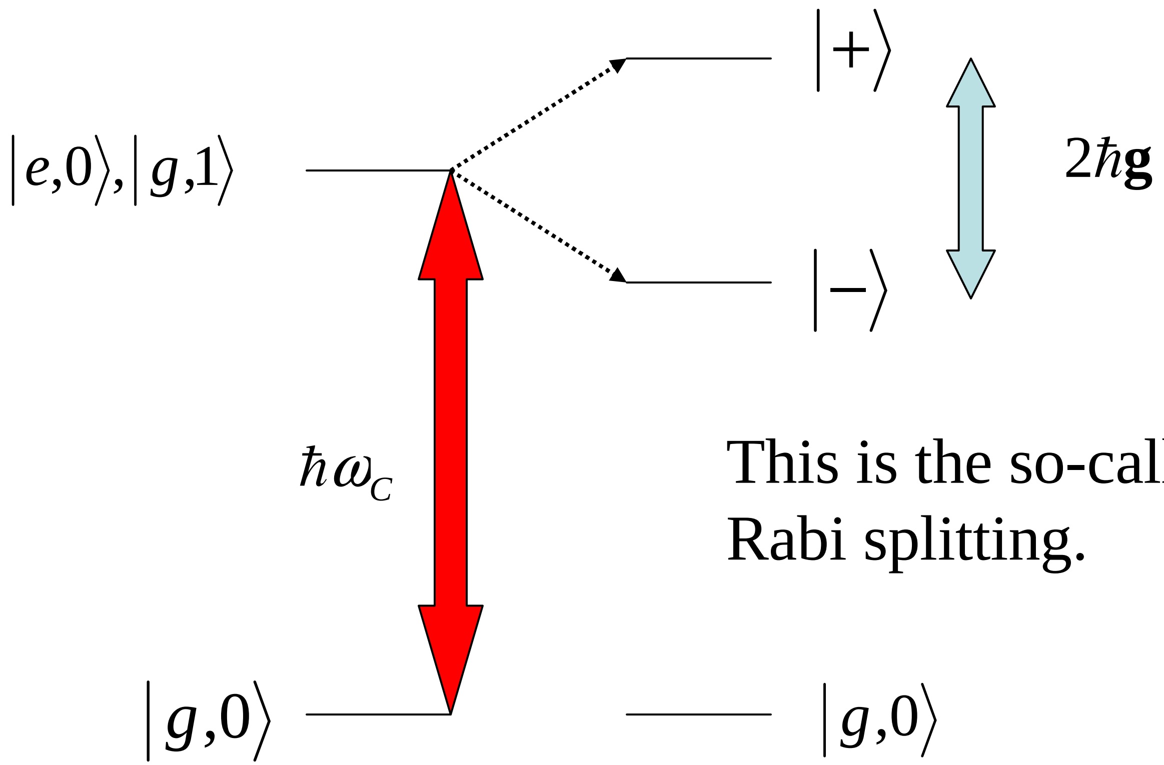
With interaction term:

$|g,0\rangle$ ground state

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|e,0\rangle \pm |g,1\rangle\right)$$

excited states

The interaction splits the degenerate excited state.



This is the so-called vacuum Rabi splitting.

Dissipative Processes

κ: Loss of a photon from the cavity due to imperfect mirrors.

γ: Spontaneous emission from an atom inside the cavity to modes other than the cavity mode.

Be careful about impedance matching!

Dissipation is not always bad. Cavity loss allows us to look inside of the cavity to study the dynamics of the system.

Can we use spontaneous emission in the same way?

Conditional dynamics from the system wavefunction

$$\left|\Psi_{ss}\right\rangle = \left|0,g\right\rangle + \lambda\left|1,g\right\rangle - \frac{2g}{\gamma}\lambda\left|0,e\right\rangle + \frac{\lambda^2 pq}{\sqrt{2}}\left|2,g\right\rangle - \frac{2g\lambda^2 q}{\gamma}\left|1,e\right\rangle$$

$$\lambda = \langle \hat{a} \rangle, \quad p = p(g,\kappa,\gamma) \text{ and } q = q(g,\kappa,\gamma)$$

A photodetection collapses the steady state into the following non-steady state from which the system evolves.

$$\hat{a}\left|\Psi_{ss}\right\rangle \Rightarrow \left|\Psi_{collapse}\right\rangle = \left|0,g\right\rangle + \lambda pq\left|1,g\right\rangle - \frac{2g\lambda q}{\gamma}\left|0,e\right\rangle$$

$$\left|\Psi(\tau)\right\rangle = \left|0,g\right\rangle + \lambda\left[\underset{\text{Field}}{f_1(\tau)}\left|1,g\right\rangle + \underset{\text{Atomic Polarization}}{f_2(\tau)}\left|0,e\right\rangle\right] + O\left(\lambda^2\right)$$

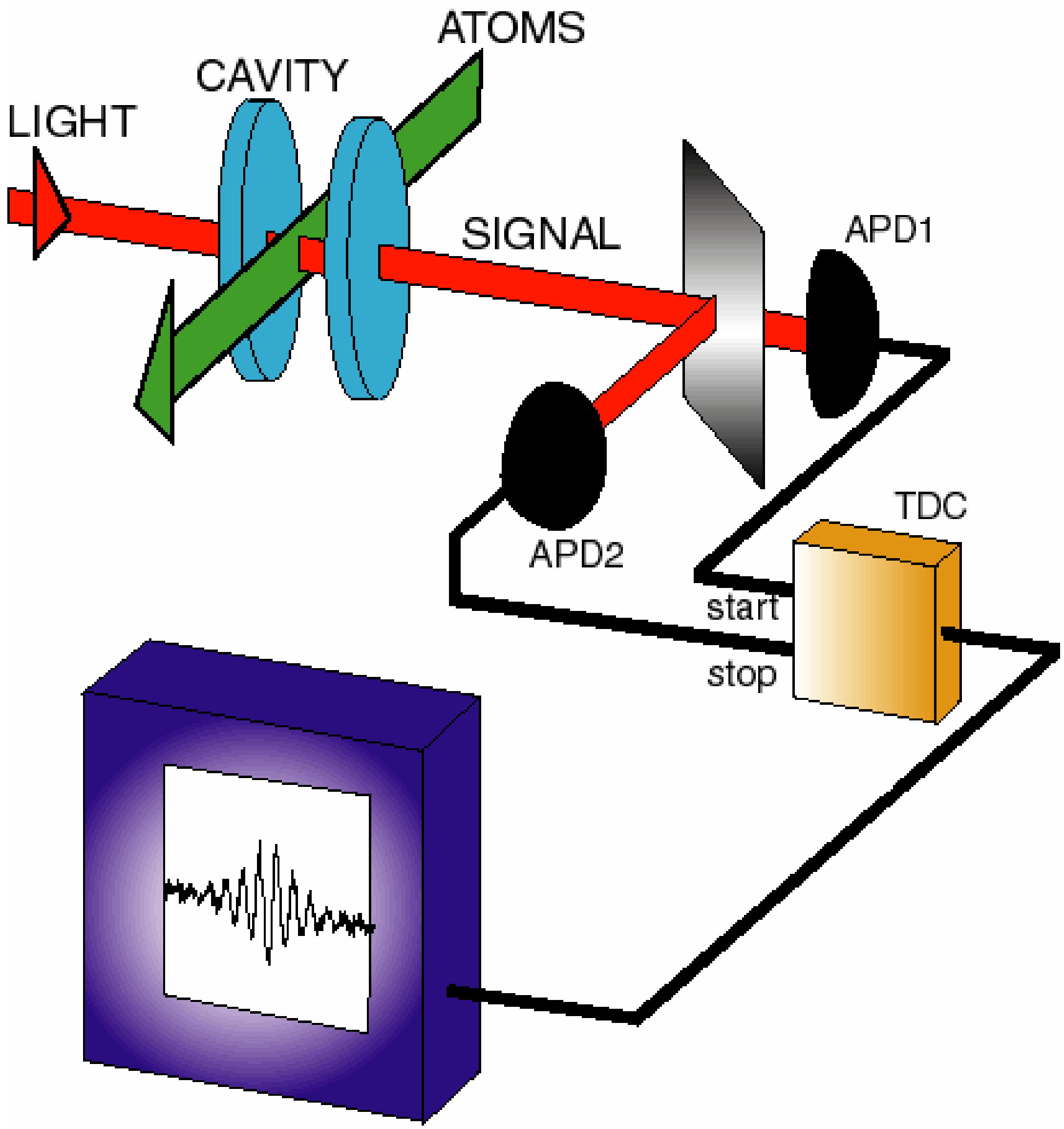
Field Atomic Polarization

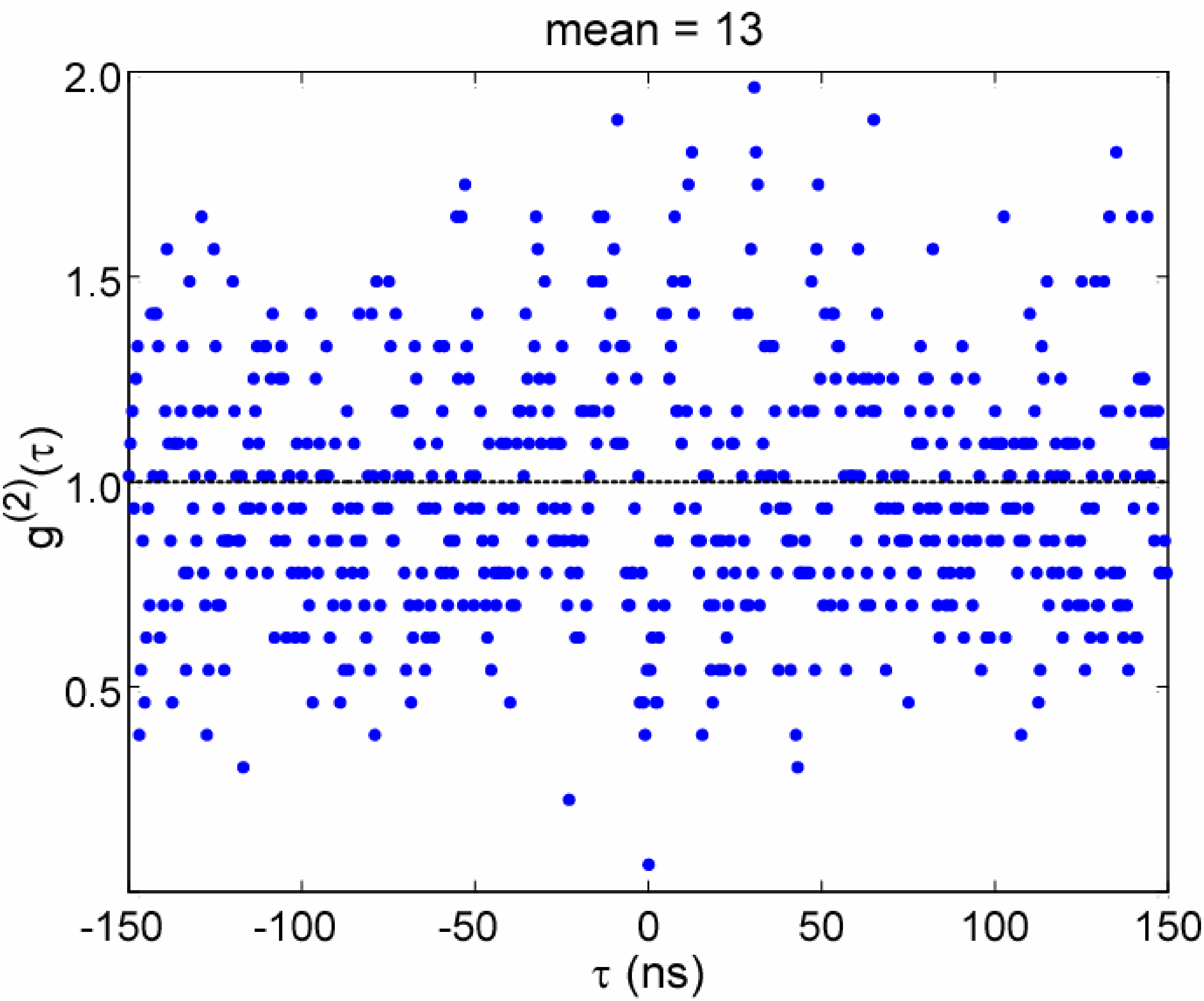
Intensity correlation function measurements:

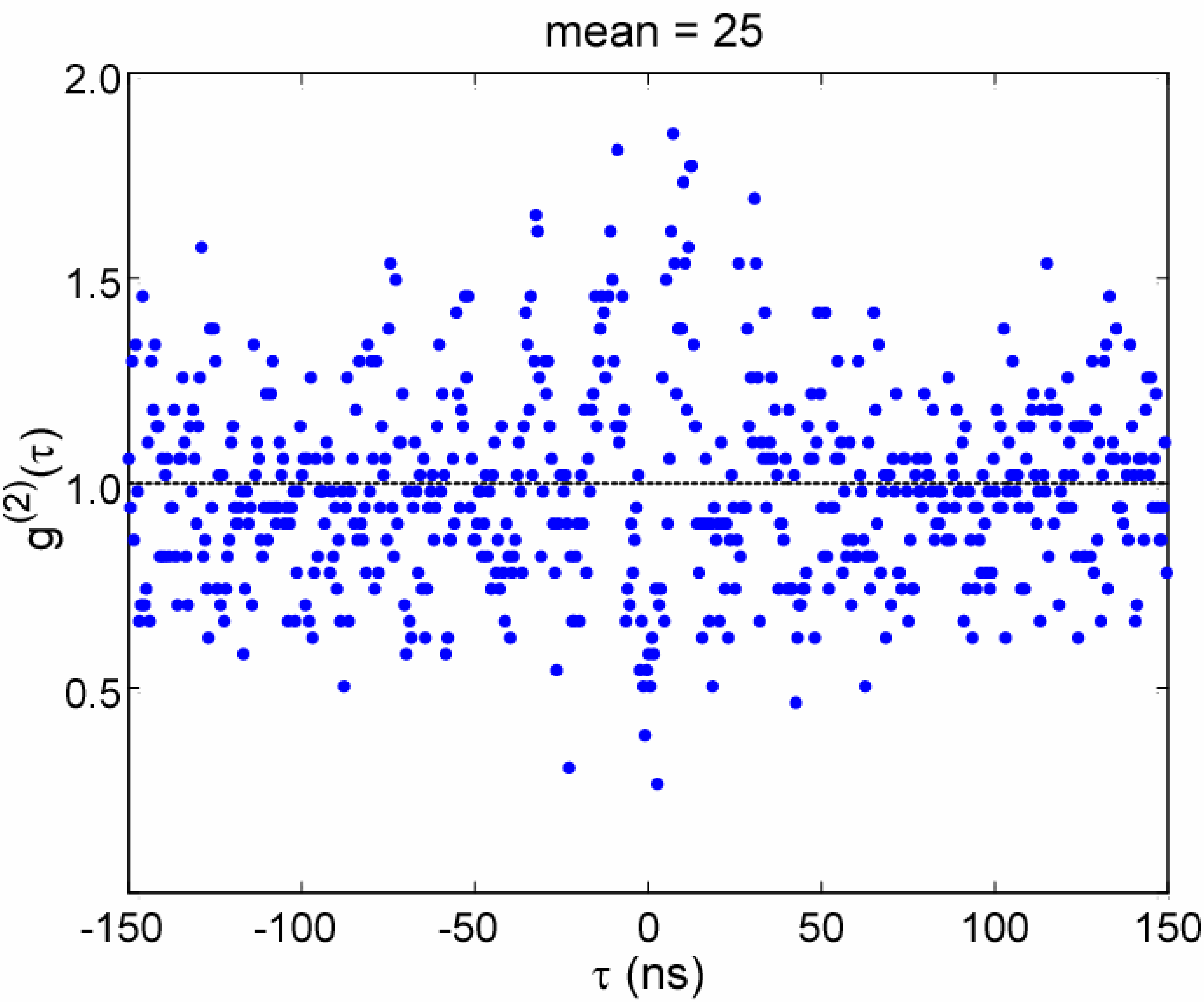
$$g^{(2)}(\tau) = \frac{\left\langle \hat{I}(t)\hat{I}(t+\tau) \right\rangle}{\left\langle \hat{I}(t) \right\rangle^2}$$

Gives the probability of detecting a photon at time t + τ given that one was detected at time t. This is a conditional measurement:

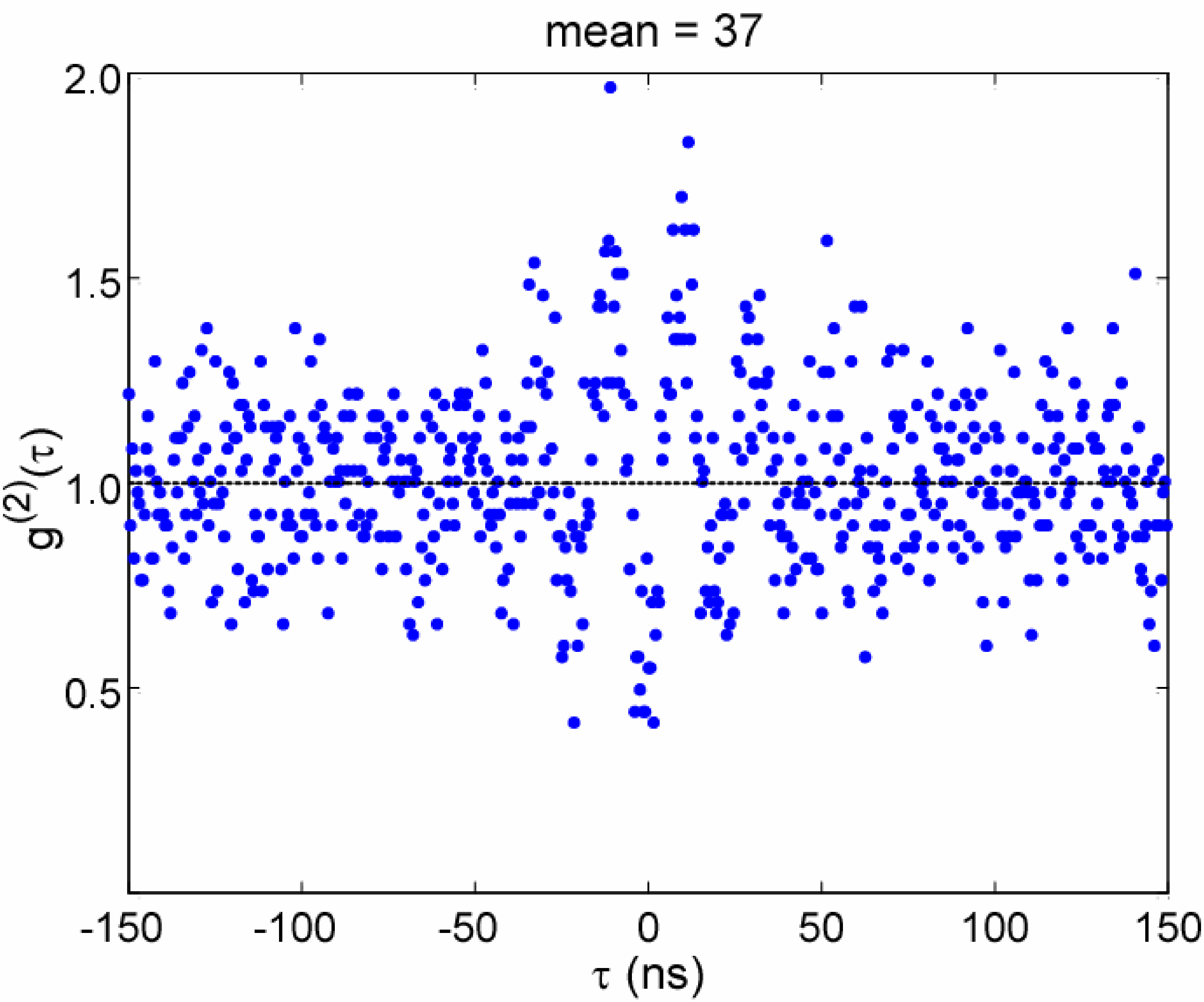
$$g^{(2)}(\tau) = \frac{\left\langle \hat{I}(\tau) \right\rangle_c}{\left\langle \hat{I} \right\rangle}$$


ATOMS
CAVITY
LIGHT
SIGNAL
APD1
APD2
TDC
start
stop

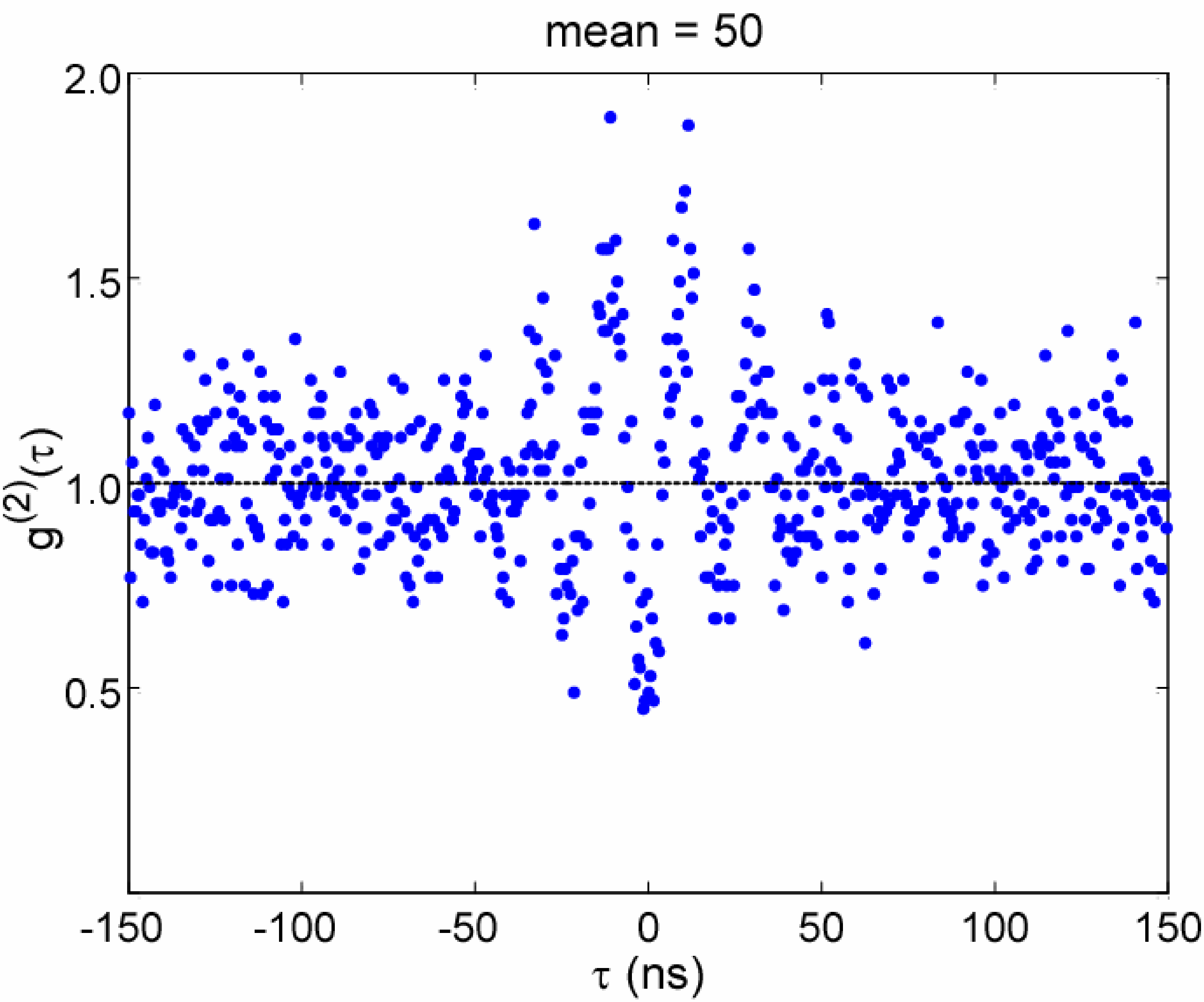

mean = 13
2.0
1.5
1.0
0.5
g(2)(τ)
-150
-100
-50
0
50
100
150
τ (ns)



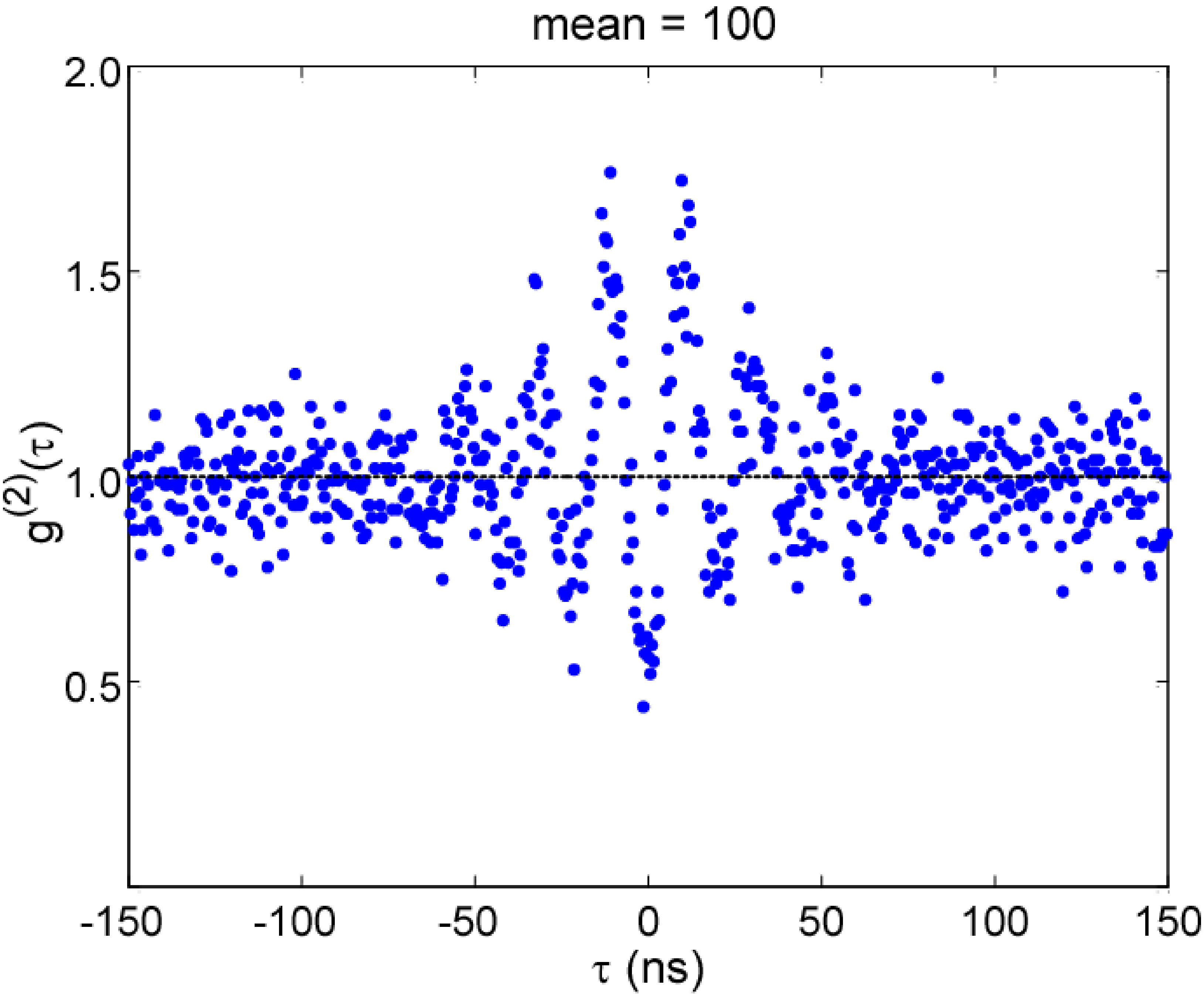
mean = 25
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



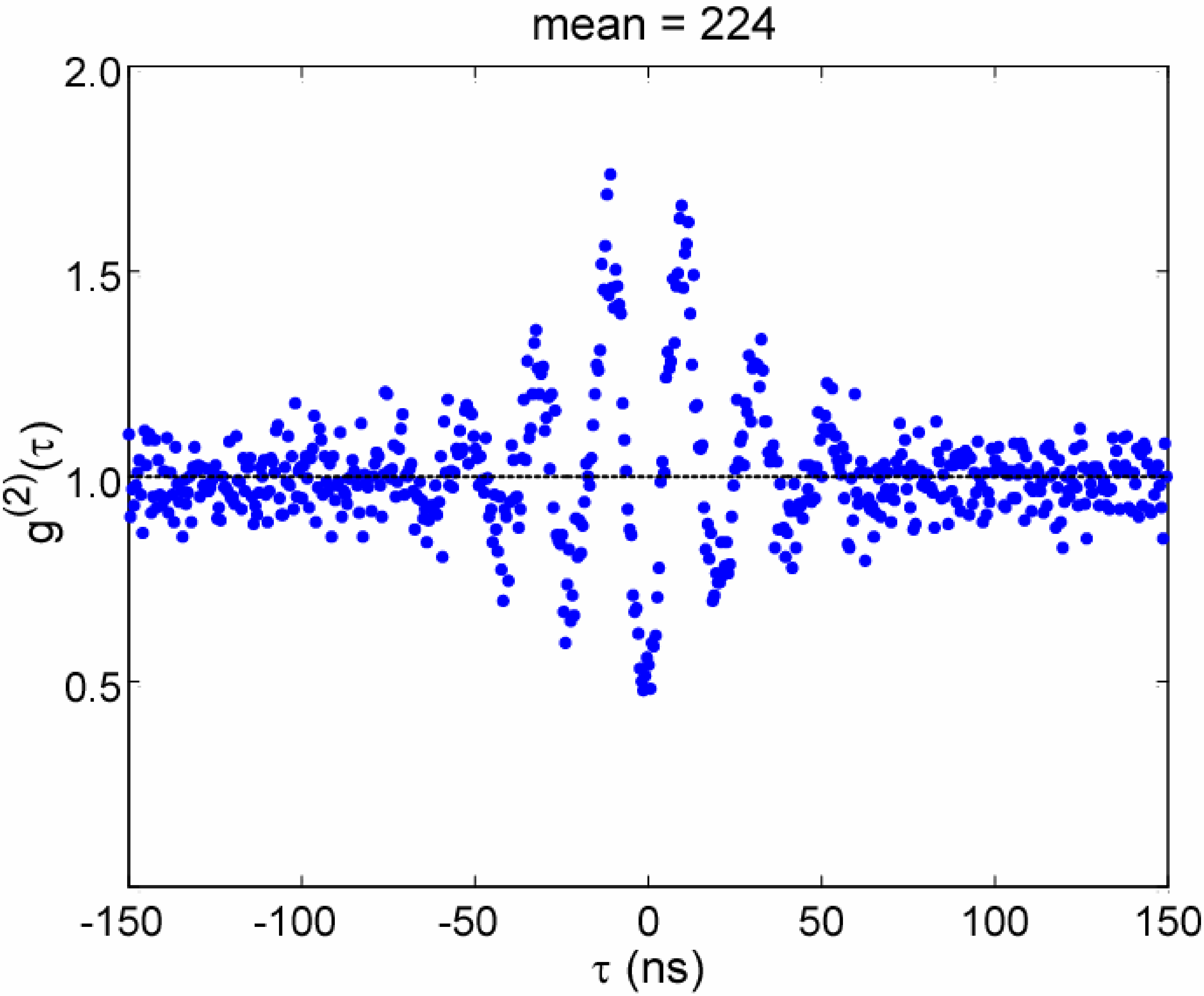
mean = 37
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



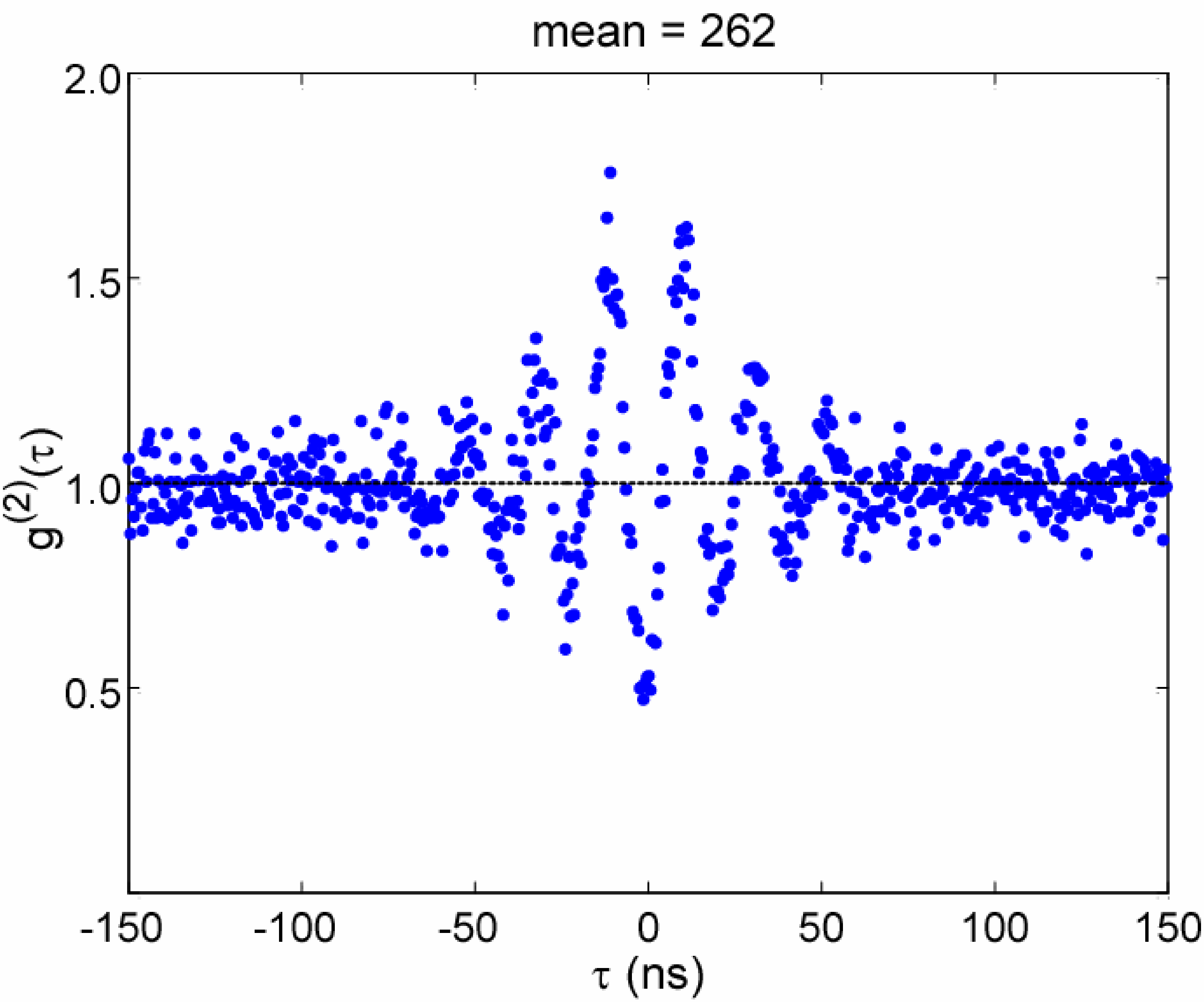
mean = 50
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



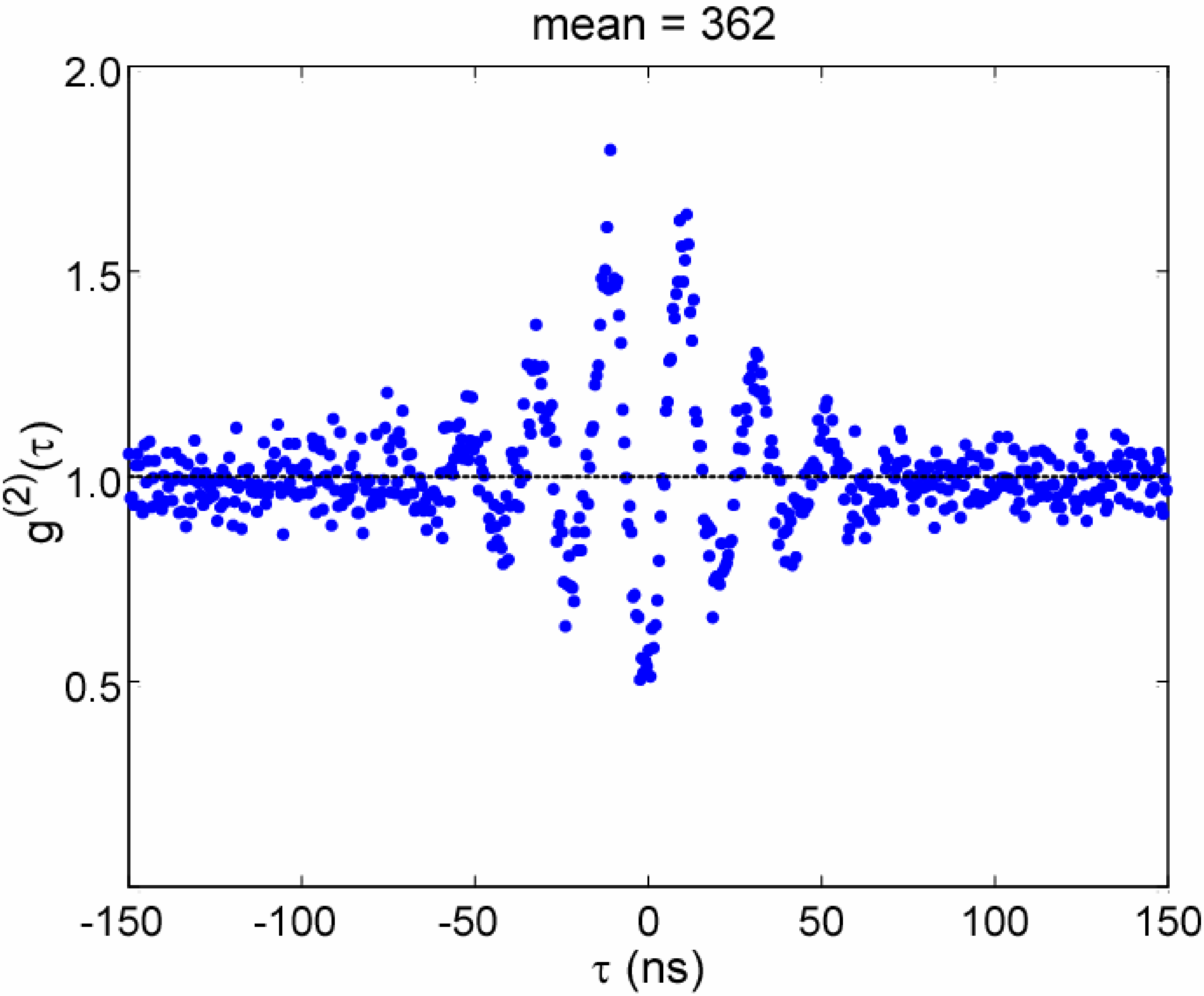
mean = 100
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



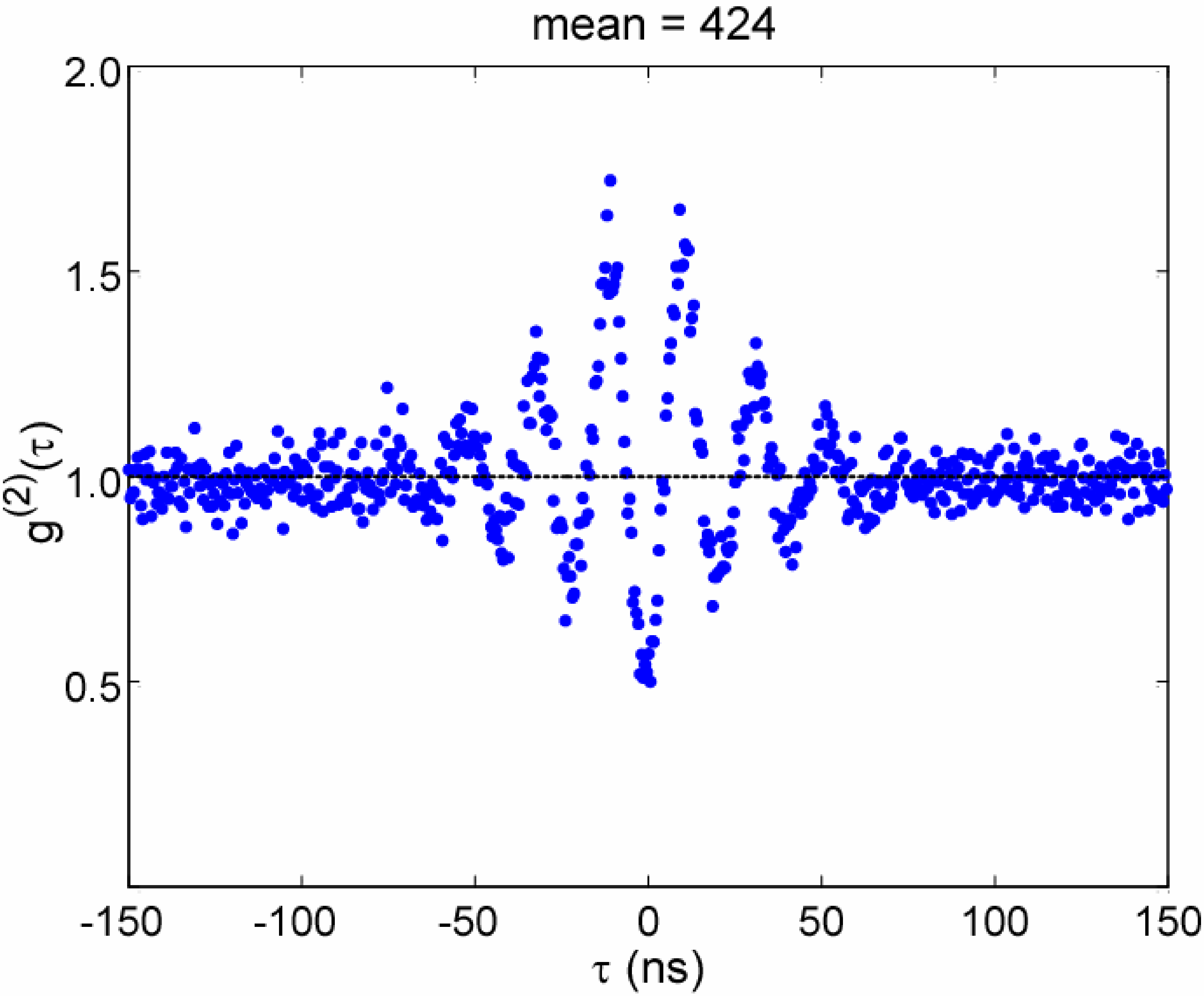
mean = 224
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



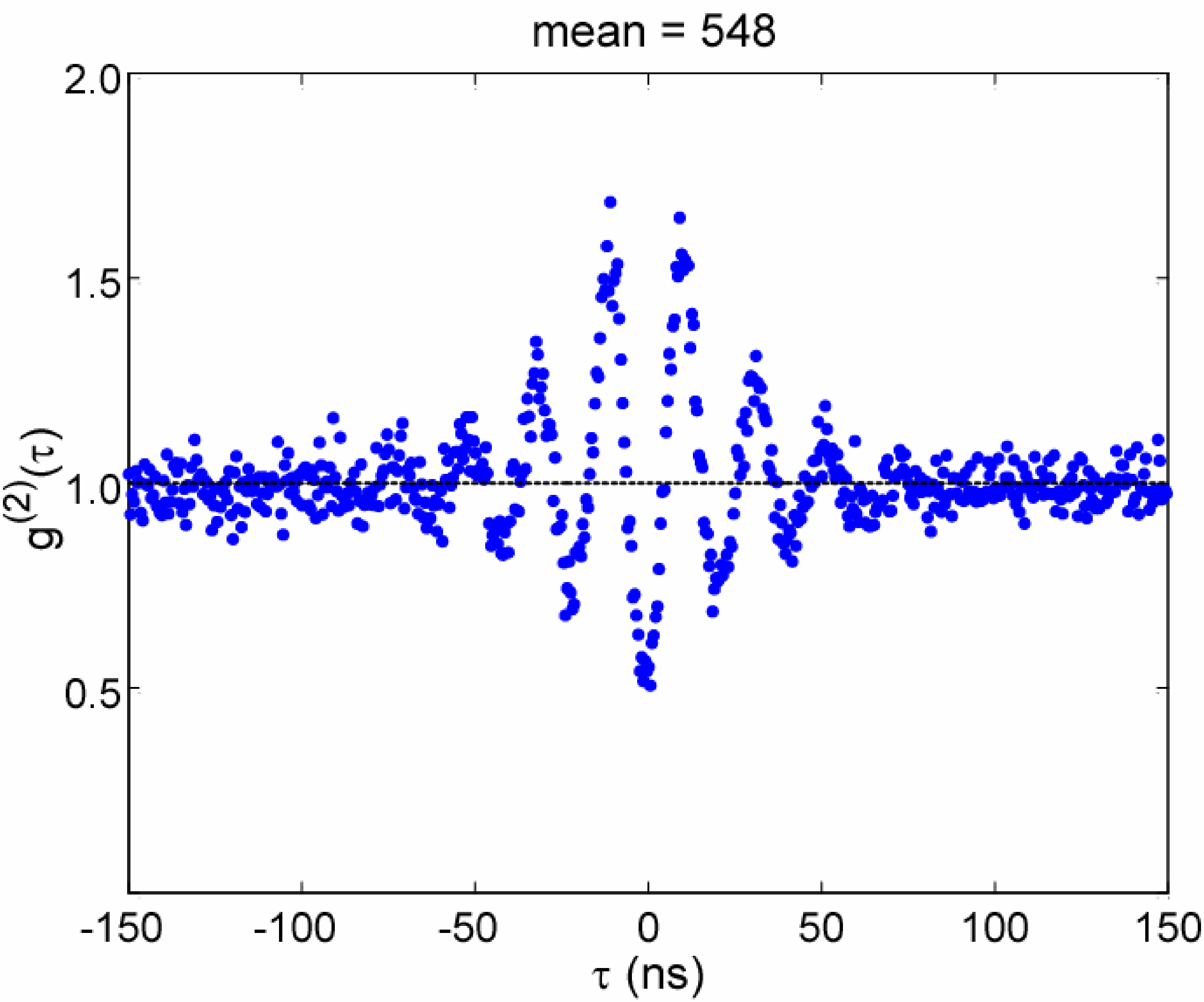
mean = 262
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



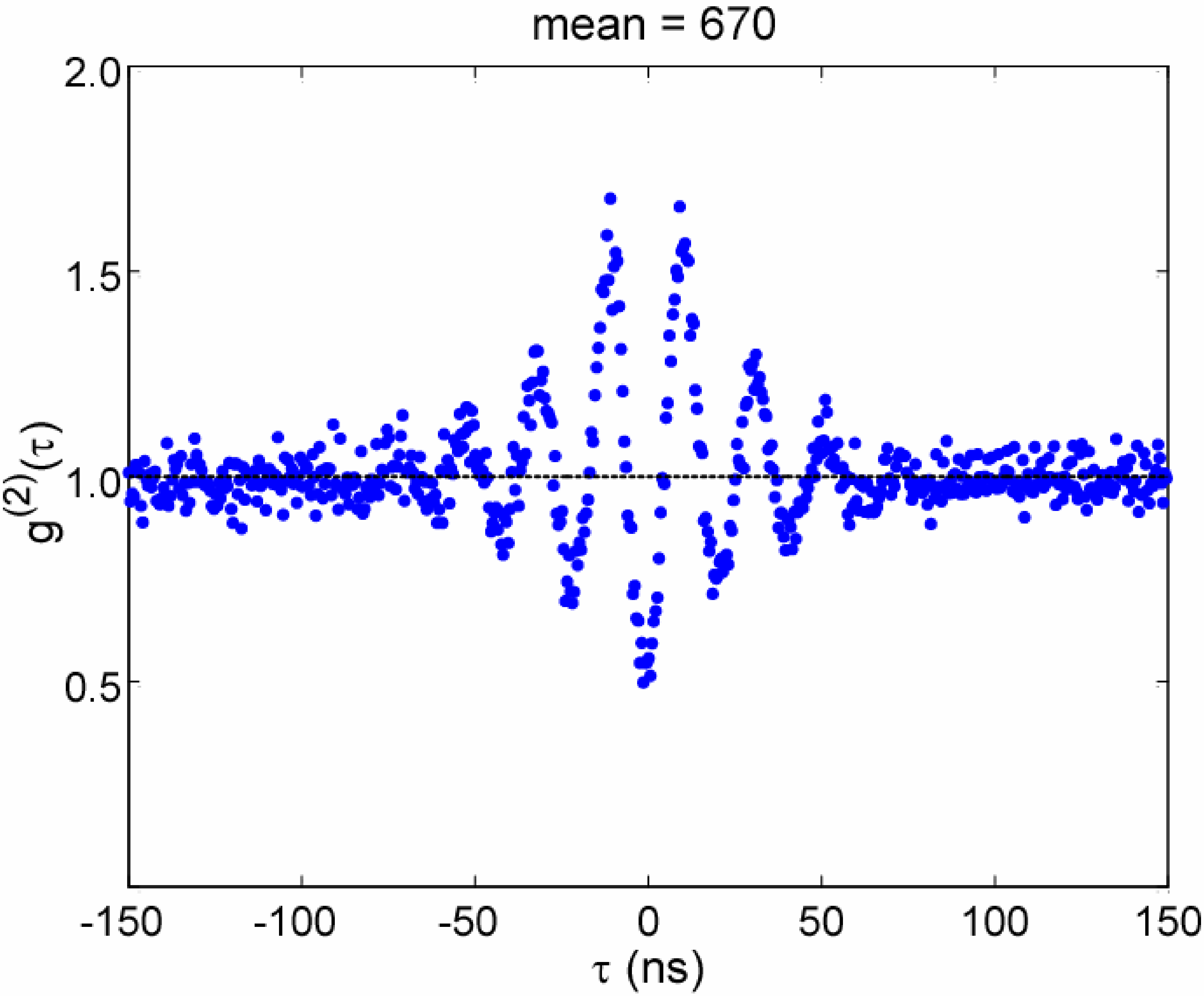
mean = 362
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

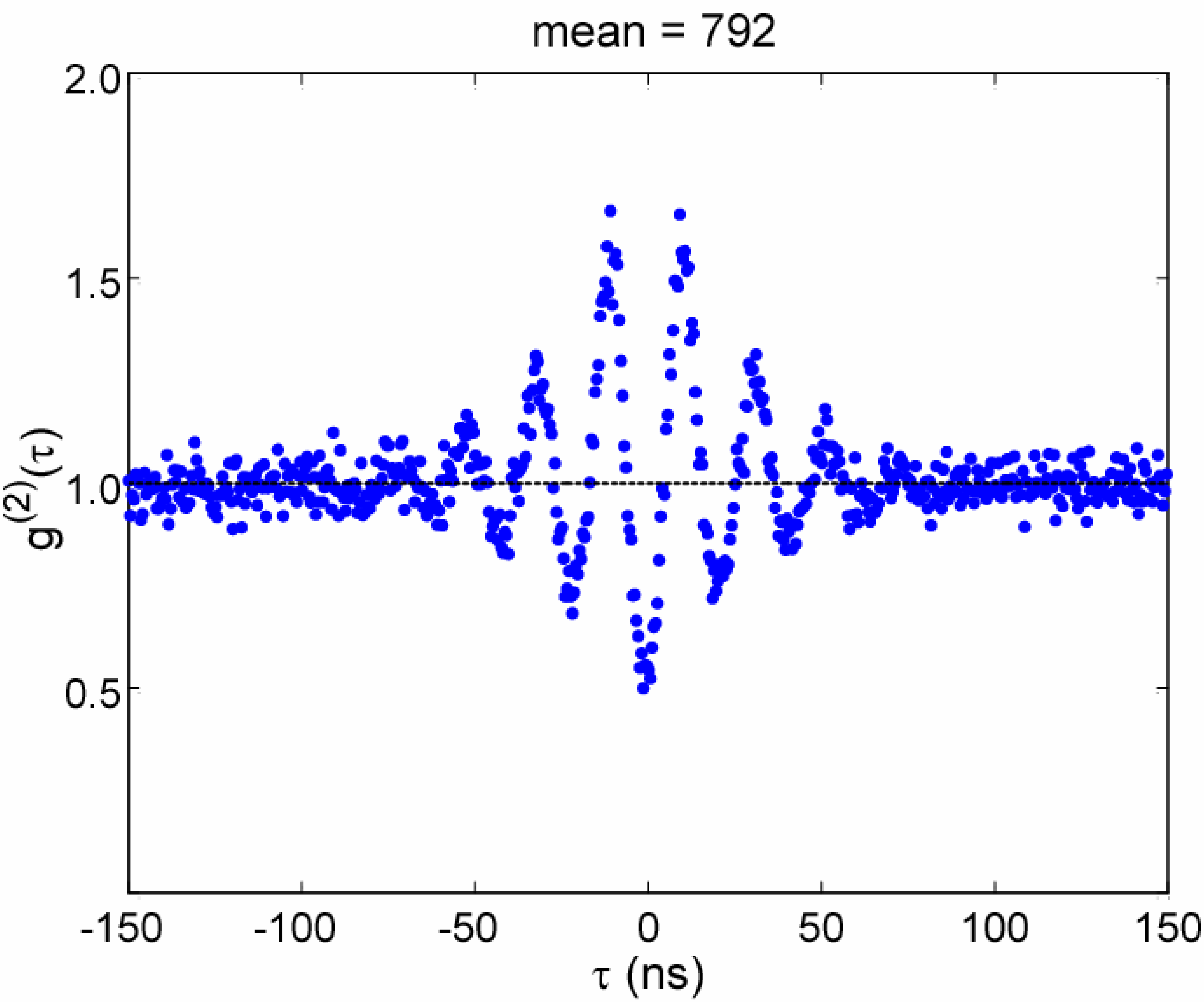


mean = 424
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

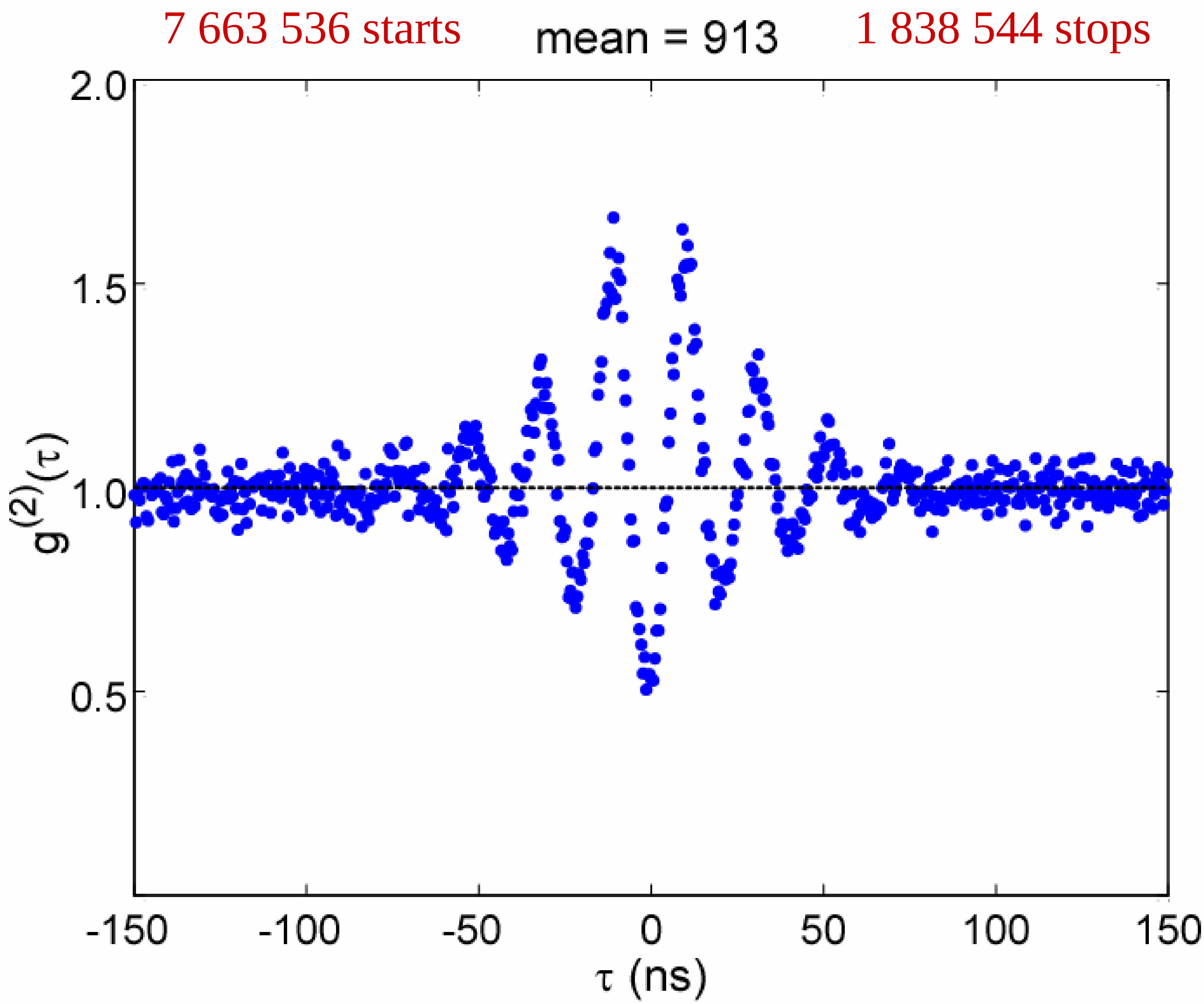


mean = 548
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

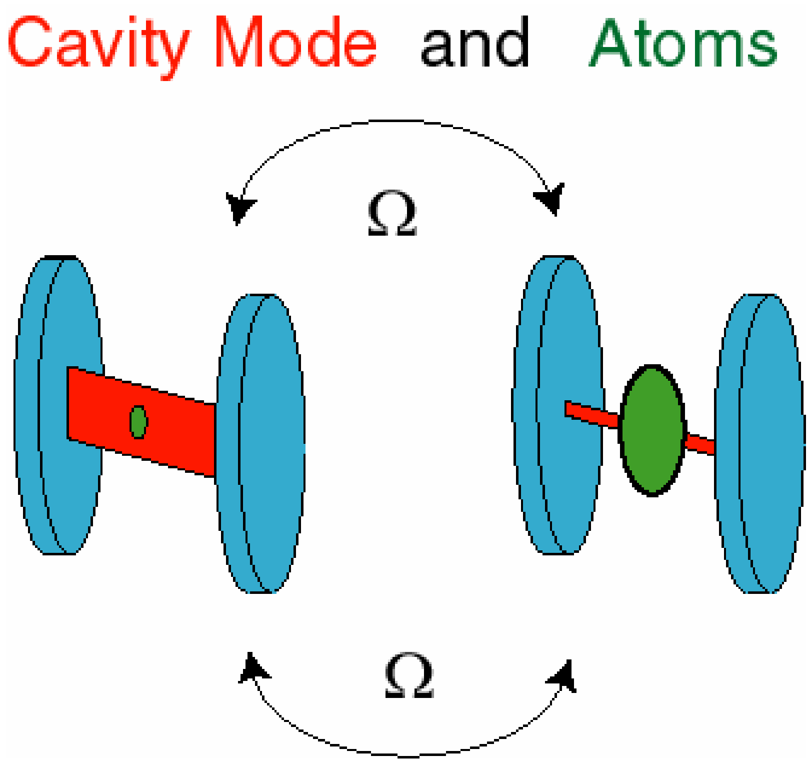

mean = 670
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)



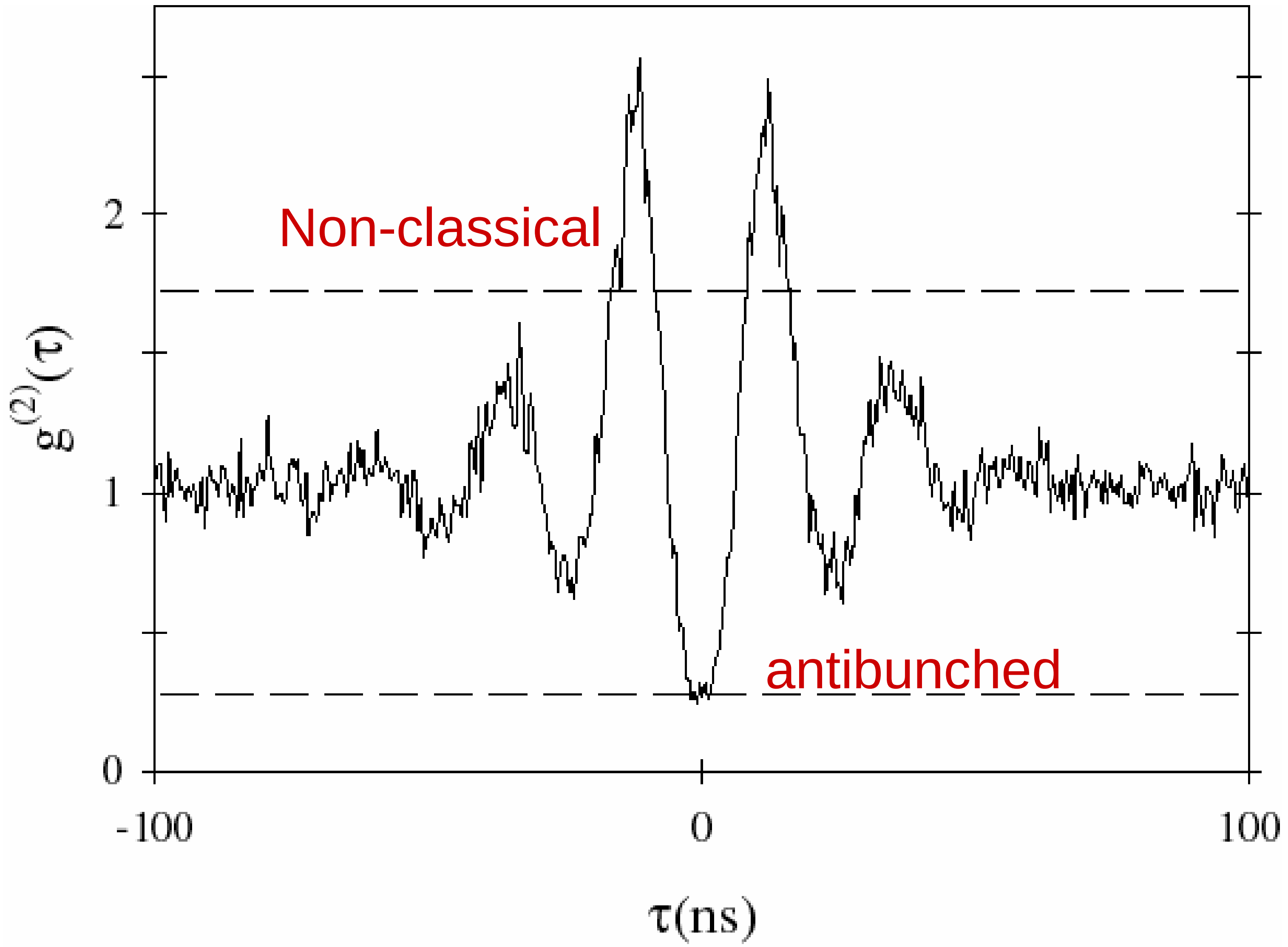
mean = 792
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)


7 663 536 starts
mean = 913
1 838 544 stops
2.0
1.5
1.0
0.5
$g^{(2)}(\tau)$
-150
-100
-50
0
50
100
150
τ (ns)

Exchange of excitation:

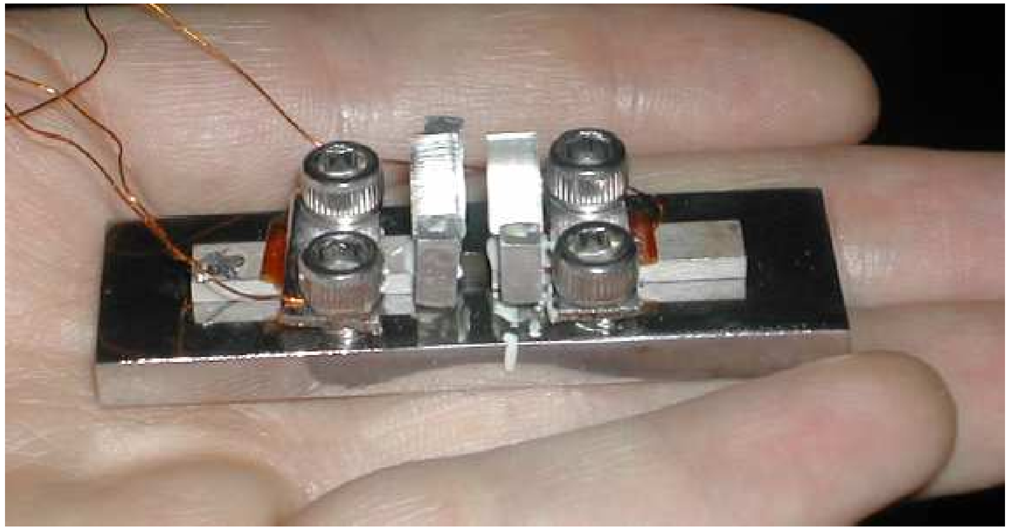


Coupled system, maybe entanglement.



Classically $g^{(2)}(0) > g^{(2)}(\tau)$ and also $|g^{(2)}(0)-1| > |g^{(2)}(\tau)-1|$

Cavity

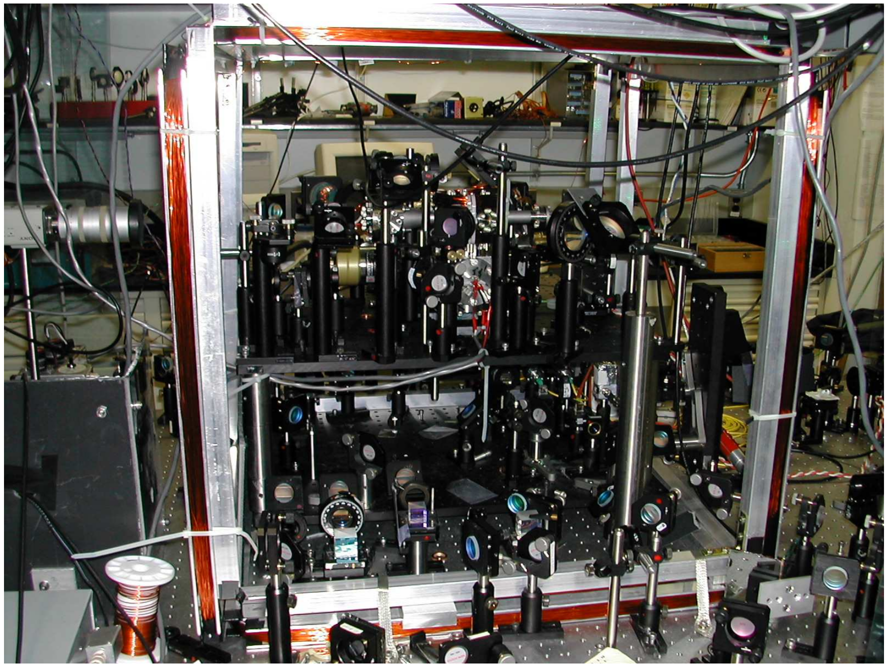


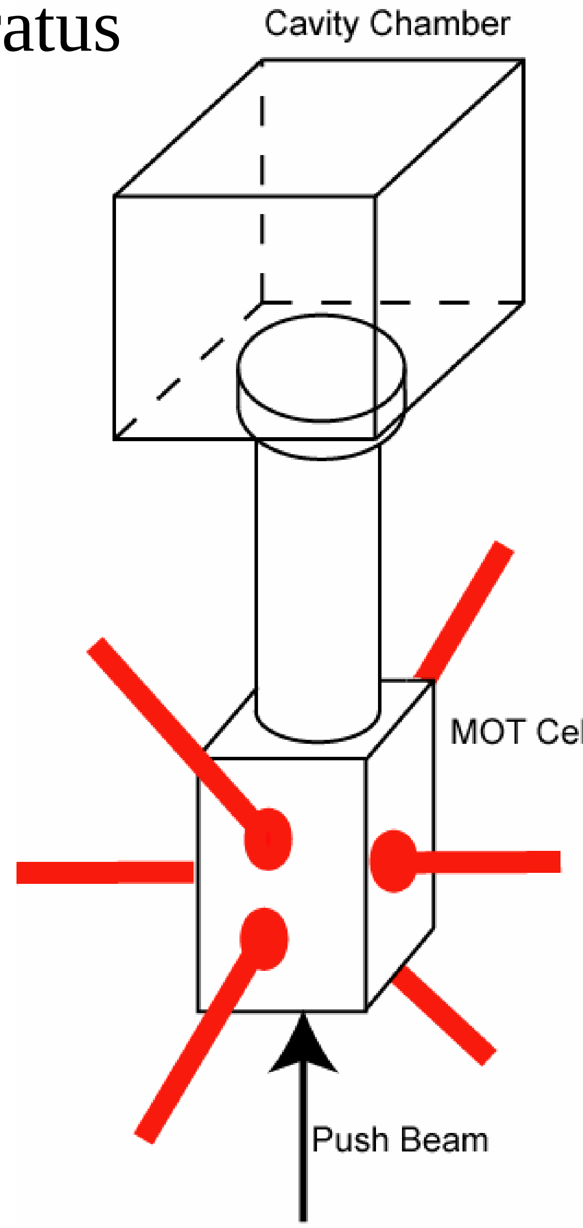
cavity waist ~ 50 microns
mirror separation ~ 2.2mm
Transmission of mirrors = 15 ppm and 300 ppm
2κ = 6.4 MHz
Finesse = 10,000
birefringence: 1:10^4, separation of modes < 1 MHz

Pushed MOT Apparatus

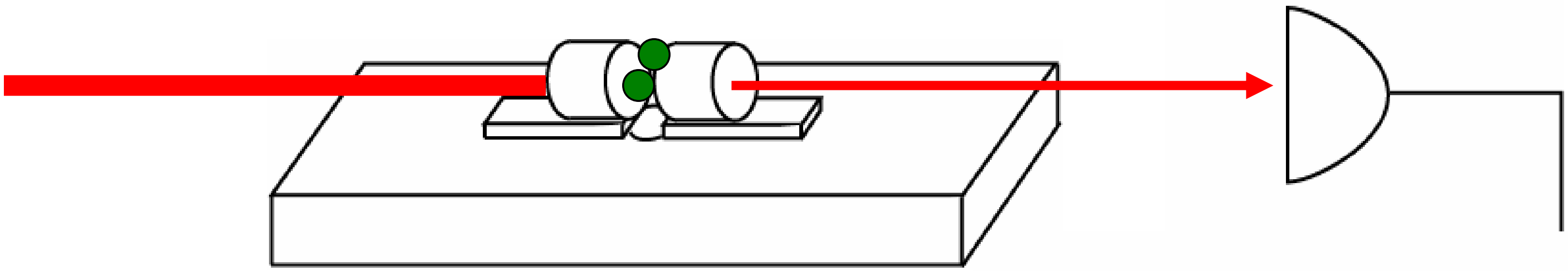
Magneto-optical trap of ^{85}Rb atoms

- N ~ 10^8 atoms.
- Loading time ~ 50 – 200 ms.
- Launched toward the cavity with a resonant push beam.

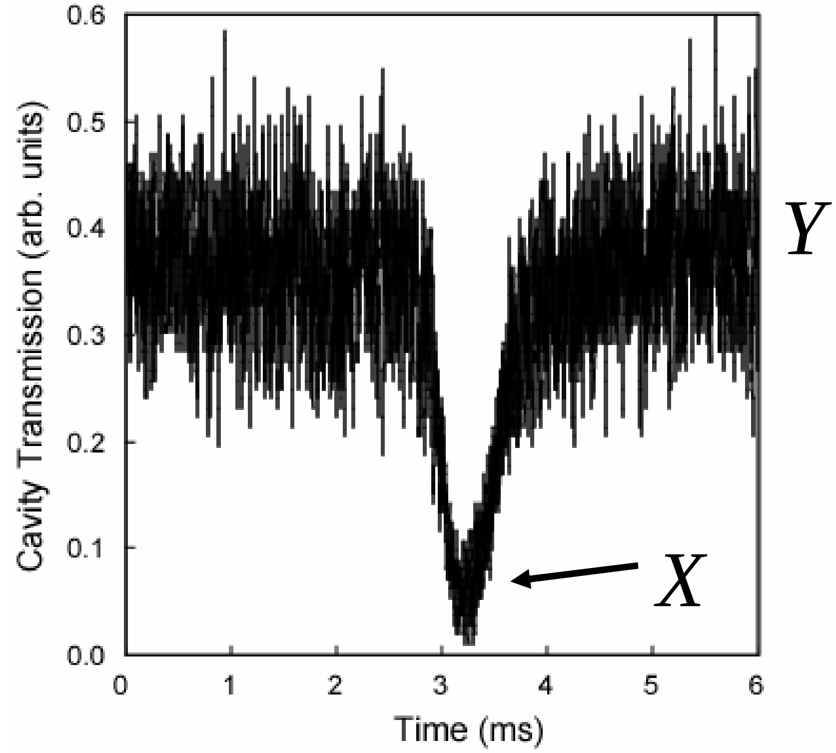




Transmission of Cavity Beam



Transmission: $$\frac{X}{Y} = \frac{1}{(1+2C)^2}$$

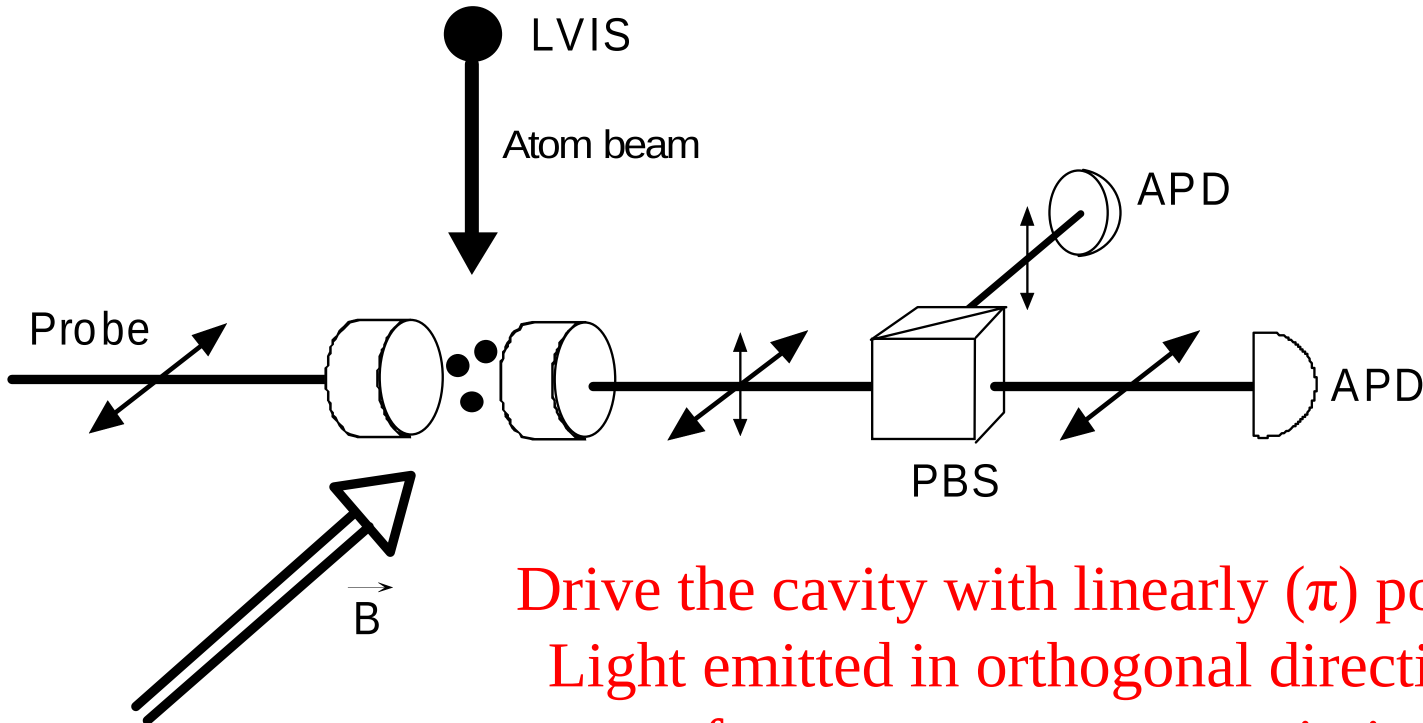


X Normalized intracavity intensity with atoms

Y Normalized intracavity intensity without atoms

$$C = NC_1 = \frac{Ng^2}{\kappa\gamma}$$ is the cooperativity parameter.

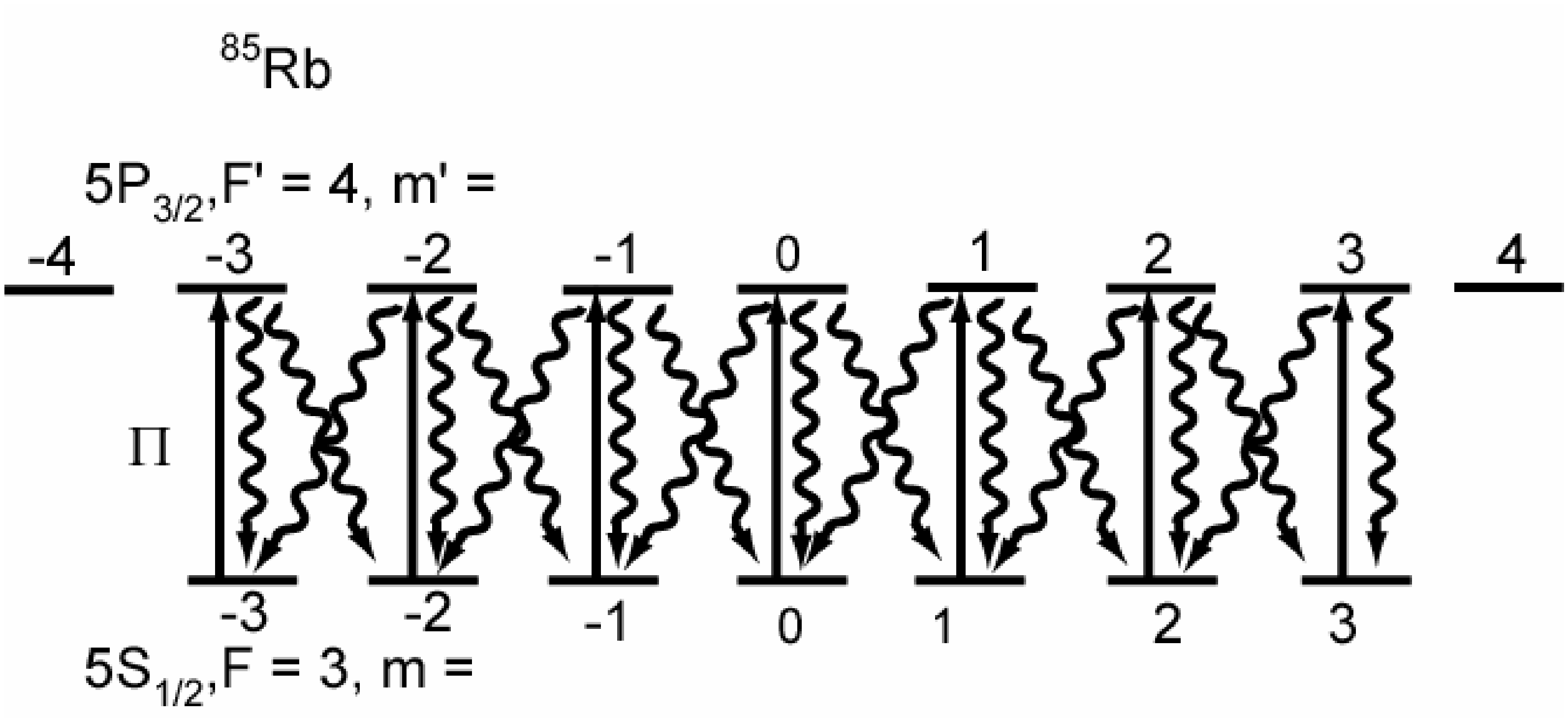
Experimental Schematic



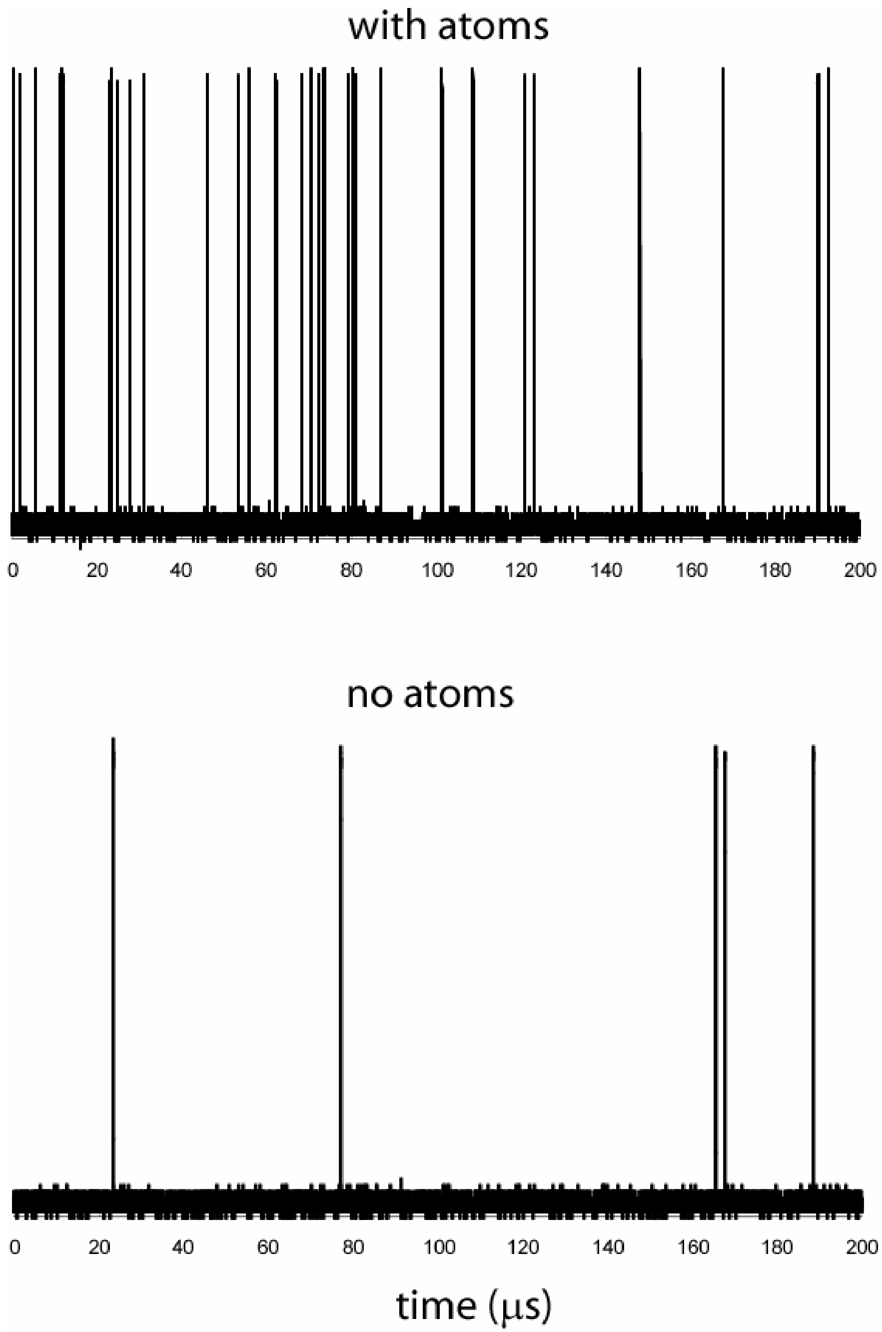
Drive the cavity with linearly (π) polarized light. Light emitted in orthogonal direction must come from a spontaneous emission.

Separate the output into two polarizations to distinguish spontaneous emission from cavity drive.

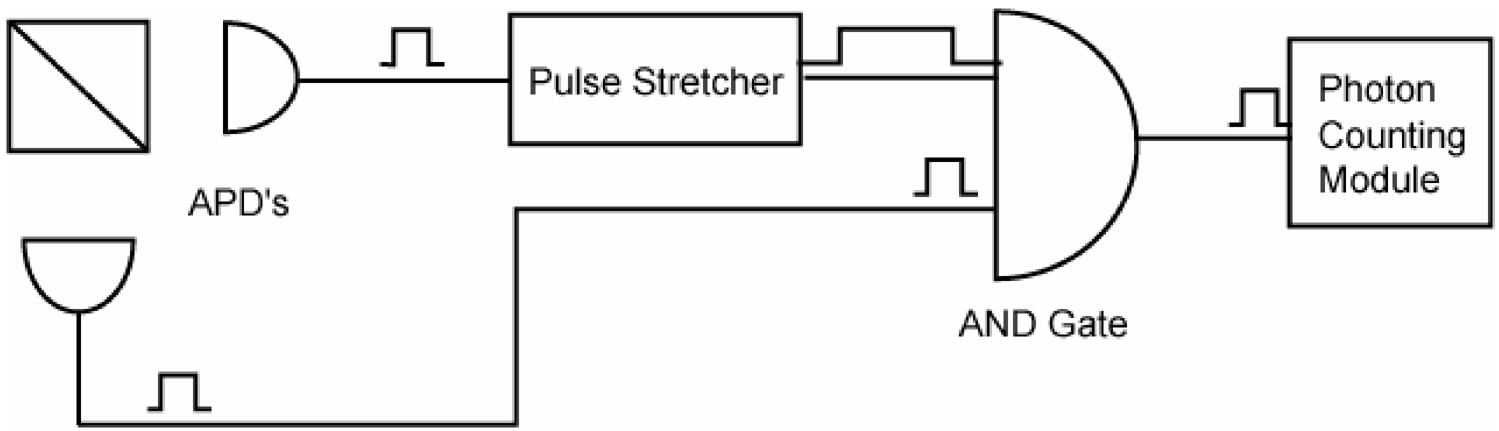
Atomic structure of Rb atoms. Quite different from 2 levels.

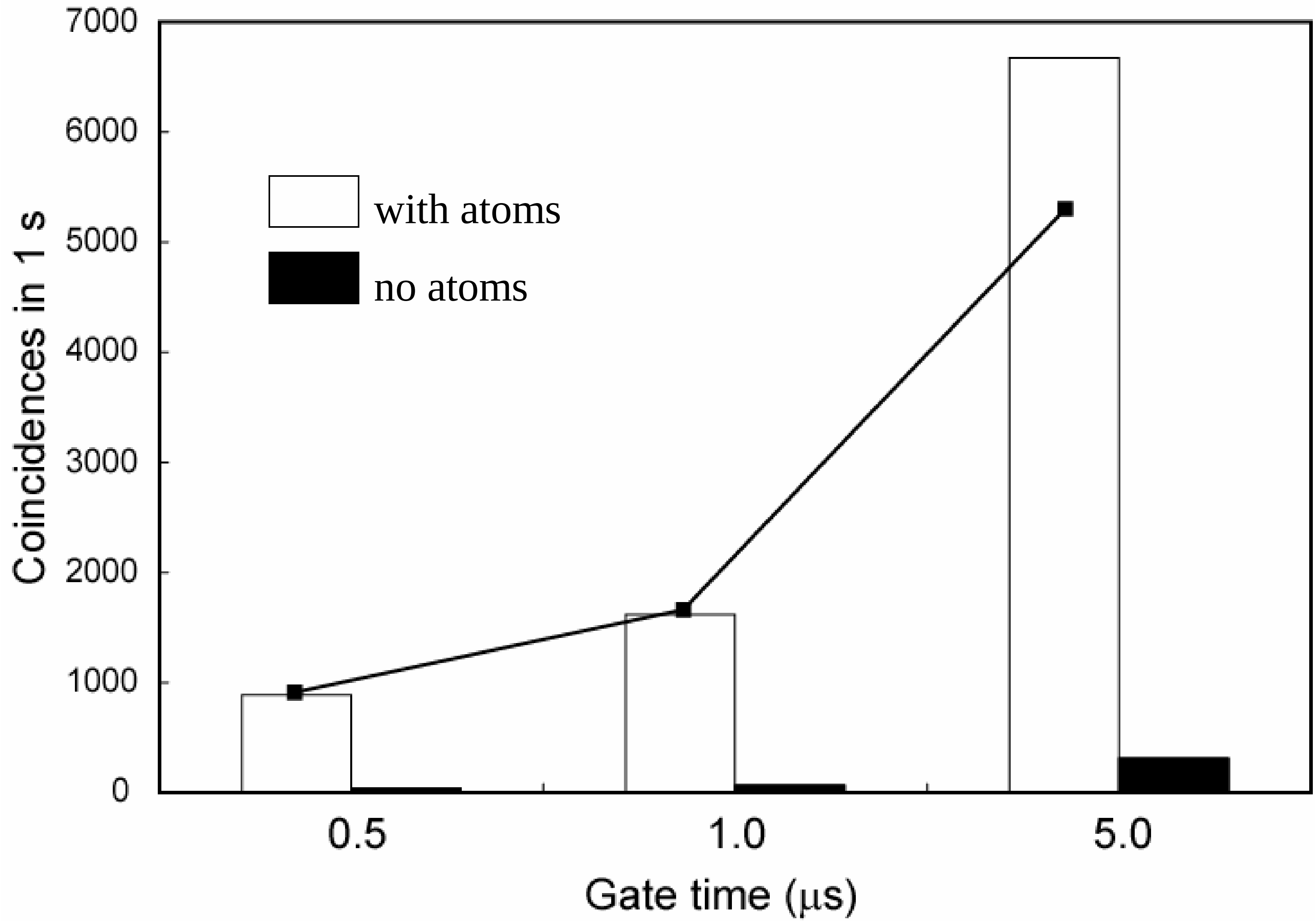


Atom transit data, measured with APD

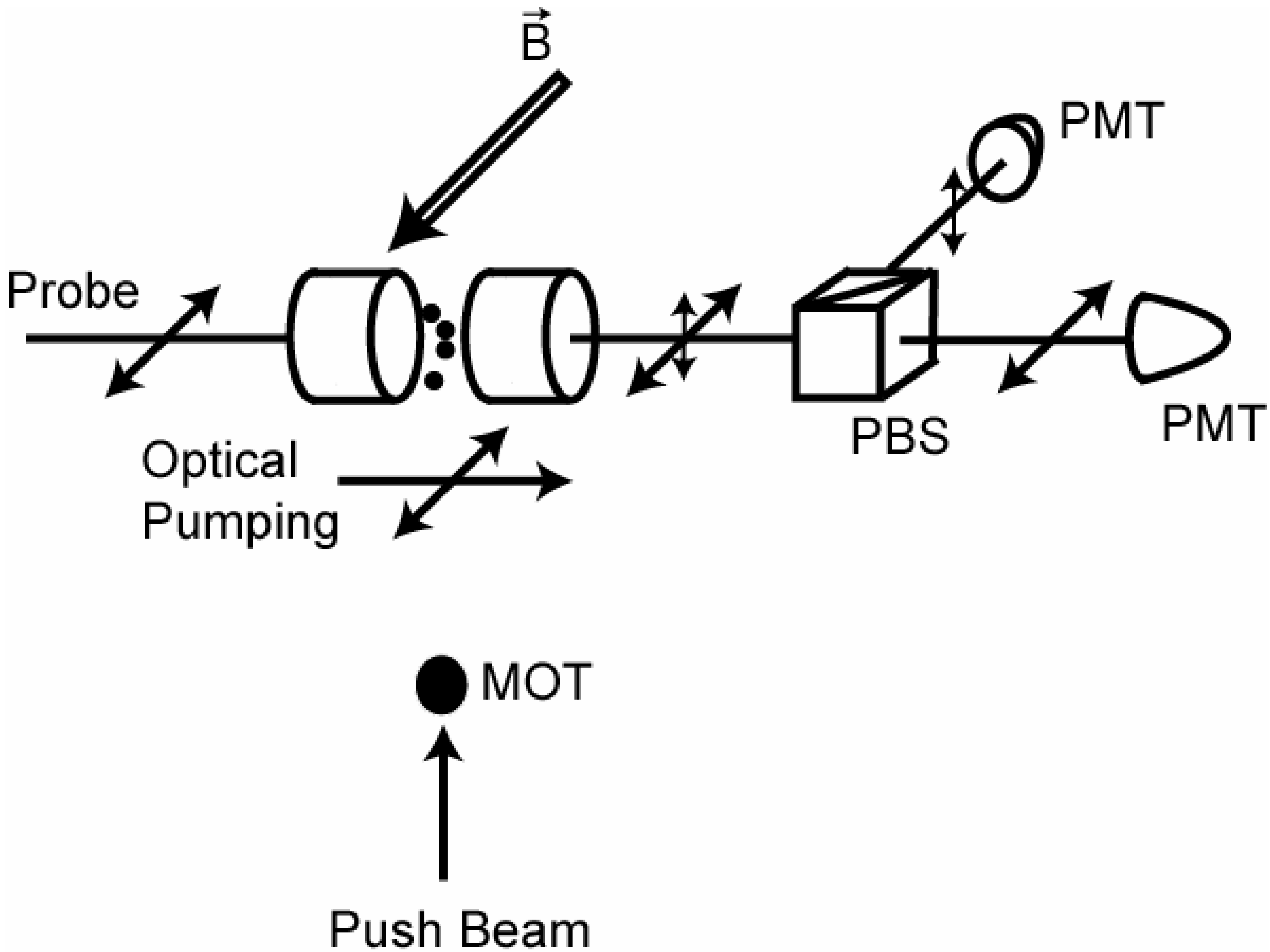


Coincidence Measurements of Atom Transits

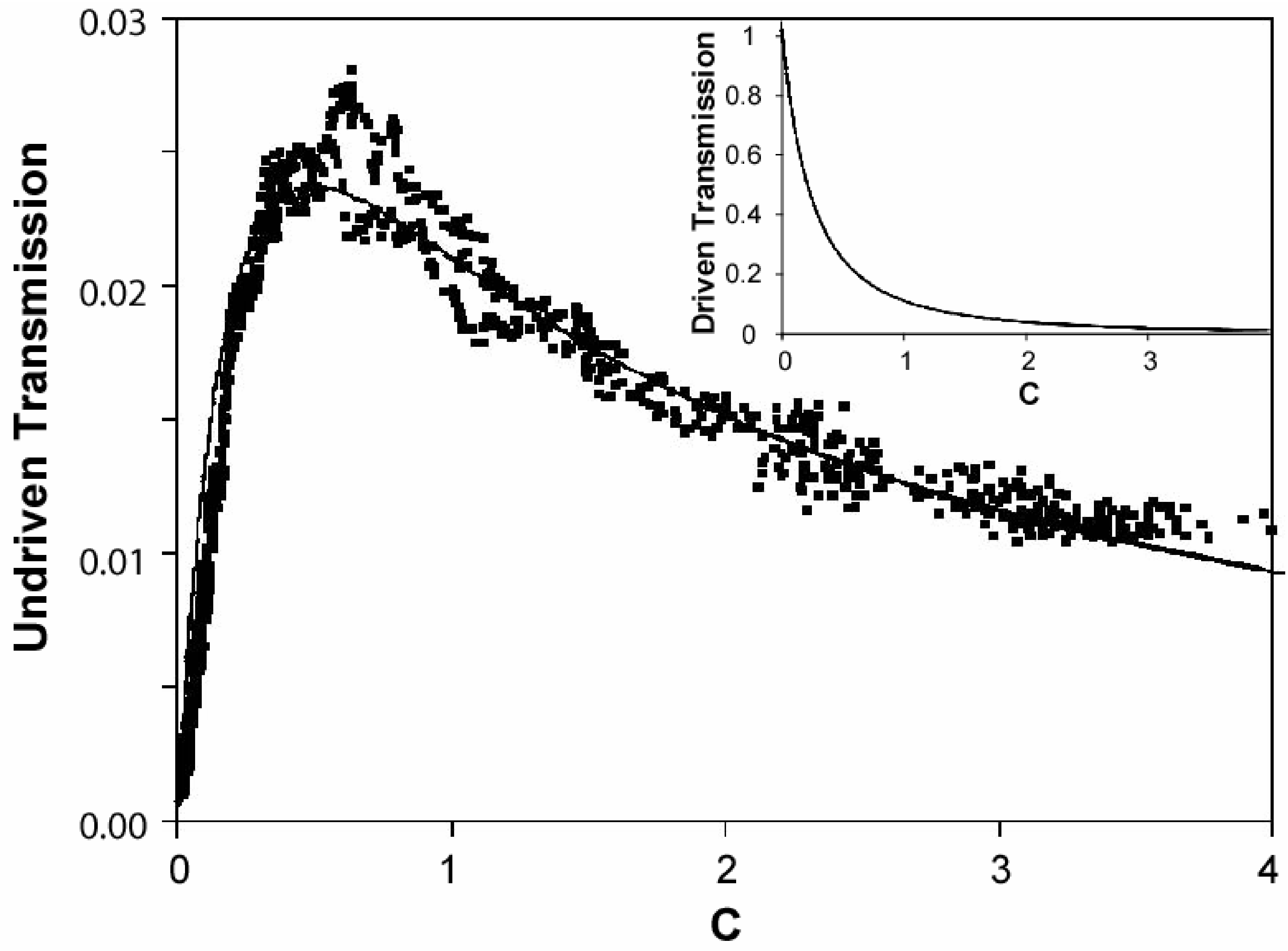




The number of coincidences for a given gate time with the expected Poissonian results (black line).



$\vec{B}$
PMT
Probe
PBS
PMT
Optical
Pumping
MOT
Push Beam



0.03
0.02
0.01
0.00
Undriven Transmission
0
1
2
3
4
C
Driven Transmission
1
0.8
0.6
0.4
0.2
0
0
1
2
3
C

$$\left|\Psi_{SS}\right\rangle = \left|0g\right\rangle + A_{1g}\left|1g\right\rangle + A_{0e}\left|0e\right\rangle + A_{2g}\left|2g\right\rangle + A_{1e}\left|1e\right\rangle + A_{0ee}\left|0ee\right\rangle$$

If the steady state wave function is separable then we should be able to write the wave function as a product state:

$$\left|\Psi_{SS}\right\rangle = \left|\psi_C\right\rangle \otimes \left|\psi_A\right\rangle$$

$$= \left(D_0\left|0\right\rangle + D_1\left|1\right\rangle + D_2\left|2\right\rangle\right) \otimes \left(C_g\left|g\right\rangle + C_e\left|e\right\rangle\right)$$

If we let $A_{1G} = D_1$, $A_{2g} = D_2$, and $A_{0e} = C_e$, then the following condition must be satisfied for a product state: $A_{1e} = D_1C_E = A_{Oe}A_{1g}$

If this is not true $\Longrightarrow$ There is entanglement!

The function, $j^{(2)}(\tau)$, measures correlations between transmitted and fluorescent clicks.

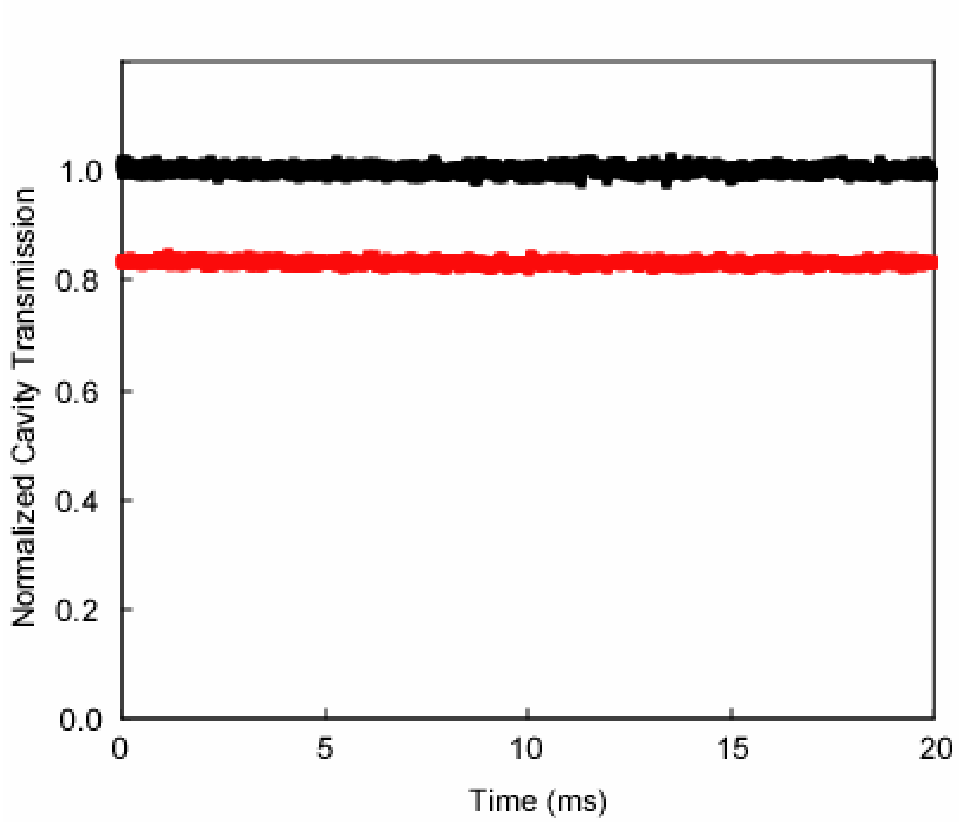
$$j^{(2)}(\tau) = \frac{\langle a^{+}(0)\sigma_{+}(\tau)\sigma_{-}(\tau)a(0)\rangle}{\langle a^{+}a\rangle\langle\sigma_{+}\sigma_{-}\rangle} = \frac{\langle\sigma_{+}(\tau)\sigma_{-}(\tau)\rangle_{c}}{\langle\sigma_{+}\sigma_{-}\rangle}$$

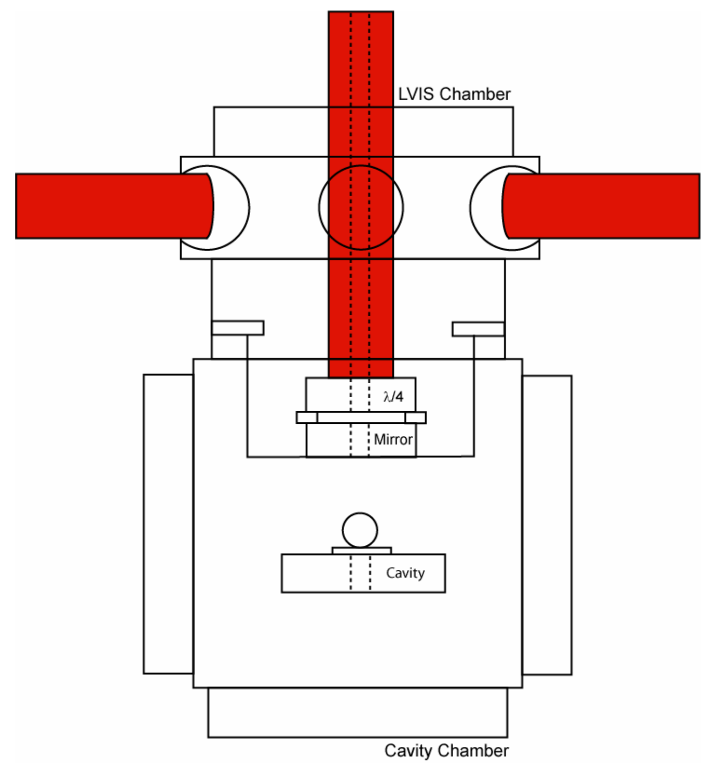
$$j^{(2)}(0) = \frac{\left|A_{1e}\right|^{2}}{\left|A_{1g}A_{0e}\right|^{2}} = \frac{\left\langle a^{+}\sigma_{+}\sigma_{-}a\right\rangle}{\left\langle a^{+}a\right\rangle\left\langle\sigma_{+}\sigma_{-}\right\rangle}$$

A measurement of $j^{(2)}(0)$ that differs from unity, is a witness of entanglement.

LVIS (Low Velocity Intense Source)

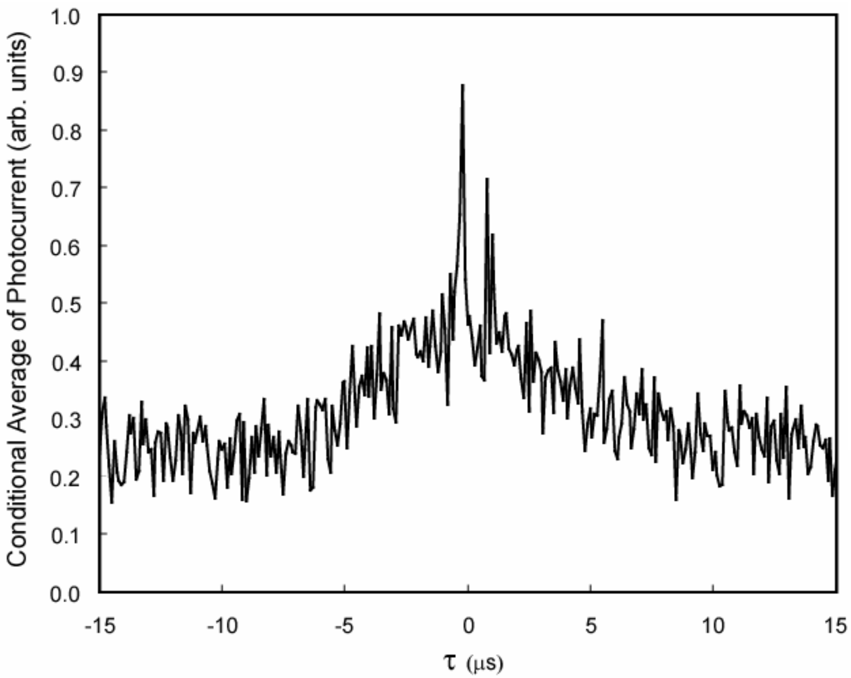
- Continuous source of cold atoms ~ 1 atom in cavity at all times!
- Similar to MOT but with a retro optic in the vacuum with a hole for extracting a beam of atoms.
- Can be pulsed by plugging hole in retro optic with a MOT beam and unplugging after a MOT forms.





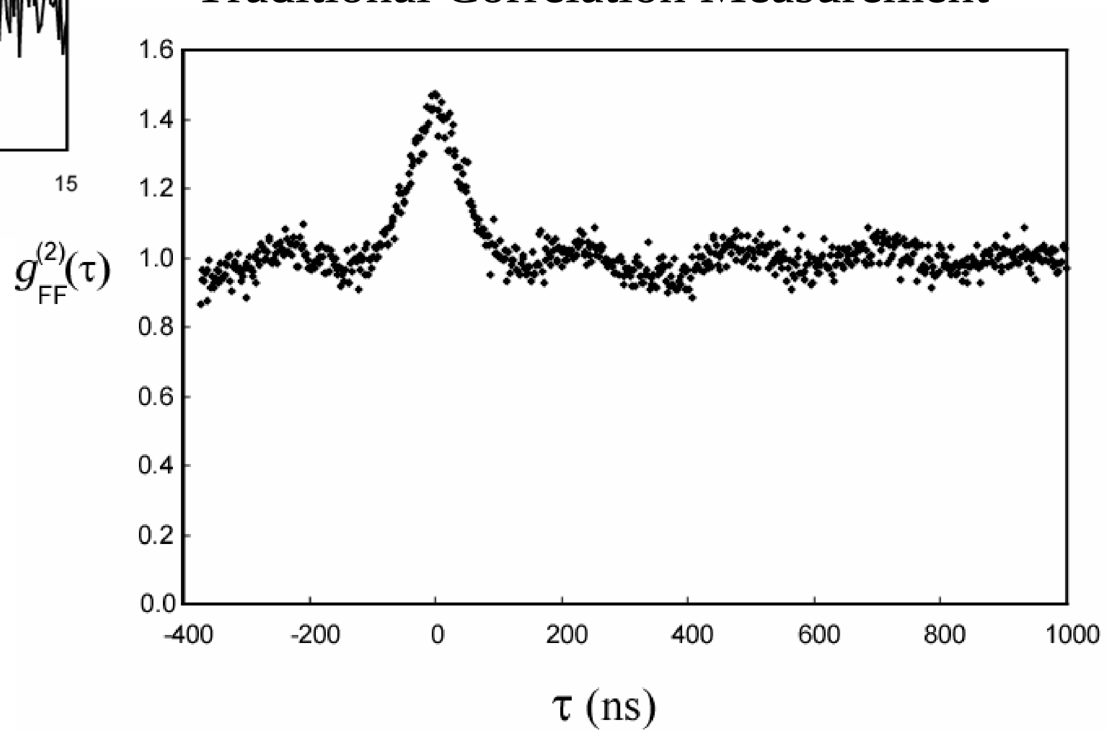
Autocorrelation of Fluorescence Mode

Conditional Average on Oscilloscope

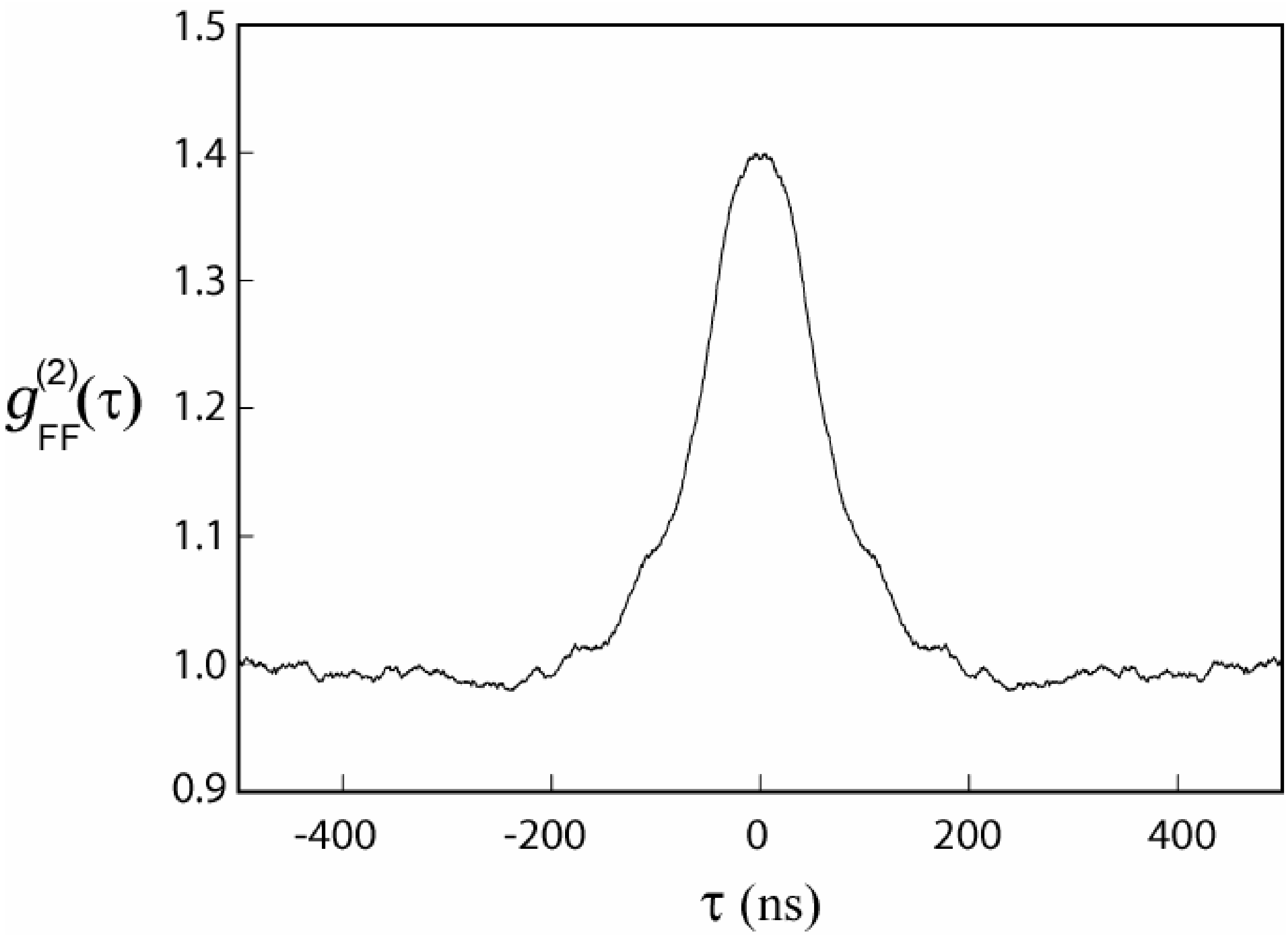


$$g_{FF}^{(2)}(\tau) = \frac{\langle b^+(0) b^+(\tau) b(\tau) b(0) \rangle}{\langle b^+ b \rangle^2}$$

Traditional Correlation Measurement

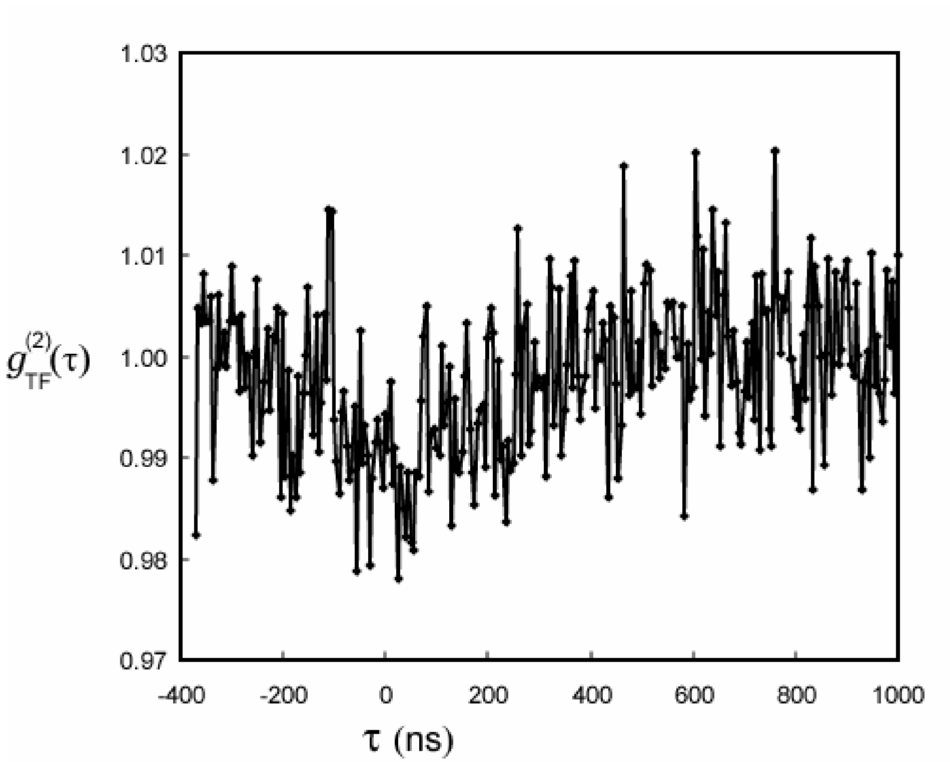


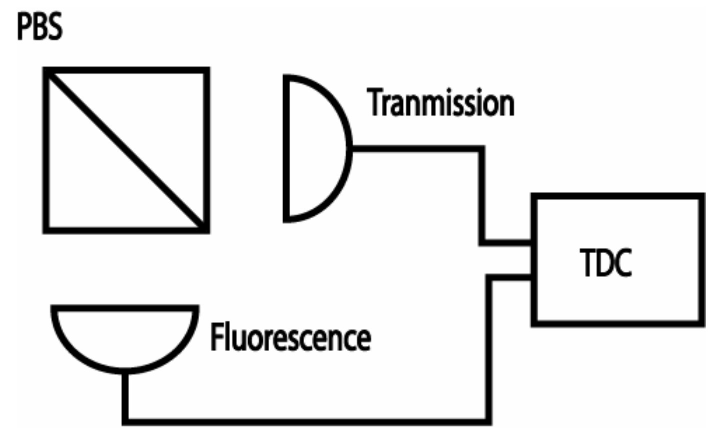
Quantum Trajectory Theory



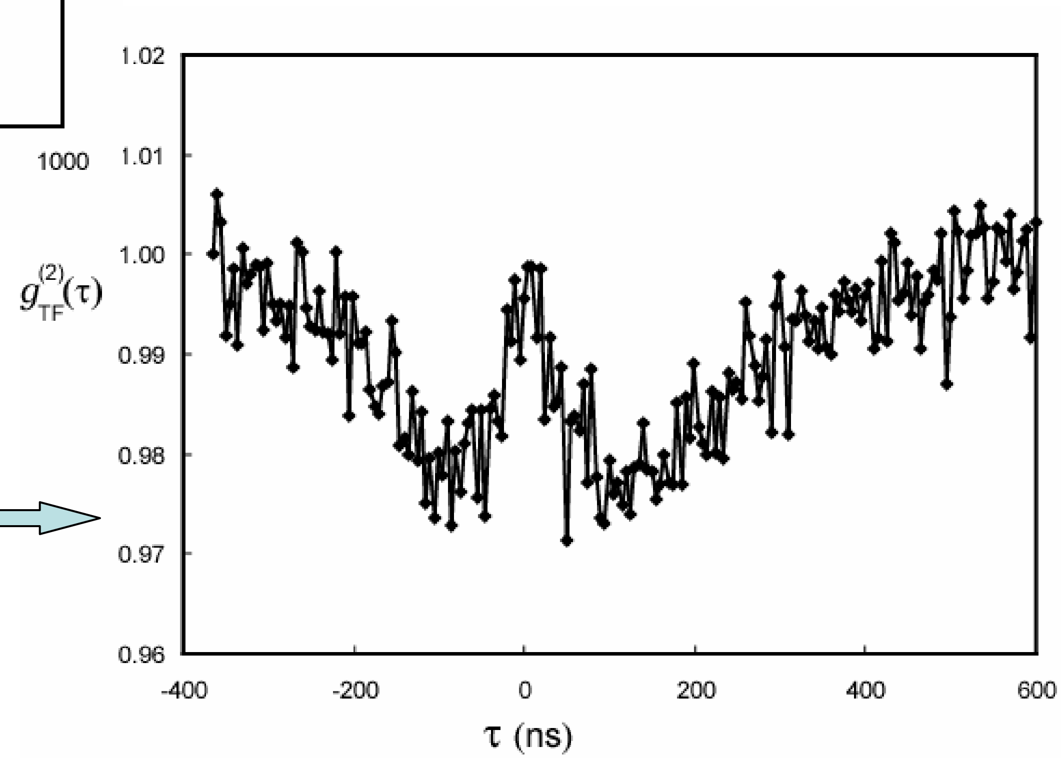
15 photons in driven mode

Measured Cross-Correlations

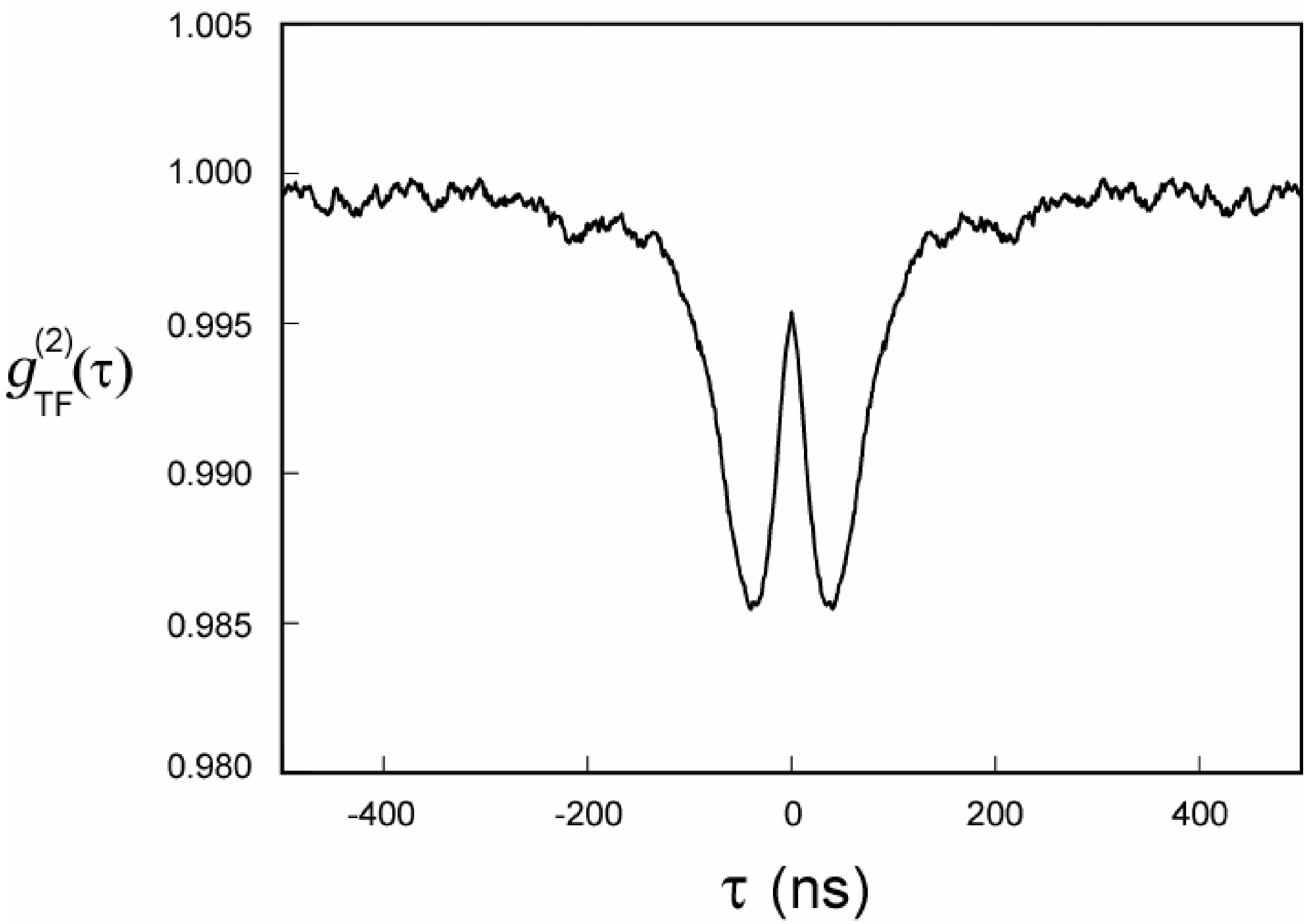




Weaker driving field



Quantum Trajectory Theory



3 photons in the driven mode, 0.1 in the undriven mode

Measuring the Concurrence

Recall that the concurrence is given by:

$$Con = \sqrt{A_{1g}^2 A_{0e}^2 \left(1 - 2\sqrt{j^{(2)}(0)} + j^{(2)}(0)^2\right)}$$

$j^{(2)}(0)$ can be extracted from the cross-correlation histogram, if we assume

the mapping $j^{(2)}(0) \longrightarrow g_{TF}^{(2)}(0)$ is valid.

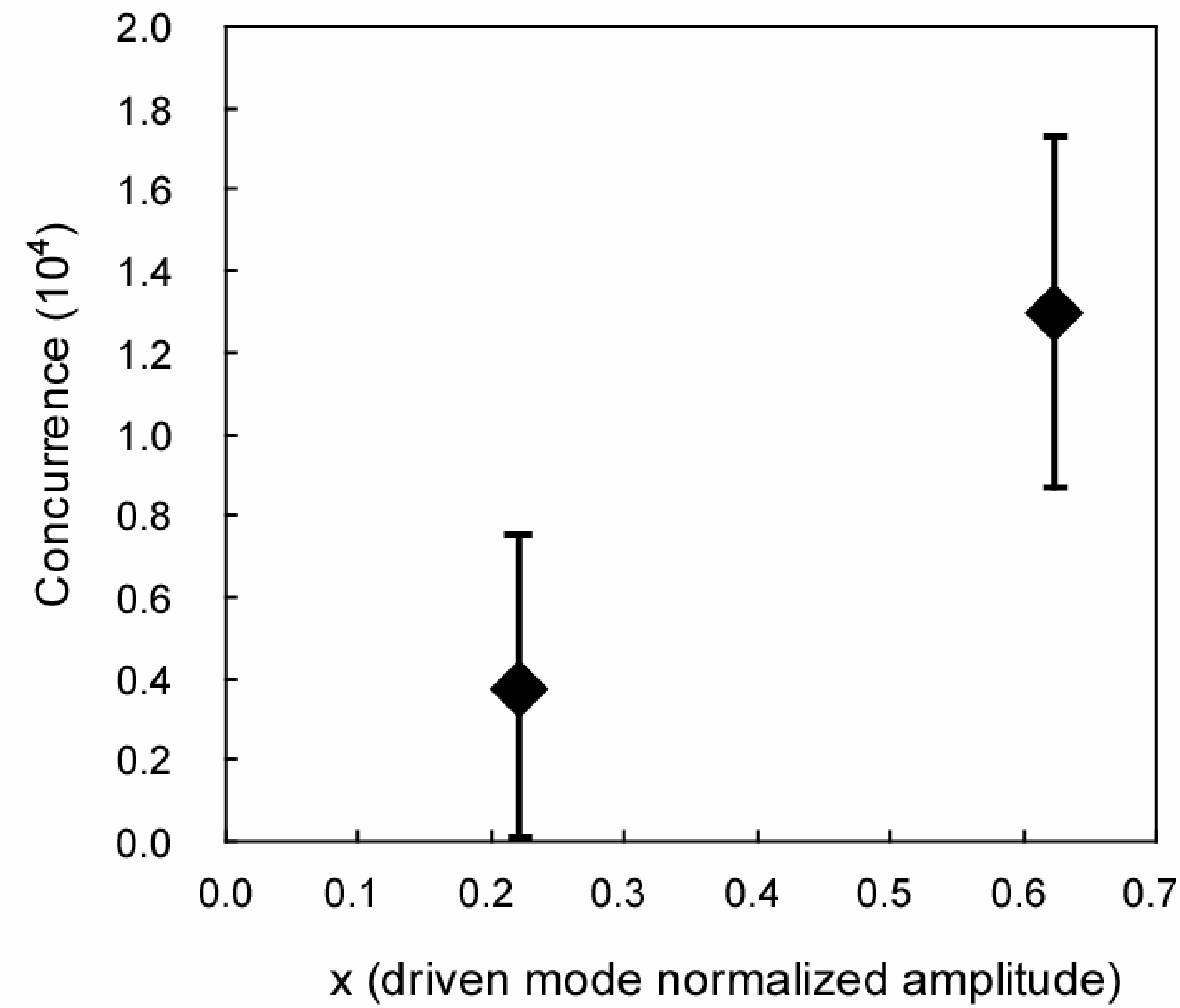
$$Con \sim \sqrt{A_{1g}^2 A_{0e}^2 \left(1 - 2\sqrt{g_{TF}^{(2)}(0)} + g_{TF}^{(2)}(0)^2\right)}$$

where $A_{1g(0e)} = \sqrt{\dfrac{X_{1g(0e)}}{n_0}} = \sqrt{\dfrac{R_{1g(0e)}}{2\kappa n_0}}$

R is the flux of photons emitted from the cavity and n_0 is the saturation photon number.

$$n_0 = \frac{\gamma^2}{3g^2}$$

Concurrence Measurements



Summary

• We have devised a unique way of observing spontaneous emission from our cavity QED system.

We have correlated spontaneous emission photons with transmitted photons to measure entanglement.